Advanced Nuclear Physics

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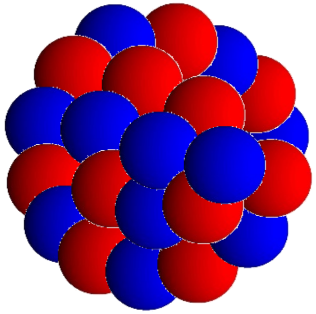
Nuclei: aggregate of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems***



- Liquid drop model
- Single-particle motion and Shell structure
- Hartree-Fock approximation
- Bruckner Theory
- Pairing correlations and Superfluid Nuclei
- Angular momentum and number projections
- 1n and 2n halo nuclei
- Random Phase Approximation
- Nuclear Reactions

References

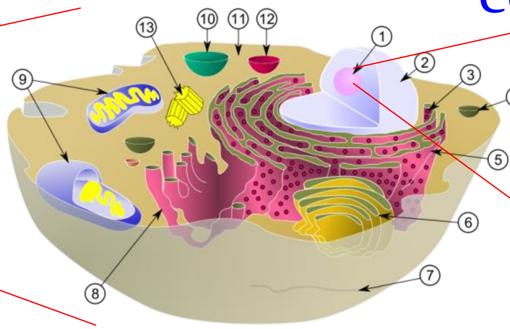
- P. Ring and P. Schuck, “The Nuclear Many-Body Ploblem”
- A. Bohr and B.R. Mottelson, “Nuclear Structure” Vol. 1 and 2
- G.E. Brown, “Unified Theory of Nuclear Models and Forces”
- D.J. Rowe, “Nuclear Collective Motion”

- J. Lilley, “Nuclear Physics”
- K.S. Krane, “Introductory Nuclear Physics”
- R.F. Casten, “Nuclear Structure from a Simple Perspective”
- S.G. Nilsson and I. Ragnarsson, “Shapes and Shells in Nuclear Structure”

Introduction: atoms and atomic nuclei

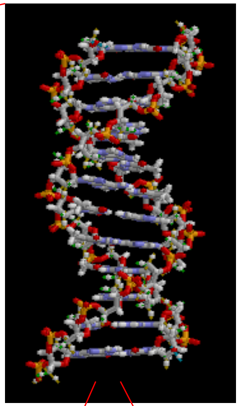
~ 50 cm

cells



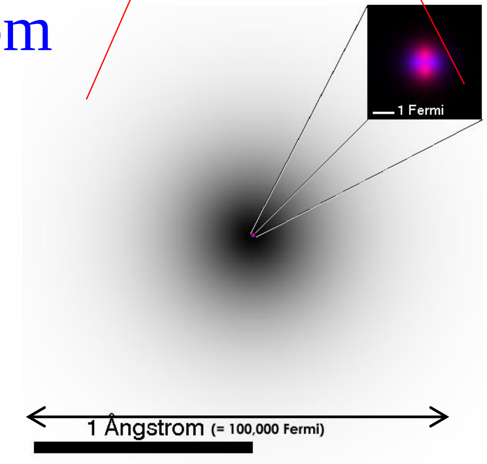
~ µm = 10^{-6} m

DNA



~ 10^{-8} m

atom



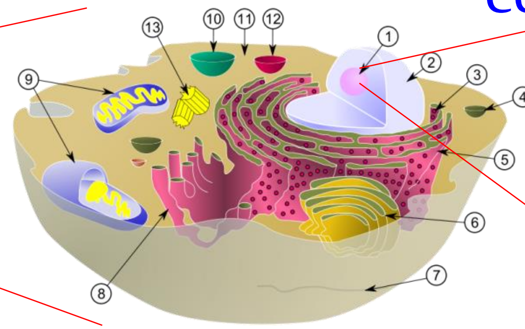
~ 10^{-10} m

All things are made of atoms.

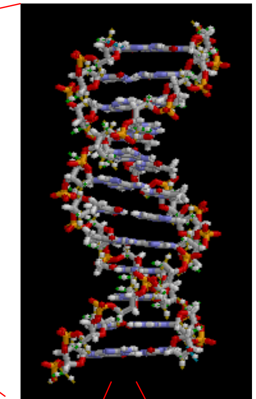
Introduction: atoms and atomic nuclei

~ 50 cm

cells

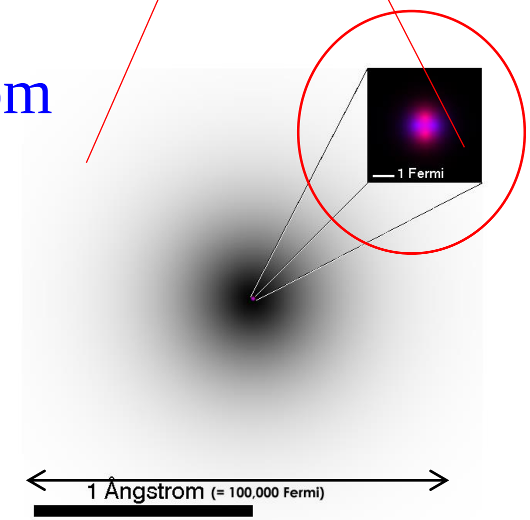


DNA



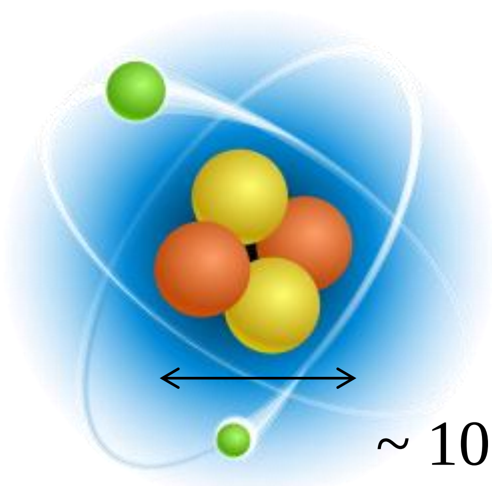
~ 10^{-8} m

atom

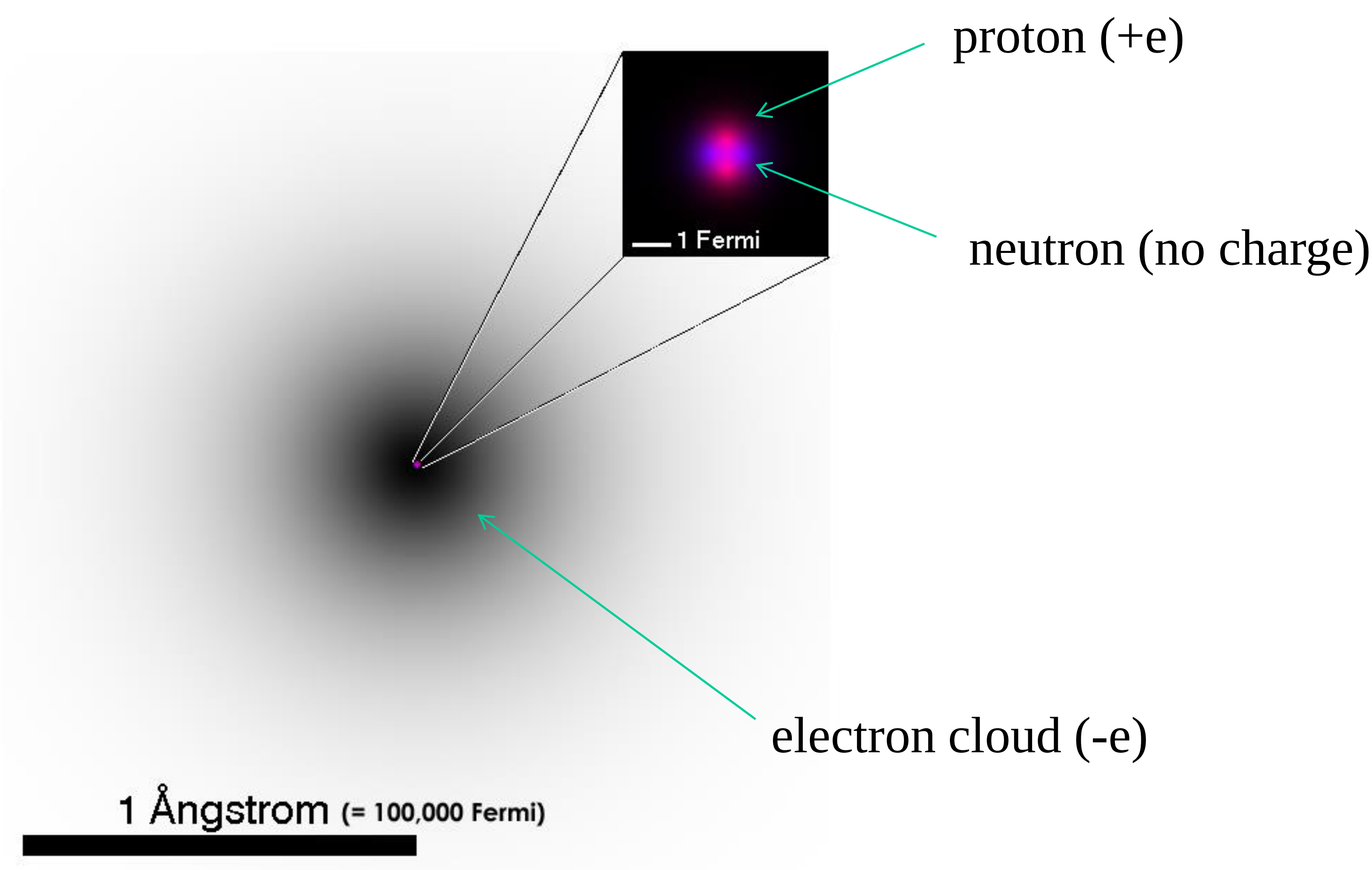


~ 10^{-10} m

atomic nucleus

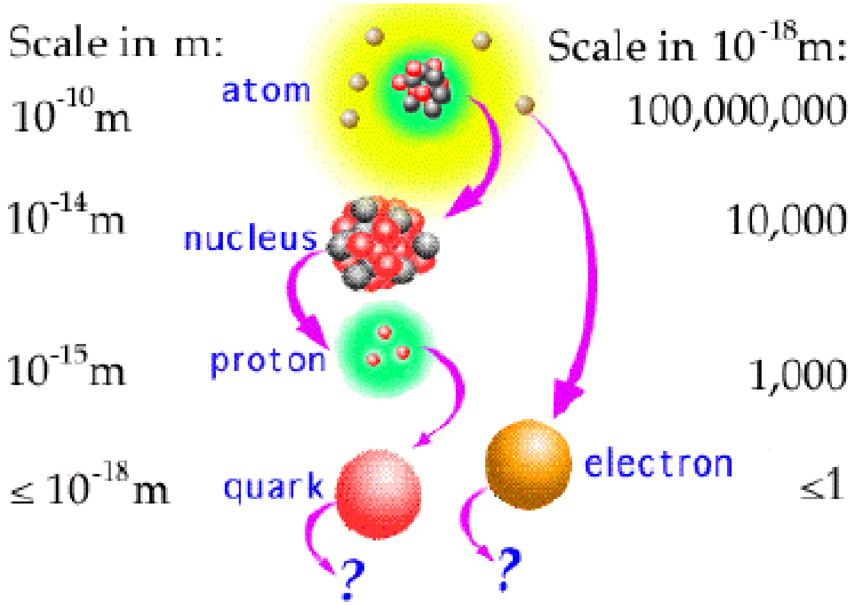


~ 10^{-15} m



- Neutral atoms: # of protons = # of electrons
- Chemical properties of atoms $\longrightarrow$ # of electrons
- $M_p \sim M_n \sim 2000\ M_e$ $\longrightarrow$ the mass of atom ~ the mass of nucleus

Nuclear Physics



1 fm = 10^{-15} m

Nucleus as a *quantum many body system*

Basic ingredients:

	charge	mass (MeV)	spin
Proton	+e	938.256	½+
Neutron	0	939.550	½+

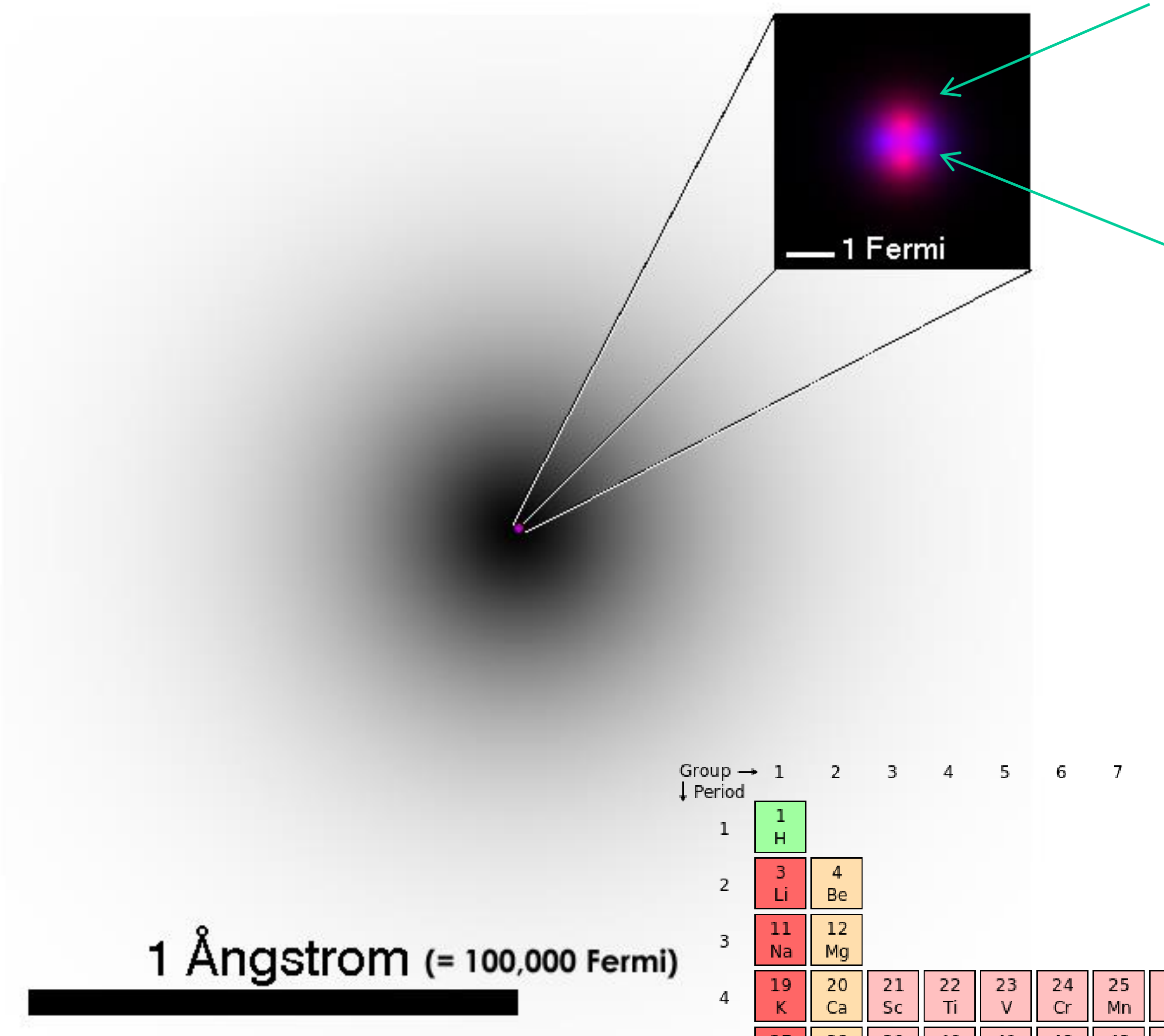
(note) $n \longrightarrow p + e^- + \bar{\nu}$ (10.4 min)

Periodic table of chemical elements

Group → / ↓ Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

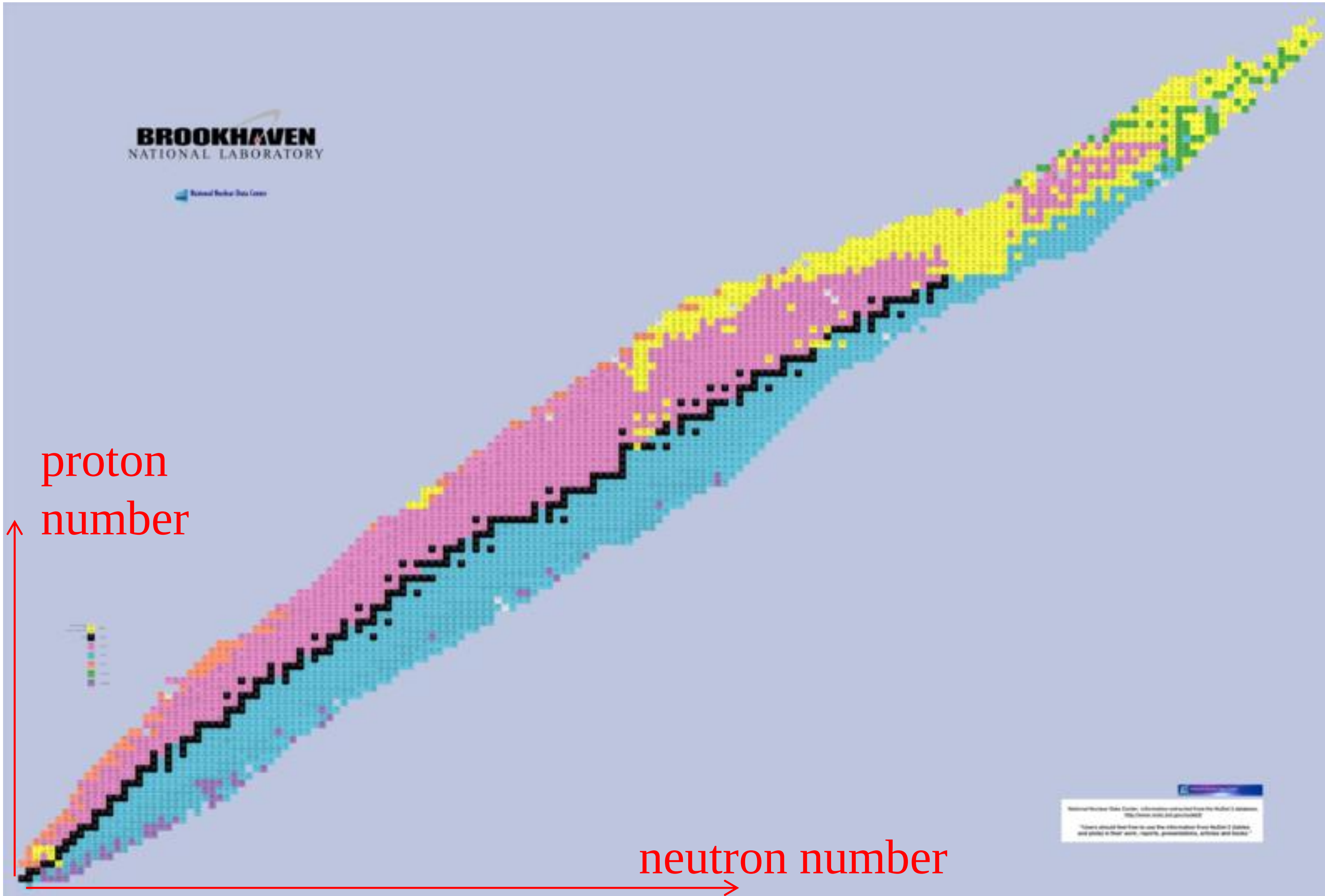
tabular arrangement of chemical elements based on the atomic numbers (= # of electrons = # of protons)



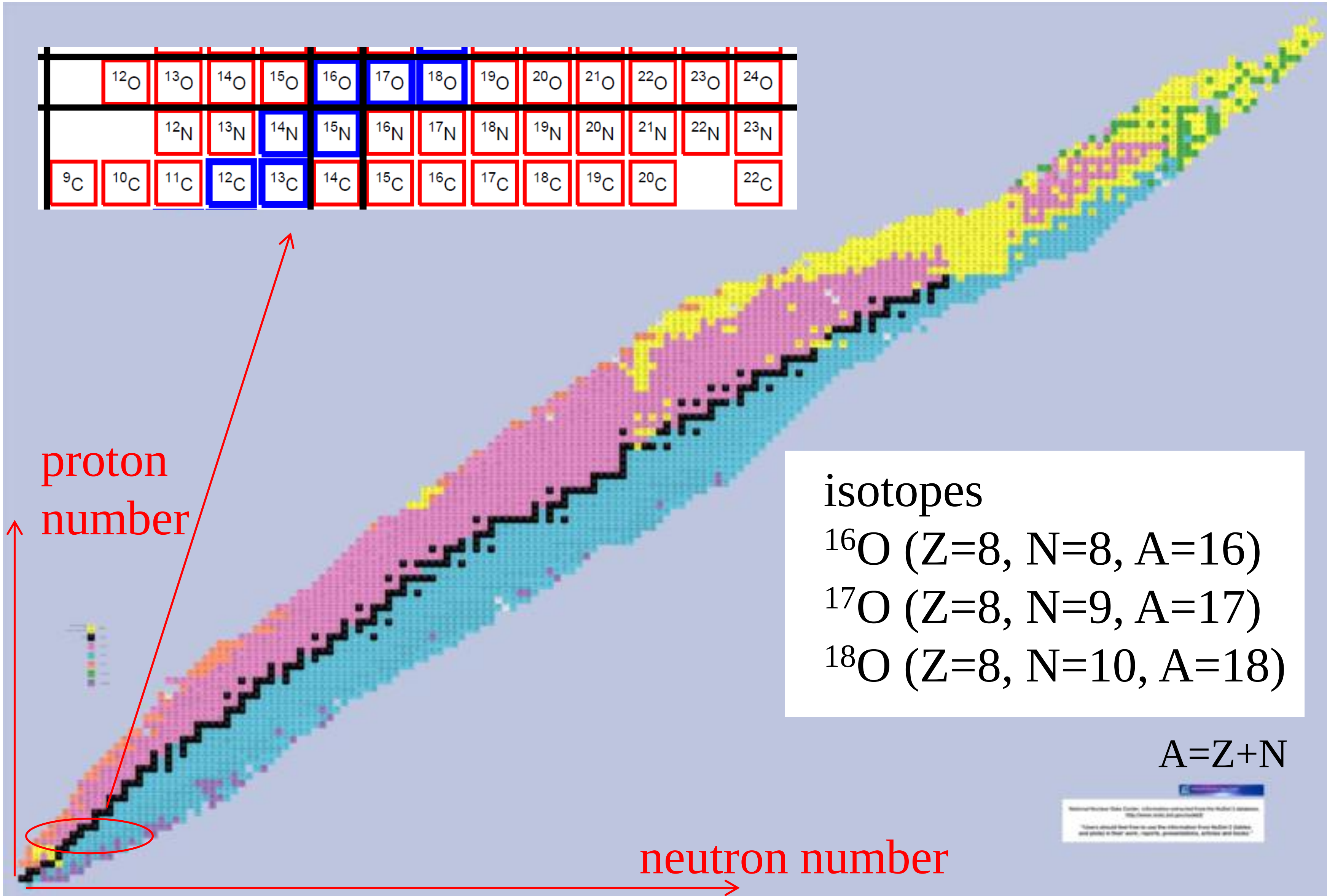
Where are neutrons?

Period ↓ / Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
Lanthanides				57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides				89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Nuclear Chart: 2D map of atomic nuclei



Nuclear Chart: 2D map of atomic nuclei



isotopes

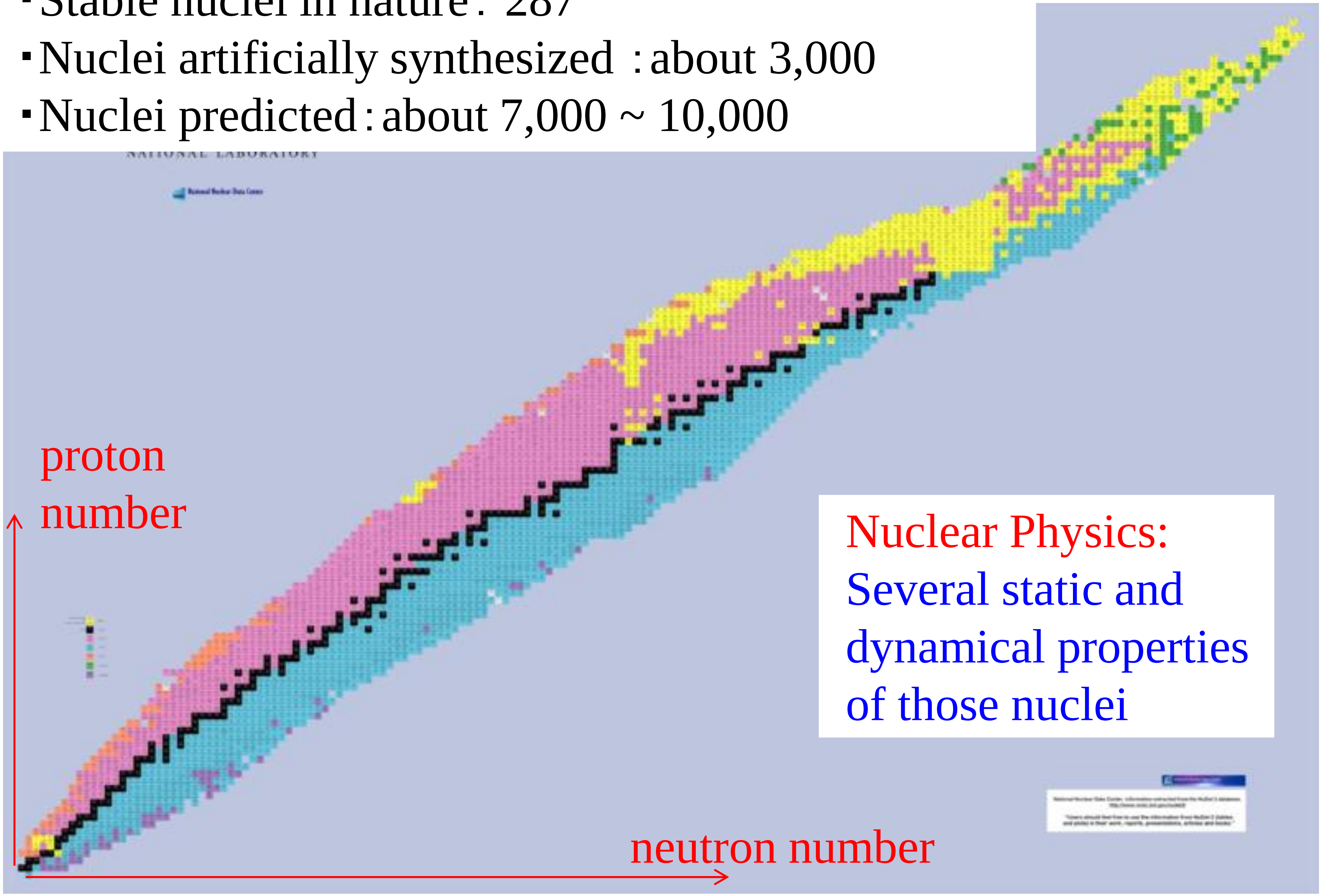
^{16}O (Z=8, N=8, A=16)

^{17}O (Z=8, N=9, A=17)

^{18}O (Z=8, N=10, A=18)

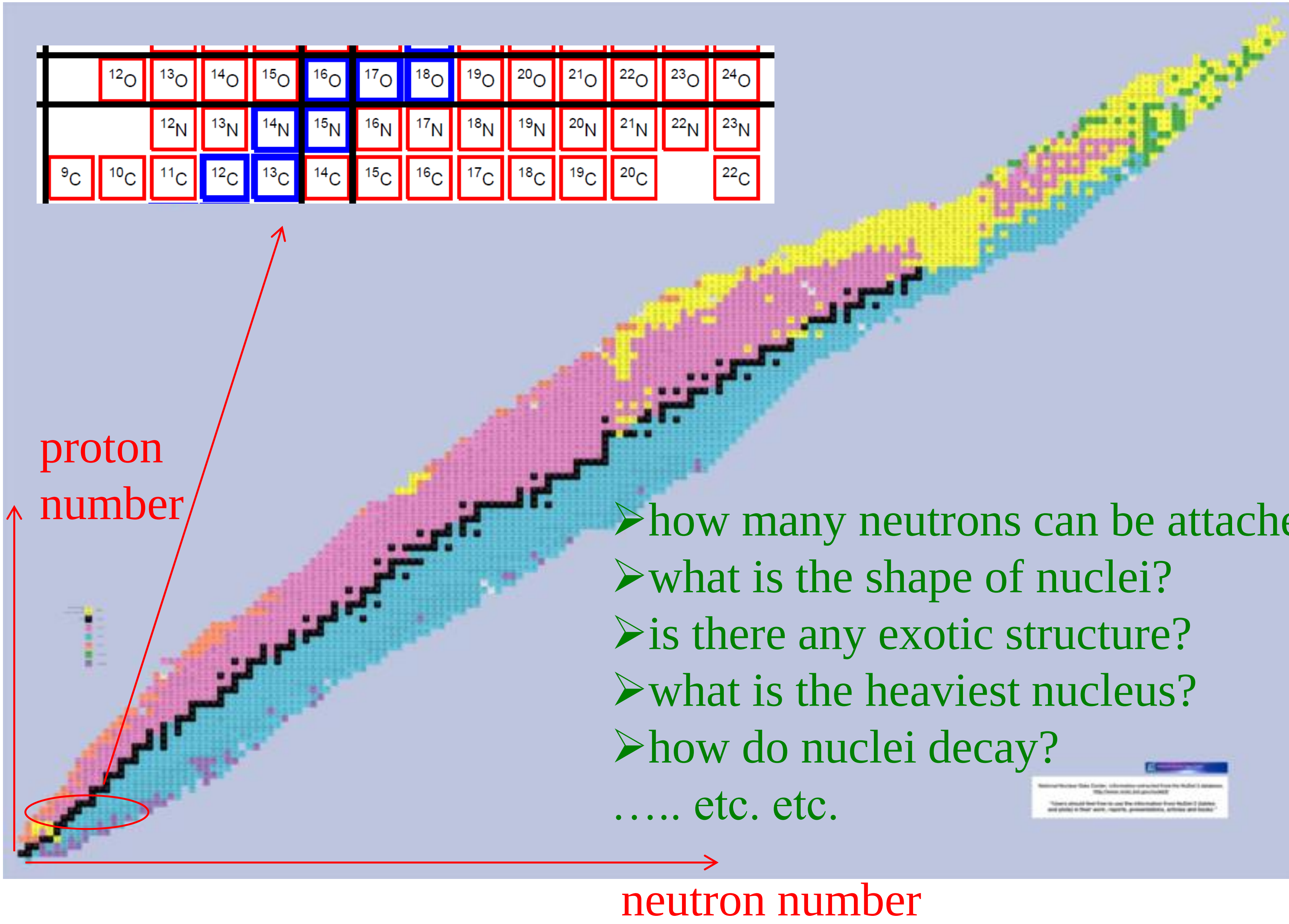
A=Z+N

- Stable nuclei in nature： 287
- Nuclei artificially synthesized ：about 3,000
- Nuclei predicted：about 7,000 ~ 10,000



Nuclear Physics:
Several static and dynamical properties of those nuclei

Nuclear Chart: 2D map of atomic nuclei

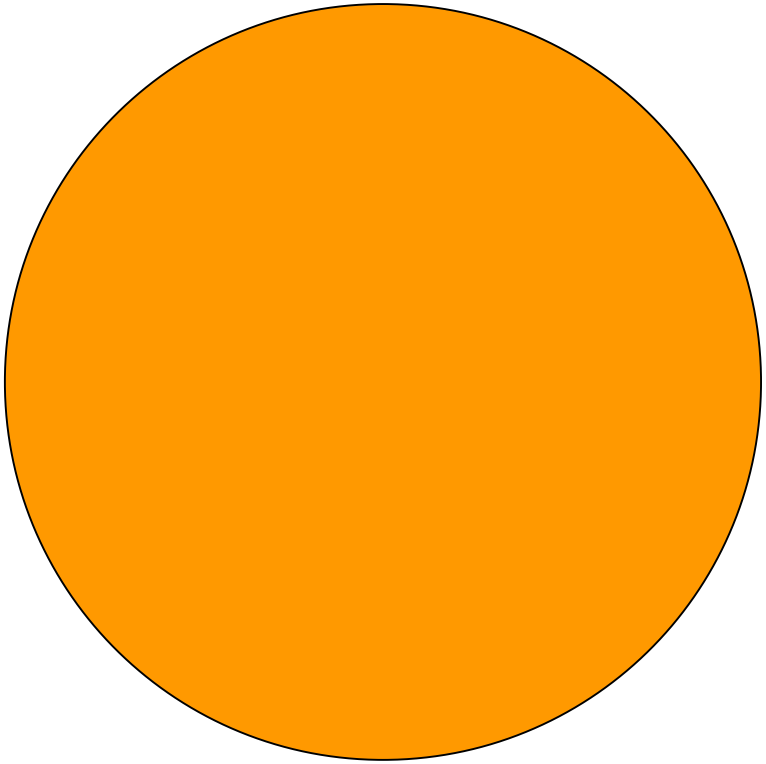


- how many neutrons can be attached?
- what is the shape of nuclei?
- is there any exotic structure?
- what is the heaviest nucleus?
- how do nuclei decay?

….. etc. etc.

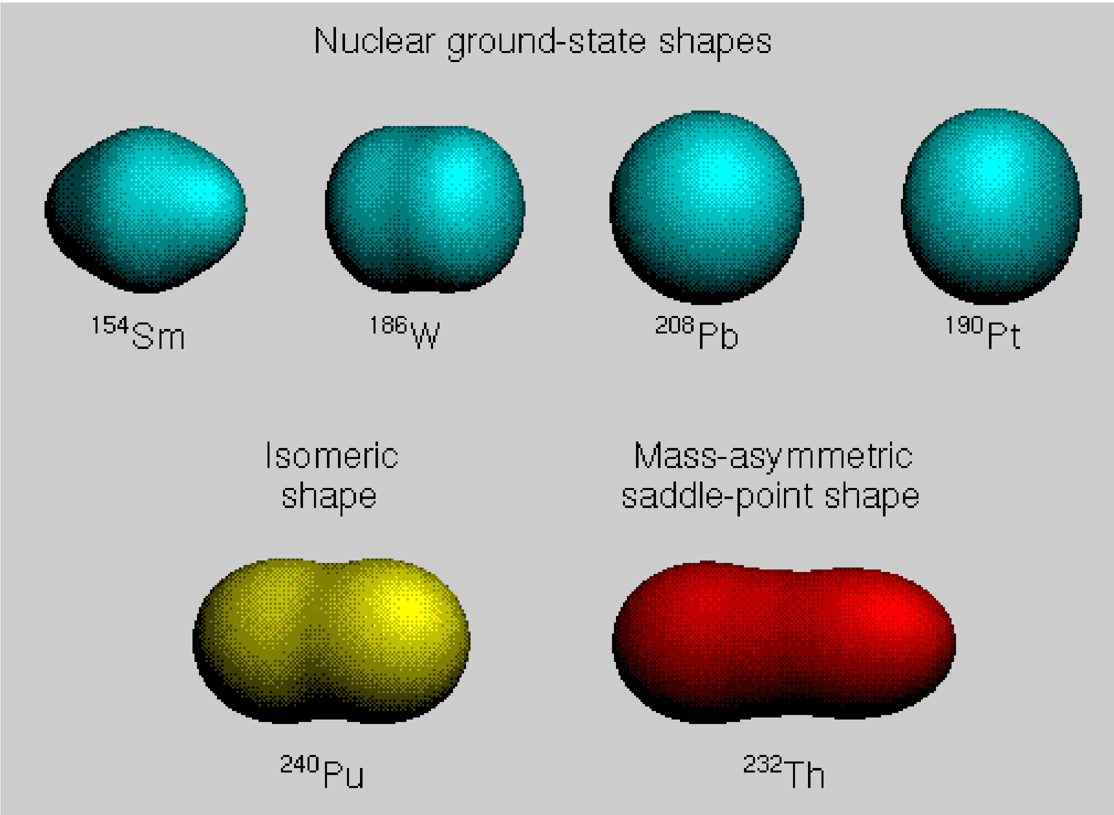
An example of what we investigate in nuclear physics

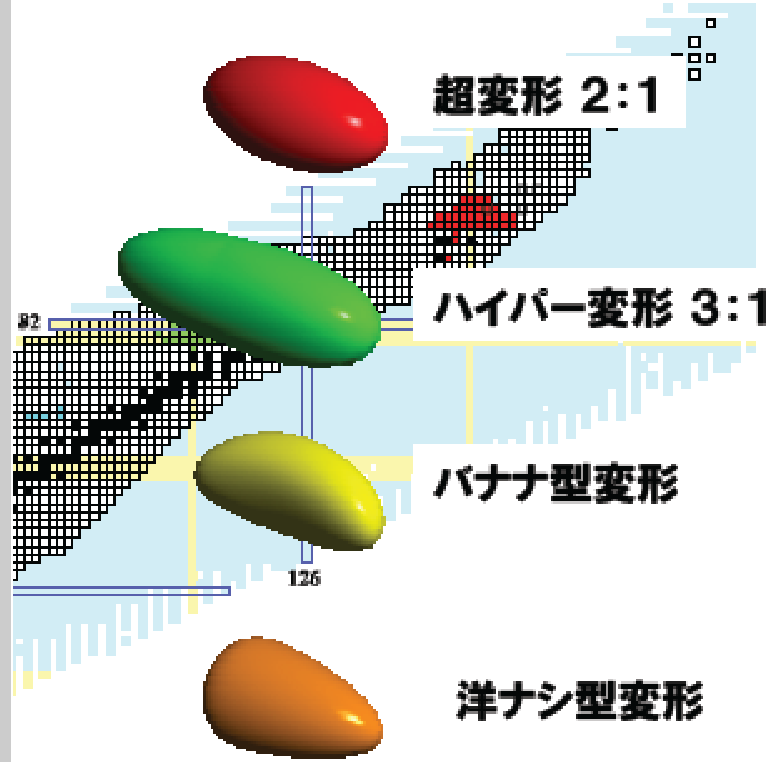
- what is the shape of a nucleus?



Are nuclei all spherical?

➢what is the shape of nucleus?



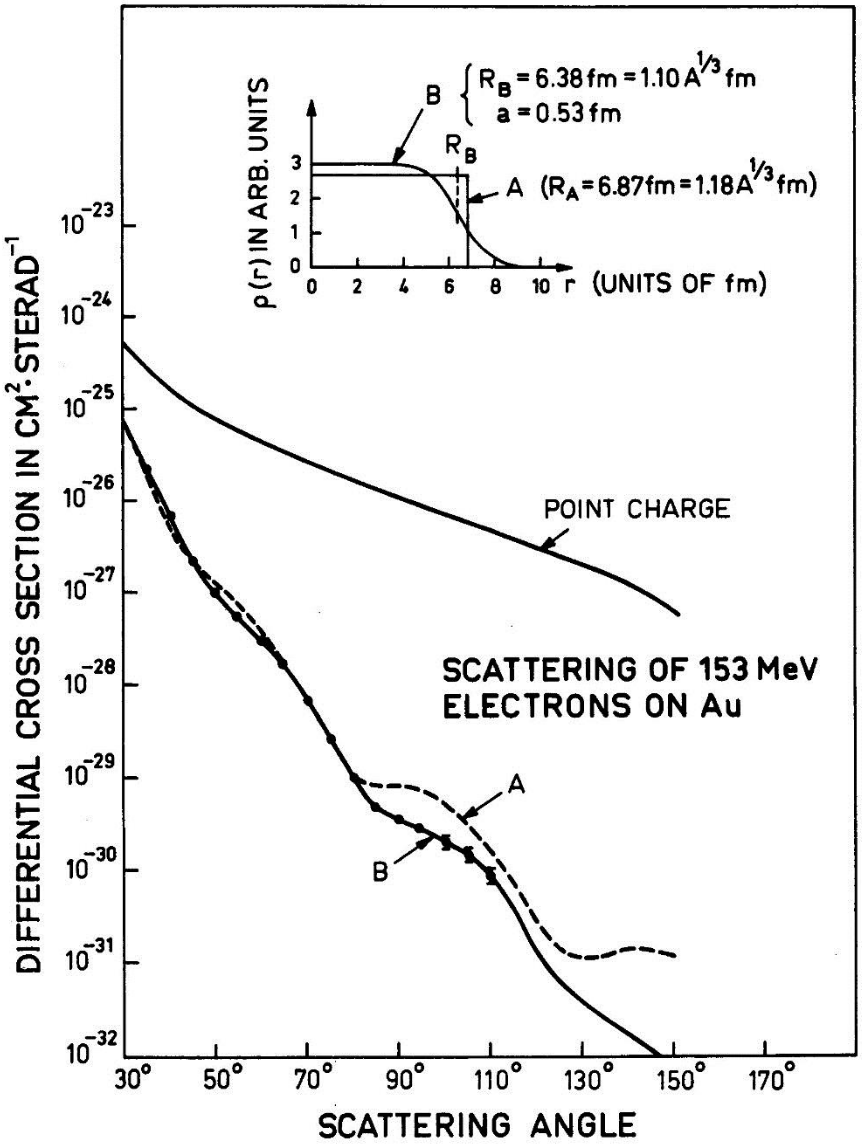


http://t2.lanl.gov/tour/sch001.html

Some nuclei are deformed in the ground state!

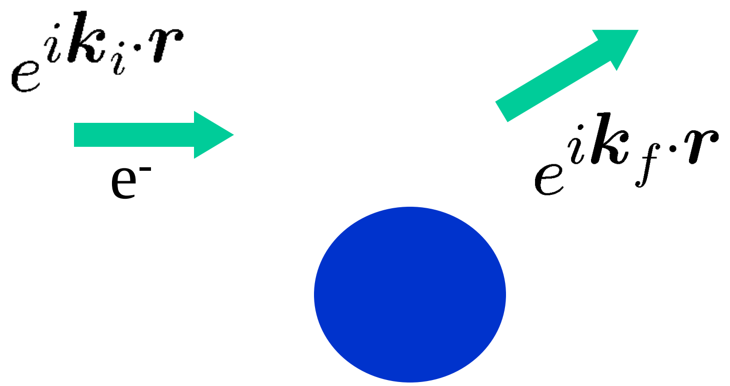
what are combinations of (Z,N) which yield a deformation?

Density Distribution



High energy electron scattering

Born approximation:

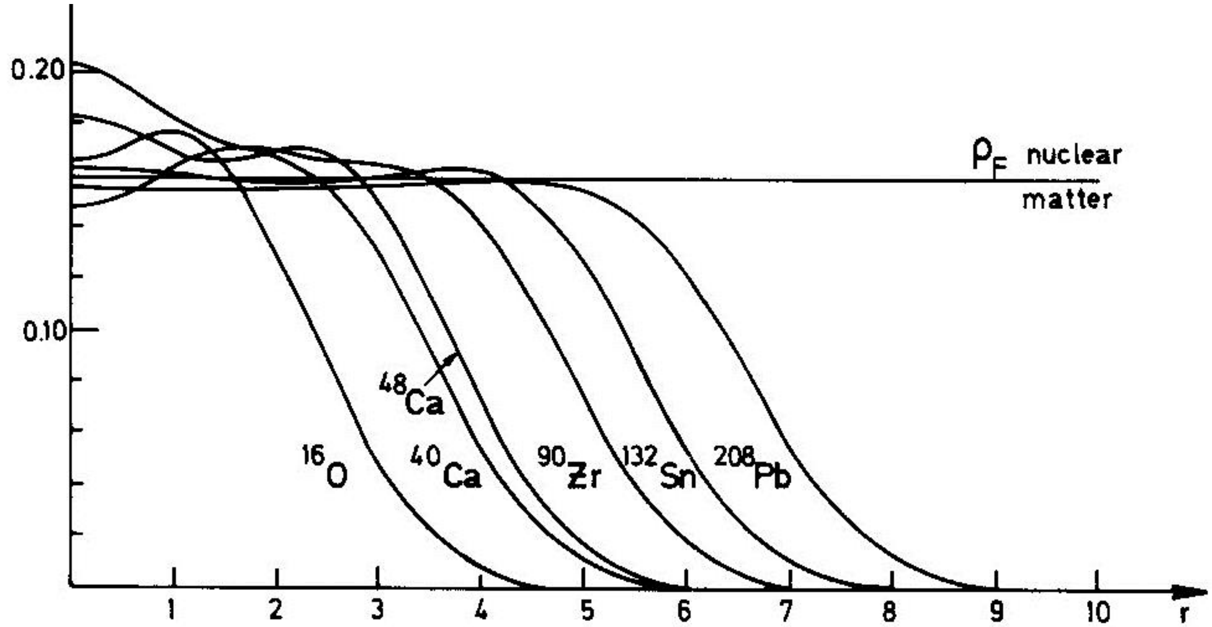


$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2$$

Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

(Fourier transform of the density)



Fermi distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp((r - R_0)/a)}$$

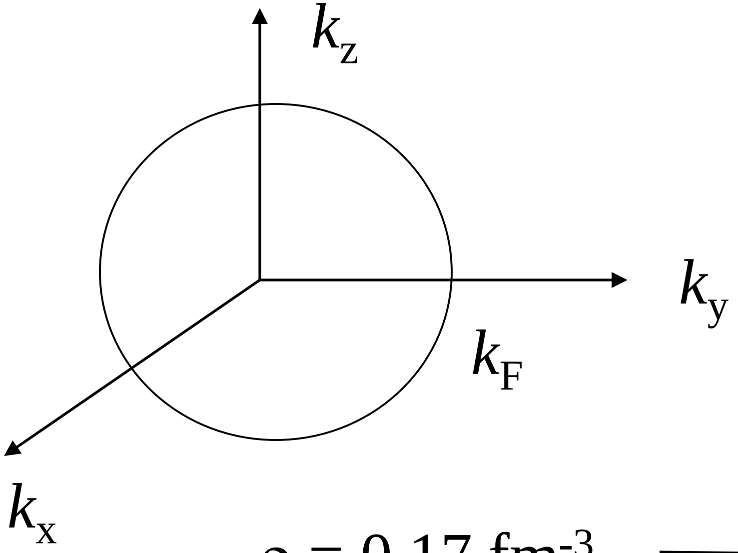
$$\rho_0 \sim 0.17 \ (\text{fm}^{-3})$$ ← Saturation property

$$R_0 \sim 1.1 \times A^{1/3} \ (\text{fm})$$

$$a \sim 0.57 \ (\text{fm})$$

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3} = \frac{2}{3\pi^2} k_F^3$$

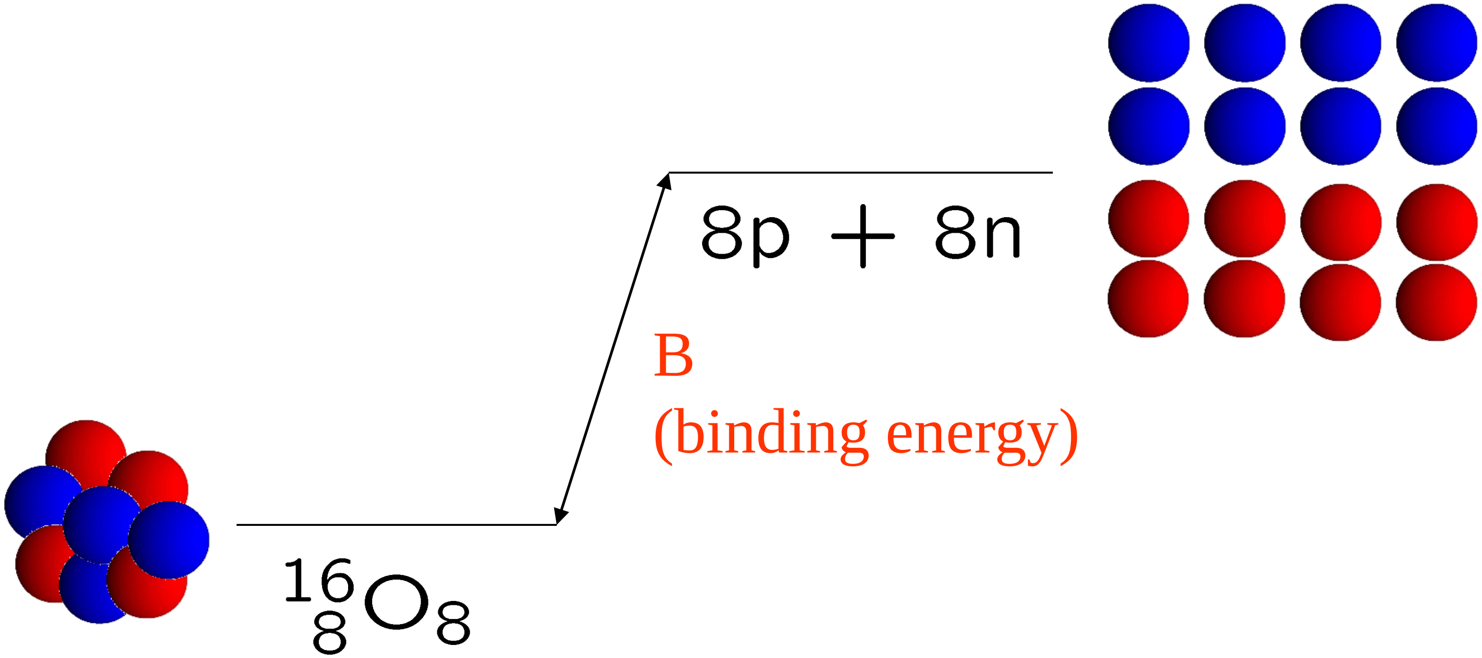
(note: spin-isospin degeneracy)

$\rho = 0.17\ \mathrm{fm}^{-3}$ ⟶ $k_F \sim 1.36\ \mathrm{fm}^{-1}$

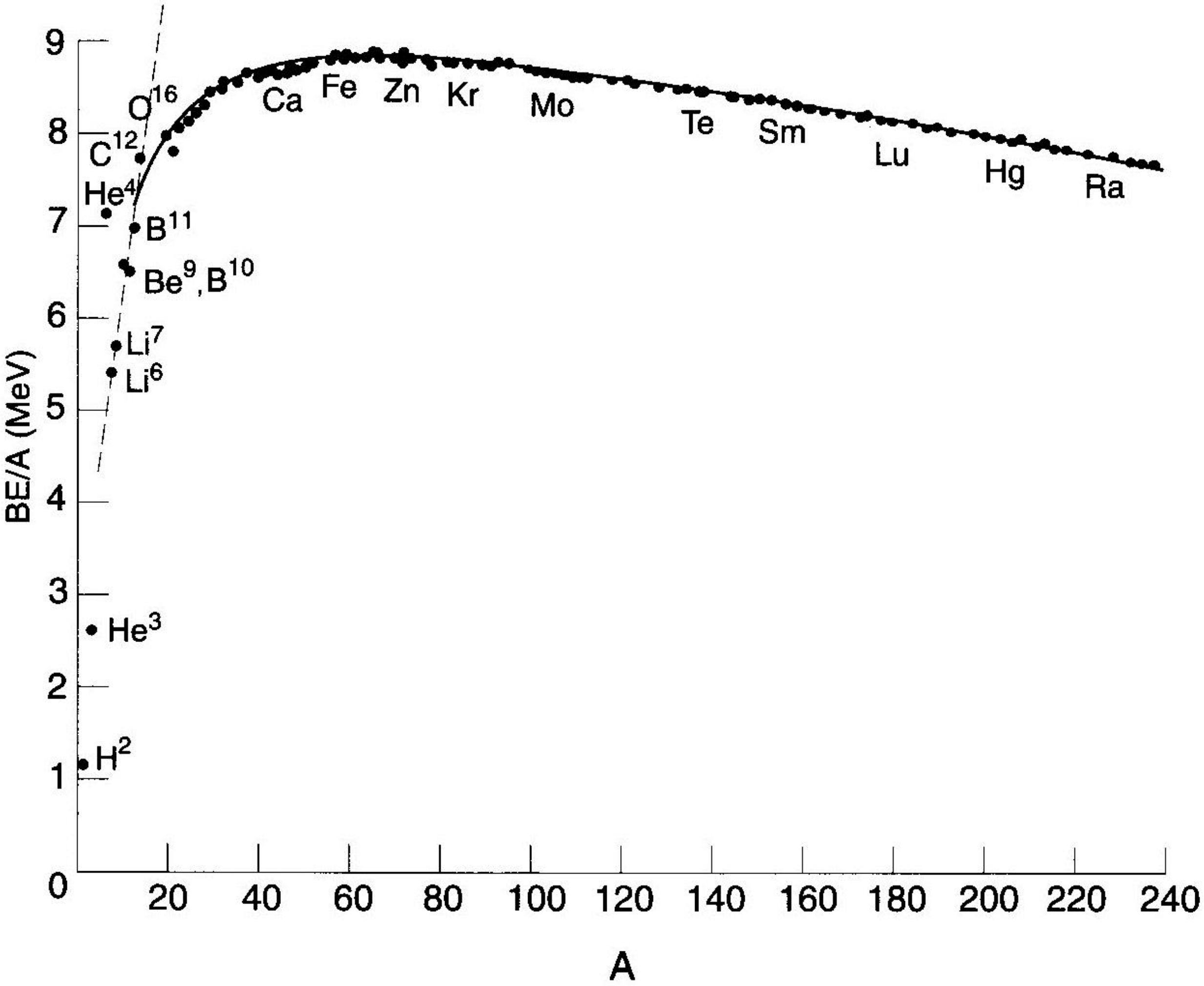
$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

Fermi energy: $\epsilon_F = \dfrac{k_F^2 \hbar^2}{2m} \sim 37 \quad \text{(MeV)}$

Nuclear Mass



$$m(N, Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$

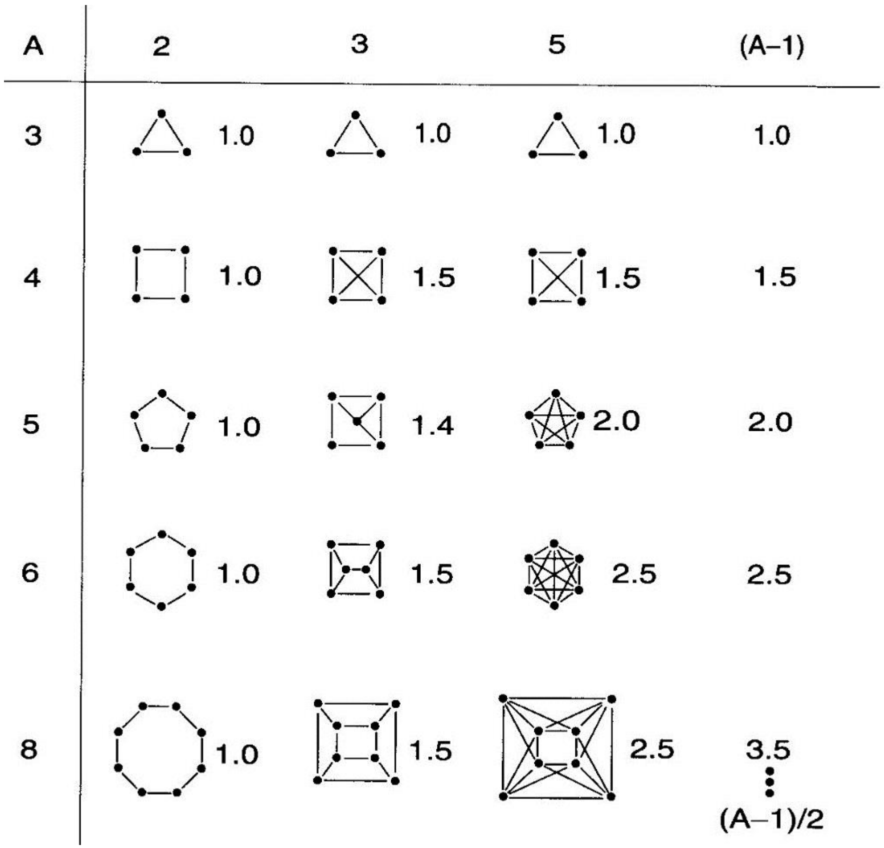


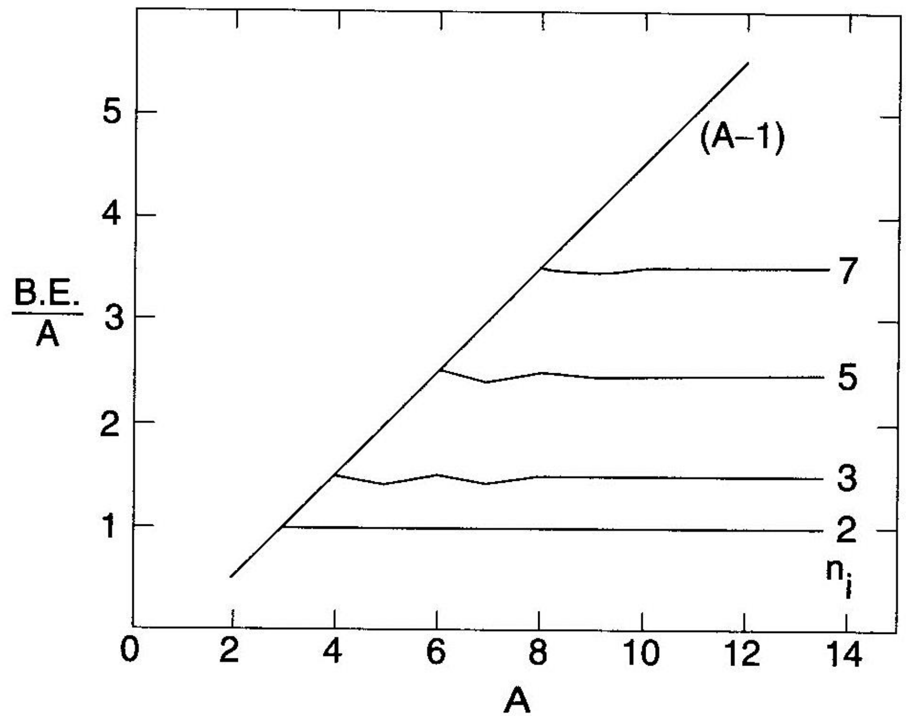
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force

Long vs short range interaction

Long range force: $B \propto A(A-1)/2 \quad \curvearrowright \quad B/A \propto A$

Short range force: saturation



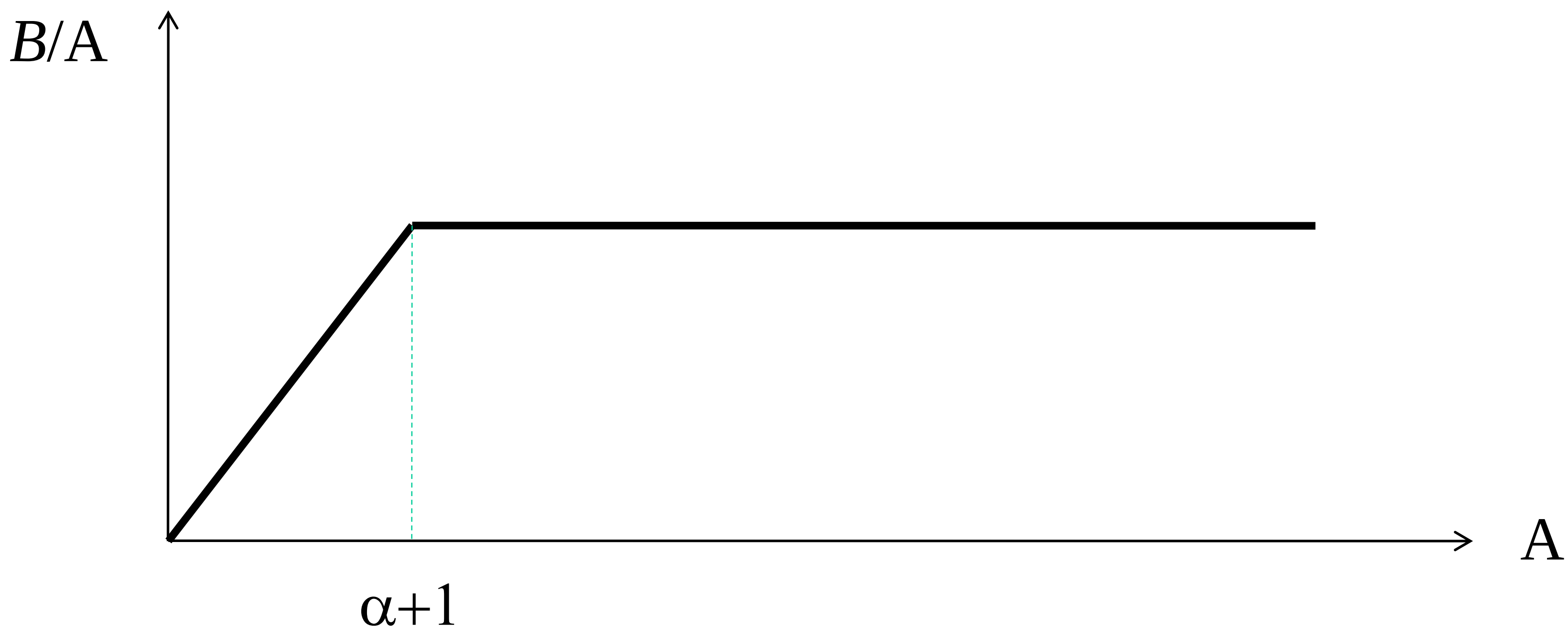


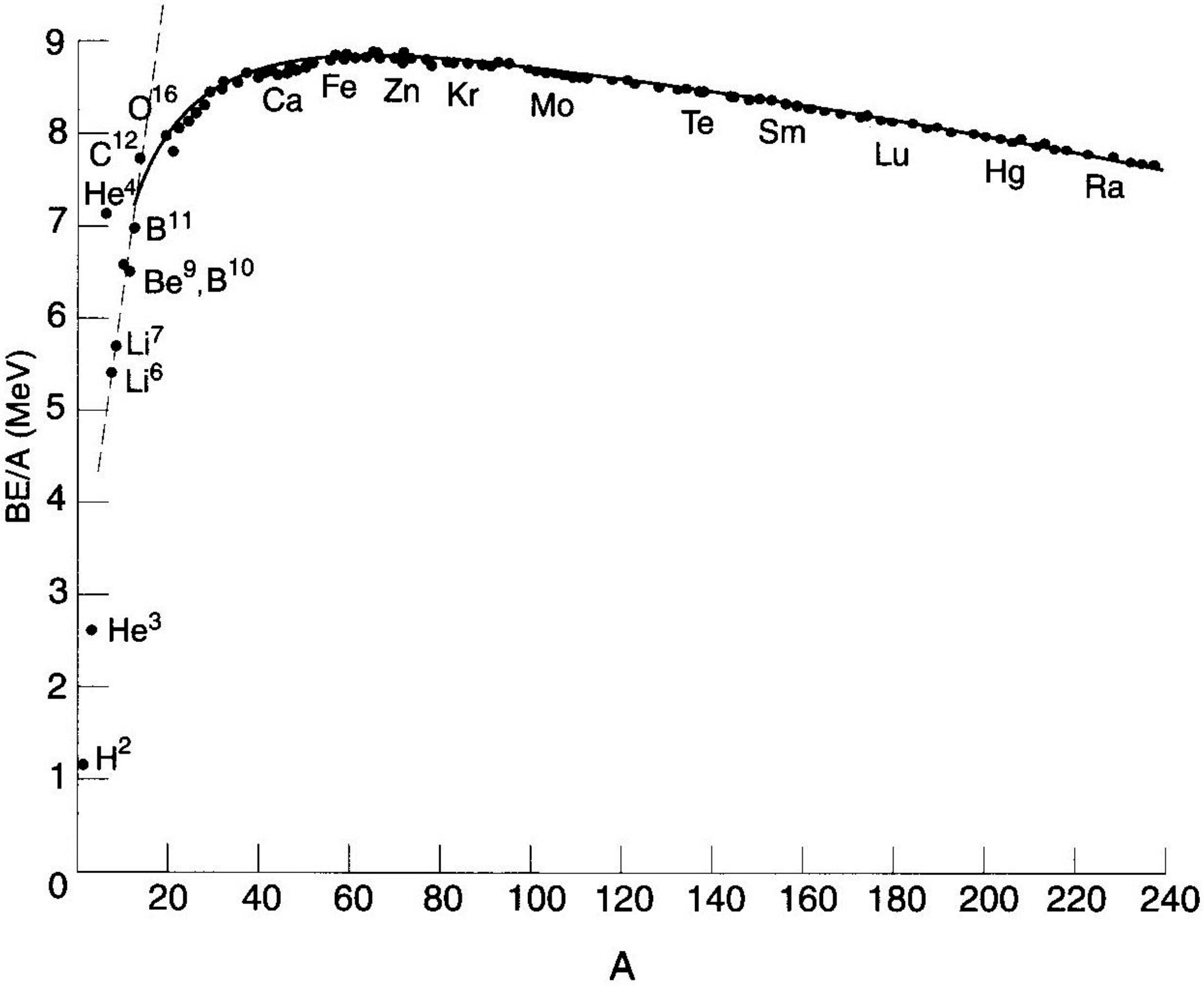
If one nucleon interacts only with surrounding α nucleons

$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

For $A < \alpha+1$, one nucleon interacts with all the other nucleons

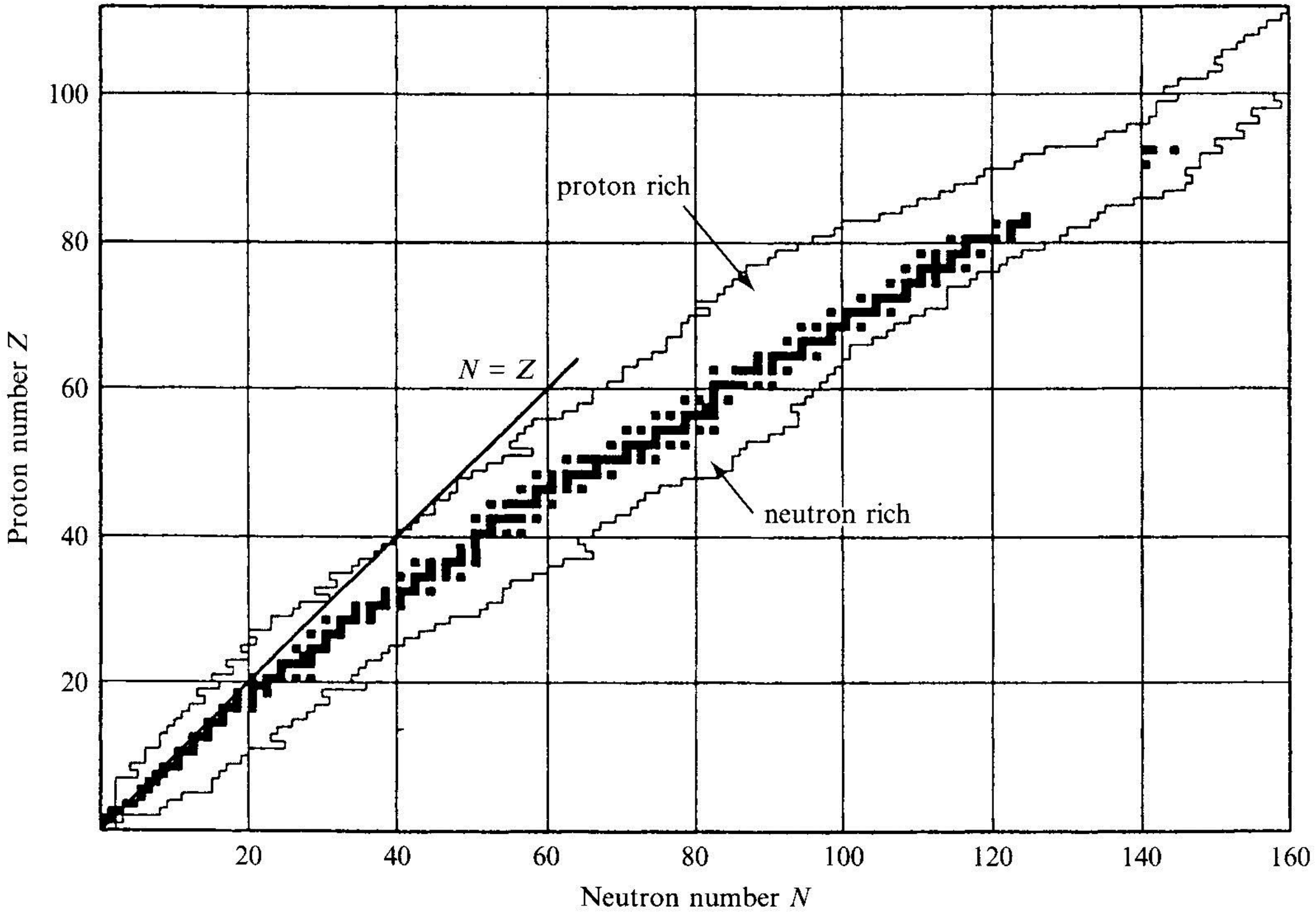
$$\longrightarrow B/A \propto A$$



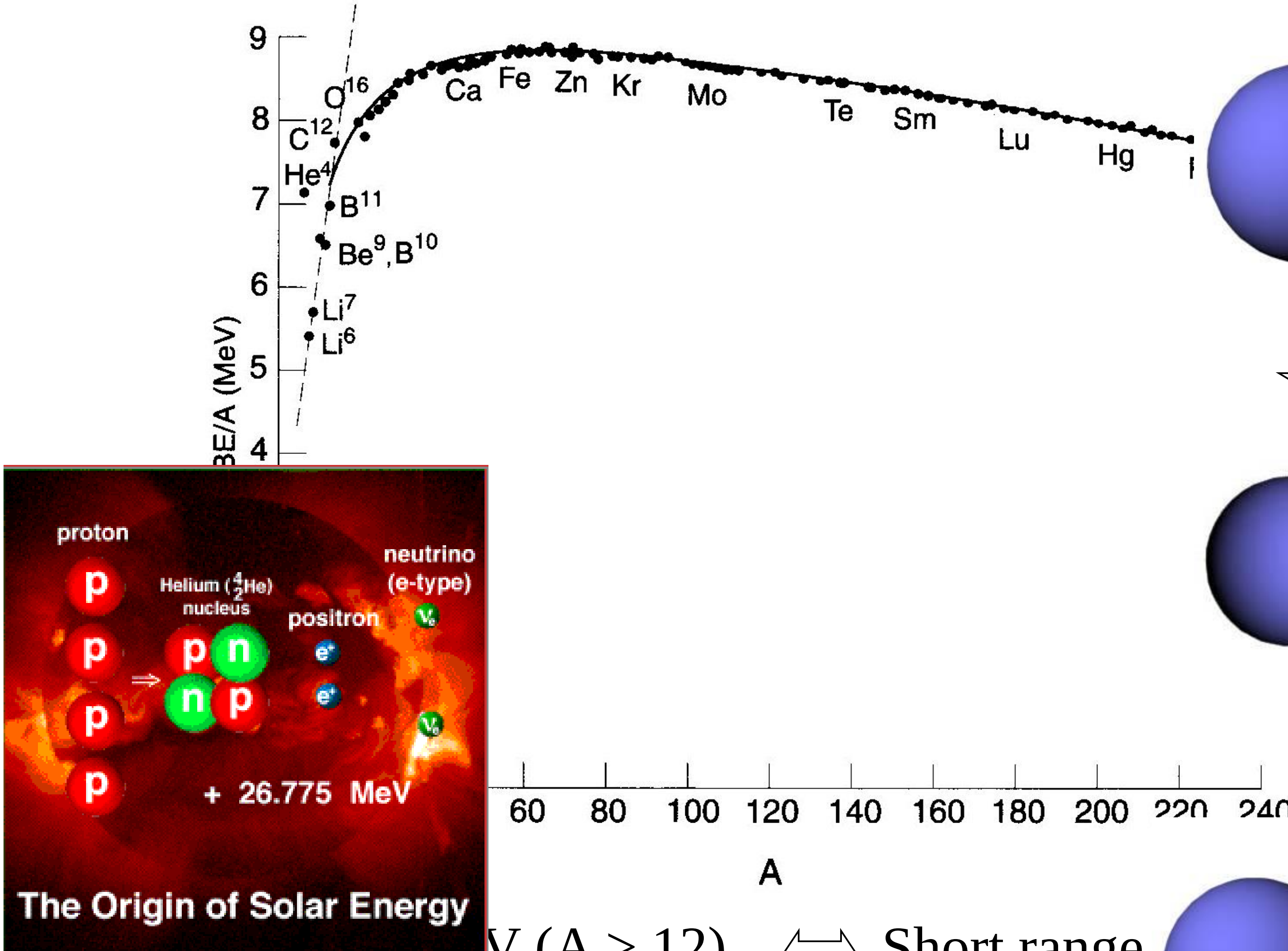


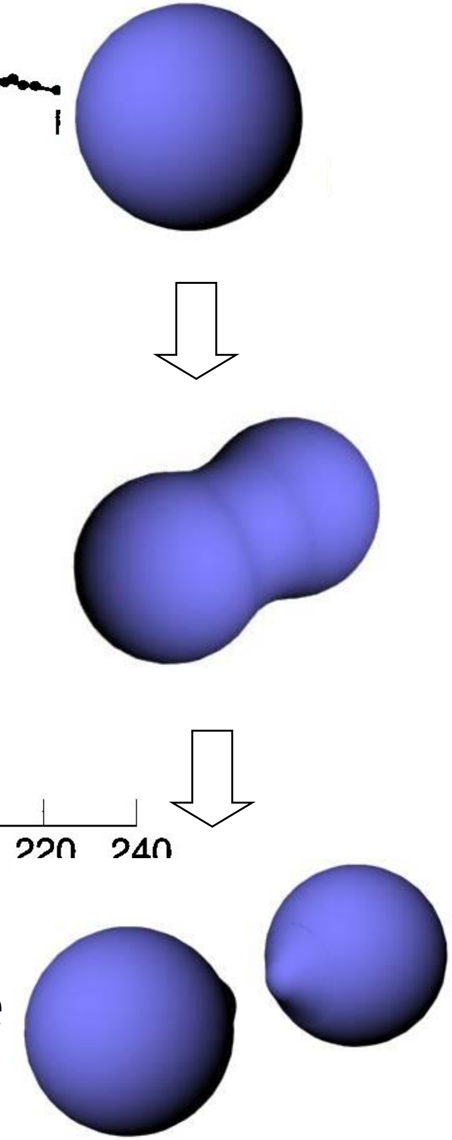
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$





1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⇔ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei