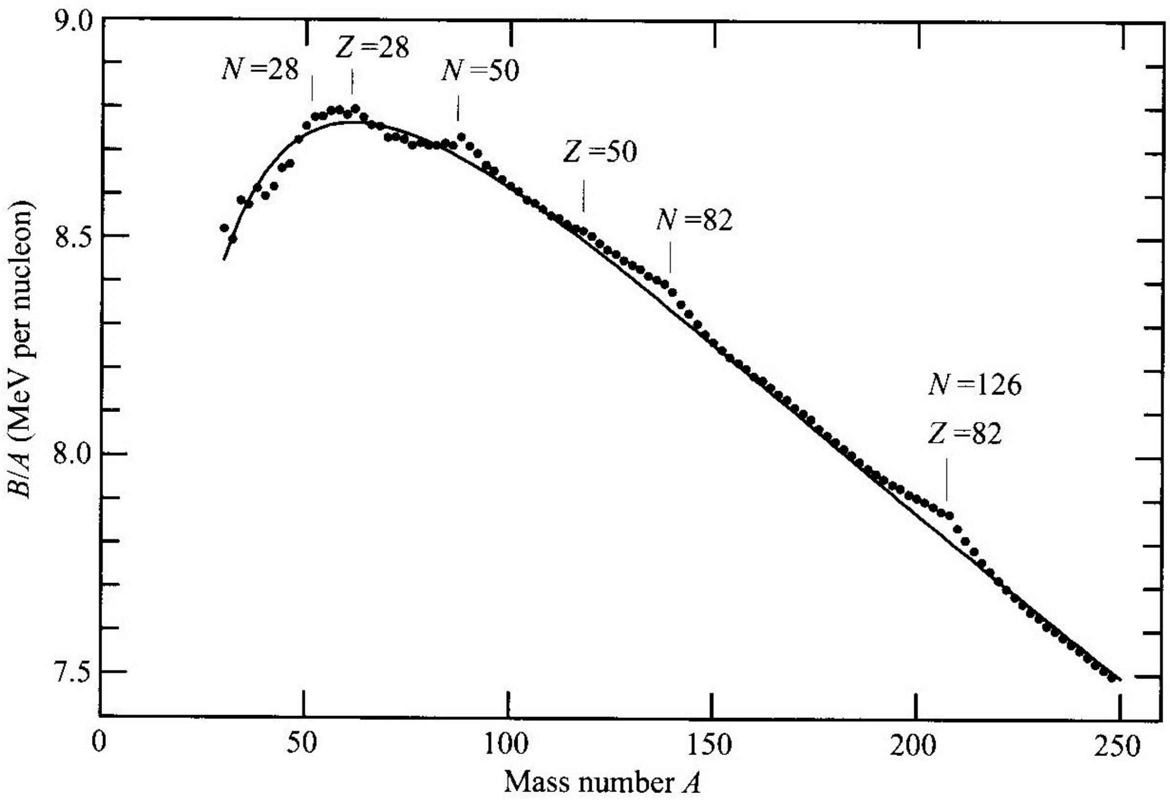
Shell Energy



Extra binding for N,Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

⇨ Very stable

${}^{4}_{2}He_{2}, {}^{16}_{8}O_{8}, {}^{40}_{20}Ca_{20}, {}^{48}_{20}Ca_{28}, {}^{208}_{82}Pb_{126}$

✓ asymmetric fission

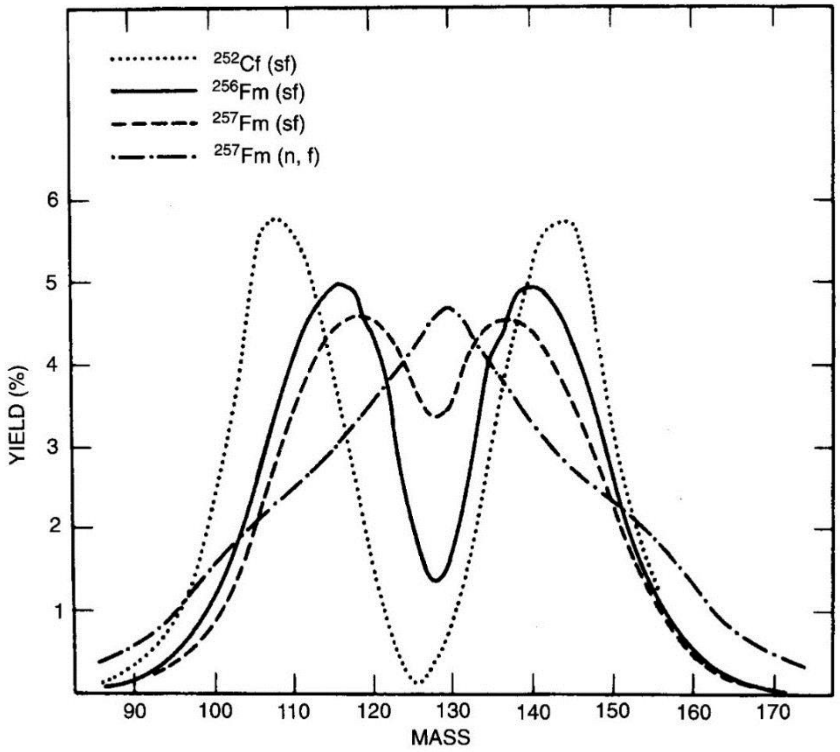


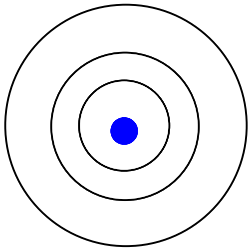
Fig. 4.1. Mass distributions in terms of the fission fragment masses for spontaneous fission of $^{252}_{98}$Cf, $^{256}_{100}$Fm and $^{257}_{100}$Fm and for neutron-induced fission of ^{257}Fm. Note the trend toward symmetric fission with increasing mass and in addition the larger number of symmetric events for neutron-induced than for spontaneous fission (from R. Vandenbosch and J.R. Huizenga, *Nuclear Fission* (Academic Press, New York and London, 1973)).

cf. $^{120}_{50}$Sn

✓ stability of superheavy elements

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



shell structure

(note) Atomic magic numbers (Noble gas)

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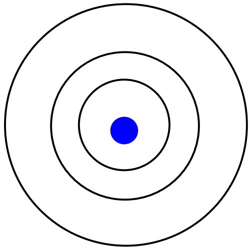
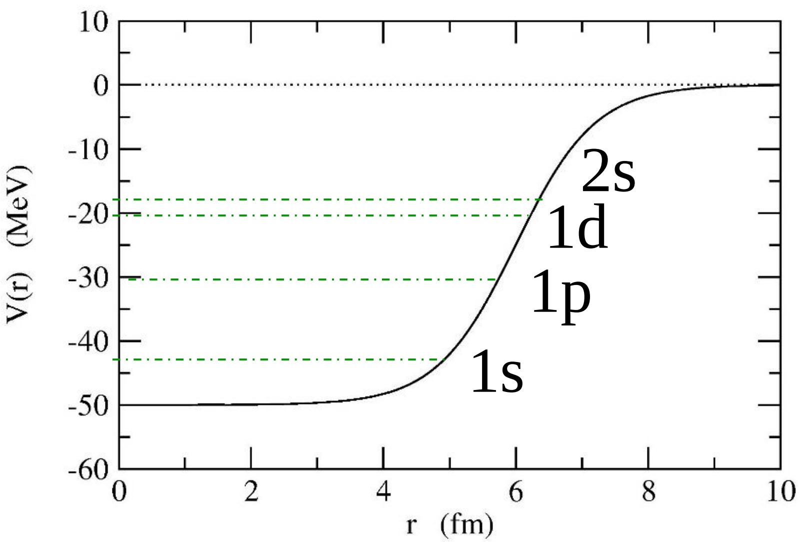


Shell structure

Similar attempt in nuclear physics: independent particle motion in a potential well

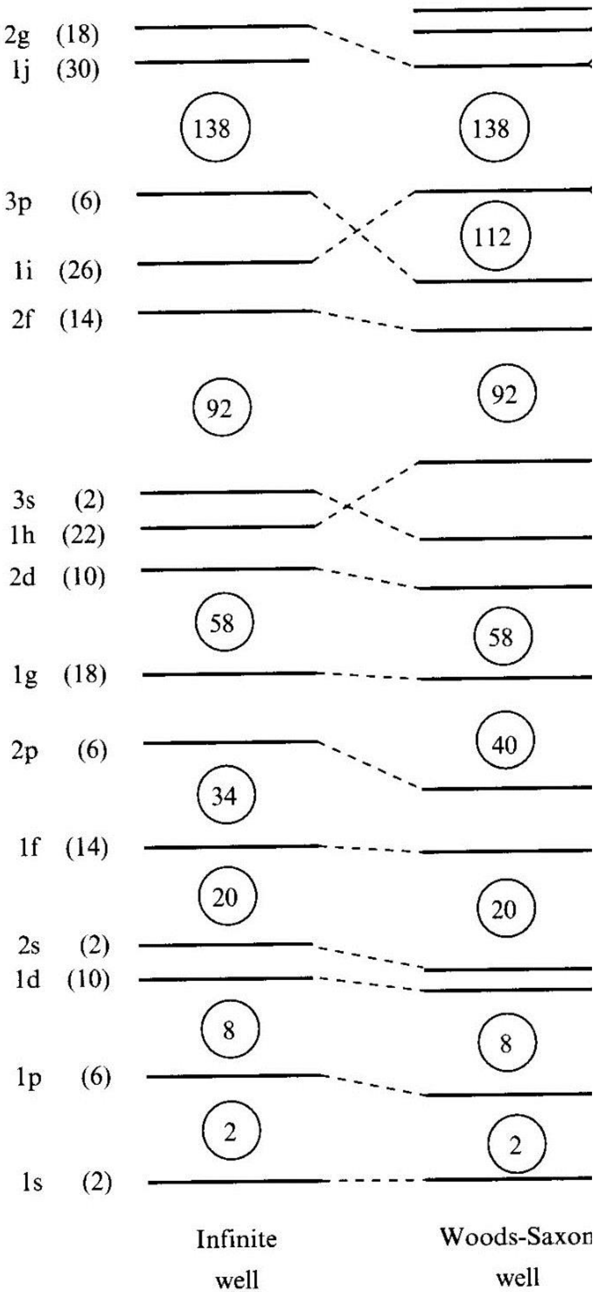
Woods-Saxon potential

$$V(r) = -V_0/[1 + \exp((r - R_0)/a]$$

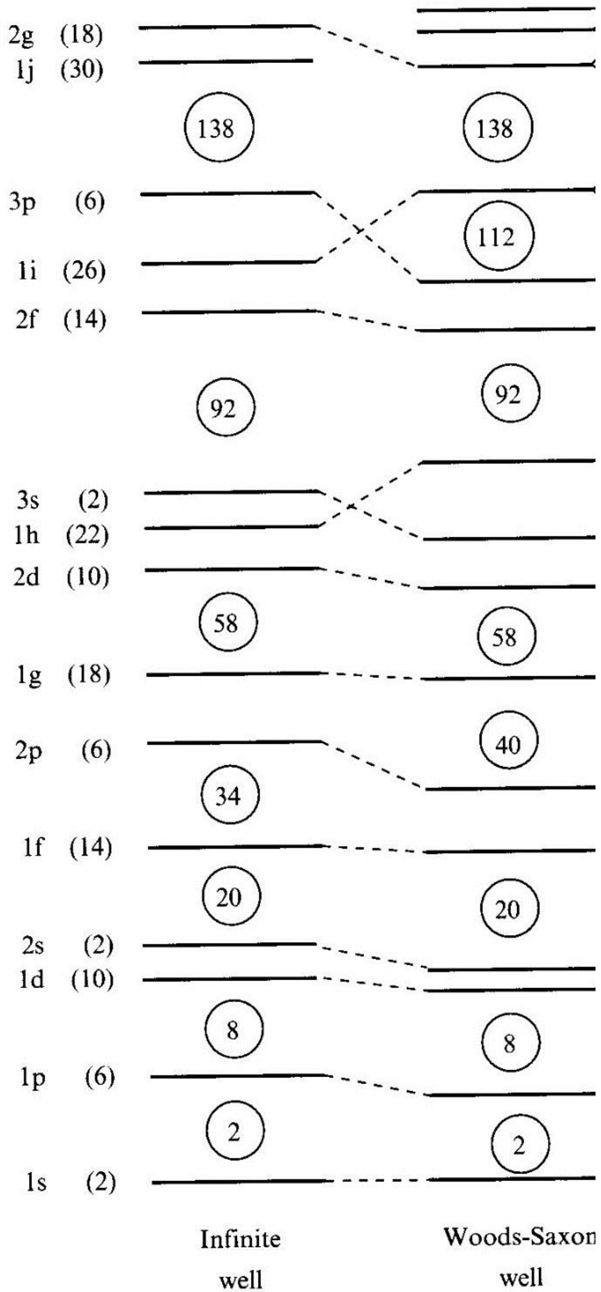


$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r}Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$



2g (18)
1j (30)
138
138
3p (6)
112
1i (26)
2f (14)
92
92
3s (2)
1h (22)
2d (10)
58
58
1g (18)
40
2p (6)
34
1f (14)
20
20
2s (2)
1d (10)
8
8
1p (6)
2
2
1s (2)
Infinite well
Woods-Saxon well



Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).

↓

Mayer and Jensen (1949):

Strong spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$V_{ls}(r) \sim -\lambda\frac{1}{r}\frac{dV}{dr} \qquad (\lambda > 0)$$

jj coupling shell model

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0 \Longrightarrow \psi_{lmm_s}(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

Spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \Longrightarrow \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\psi_{jlm}(\boldsymbol{r}) = \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l, m_s} \langle l\ m_l\ 1/2\ m_s | j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

jj coupling shell model

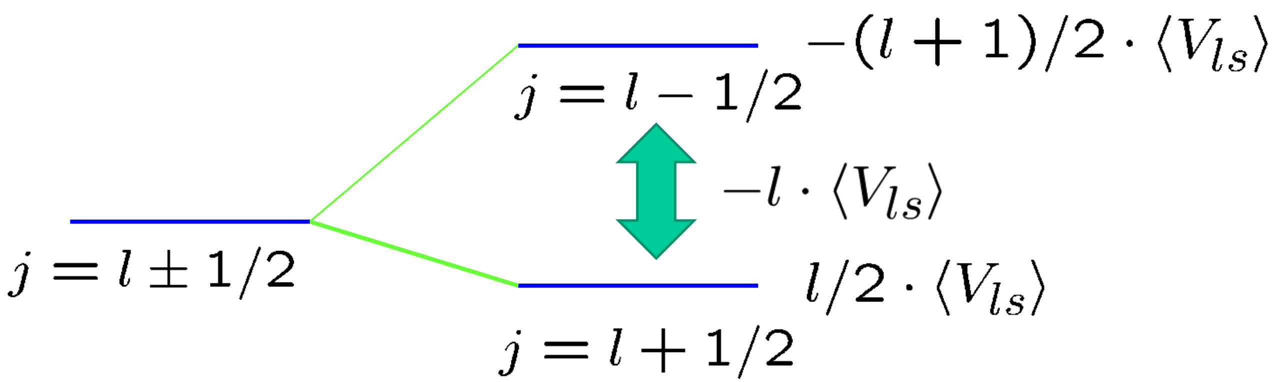
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

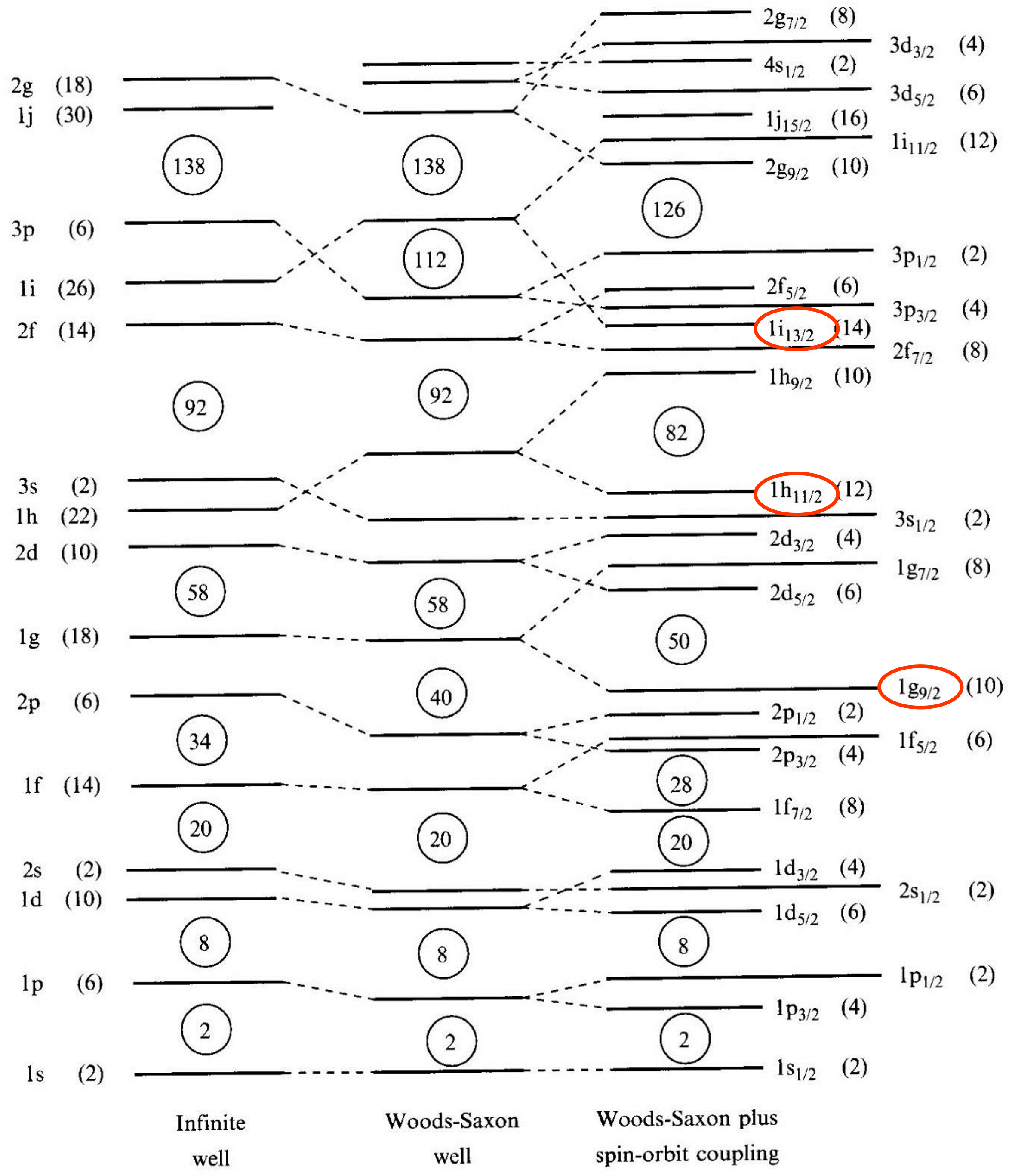
(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \quad \Longrightarrow \quad \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\psi_{jlm}(\boldsymbol{r}) = \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l,m_s}\langle l\ m_l\ 1/2\ m_s|j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

$$\boldsymbol{l}\cdot\boldsymbol{s} = l/2\ (j = l + 1/2), \quad -(l+1)/2\ (j = l - 1/2)$$





intruder states
unique parity states

Single particle spectra

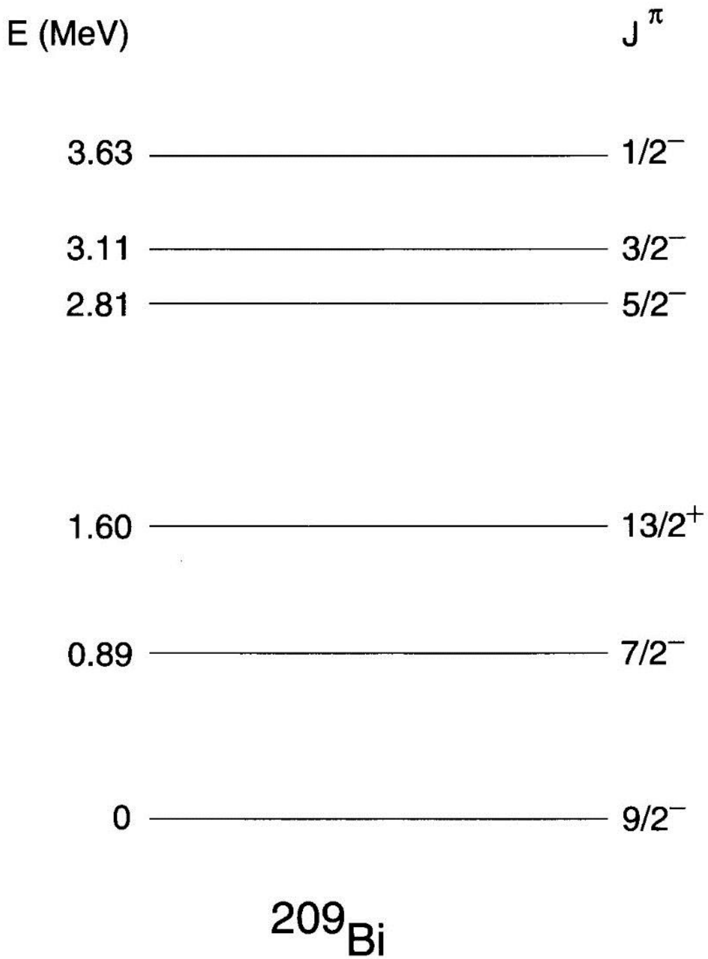
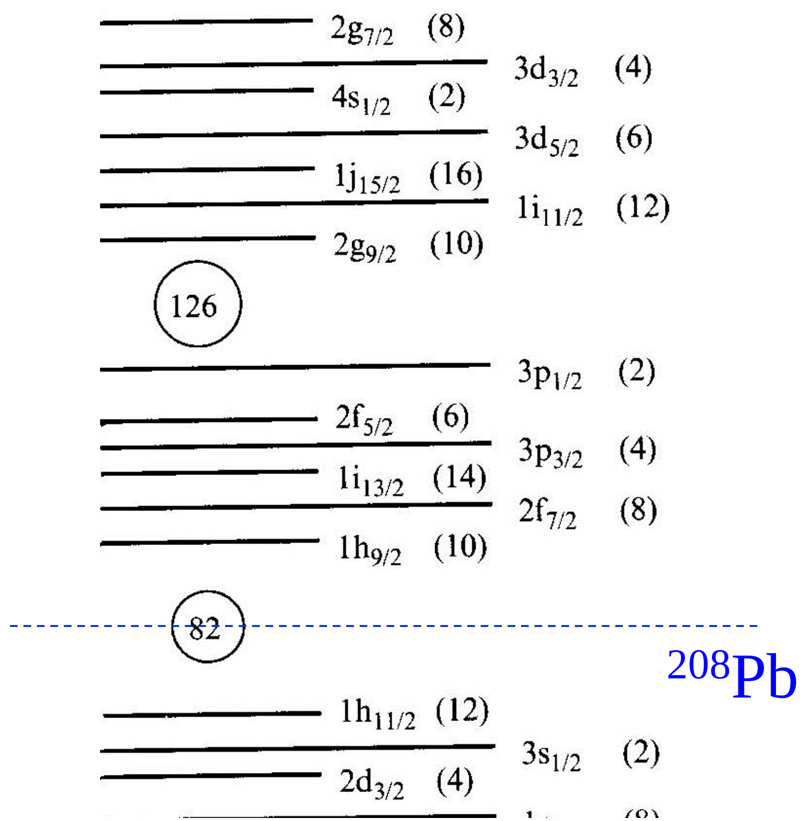


FIG. 3.6. Low-lying single-particle levels of ^{209}Bi.

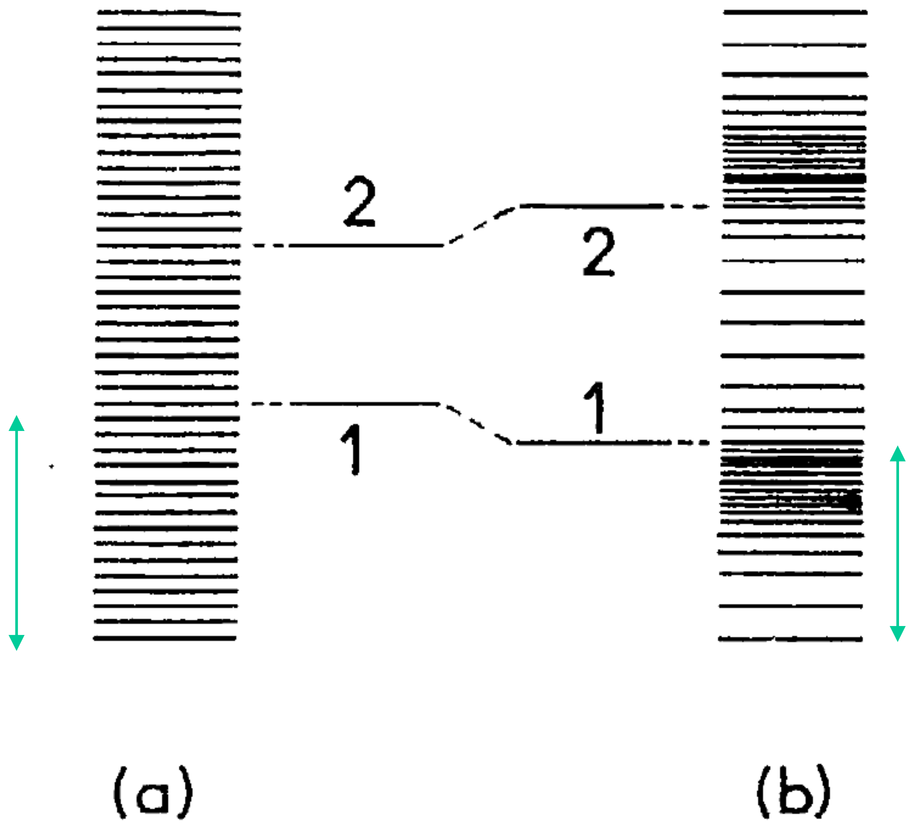


- How to construct V(r) microscopically?
- Does the independent particle picture really hold?

⇨ Later in this lecture

Why do closed-shell-nuclei become stable?

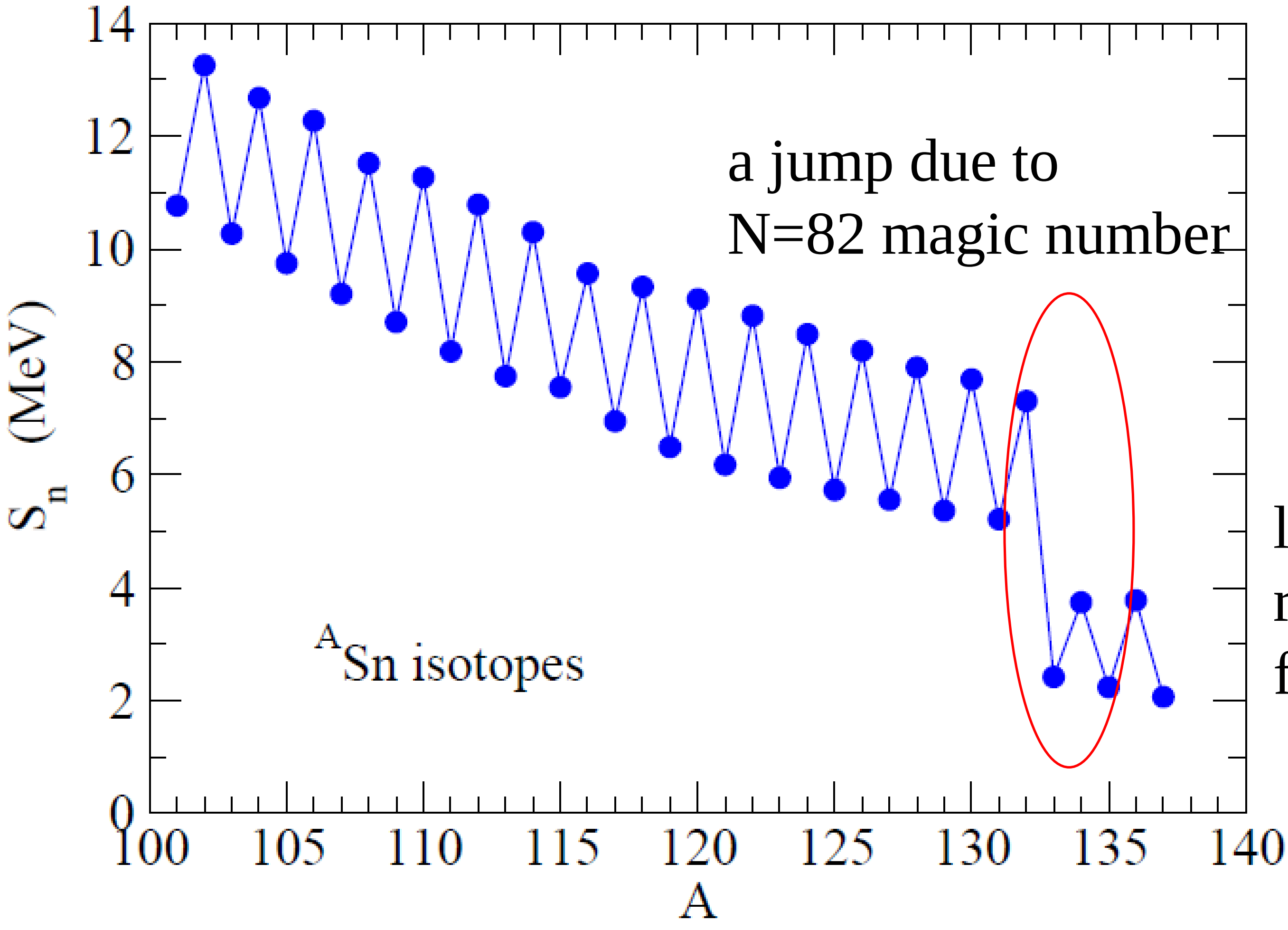
level density



smaller total energy (more stable)

uniform

non-uniform



less energy to
remove a neutron
from a higher orbit

1n separation energy: S_n (A,Z) = B(A,Z) – B(A-1,Z)

Lucky accident for the origin of life

Atomic magic numbers

electron #: 2, 10, 18, 36, 54, 86

inert gas：He, Ne, Ar, Kr, Xe, Rn

Nuclear magic numbers

proton # or neutron #

2, 8, 20, 28, 50, 82, 126

→ e.g., $^{16}_{8}O_{8}$ (double magic)

→ many oxygen nuclei: produced during nucleosynthesis

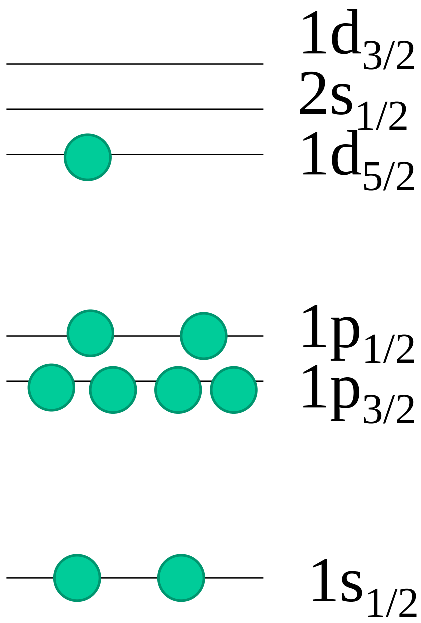
→ oxygen: chemically active

→ several complex chemical reactions, leading to the birth of life

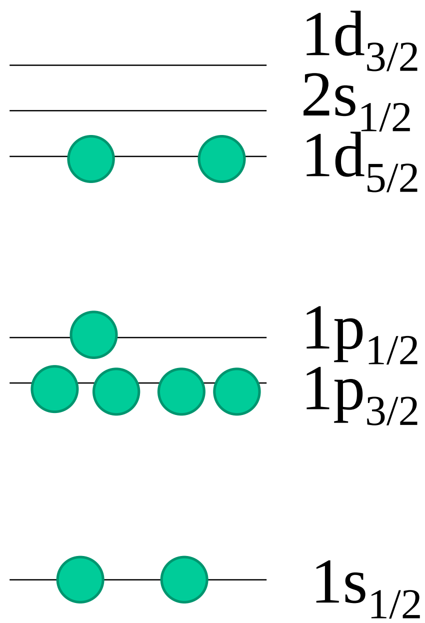
参考：望月優子　ビデオ「元素誕生の謎にせまる」　http://rarfaxp.riken.go.jp/~motizuki/contents/genso.html

single-j model

shell model



configuration 1



configuration 2

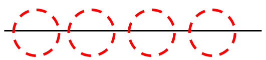
…… several others

angular momentum (spin) and parity for each configuration?

→ let us first investigate a single-j case single-j level: one level with an angular momentum j

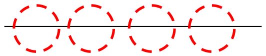
j

example: j = $p_{3/2}$



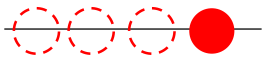
$p_{3/2}$

can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)



$p_{3/2}$ can accommodate 4 nucleons (j_z= +3/2, +1/2, -1/2, -3/2)

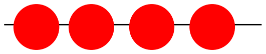
i) 1 nucleon



$p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons



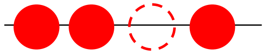
$p_{3/2}$ → $I^\pi = 0^+$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons



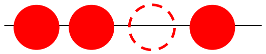
$p_{3/2}$ → $I^\pi = 3/2^-$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iii) 3 nucleons

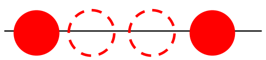
 $p_{3/2}$ → $I^{\pi} = 3/2^{-}$

$$I = j_1 + j_2 + j_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iv) 2 nucleons

 $p_{3/2}$

$$I = j_1 + j_2$$

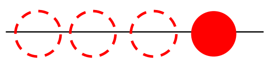
there are 4 x 3/2=6 ways to occupy this level with 2 nucleons.

→ $I^{\pi} = 0^{+}$ or 2^{+}

3/2 + 3/2 → I = 0, ~~1~~, 2, ~~3~~

anti-symmetrization

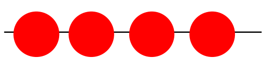
i) 1 nucleon



$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

ii) 4 nucleons

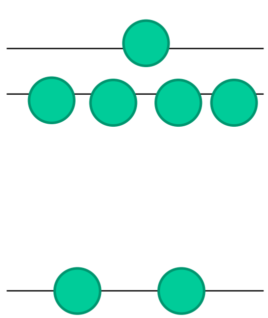


$p_{3/2}$ → $I^{\pi} = 0^{+}$

$$I = j_1 + j_2 + j_3 + j_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1



$1p_{1/2}$ → $I^{\pi} = 1/2^{-}$

$1p_{3/2}$ → $I^{\pi} = 0^{+}$

$1s_{1/2}$ → $I^{\pi} = 0^{+}$

in total,
$I^{\pi} = 1/2^{-}$

example: (main) shell model configurations for ^{11}B

cf. $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ (=$^{11}B+\Lambda$)

MeV

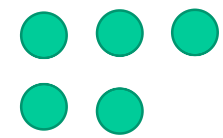
5.02	$3/2^-$
4.44	$5/2^-$
2.12	$1/2^-$
0	$3/2^-$

$^{11}_{5}B_6$

$1p_{1/2}$

$1p_{3/2}$

$1s_{1/2}$



single-j

$p_{3/2}$ → $I^\pi = 3/2^-$

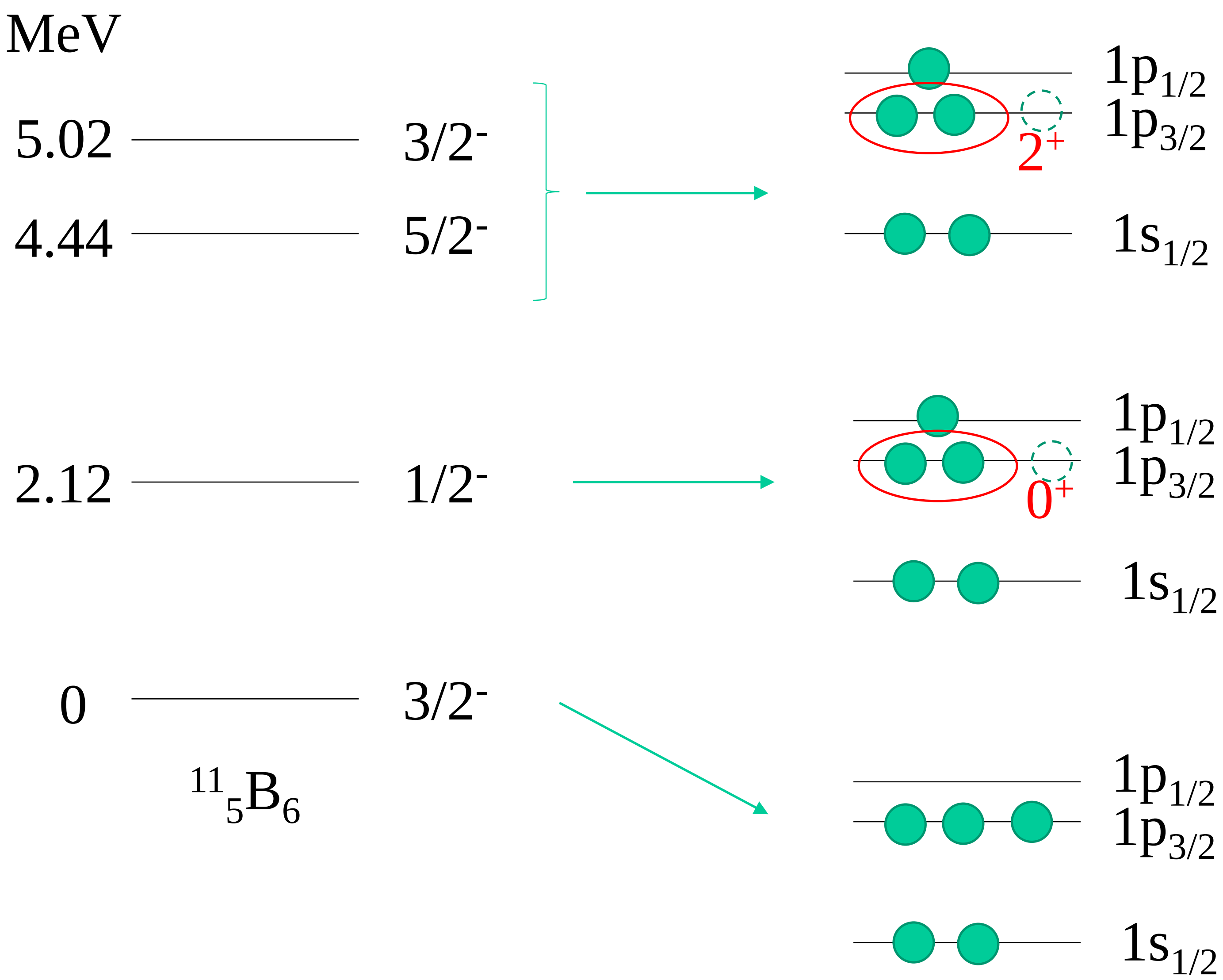
$p_{3/2}$ → $I^\pi = 0^+$ or 2^+

$p_{3/2}$ → $I^\pi = 3/2^-$

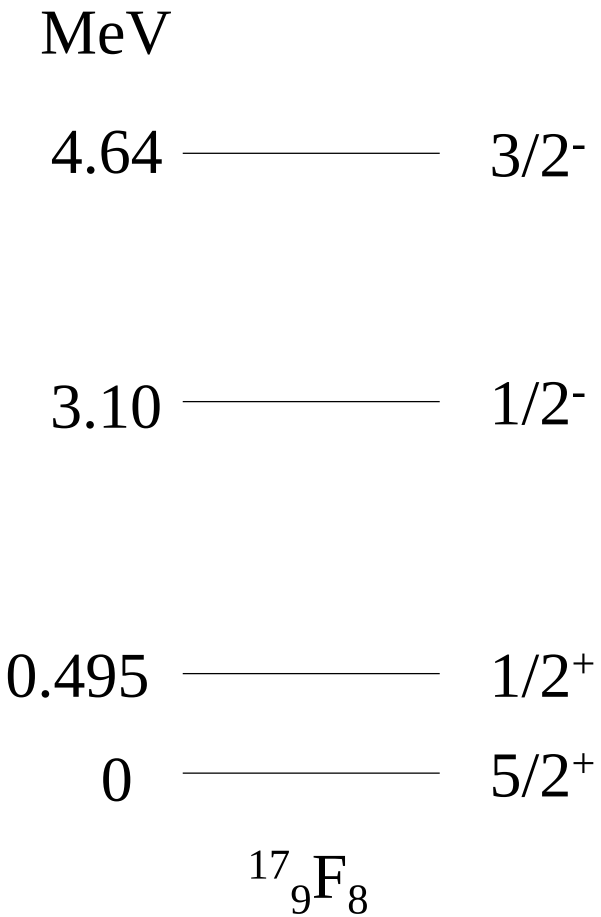
$p_{3/2}$ → $I^\pi = 0^+$

example: (main) shell model configurations for ^{11}B

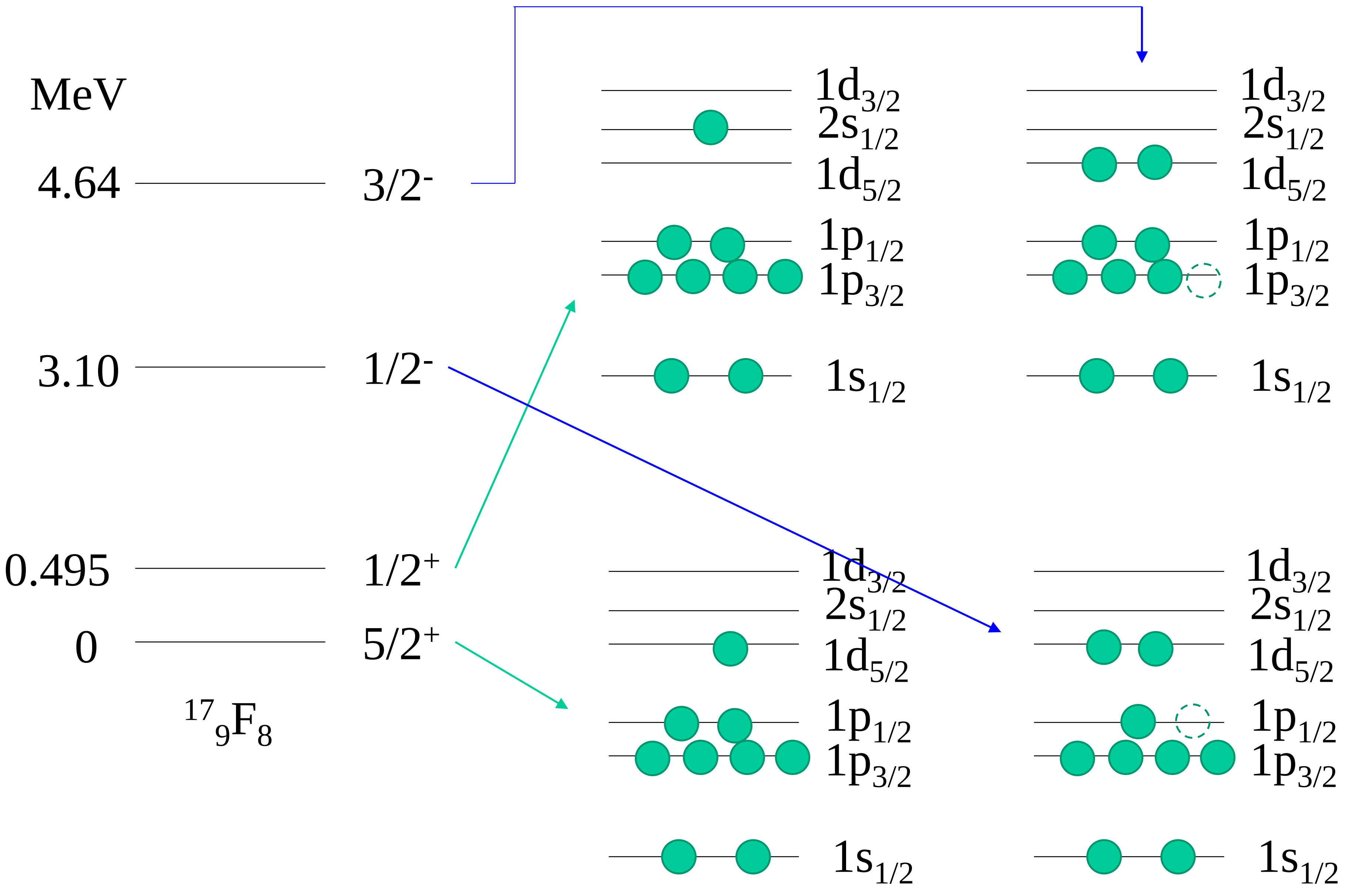
cf. $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ (=$^{11}B+\Lambda$)



another example: (main) shell model configurations for ^{17}F

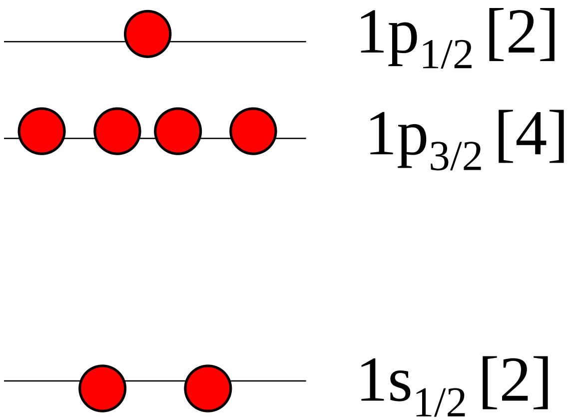


another example: (main) shell model configurations for ^{17}F



Level scheme of $^{11}_{4}Be_7$

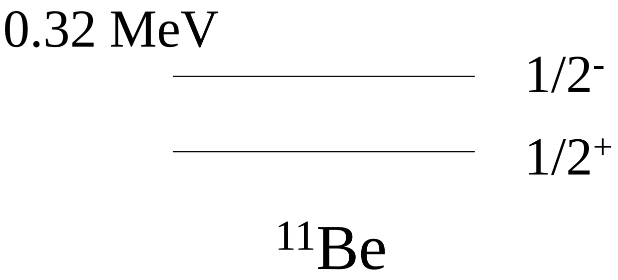
With a spherical potential :



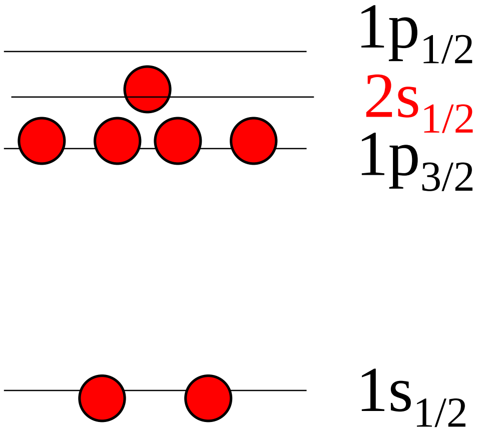
The g.s. of ^{11}Be : $I^{\pi} = 1/2^-$

very artificial

In reality…..



?



"parity inversion"

What happens if ^{11}Be is deformed?