Properties of one-neutron halo nuclei

- -bound states
- -Effects of angular momentum
- -Coulomb excitations
- -Deformation



Nuclear Chart



Nuclear Physics: developed for stable nuclei (until mid 1980's)

saturation, radii, binding energy, magic numbers and independent particle....

Nuclear Chart



Nuclear Physics: developed for stable nuclei (until mid 1980's) natural questions:

- how many neutrons can be put into a nucleus when the number of proton is fixed?
- what are the properties of nuclei far from the stability line?

Start of a research on unstable nuclei: interaction cross sections (1985)



$$\sigma_I \sim \pi [R_I(P) + R_I(T)]^2$$
$$\longrightarrow R_I(P)$$

stable nuclei



$$ho_0 \sim 0.17 \ ({\rm fm}^{-3})$$

 $R_0 \sim 1.1 \times A^{1/3} \ ({\rm fm})$ \clubsuit
 $a \sim 0.57 \ ({\rm fm})$

Start of a research on unstable nuclei: interaction cross sections (1985)



 $\longrightarrow R_{\rm I}({\rm P})$

One neutron halo nuclei

A typical example: ${}^{11}_4\text{Be}_7$



I. Tanihata et al., PRL55('85)2676; PLB206('88)592 One neutron separation energy



cf.
$$S_n = 6.81 \text{ MeV}$$

for ¹⁰Be

One neutron halo nuclei



Interpretation : a weakly bound neutron surrounding ¹⁰Be



$$\psi(r) \sim \exp(-\kappa r)$$
 $\kappa = \sqrt{2m|\epsilon|/\hbar^2}$

weakly bound system

large spatial extension of density (halo structure)

Interpretation : a weakly bound neutron surrounding ¹⁰Be

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weakly bound system

large spatial extension of density (halo structure)

Density distribution which explains the experimental reaction cross section



 ^{10}Be

n

lunar halo (a thin ring around moon)



r (fm) M. Fukuda et al., PLB268('91)339

Momentum distribution



FIG. 1. Transverse-momentum distributions of (a) ⁶He fragments from reaction ⁸He+C and (b) ⁹Li fragments from reaction ¹¹Li+C. The solid lines are fitted Gaussian distributions. The dotted line is a contribution of the wide component in the ⁹Li distribution.

T. Kobayashi et al., PRL60 ('88) 2599

Properties of single-particle motion: bound state



assume a 2body system with a core nucleus and a valence neutron



consider a spherical potential V(r) as a function of r

cf. mean-field potential:

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}, \boldsymbol{r}') \rho(\boldsymbol{r}') d\boldsymbol{r}'$$

Hamiltonian for the relative motion

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$$



For simplicity, let us ignore the spin-orbit interaction (the essence remains the same even if no spin-orbit interaction)

$$\Psi_{lm}(r) = \frac{u_l(r)}{r} Y_{lm}(\hat{r})$$

$$\int \left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - \epsilon_l \right] u_l(r) = 0$$

Boundary condition for bound states

$$u_l(r) \sim r^{l+1} \quad (r \sim 0)$$

 $\rightarrow e^{-\kappa r} \quad (r \rightarrow \infty)$

* For a more consistent treatment, a modified spherical Bessel function has to be used Angular momentum and halo phenomenon

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - \epsilon_l\right]u_l(r) = 0$$



Height of centrifugal barrier: 0 MeV (l = 0), 0.69 MeV (l = 1), 2.94 MeV (l = 2) Wave function

Change V_0 for each *l* so that $\varepsilon = -0.5$ MeV



l = 0: a long tail l = 2: localization l = 1: intermediate

root-mean-square radius

$$\sqrt{\langle r^2 \rangle} = \sqrt{\int_0^\infty dr \, r^2 u_l(r)^2}$$

7.17 fm (*l* = 0) 5.17 fm (*l* = 1) 4.15 fm (*l* = 2) Wave function

For $\varepsilon = -7$ MeV



Wave function: localized for all *l*

root-mean-square radius:

$$\sqrt{\langle r^2 \rangle} = \sqrt{\int_0^\infty dr \, r^2 u_l(r)^2}$$

3.58 fm (*l* = 0) 3.05 fm (*l* = 1) 3.14 fm (*l* = 2)

Wave functions





Other candidates for 1n halo nuclei



Coulomb breakup of ¹⁹C T. Nakamura et al., PRL83('99)1112

³¹Ne:
$$S_n = 0.29 + - 1.64 \text{ MeV}$$



Large Coulomb breakup cross sections

T. Nakamura et al., PRL103('09)262501

Coulomb breakup of 1n halo nuclei

$$^{A}Z \rightarrow ^{A}Z^{*} \rightarrow ^{A-1}Z + n$$



breakup if excited to continuum states

excitations due to the Coulomb field from the target nucleus

Electromagnetic transitions



final state: $|\psi_f\rangle|n_{k\alpha}=0
angle$

(note) time-dependent perturbation theory

$$H_{\text{int}} = \frac{1}{Am} \cdot \frac{Ze}{c} \boldsymbol{A} \cdot \boldsymbol{p}$$

$$A(r,t) = \sum_{\alpha} \int \frac{dk}{2\pi} \frac{\hbar c}{\sqrt{\hbar\omega}} \left[a_{k\alpha} \epsilon_{\alpha} e^{-i\omega t} + a_{k\alpha}^{\dagger} \epsilon_{\alpha} e^{i\omega t} \right] = A(t) \quad \text{(dipole approximation)}$$

transition probability per unit time due to: $V(\mathbf{r}, t) = F(\mathbf{r})e^{\pm i\omega t}$ (for a transition to a single state)

$$\Gamma_{i \to f} = \frac{2\pi}{\hbar} |\langle f | F | i \rangle|^2 \,\delta(e_f - e_i \pm \hbar \omega)$$
 Fermi's Golden Rule

application to the present problem:

$$\Gamma_{i \to f} = \frac{1}{2\pi\hbar} \left(\frac{Ze}{A+1}\right)^2 \left(e_f - e_i\right) \left|\langle \psi_f | z | \psi_i \rangle\right|^2 \delta(e_f - e_i - \hbar\omega)$$

(note) photo-absorption cross section if Γ is devided by the photon flux $c/(2\pi)^3$:

$$\sigma_{\gamma} = \frac{4\pi^2}{\hbar c} \left(\frac{Ze}{A+1}\right)^2 \left(e_f - e_i\right) \left|\langle \psi_f | z | \psi_i \rangle\right|^2 \delta(e_f - e_i - \hbar \omega)$$

Application to the present problem:

$$\Gamma_{i \to f} = \frac{1}{2\pi\hbar} \left(\frac{Ze}{A+1}\right)^2 \left(e_f - e_i\right) \left|\langle \psi_f | z | \psi_i \rangle\right|^2 \delta(e_f - e_i - \hbar\omega)$$

$$P_{i \to f} \sim \left| \langle \psi_f | z | \psi_i \rangle \right|^2$$





Application to the present problem:

$$\Gamma_{i \to f} = \frac{1}{2\pi\hbar} \left(\frac{Ze}{A+1} \right)^2 (e_f - e_i) \left| \langle \psi_f | z | \psi_i \rangle \right|^2 \delta(e_f - e_i - \hbar\omega)$$

$$\sum_{f} P_{i \to f} = \sum_{f} \langle \psi_i | z | \psi_f \rangle \langle \psi_f | z | \psi_i \rangle$$
$$= \langle \psi_i | z^2 | \psi_i \rangle$$

large transition probability if the spatial extention in z is large

Wigner-Eckart theorem and reduced transition probability

$$\sigma_{\gamma} = \frac{16\pi^3}{3\hbar c} \left(\frac{Ze}{A+1}\right)^2 E_{\gamma} \left| \langle \psi_f | rY_{10} | \psi_i \rangle \right|^2 \delta(e_f - e_i - E_{\gamma}) \qquad \qquad E_{\gamma} = e_f - e_i = \hbar \omega$$

$$\begin{aligned} \left| \langle \psi_{f} | rY_{10} | \psi_{i} \rangle \right|^{2} &\to \frac{1}{2l+1} \sum_{m,m'} |\langle \psi_{l'm'} | rY_{10} | \psi_{lm} \rangle|^{2} \\ &= \frac{1}{3} \cdot \frac{1}{2l+1} |\langle \psi_{l'} | | rY_{1} | | \psi_{l} \rangle|^{2} \end{aligned}$$

$$\sigma_{\gamma} = \frac{16\pi^{3}}{9\hbar c} E_{\gamma} \cdot \frac{1}{2l+1} \left| \langle \psi_{f} || e_{E1} r Y_{1} || \psi_{i} \rangle \right|^{2} \delta(e_{f} - e_{i} - E_{\gamma})$$
$$= \frac{16\pi^{3}}{9\hbar c} E_{\gamma} \cdot \frac{dB(E1)}{dE_{\gamma}}$$

Reduced transition probability

$$\frac{dB(E1)}{dE_{\gamma}} = \frac{1}{2l+1} \left| \langle \psi_f || e_{\mathsf{E}1} r Y_1 || \psi_i \rangle \right|^2 \delta(e_f - e_i - E_{\gamma})$$

E1 effective charge

$$\sigma_{\gamma} = \frac{16\pi^3}{3\hbar c} \left(\frac{Ze}{A+1}\right)^2 \left(e_f - e_i\right) \left|\langle \psi_f | rY_{10} | \psi_i \rangle\right|^2 \delta(e_f - e_i - \hbar \omega)$$

dipole operator:

$$\widehat{D}_{\mu} = e_{\mathsf{E}1} \cdot rY_{1\mu}(\theta, \phi) \qquad e_{\mathsf{E}1}$$

$$Z_1(r_1 - R) + Z_2(r_2 - R)$$

 $R - \frac{A_1r_1 + A_2r_2}{A_1r_1 + A_2r_2}$

$$\mathbf{R} = \frac{1}{A_{1} + A_{2}}$$

$$\mathbf{R} = \frac{1}{A_$$

(a general formula for 2-body)

 $=\frac{Z}{A+1}e$

Coulomb breakup cross sections

$$\sigma_{\gamma} = \frac{16\pi^{3}}{9\hbar c} E_{\gamma} \cdot \frac{dB(E1)}{dE_{\gamma}}$$

$$\frac{d\sigma_{\gamma}}{dE_{\gamma}} \sim \frac{16\pi^3}{9\hbar c} \cdot \frac{dB(E1)}{dE_{\gamma}}$$

In actual nuclear reactions, absorption of virtual photons rather than real photons

$$\frac{d\sigma}{dE_{\text{ex}}} \sim \frac{16\pi^3}{9\hbar c} \cdot N_{\text{E1}}(E_{\text{ex}}) \cdot \frac{dB(E1)}{dE_{\text{ex}}}$$

of virtual photon

See: C.A. Bertulani and P. Danielwicz, "Introduction to Nuclear Reactions" for more details



Simple estimate of E1 strength distribution (analytic model)

Transition from an l = 0 to an l = 1 states:

WF for the initial state:
$$\Psi_i(r) = \sqrt{2\kappa} \frac{e^{-\kappa r}}{r} Y_{00}(\hat{r})$$
 $\kappa = \sqrt{\frac{2\mu|E_b|}{\hbar^2}}$
WF for the final state: $\Psi_f(r) = \sqrt{\frac{2\mu k}{\pi \hbar^2}} j_1(kr) Y_{1m}(\hat{r})$ $j_1(kr)$: spherica Bessel function

$$\frac{dB(E1)}{dE} = \frac{3}{4\pi} e_{\text{E1}}^2 \left| \int_0^\infty r^2 dr \, r \cdot \frac{\sqrt{2\kappa}e^{-\kappa r}}{r} \cdot \sqrt{\frac{2\mu k}{\pi\hbar^2}} j_1(kr) \right|^2$$

The integral can be performed analytically

$$\frac{dB(E1)}{dE} = \frac{3\hbar^2}{\pi^2 \mu} e_{\mathsf{E}1}^2 \frac{\sqrt{|E_b|} E_c^{3/2}}{(|E_b| + E_c)^4}$$

Refs. (for more general l_i and l_f)

• M.A. Nagarajan, S.M. Lenzi, A. Vitturi, Eur. Phys. J. A24('05)63

 $k = \sqrt{\frac{2\mu E_c}{\hbar^2}}$

• S. Typel and G. Baur, NPA759('05)247





peak position: $E_c = \frac{3}{5} |E_b|$ $\left(E_x = E_c - E_b = \frac{8}{5} \left|E_b\right|\right)$ $\propto 1/|E_b|^2$ peak height: Total transition probability: $B(E1) = S_0 = \frac{3\hbar^2 e_{\text{E1}}^2}{16\pi^2 \mu |E_1|}$

≻ a high and sharp peak as the bound state energy, $|E_b|$, becomes small

As the bound state energy, $|E_b|$, gets small, the peak appears at a low energy

 $E_{\text{peak}} = 0.28 \text{ MeV} (E_{\text{b}} = -0.5 \text{ MeV})$

cf.
$$\frac{3}{5}|E_b| = 0.3$$
 MeV



Actual numerical calculations with

a Woods-Saxon potential

 ${}^{11}\text{Be} = {}^{10}\text{Be} + n$

transition from the $2s_{1/2}$ state (bound) to the p-wave (l = 1) state

Comparison between a weakly-bound case and a strongly-bound case





$$^{11}\text{Be} = {}^{10}\text{Be} + n \qquad 2\text{s}_{1/2} \longrightarrow \text{p state}$$

≻ a high and sharp peak as the bound state energy, $|E_{\rm b}|$, becomes small

$$S_{0} = \int_{0}^{\infty} dE_{c} \frac{dB(E1)}{dE_{c}}$$

=1.53 e²fm² (E_b = -0.5 MeV)
0.32 e²fm² (E_b = -7 MeV)

As the bound state energy, $|E_b|$, gets small, the peak appears at a low energy

$$E_{\text{peak}} = 0.28 \text{ MeV} (E_{\text{b}} = -0.5 \text{ MeV})$$

0.96 MeV ($E_{\text{b}} = -7 \text{MeV}$)

Weak $E_{\rm b}$ dependence when the transition strength is multiplied by $(E_{\rm c} - E_{\rm b})$

$$S_1 = \int_0^\infty dE_c \left(E_c - E_b \right) \frac{dB(E1)}{dE_c}$$

=2.79 $e^{2}fm^{2} MeV (E_{b} = -0.5 MeV)$ 3.18 $e^{2}fm^{2} MeV (E_{b} = -7 MeV)$ Sum Rule

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_{\mathsf{E}1}^2 \langle r^2 \rangle_i$$

Total E1 transition probability: proportional to the g.s. expectation value of r^2



Value of 7²

$$S_{0} = \int_{0}^{\infty} dE_{c} \frac{dB(E1)}{dE_{c}}$$

$$= 1.53 \ e^{2} \text{fm}^{2} \ (E_{b} = -0.5 \text{ MeV})$$

$$0.32 \ e^{2} \text{fm}^{2} \ (E_{b} = -7 \text{ MeV})$$

$$\frac{3}{4\pi} e^{2}_{\text{E1}} \langle r^{2} \rangle_{i}$$

$$= 1.62 \ e^{2} \text{fm}^{2} \ (E_{b} = -0.5 \text{ MeV})$$

$$0.41 \ e^{2} \text{fm}^{2} \ (E_{b} = -7 \text{ MeV})$$

* almost coincide with each other. Small difference due to Pauli forbidden transitons (the transition from 2s to 1p) Sum Rule

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_{\text{E1}}^2 \langle r^2 \rangle_i$$

Total E1 transition probability: proportional to the g.s. expectation value of r^2



If the initial state is l=0 or l=1, the radius increases forweakly bound



Inversely, if a large E1 prob. (or a large Coul. b.u. cross sections) are observed, this indicates l=0 or $l=1 \longrightarrow$ halo structure

Other candidates for 1n halo nuclei



Coulomb breakup of ¹⁹C T. Nakamura et al., PRL83('99)1112

³¹Ne:
$$S_n = 0.29 + - 1.64 \text{ MeV}$$



Large Coulomb breakup cross sections

T. Nakamura et al., PRL103('09)262501 Nuclear deformation



What happens if ¹¹Be is deformed?

s.p. motion in a deformed potential

halo : only for l = 0 or 1

 \Rightarrow however, a possibility is enlarged for a deformed nucleus

deformed potential $V(r,\theta) \longrightarrow$ mixture of angular momenta

e.g.,

$$|d_{5/2}\rangle \rightarrow |d_{5/2}\rangle + |s_{1/2}\rangle + |g_{7/2}\rangle + \cdots$$

 $|f_{7/2}\rangle \rightarrow |f_{7/2}\rangle + |p_{3/2}\rangle + |p_{1/2}\rangle + \cdots$

(note) $s_{1/2}: \Omega^{\pi} = 1/2^+$ only $p_{1/2}: \Omega^{\pi} = 1/2^-$ only $p_{3/2}: \Omega^{\pi} = 3/2^-$ and $1/2^-$ only $\int \cdots \quad \text{possibility of halo}$ $\longrightarrow \quad \text{only for s.p. states}$ with $\Omega^{\pi} = 1/2^+, 1/2^-, 3/2^-$ s.p. motion in a deformed potential

$$\begin{array}{rcl} |d_{5/2}\rangle & \rightarrow & |d_{5/2}\rangle + |s_{1/2}\rangle + |g_{7/2}\rangle + \cdots \\ & & \rightarrow & |s_{1/2}\rangle & (|\epsilon| \rightarrow 0) \end{array} \end{array}$$

T. Misu, W. Nazarewicz, and S. Aberg, NPA614('97)44 (deformed square well)

When weakly bound, the l=0 terms $_{0.2}^{0.5}$ becomes dominant even for a very $_{0.1}^{0.1}$ small deformation (in the zero binding limit, 100% of l=0 component)



s-wave dominance phenomenon





l = 1 component is also enhanced when weakly bound(but, always less than 100%)

a possibility of deformed halo nucleus: ³¹Ne

Nilsson model analysis [I. Hamamoto, PRC81('10)021304(R)]



³¹Ne



Large Coulomb breakup cross section

T. Nakamura et al., PRL103('09)262501



Y. Urata, K.Hagino, and H. Sagawa, PRC83('11)041303(R)

Another example: ³⁷Mg

PRL 112, 242501 (2014)

Observation of a *p*-Wave One-Neutron Halo Configuration in ³⁷Mg

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