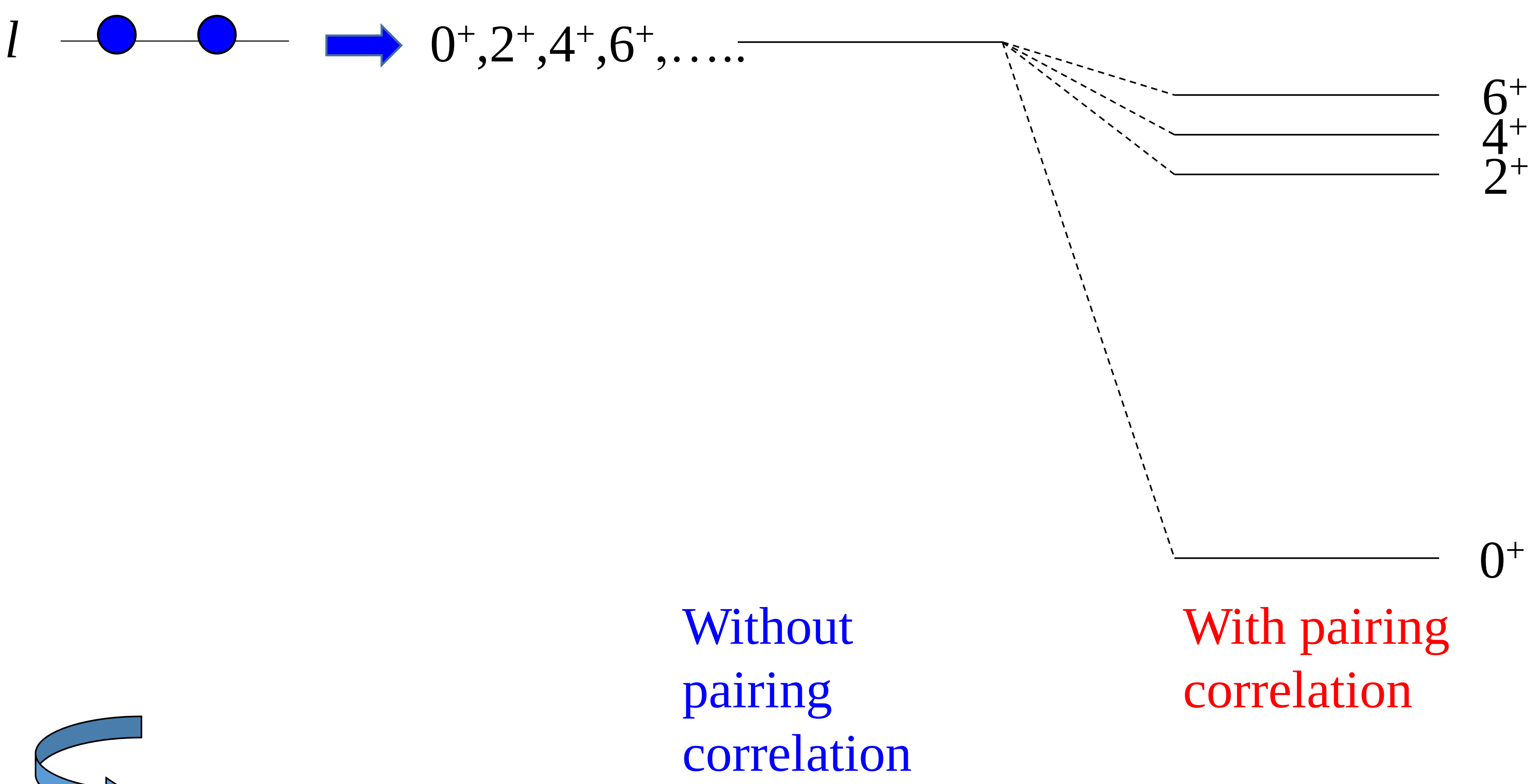
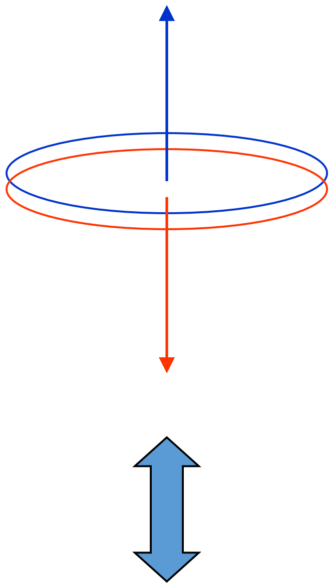
Pairing correlations

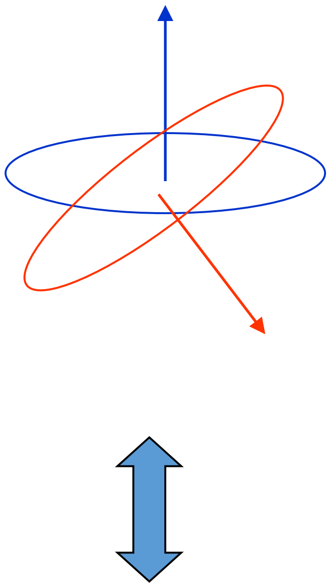


Spin and parity of the ground state of nuclei

- even-even nuclei: always 0^+ (no exception)

Simple interpretation:



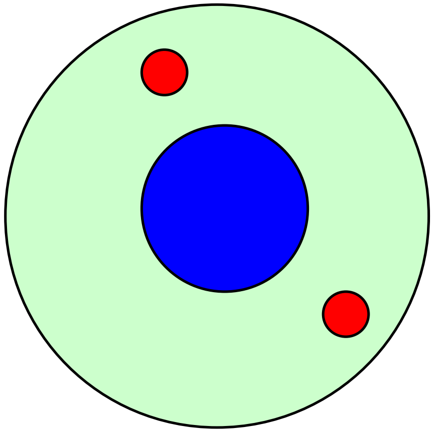


I=0 pair $\qquad$ $I \neq 0$ pair

The spatial overlap is the largest for the I=0 pair.

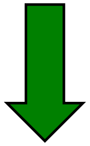
"Pairing Correlation"

Di-neutron correlation



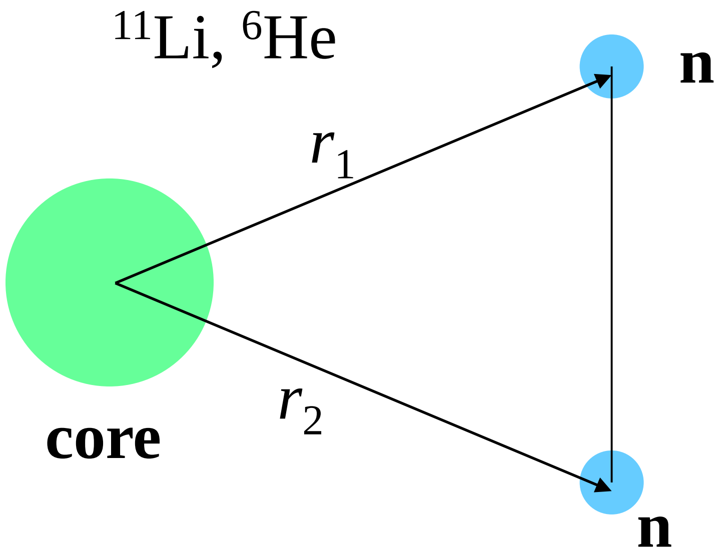
What is the spatial structure of the two-valence neutrons?

If the two neutrons moved independently, one neutron does not care where the other neutron is.



How does this change due to the pairing correlation?

Three-body model : microscopic understanding of di-neutron correlation



$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn}(\boldsymbol{r}_1, \boldsymbol{r}_2) + \frac{\boldsymbol{P}_{\rm core}^2}{2A_c m}$$

⇨ the ground state of this three-body Hamiltonian and also the density distribution

(e.g.,) expand the wf with the eigen-functions for H without V_{nn} and determine the expansion coefficients

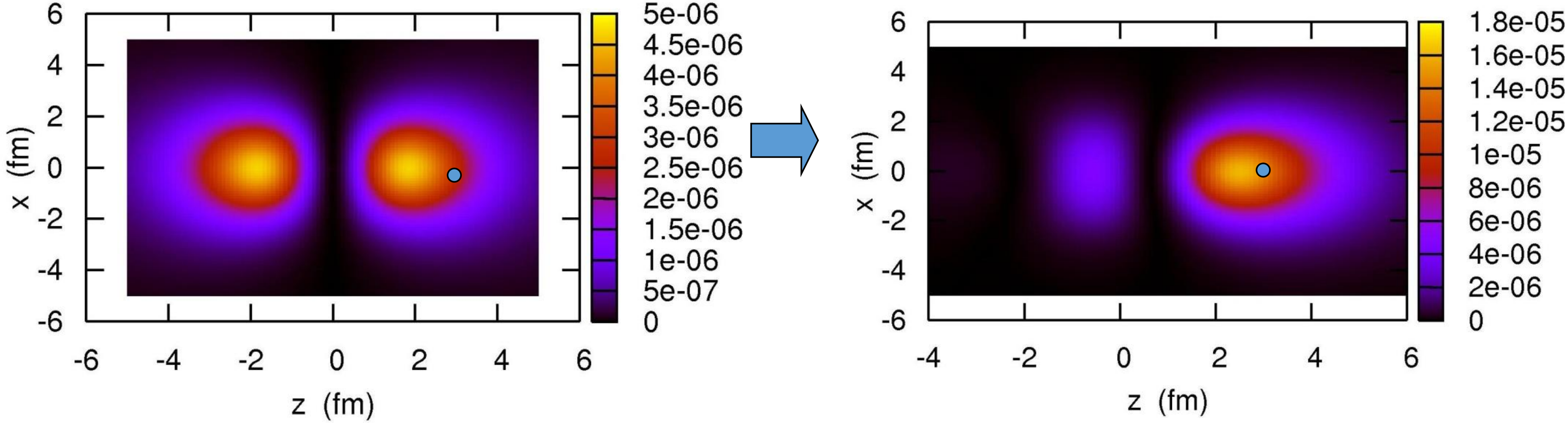
$$\boxed{\Psi_{gs}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \mathcal{A} \sum_{nn'lj} \alpha_{nn'lj} \Psi^{(2)}_{nn'lj}(\boldsymbol{r}_1, \boldsymbol{r}_2)}$$

$$\Psi^{(2)}_{nn'lj}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_m \langle jmj-m|00\rangle \psi_{nljm}(\boldsymbol{r}_1)\psi_{n'lj-m}(\boldsymbol{r}_2)$$

Comparison between with and without paring correlations

^{11}Li a distribution of one of the neutrons when the other neutron is at (z_1, x_1)=(3.4 fm, 0)

Without pairing $[1p_{1/2}]^2$ With pairing



- When no pairing, symmetric between z and $-z$.
  The distribution does not change whereever the 2nd neutron is.
- When with pairing, the nearside density is enhanced.
  The distribution changes when the 2nd neutron moves.

What is Di-neutron correlation?

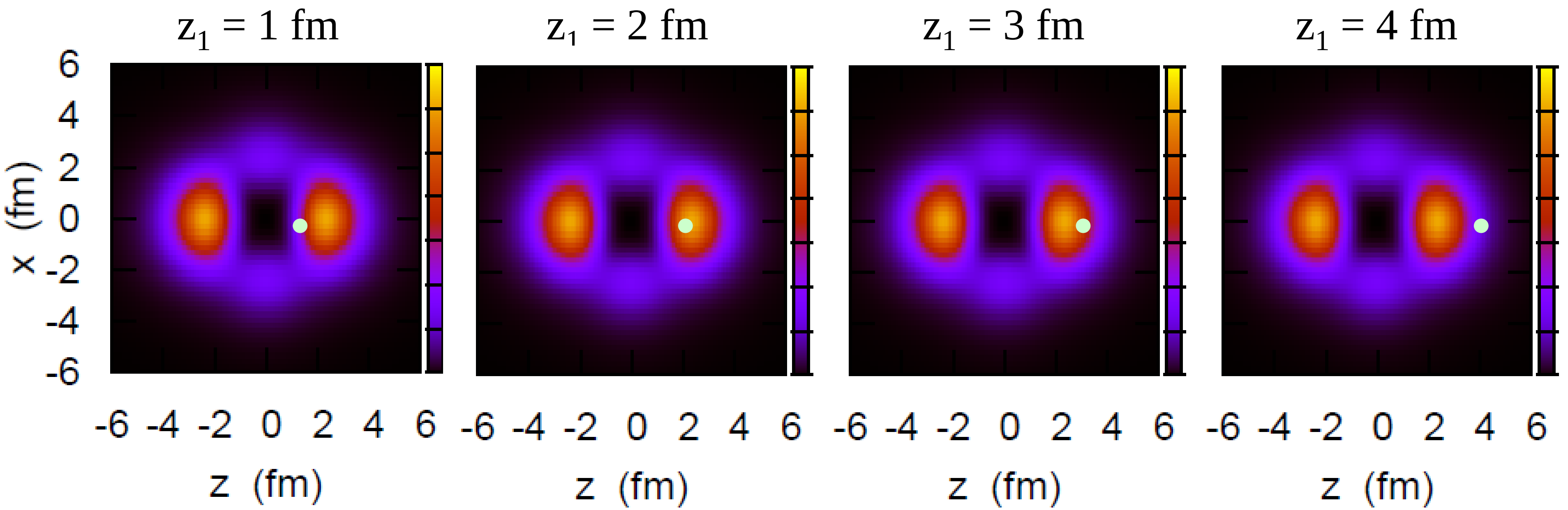
Correlation : $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: $^{18}O = {}^{16}O + n + n$

cf. $^{16}O + n$: 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

i) Without nn interaction: $|nn\rangle = |(1d_{5/2})^2\rangle$

Distribution of the 2nd neutron when the 1st neutron is at z_1 :



✓Two neutrons move independently

✓No influence of the 2nd neutron from the 1st neutron

$$\langle AB \rangle = \langle A \rangle \langle B \rangle$$

What is Di-neutron correlation?

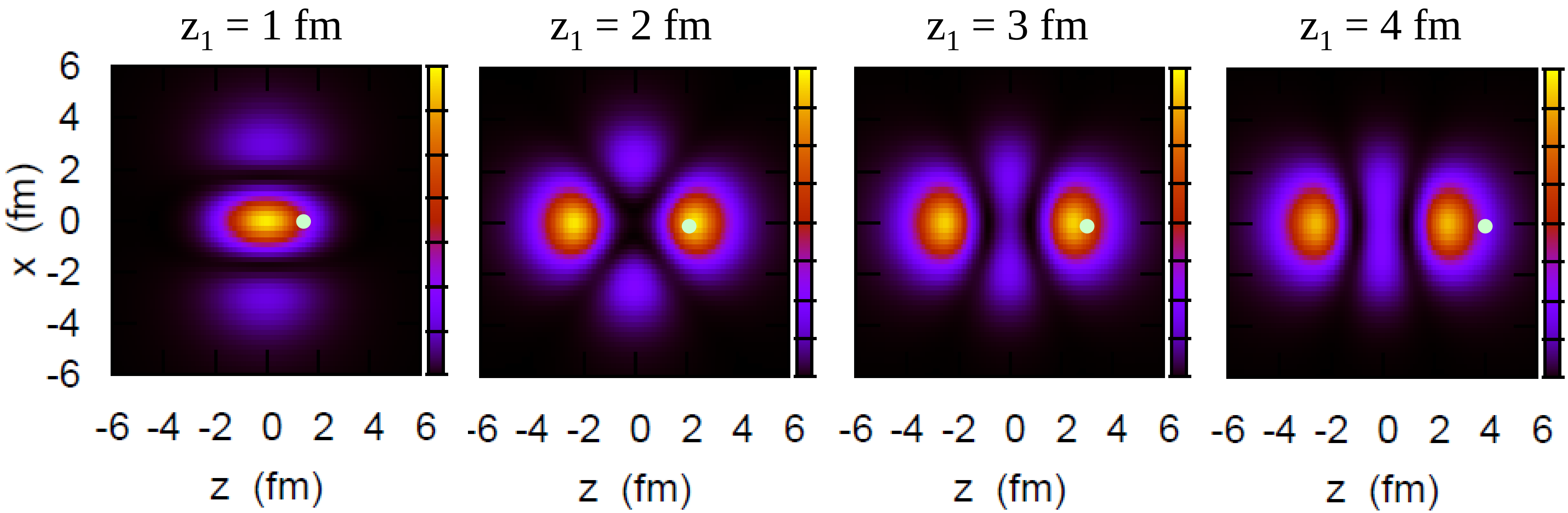
Correlation : $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: $^{18}O = {}^{16}O + n + n$

cf. $^{16}O + n$: 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

ii) nn interaction: works only on the positive parity (bound) states

$$|nn\rangle = \alpha|(1d_{5/2})^2\rangle + \beta|(2s_{1/2})^2\rangle + \gamma|(1d_{3/2})^2\rangle$$



✓distribution changes according to the 1st neutron (nn correlation)

✓but, the distribution of the 2nd neutron has peaks both at z_1 and $-z_1$

⟶ this is NOT called the di-neutron correlation

What is Di-neutron correlation?

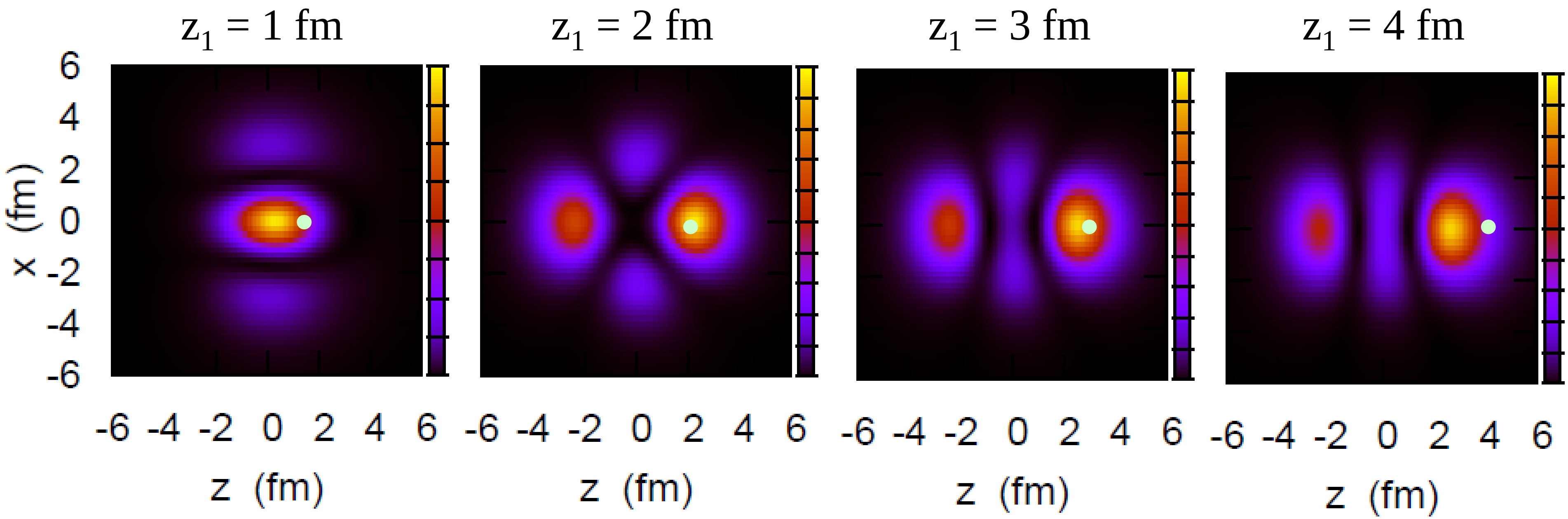
Correlation: $\langle AB\rangle \neq \langle A\rangle\langle B\rangle$

Example: ^{18}O = ^{16}O + n + n

cf. ^{16}O + n : 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

iii) nn interaction: works also on the continuum states

$$|nn\rangle = \sum_{n,n',j,l} C_{nn'jl}|(nn'jl)^2\rangle$$

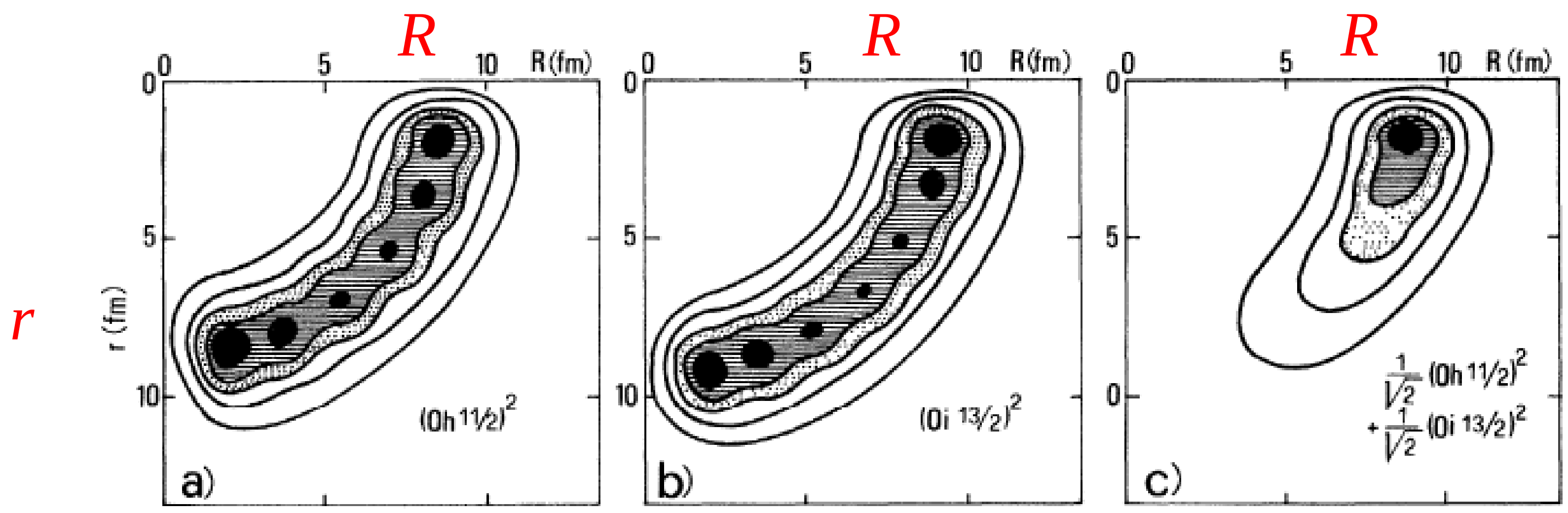


✓spatial correlation: the density of the 2nd neutron localized close to the 1st neutron (dineutron correlation)

✓parity mixing: essential role

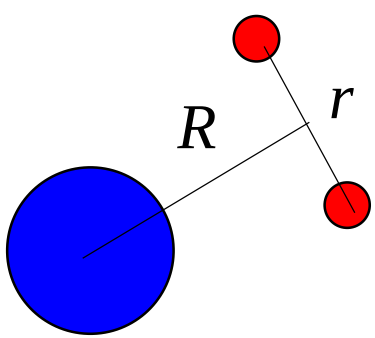
cf. F. Catara et al., PRC29('84)1091

dineutron correlation: caused by the admixture of different parity states



$(0h_{11/2})^2$ $(0i_{13/2})^2$ $(0h_{11/2})^2 + (0i_{13/2})^2$

F. Catara, A. Insolia, E. Maglione,
and A. Vitturi, PRC29(‘84)1091



interference of even and odd partial waves

$$\rho_2(x_1,x_2) = |\Psi_{ee}(x_1,x_2)|^2 + |\Psi_{oo}(x_1,x_2)|^2 + \boxed{2\Psi_{ee}(x_1,x_2)\Psi_{oo}(x_1,x_2)}$$

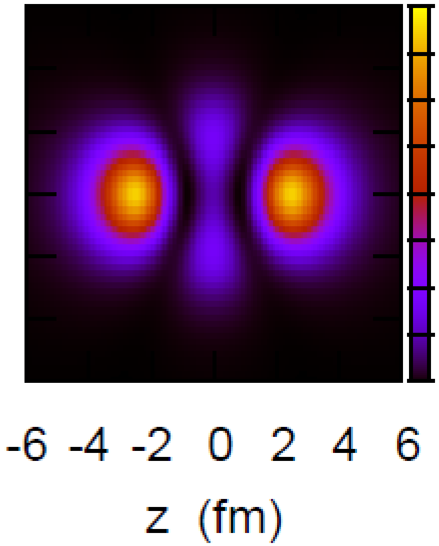
spatial localization of two neutrons

(dineutron correlation)

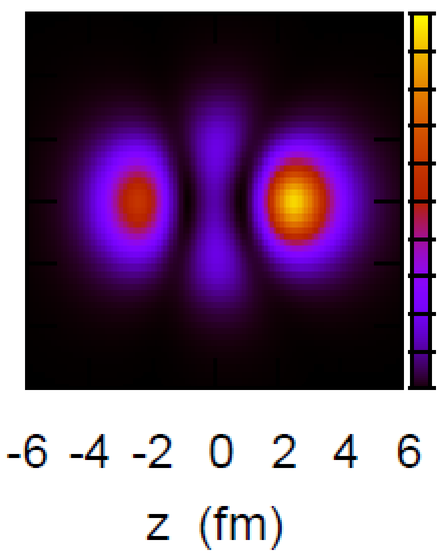
cf. Migdal, Soviet J. of Nucl. Phys. 16 ('73) 238
Bertsch, Broglia, Riedel, NPA91('67)123

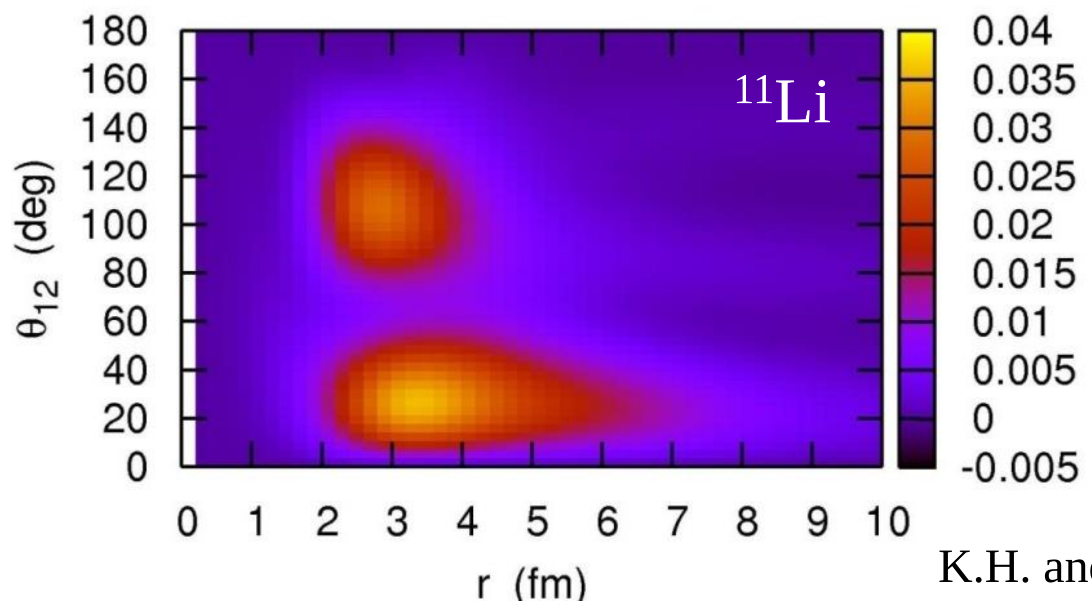
weakly bound systems

→easy to mix different parity states due to the continuum couplings
+ enhancement of pairing on the surface



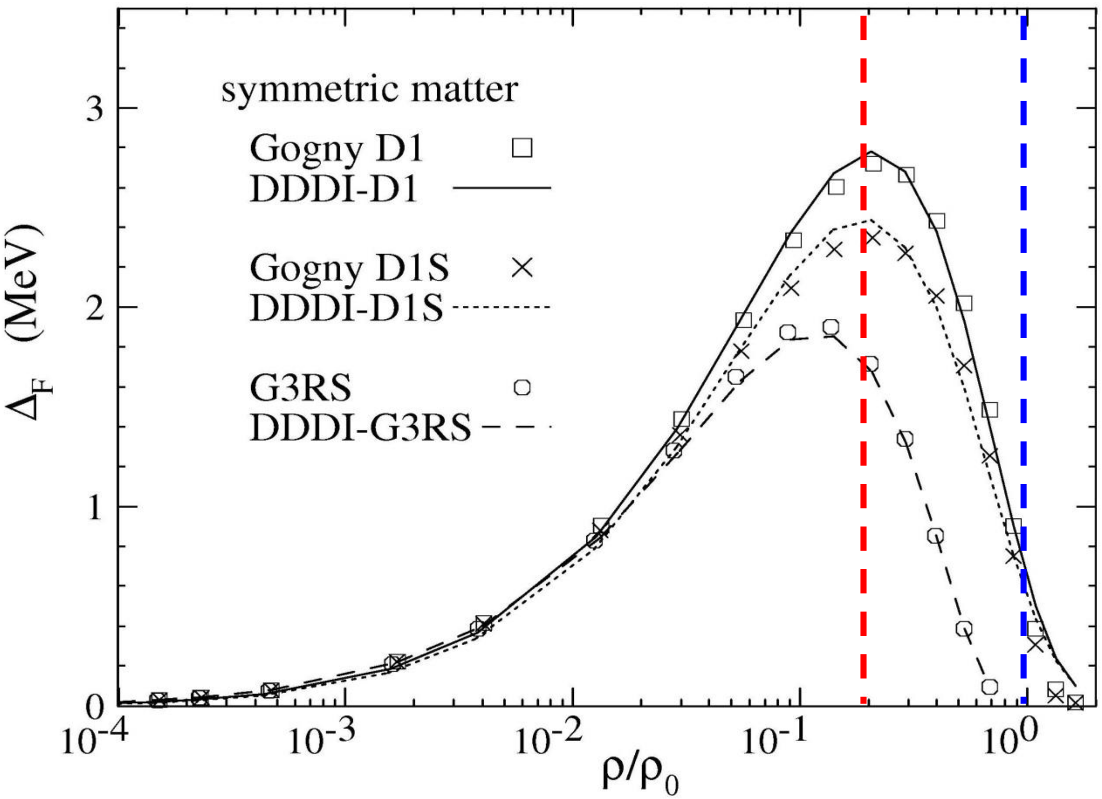
parity mixing





K.H. and H. Sagawa, PRC72('05)044321

pairing gap in infinite nuclear matter



M. Matsuo, PRC73('06)044309

spatial localization of two neutrons

(dineutron correlation)

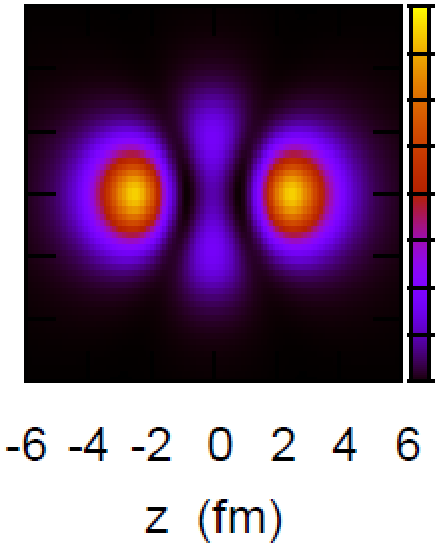
cf. Migdal, Soviet J. of Nucl. Phys. 16 (‘73) 238
Bertsch, Broglia, Riedel, NPA91(‘67)123

weakly bound systems

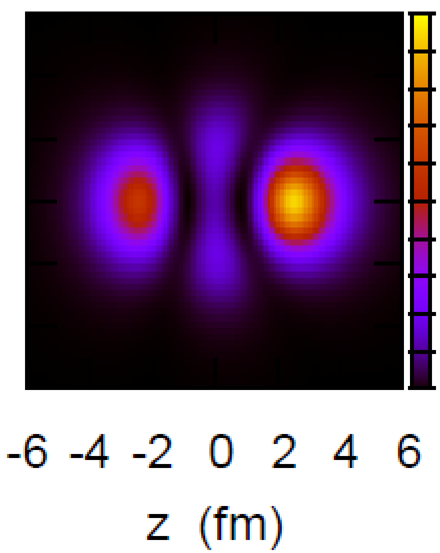
→easy to mix different parity states due to the continuum couplings
+ enhancement of pairing on the surface

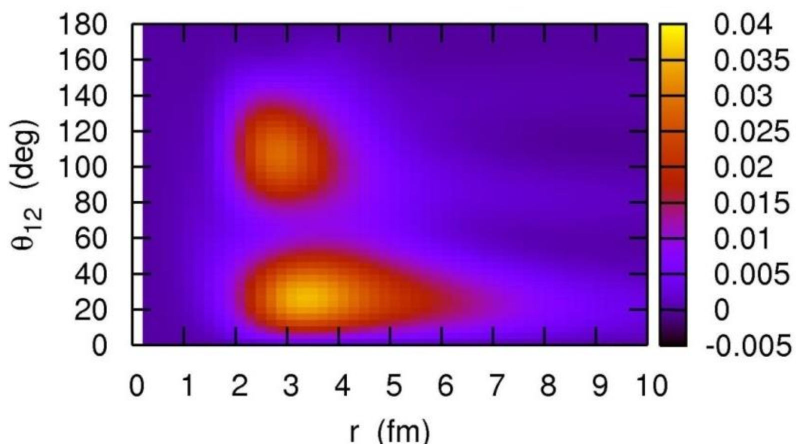
→ dineutron correlation: enhanced

cf. - Bertsch, Esbensen, Ann. of Phys. 209(’91)327
- M. Matsuo, K. Mizuyama, Y. Serizawa, PRC71(‘05)064326



parity mixing





K.H. and H. Sagawa, PRC72(’05)044321

The BCS theory

Many-particles in non-degenerate levels
~ mean-field approx. for the pairing channel ~

Simplified pairing interaction

$$V = -G\, P^\dagger P; \qquad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

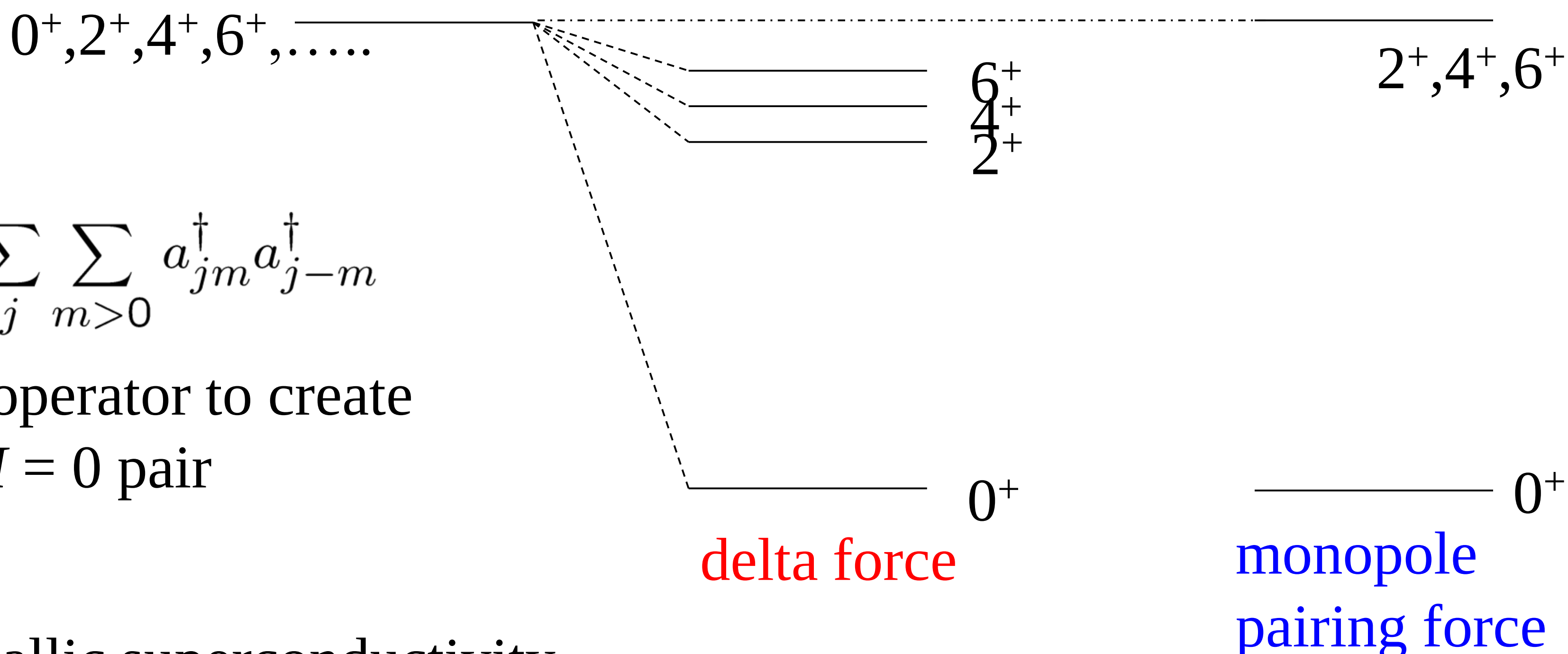
$\bar{\nu}$: the time reversed state of ν

e.g.,

$$|\nu\rangle = |njlm\rangle, \quad |\bar{\nu}\rangle = |njl-m\rangle$$

$$P^\dagger = \sum_j \sum_{m>0} a^\dagger_{jm} a^\dagger_{j-m}$$

: an operator to create an $I = 0$ pair



Cf. Metallic superconductivity

Solve the pairing Hamiltonian

$$\boxed{H=\sum_\nu \epsilon_\nu(a_\nu^\dagger a_\nu + a_{\bar{\nu}}^\dagger a_{\bar{\nu}}) - G\left(\sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger\right)\left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu\right)}$$

in the mean-field approximation

- Mean-field approximation:

$$V=-G\,P^\dagger P \to -G\left(\langle P^\dagger\rangle P + P^\dagger\langle P\rangle\right) = -\Delta(P^\dagger + P)$$

$$\Delta \equiv G\langle P^\dagger\rangle = G\langle P\rangle$$

$\Longleftrightarrow$ particle number violation

we consider $H' = H - \lambda\hat{N}$ instead of H :

$$H' \to \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta\sum_{k>0}(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

$$H' \to \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

Bogoliubov transformation

$$\boxed{\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu}$$

(Quasi-particle operator)

● Transform *H'* in a form of

$$H' = const. + \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

→ g.s.: $\alpha_k |BCS\rangle = 0$

1st excited state: $|1_k\rangle = \alpha_k^\dagger |BCS\rangle$ at E_k

…. and so on.

Ground state wave function: $\alpha_k|BCS\rangle = 0$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|\,0\Big\rangle$$

(note) $\left\langle BCS|a^\dagger_\nu a_\nu|BCS\right\rangle = |v_\nu|^2$: occupation probability

(note) $E'_{\rm BCS} = \langle BCS|H'|BCS\rangle \sim 2\sum_{\nu>0}(\epsilon_\nu-\lambda)v_\nu^2 - \frac{\Delta^2}{G}$

$$H' = const. + \sum_{k>0} E_k(\alpha^\dagger_k\alpha_k + \alpha^\dagger_{\bar{k}}\alpha_{\bar{k}})$$

$$u_\nu^2 = \frac{1}{2}\left(1+\frac{\epsilon_\nu-\lambda}{\sqrt{(\epsilon_\nu-\lambda)^2+\Delta^2}}\right)$$

$$v_\nu^2 = \frac{1}{2}\left(1-\frac{\epsilon_\nu-\lambda}{\sqrt{(\epsilon_\nu-\lambda)^2+\Delta^2}}\right)$$

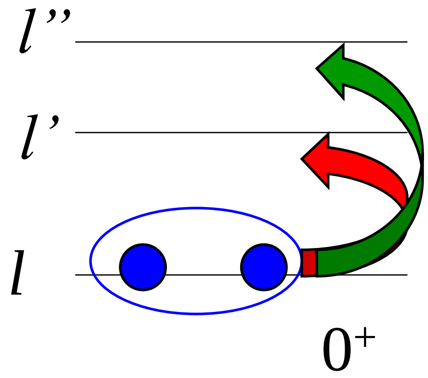
$$E_k = \sqrt{(\epsilon_k-\lambda)^2+\Delta^2}$$

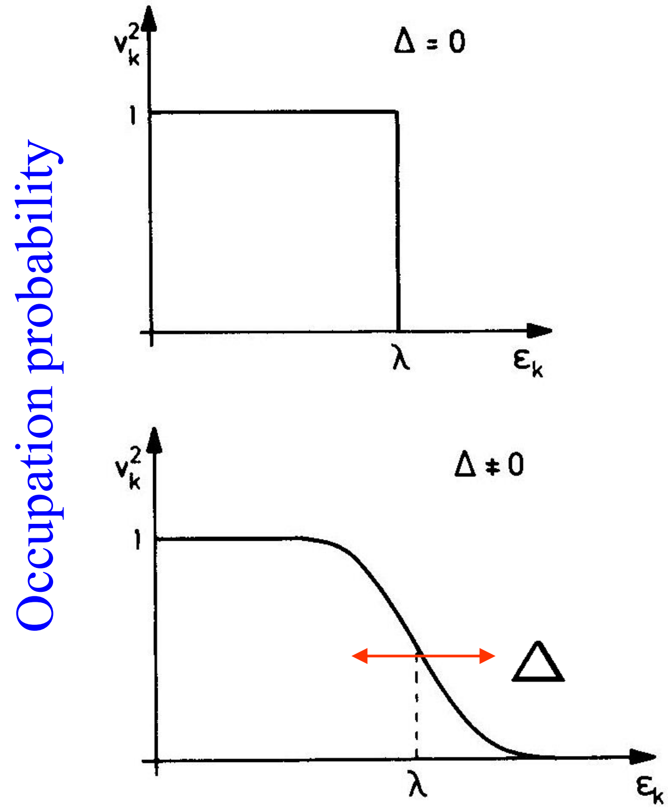
Self-consistency condition:

$$\Delta = G\langle BCS|\hat{P}|BCS\rangle = G\sum_{\nu>0}u_\nu v_\nu = \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{E_\nu}$$

Gap equation

Wave function:





$$|\Psi_{0+}\rangle = |(ll)L=0\rangle + \sum_{l'} \frac{\langle (l'l')L=0|v_{\rm res}|(ll)L=0\rangle}{2\epsilon_l - 2\epsilon_{l'}} |(l'l')L=0\rangle + \cdots$$

Each orbit is occupied only partially.

cf. BCS theory

i) Trivial solution: always exists

$$\Delta = 0$$

$$\begin{aligned} v_\nu^2 &= 1 \quad (\epsilon_\nu \leq \lambda) \\ &= 0 \quad (\epsilon_\nu > \lambda) \end{aligned}$$

$$|\Psi\rangle = \prod_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger |0\rangle$$

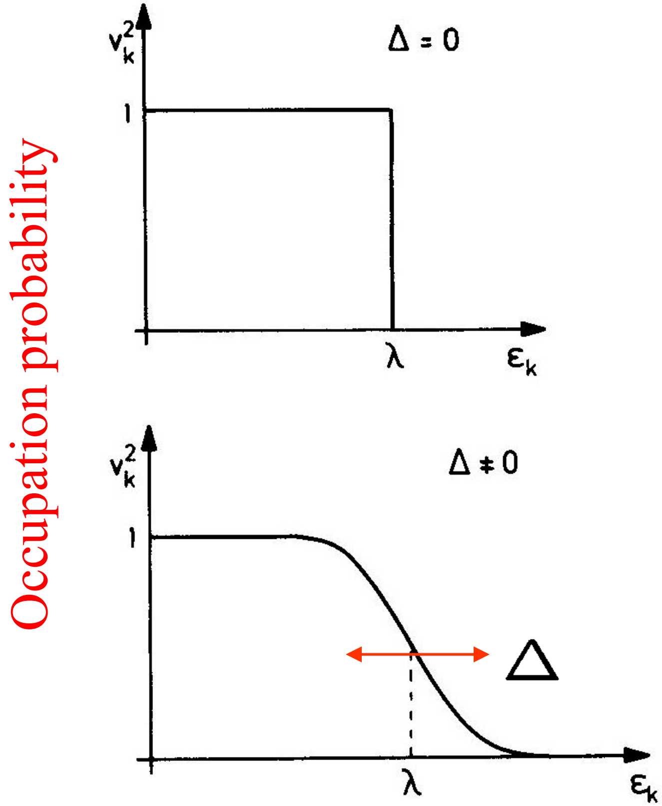
G a/o $N \longrightarrow$ large

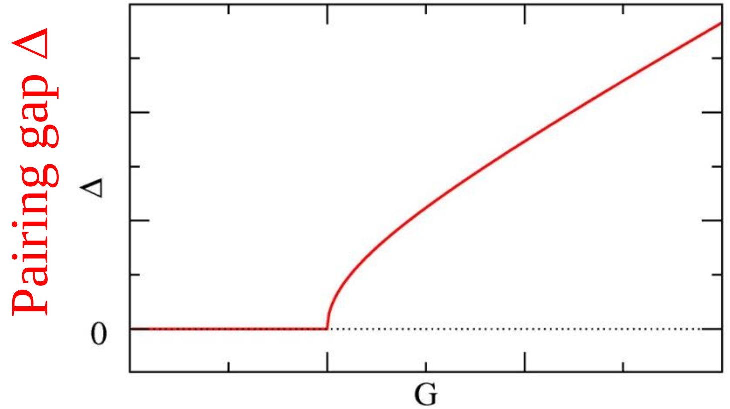
ii) Superfluid solution

$$\Delta \neq 0$$

$$v_\nu^2 < 1$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right) \Big| 0 \Big\rangle$$





Number fluctuation

Normal-Superfulid phase transition

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_\nu E_\nu\, \alpha^\dagger_\nu \alpha_\nu$$

- g.s. of even-even nuclei: $|BCS\rangle$
- One quasi-particle states:

$$|\nu_1\rangle = \alpha^\dagger_{\nu_1}|BCS\rangle = a^\dagger_{\nu_1} \prod_{\nu\neq\nu_1} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar\nu}\right)\Big|0\Big\rangle$$

Wave function for odd-mass nuclei

$$\langle\nu_1|H|\nu_1\rangle = \langle H\rangle + E_{\nu_1}$$

- Two quasi-particle states:

$$|\nu_1\nu_2\rangle = \alpha^\dagger_{\nu_1}\alpha^\dagger_{\nu_2}|BCS\rangle$$

Excited state of the even-even nuclei

$$\begin{aligned}\langle\nu_1\nu_2|H|\nu_1\nu_2\rangle - \langle H\rangle &= E_{\nu_1} + E_{\nu_2} \\ &\geq 2\Delta \longleftarrow \text{Energy gap}\end{aligned}$$

(note) no pairing limit:

$$\alpha^\dagger_p\alpha^\dagger_h \to a^\dagger_p a_h, \qquad E_p + E_h \to (\epsilon_p - \lambda) + (\lambda - \epsilon_h)$$

(particle-hole excitation)

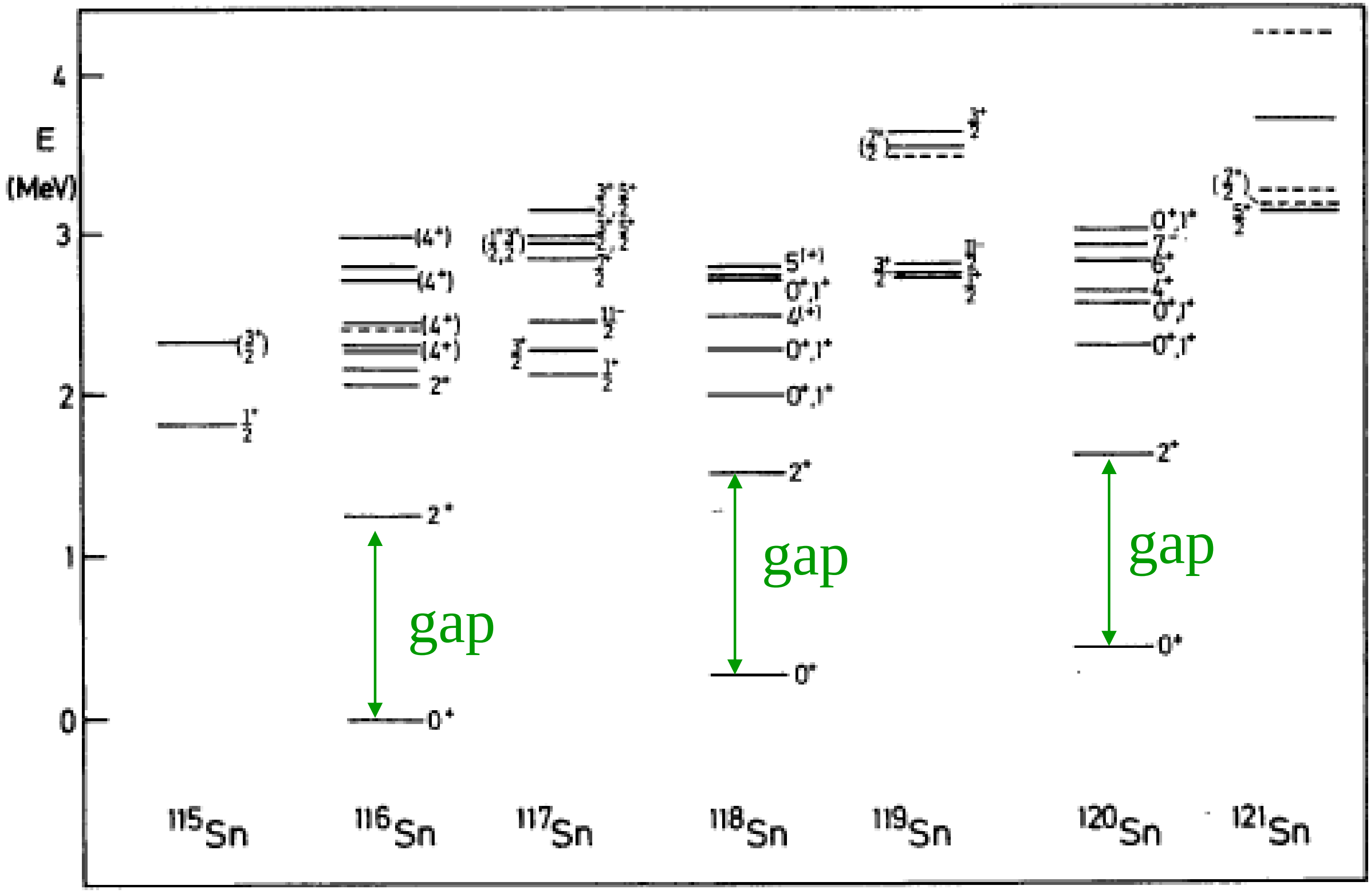


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Ring-Schuck

Effects of pairing on moment of inertia

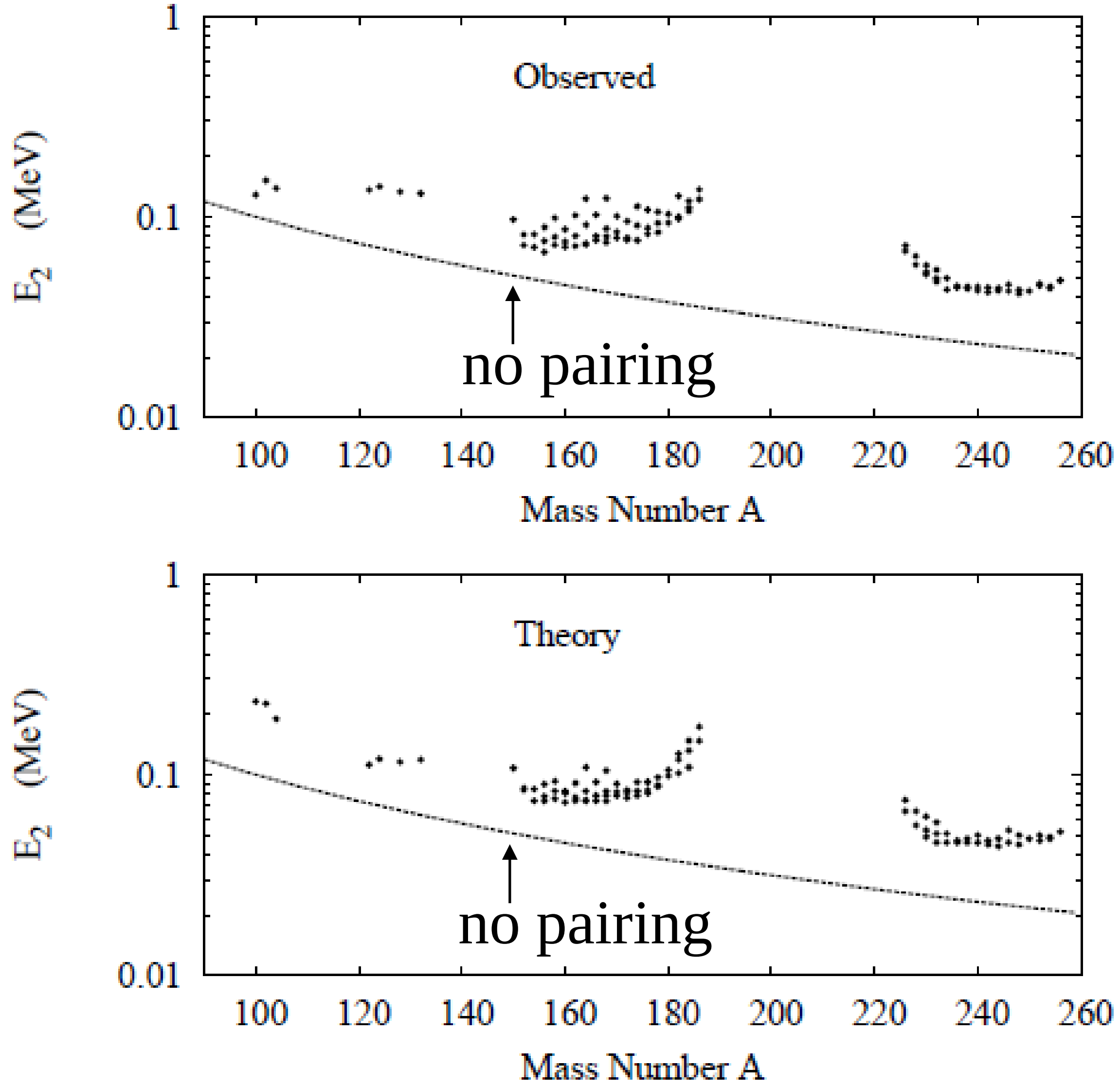


Fig. 9. Excitation energy of the first 2^+ state in deformed nuclei. The line shows the prediction assuming a rigid rotor.

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

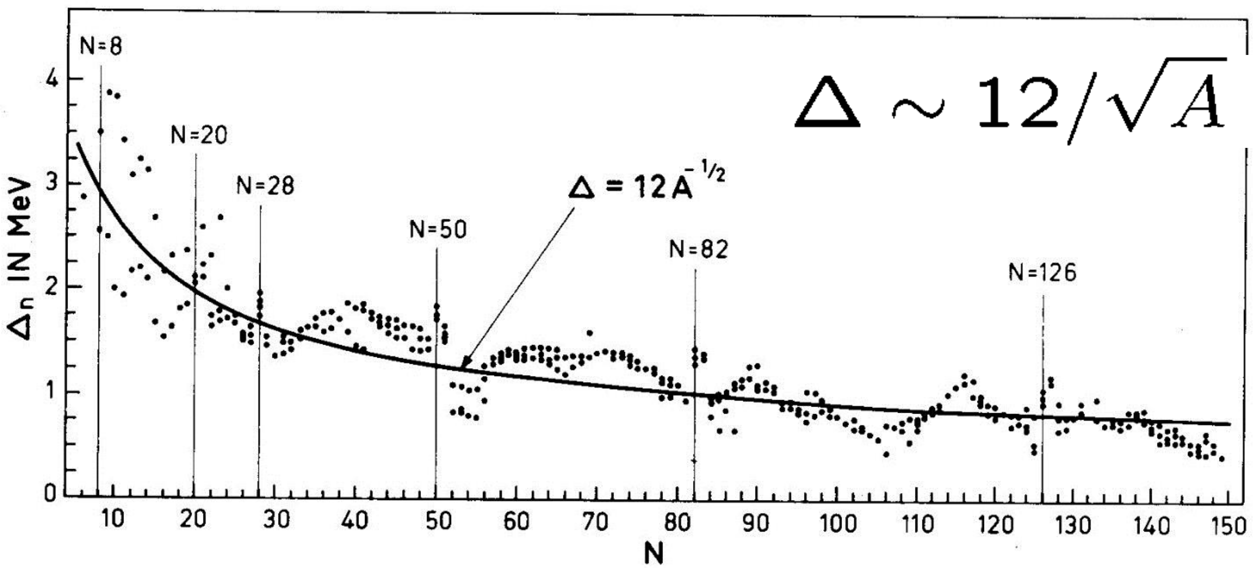
G.F. Bertsch,
in "Fifty years of
nuclear BCS"

Even-odd mass difference and pairing gap

$$
\begin{aligned}
B_{\text{pair}} &= \Delta \qquad (\text{for even} - \text{even}) \\
&= 0 \qquad (\text{for even} - \text{odd}) \\
&= -\Delta \qquad (\text{for odd} - \text{odd})
\end{aligned}
$$

$$
\begin{aligned}
E(N+2,Z) &= E(N,Z) + 2\lambda \\
E(N+1,Z) &= E(N,Z) + \lambda + \Delta
\end{aligned}
$$

$$\boxed{-\Delta_n \sim [E(N+2,Z) - 2E(N+1,Z) + E(N,Z)]/2}$$



$$\Delta \sim 12/\sqrt{A} \quad (\text{MeV})$$

Bohr-Mottelson ('69)

Hartree-Fock-Bogoliubov (HFB) Theory

HF+BCS method: first solve HF, and then solve the gap equation

s.p. wave functions, occupation probabilities,
chemical potential, pairing gaps

$$\psi_k(\boldsymbol{r}), u_k, v_k$$

→ Hartree-Fock-Bogoliubov (HFB) theory:

both wave functions and occupation probabilities
at the same time

$$U_k(\boldsymbol{r}), V_k(\boldsymbol{r})$$

cf. weakly bound systems

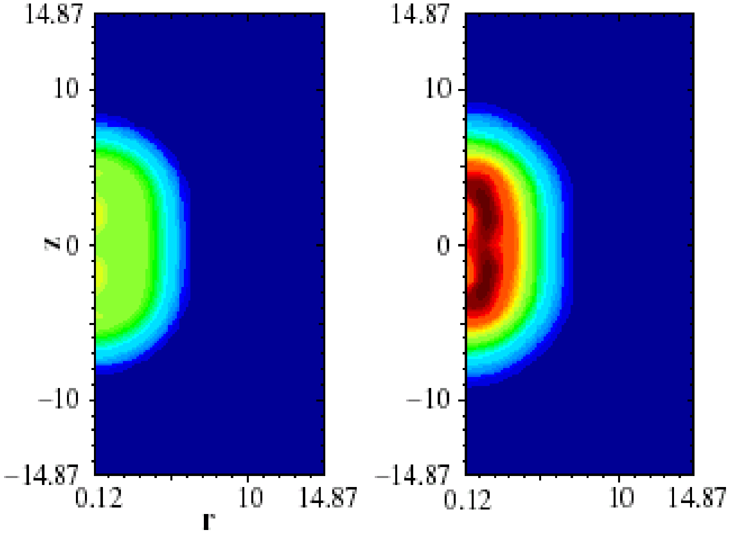
$$\boxed{\begin{pmatrix} \widehat{h}(\boldsymbol{r})-\lambda & \tilde{\Delta}(\boldsymbol{r}) \\ \tilde{\Delta}(\boldsymbol{r})^{*} & -\widehat{h}(\boldsymbol{r})+\lambda \end{pmatrix} \begin{pmatrix} U_k(\boldsymbol{r}) \\ V_k(\boldsymbol{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\boldsymbol{r}) \\ V_k(\boldsymbol{r}) \end{pmatrix}}$$

$$\begin{aligned} \widehat{h}(\boldsymbol{r}) &= -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathsf{HF}}(\boldsymbol{r}) \\ \rho(\boldsymbol{r}) &= \sum_k |V_k(\boldsymbol{r})|^2 \end{aligned}$$

u,*v* factors → *u*, *v* functions

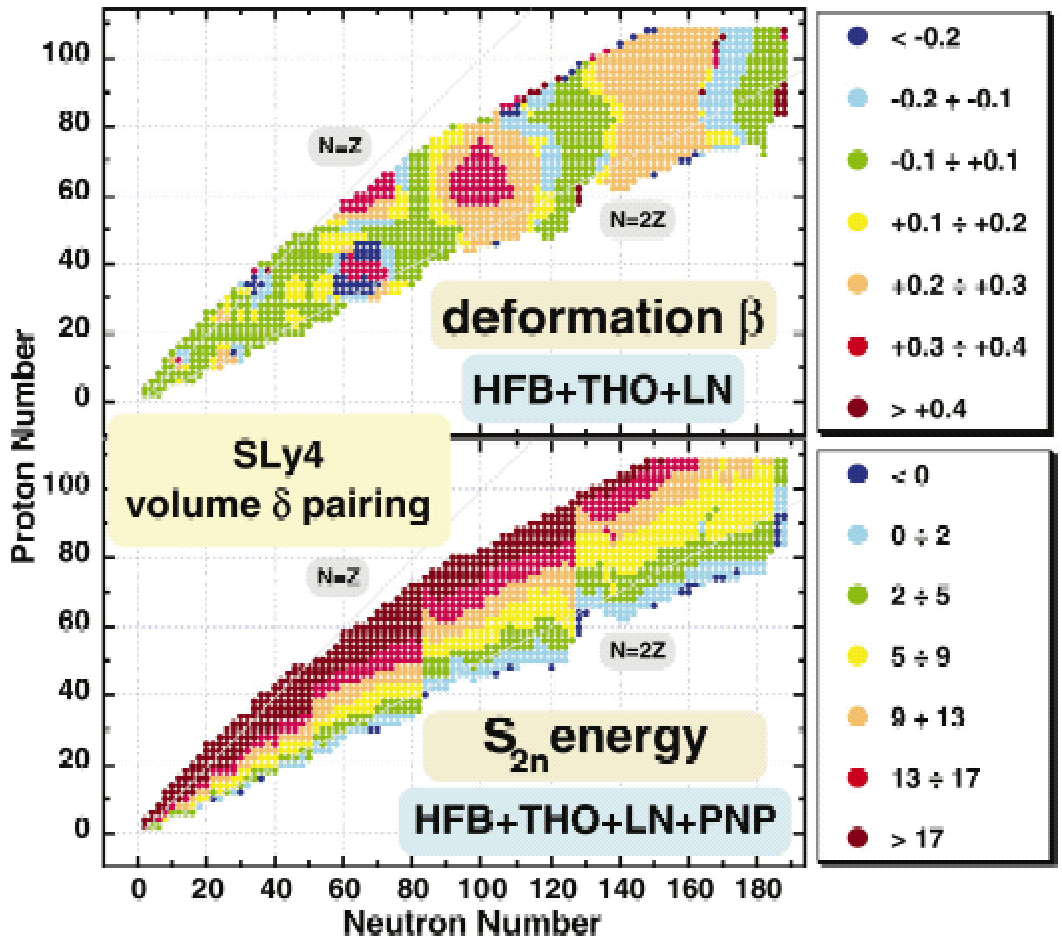
Application of the HFB method

Density of ^{110}Zr (SHFB-SLy4)

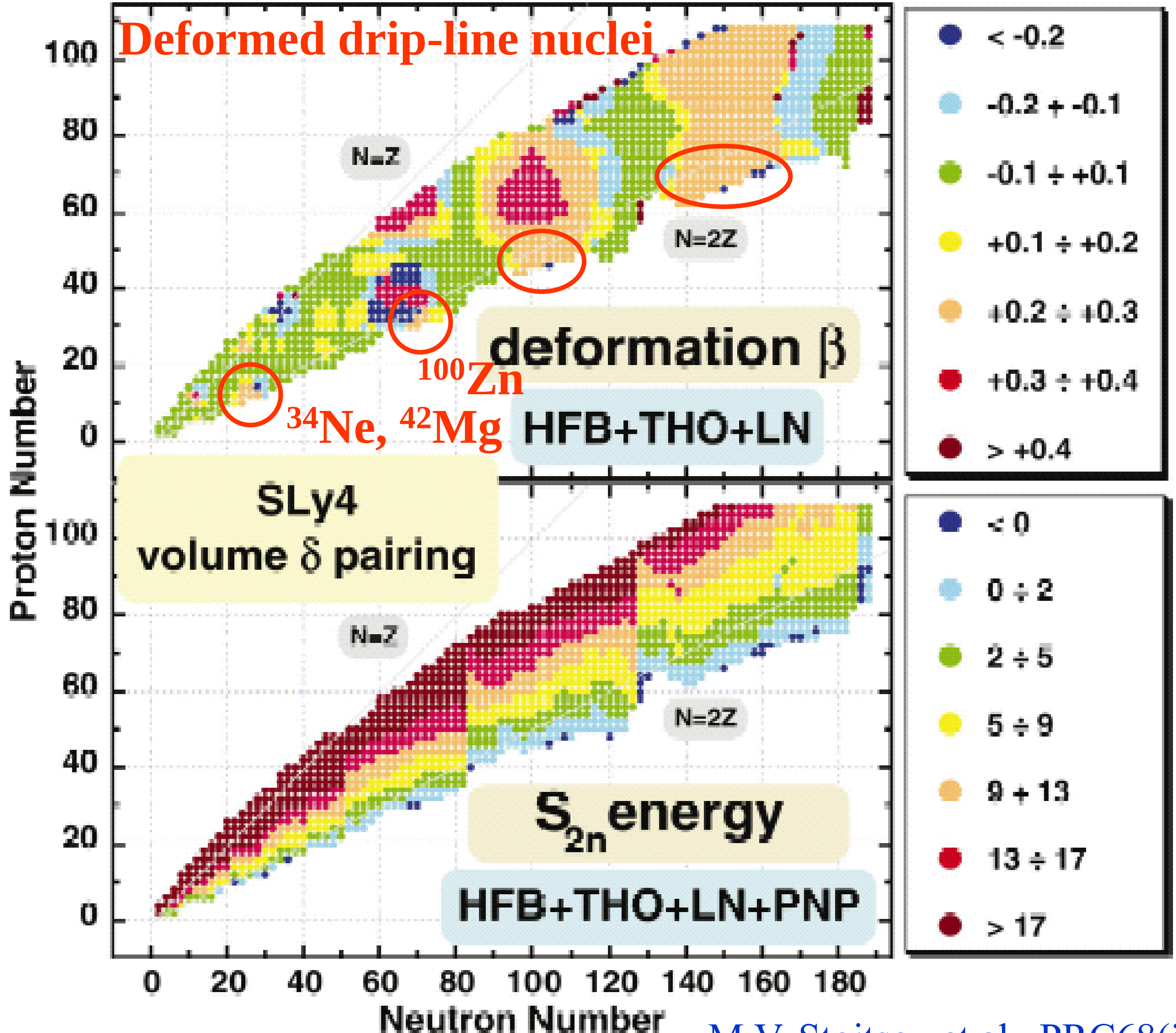


A. Blazkiewicz et al., PRC71('05)054231

Systematics of β_2 and S_{2n}

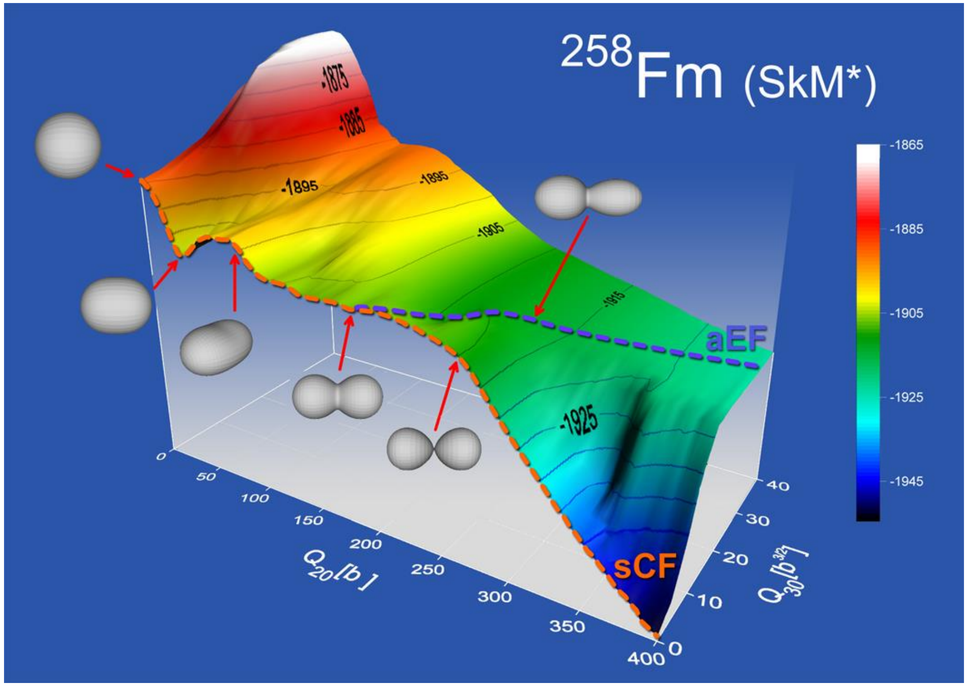


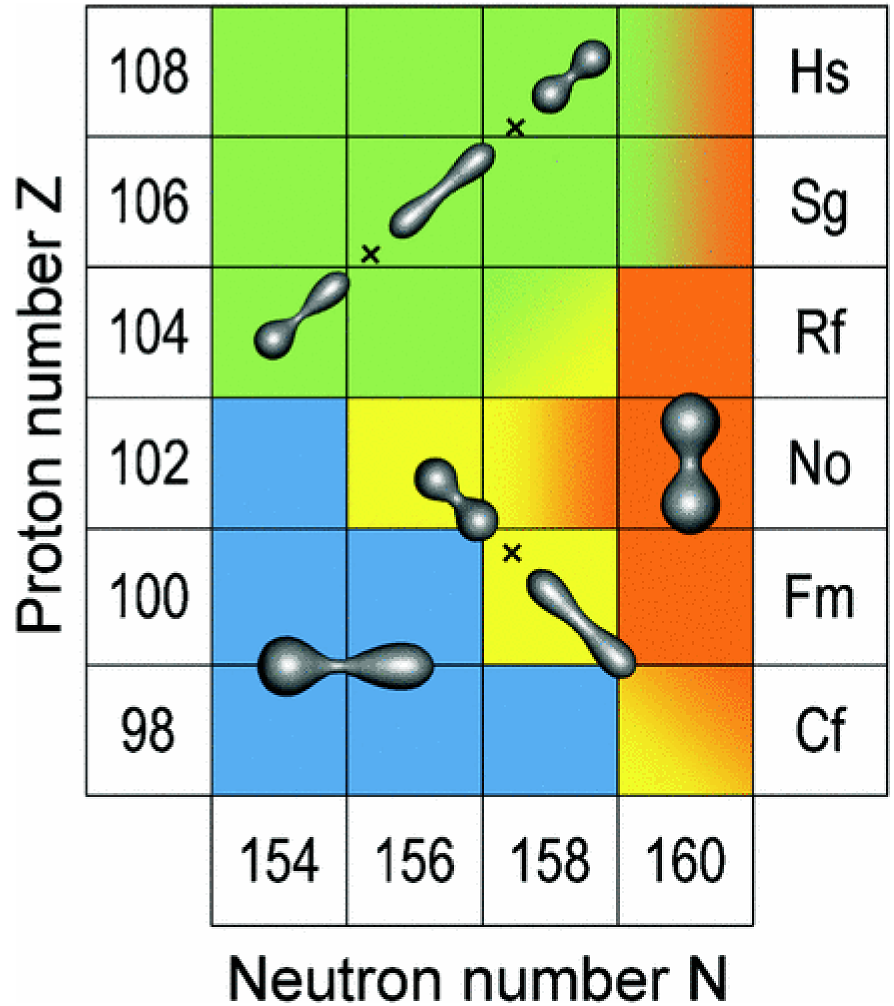
M.V. Stoitsov et al., PRC68('03)054312



M.V. Stoitsov et al., PRC68('03)054312

potential energy surface for fission process





A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, PRC80 (‘09) 014309