

Advanced Nuclear Physics

Nuclear Theory Group,
Tohoku University
Kouichi Hagino

原子核理論特論

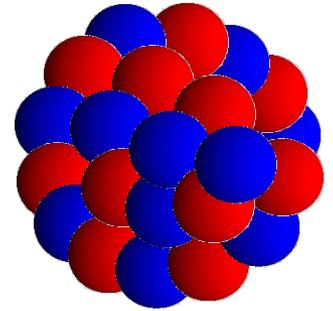
東北大学
原子核理論研究室
萩野浩一

Contents

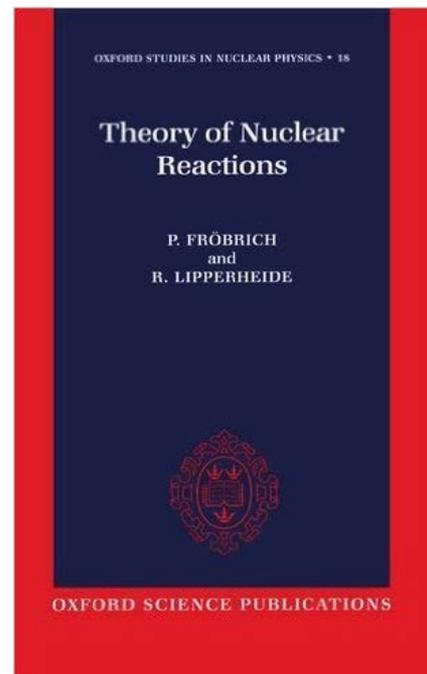
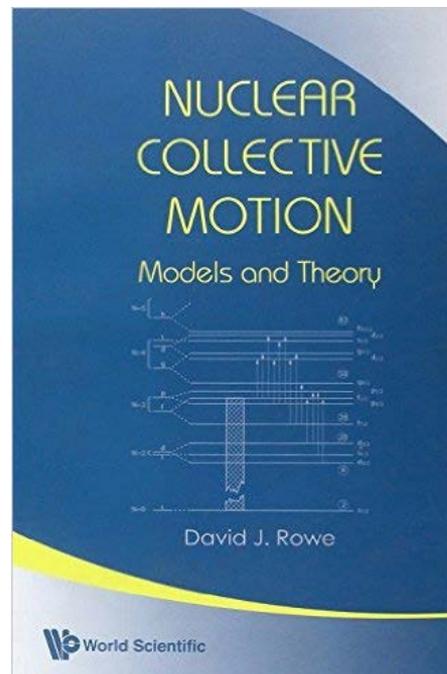
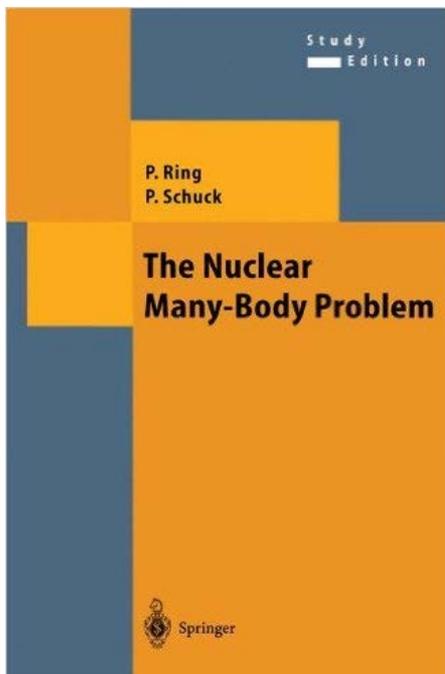
Nuclei: aggregate of nucleons (protons and neutrons)

→ *Nuclear Many-Body Problems*

- Liquid drop model
- Single-particle motion and Shell structure
- **Hartree-Fock approximation**
- Bruckner Theory
- Pairing correlations and Superfluid Nuclei
- Angular momentum and number projections
- **1n and 2n halo nuclei**
- **Random Phase Approximation**
- **Nuclear Reactions**

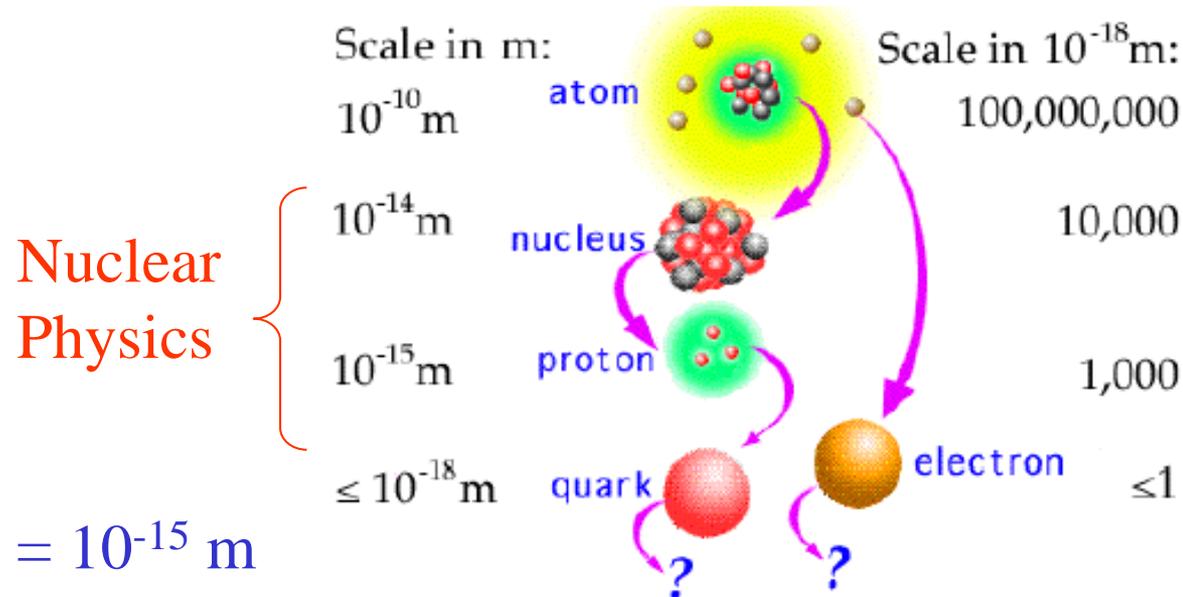


References



Basic Properties of Nuclei

すべてのものは原子からできている

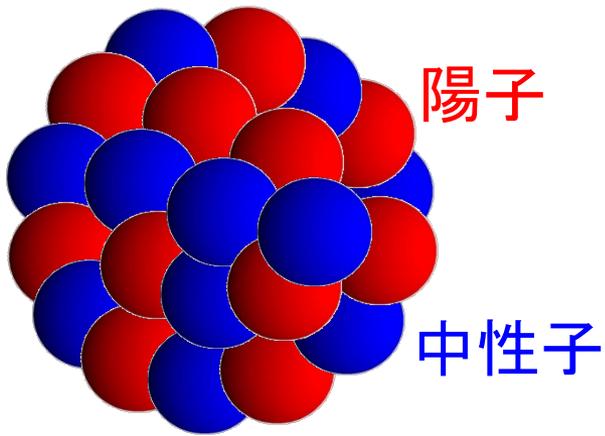


Nucleus as a *quantum many body system*

Basic ingredients:

	charge	mass (MeV)	spin
Proton	+e	938.256	$1/2^+$
Neutron	0	939.550	$1/2^+$

(note) $n \rightarrow p + e^- + \bar{\nu}$ (10.4 min)

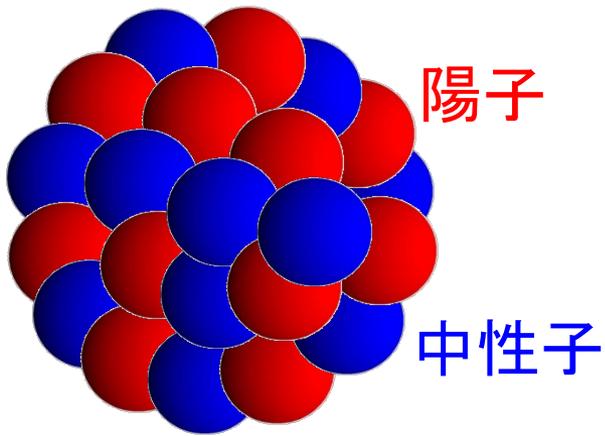


陽子

中性子

- 核子はじっとしているわけではない
(比較的自由に動きまわっている)
- ただし、完全に自由というわけではない
お互いに飛び出すことのないよう
に引っ張りあいながら一定の形
を保っている

自己束縛系



陽子

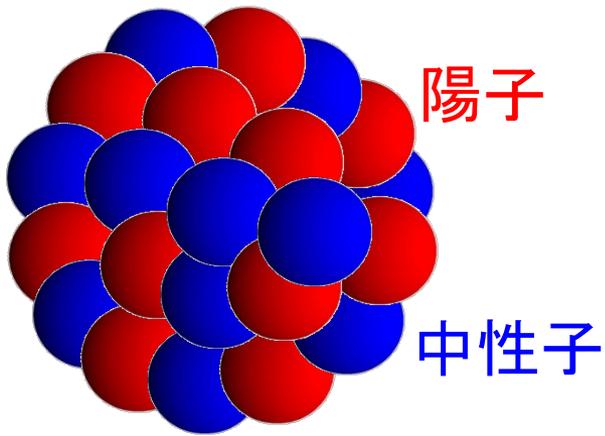
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自己束縛系

ここに外から光を当てて熱くしてやったら核子は
どんな振る舞いをする?

核子の動きが激しくなるだけ?

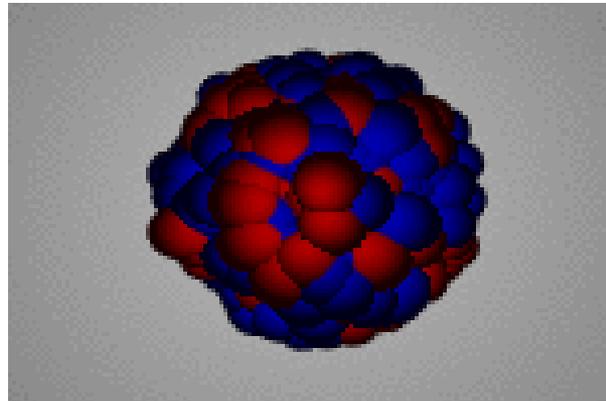


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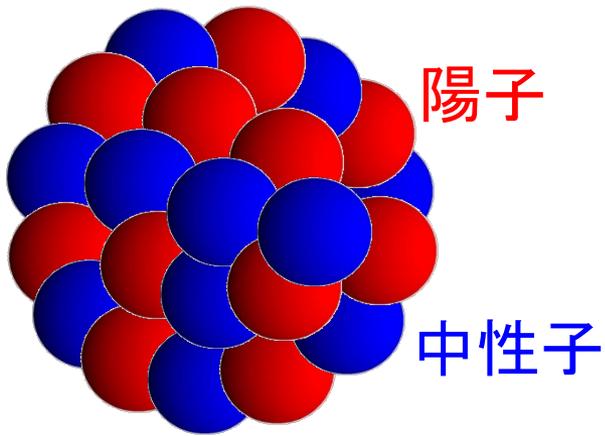
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規則正しい運動
をすることがある

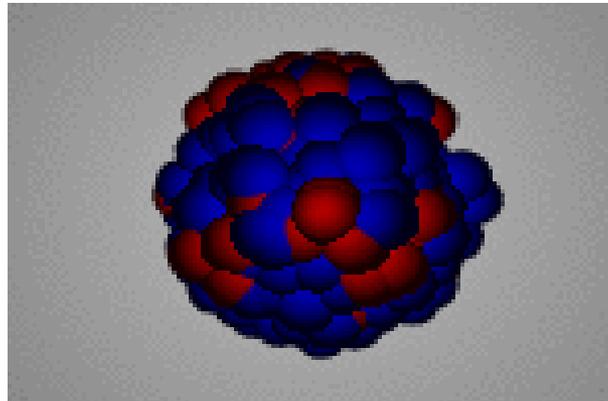
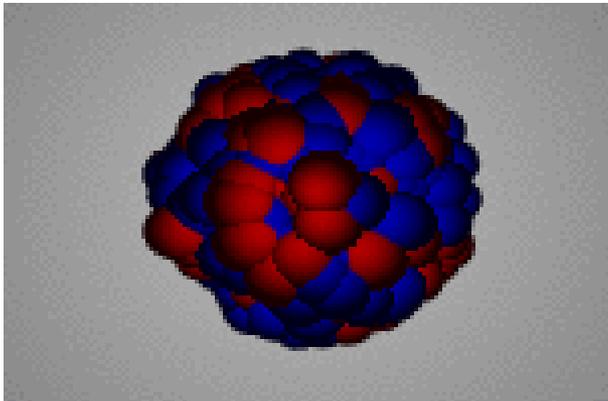
集団運動



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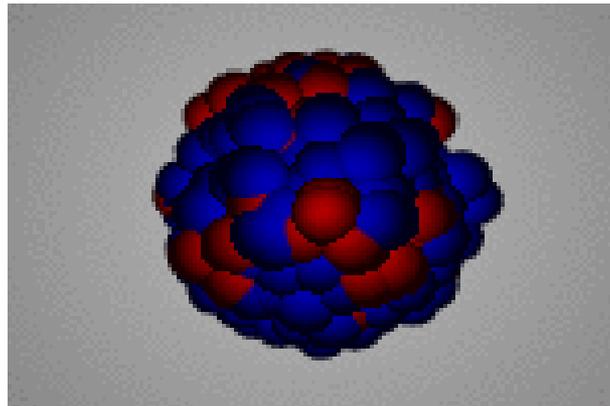
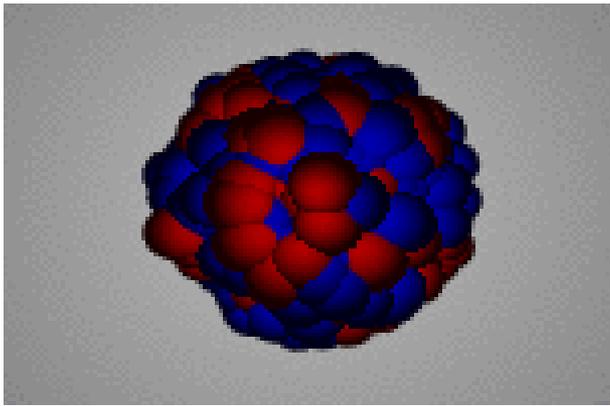
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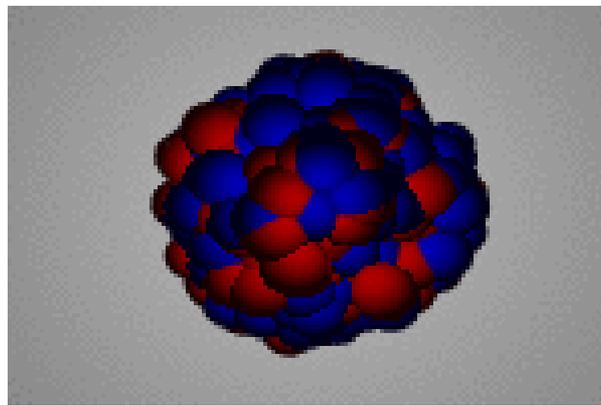
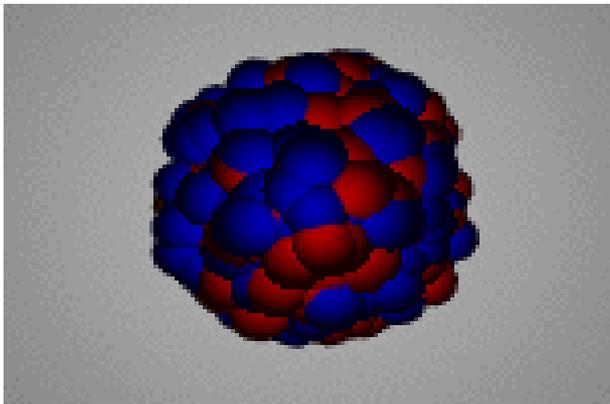
規則正しい運動
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集団運動

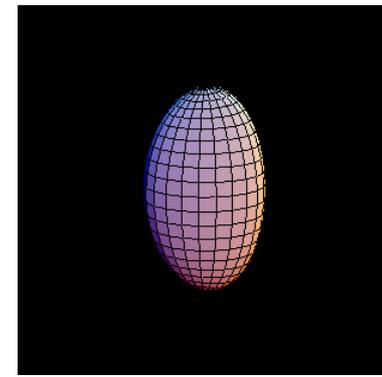
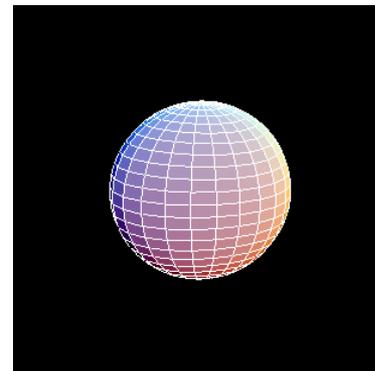
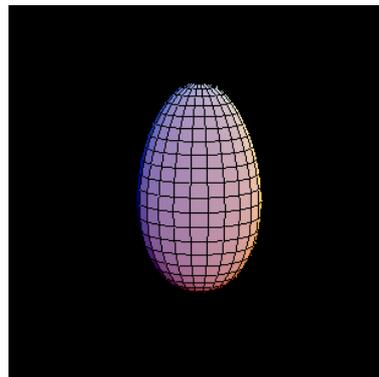
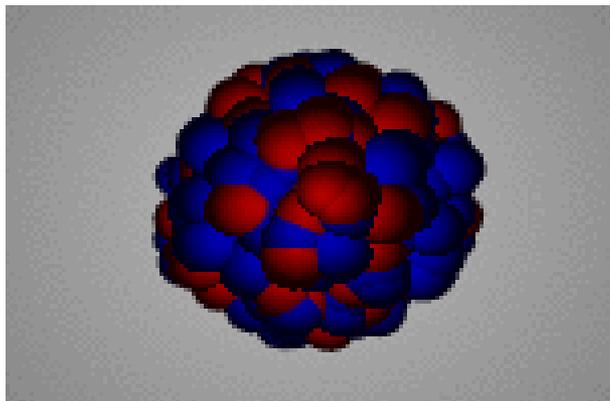


規則正しい運動
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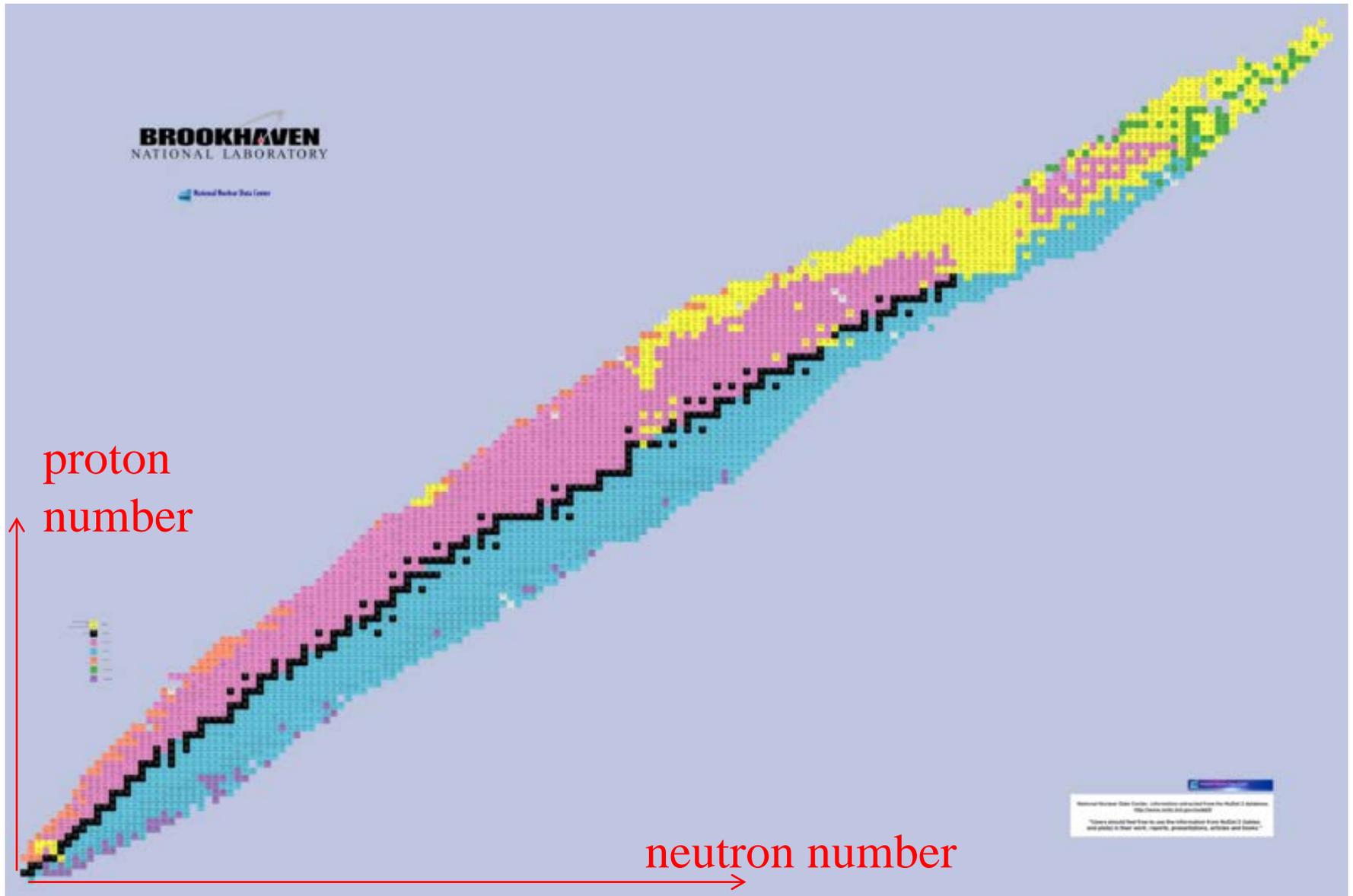
集団運動



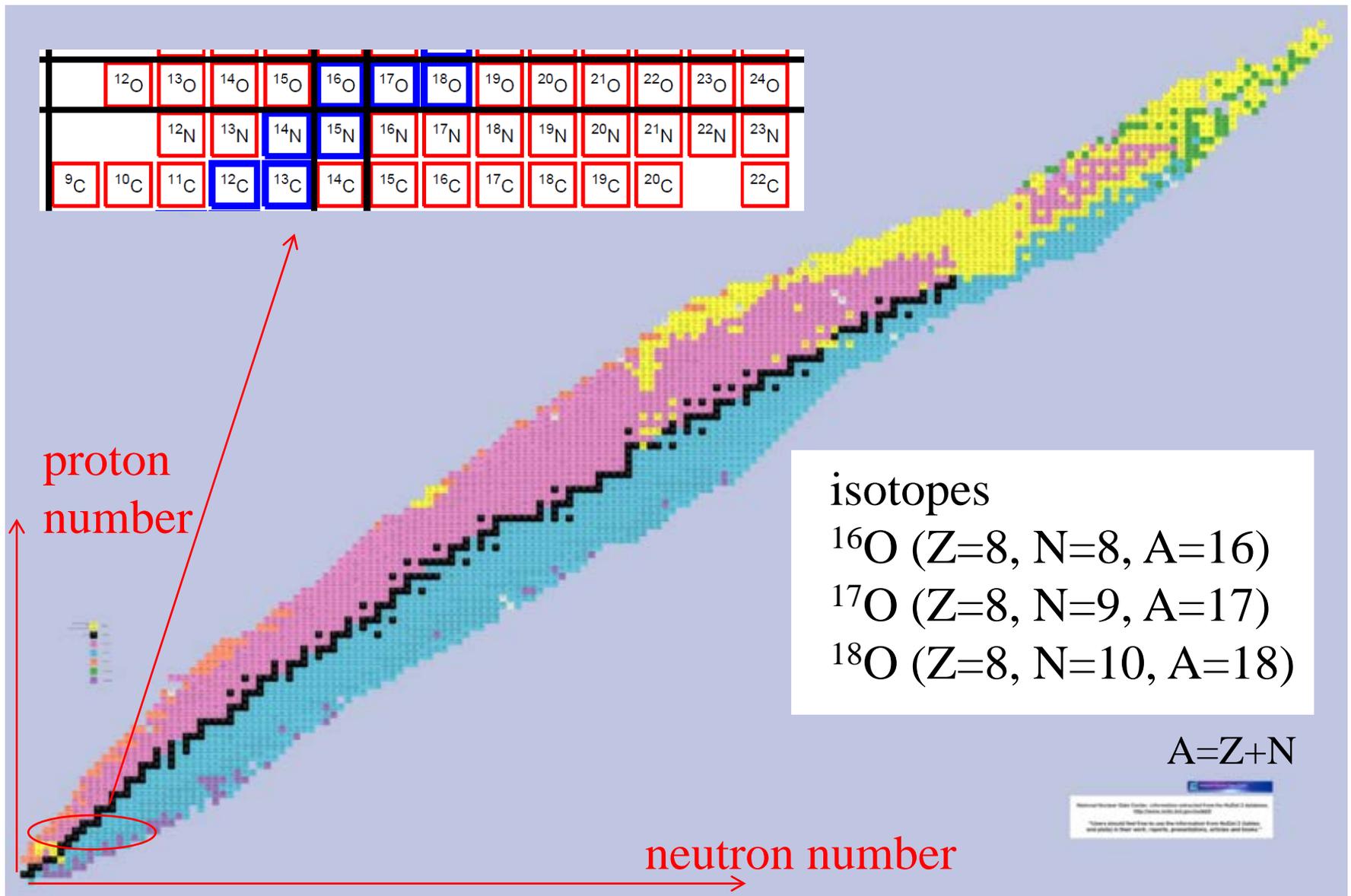
バラエティー
豊か



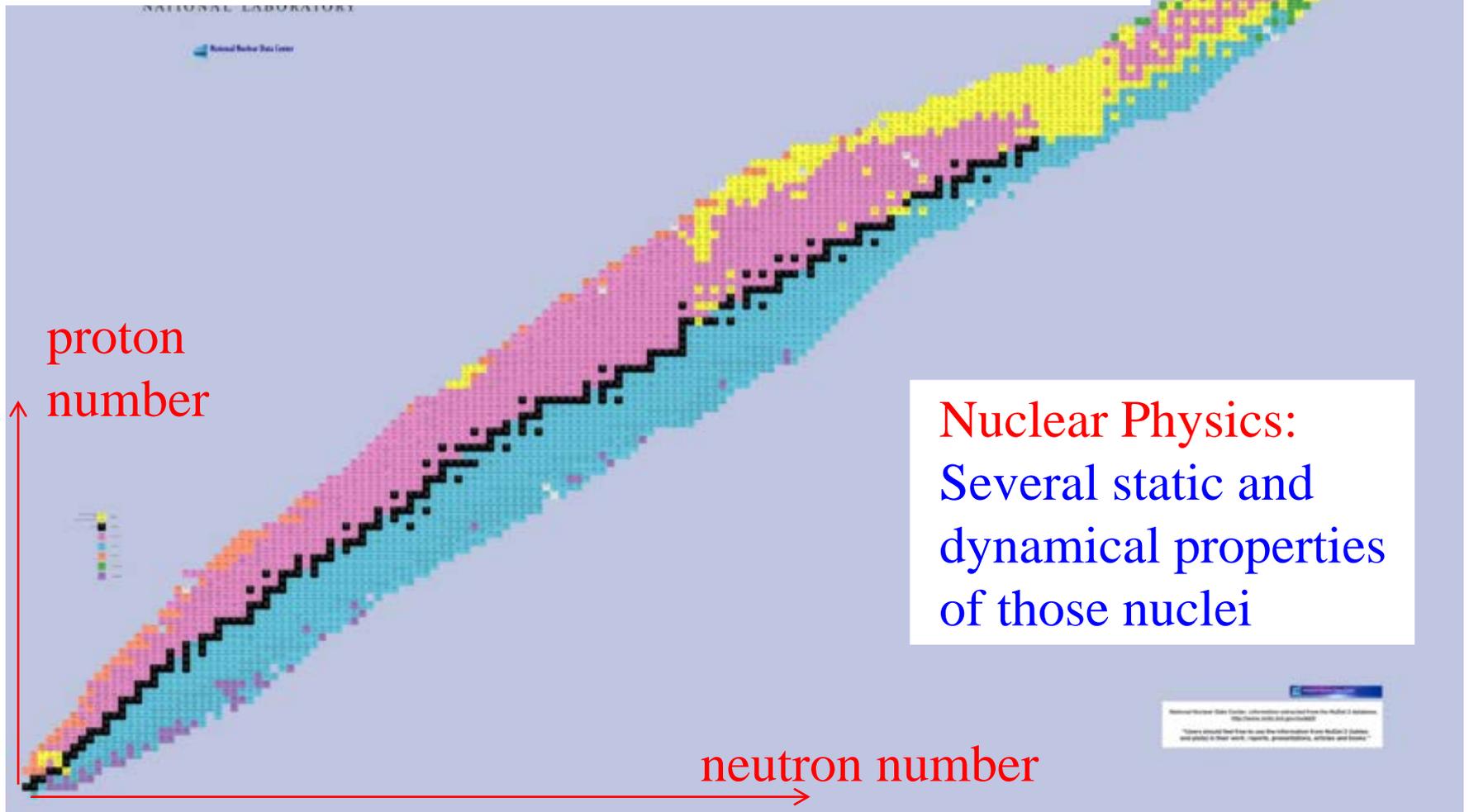
Nuclear Chart: 2D map of atomic nuclei



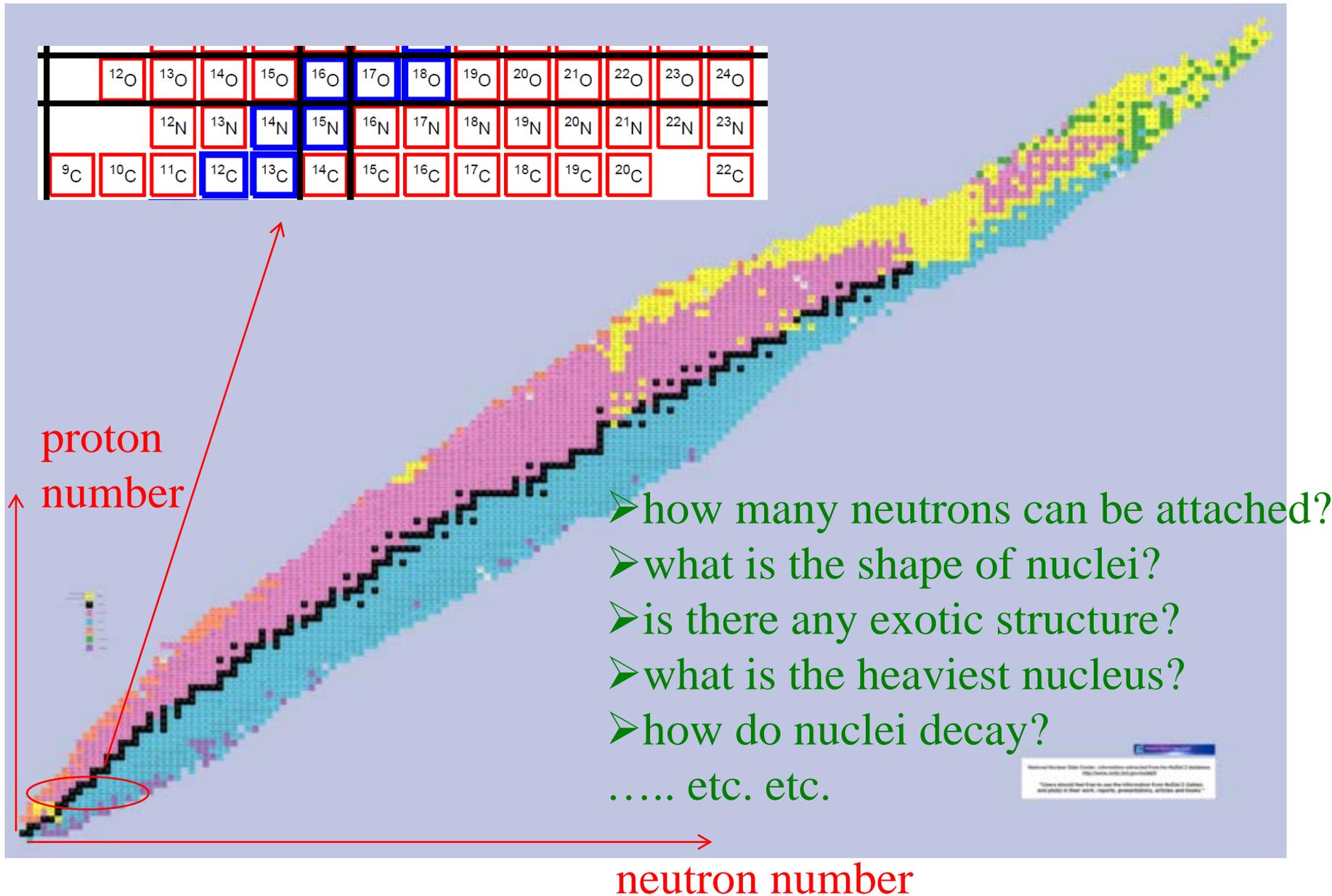
Nuclear Chart: 2D map of atomic nuclei



- Stable nuclei in nature: 287
- Nuclei artificially synthesized : about 3,000
- Nuclei predicted : about 7,000 ~ 10,000

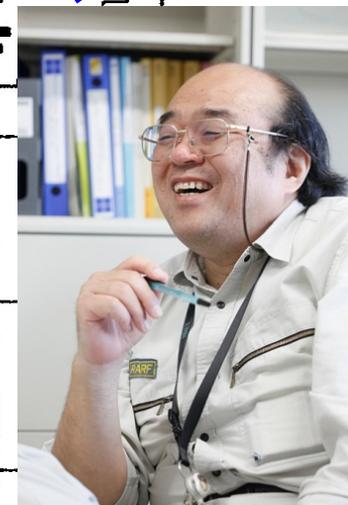
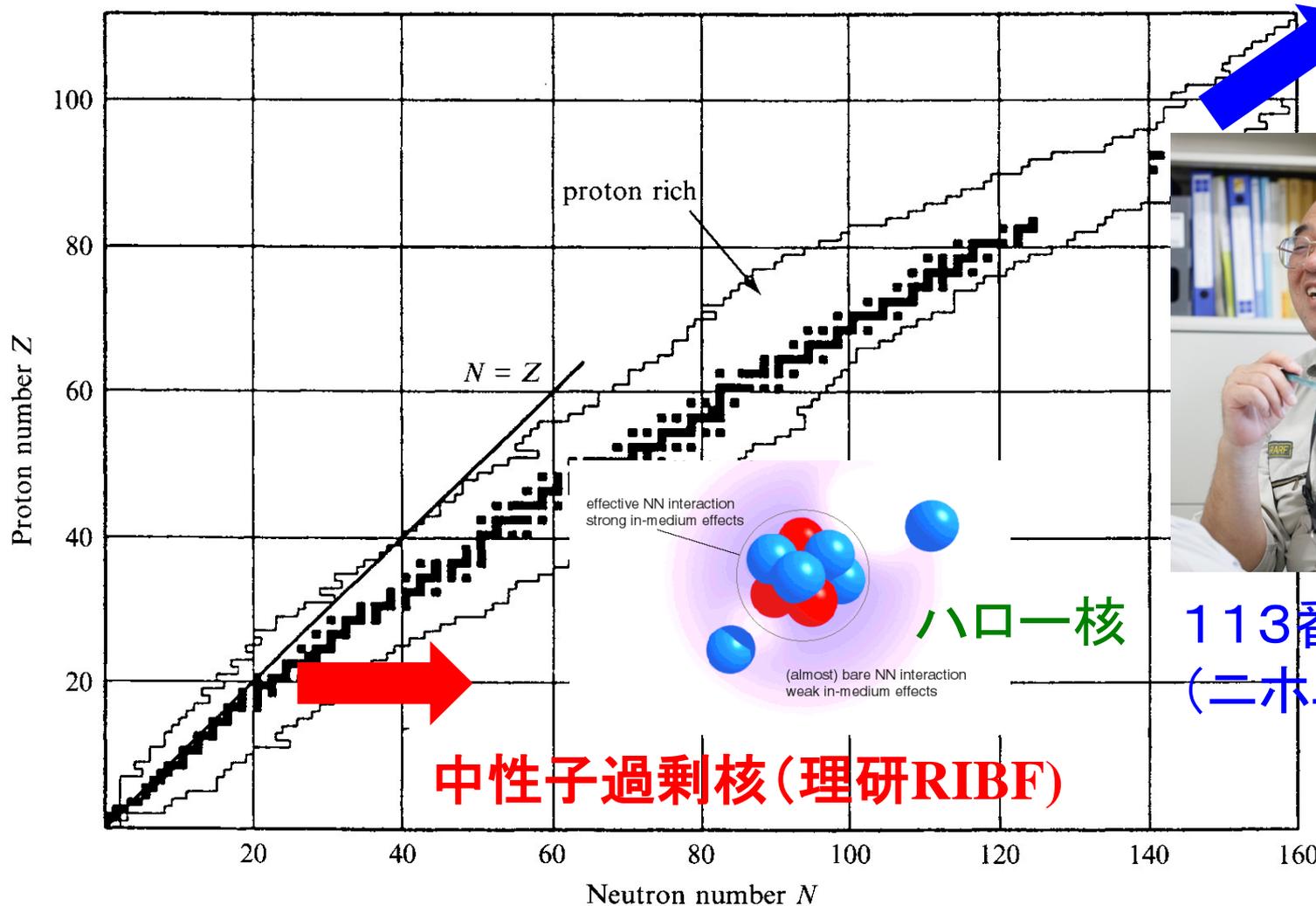


Nuclear Chart: 2D map of atomic nuclei



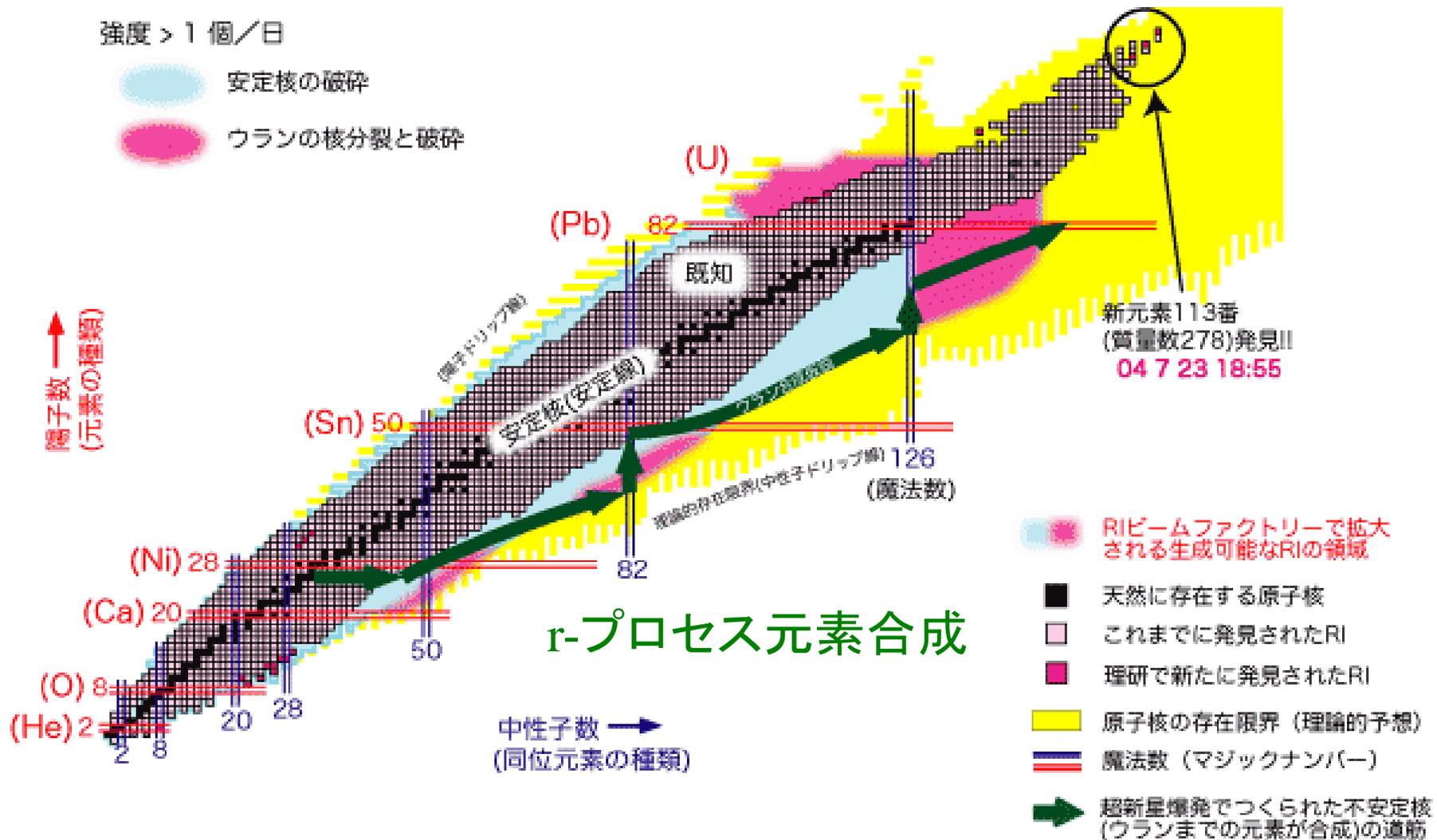
核図表の拡大: 原子核物理のフロンティア

超重核

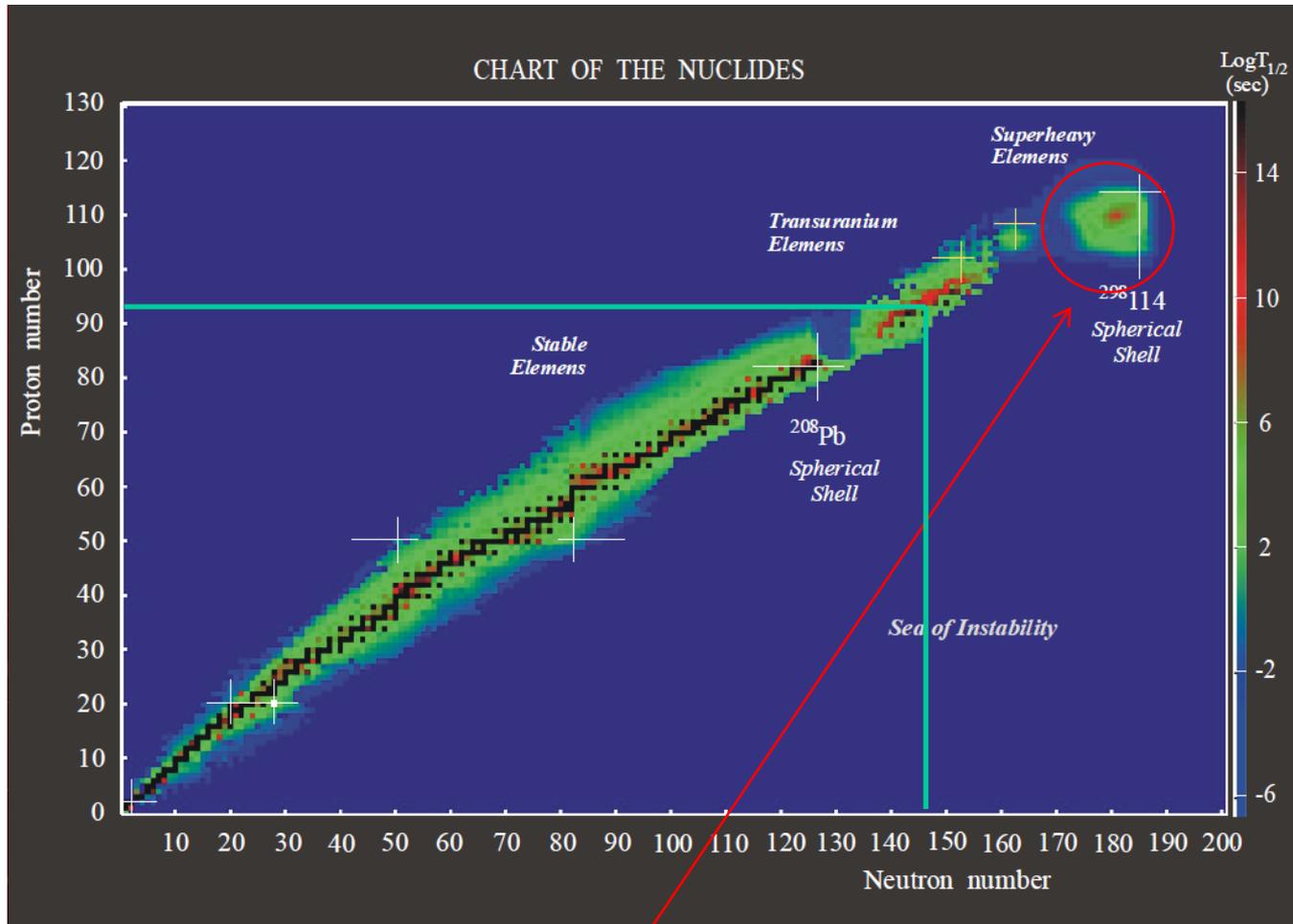


113番元素
(ニホニウム)

中性子過剰核(理研RIBF)



Prediction of island of stability: an important motivation of SHE study



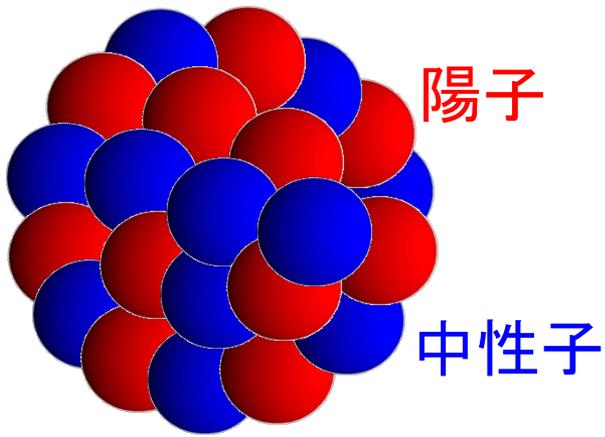
island of stability around Z=114, N=184

Yuri Oganessian

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

→ modern calculations: Z=114,120, or 126, N=184

e.g., H. Koura et al. (2005)

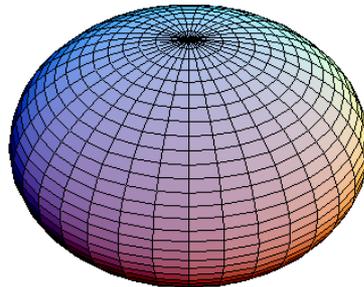
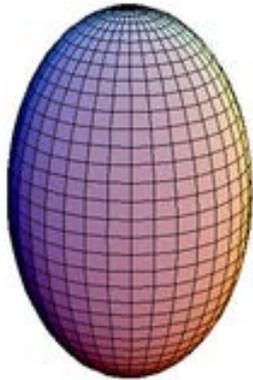


陽子

中性子

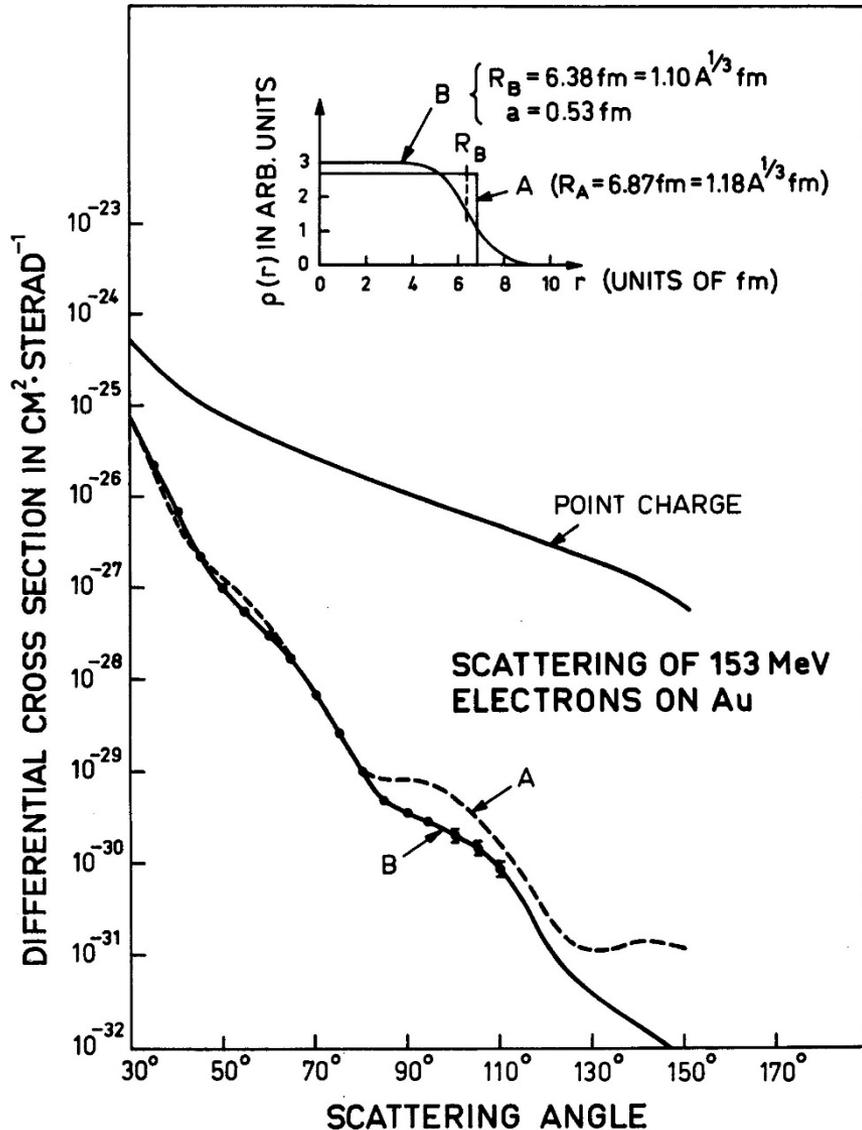
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球形とは限らない



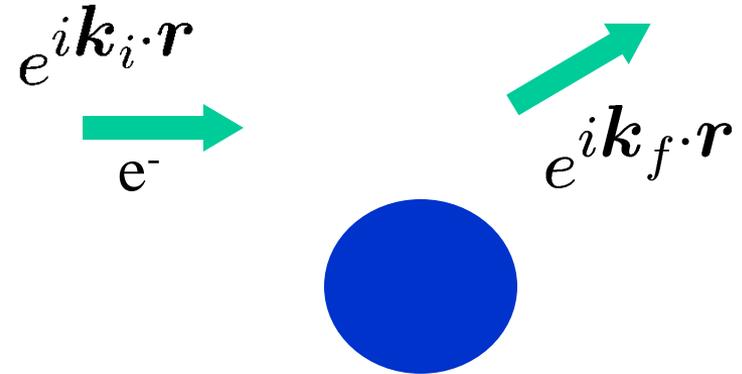
「形の量子論」

Density Distribution



High energy electron scattering

Born approximation:



$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E \sin^2 \theta/2)^2} |F(\mathbf{q})|^2$$

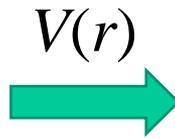
Form factor

$$F(\mathbf{q}) = \int e^{-i\mathbf{q} \cdot \mathbf{r}} \rho(\mathbf{r}) d\mathbf{r}$$

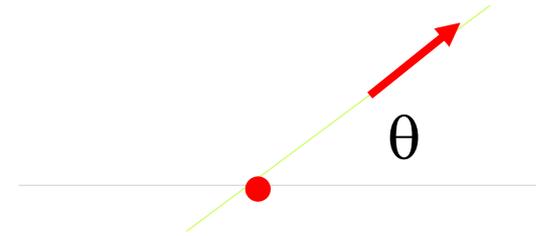
(Fourier transform of the density)

Born approximation

$$\psi_i(\mathbf{r}) = e^{i\mathbf{p}_i \cdot \mathbf{r} / \hbar}$$



$$\psi_f(\mathbf{r}) = e^{i\mathbf{p}_f \cdot \mathbf{r} / \hbar}$$



$$W_{fi} = \frac{\mu p_i}{4\pi^2 \hbar^4} \int d\Omega |\tilde{V}(\mathbf{q})|^2$$

$$\tilde{V}(\mathbf{q}) = \int d\mathbf{r} e^{i(\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{r} / \hbar} V(r) \equiv \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} V(r)$$

momentum transfer

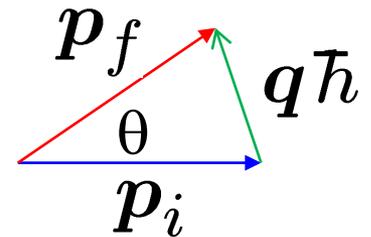


incident flux: $j_{\text{inc}} = \rho_i v = p_i / \mu$



$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \frac{\mu^2}{4\pi^2 \hbar^4} |\tilde{V}(\mathbf{q})|^2$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin \frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

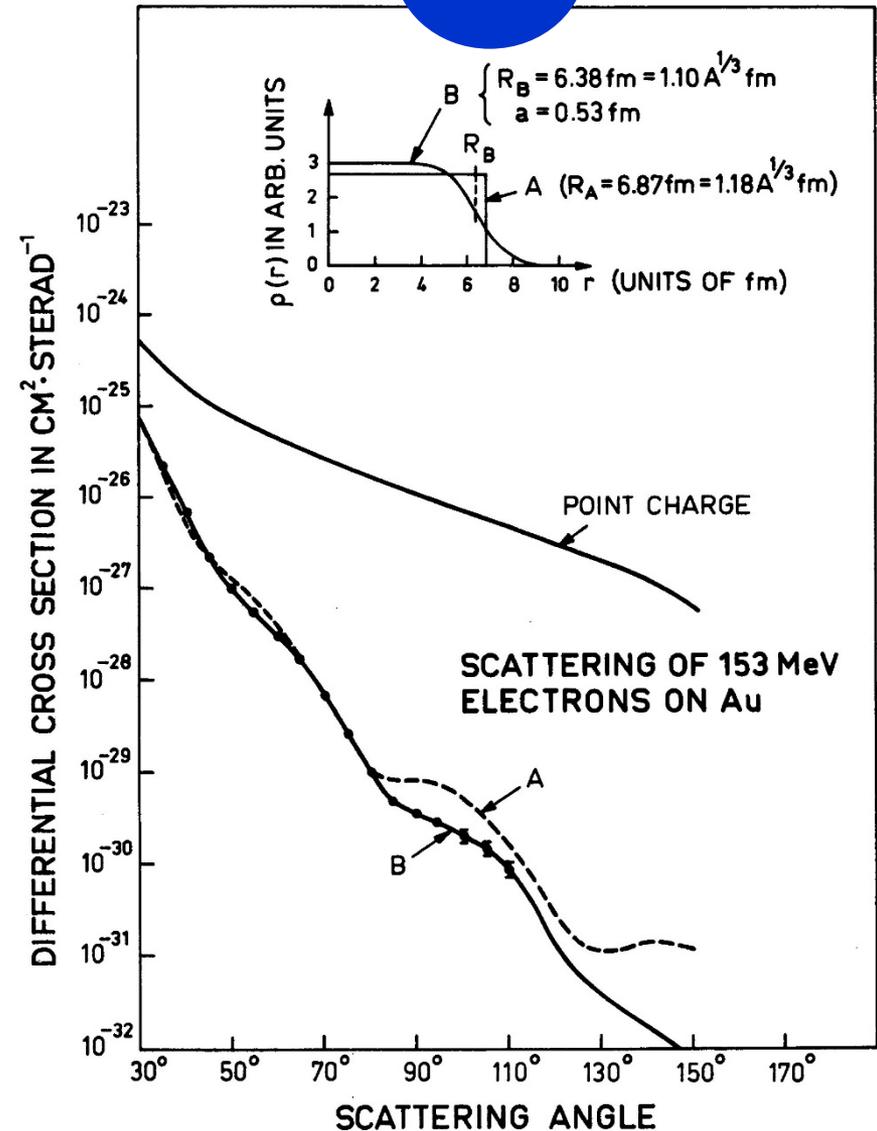
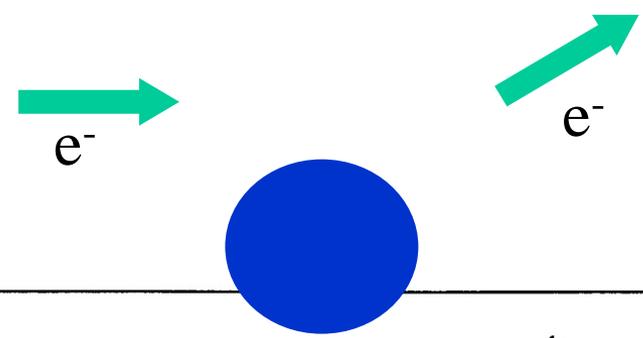
$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{Z_P^2 e^4}{(4E \sin^2 \theta/2)^2} |F(\mathbf{q})|^2 \\ &= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega} \right) |F(\mathbf{q})|^2 \end{aligned}$$

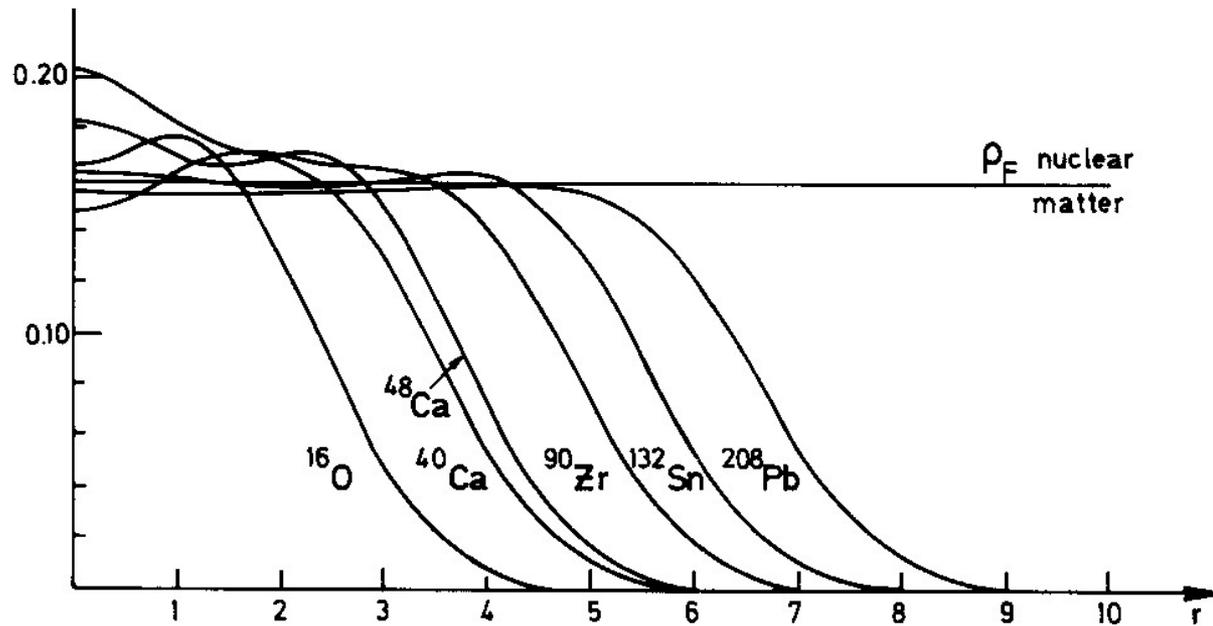
Form factor

$$F(\mathbf{q}) = \int e^{-i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r}) d\mathbf{r}$$

* relativistic correction:

$$\begin{aligned} \frac{d\sigma_{\text{Ruth}}}{d\Omega} &\rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega} \\ &= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2 \frac{\theta}{2} \right) \\ &\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2 \frac{\theta}{2} \quad (v \rightarrow c) \end{aligned}$$





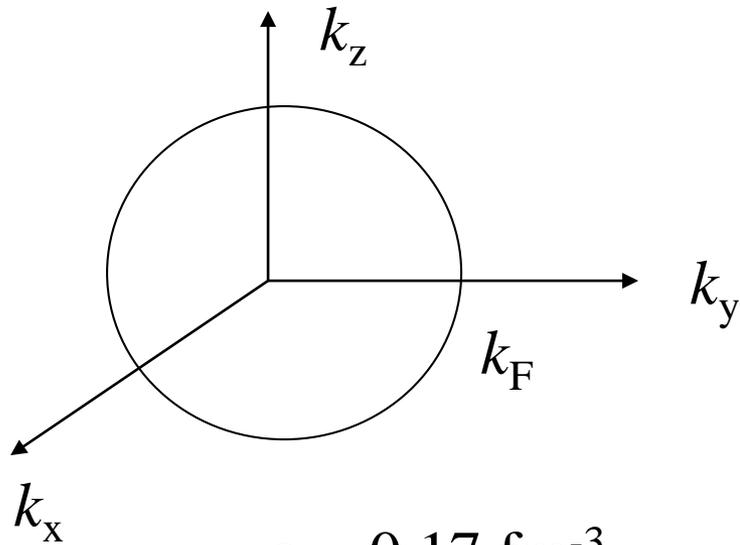
Fermi distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp((r - R_0)/a)}$$

$$\begin{aligned} \rho_0 &\sim 0.17 \text{ (fm}^{-3}\text{)} && \leftarrow \text{Saturation property} \\ R_0 &\sim 1.1 \times A^{1/3} \text{ (fm)} \\ a &\sim 0.57 \text{ (fm)} \end{aligned}$$

Momentum Distribution

Fermi gas approximation



$$\begin{aligned}\rho &= 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3} \\ &= \frac{2}{3\pi^2} k_F^3\end{aligned}$$

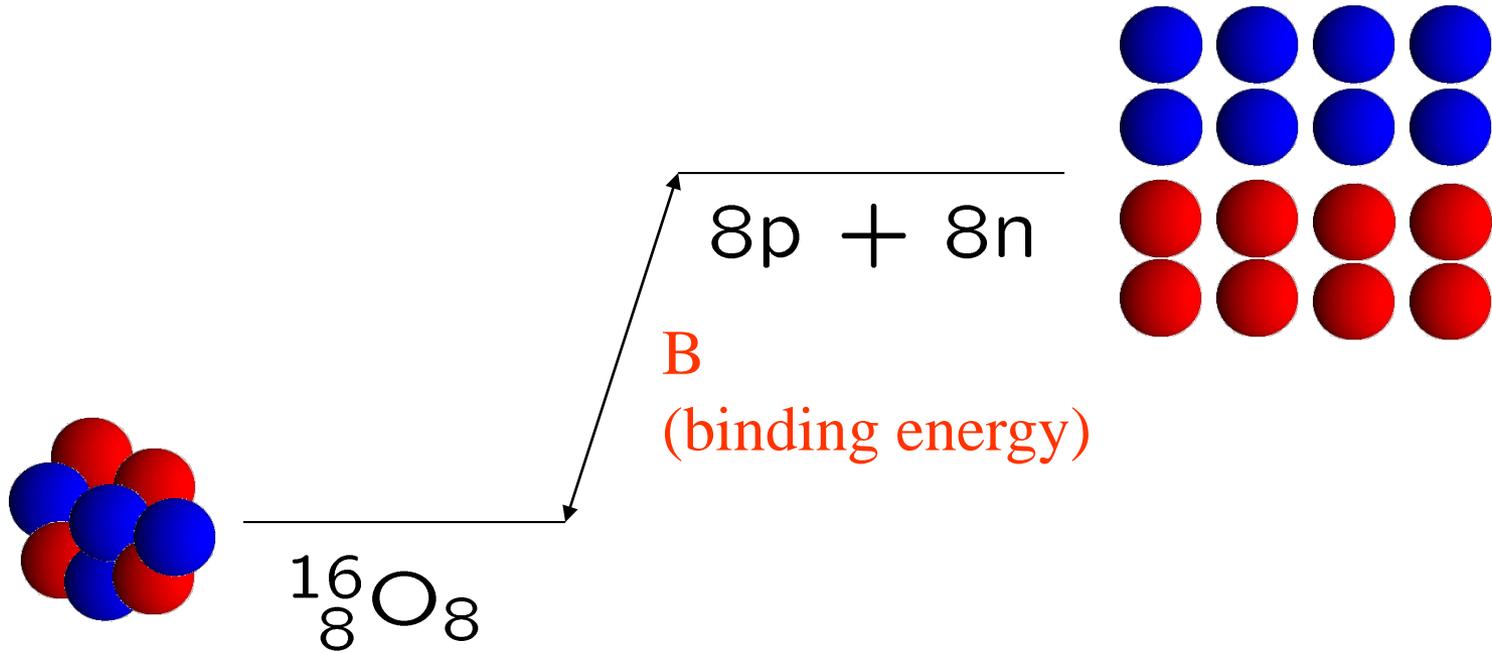
(note: spin-isospin degeneracy)

$$\rho = 0.17 \text{ fm}^{-3} \longrightarrow k_F \sim 1.36 \text{ fm}^{-1}$$

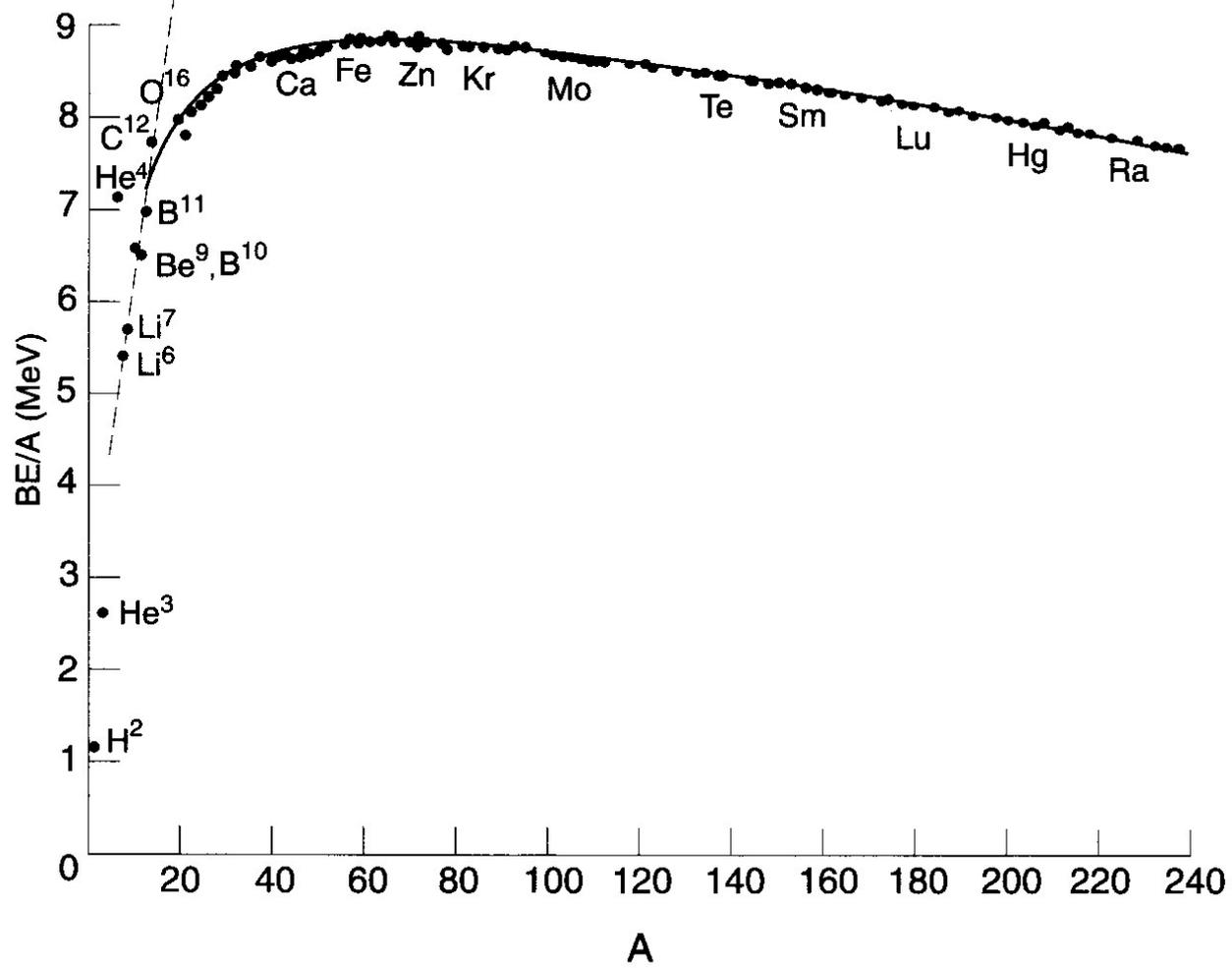
$$\iff \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

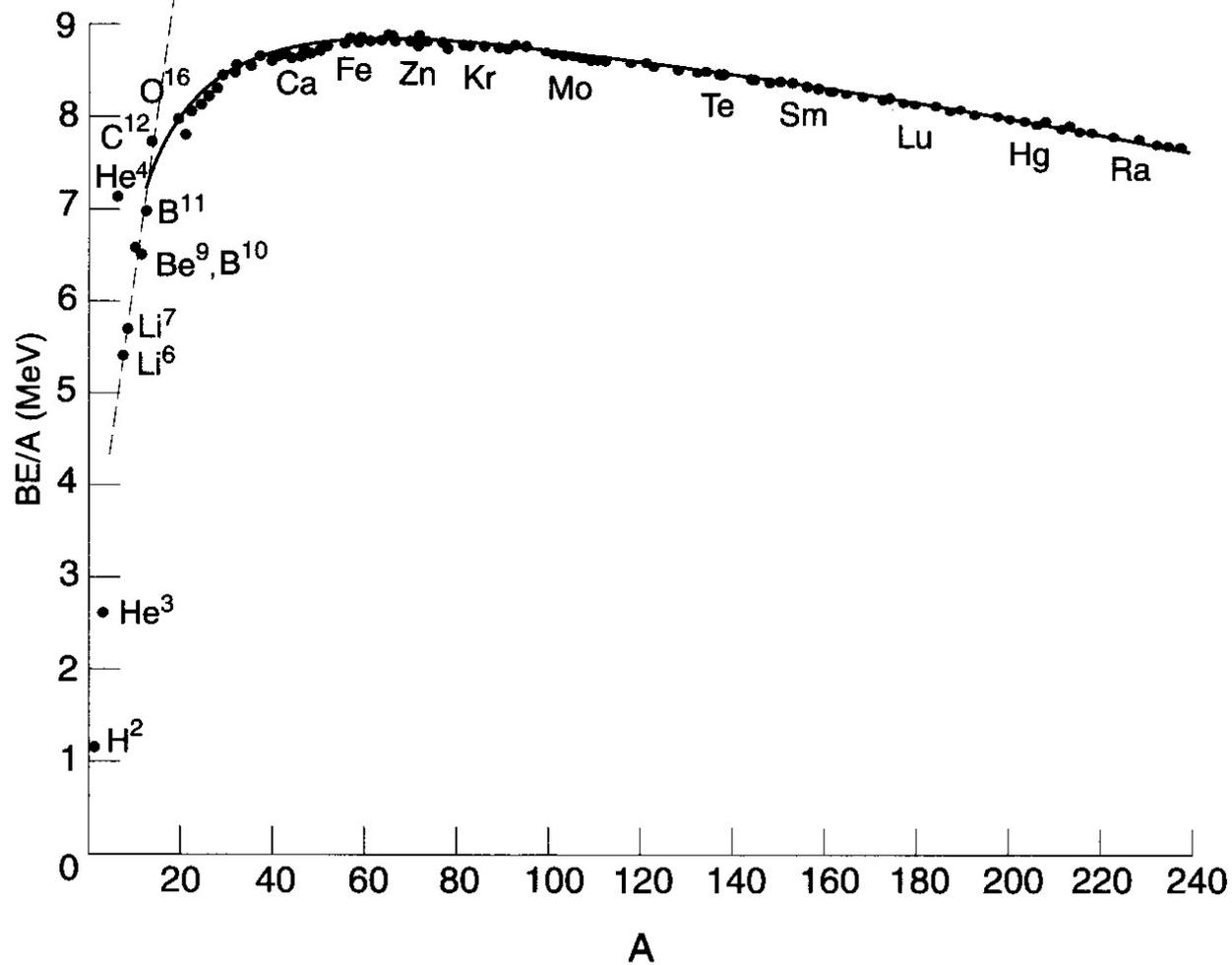
$$\text{Fermi energy: } \epsilon_F = \frac{k_F^2 \hbar^2}{2m} \sim 37 \text{ (MeV)}$$

Nuclear Mass

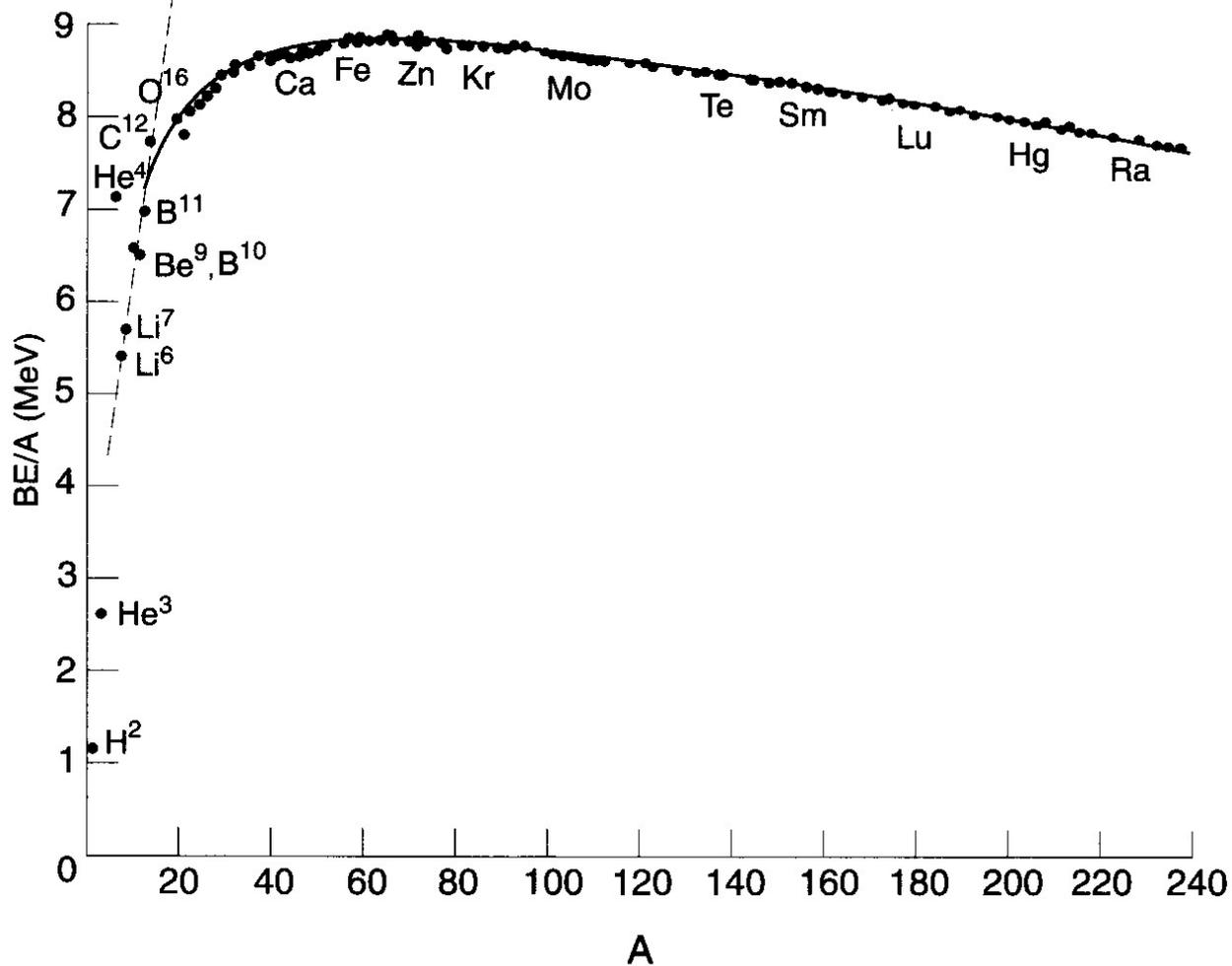


$$m(N, Z)c^2 = Zm_p c^2 + Nm_n c^2 - B$$





1. $B(N,Z)/A \sim 8.5 \text{ MeV} (A > 12) \iff$ Short range nuclear force



1. $B(N,Z)/A \sim -8.5$ MeV ($A > 12$) \iff Short range nuclear force
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

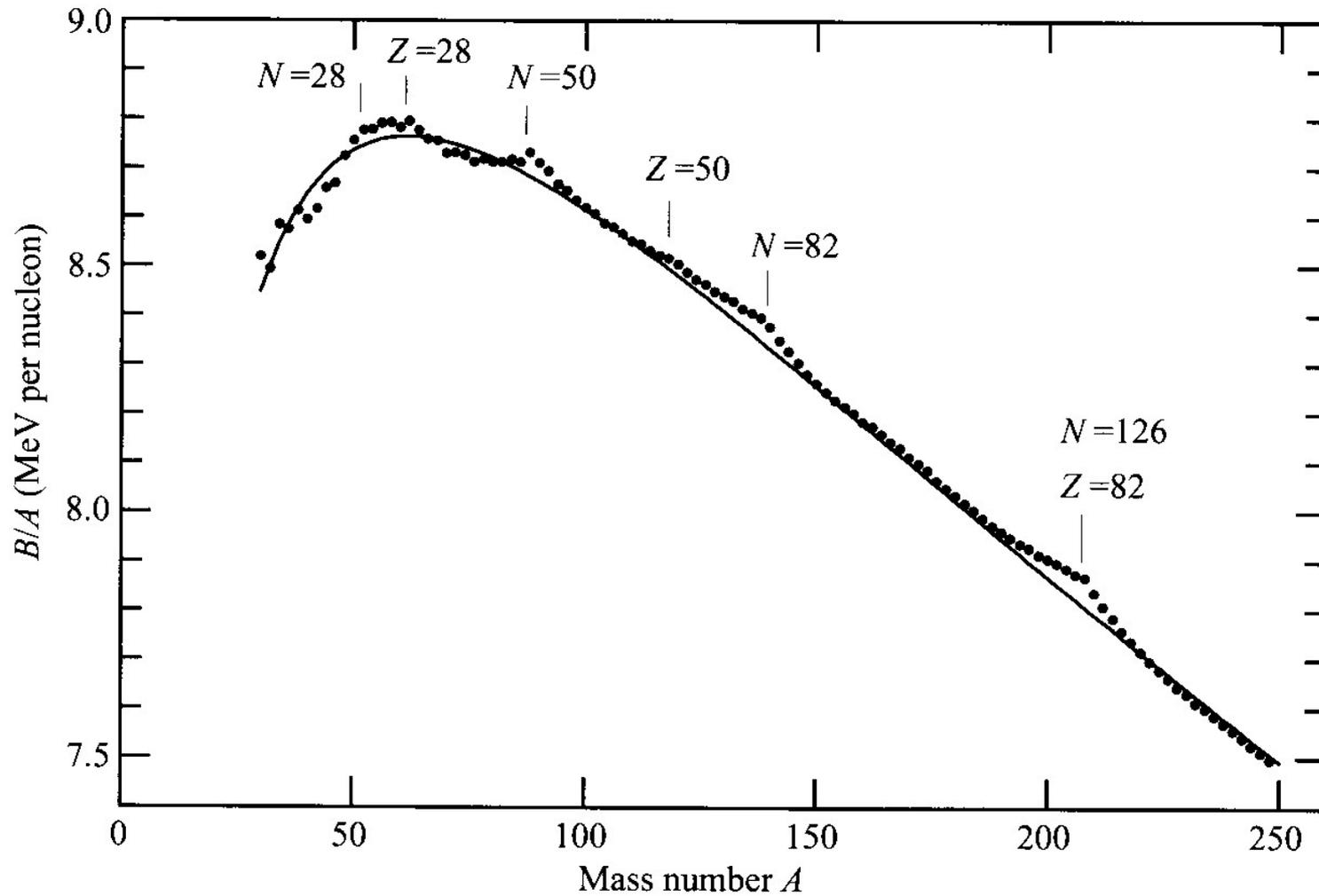
Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

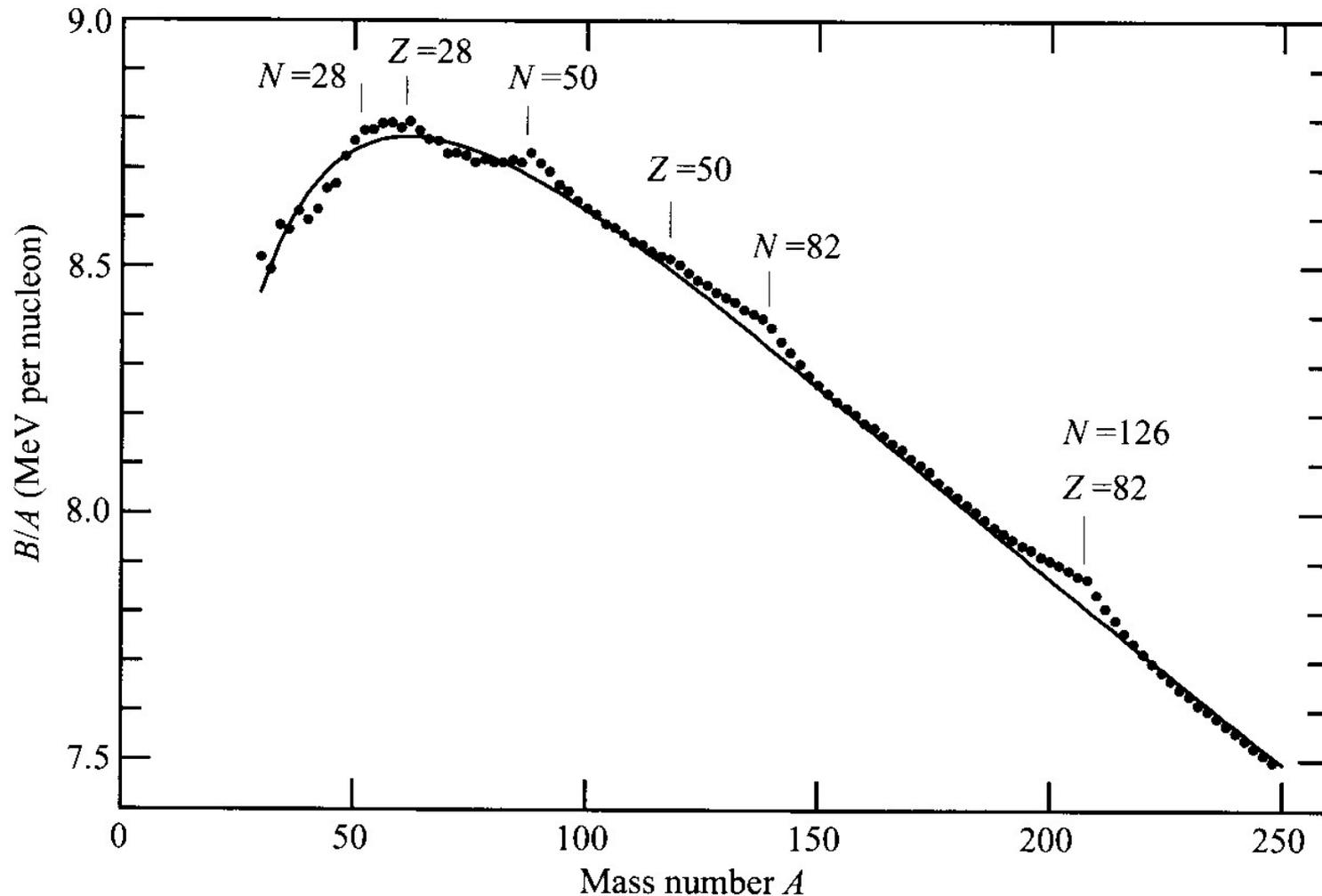
$$B(N, Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N - Z)^2}{A}$$

- Volume energy: $a_v A$
- Surface energy: $-a_s A^{2/3}$
- Coulomb energy: $-a_C Z^2 / A^{1/3}$
- Symmetry energy: $-a_{\text{sym}} (N - Z)^2 / A$

How well does the Bethe-Weizacker formula reproduce the data?



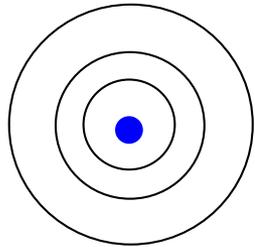
How well does the Bethe-Weizacker formula reproduce the data?



cf. $N, Z = 2, 8, 20, 28, 50, 82, 126$: large binding energy
“magic numbers”

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



shell structure

元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8	1B	2B	3B	4B	5B	6B	7B	0		
1	H															He		
2	Li	Be									B	C	N	O	F	Ne		
3	Na	Mg									Al	Si	P	S	Cl	Ar		
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	Cs	Ba	L	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	Fr	Ra	A															
	L	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
	A	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Legend:

- 典型金属元素 (Orange)
- 半金属元素 (Light Green)
- 非金属元素 (Cyan)
- 遷移金属元素 (Yellow)
- 希ガス (Pink)

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Why do closed-shell-nuclei become stable?

level density

