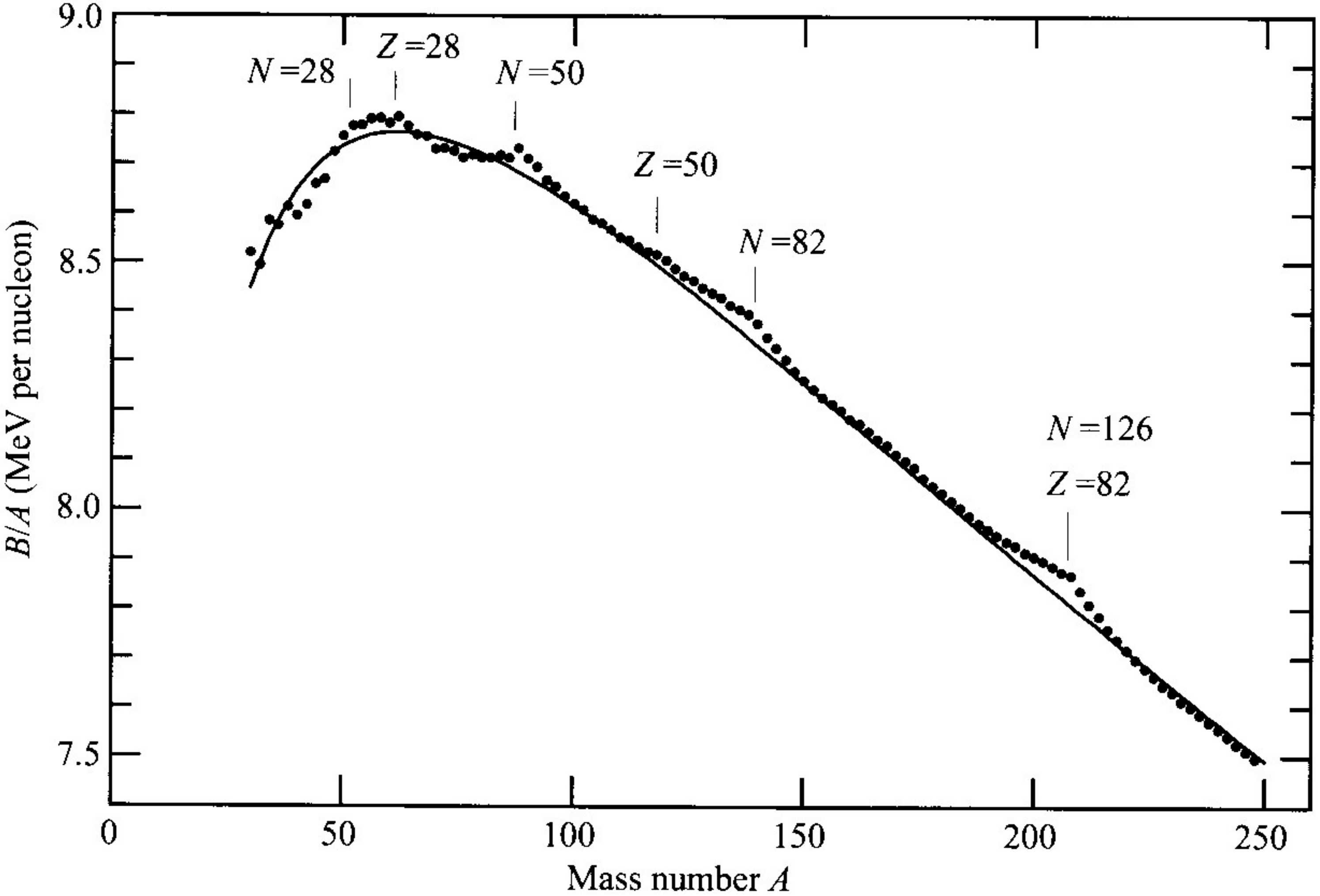
# Shell Energy




cf. N,Z = 2, 8, 20, 28, 50, 82, 126 (魔法数)に対して束縛エネルギー大

# 成績のつけかた

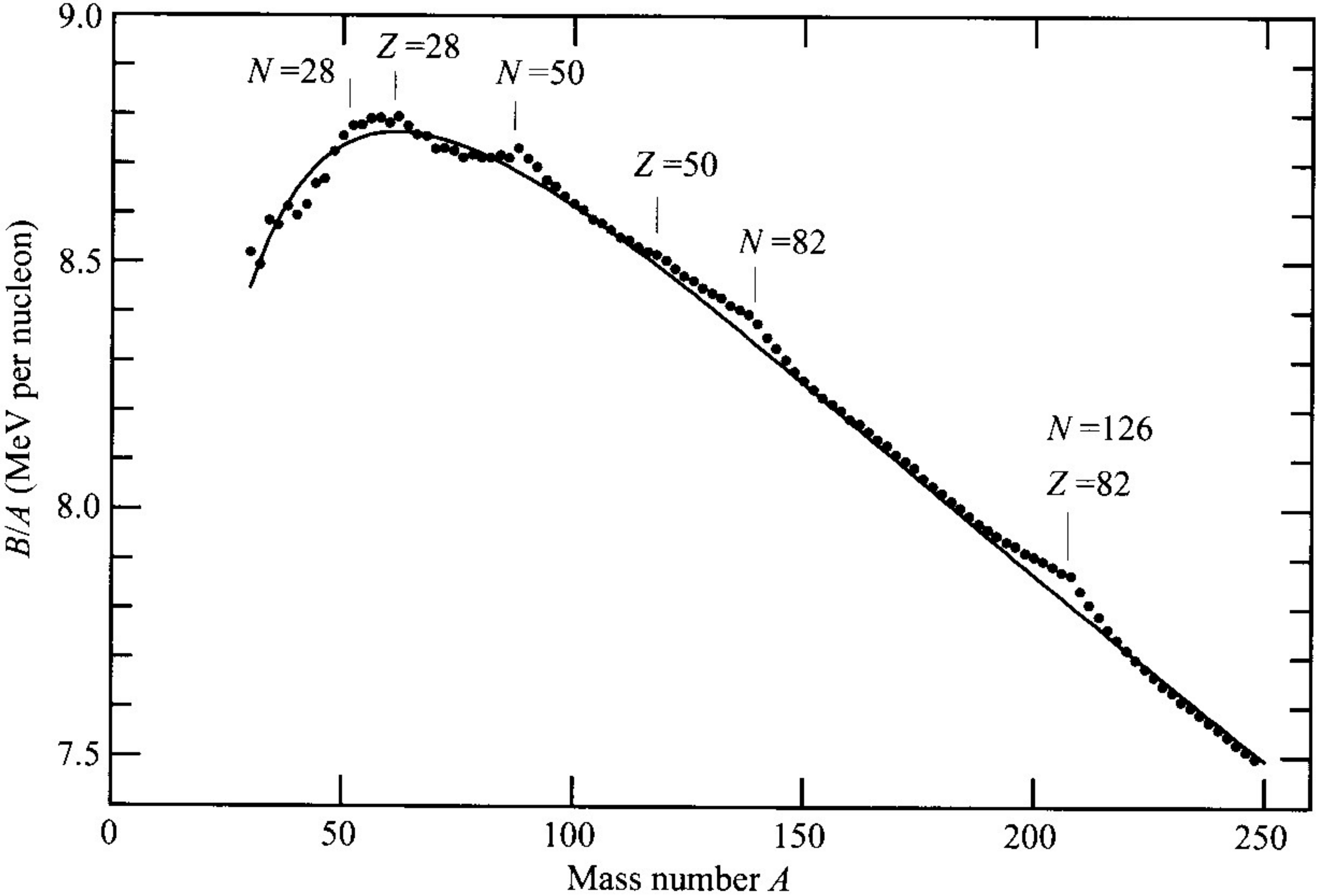
期末レポート（必須）＋出席点

質問をした日は出席点１。
質問を考えながら講義を聴いてください。

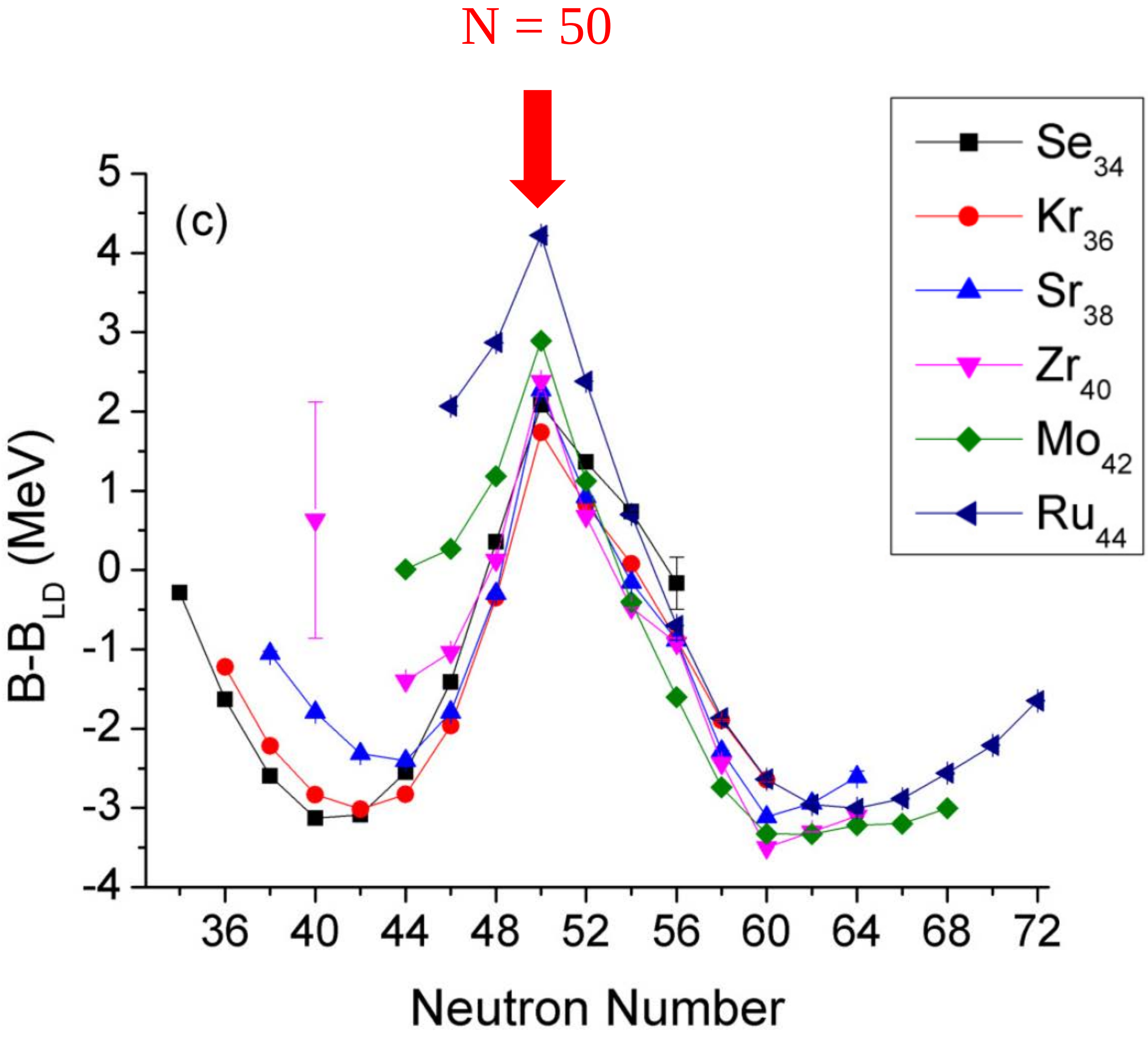
成績の基準：レポートがOKで、出席点２以上
（2回以上質問をする） → A

AAが欲しい場合は2回以上質問してください。

# Shell Energy

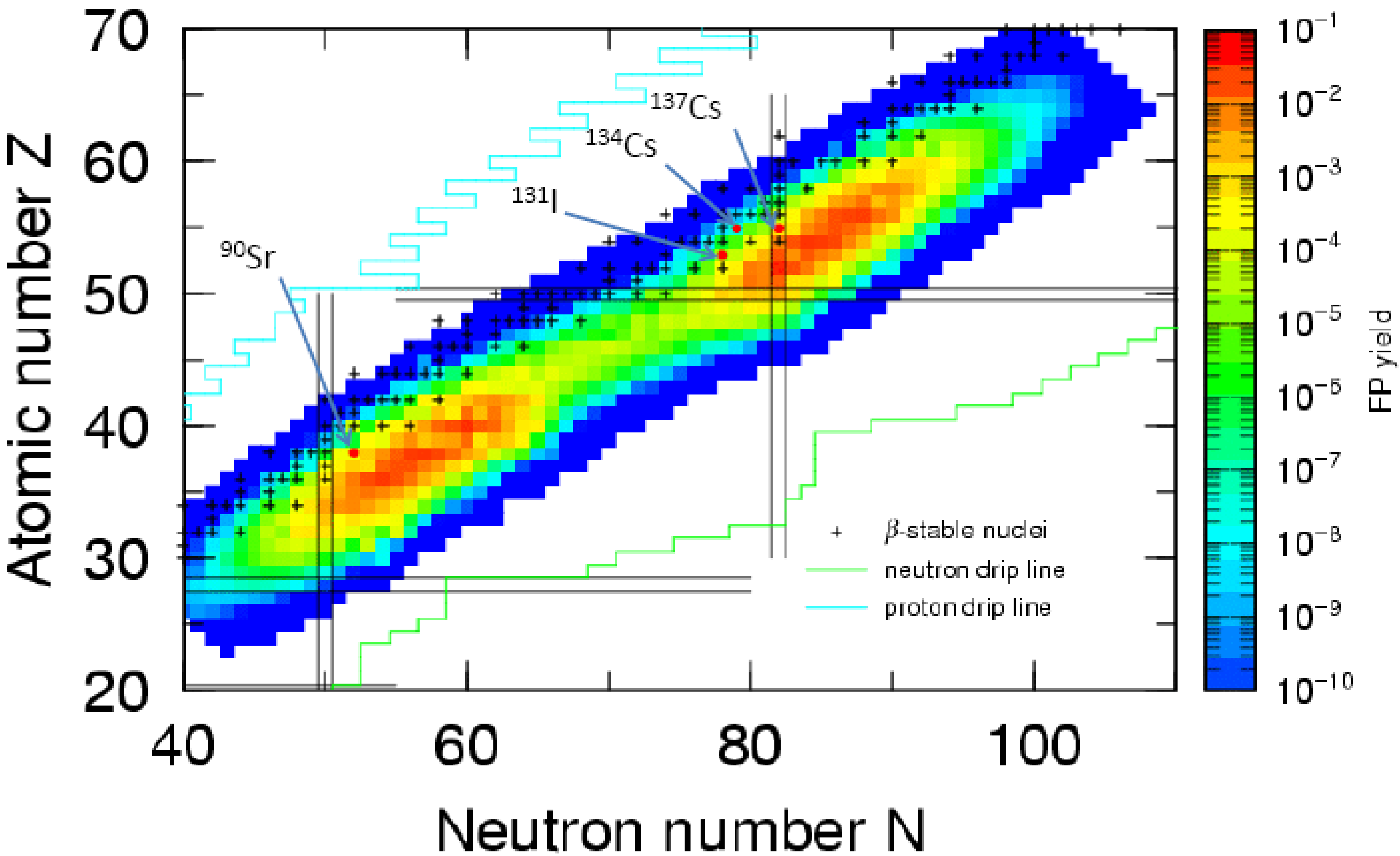



cf. N,Z = 2, 8, 20, 28, 50, 82, 126 (魔法数)に対して束縛エネルギー大



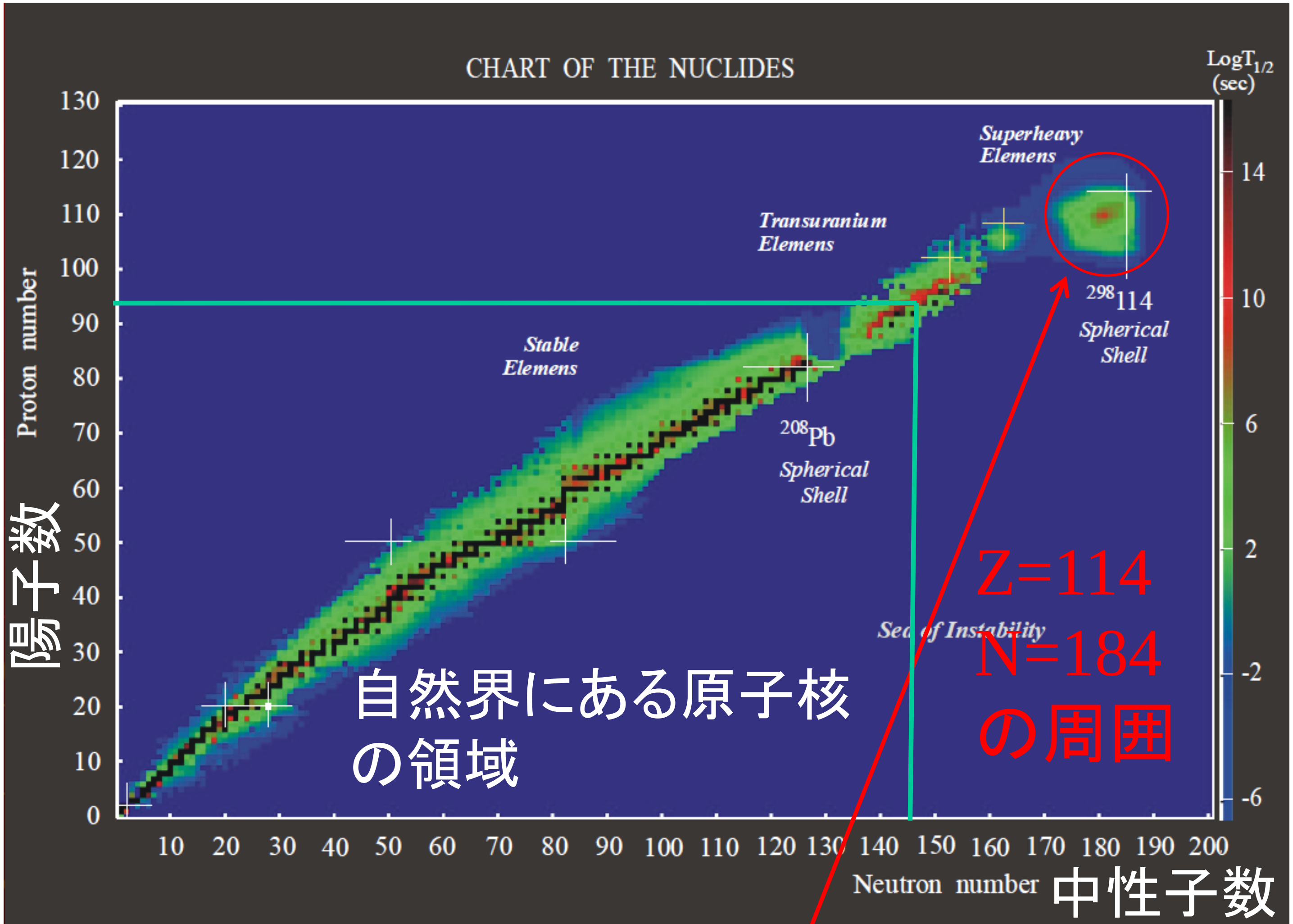

I. Bentley et al., PRC93 (‘16) 044337

# Fission fragment mass distribution for $n_{th}$ + $^{235}U$ reaction




非对称核分裂

# 超重元素(超重原子核)

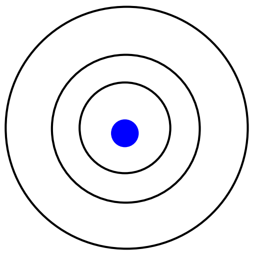



Yuri Oganessian

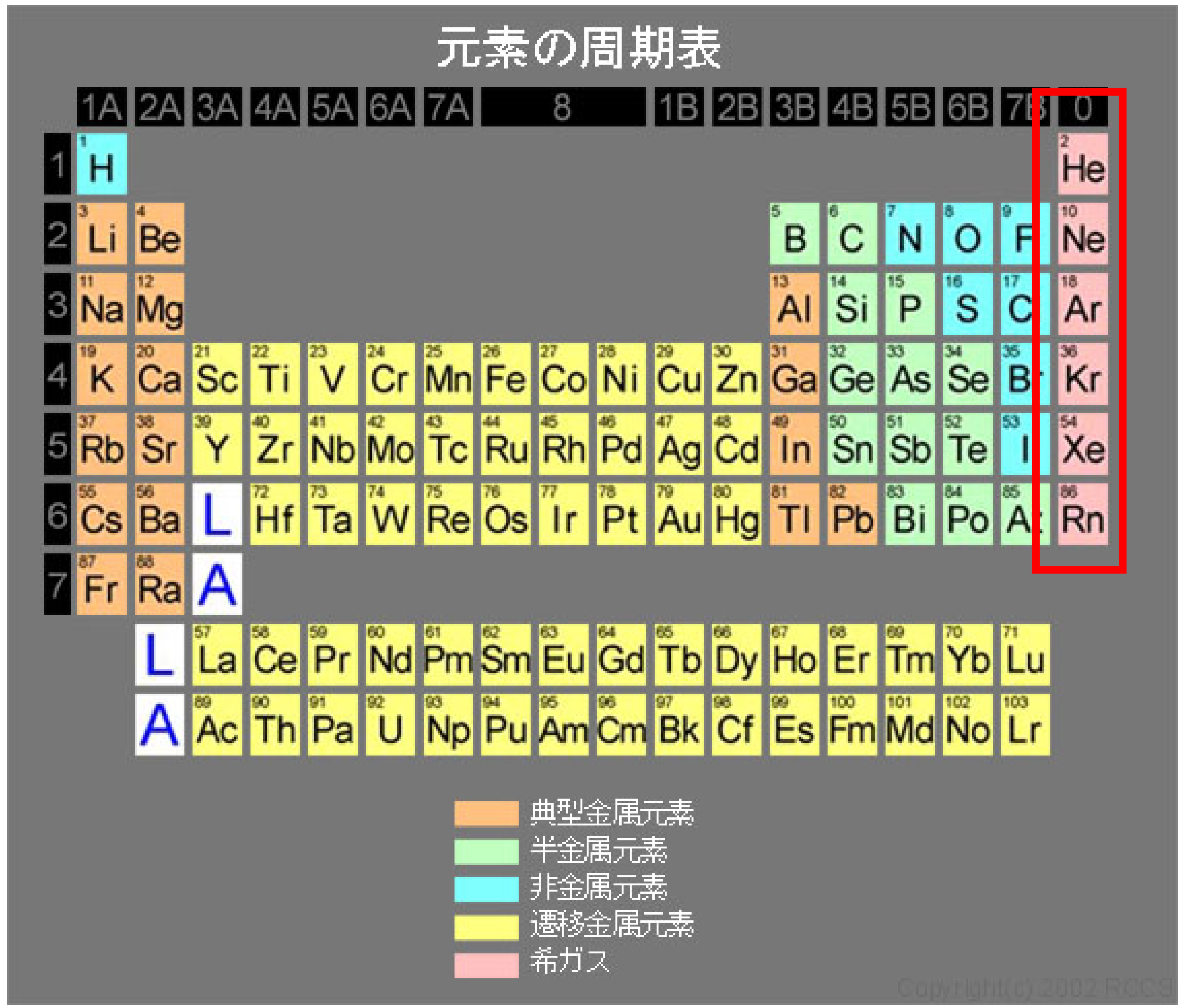
**原子核の安定領域の理論的予言**
**「安定の島」**

# (note) 原子の魔法数 (貴ガス･希ガス)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

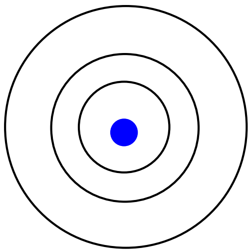


電子の殻構造



(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

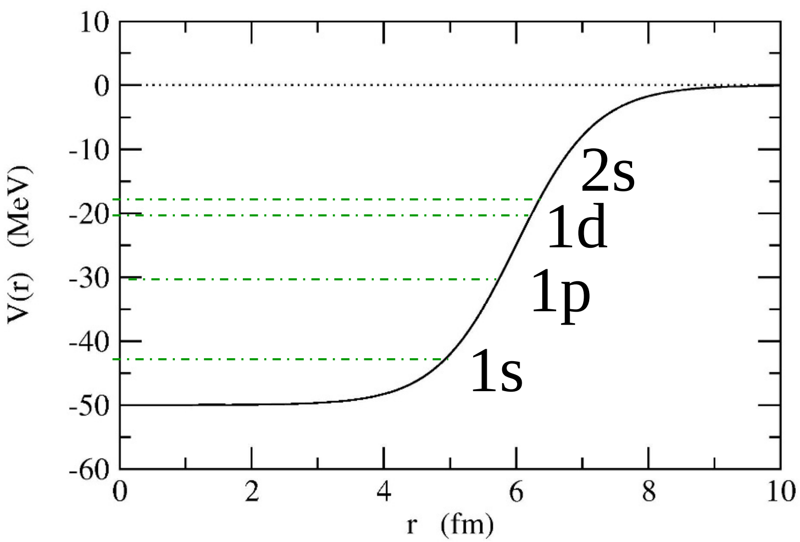


Shell structure

Similar attempt in nuclear physics: independent particle motion in a potential well

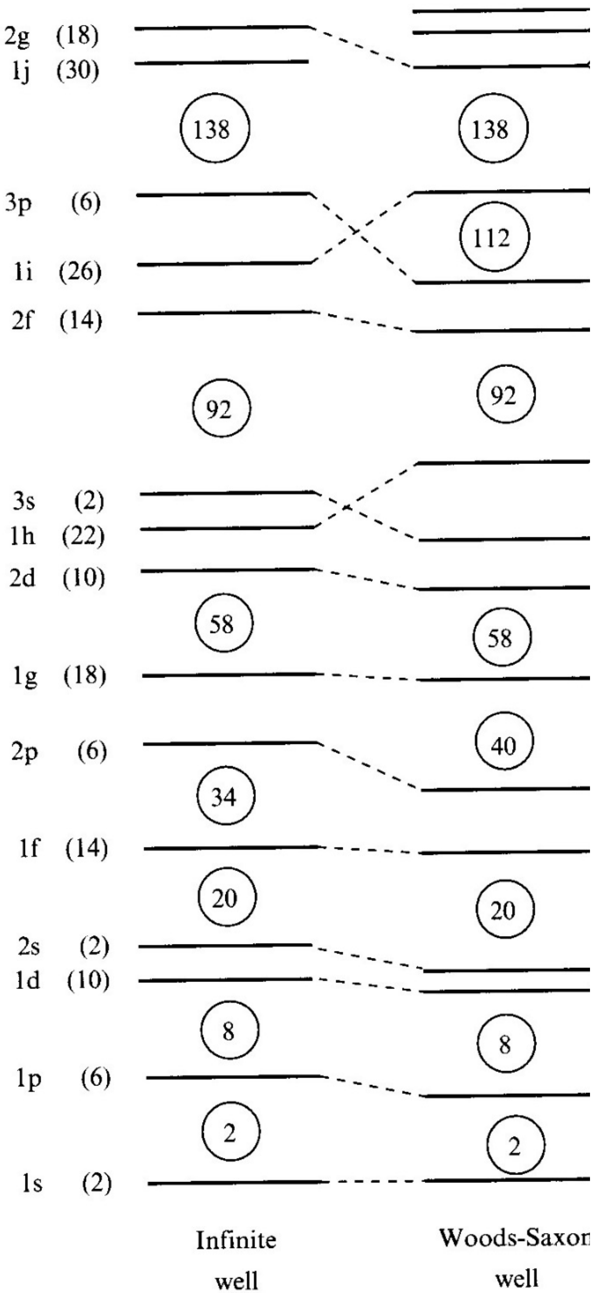
Woods-Saxon potential

$$V(r) = -V_0/[1 + \exp((r - R_0)/a]$$

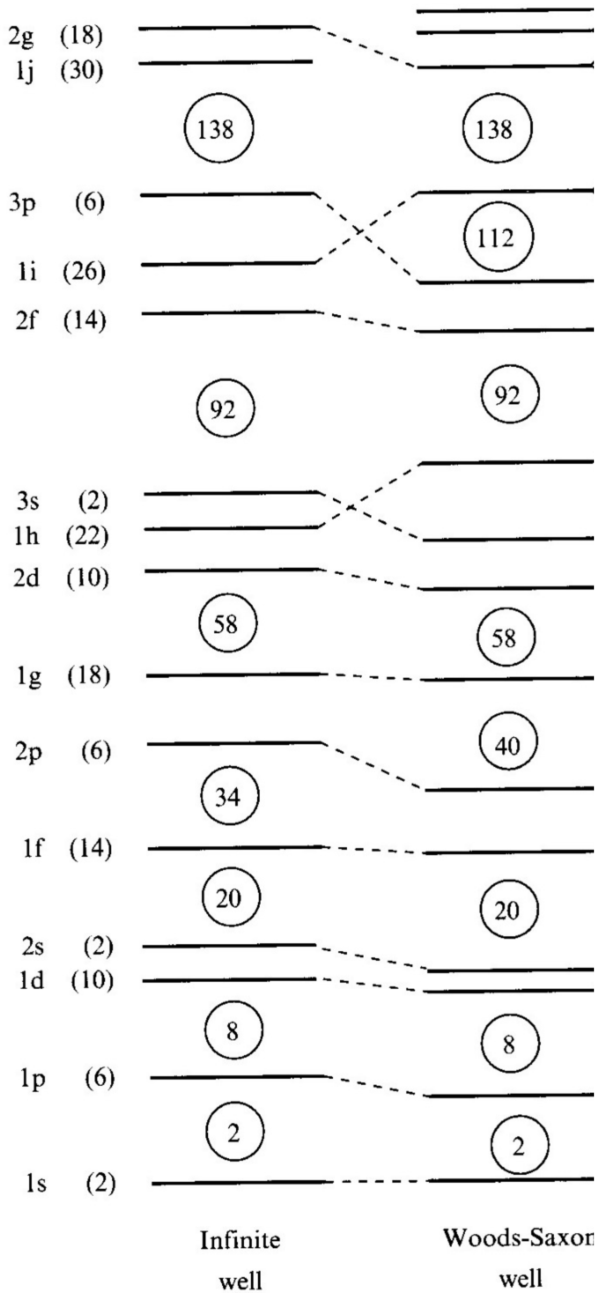



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$




Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).




Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).

Meyer and Jensen (1949):
Strong spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$V_{ls}(r) \sim -\lambda \frac{1}{r}\frac{dV}{dr} \qquad (\lambda > 0)$$

# jj coupling shell model

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0 \quad \Longrightarrow \quad \psi_{lmm_s}(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

## Spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \quad \Longrightarrow \quad \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\psi_{jlm}(\boldsymbol{r}) = \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l, m_s} \langle l \ m_l \ 1/2 \ m_s | j \ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

## jj coupling shell model

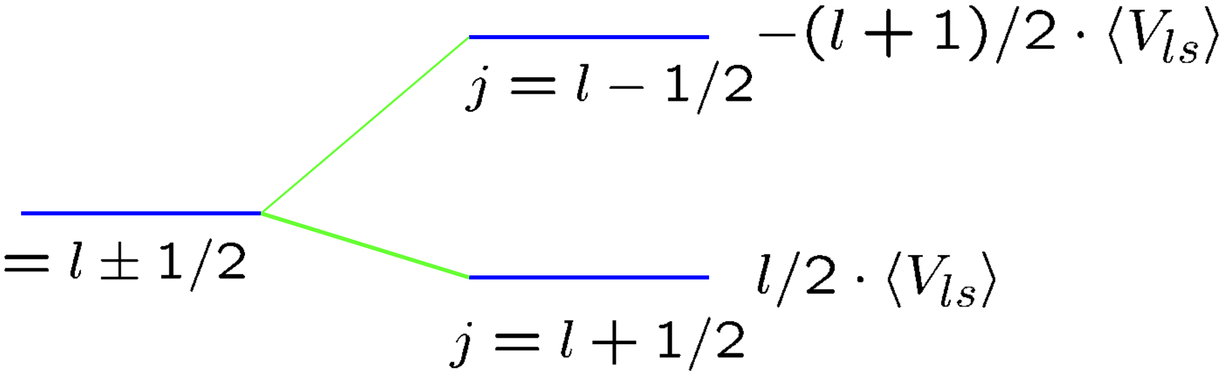
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

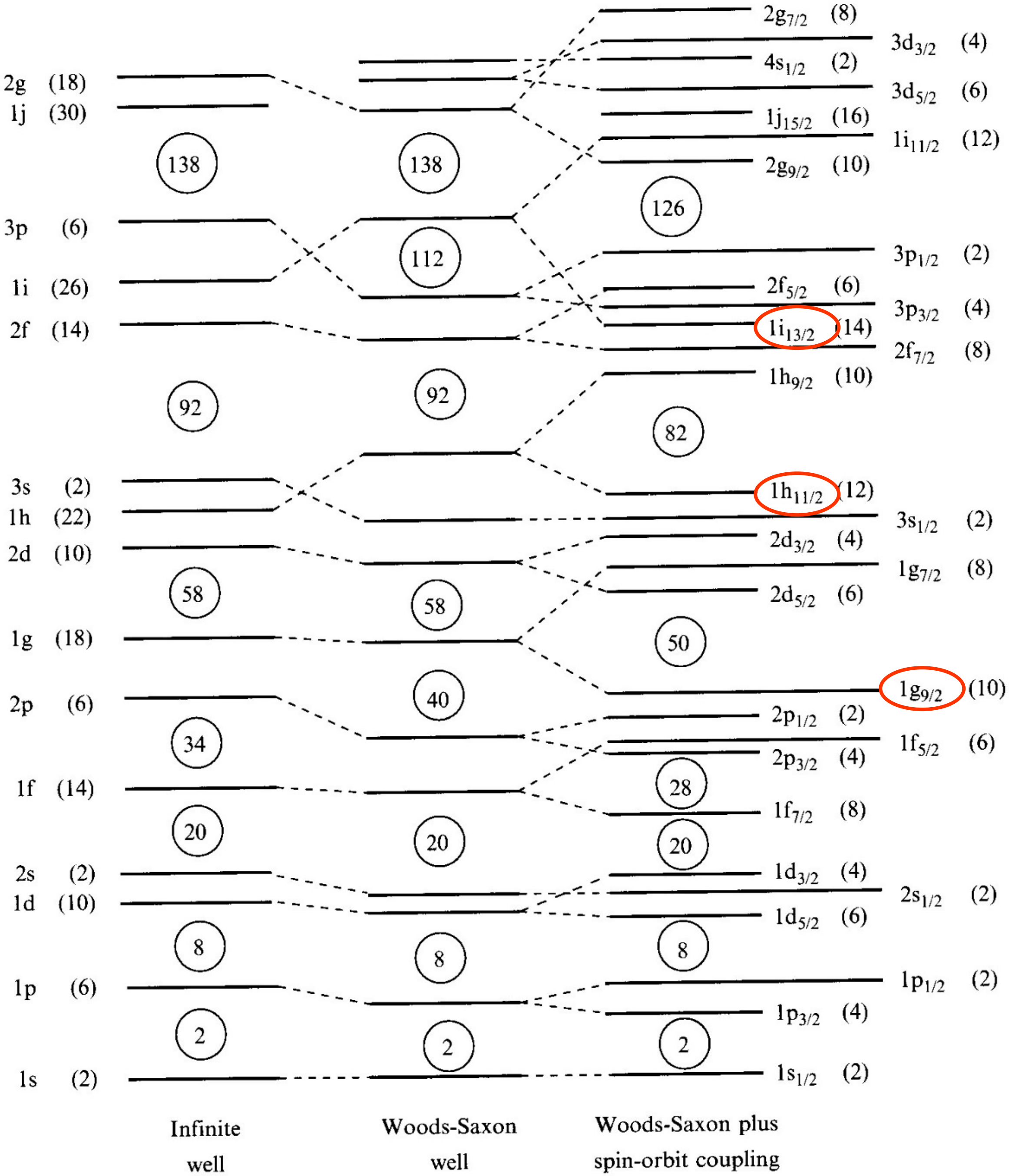
(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \quad \Longrightarrow \quad \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\psi_{jlm}(\boldsymbol{r}) = \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l,m_s}\langle l\ m_l\ 1/2\ m_s|j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

$$\boldsymbol{l}\cdot\boldsymbol{s} = l/2\ (j = l + 1/2), \quad -(l+1)/2\ (j = l - 1/2)$$






intruder states
unique parity states

# Single particle spectra

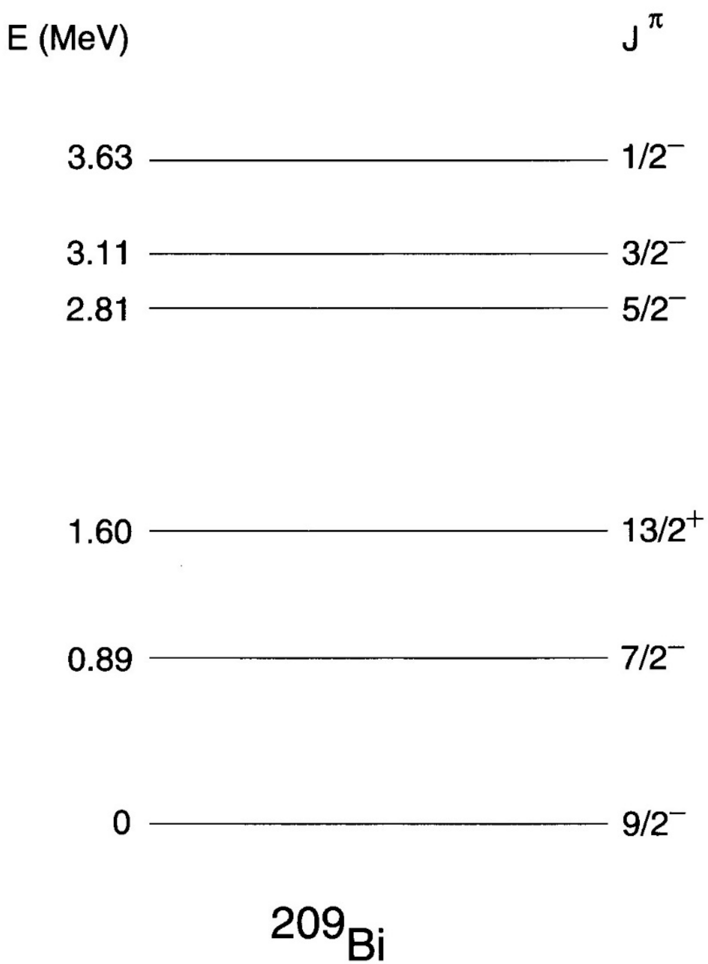
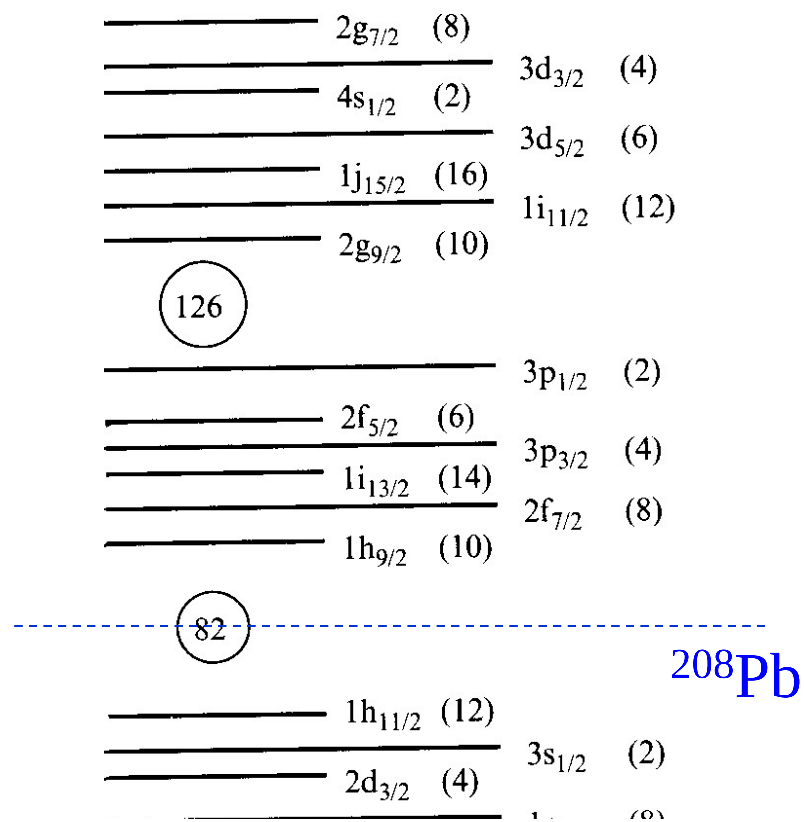



FIG. 3.6. Low-lying single-particle levels of $^{209}$Bi.

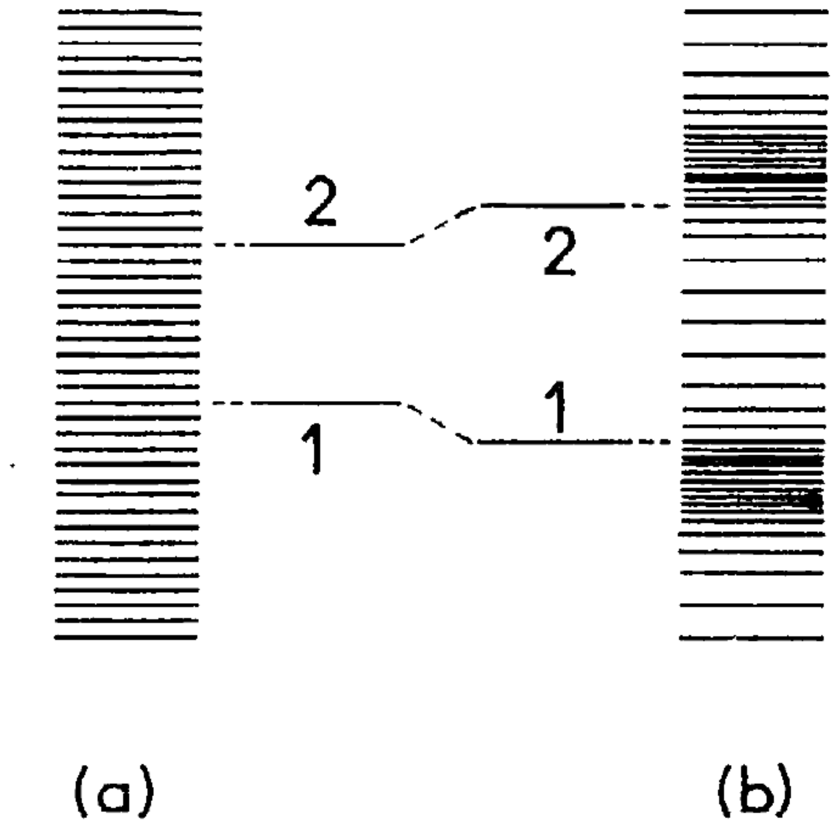



- How to construct V(r) microscopically?
- Does the independent particle picture really hold?

⇒ Later in this lecture

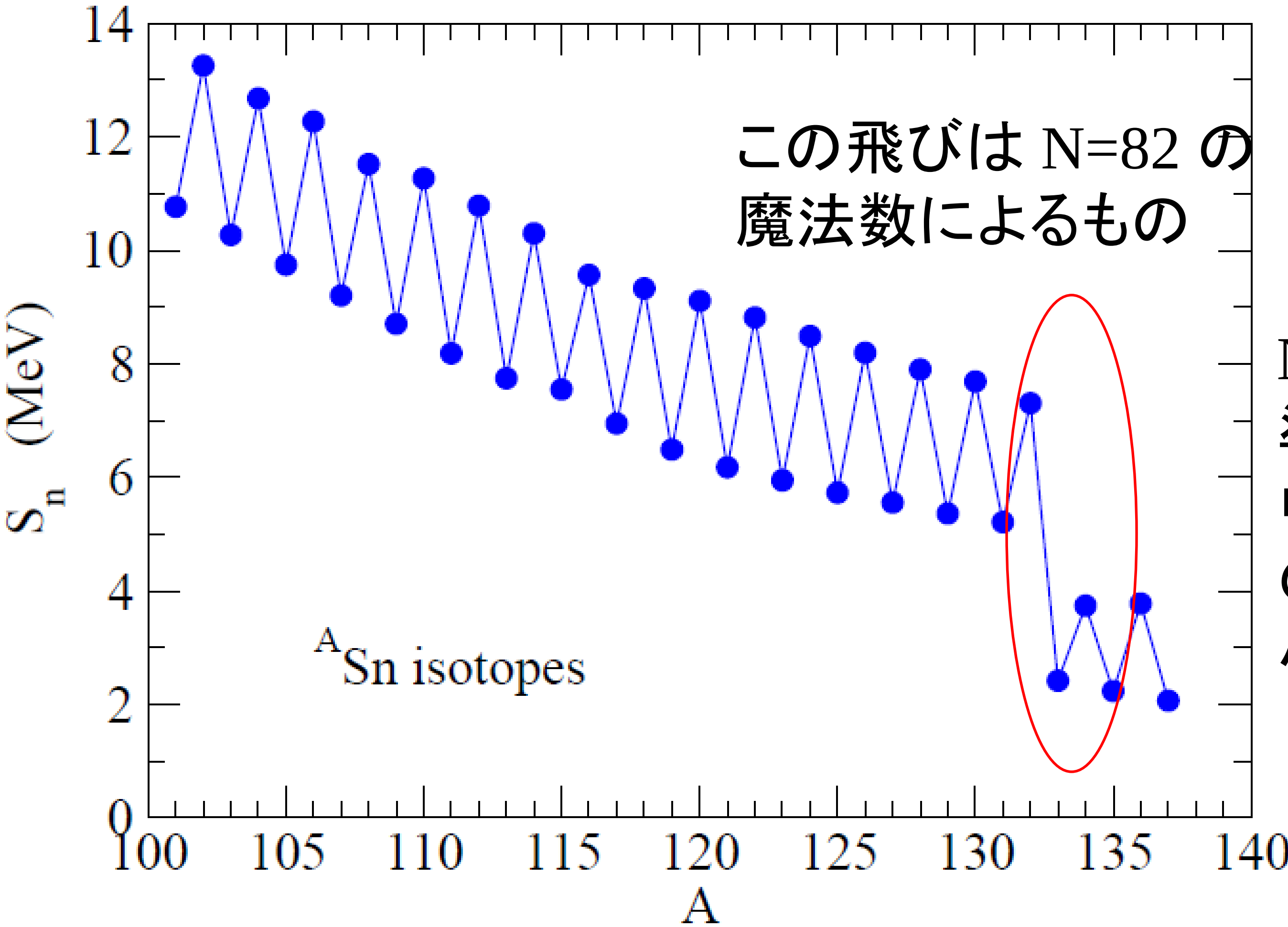
# 何故、閉殻の原子核は安定になるのか?

準位密度




均一の場合　　　濃淡がある場合

準位密度に濃淡があれば、下から数えて濃淡の終わりまで準位がつまると(図の1の場合)、均一の場合に比べてエネルギーが小さい

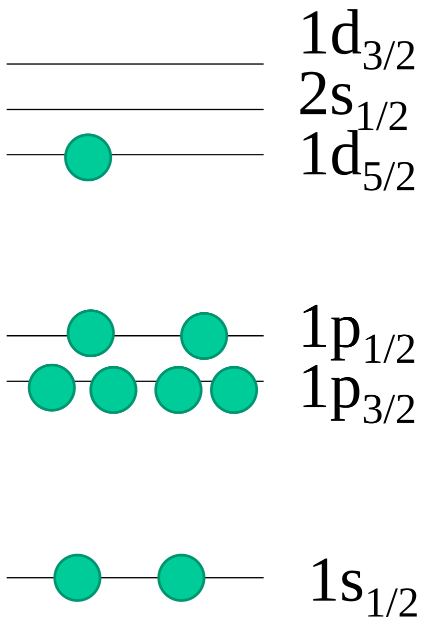



N=83から上の
準位がつまるため
中性子をとりのぞく
のにエネルギーが
小さくてすむ

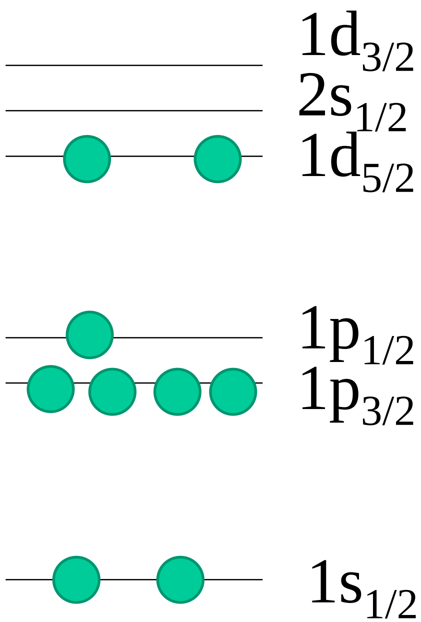
1n separation energy: $S_n$ (A,Z) = B(A,Z) – B(A-1,Z)

# single-j model

shell model




configuration 1




configuration 2

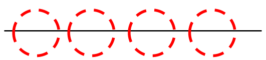
…… several others

angular momentum (spin) and parity for each configuration?

→ let us first investigate a single-j case single-j level: one level with an angular momentum j

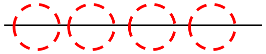
j

example: $j = p_{3/2}$

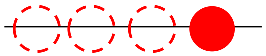


$p_{3/2}$

can accommodate 4 nucleons
($j_z$= +3/2, +1/2, -1/2, -3/2)

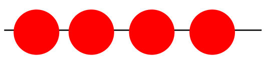
 $p_{3/2}$ can accommodate 4 nucleons ($j_z$= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon

 $p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons

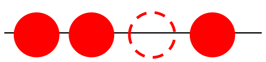
 $p_{3/2}$ → $I^\pi = 0^+$

(there is only 1 way to occupy this level)

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons

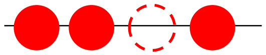
 $p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to make a hole)

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

parity: (-1) x (-1) x (-1) = -1

iii) 3 nucleons



$p_{3/2}$

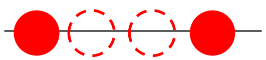
$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

→ $I^{\pi} = 3/2^{-}$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iv) 2 nucleons



$p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2$$

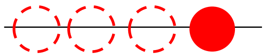
there are 4 x 3/2=6 ways to occupy this level with 2 nucleons.

→ $I^{\pi} = 0^{+}$ or $2^{+}$

3/2 + 3/2 → I = 0, ~~1~~, 2, ~~3~~

anti-symmetrization

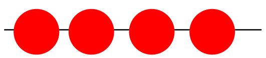
i) 1 nucleon



$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

ii) 4 nucleons

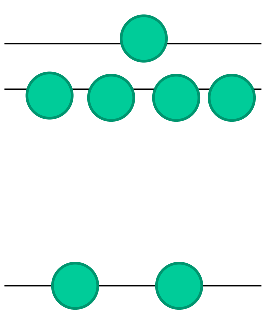


$p_{3/2}$ → $I^{\pi} = 0^{+}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1



$1p_{1/2}$ → $I^{\pi} = 1/2^{-}$

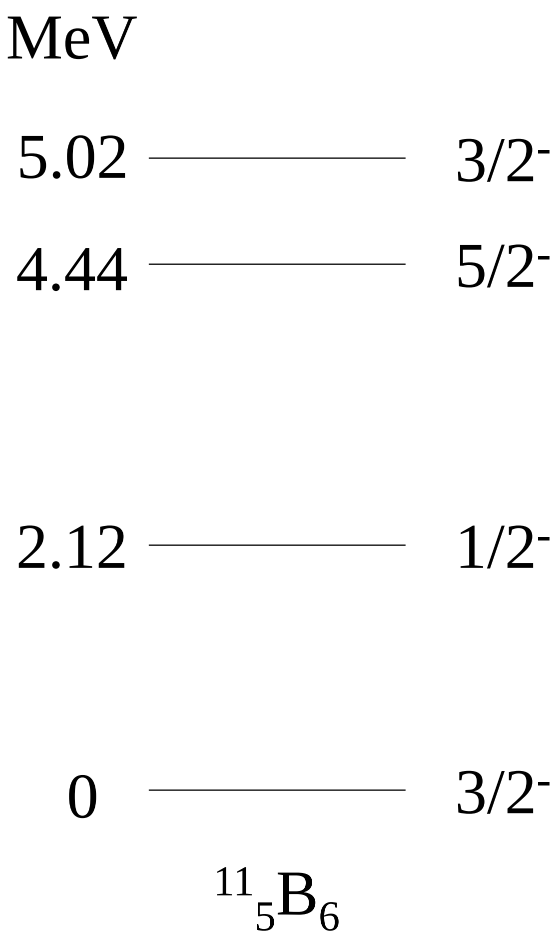
$1p_{3/2}$ → $I^{\pi} = 0^{+}$

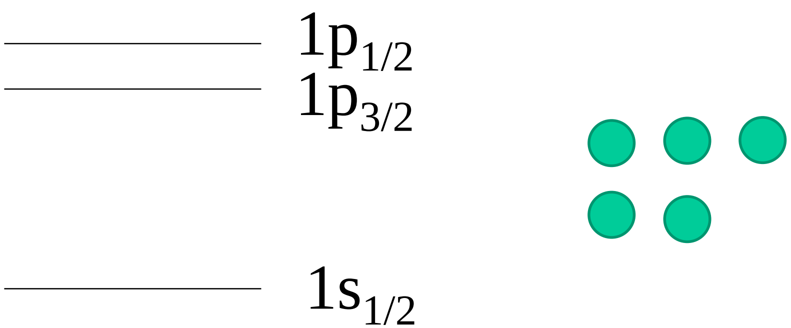
$1s_{1/2}$ → $I^{\pi} = 0^{+}$

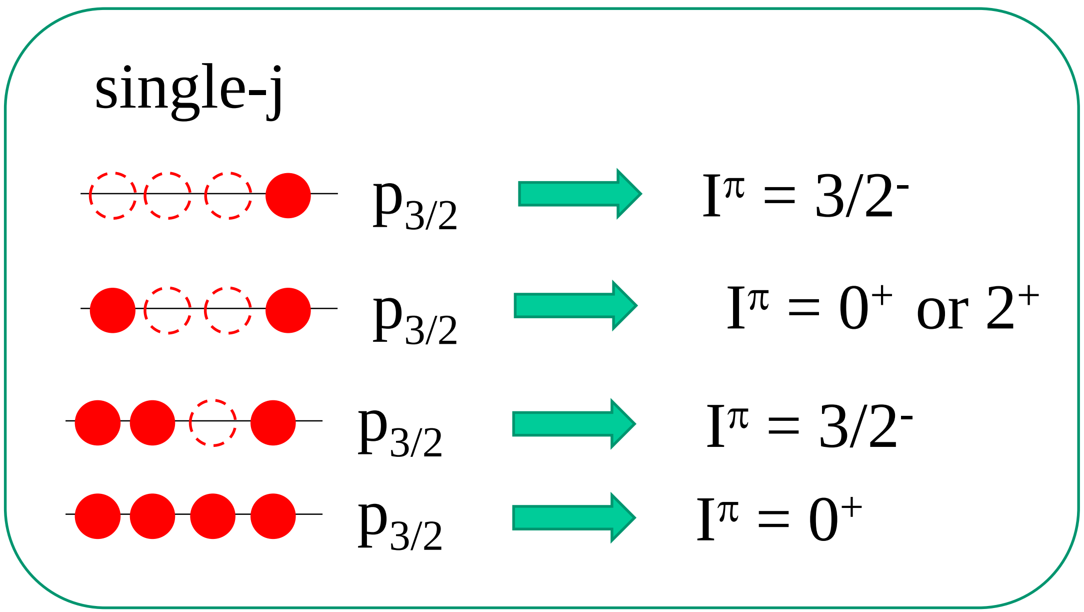
in total,
$I^{\pi} = 1/2^{-}$

example: (main) shell model configurations for $^{11}B$

cf. $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ (= $^{11}B+\Lambda$)



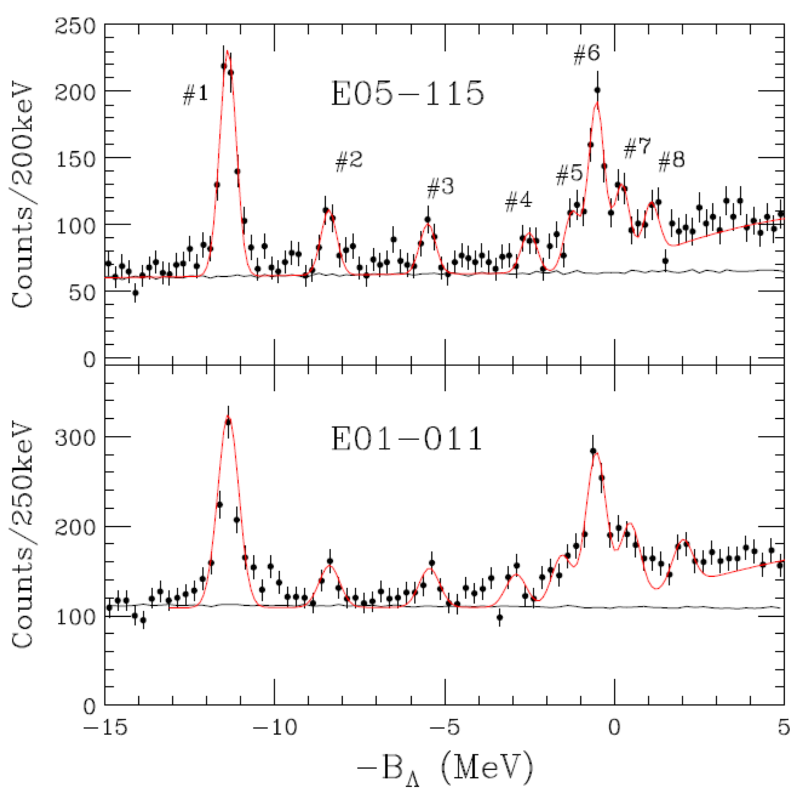



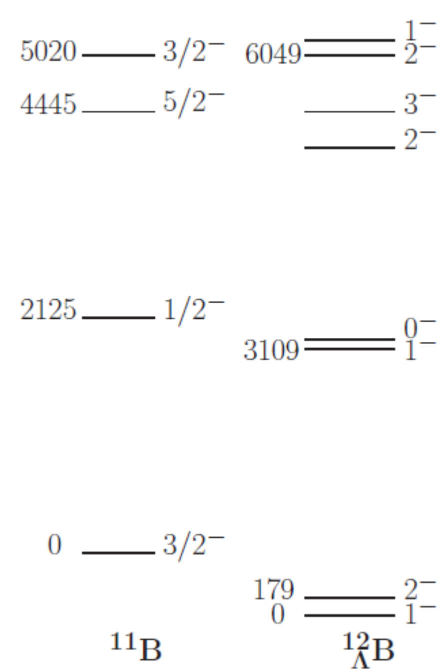
# cf. $^{12}C(e,e'K^+)^{12}{}_{\Lambda}B$ (=$^{11}B+\Lambda$)

PHYSICAL REVIEW C **90**, 034320 (2014)

**Experiments with the High Resolution Kaon Spectrometer at JLab Hall C and the new spectroscopy of $^{12}_{\Lambda}B$ hypernuclei**

L. Tang,[1,2,*] C. Chen,[1] T. Gogami,[3] D. Kawama,[3] Y. Han,[1] L. Yuan,[1] A. Matsumura,[3] Y. Okayasu,[3] T. Seva,[4] V. M. Rodriguez,[5,6] P. Baturin,[7] A. Acha,[7] P. Achenbach,[8] A. Ahmidouch,[9] I. Albayrak,[5] D. Androic,[4] A. Asaturyan,[10] R. Asaturyan,[10,†] O. Ates,[1] R. Badui,[7] O. K. Baker,[1] F. Benmokhtar,[11] W. Boeglin,[7] J. Bono,[7] P. Bosted,[2] E. Brash,[12] P. Carter,[12] R. Carlini,[2] A. Chiba,[3] M. E. Christy,[1] L. Cole,[1] M. M. Dalton,[2,13] S. Danagoulian,[9] A. Daniel,[5] R. De Leo,[14] V. Dharmawardane,[2] D. Doi,[3] K. Egiyan,[10] M. Elaasar,[15] R. Ent,[2] H. Fenker,[2] Y. Fujii,[3] M. Furic,[4] M. Gabrielyan,[7] L. Gan,[16] F. Garibaldi,[17] D. Gaskell,[2] A. Gasparian,[9] E. F. Gibson,[18] P. Gueye,[1] O. Hashimoto,[3,†] D. Honda,[3] T. Horn,[2,11] B. Hu,[19] Ed V. Hungerford,[5] C. Jayalath,[1] M. Jones,[2] K. Johnston,[20] N. Kalantarians,[5] H. Kanda,[3] M. Kaneta,[3] F. Kato,[3] S. Kato,[21]
er,[7] K. J. Lan,[5] … K. Maeda,[3]
owitz,[7] T. Maru … llener,[22]
otoba,[23,24] S. Na … Neville,[7]
ian,[25] H. Nomu … N. Perez,[7]
3. Raue,[7] J. Rei … Y. Sato,[28]
ichijo,[3] N. Simi … na,[3]
Tsukada,[3] V. Tv … s,[20]
[3] C. Yan,[2] Z. Ye … Zhu[1]
15 and E01-011

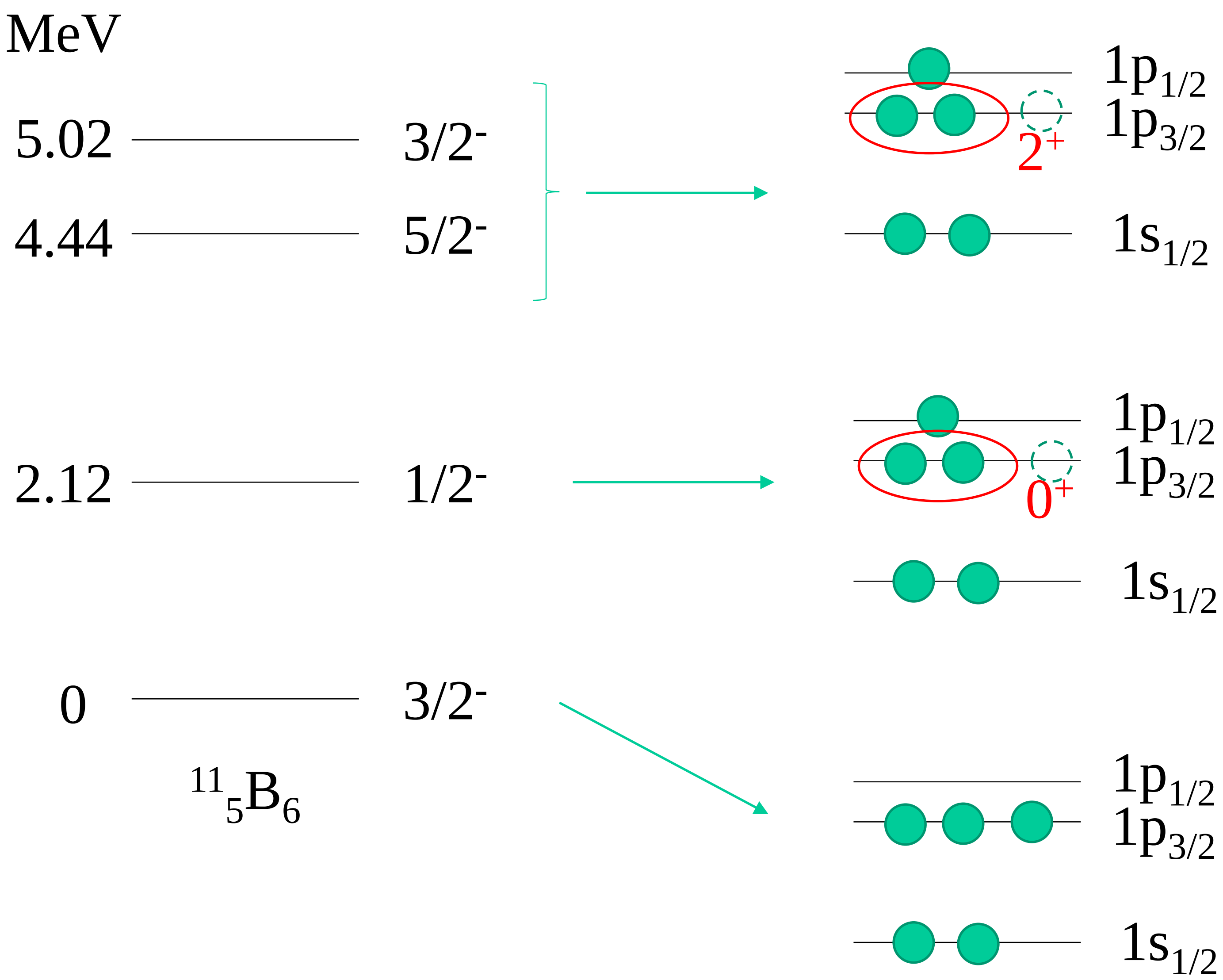


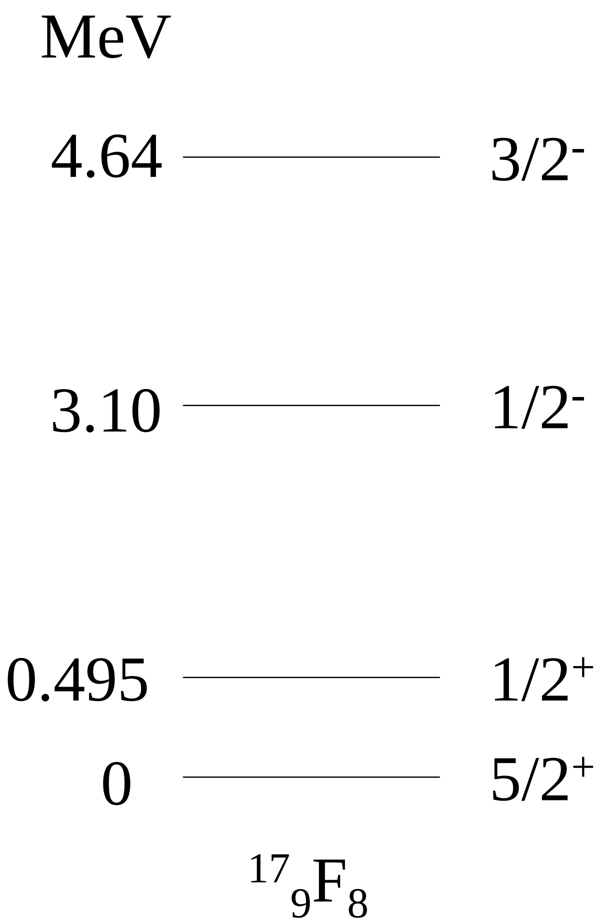

(b)

# example: (main) shell model configurations for $^{11}B$

cf. $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ (=$^{11}B+\Lambda$)



another example: (main) shell model configurations for $^{17}F$



another example: (main) shell model configurations for $^{17}F$