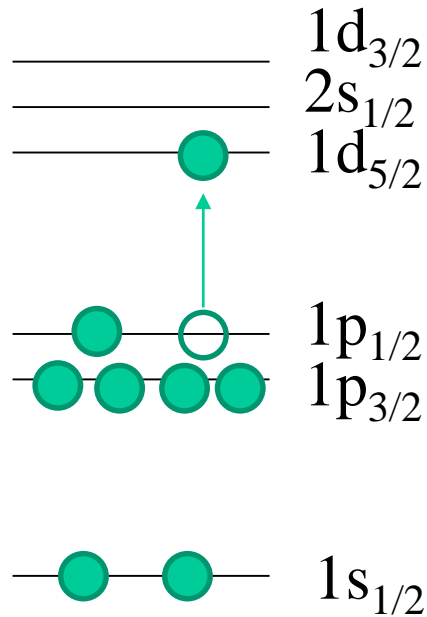


原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に参与)
- ✓ 集団励起(多くの核子が集団として励起に参与)



一粒子励起の例

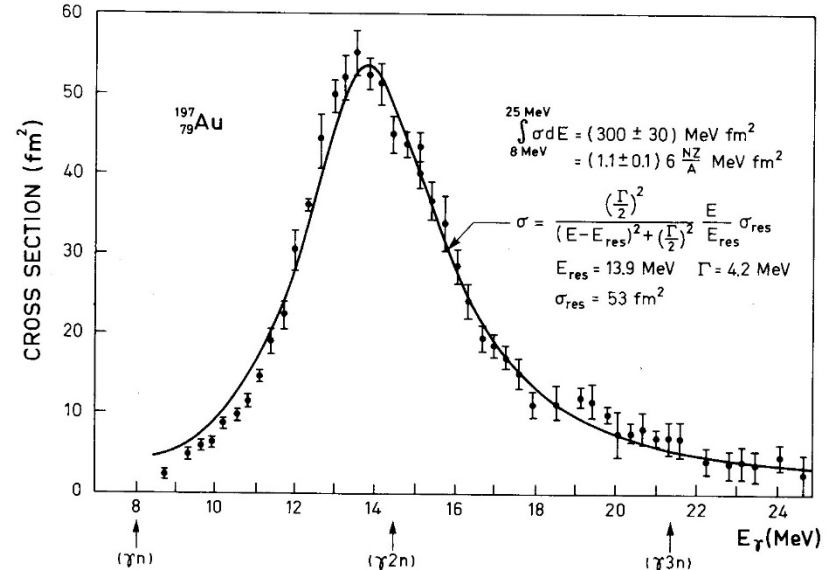
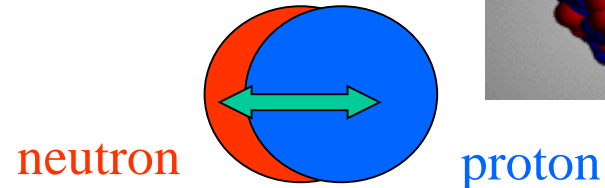


Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



集団励起の例: 巨大双極子共鳴

原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に参与)
- ✓ 集団励起(多くの核子が集団として励起に参与)

集団励起を微視的に理解
 してみる
 (集団励起を細かく見て
 みるとどうなっているのか?)

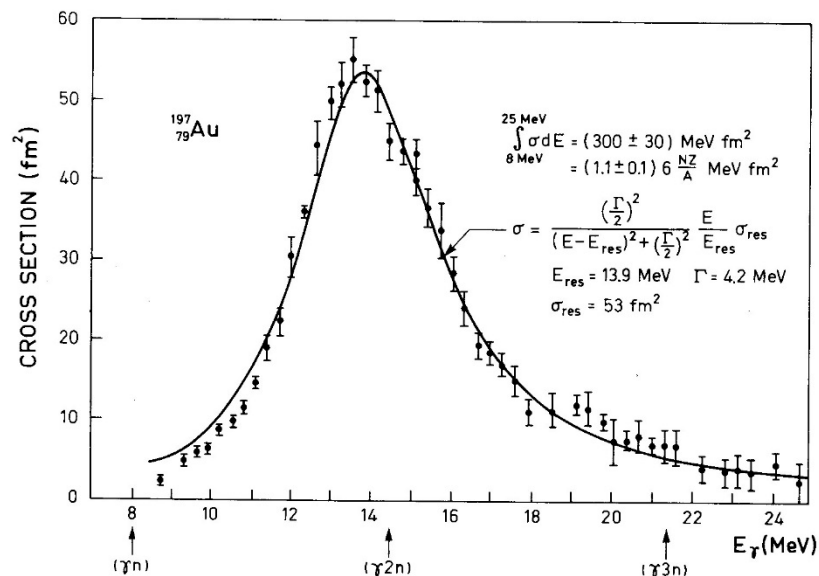
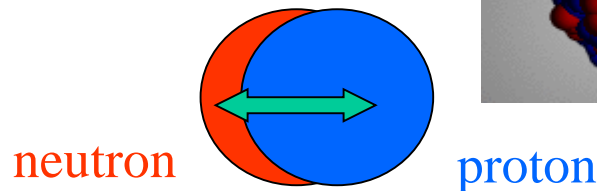


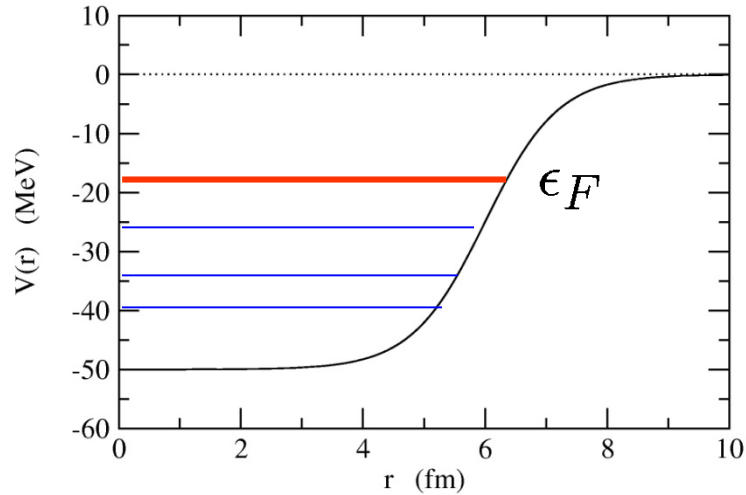
Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



集団励起の例: 巨大双極子共鳴

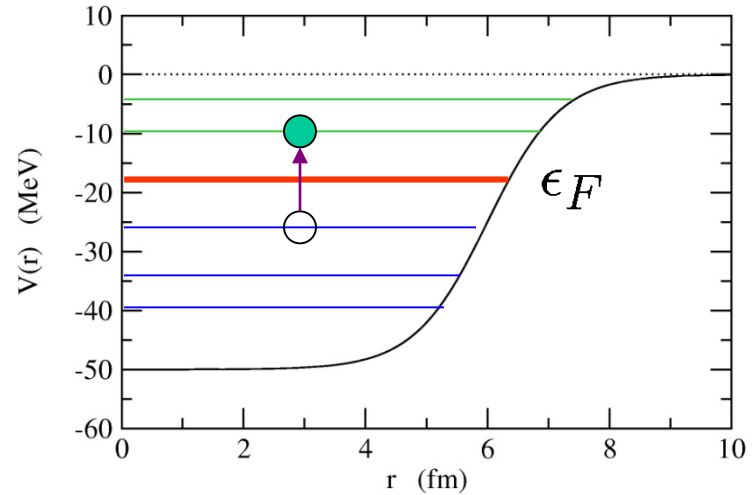
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle$$

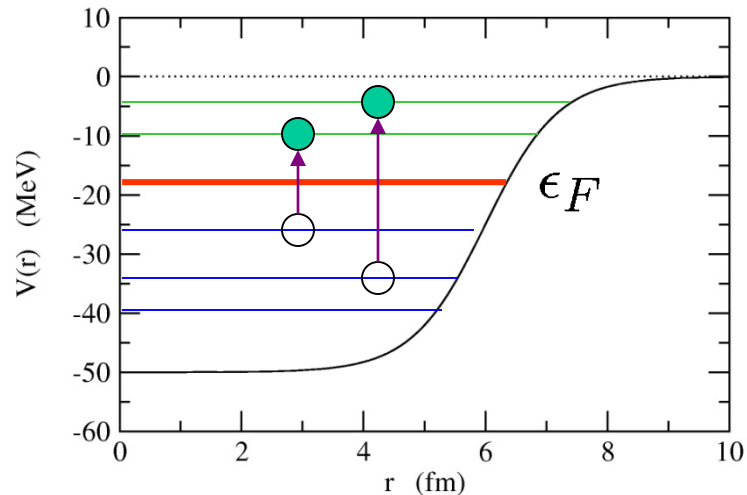
1 particle-1 hole (1p1h) state



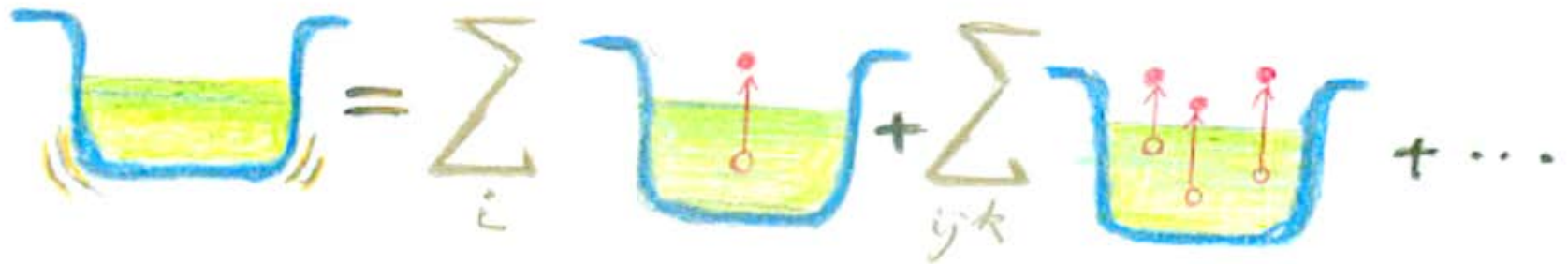
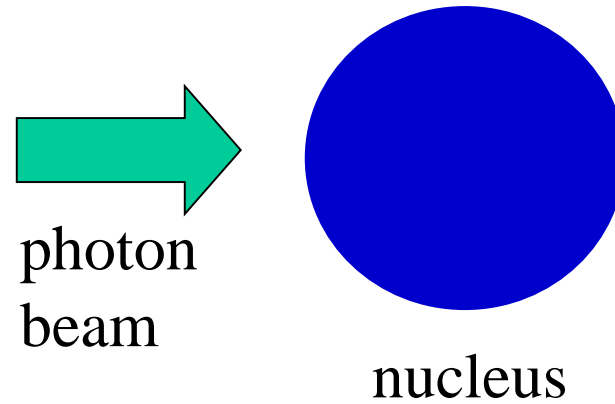
$$a_p^\dagger a_h |HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$



原子核を外場により揺らしてみると何が起こるのか？



スライド：松柳研一氏

Tamm-Dancoff Approximation

$$\begin{aligned}\text{Assume: } |\nu\rangle = Q_\nu^\dagger |HF\rangle &= \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \\ &= \sum_{ph} X_{ph} |ph^{-1}\rangle\end{aligned}$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

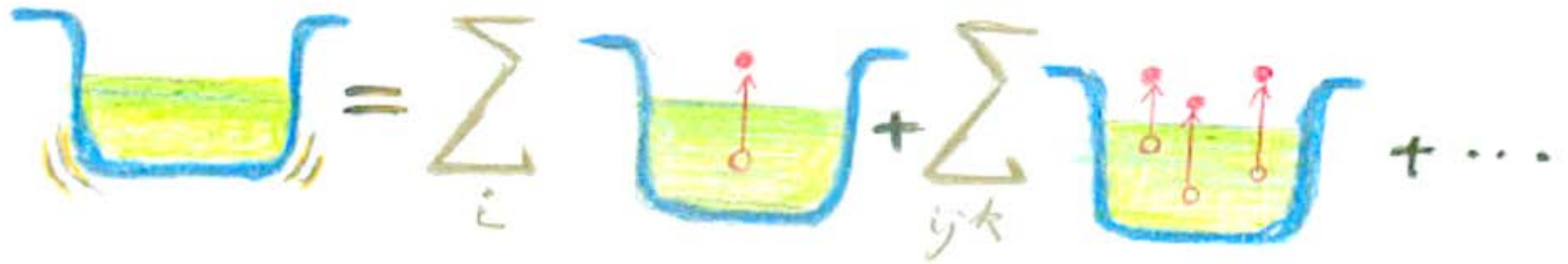


$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

residual
interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph^{-1} | \bar{v} | hp'^{-1} \rangle$$

Tamm-Dancoff equation



スライド: 松柳研一氏

$$V(\mathbf{r}) \sim \int d\mathbf{r}' v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}')$$

vibration: $\rho = \rho_0(\mathbf{r}) \rightarrow \rho_0(\mathbf{r}) + \delta\rho(\mathbf{r}, t)$

↓


residual
interaction


TDA on a schematic model


Separable interaction: $\langle ph^{-1} | \bar{v} | hp'^{-1} \rangle = \langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$

Tamm-Dancoff equation: $\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$


$$(\epsilon_{ph} - E) X_{ph} + \lambda D_{ph} \cdot T = 0 \quad T \equiv \sum_{ph} D_{ph}^* X_{ph}$$


$$X_{ph} = -\lambda \frac{D_{ph} T}{\epsilon_{ph} - E}$$


$$T = \sum_{ph} D_{ph}^* X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$


(TDA dispersion relation)

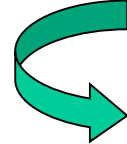
(separable interaction)


$$H\psi = E\psi; \quad \psi = \sum_i C_i \phi_i$$


$$\longrightarrow \sum_j H_{ij} C_j = E C_i; \quad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j$ (separable form)


$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$


$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$


$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T$$


$$\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}$$

Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

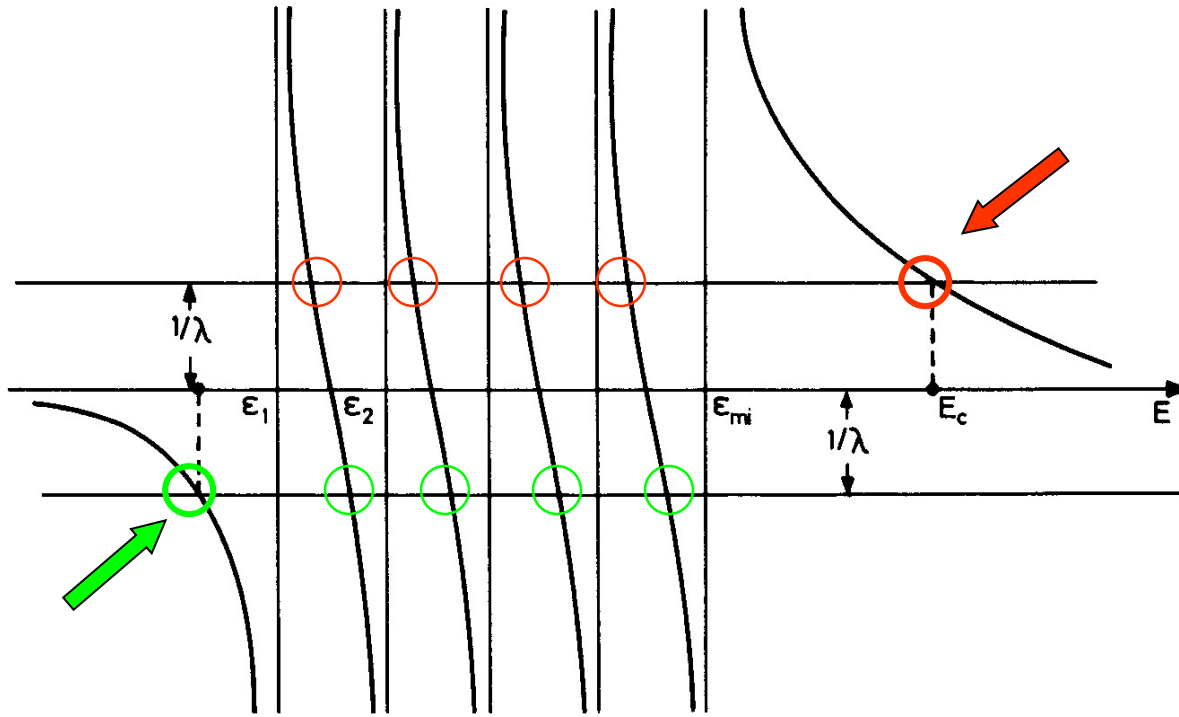


Figure 8.4. Graphical solution of Eq. (8.18).

(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon + \lambda \sum_{ph} |D_{ph}|^2, \quad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superposition of 1p1h states

Giant Dipole Resonance (GDR) 巨大双極子共鳴

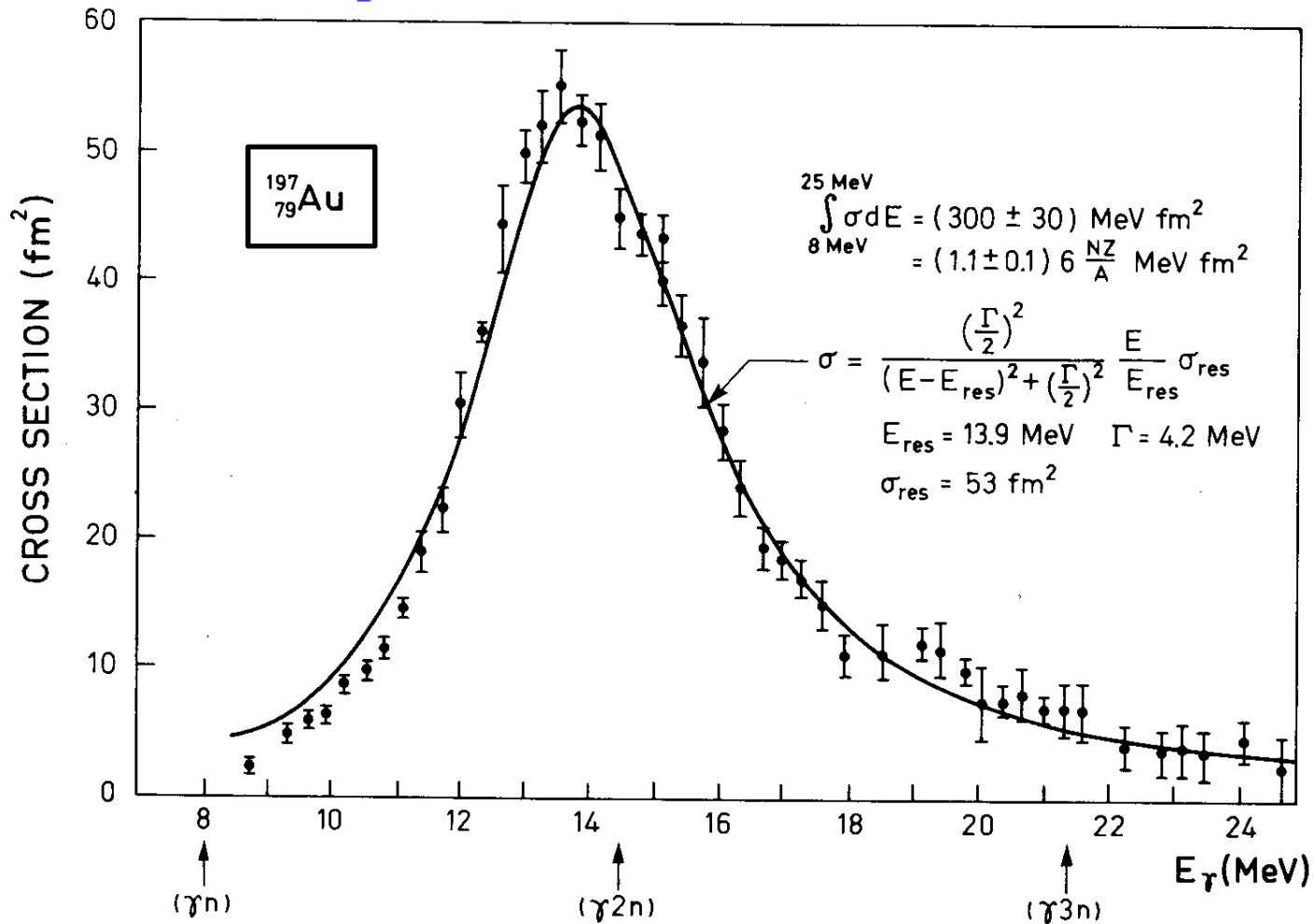


Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

$$\text{cf. } 41 \times 197^{-1/3} = 7.05 \text{ MeV}$$

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

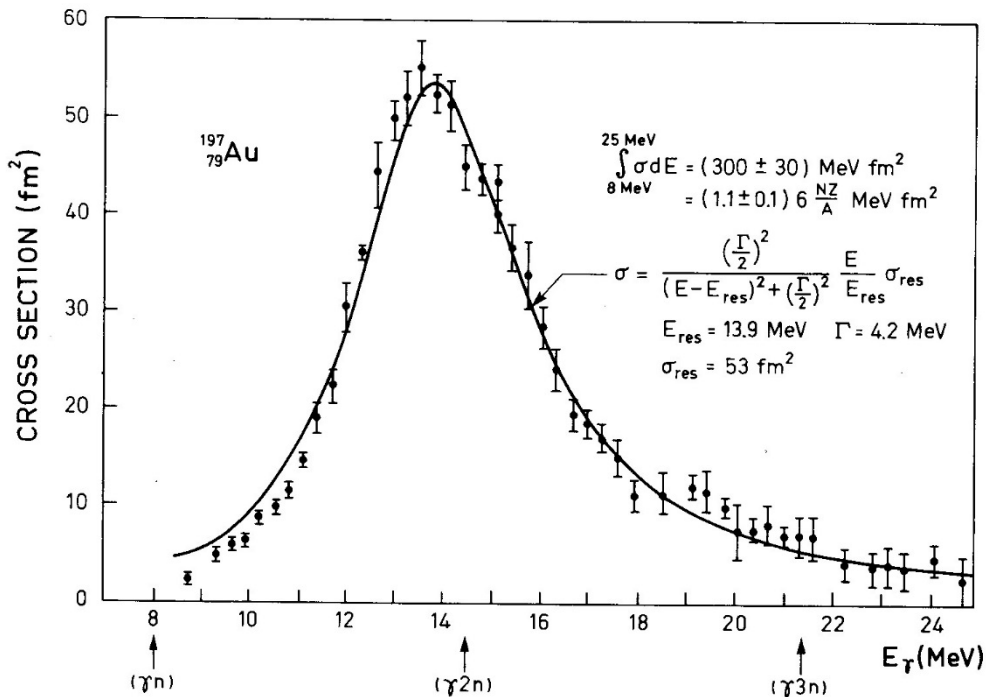
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 41 A^{-1/3}$

IS GQR: $E \sim 65 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41 A^{-1/3}$ (MeV)



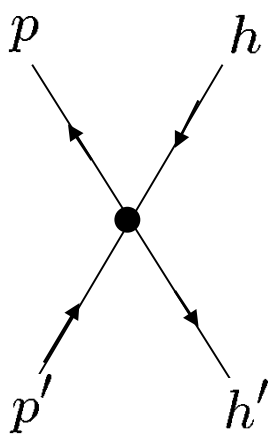
¹⁹⁷Au

$$E_{\text{GDR}} = 14 \text{ (MeV)}$$

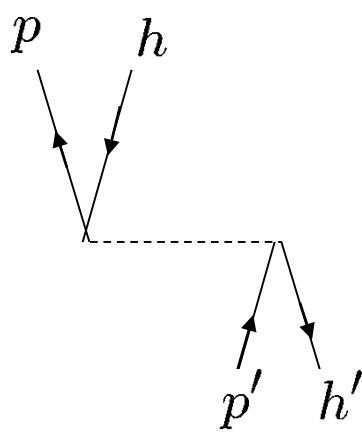
$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

$$\sim 7 \text{ (MeV)}$$

$$\langle ph^{-1} | \bar{v} | p'h'^{-1} \rangle = \langle ph' | \bar{v} | hp' \rangle = \langle ph' | v | hp' \rangle - \langle ph' | v | p'h \rangle$$

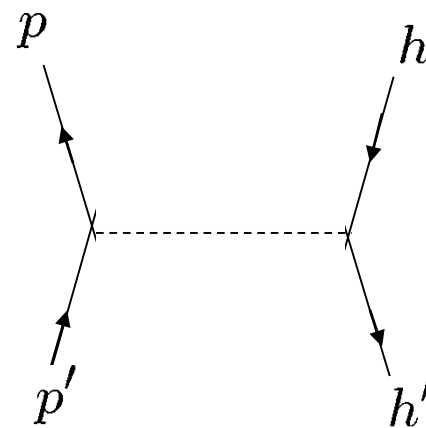


=



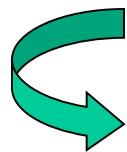
Direct term

-



Exchange term

$$\left\{ \begin{array}{l} \langle PP^{-1} | \bar{v} | PP^{-1} \rangle \sim \langle NN^{-1} | \bar{v} | NN^{-1} \rangle = D - E \\ \langle PP^{-1} | \bar{v} | NN^{-1} \rangle = D \quad (\text{no charge exchange}) \end{array} \right.$$



$$\langle IS | \bar{v} | IS \rangle = 2D - E \sim D$$

$$\langle IV | \bar{v} | IV \rangle = -E \sim -D$$

$$|IS\rangle \propto |NN^{-1}\rangle + |PP^{-1}\rangle$$

$$|IV\rangle \propto |NN^{-1}\rangle - |PP^{-1}\rangle$$

Random Phase Approximation

Tamm-Dancoff Approximation: $|\nu\rangle = Q_\nu^\dagger |HF\rangle = \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \approx E_\nu Q_\nu^\dagger$$

$$\iff \langle HF | [\delta Q, [H, Q_\nu^\dagger]] | HF \rangle = E_\nu \langle HF | [\delta Q, Q_\nu^\dagger] | HF \rangle$$

Drawbacks:

➤ No influence of ν in the ground state

$$E_{coll} = \epsilon + \lambda \sum_{ph} |D_{ph}|^2 \quad \longleftarrow \text{Interaction is essential in describing collective excitations}$$

➤ Energy Weighted Sum Rule is violated in TDA

➤ Admixture of the spurious modes with the physical excitation modes

HF \longleftrightarrow Broken Symmetries (CM localization, rotation,.....)

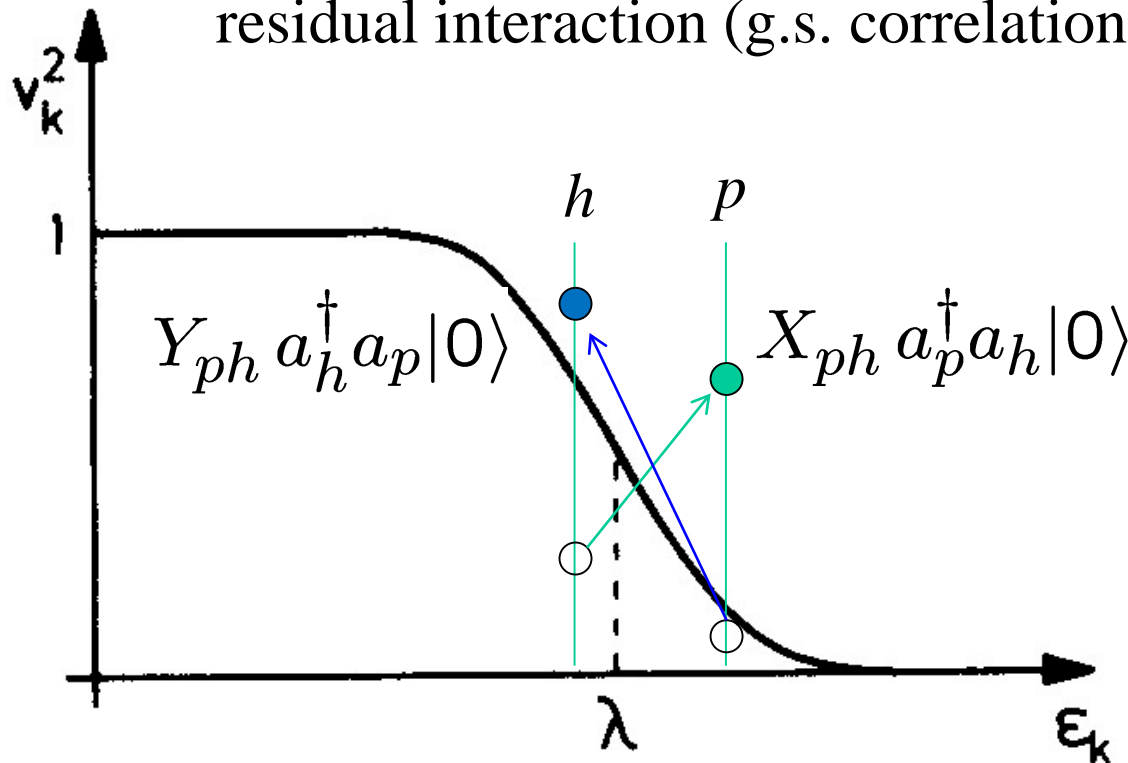
Restoration of broken symmetries \longrightarrow Goldstone mode
(spurious motion)

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left(X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



RPA on a schematic model

Separable interaction: $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}} \quad (\text{RPA dispersion relation})$$

Cf. TDA dispersion relation: $\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$

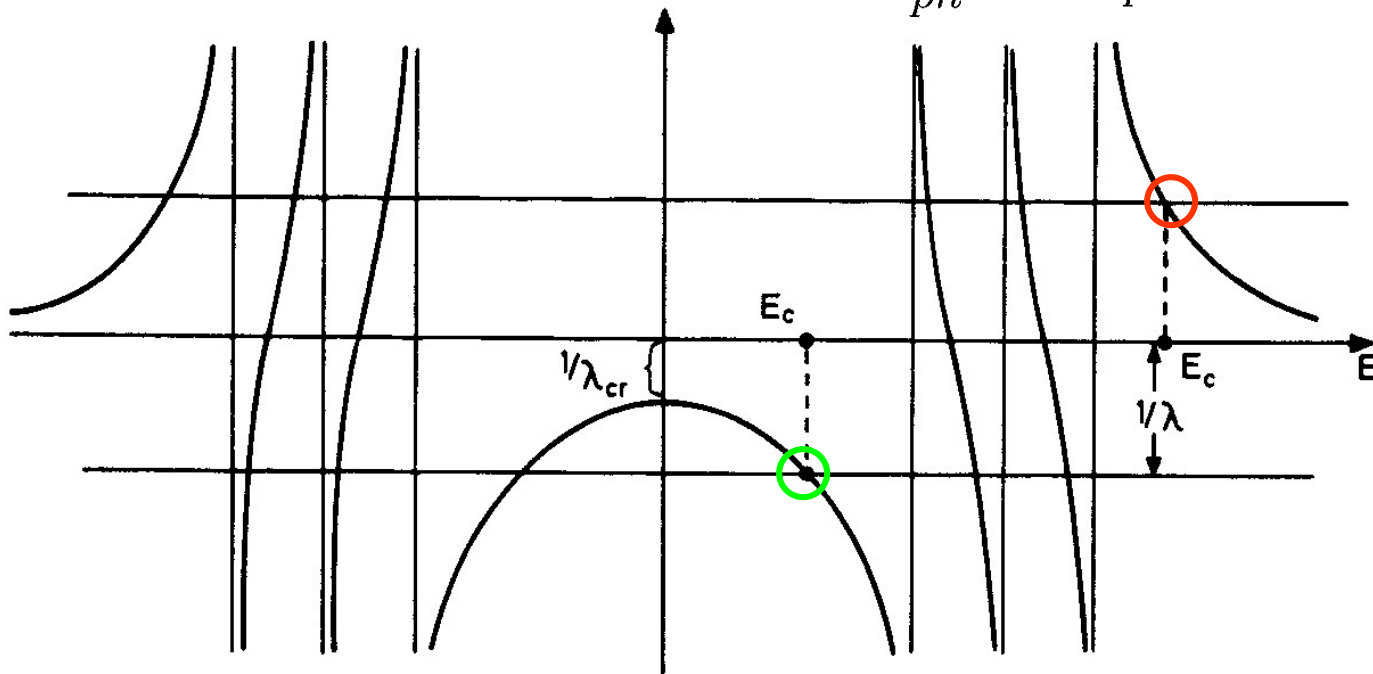


Figure 8.11. Graphical solution of the dispersion relation (8.135).

Spurious motion in RPA

Mean-Field Approximation \longleftrightarrow Broken symmetries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

\longrightarrow Zero mode (Nambu-Goldstone mode)

RPA $\langle HF | [\delta Q, [H, Q_\nu^\dagger]] | HF \rangle = E_\nu \langle HF | [\delta Q, Q_\nu^\dagger] | HF \rangle$

\curvearrowright if $[H, \hat{O}] = 0$

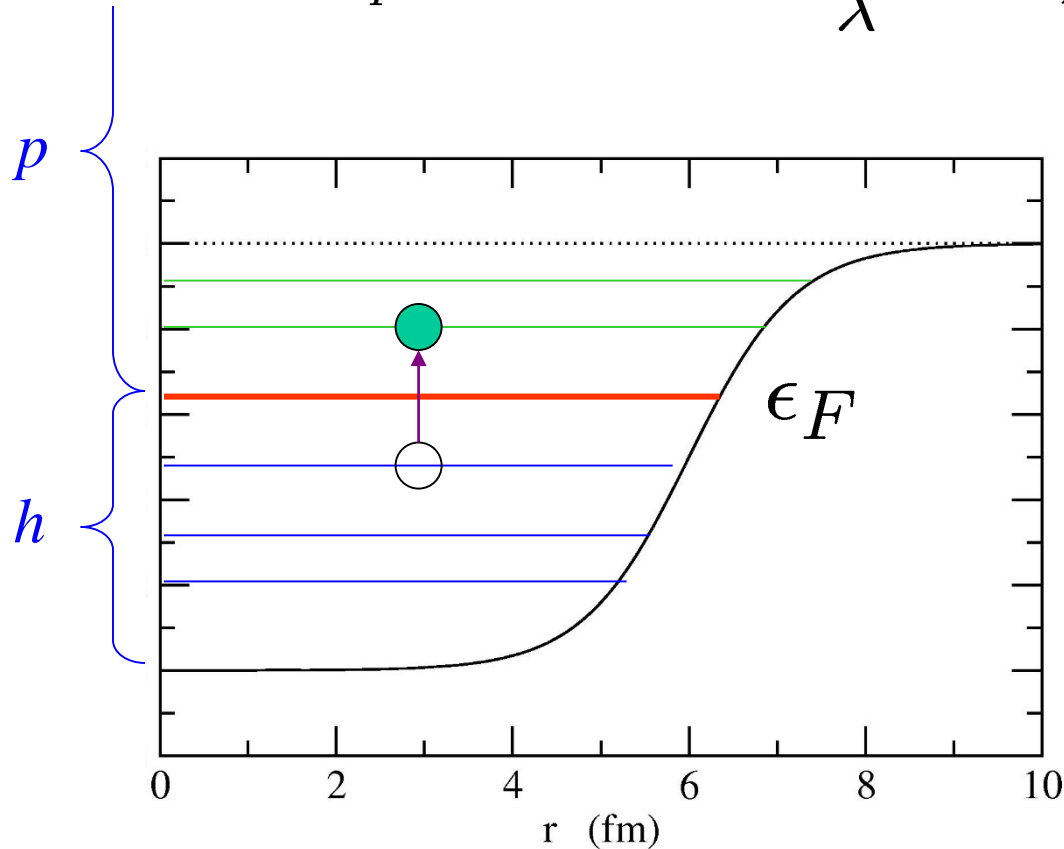
Then \hat{O} is a solution of RPA with $E=0$

$$\hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

\curvearrowright The physical solutions are exactly separated out from the spurious modes.

Continuum Excitations

$$\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^* \longrightarrow \frac{1}{\lambda} = - \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$



h : all the occupied (bound) states

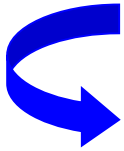
p : the bound excited states + continuum states

$$\frac{1}{\lambda} = - \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E} = - \sum_{ph} \underbrace{\langle \phi_h | D | \phi_p \rangle \frac{1}{\epsilon_p - \epsilon_h - E} \langle \phi_p | D | \phi_h \rangle}_{\frac{1}{\hat{h} - \epsilon_h - E}} = \frac{1}{\hat{h} - \epsilon_h - E} |\phi_p\rangle \langle \phi_p|$$

(note)

$$\hat{h}\phi_p = \epsilon_p\phi_p$$

$$1 = \sum_i |\phi_i\rangle \langle \phi_i| = \sum_p |\phi_p\rangle \langle \phi_p| + \sum_h |\phi_h\rangle \langle \phi_h|$$



$$\sum_p |\phi_p\rangle \langle \phi_p| = 1 - \sum_h |\phi_h\rangle \langle \phi_h|$$

$$\frac{1}{\lambda} = - \sum_h \langle \phi_h | D \frac{1}{\hat{h} - \epsilon_h - E} D | \phi_h \rangle + \sum_{h,h'} \langle \phi_h | D | \phi_{h'} \rangle \frac{1}{\epsilon_{h'} - \epsilon_h - E} \langle \phi_{h'} | D | \phi_h \rangle$$

Comparison between Skyrme-(Q)RPA calculation and exp. data

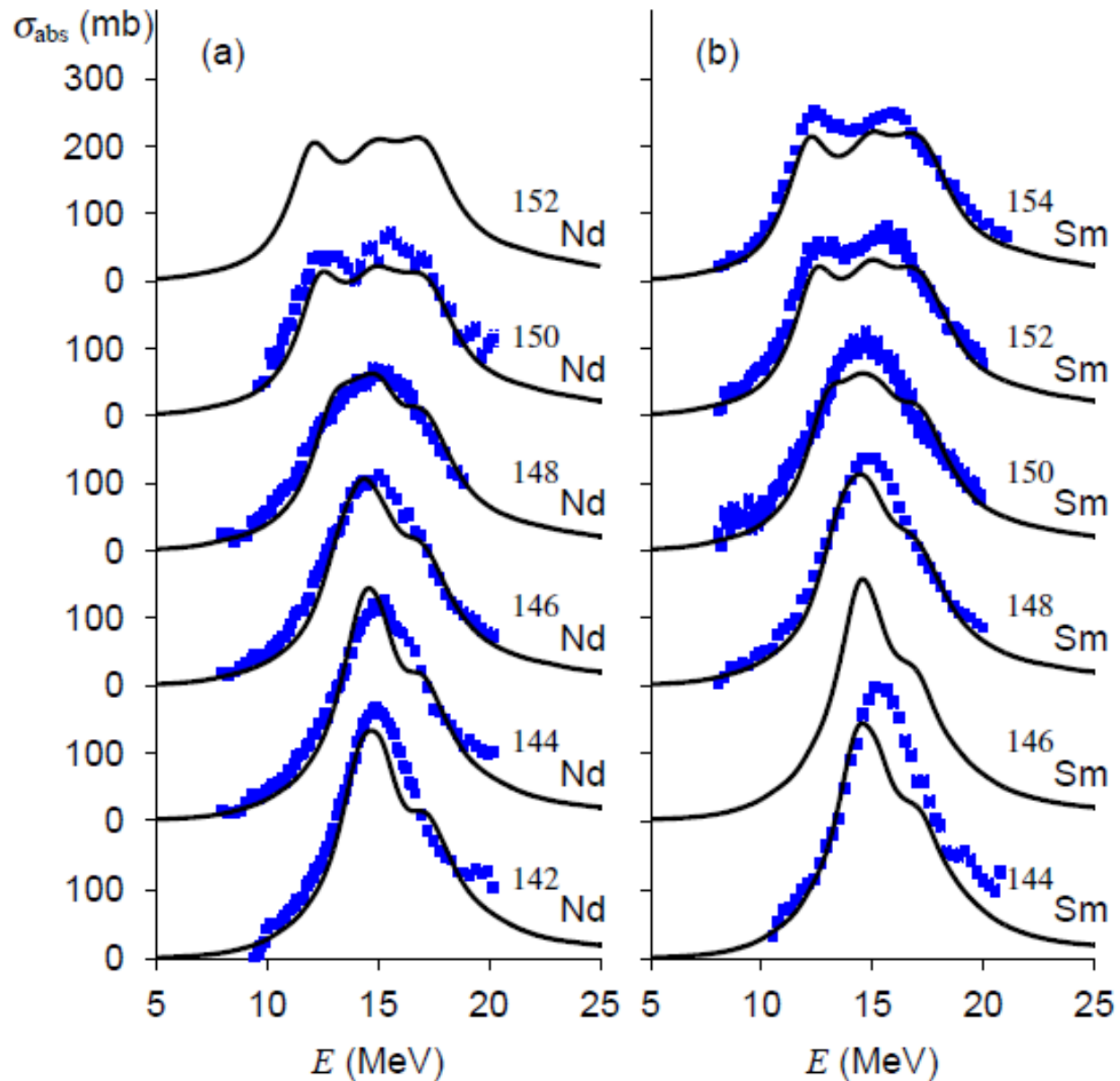
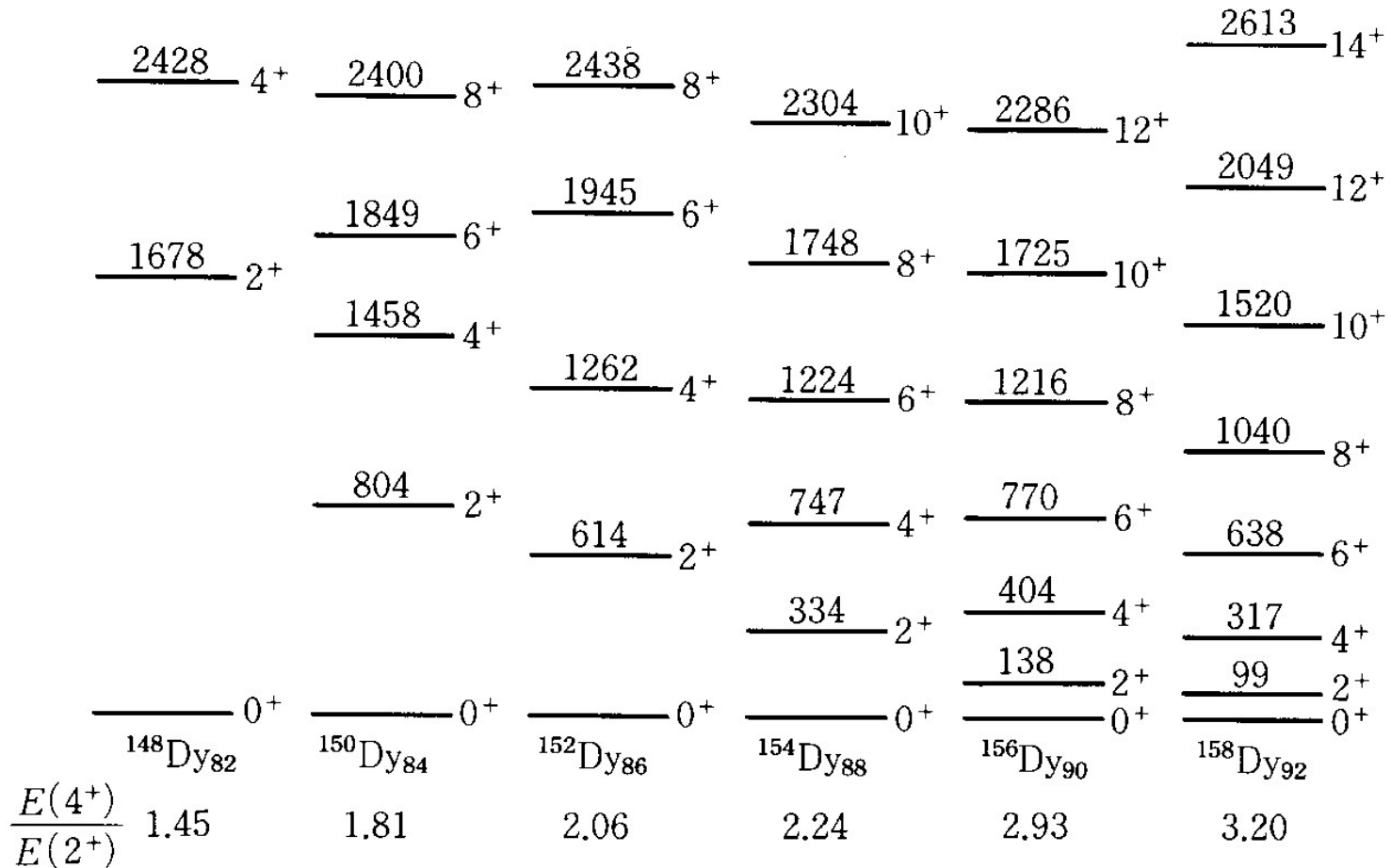


photo-absorption
cross section
(GDR)

K. Yoshida
and T. Nakatsukasa,
PRC83('11)021304

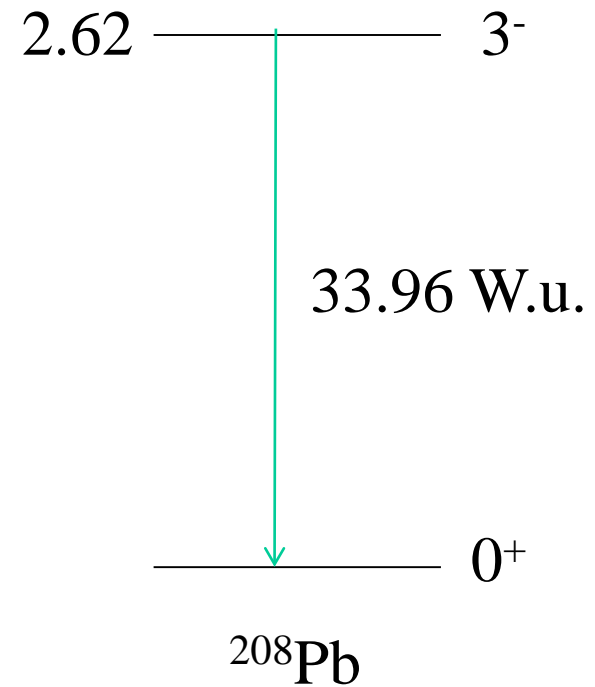
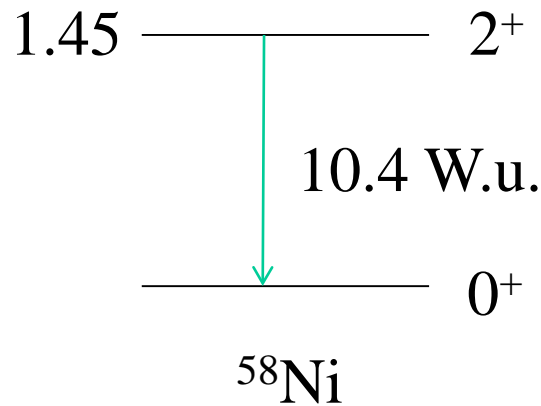
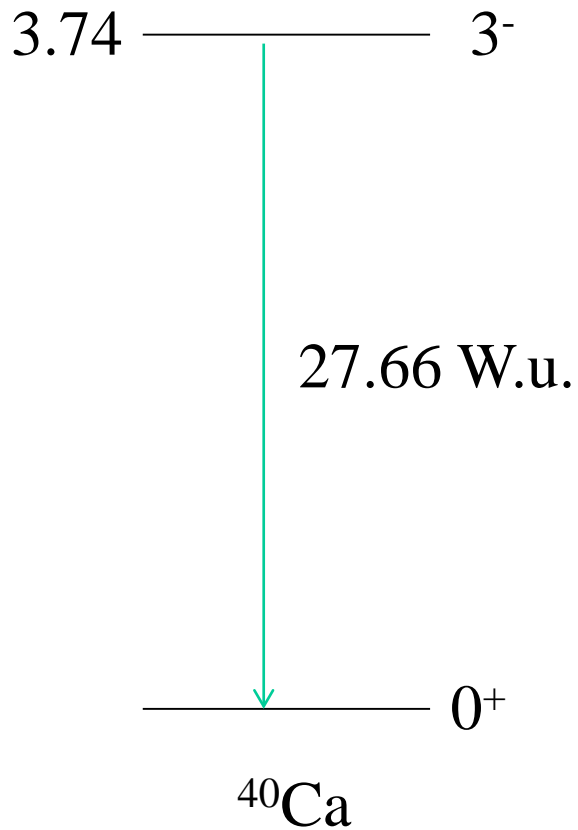
Low-lying collective states

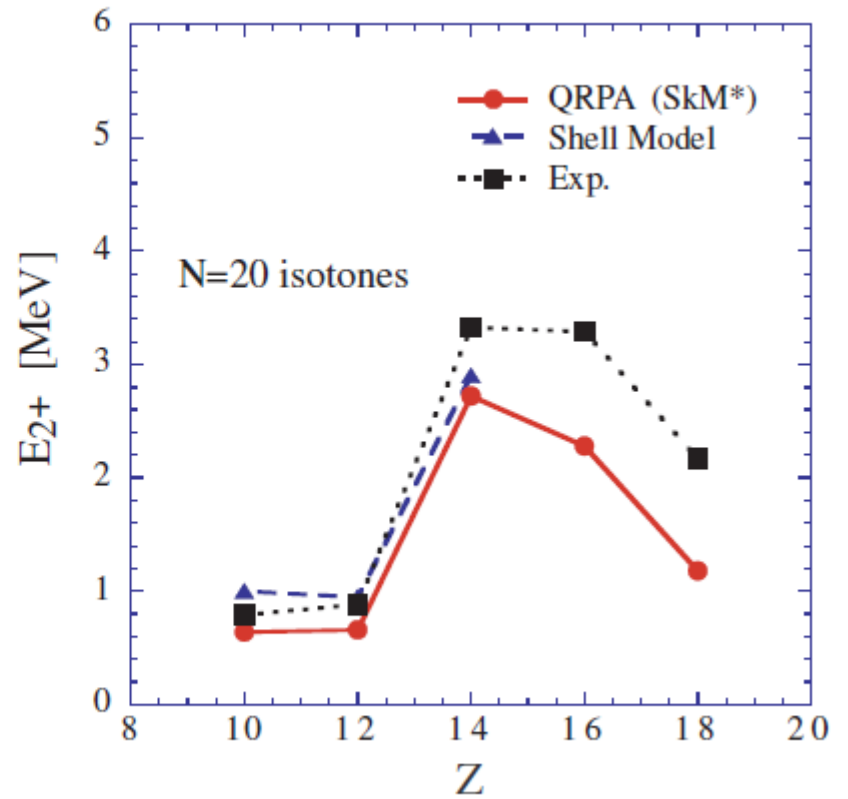
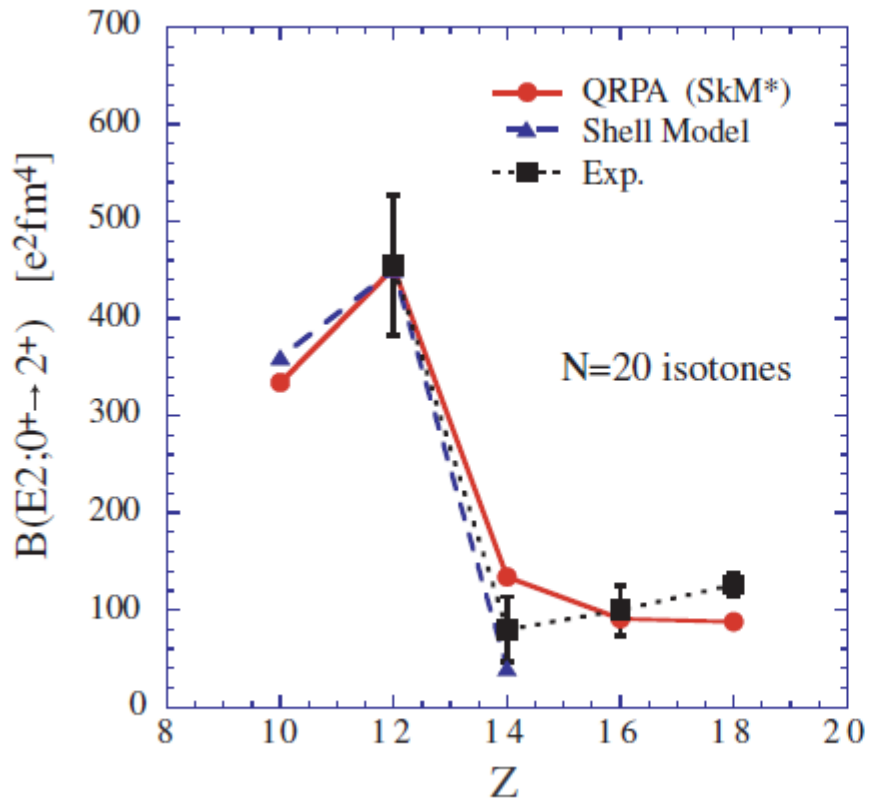
Low-lying excited states in even-even nuclei are collective excitations, and strongly reflect the pairing correlation and shell structure



Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \rightarrow I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi} \left(\frac{3}{\lambda + 3} \right)^2 (e^2 \text{fm}^{2\lambda})$$





M. Yamagami and Nguyen Van Giai, PRC69 ('04) 034301