Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

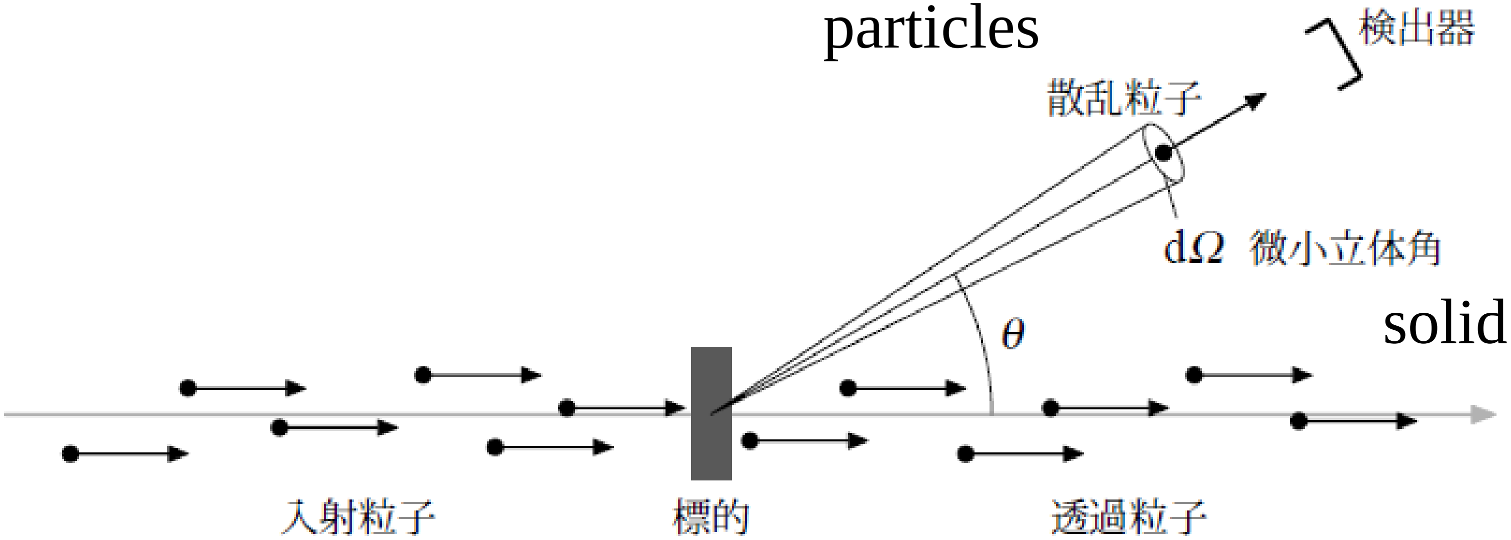


図 21.1: 散乱実験

scattered particles

detector

solid angle

projectile

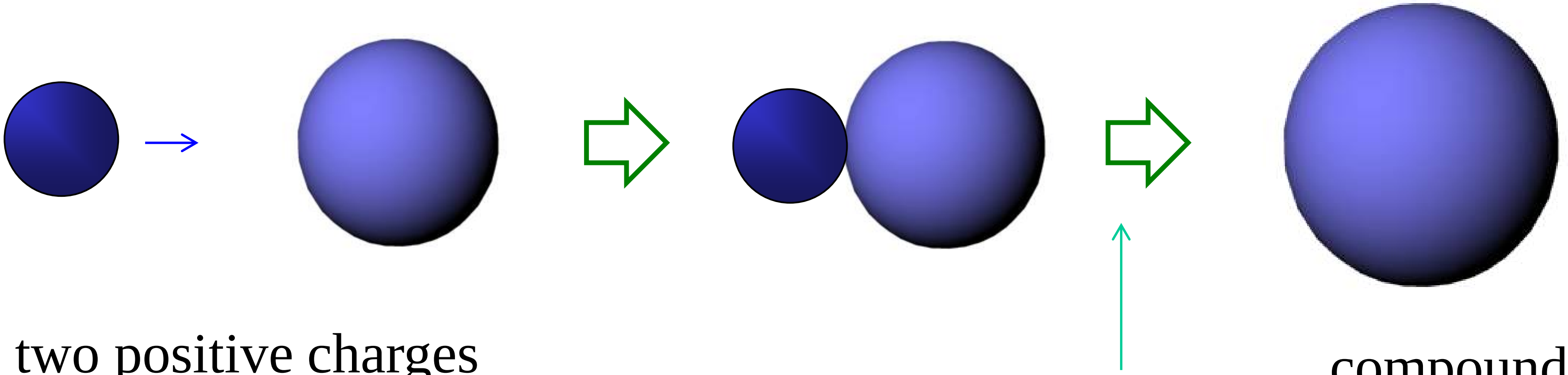
target

transmitted particles

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

K. Muto (TIT)

Nuclear fusion reactions



two positive charges
repel each other

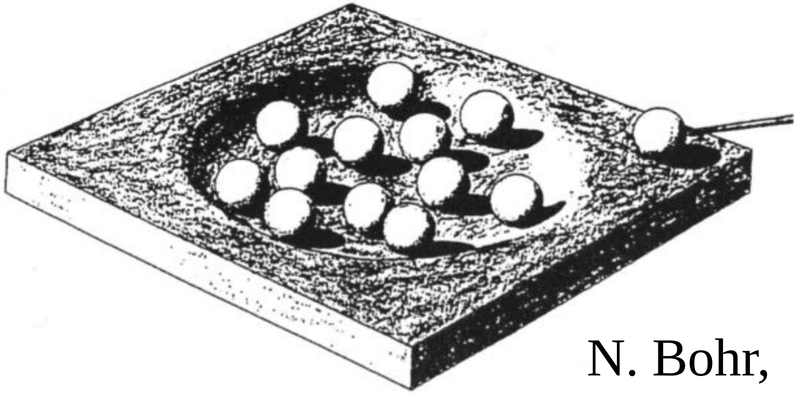
nuclear *attractive*
intraction

compound
nucleus

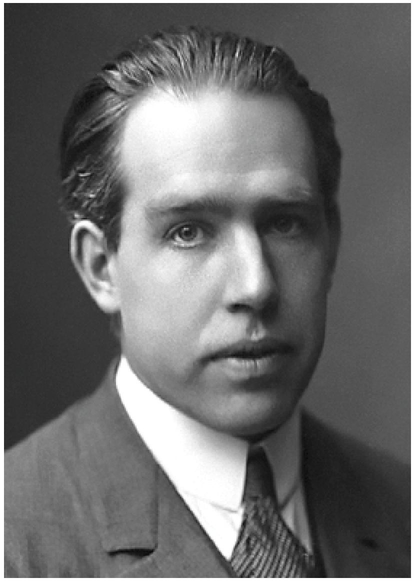
Fusion reactions: compound nucleus formation

Niels Bohr (1936)

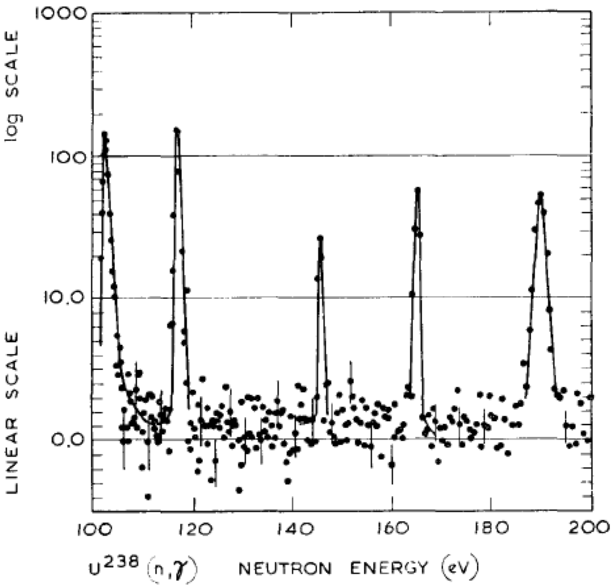
Neutron capture of nuclei → compound nucleus



N. Bohr,
Nature 137 (‘36) 351



Wikipedia



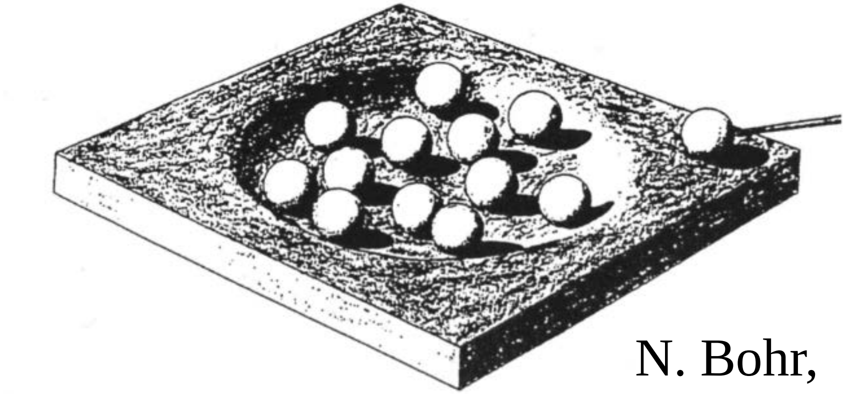
cf. Experiment of Enrico Fermi (1935)
many very narrow (=long life-time)
resonances (width ~ eV)

M. Asghar et al., Nucl. Phys. 85 (‘66) 305

Fusion reactions: compound nucleus formation

Niels Bohr (1936)

Neutron capture of nuclei → compound nucleus



N. Bohr,
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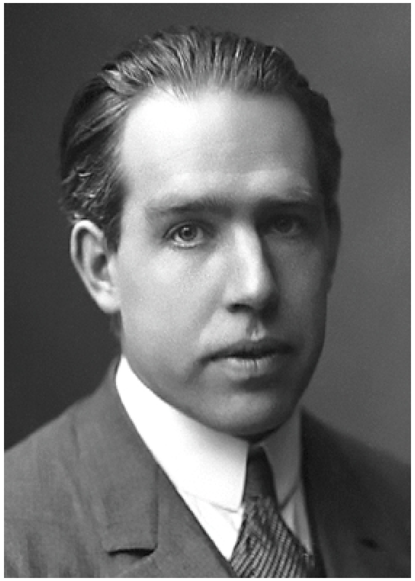
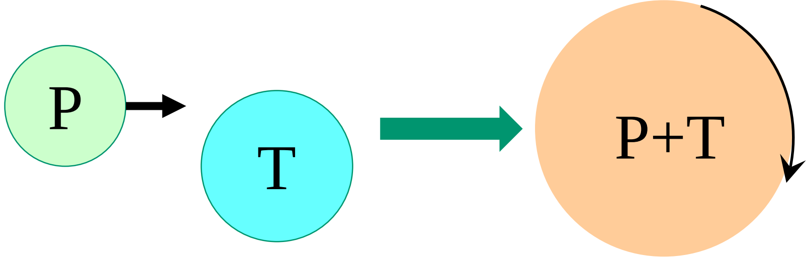


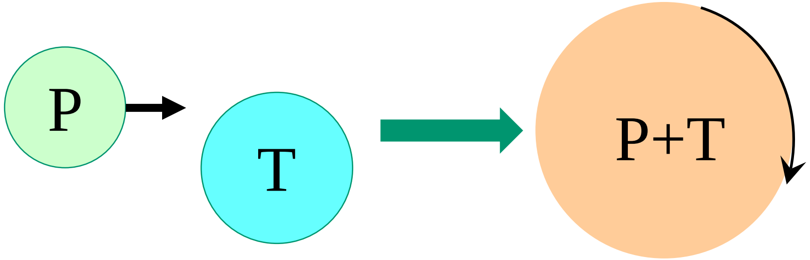
Wikipedia

forming a compound nucleus with heavy-ion reactions = H.I. fusion

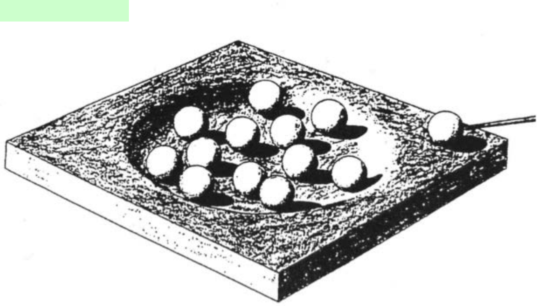


compound nucleus

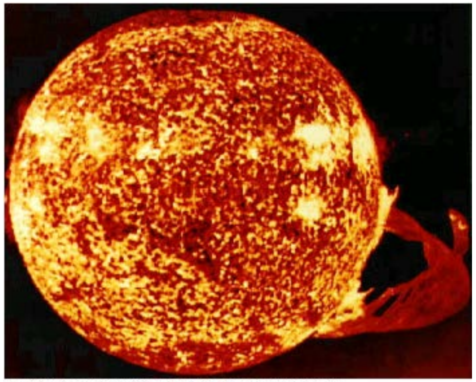
Fusion reactions: compound nucleus formation



compound
nucleus

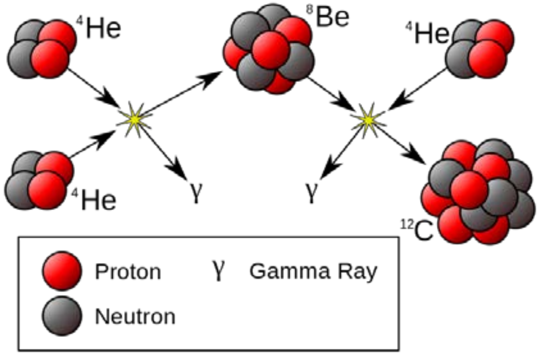


cf. Bohr '36

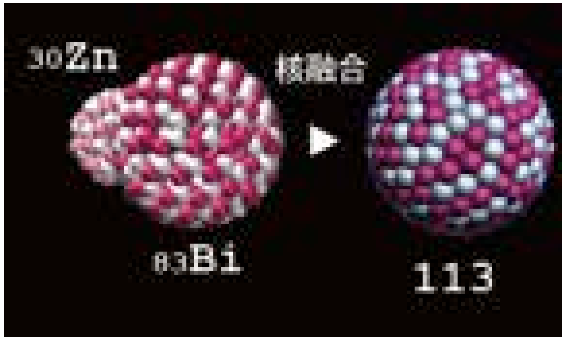


NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

energy production
in stars (Bethe '39)



nucleosynthesis

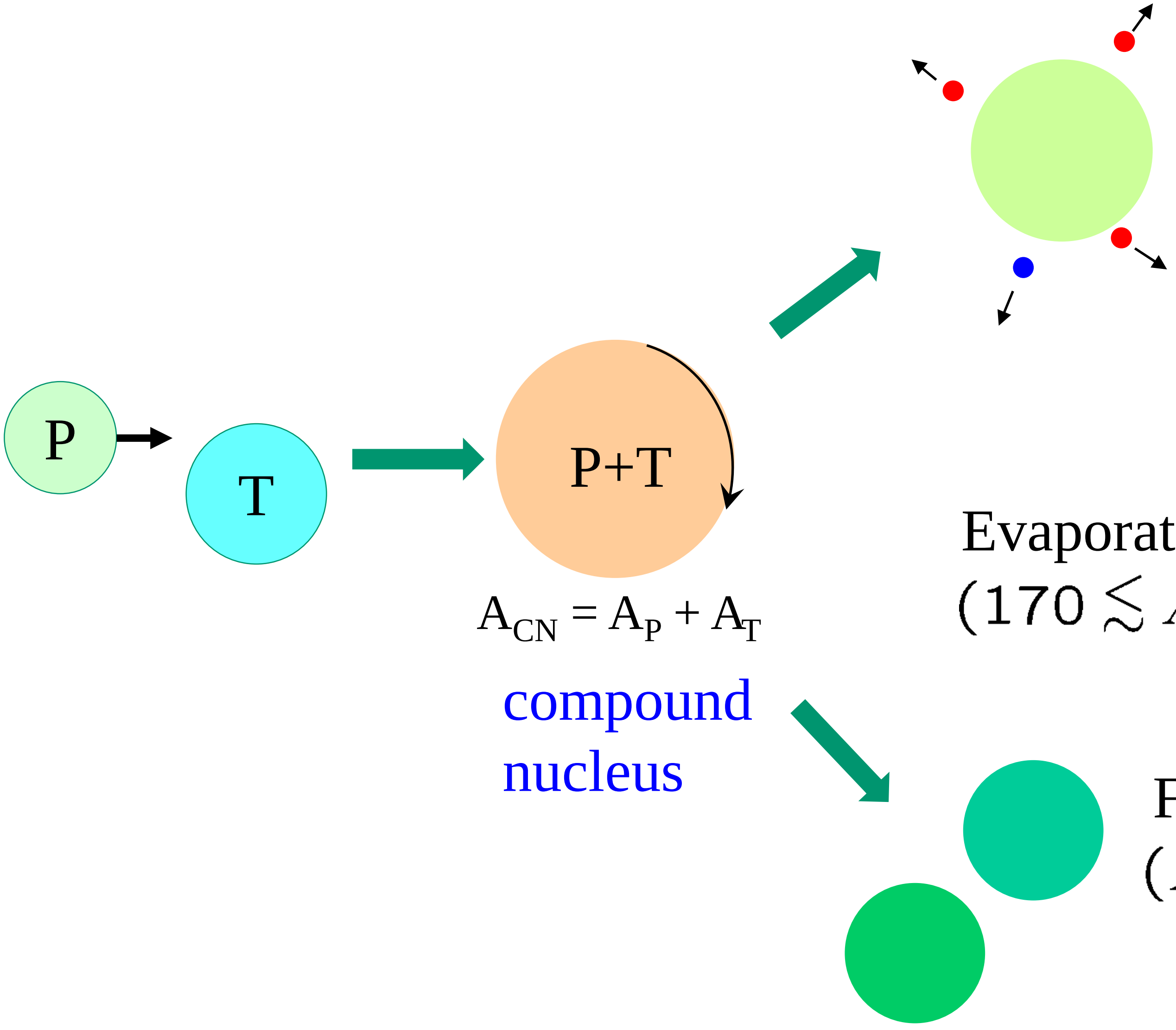


superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

⟵ microscopic understanding: an ultimate goal of nuclear physics

Fusion reactions: compound nucleus formation



$A_{CN} = A_P + A_T$

compound nucleus

Evaporation
$(A_{CN} \lesssim 170)$

n,p,α emissions
γ decay

Evaporation + Fission
$(170 \lesssim A_{CN} \lesssim 220)$

Fission
$(A_{CN} \gtrsim 220)$

cf. superheavy elements

Inter-nucleus potential

Two interactions:

1. Coulomb force
   long range repulsion
2. Nuclear force
   short range attraction

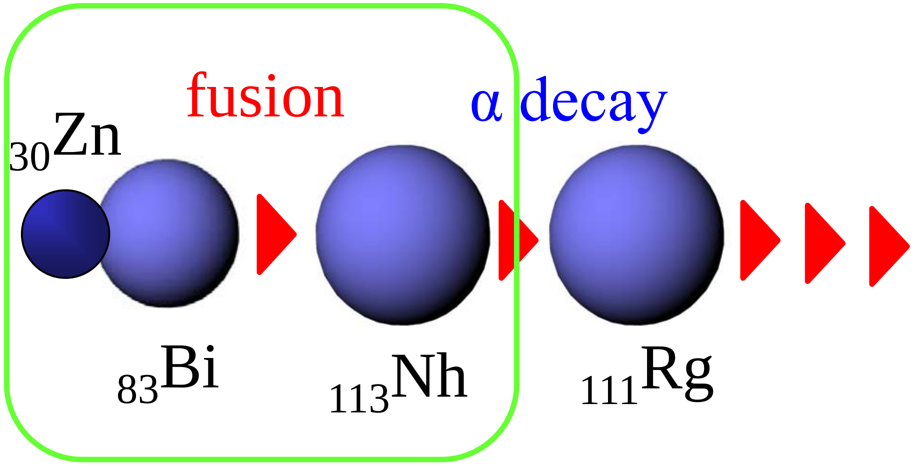
potential barrier
due to a cancellation
between the two
(Coulomb barrier)

- Above-barrier energies
- Sub-barrier energies
  (energies around the Coulomb barrier)
- Deep sub-barrier energies

Why sub-barrier fusion?

two obvious reasons:

superheavy elements



cf. ^{209}Bi (^{70}Zn,n) ^{278}Nh

$V_B \sim 260$ MeV

$E_{cm}^{(exp)} \sim 262$ MeV

Why sub-barrier fusion?

two obvious reasons:

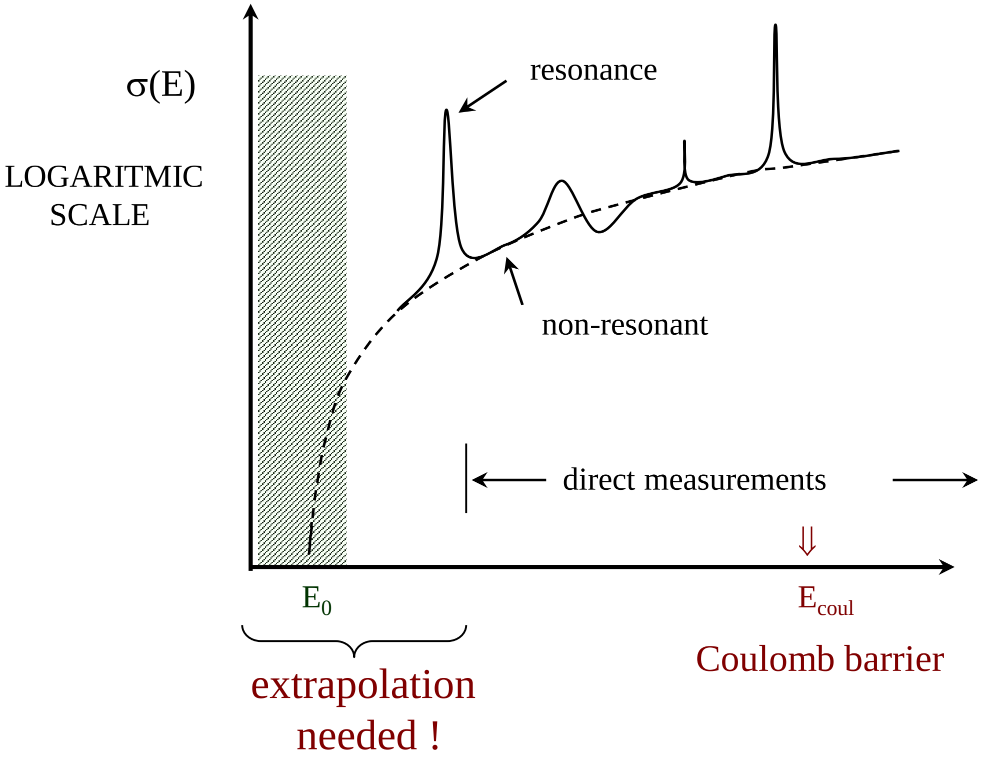
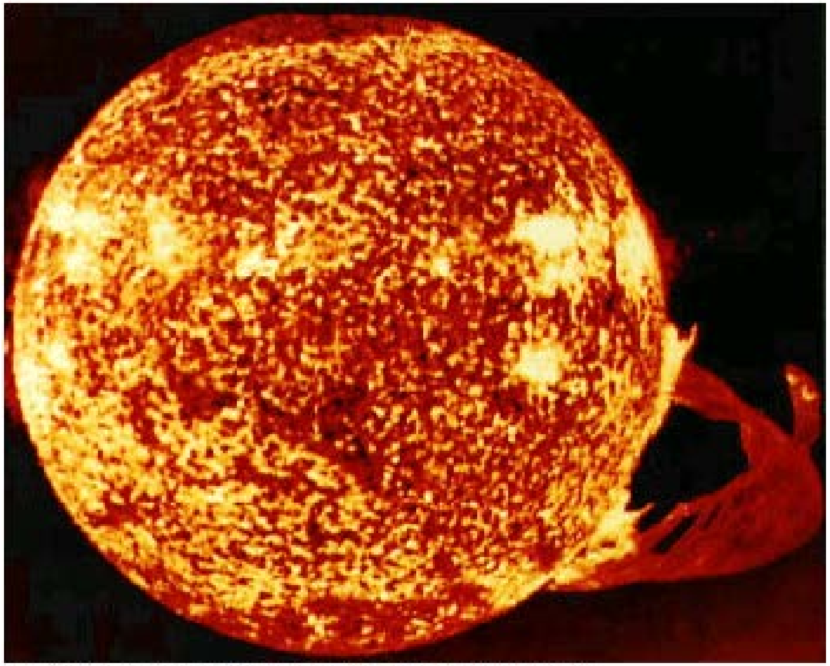


figure: M. Aliotta



NASA, Skylab space station December 19. 1973, solar flare reaching 588 000 km off solar surface

nuclear astrophysics
(nuclear fusion in stars)

cf. extrapolation of data

Why sub-barrier fusion?

two obvious reasons:

✓superheavy elements ✓ nuclear astrophysics

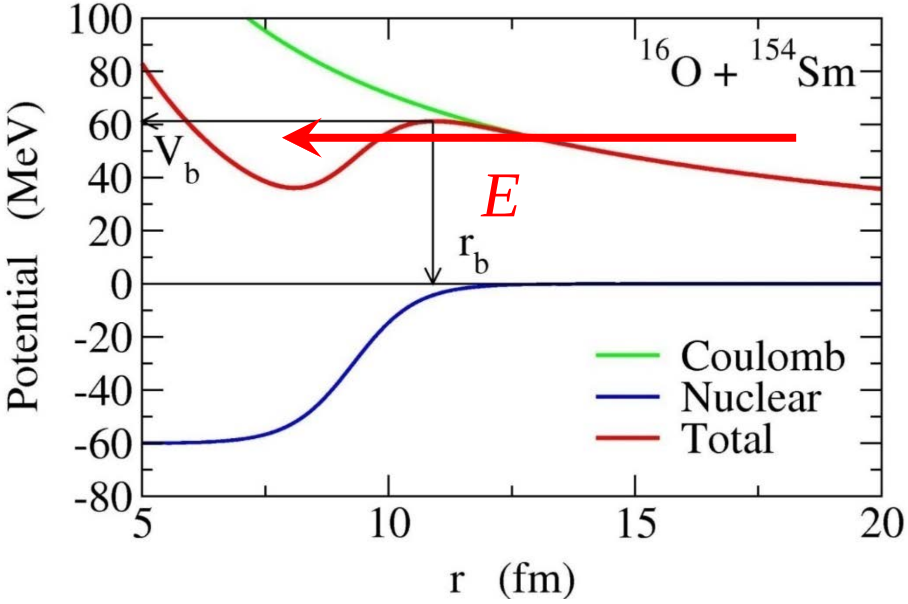
other reasons:

✓reaction dynamics

strong interplay between reaction and structure

cf. high *E* reactions: much simpler reaction mechanisms

✓many-particle tunneling



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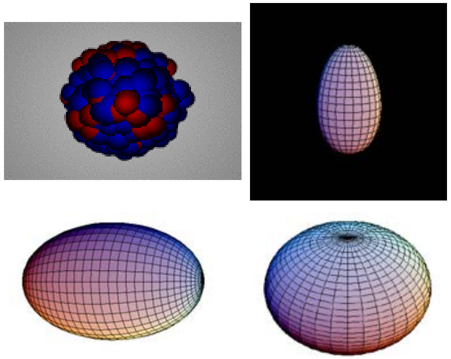
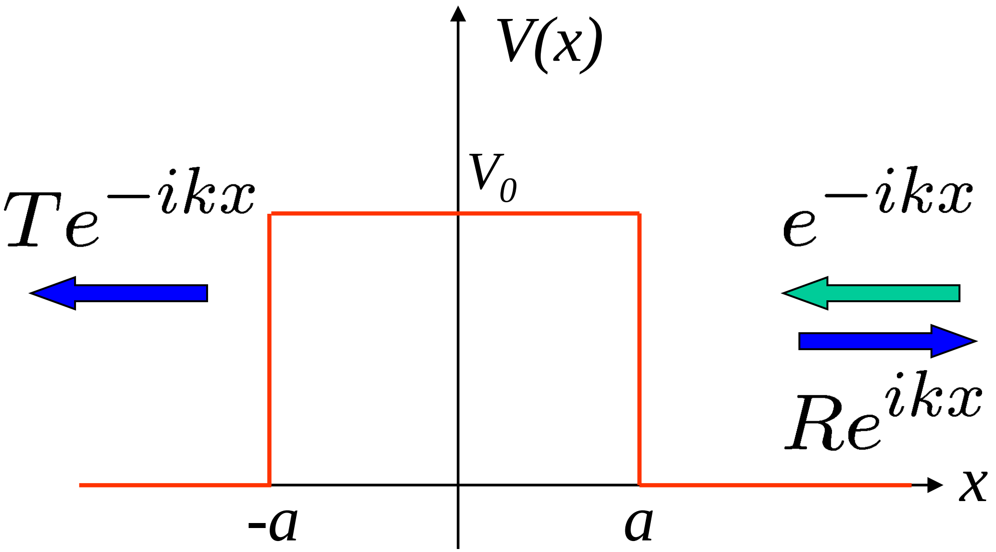


- many types of intrinsic degrees of freedom (several types of collective vibrations, deformation with several multipolarities)
- energy dependence of tunneling probability
  cf. alpha decay: fixed energy

H.I. fusion reaction = an ideal playground to study quantum tunneling with many degrees of freedom

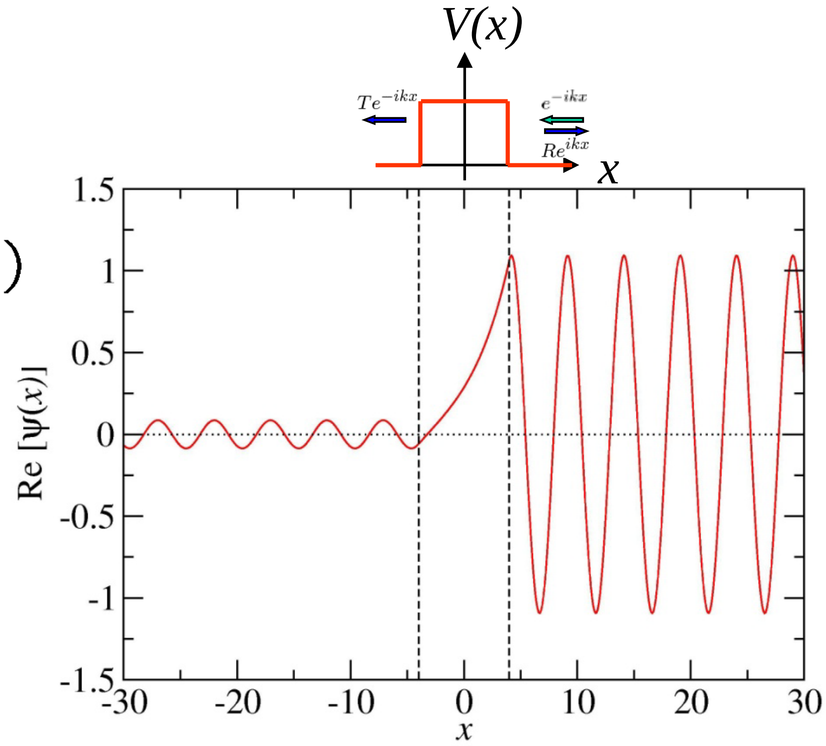
Quantum Tunneling Phenomena



$$
\begin{aligned}
\psi(x) &= T\,e^{-ikx} && (x \le -a)\\
&= A\,e^{-\kappa x} + B\,e^{\kappa x} && (-a < x < a)\\
&= e^{-ikx} + R\,e^{ikx} && (x \ge a)
\end{aligned}
$$

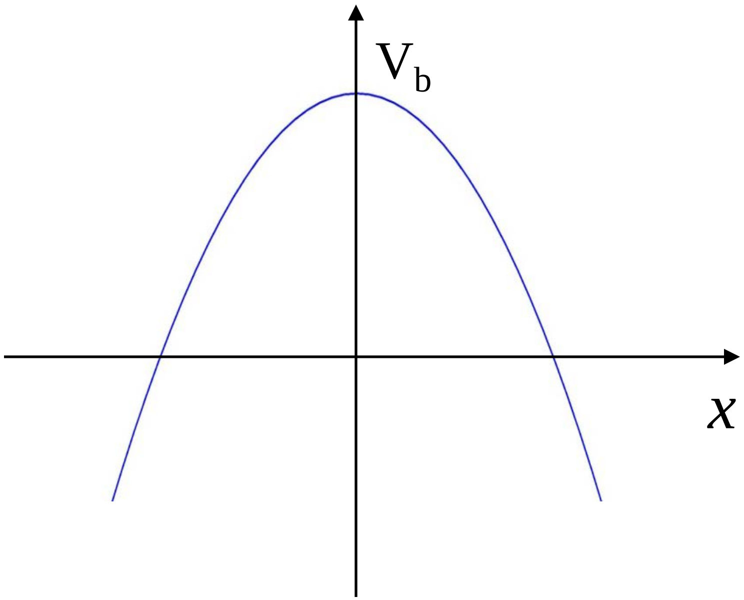
$$
\begin{aligned}
k &= \sqrt{2mE/\hbar^2}\\
\kappa &= \sqrt{2m(V_0 - E)/\hbar^2}
\end{aligned}
$$

Tunnel probability: $P(E) = |T|^2$

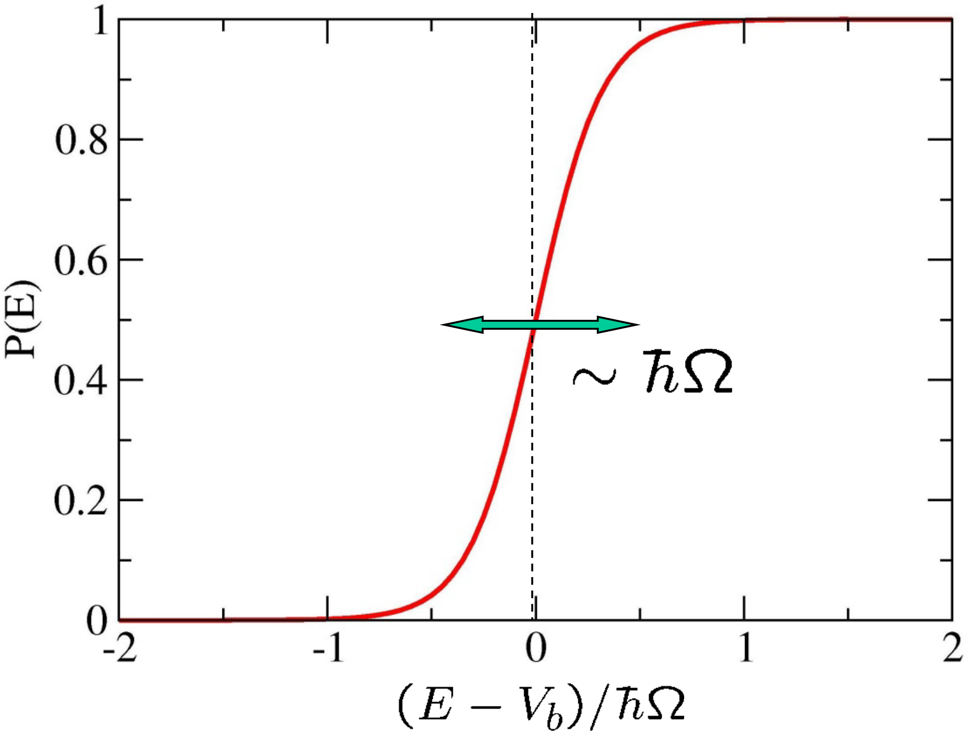


For a parabolic barrier……

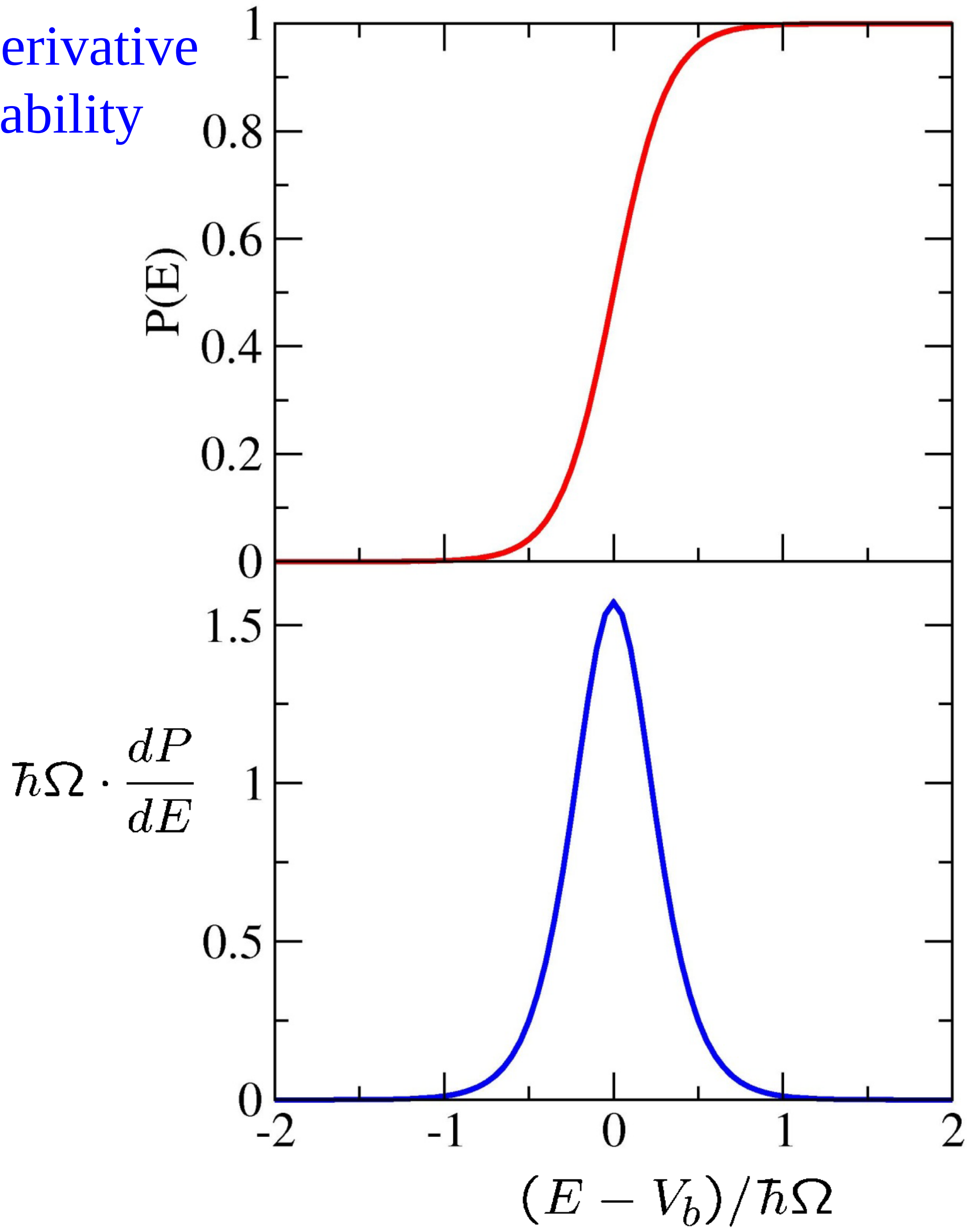
$$V(x) = V_b - \frac{1}{2}m\Omega^2 x^2$$



$$\boxed{P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}}$$



Energy derivative of penetrability



(note) Classical limit

$$P(E) = \theta(E - V_b)$$
$$dP/dE = \delta(E - V_b)$$

cf. WKB approximation

One dimensional Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + (V(x) - E)\psi(x) = 0$$

$$\Rightarrow \quad \frac{d^2}{dx^2}\psi(x) + \frac{p^2(x)}{\hbar^2}\psi(x) = 0, \qquad p(x) \equiv \sqrt{2m(E - V(x))}$$

Assume $\boxed{\psi(x) = e^{iS(x)/\hbar}}$

$$\longrightarrow \quad \begin{aligned} \psi' &= \frac{i}{\hbar}S'\psi \\ \psi'' &= \frac{i}{\hbar}S''\psi - \frac{1}{\hbar^2}(S')^2\psi \end{aligned}$$

$$\longrightarrow \quad \boxed{\frac{i}{\hbar}S'' - \frac{1}{\hbar^2}(S')^2 + \frac{p(x)^2}{\hbar^2} = 0}$$

cf. WKB approximation

$$\boxed{i\hbar S'' - (S')^2 + p(x)^2 = 0}$$

Expand *S* as: $S(x) = S_0(x) + \hbar S_1(x) + \hbar^2 S_2(x) + \cdots$

$$\begin{aligned} S_0(x) &= \pm \int^x p(x')dx' \\ S_1(x) &= \frac{i}{2} \ln p(x) + const. \end{aligned}$$

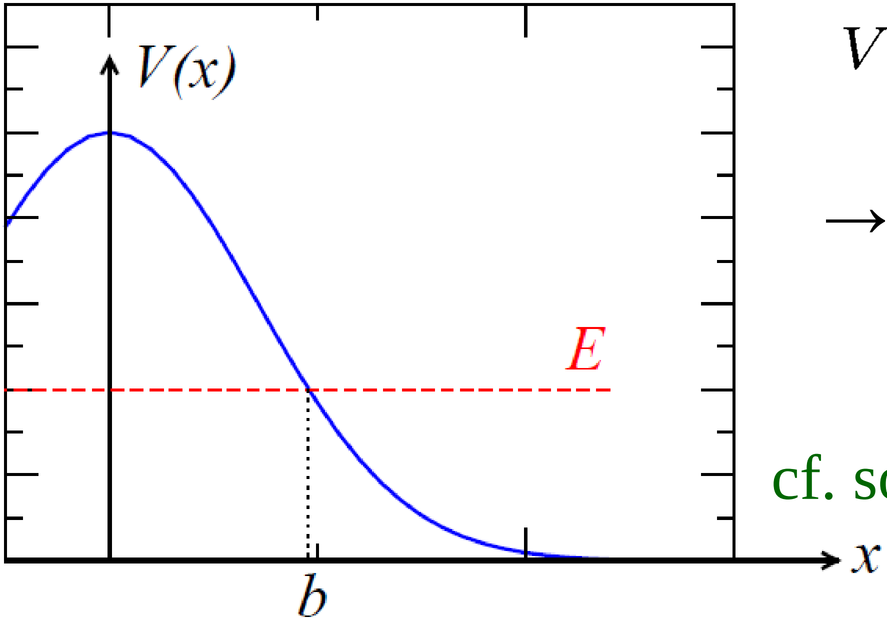
$$\boxed{\psi(x) = e^{iS(x)/\hbar} \sim \frac{1}{\sqrt{p(x)}} e^{\pm \frac{i}{\hbar} \int^x p(x')dx'}}$$

cf. WKB approximation

$$\boxed{\psi(x) = e^{iS(x)/\hbar} = \frac{1}{\sqrt{p(x)}} e^{\pm\frac{i}{\hbar}\int^x p(x')dx'}}$$

this wave function breaks down at x which satisfies $p(x) = 0$.

$$p(x) = \sqrt{2m(E - V(x))} \to V(x) = E \qquad \text{(a turning point)}$$



$$V(x) \sim V(b) + V'(b)(x-b)$$

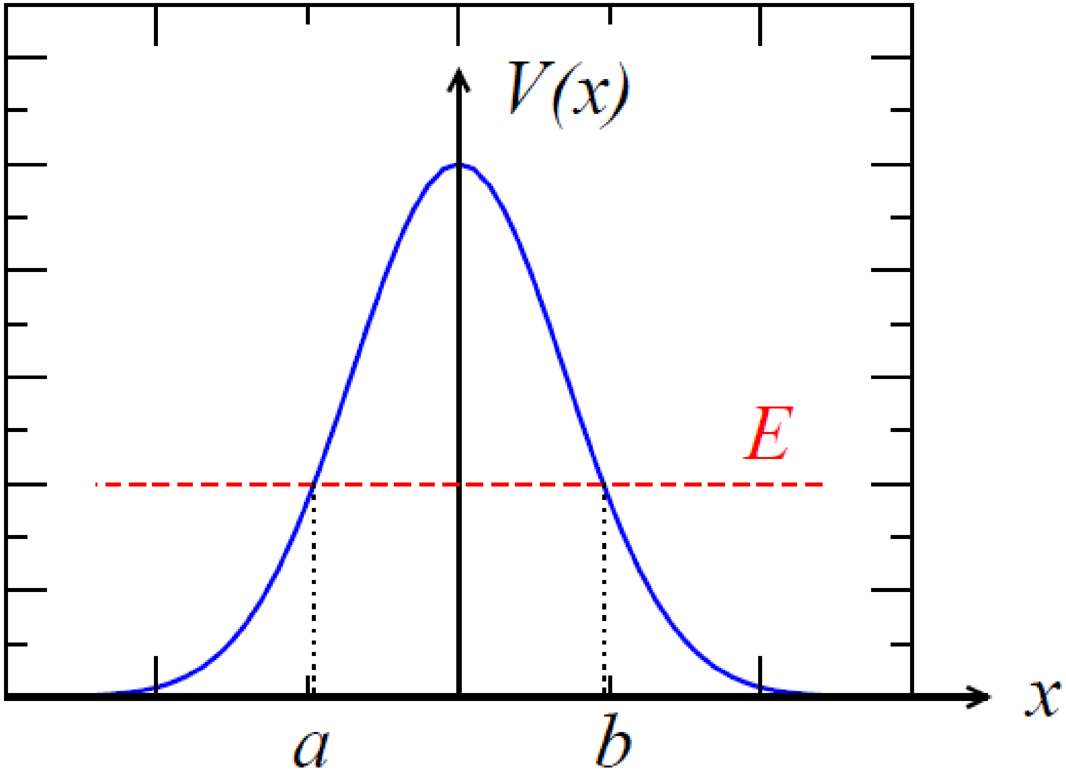
around $x \sim b$

→ connect wave function from $x > b$
to $x < b$
(WKB connection formula)

cf. solution for a linear potential
: Airy function

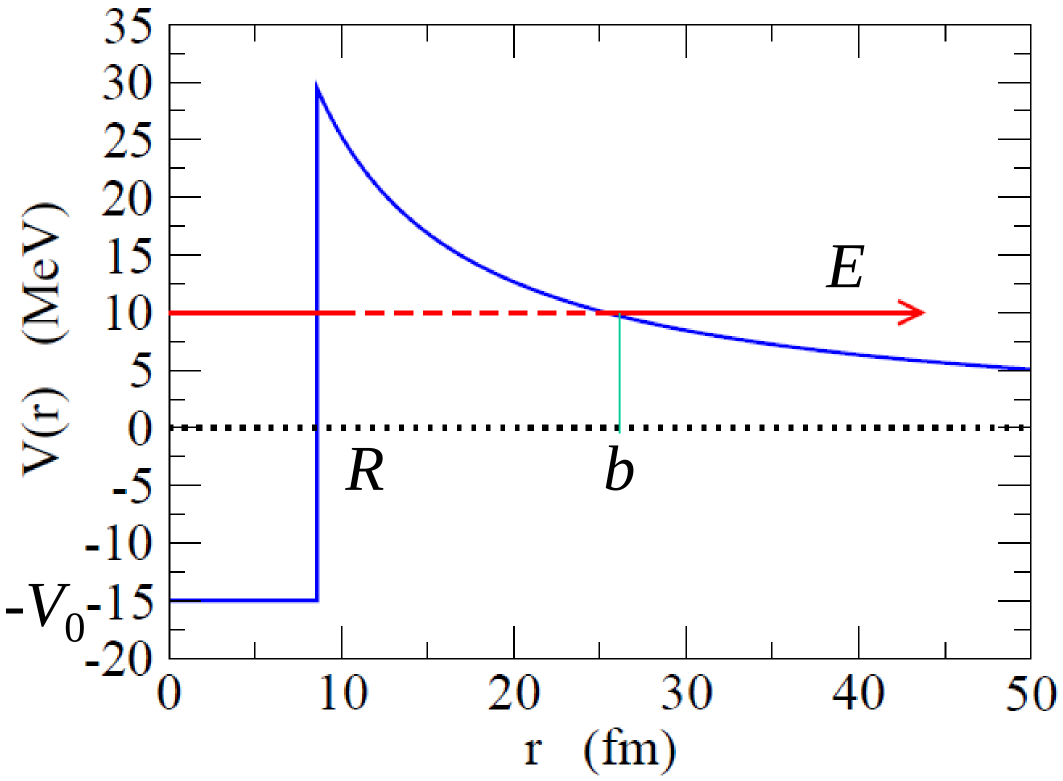
cf. WKB approximation

if applied to a tunneling problem:



$$\boxed{P(E) = \exp\left[-2\int_a^b dx\sqrt{\frac{2m}{\hbar^2}(V(x)-E)}\right]}$$

for a Coulomb potential



$$
\begin{aligned}
V(r) &= -V_0 & (r < R) \\
&= \frac{Z_1 Z_2 e^2}{r} & (r \geq R)
\end{aligned}
$$

$$
\boxed{P = \exp\left[-2\int_R^b dr\,\sqrt{\frac{2\mu}{\hbar^2}\left(\frac{Z_1 Z_2 e^2}{r} - E\right)}\right]}
$$

$$\boxed{P = \exp\left[-2\int_R^b dr\sqrt{\frac{2\mu}{\hbar^2}\left(\frac{Z_1 Z_2 e^2}{r} - E\right)}\right]}$$

$$P = e^{-G} \qquad G \sim \sqrt{\frac{2\mu R}{\hbar^2} Z_1 Z_2 e^2}\left(\pi\sqrt{\frac{Z_1 Z_2 e^2}{RE}} - 4\right)$$

(note) for $R \rightarrow 0$

$$G \sim \sqrt{\frac{2\mu R}{\hbar^2} Z_1 Z_2 e^2}\left(\pi\sqrt{\frac{Z_1 Z_2 e^2}{RE}}\right) = \pi Z_1 Z_2 e^2\sqrt{\frac{2\mu}{\hbar^2 E}} = 2\pi\frac{Z_1 Z_2 e^2}{\hbar v}$$

$$\equiv \eta(E)$$

$$\boxed{P(E) \sim e^{-2\pi\eta(E)}}$$

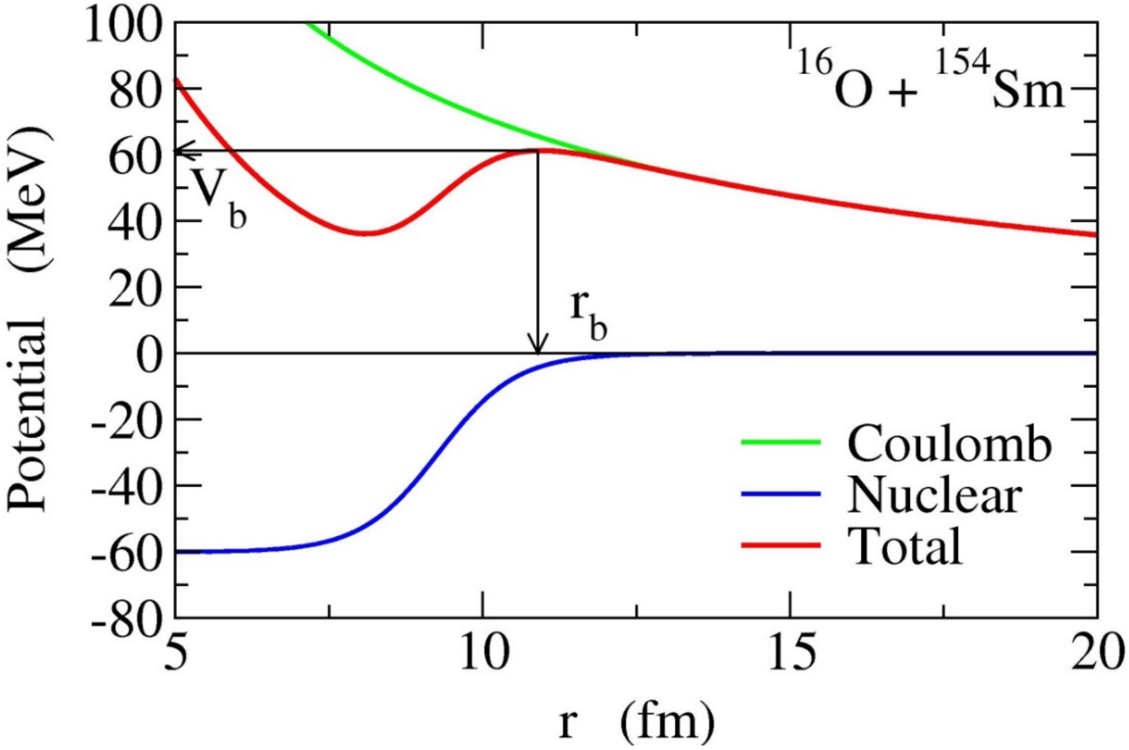
Sommerfeld parameter

The simplest approach to fusion: potential model

Potential model: $V(r)$ + absorption

$$\boxed{\sigma_{\mathsf{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)}$$

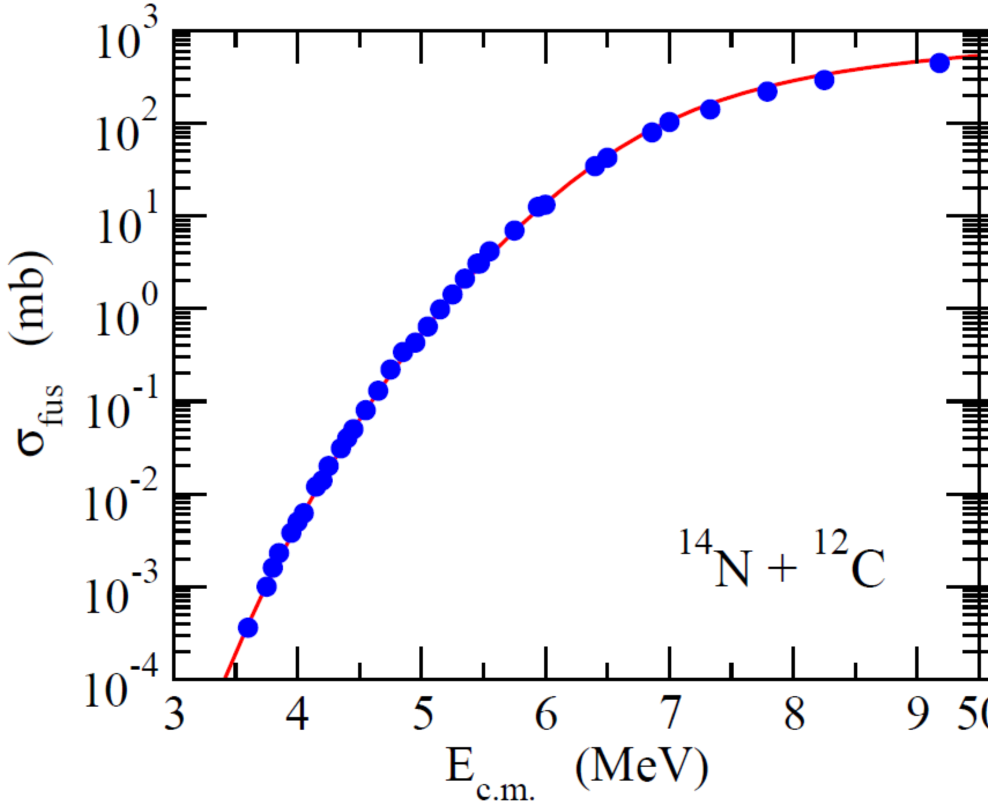
$P_l\,(E)$: barrier penetrability



Comparison with experimental data: large enhancement of $\sigma_{\rm fus}$

Potential model: $V(r)$ + absorption

$$\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$



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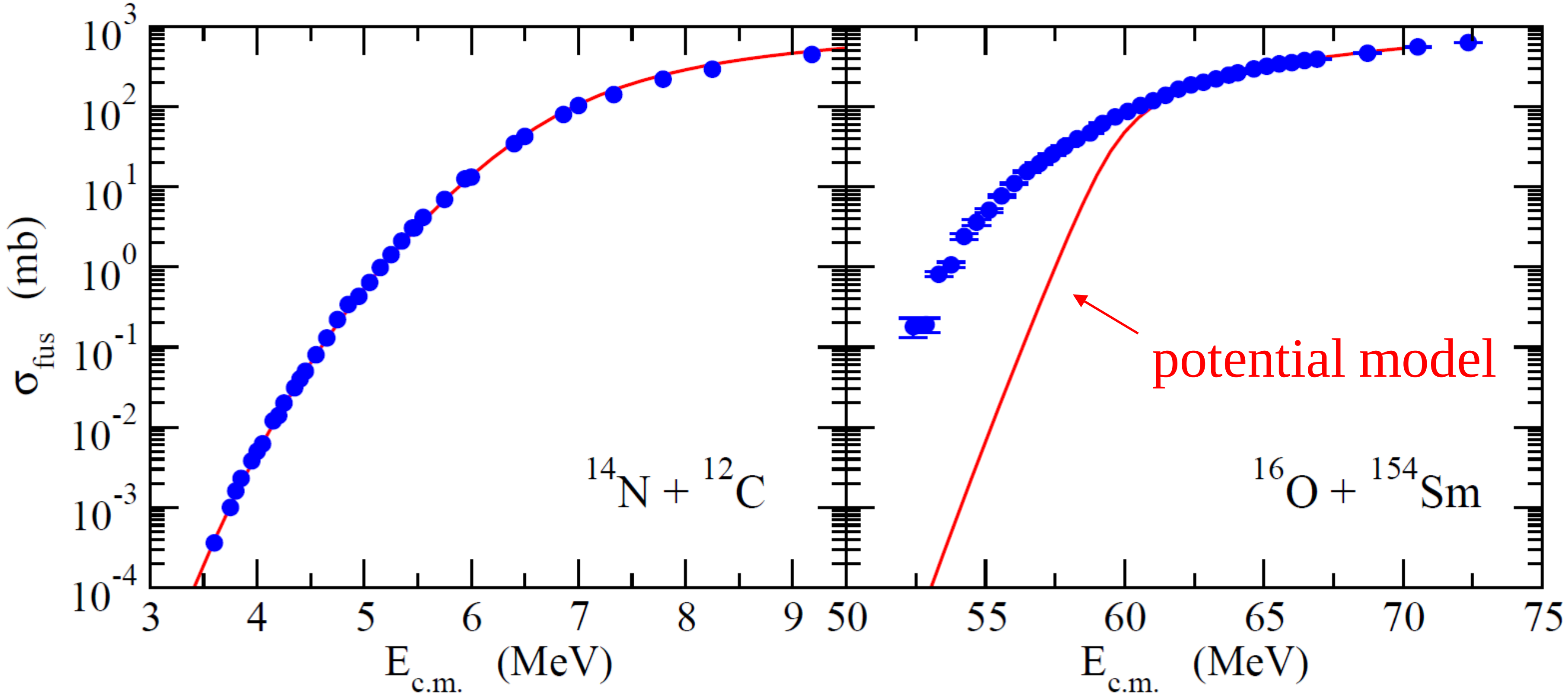
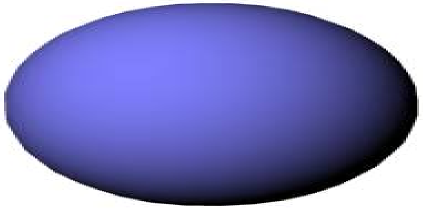


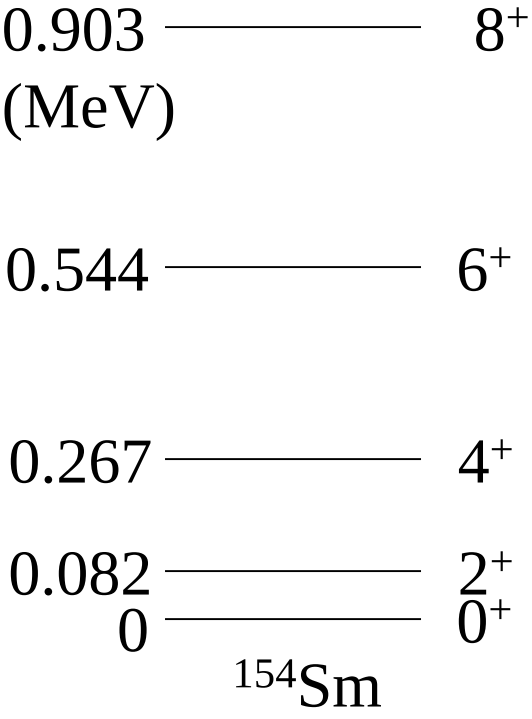
cf. seminal work:

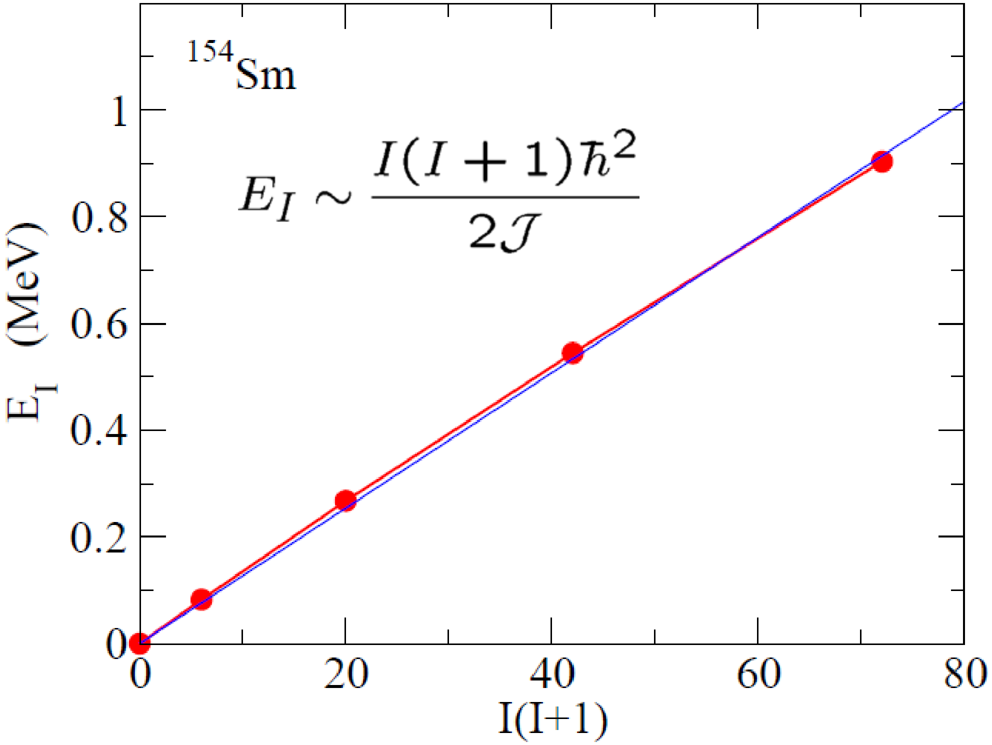
R.G. Stokstad et al., PRL41('78) 465

^{154}Sm : a typical deformed nucleus



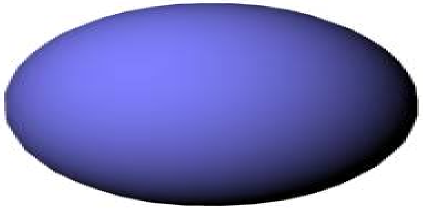
^{154}Sm



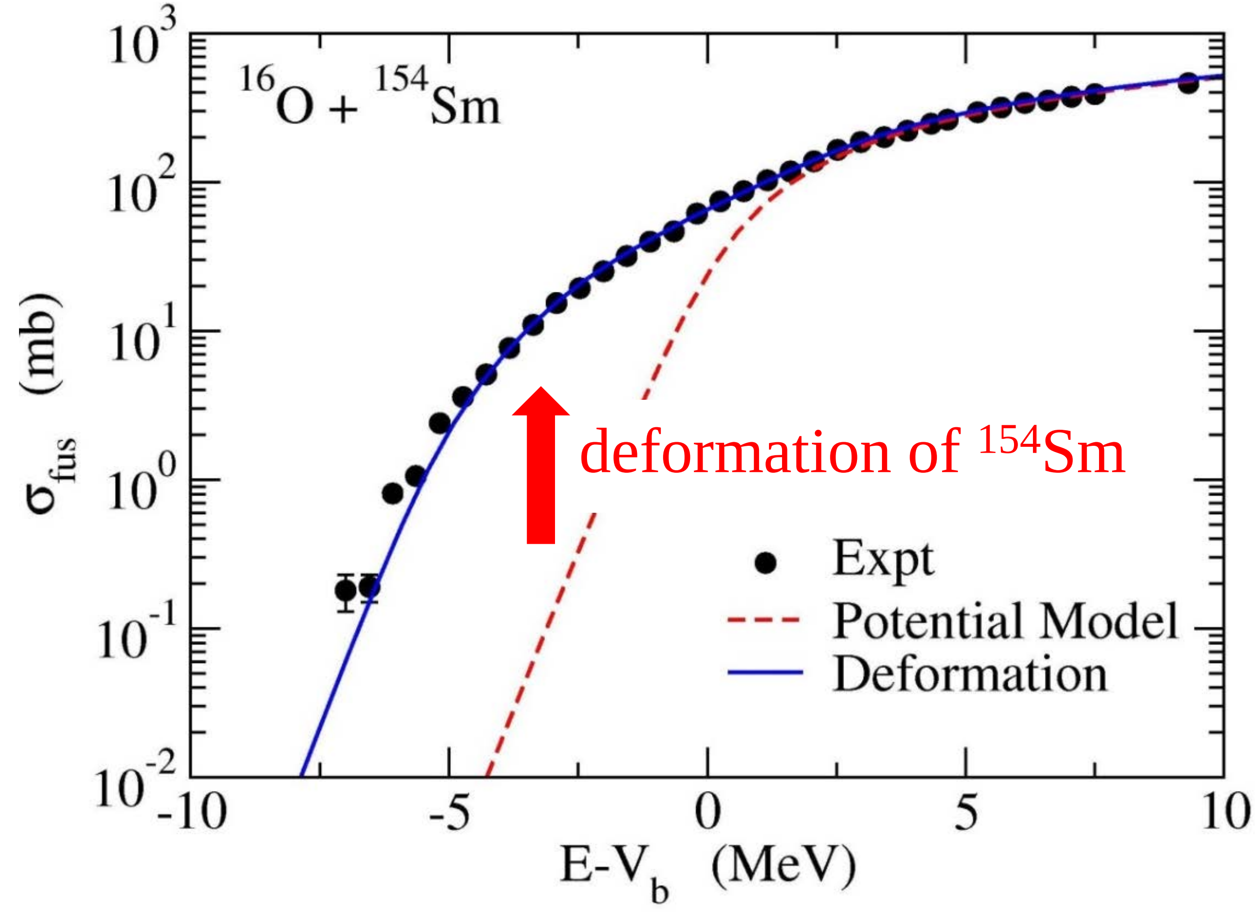


$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

^{154}Sm : a typical deformed nucleus



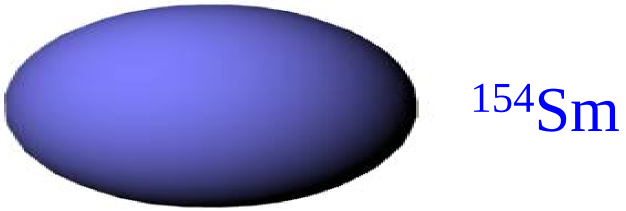
^{154}Sm

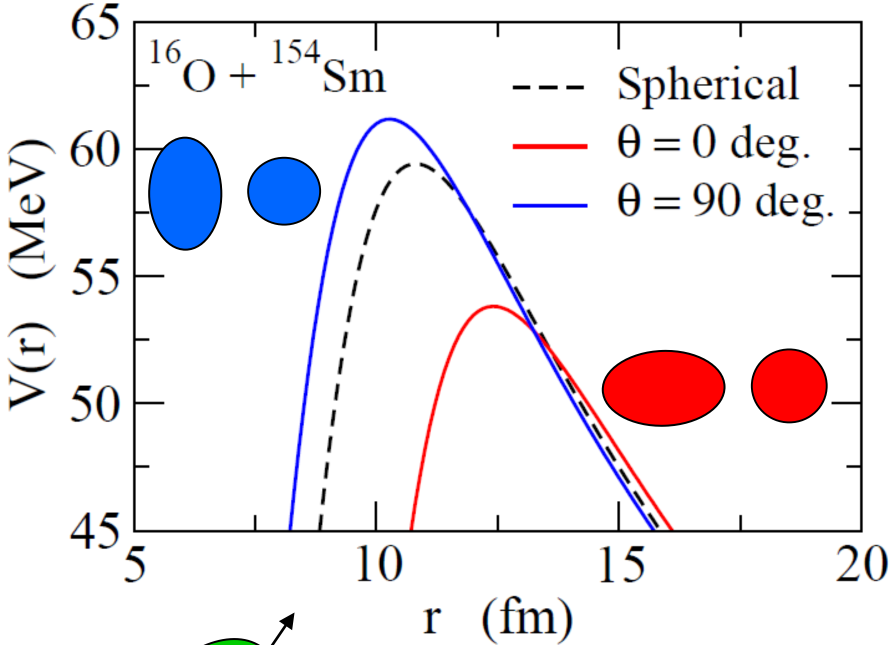


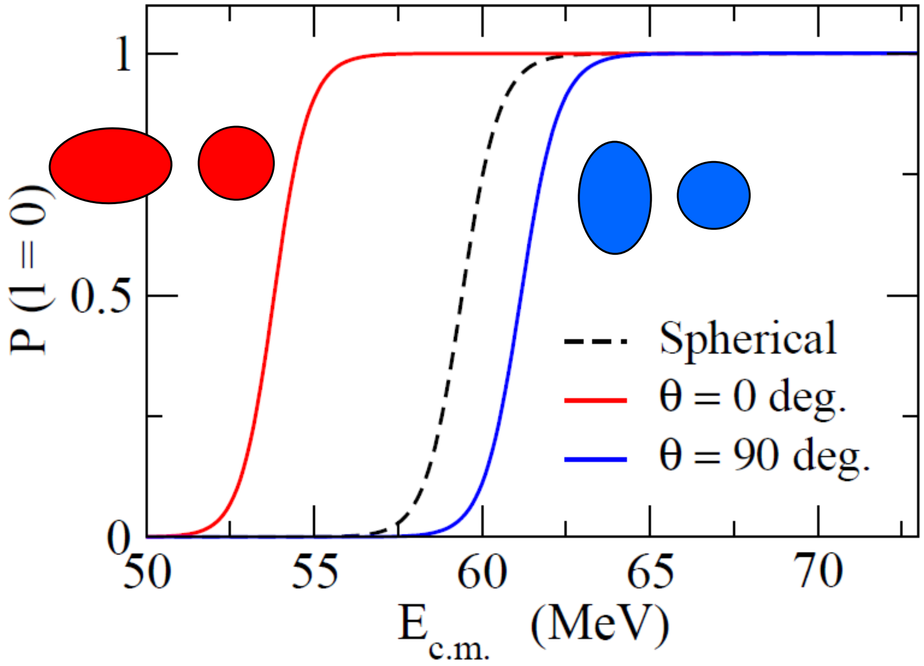
deformation of ^{154}Sm

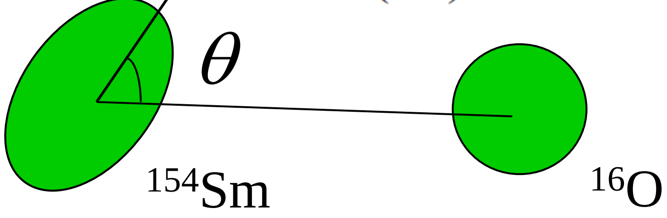
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus









Effects of nuclear deformation

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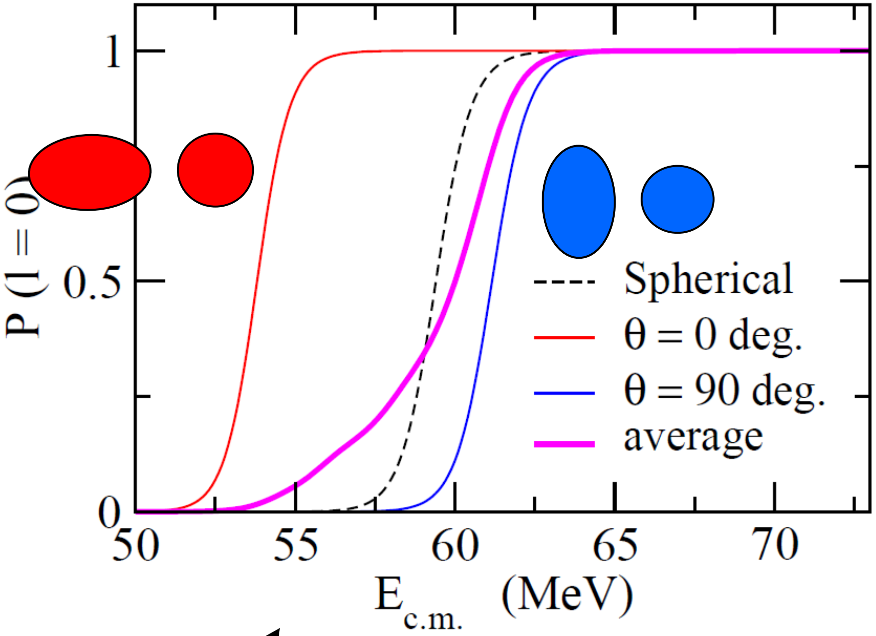
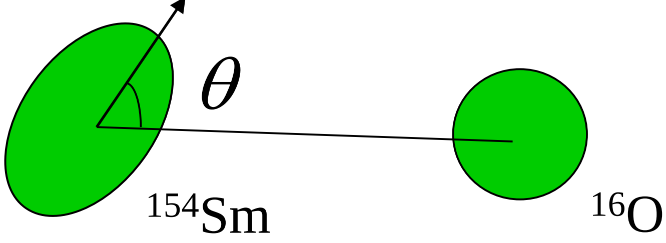
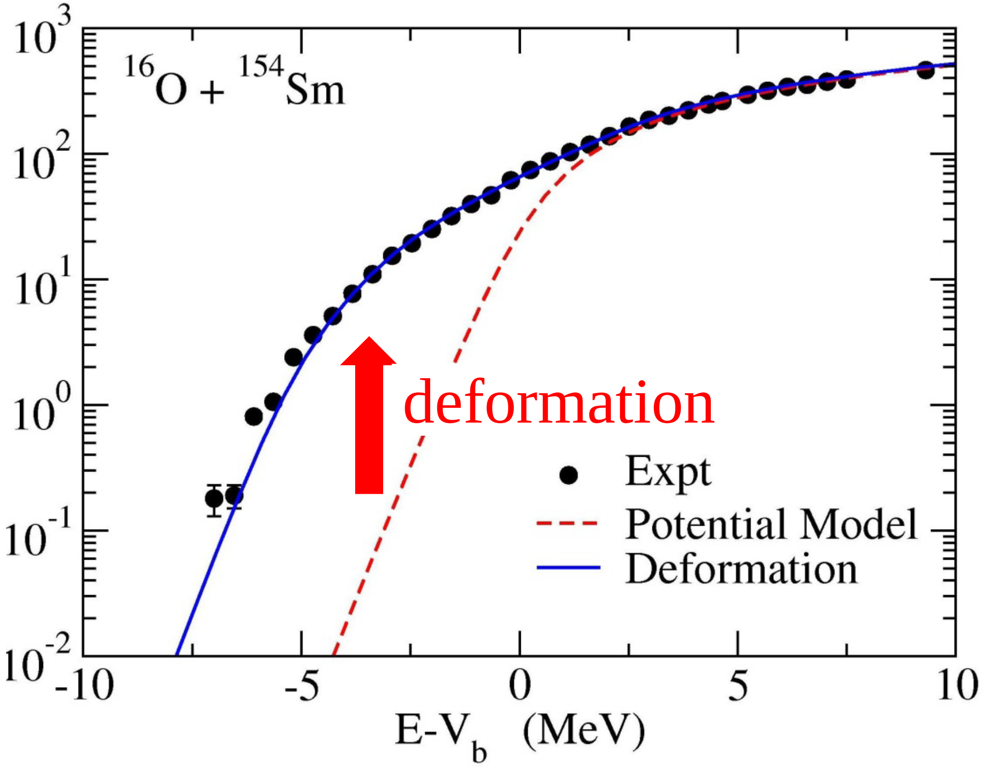




$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

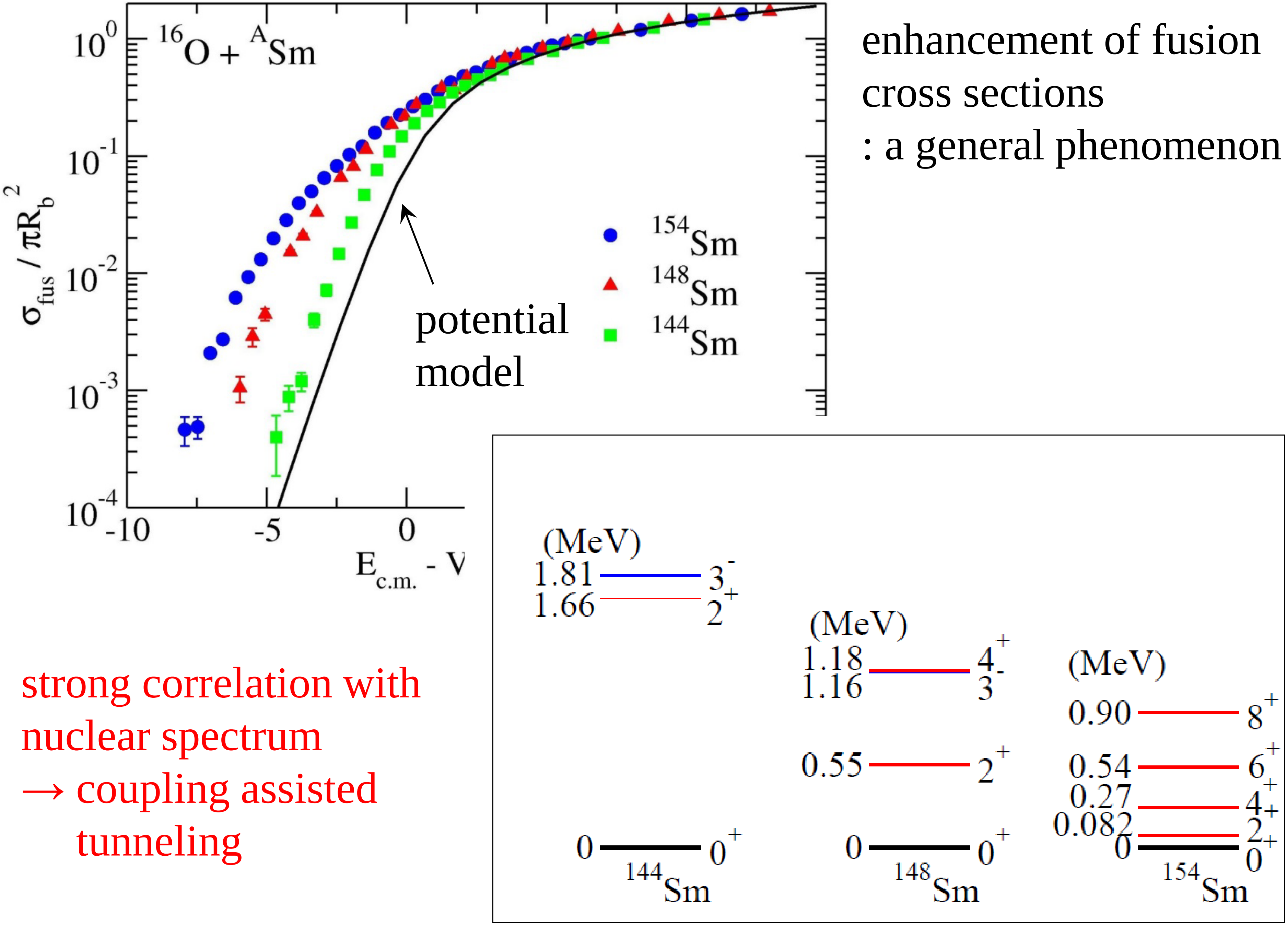


Fusion: strong interplay between nuclear structure and reaction

* Sub-barrier enhancement also for non-deformed targets:

couplings to low-lying collective excitations → coupling assisted tunneling

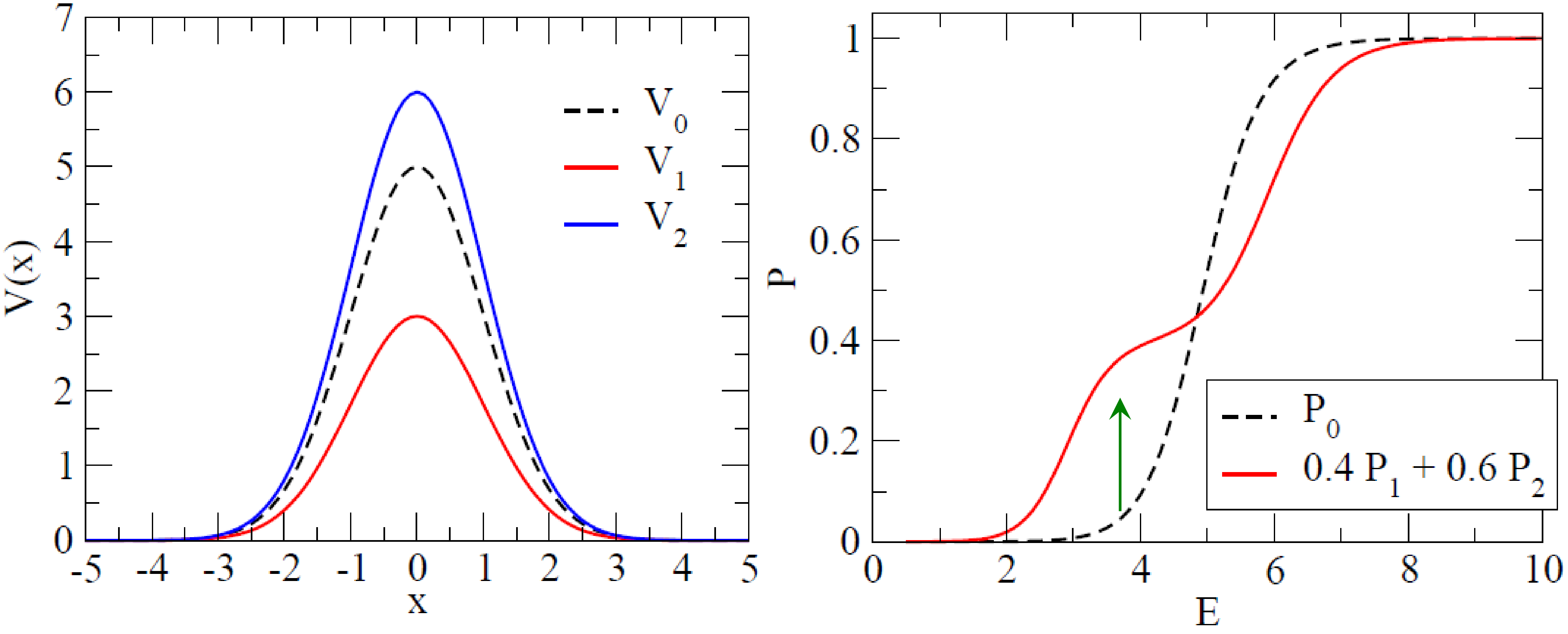
enhancement of fusion cross sections
: a general phenomenon



strong correlation with
nuclear spectrum
→ coupling assisted
tunneling

Enhancement of tunneling probability
: a problem of two potential barriers

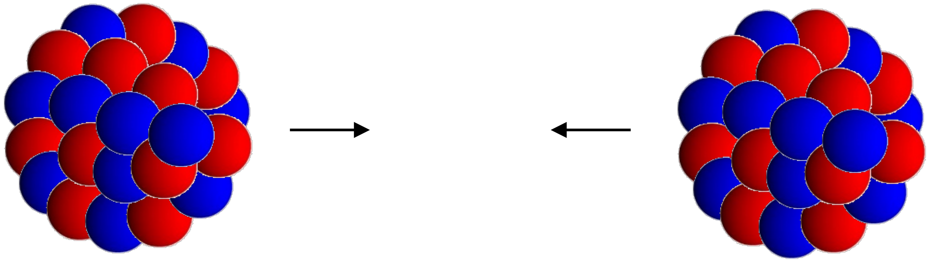
$$P(E) = P(E;V_0) \to w_1 P(E;V_1) + w_2 P(E;V_2)$$



“barrier distribution” due to couplings to excited states in projectile/target nuclei

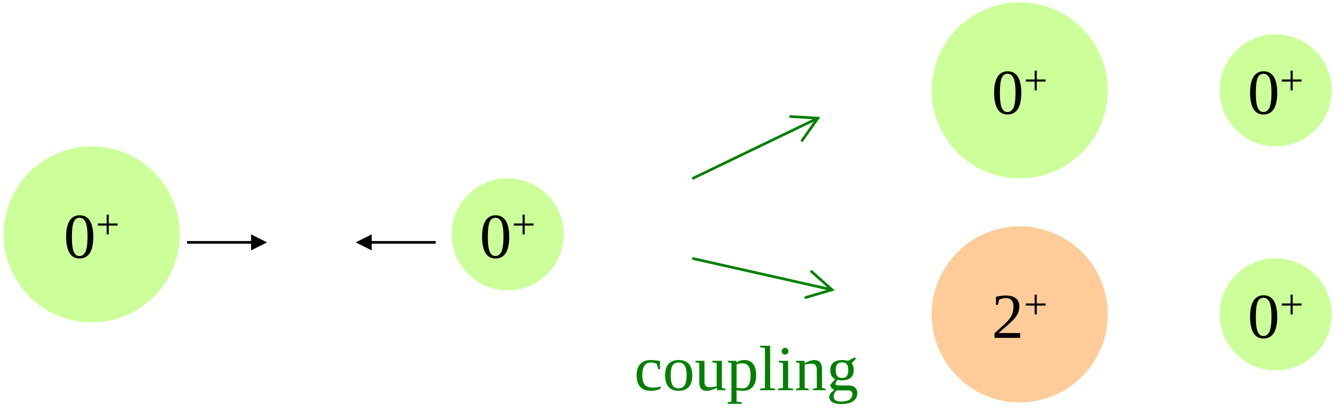
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

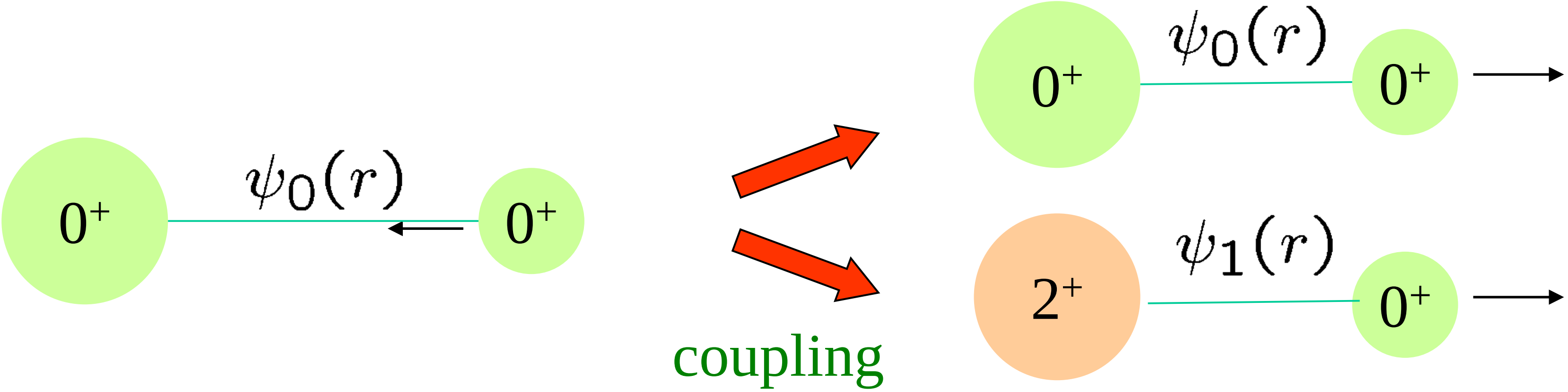


still very challenging

two-body problem, but with excitations
(coupled-channels approach)



Coupled-channels method: a quantal scattering theory with excitations

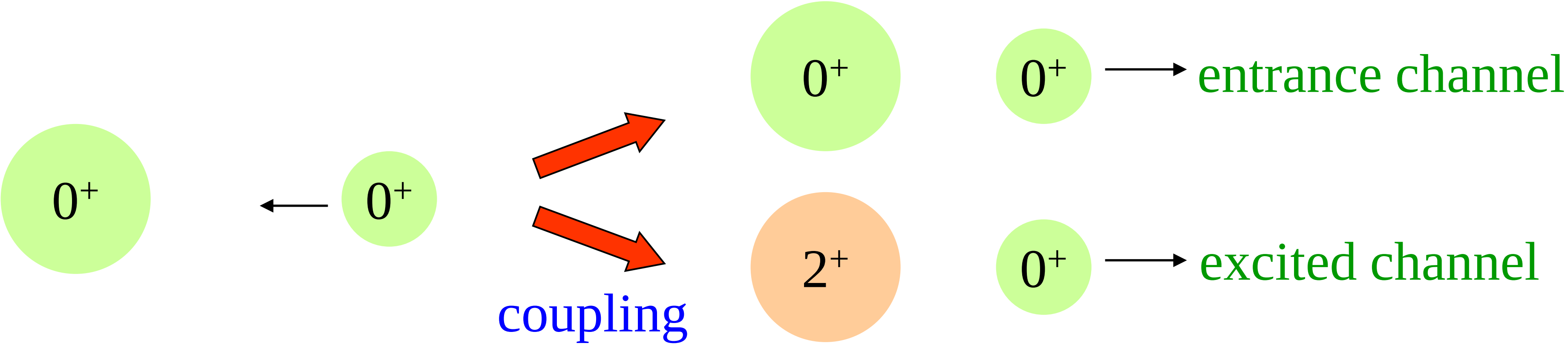


$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2+\overleftrightarrow{V}(r)-\overleftrightarrow{E}\right]\vec{\psi}(\boldsymbol{r})=0}$$

if written down more explicitly:

$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2+V_0(r)+\underset{\underset{\text{excitation energy}}{\uparrow}}{\epsilon_k}-E\right]\psi_k(\boldsymbol{r})+\sum_{k'}\langle\phi_k|\underset{\underset{\text{excitation operator}}{\uparrow}}{V_{\rm coup}}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r})=0}$$

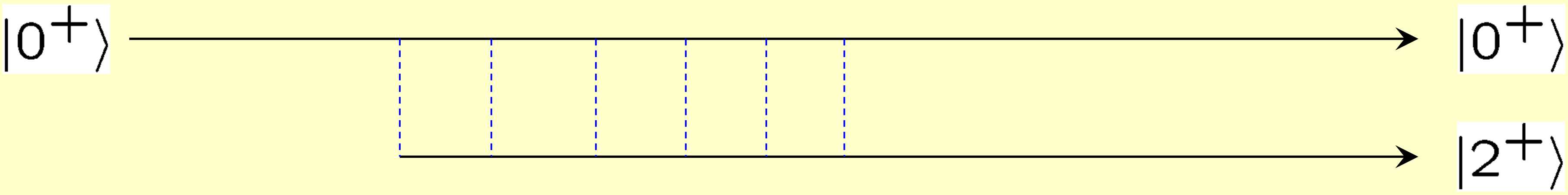
Coupled-channels method: a quantal scattering theory with excitations



$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|V_{\mathrm{coup}}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0$$

excitation energy

excitation operator



full order treatment of excitation/de-excitation dynamics during reaction

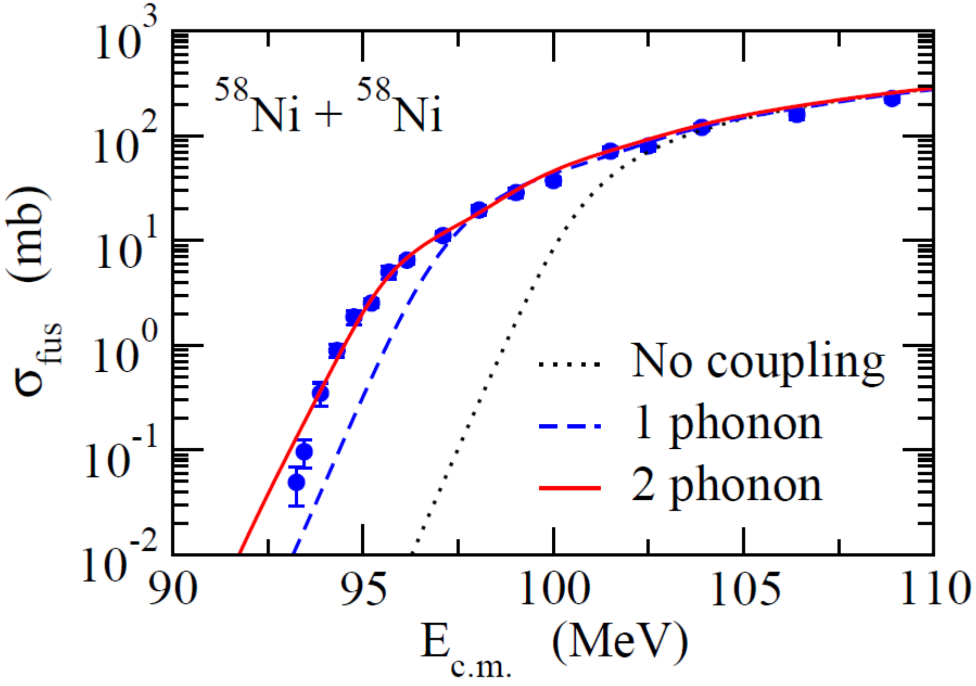
Inputs for C.C. calculations

i) Inter-nuclear potential

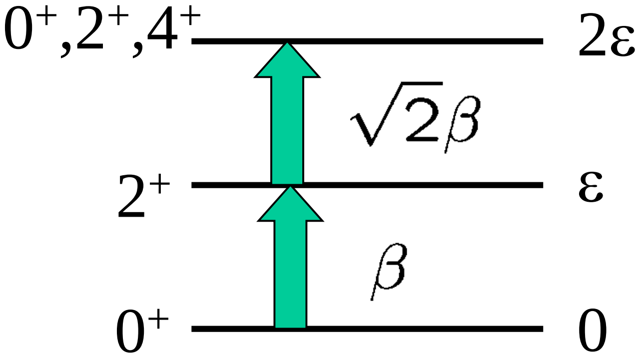
a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom

in most of cases, (macroscopic) collective model
(rigid rotor / harmonic oscillator)



simple harmonic oscillator

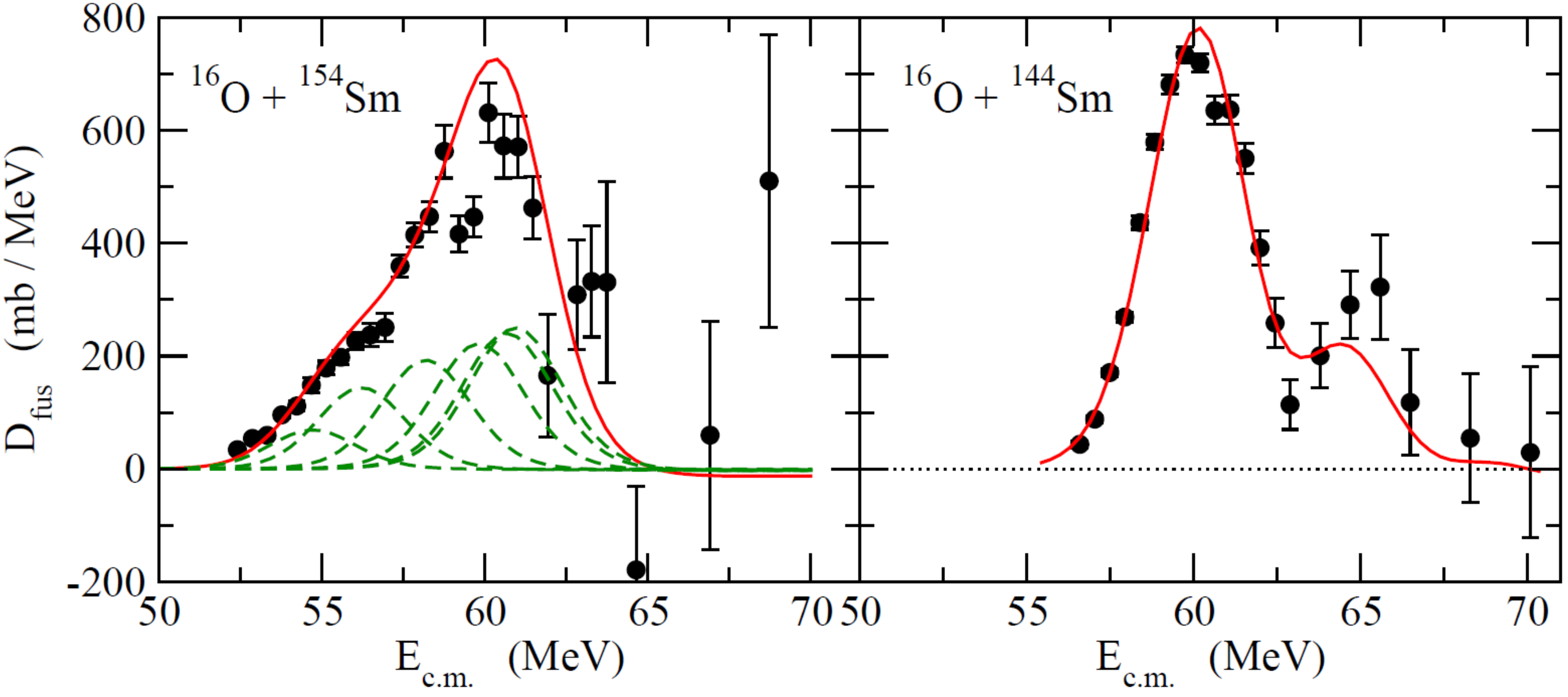


C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

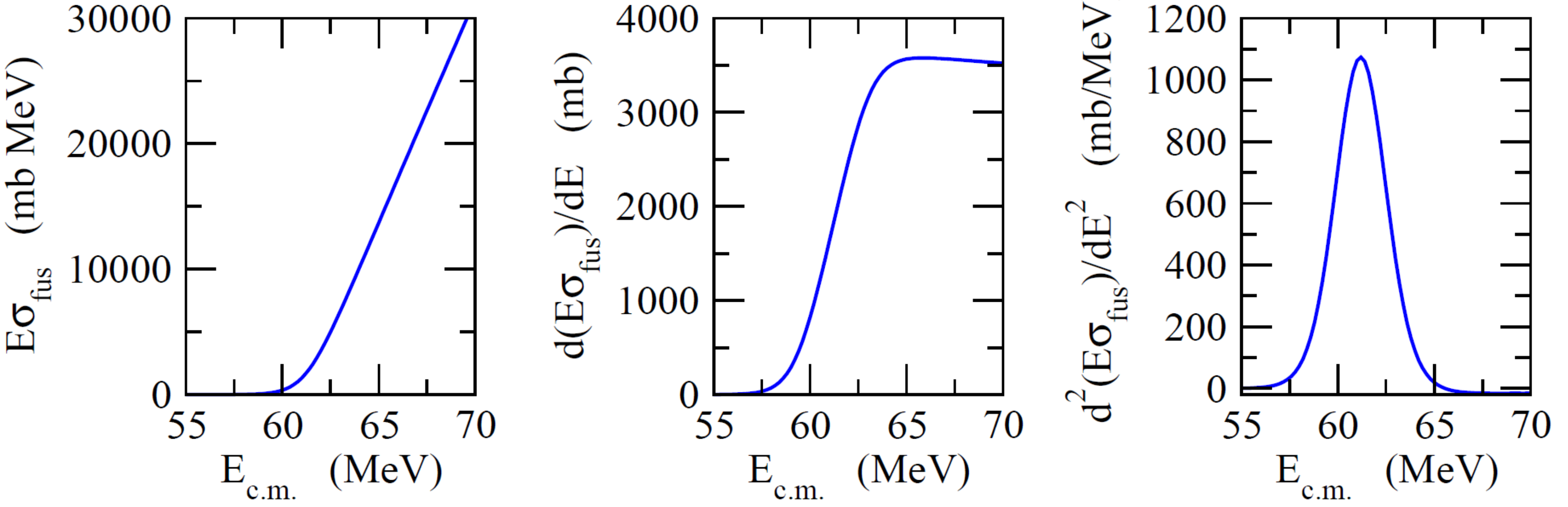
✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

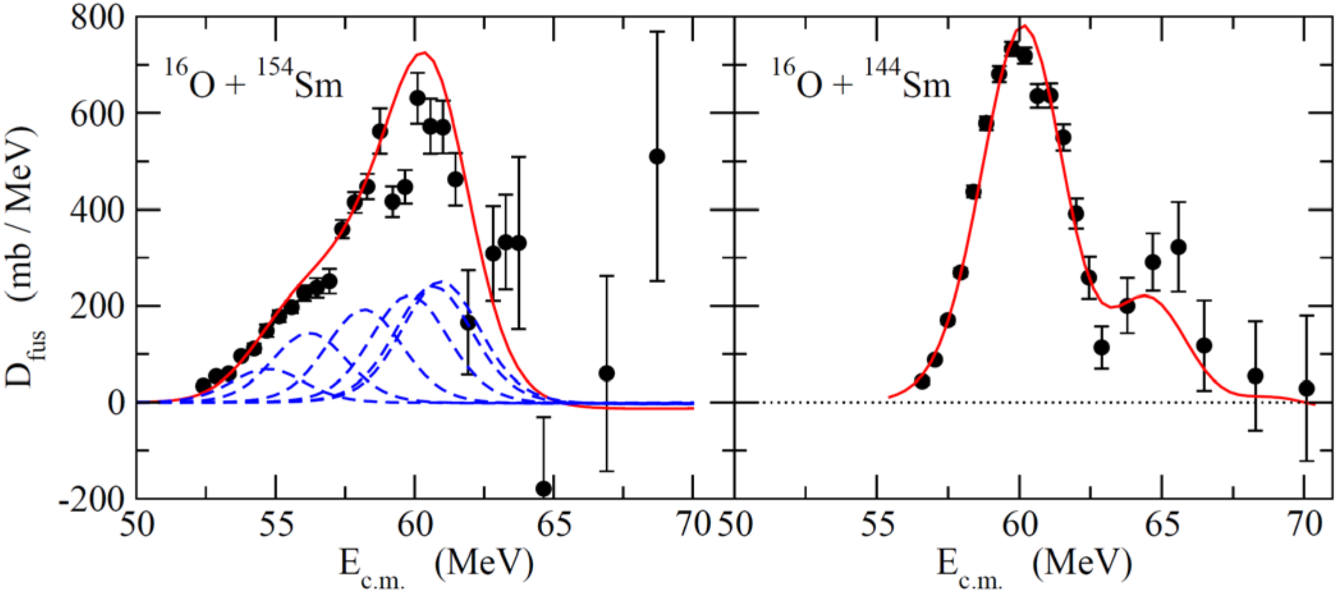
$$D_{\mathsf{fus}}(E) = \frac{d^2(E\sigma_{\mathsf{fus}})}{dE^2}$$



K.H., N. Takigawa, PTP128 (‘12) 1061

$\sim \pi R_b^2 \, P_{l=0}(E)$ $\sim \pi R_b^2 \cdot dP_{l=0}/dE$





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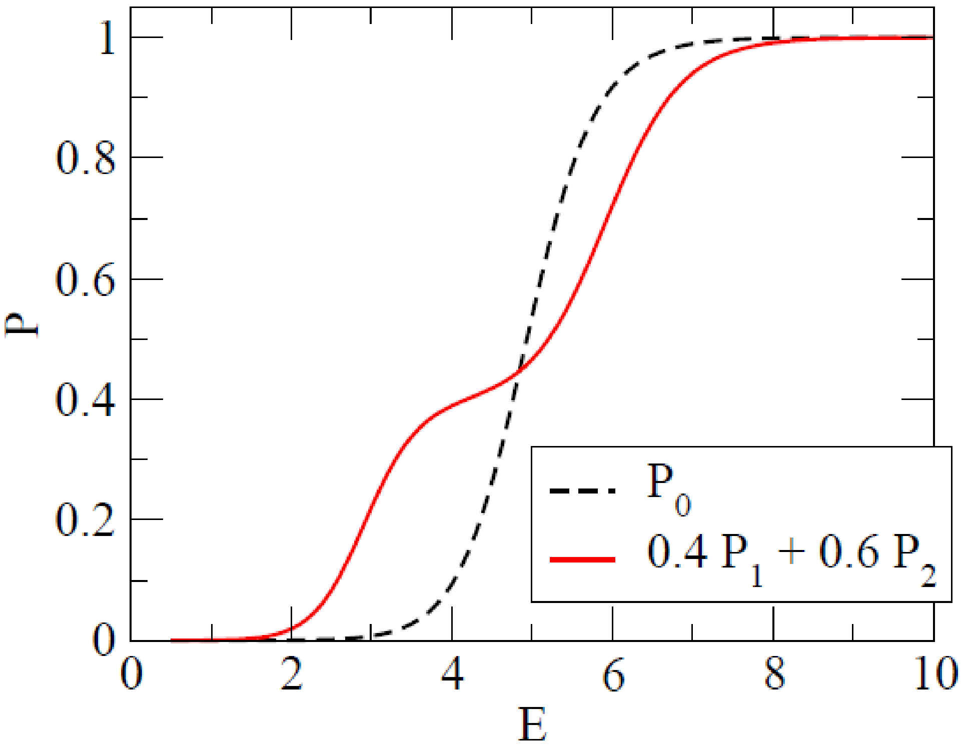
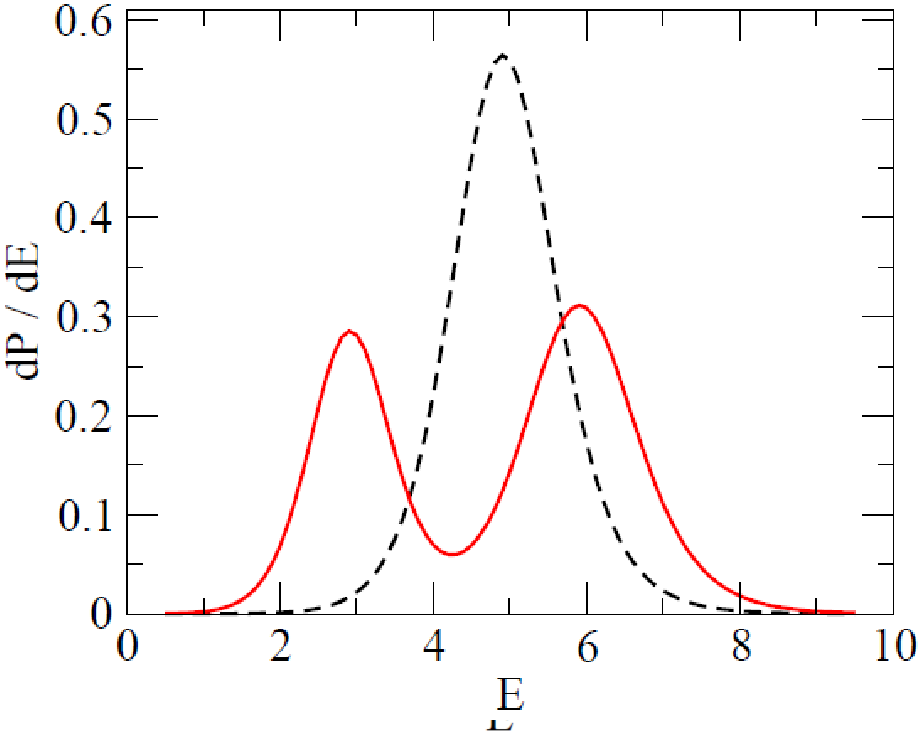
barrier distribution: a problem of two potential barriers

$$P(E) = P(E; V_0) \rightarrow w_1 P(E; V_1) + w_2 P(E; V_2)$$

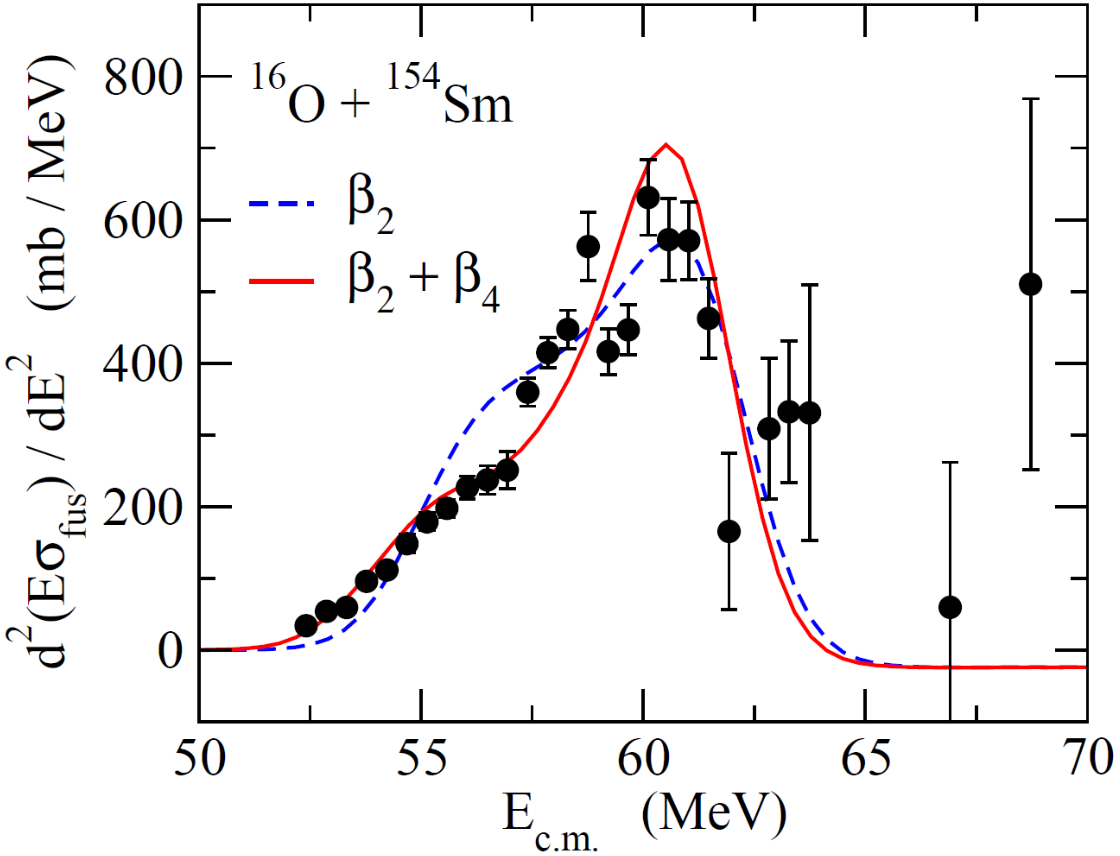




Fusion barrier distribution

$$D_{\mathsf{fus}}(E) = \frac{d^2(E\sigma_{\mathsf{fus}})}{dE^2}$$

- N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 (‘91) 25
- J.X. Wei, J.R. Leigh et al., PRL67 (’91) 3368
- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48 (’98) 401
- A.M. Stefanini et al., Phys. Rev. Lett. 74 (‘95) 864



sensitive to nuclear structure