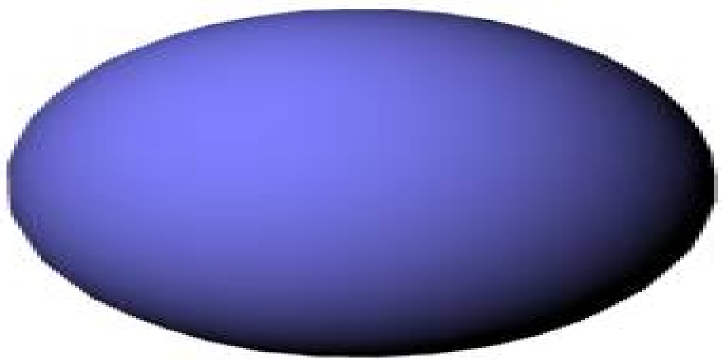
Nuclear Deformation



Nuclear Deformation

cf. Rotational energy of a rigid body
(Classical mechanics)

Excitation spectra of ^{154}Sm

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \;\; \omega = \dot{\theta})$$

0.903 (MeV) — 8^+

0.544 — 6^+

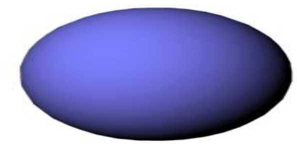
0.267 — 4^+

0.082 — 2^+

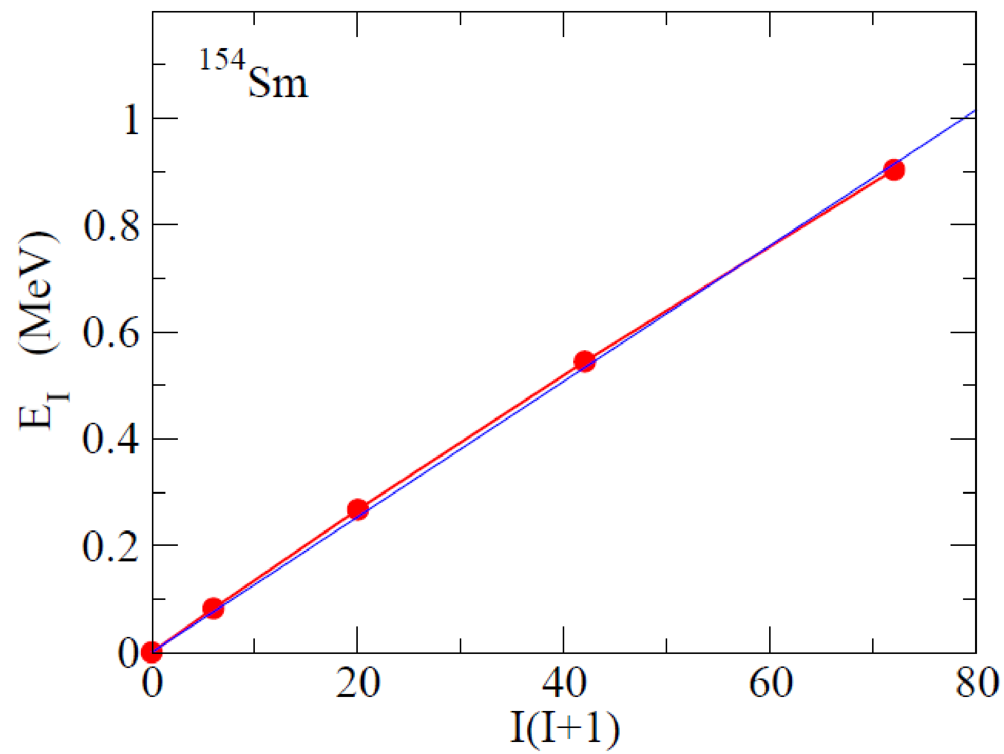
0 — 0^+

^{154}Sm

^{154}Sm is deformed



$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

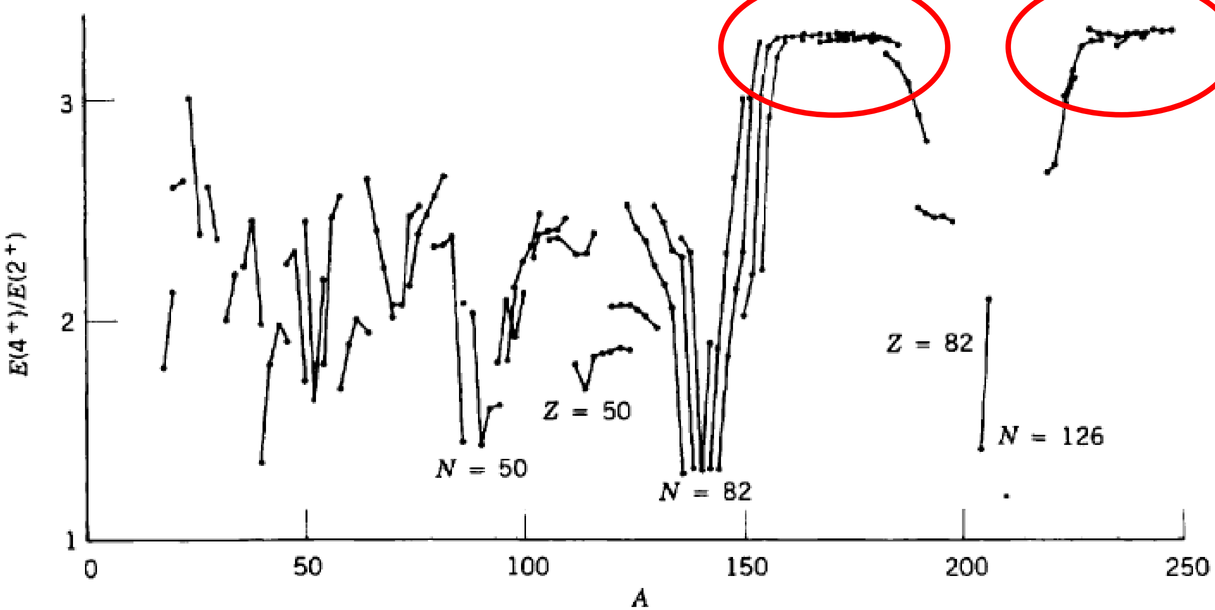


$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

$$\Rightarrow E_2 \propto 2 \times 3 = 6, \quad E_4 \propto 4 \times 5 = 20$$

$$\Rightarrow E_4/E_2 = 20/6 = 3.3333\cdots$$

E(4⁺)/E(2⁺)

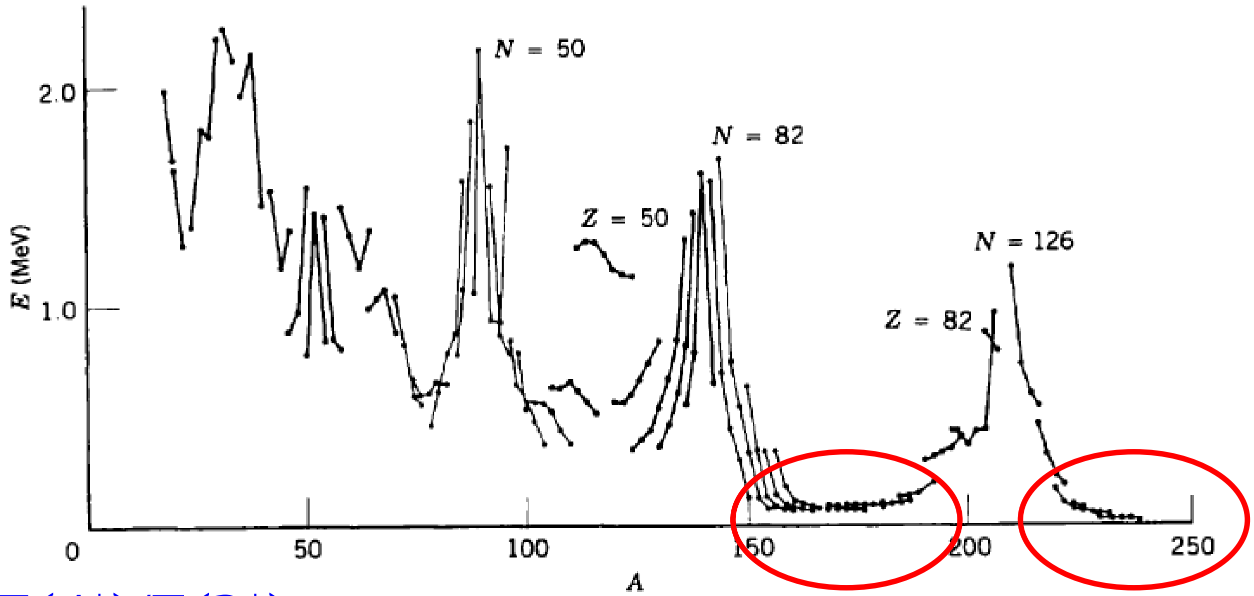


deformed nuclei:
$E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:
$E(4^+)/E(2^+) \sim 2$

K.S. Krane, "Introductory Nuclear Physics"

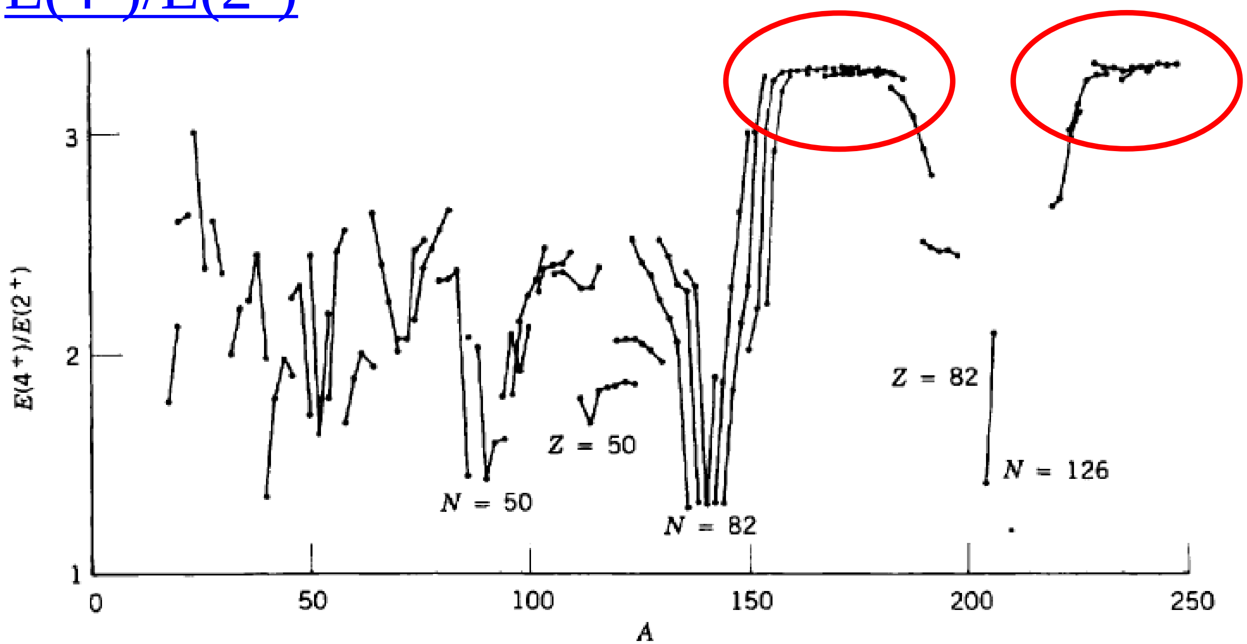
The energy of the first 2^+ state in even-even nuclei



a small energy
→ spontaneously
symm. breaking

deformed nuclei

$E(4^+)/E(2^+)$



deformed nuclei:
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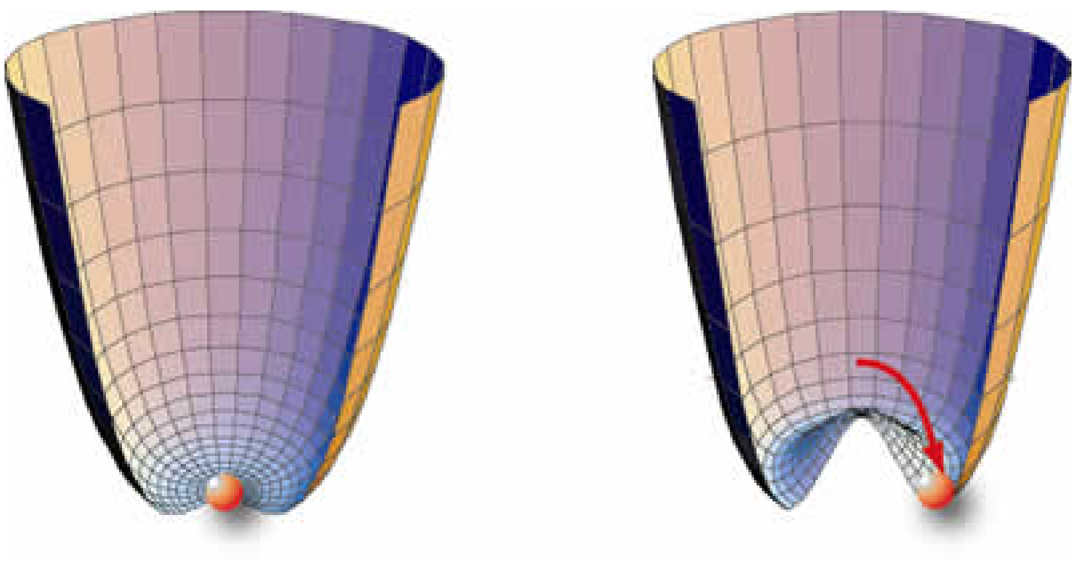
K.S. Krane, "Introductory Nuclear Physics"

Spontaneous symmetry breaking

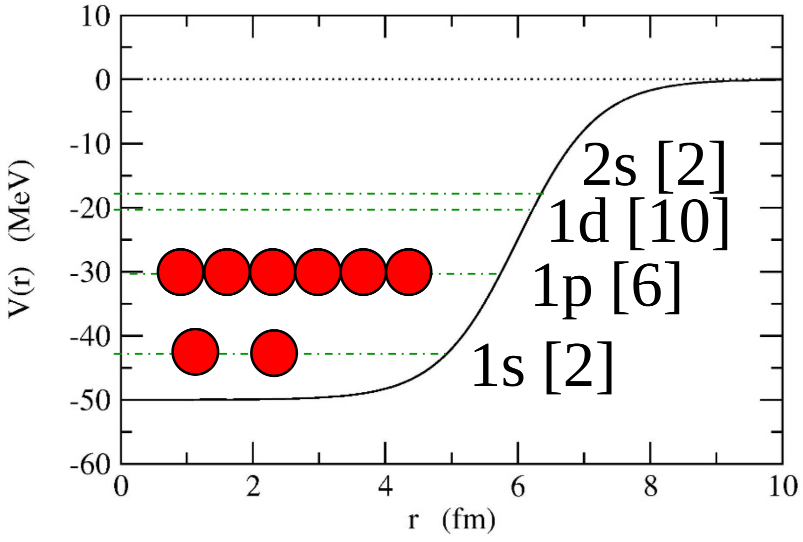
The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.



Nambu-Goldstone mode (zero energy mode) to restore the symmetry

Mean-field approximation and deformation

Mean-field approximation



$$H \sim \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\mathsf{MF}}(\boldsymbol{r}_i) \right)$$

Slater determinant

$$\Psi_{\mathsf{MF}}(1,2,\cdots,A) = \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)]$$

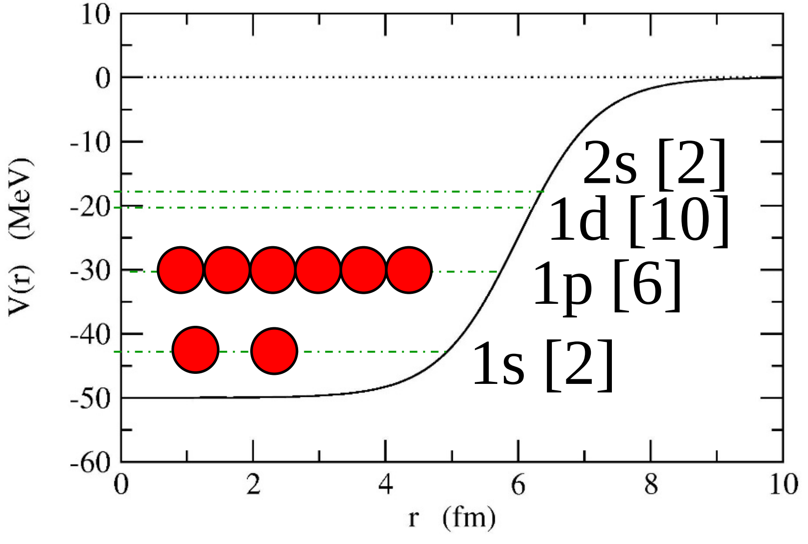
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathsf{MF}}(\boldsymbol{r}) \right)\psi_k(\boldsymbol{r}) = \epsilon_k\psi_k(\boldsymbol{r})$$

the original many-body *H*:

$$\begin{aligned} H &= -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) \\ &= \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\mathsf{MF}}(\boldsymbol{r}_i) \right) + \cancel{\frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) - \sum_i V_{\mathsf{MF}}(\boldsymbol{r}_i)} \end{aligned}$$

Mean-field approximation and deformation

Mean-field approximation



$$H \sim \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\mathrm{MF}}(\boldsymbol{r}_i) \right)$$

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$$H = \sum_{i=1}^{A} \left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\mathrm{MF}}(\boldsymbol{r}_i) \right) + \cancel{\frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) - \sum_i V_{\mathrm{MF}}(\boldsymbol{r}_i)}$$

Ψ_{MF} : does not necessarily possess the symmetries that H has.

"Symmetry-broken solution"
"Spontaneous Symmetry Broken"

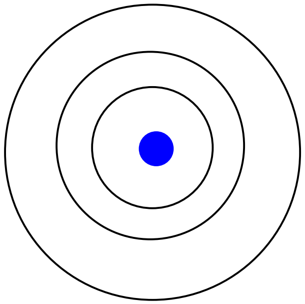
Ψ_{MF} : does not necessarily possess the symmetries that H has.

Typical Examples

- Translational symmetry: always broken in nuclear systems

$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i - \boldsymbol{r}_j) \to \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \underline{\underline{V_{\mathrm{MF}}(\boldsymbol{r}_i)}}\right)$$

(cf.) atoms

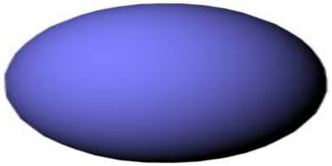


nucleus in the center

$\longrightarrow$ translational symmetry: broken from the begining

- Rotational symmetry

Deformed solution



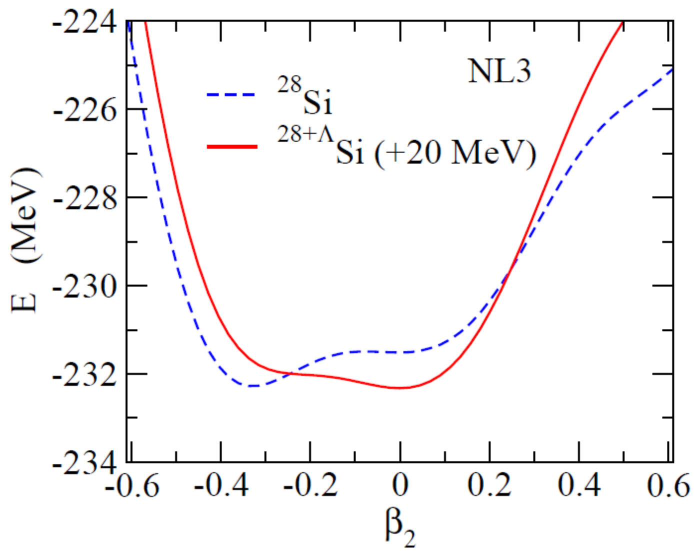
Constrained Hartree-Fock method

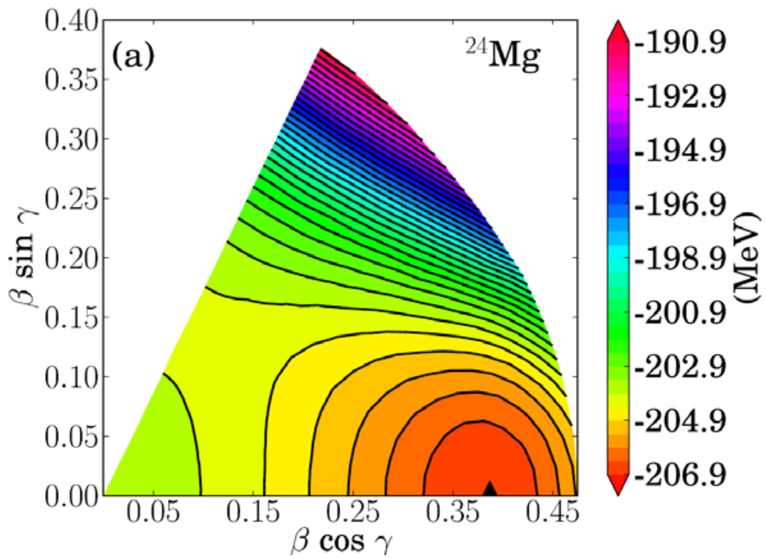
minimize $H' = H - \lambda \hat{Q}_{20}$ with a Slater determinant w.f.

$\hat{Q}_{20} = \sum_i r_i^2 Y_{20}(\hat{\boldsymbol{r}}_i)$: quadrupole operator

λ : Lagrange multiplier, to be determined so that $\langle \hat{Q}_{20} \rangle = Q \propto R^2\beta$

$\longrightarrow$ $E(\beta)$: potential energy curve





$E(\beta,\gamma)$: potential energy surface

One-particle motion in a deformed potential

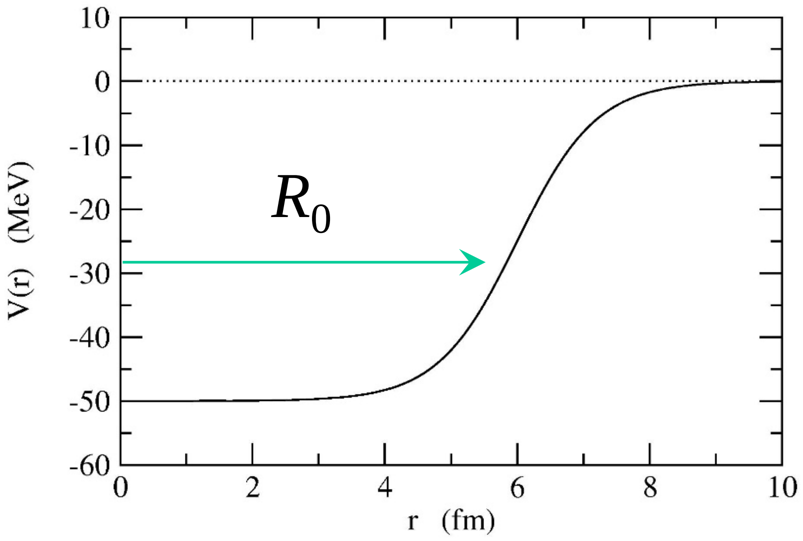
$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$

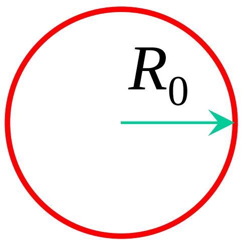
if the density is deformed, so is the mean-field potential

(example) a deformed Woods-Saxon potential

a spherical WS potential:

$$V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R_0}{a}\right)}$$





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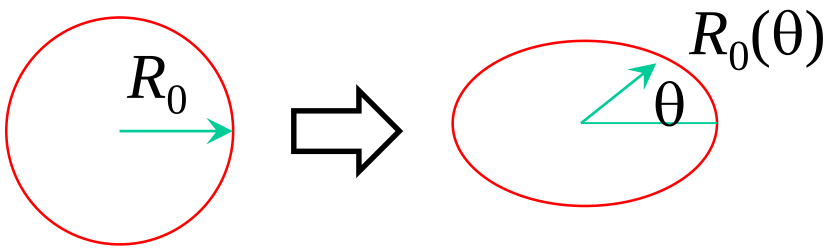
a WS potential:

$$V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R_0}{a}\right)}$$

$$\boxed{\begin{aligned} &V(r) \\ &\to V(r,\theta) = -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)} \end{aligned}}$$



$$R_0 \to R_0(1+\beta_2 Y_{20}(\theta))$$

One-particle motion in a deformed potential

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$

if the density is deformed, so is the mean-field potential

(example) a deformed Woods-Saxon potential

$$V(r,\theta) = -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)} = -\frac{V_0}{1+\exp\left(\frac{r-R_0-R_0\beta_2 Y_{20}(\theta)}{a}\right)}$$

$$\sim -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)} - R_0\beta_2 Y_{20}(\theta)\frac{d}{dr}\left[\frac{-V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)}\right]$$

$$\equiv V_0(r) + V_2(r)Y_{20}(\theta) \qquad V_2(r) = -R_0\beta_2 V_0'(r)$$

One-particle motion in a deformed potential

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$

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* non-spherical potential → angular momentum: not conserved

One-particle motion in a deformed potential

$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

■the effect of Y_{20} term

Eigen-functions for β_2=0 (spherical pot.) :

$$\psi_{nll_z}(\boldsymbol{r}) = R_{nl}(r) Y_{ll_z}(\hat{\boldsymbol{r}})$$

eigen-values: E_{nl} (no dependence on l_z)

The change of energy due to the Y_{20} term (1st order perturbation theory):

$$E_{nl} \quad \rightarrow \quad E_{nl} + \underline{\langle \psi_{nll_z} | \Delta V | \psi_{nll_z} \rangle}$$

$$\Delta V(r) = -\beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta)$$

One-particle motion in a deformed potential

$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

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$$\psi_{nll_z}(\boldsymbol{r}) = R_{nl}(r) Y_{ll_z}(\hat{\boldsymbol{r}})$$

$$\Delta E = -\beta_2 R_0 \int_0^\infty r^2 dr |R_{nl}(r)|^2 V_0'(r) \times \underline{\int d\hat{\boldsymbol{r}} Y^*_{ll_z}(\theta) Y_{20}(\theta) Y_{ll_z}(\theta)}$$

$$+ \qquad +$$

$$\propto -(3l_z^2 - l(l+1))$$

One-particle motion in a deformed potential

$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

■the effect of Y_{20} term

$$E_{nl} \rightarrow E_{nl} + \langle \psi_{nll_z} | \Delta V | \psi_{nll_z} \rangle$$

$$\Delta E = -\beta_2 R_0 \int_0^\infty r^2 dr |R_{nl}(r)|^2 V_0'(r)$$

+ +

$$\times \underline{\int d\hat{\boldsymbol{r}} Y^*_{ll_z}(\theta) Y_{20}(\theta) Y_{ll_z}(\theta)}$$

$$\propto -(3l_z^2 - l(l+1))$$

$$\equiv \beta_2 \times \alpha_{nl} (3l_z^2 - l(l+1)) \quad (\alpha_{nl} > 0)$$

One-particle motion in a deformed potential

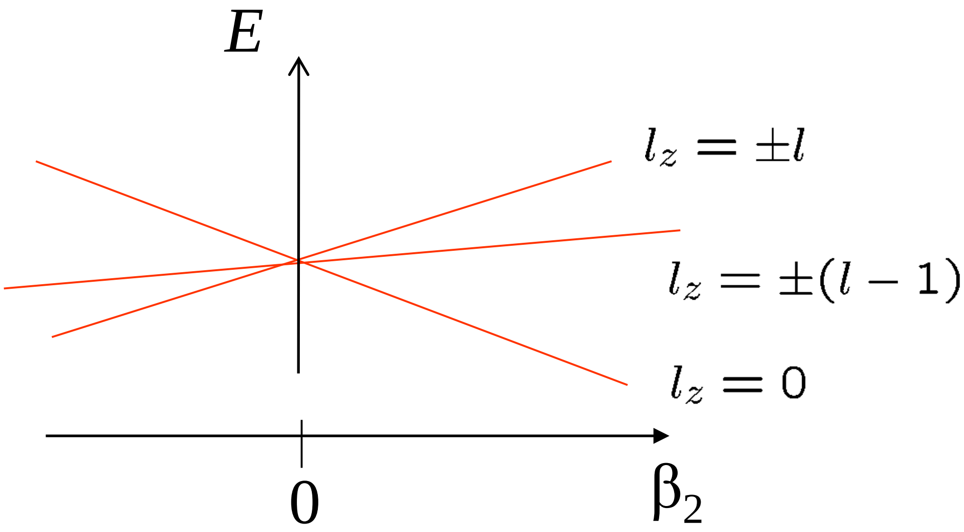
$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

$$\Delta E = \beta_2 \times \alpha_{nl} \left(3l_z^2 - l(l+1)\right) \quad (\alpha_{nl} > 0)$$

for $\beta_2 > 0$

$\Delta E < 0$ ($l_z = 0$)

$\Delta E > 0$ ($l_z = l$)

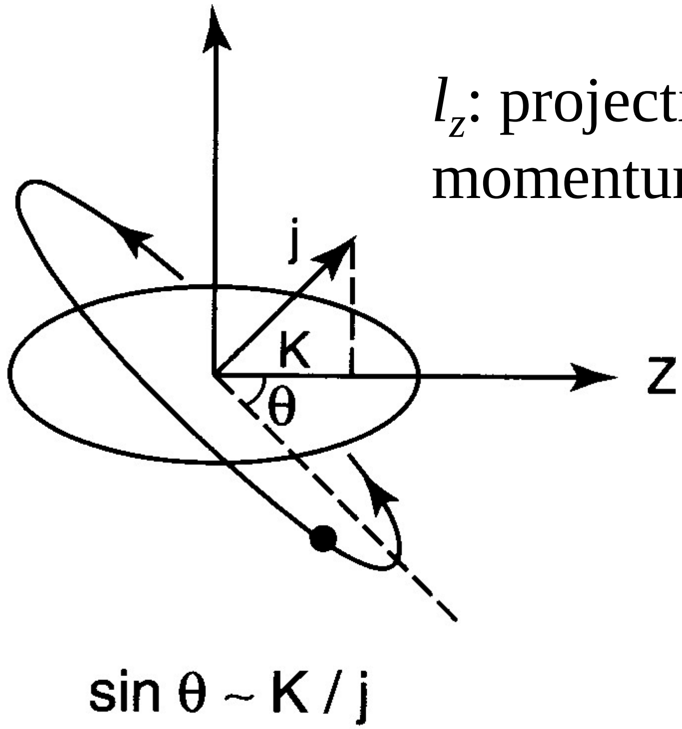


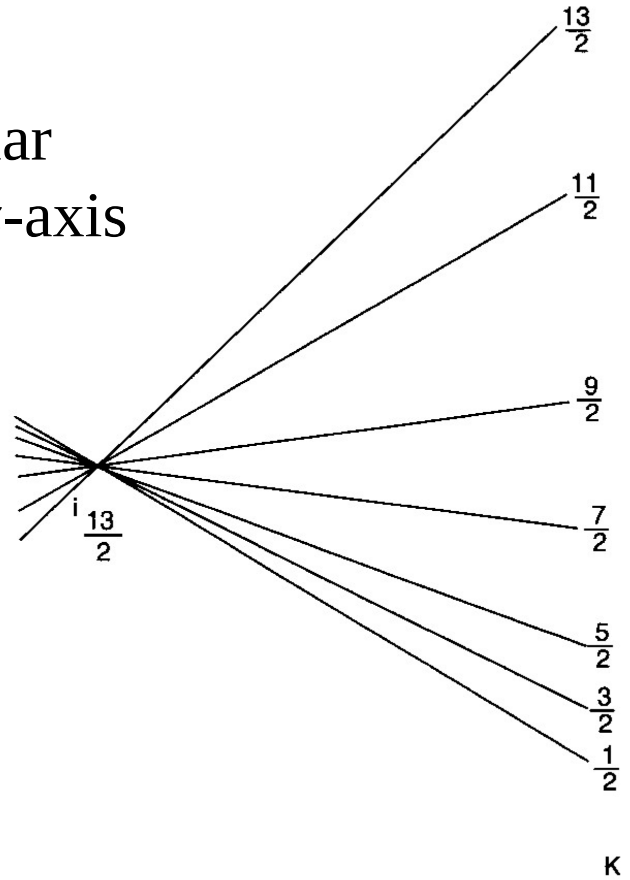
✓ degeneracy: resolved
(E: now depends on l_z)

✓ degeneracy: $+l_z$ and $-l_z$

Geometrical interpretation

l_z: projection of angular momentum onto the z-axis

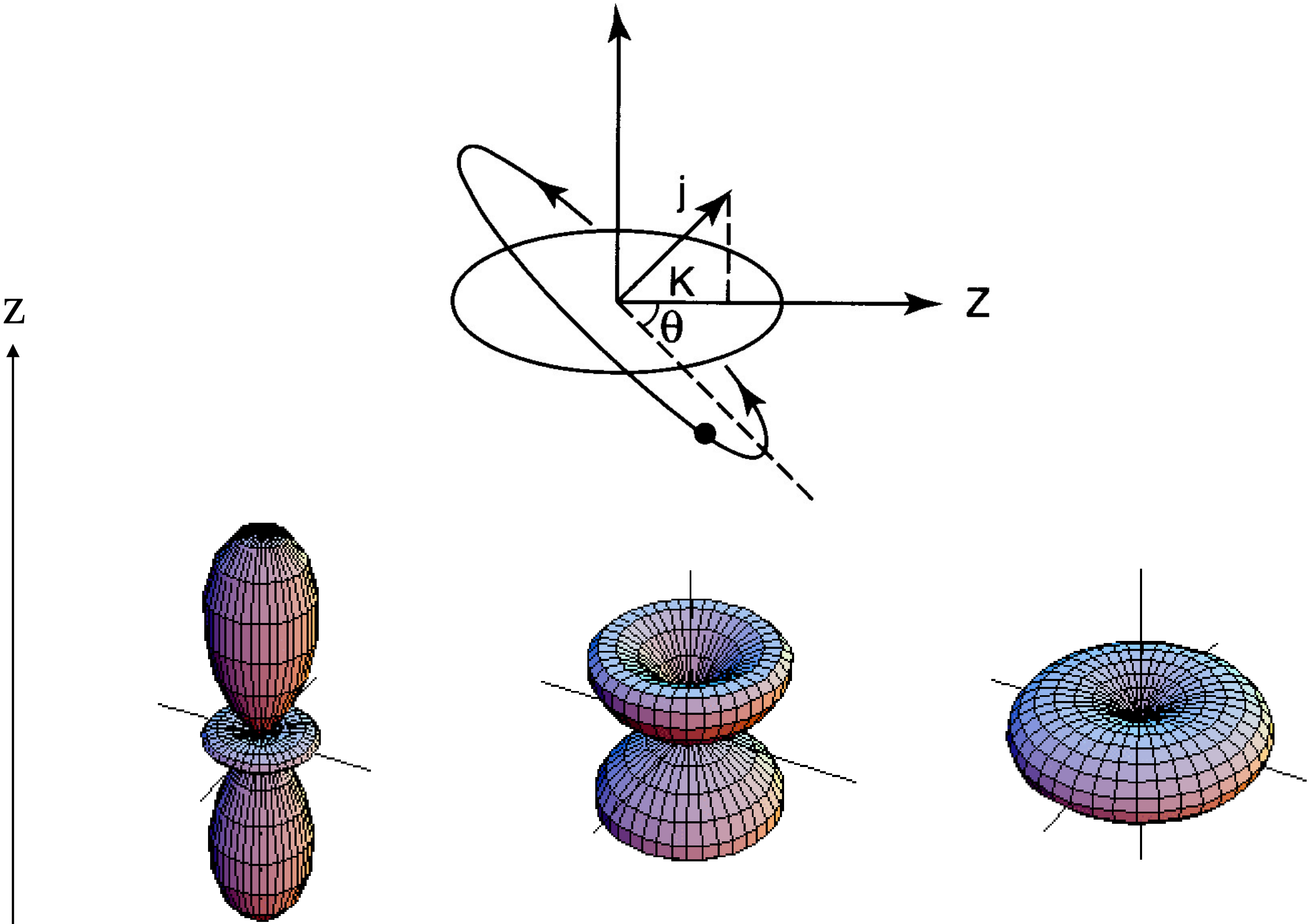




for $\beta_2 > 0$

small l_z $\longleftrightarrow$ a motion along the longer axis
$\longrightarrow$ the energy is lowered

large l_z $\longleftrightarrow$ a motion along the shorter axis
$\longrightarrow$ the energy is increased



$r = Y_{20}$ (l_z=0)

$r = Y_{21}$ (l_z=1)

$r = Y_{22}$ (l_z=2)

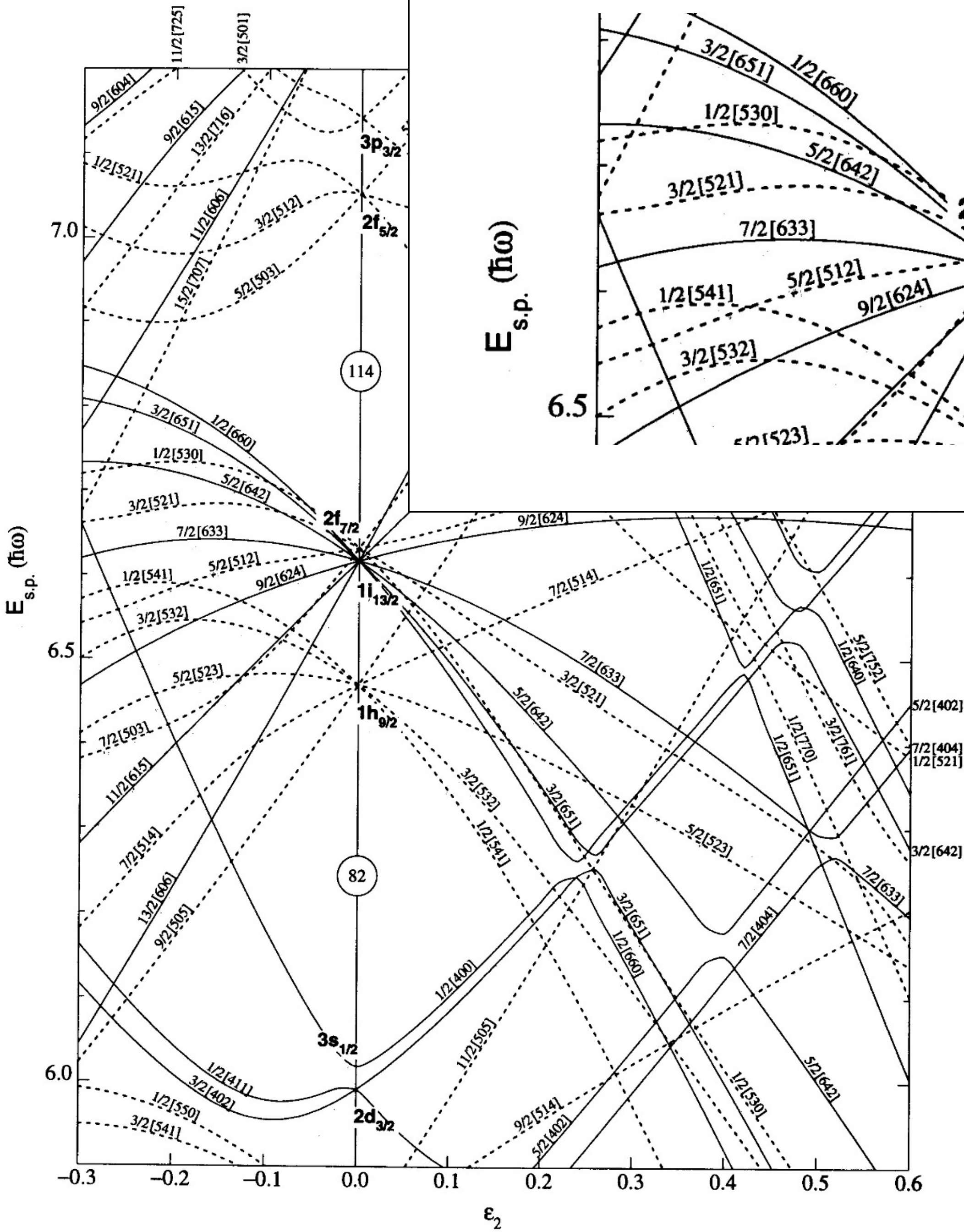
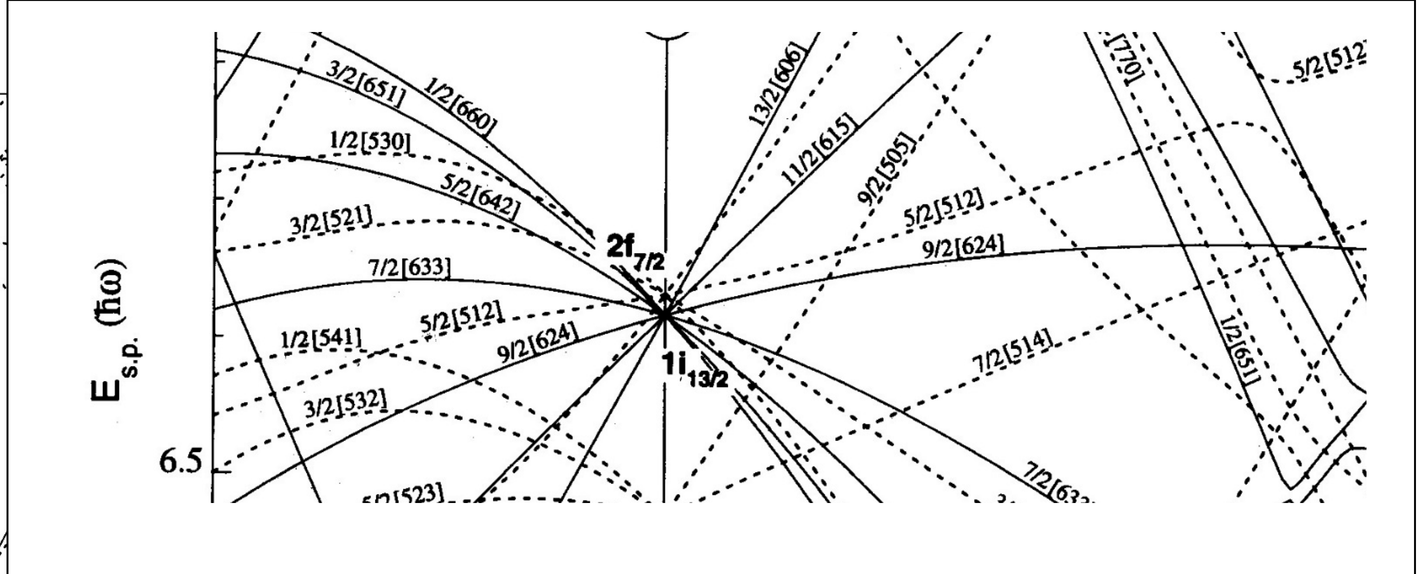


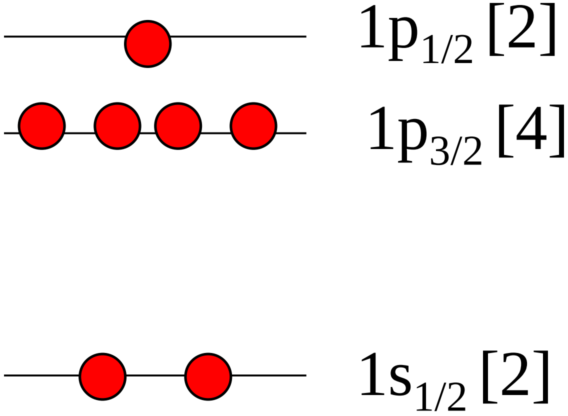
Figure 13. Nilsson diagram for protons, $Z \geq 82$ ($\varepsilon_4 = \varepsilon_2^2/6$).



Nilsson diagram
(without perturbation)

Level scheme of $^{11}_{4}Be_{7}$

With a spherical potential :

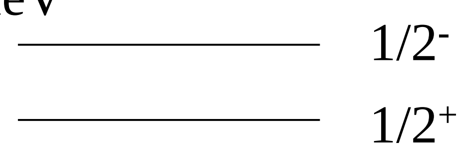


The g.s. of ^{11}Be : $I^{\pi} = 1/2^{-}$

very artificial

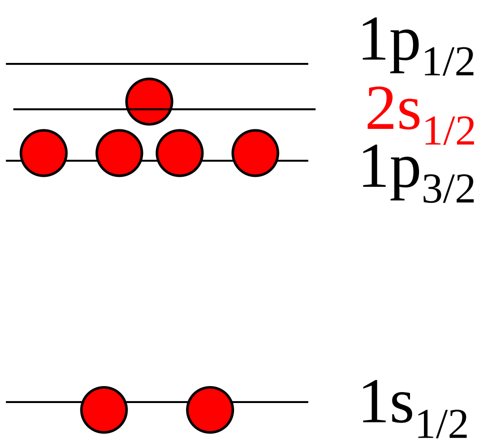
In reality…..

0.32 MeV



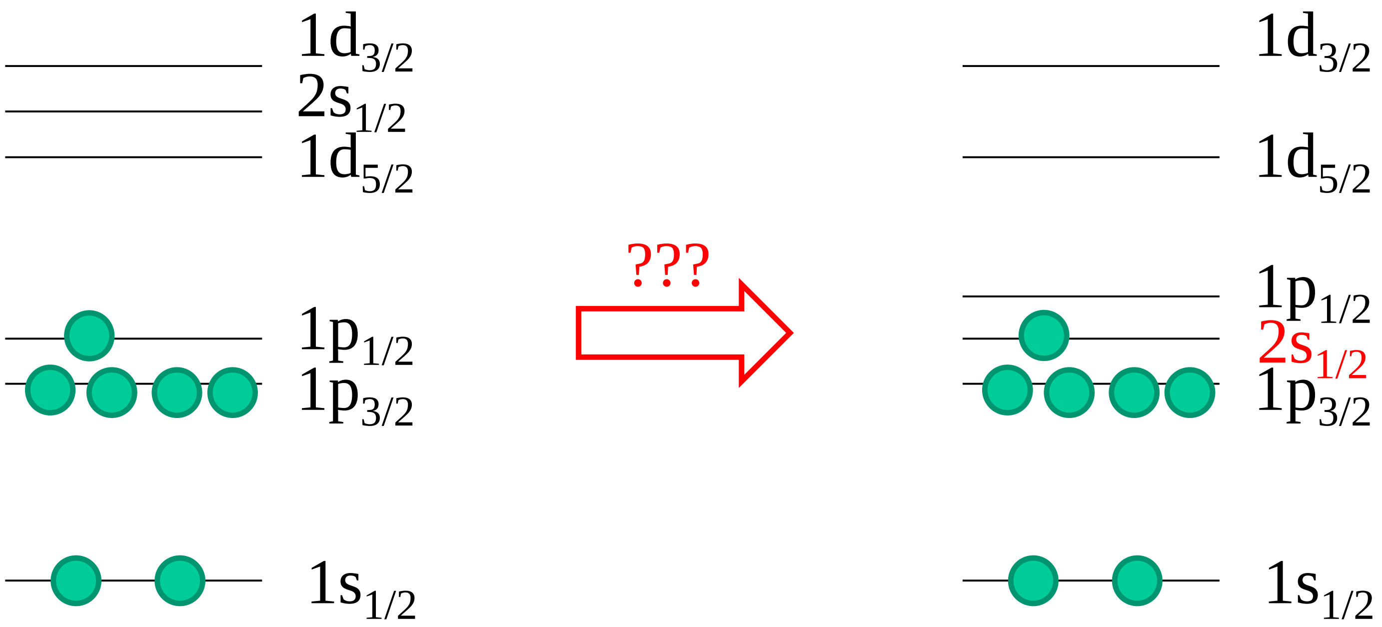
^{11}Be

?



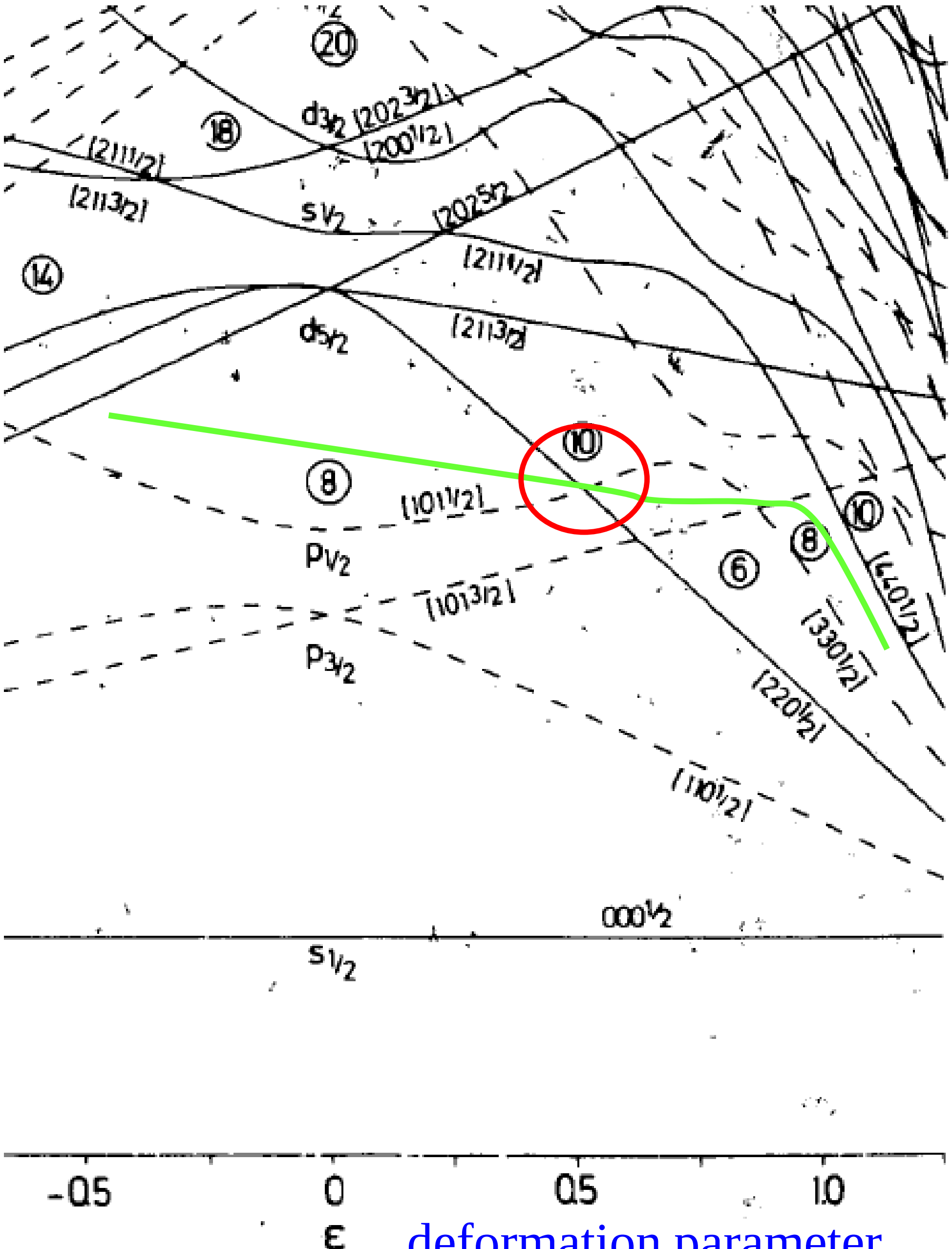
"parity inversion"

What happens if ^{11}Be is deformed?

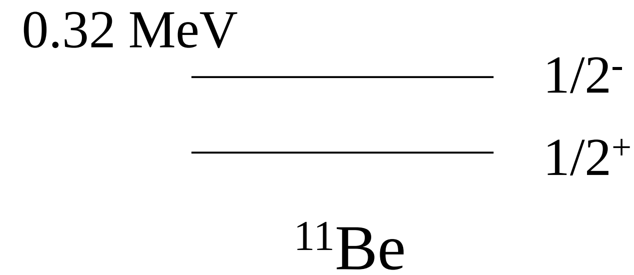


Very unnatural.
The observed $1/2^+$ state can be more naturally explained if one considers a deformation of ^{11}Be.

$^{11}_{4}Be_{7}$



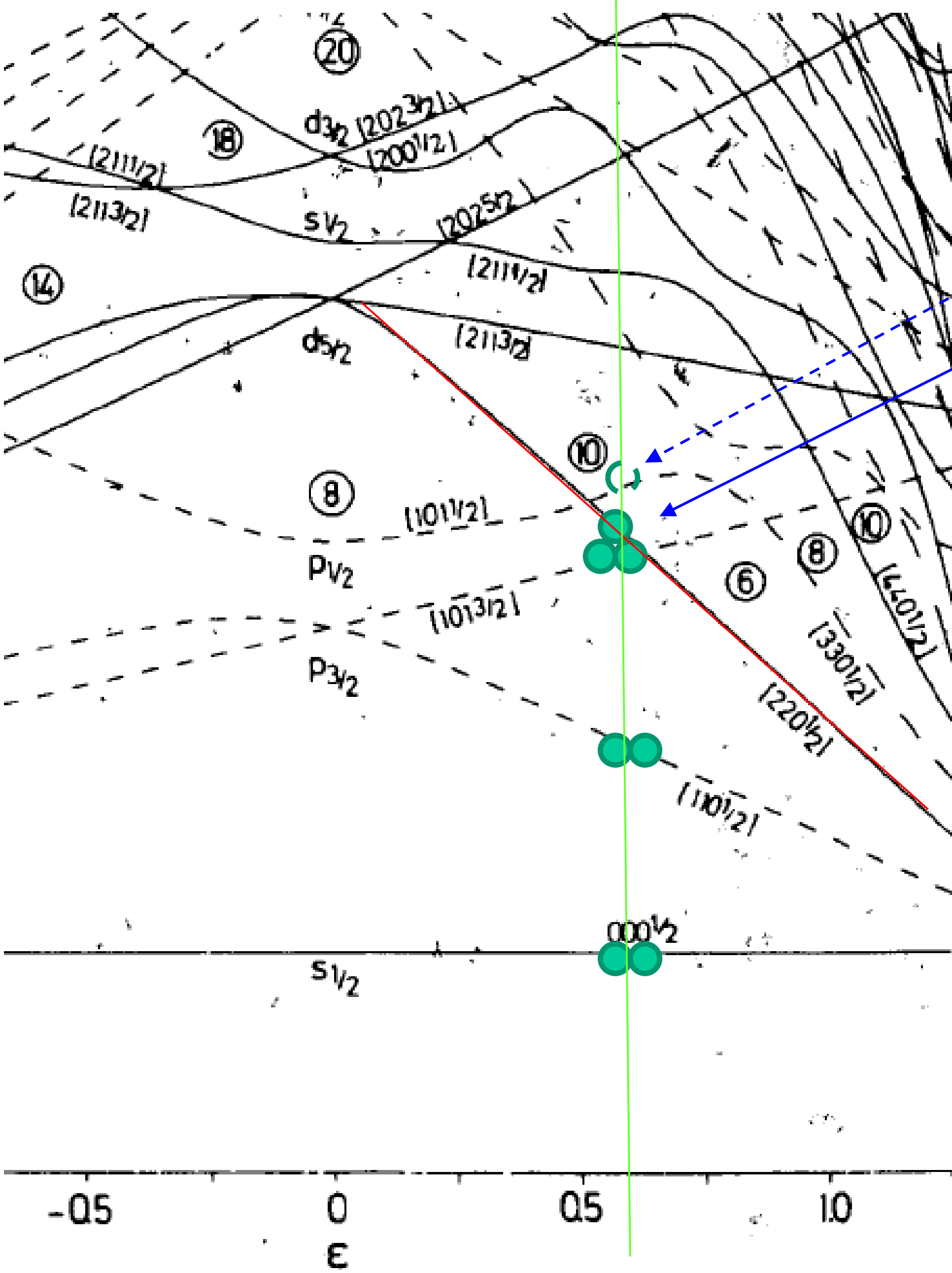
deformation parameter

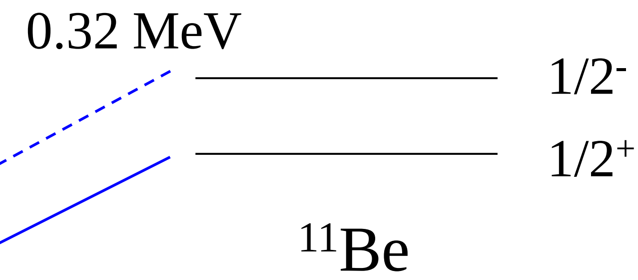


- assume some deformation, and put 2 nucleons in each level from the bottom (degeneracy of $+K$ and $-K$)
- Look for the level which is occupied by the valence nucleon (the 7th level for ^{11}Be)
- Identify the value of K^{π} for that level with the spin and parity of the whole nucleus.

cf. particle-rotor model

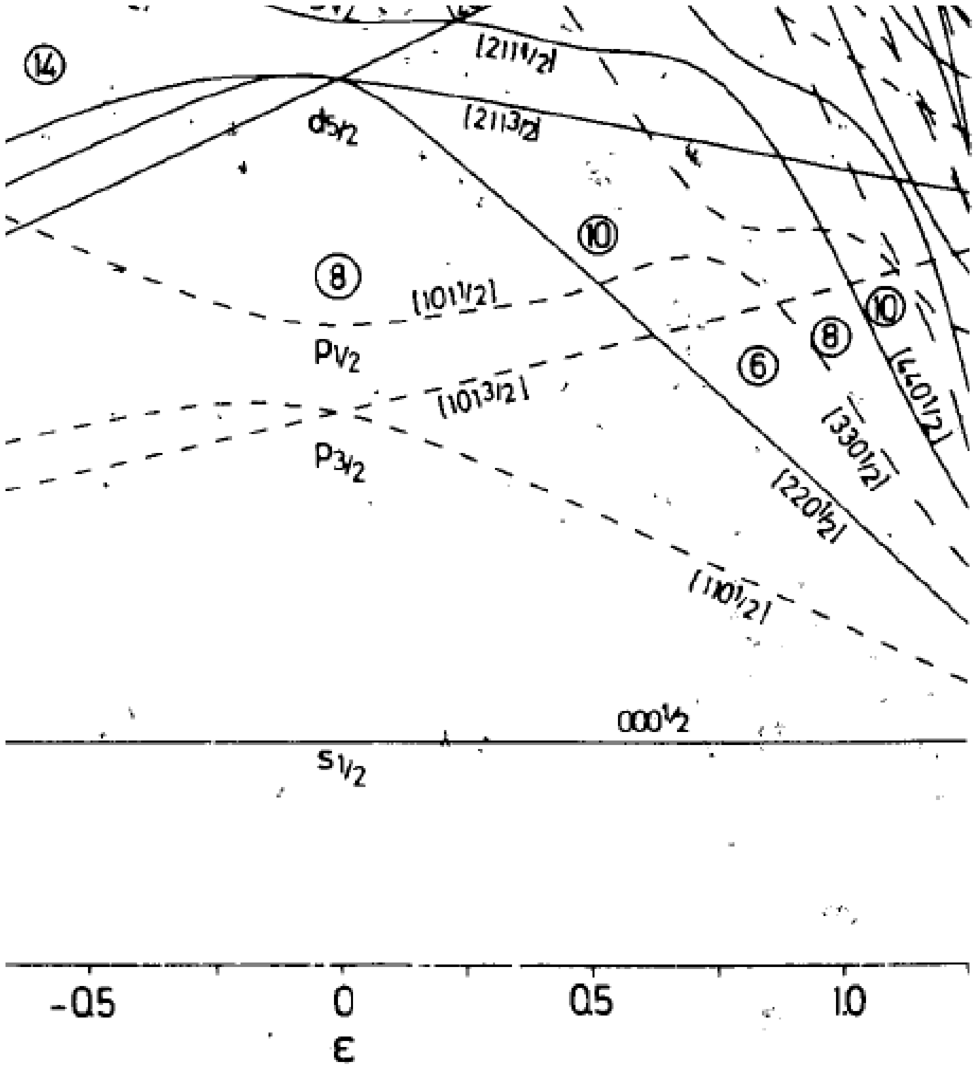
${}^{11}_{4}\mathrm{Be}_{7}$



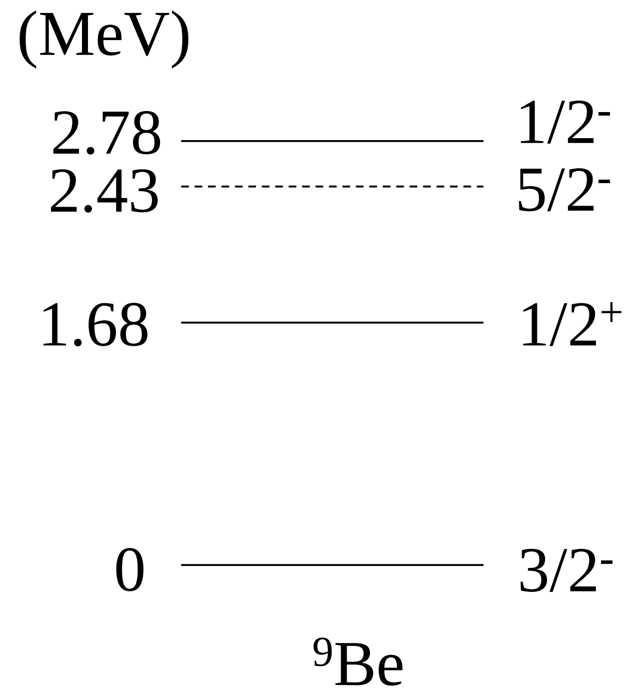


- ➢ assume some deformation, and put 2 nucleons in each level from the bottom
- ➢ Look for the level which is occupied by the valence nucleon (the 7th level for ^{11}Be)
- ➢ Identify the value of K^{π} for that level with the spin and parity of the whole nucleus.

Can the level scheme of $^{9}_{4}Be_{5}$ be explained in a similar way?



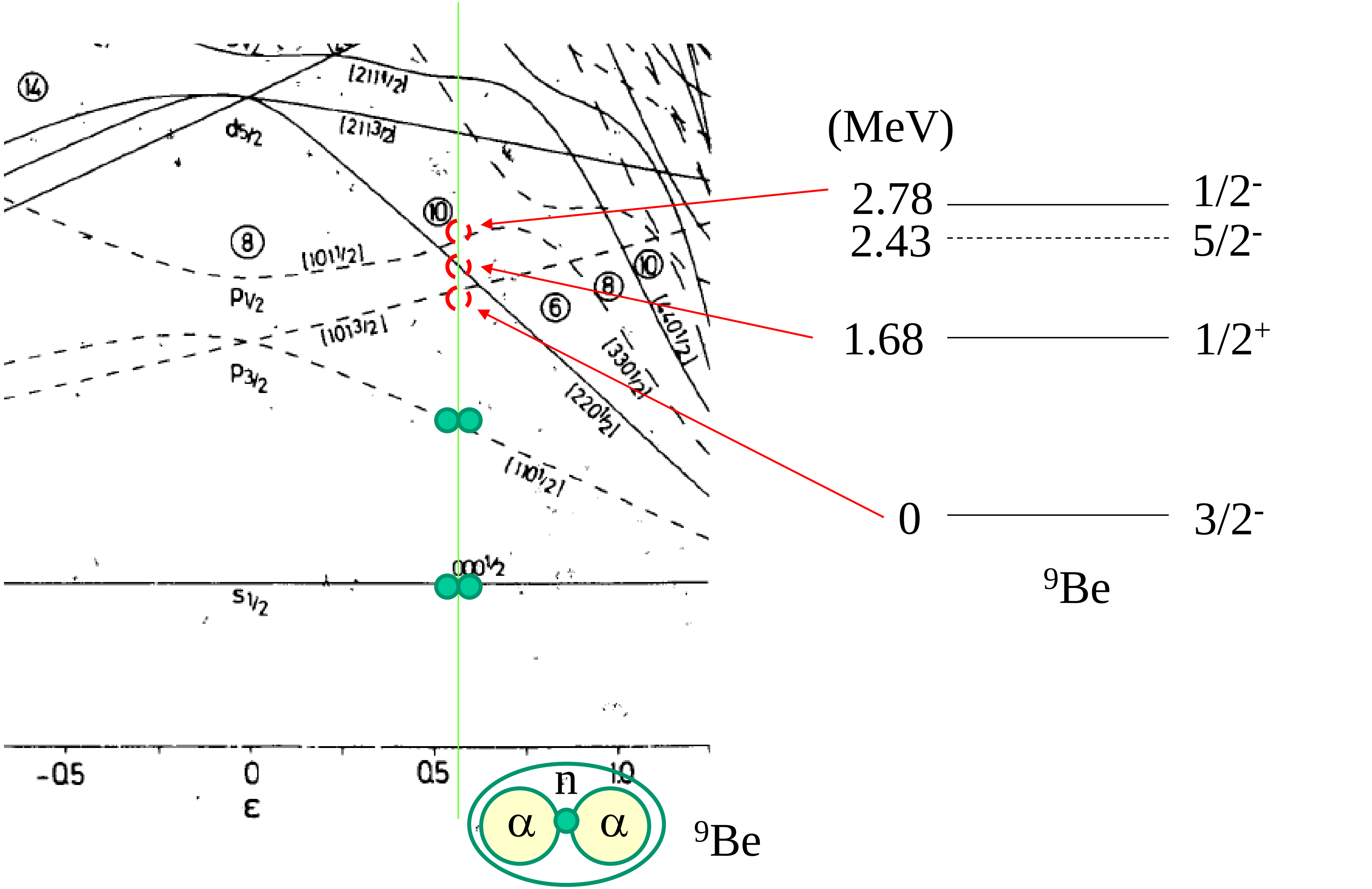
cf. $^{10}B(e,e'K^{+})^{10}{}_{\Lambda}Be$ (= $^{9}Be+\Lambda$)



The $5/2^{-}$ state at 2.43 MeV: rotational state with the same configuration as the g.s. state (not considered here)

Can the level scheme of $^{9}_{4}Be_5$ be explained in a similar way?

cf. $^{10}B(e,e'K^+)^{10}_{\Lambda}Be$ (= $^{9}Be+\Lambda$)



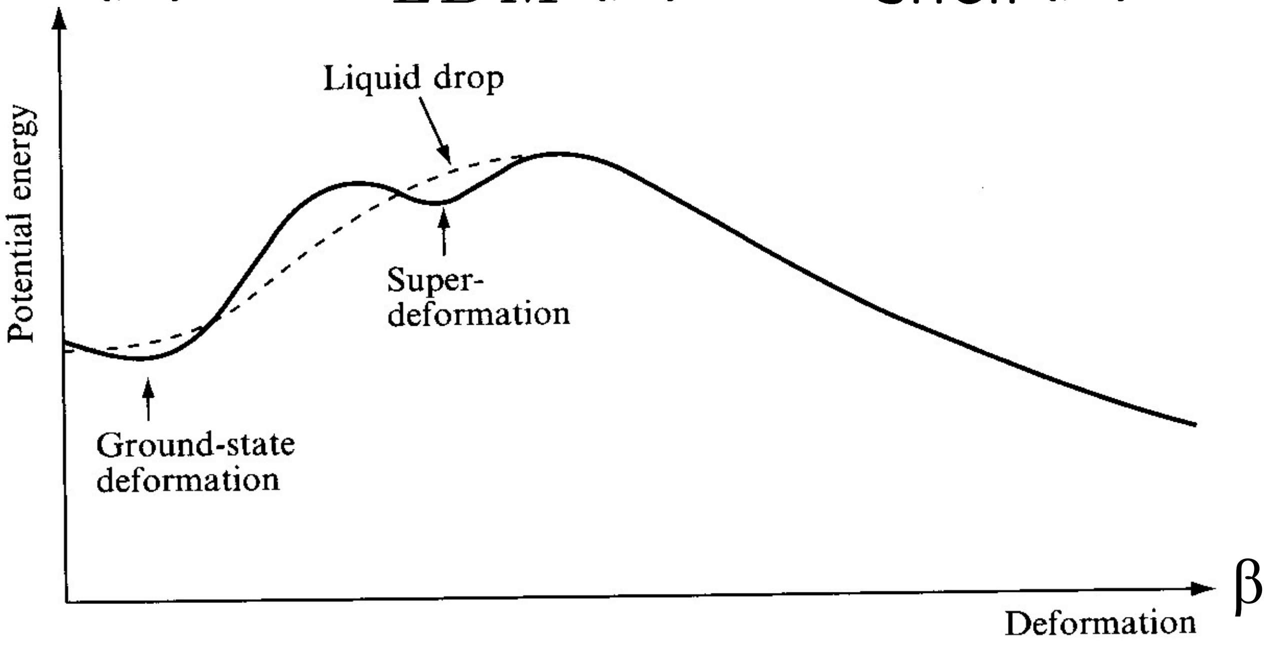
Several topics:

- deformation as a quantum effect
- RMF for deformed hypernuclei
- Quiz: spontaneous symmetry breaking

nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$



LDM only → always spherical ground state

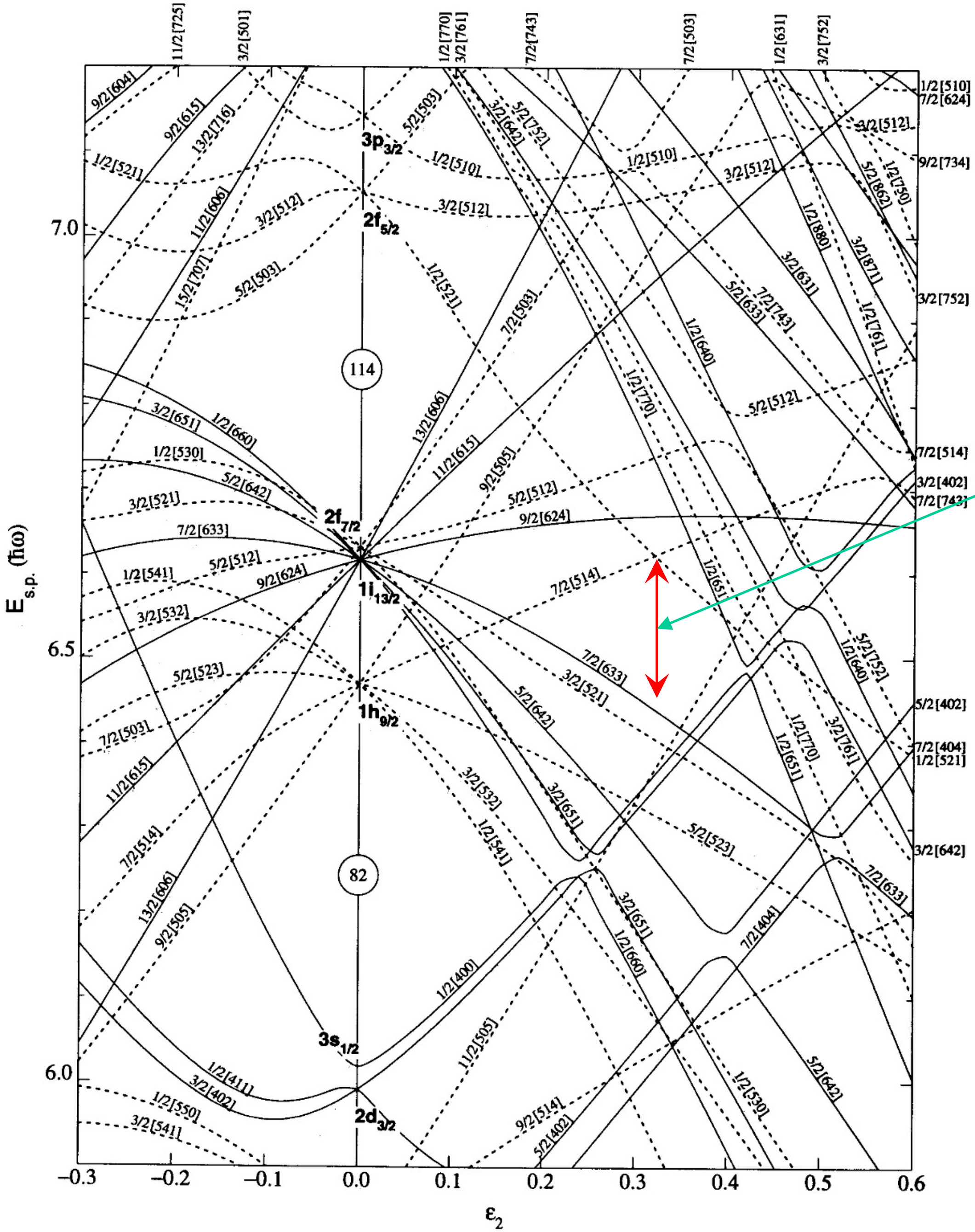


Figure 13. Nilsson diagram for protons, $Z \geq 82$ ($\varepsilon_4 = \varepsilon_2^2/6$).

energy gap if deformed

Nilsson diagram

nuclear deformation

Deformed energy surface for a given nucleus

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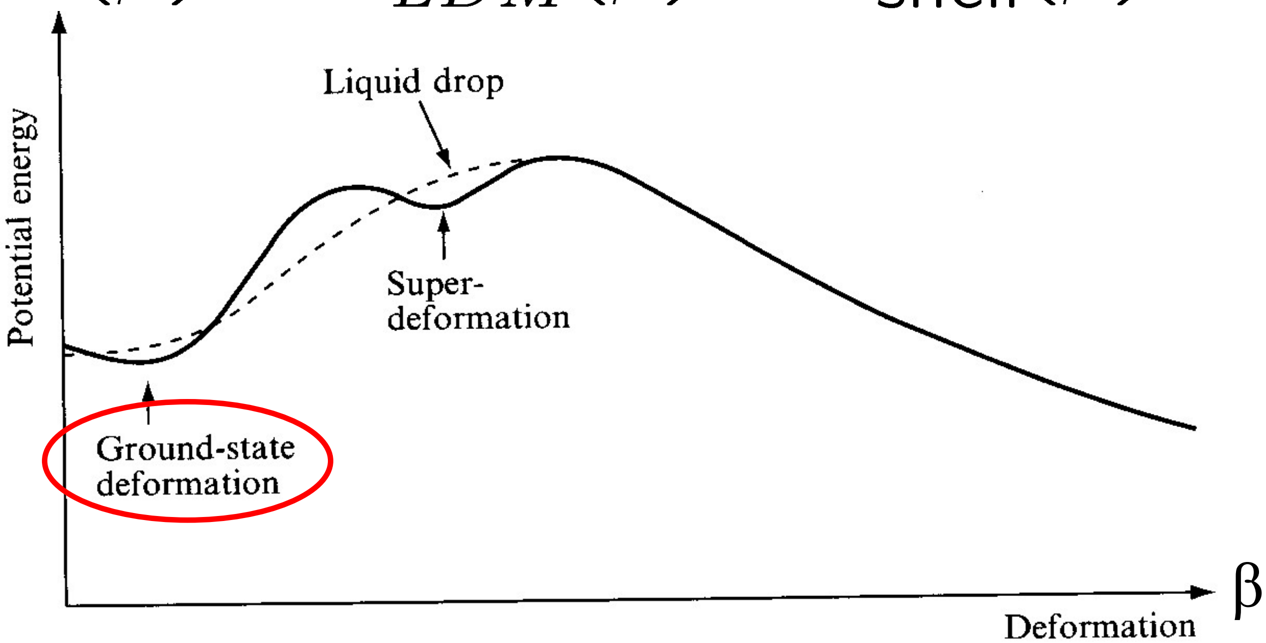
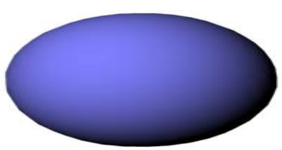




LDM only → always spherical ground state

Shell correction → may lead to a deformed g.s.

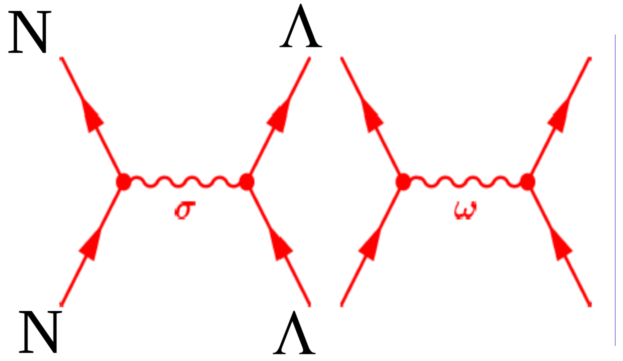
* Spontaneous Symmetry Breaking

RMF calculations for deformed hypernuclei

Hypernuclei: nucleus + Lambda particle

Effect of a Λ particle on nuclear shapes?

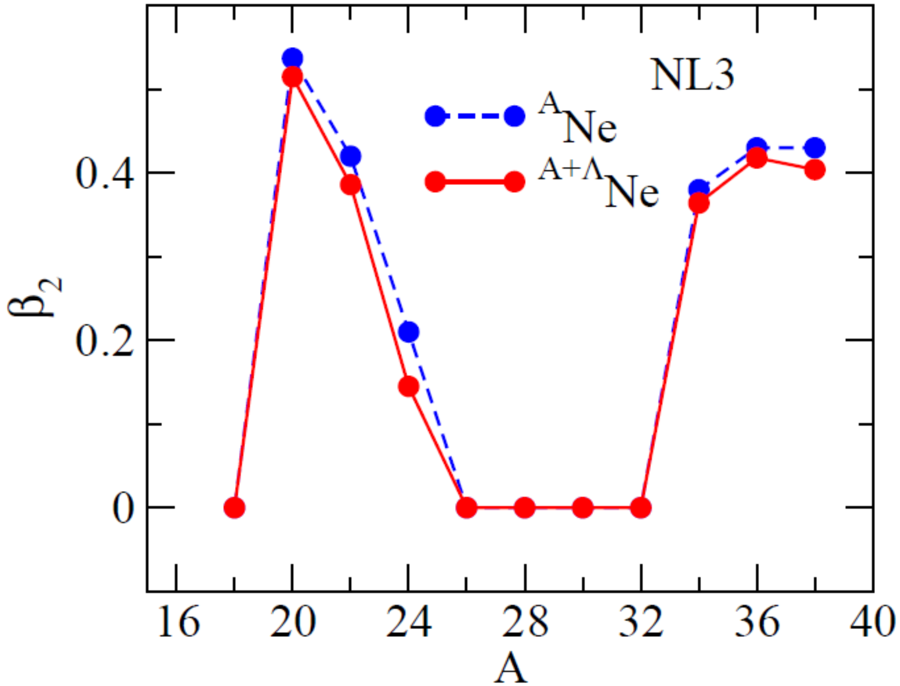
Relativistic Mean-field model



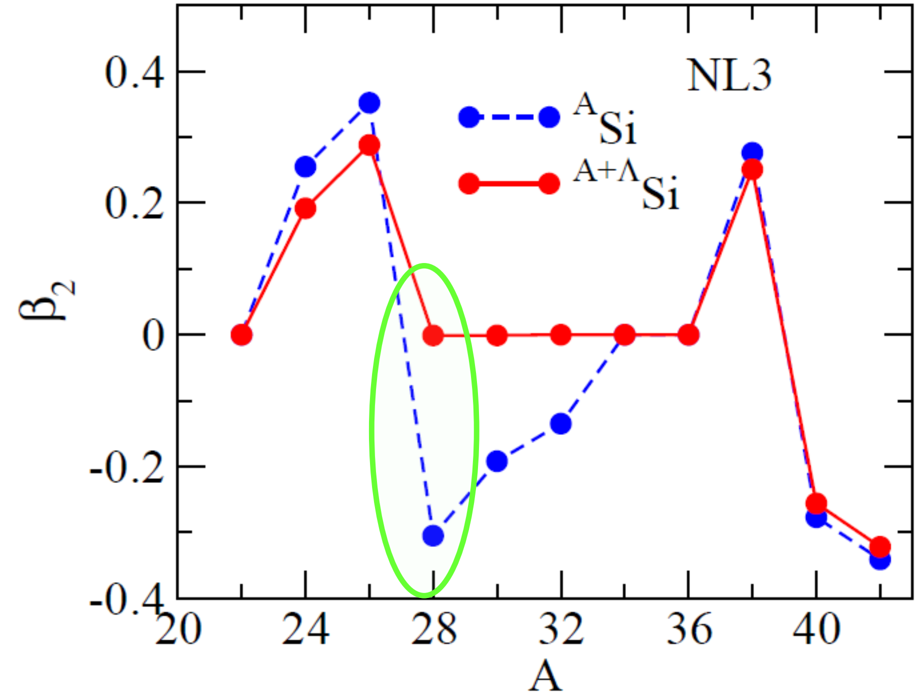
nucleon-nucleon interaction
via meson exchange

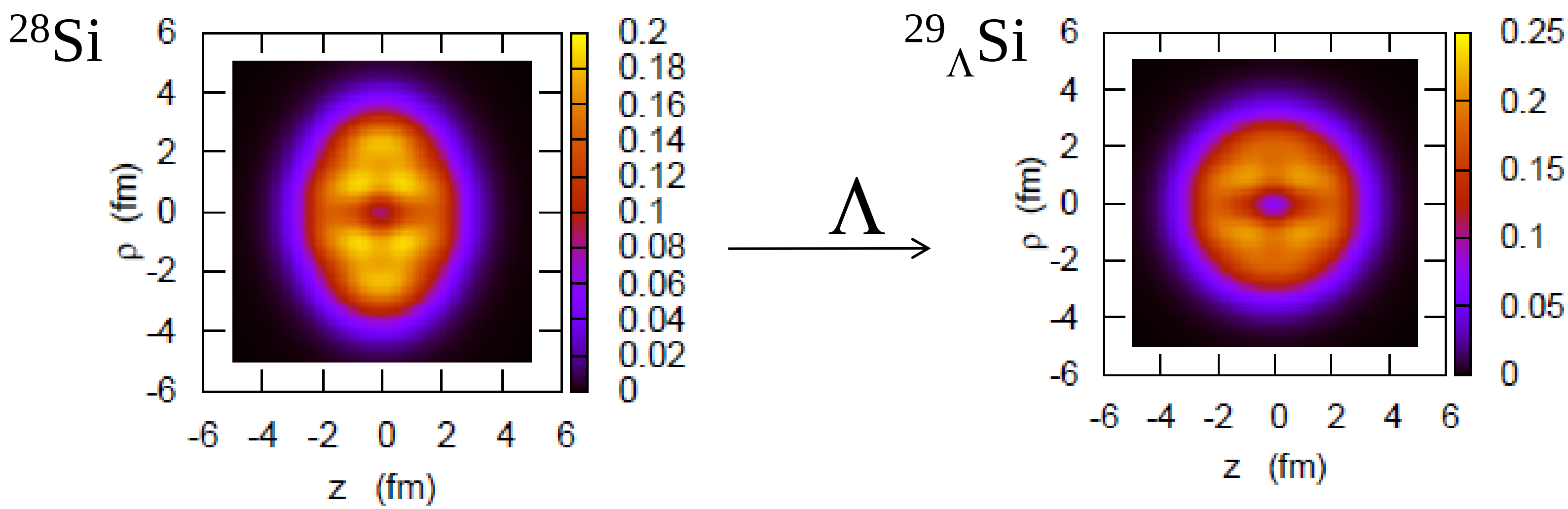
Λσ and Λω couplings

Ne isotopes



Si isotopes





Myaing Thi Win and K.Hagino, PRC78('08)054311

Quiz: spontaneous symmetry breaking

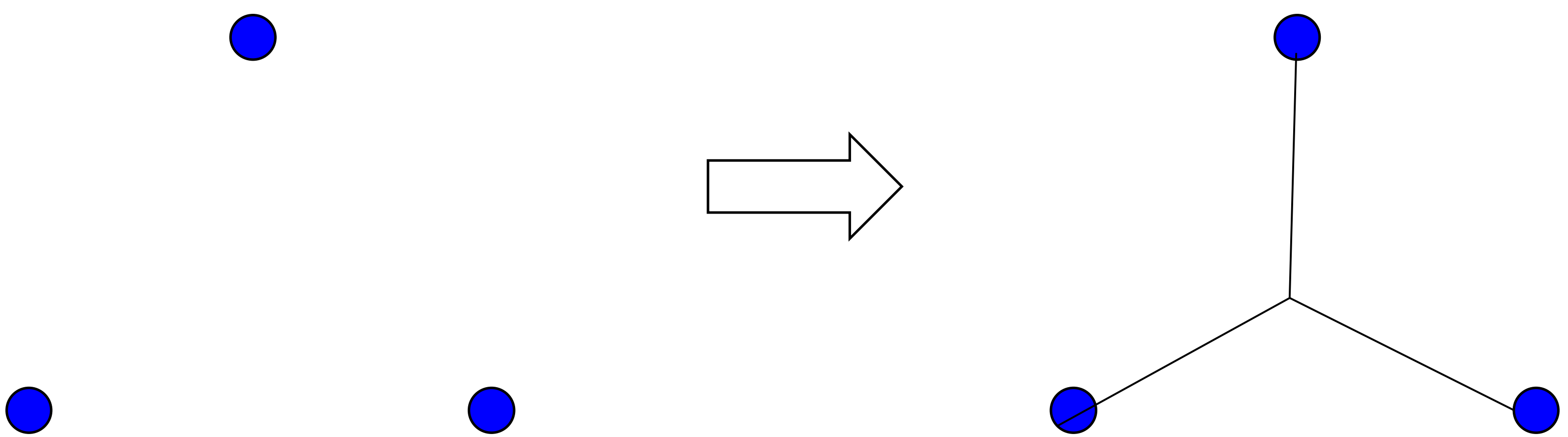
There are a few dots.

- Connect the dots.
- The number of lines is not limited.
- Two lines can cross.
- Connect the dots so that one can go from one dot to all the other dots.

How do you connect the lines if you want to make the total length of lines the shortest?

e.g.) Equilateral triangle

Connect symmetrically



Quiz: spontaneous symmetry breaking

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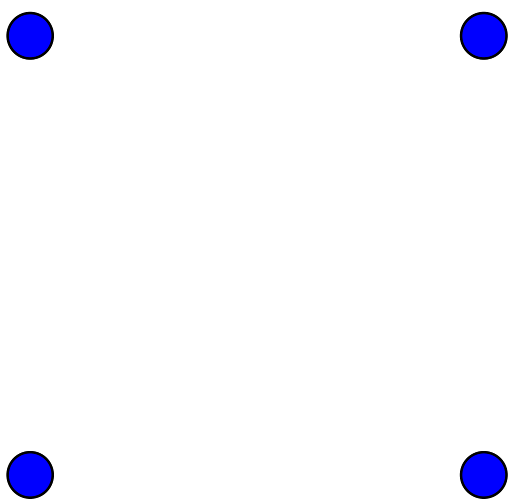
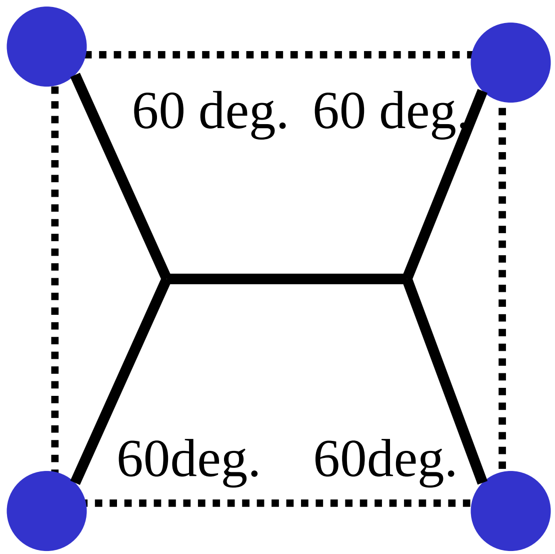
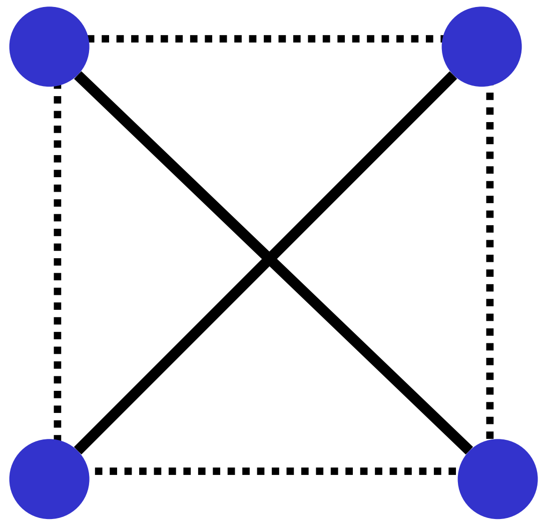
(question) how about the case for a square?



(the answer)



cf.



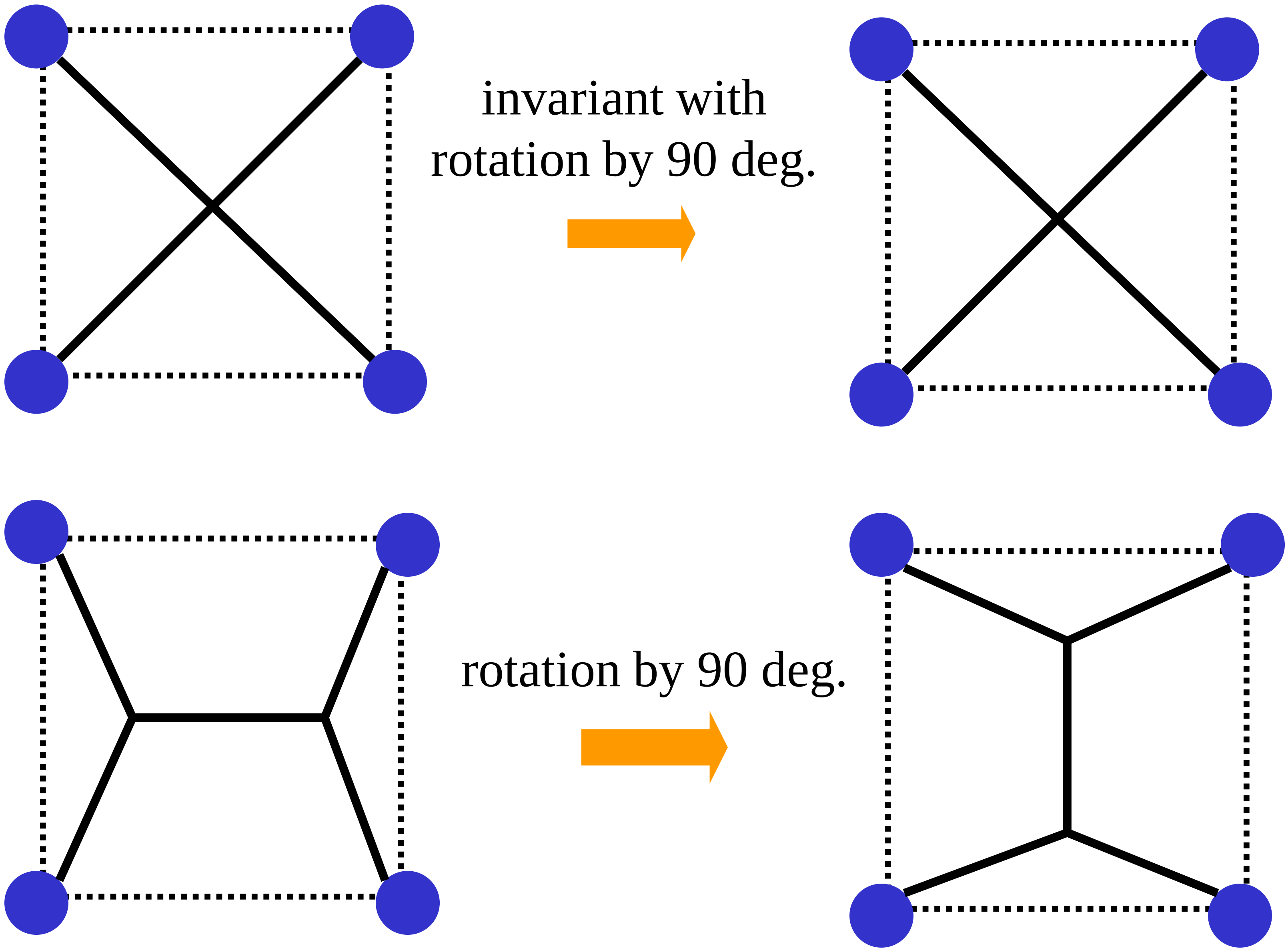
Length

$$4 \times \frac{1}{\sqrt{3}} + \left(1 - 2 \times \frac{1}{2\sqrt{3}}\right)$$
$$= 1 + \sqrt{3}$$
$$= 2.732\cdots$$

Length

$$2 \times \sqrt{2} = 2.828\cdots$$

Ref. Takeshi Koike,
"Genshikaku Kenkyu" Vol. 52 No. 2, p. 14



a good example of spontaneous symm. breaking

Courtesy: Takeshi Koike