

# 原子核反应

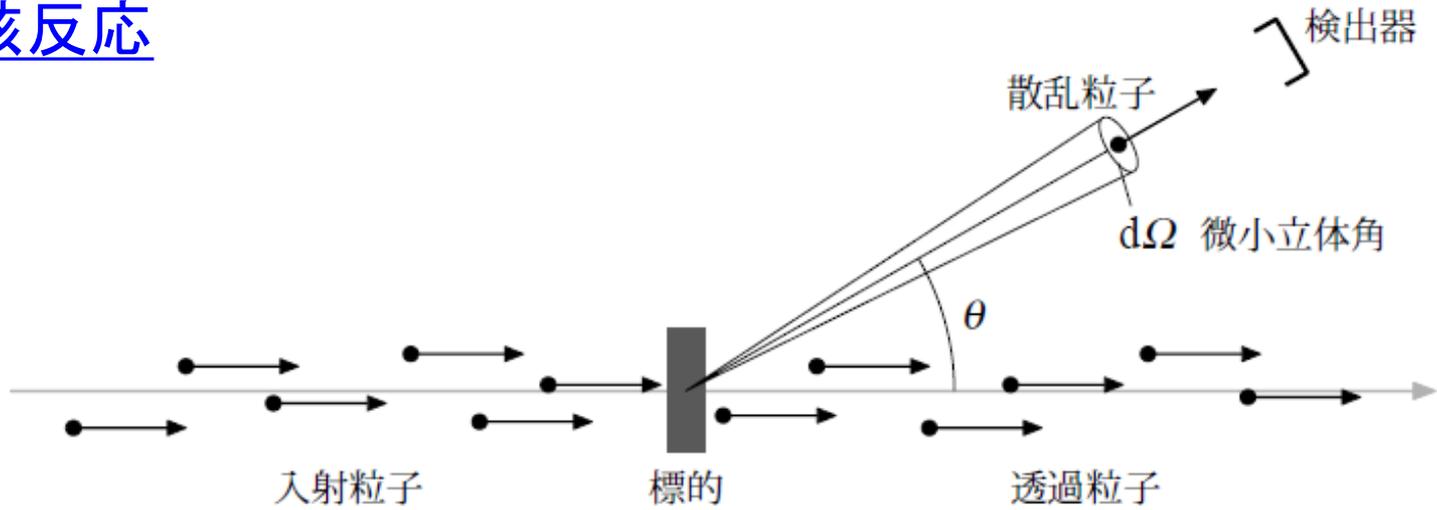
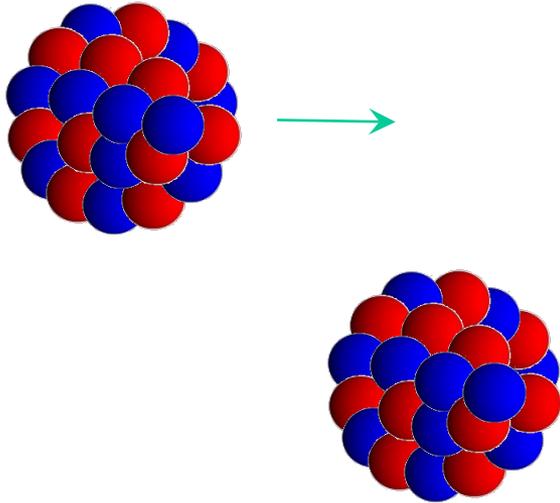
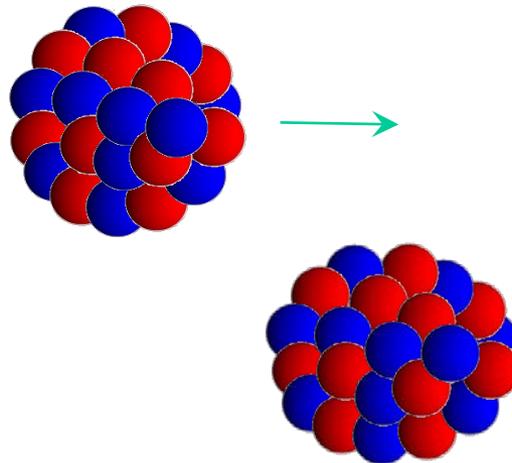


图 21.1: 散乱实验

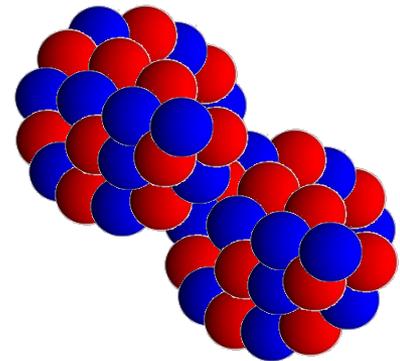
弹性散乱



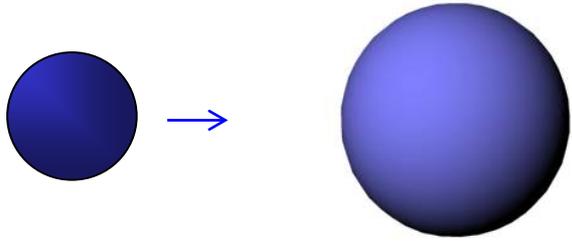
非弹性散乱



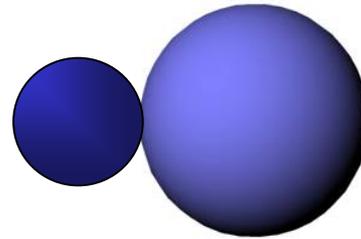
核融合



# Nuclear fusion reactions



two positive charges  
repel each other

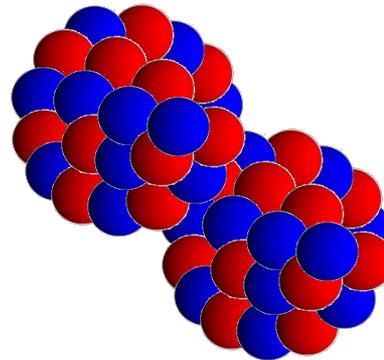


nuclear *attractive*  
interaction



compound  
nucleus

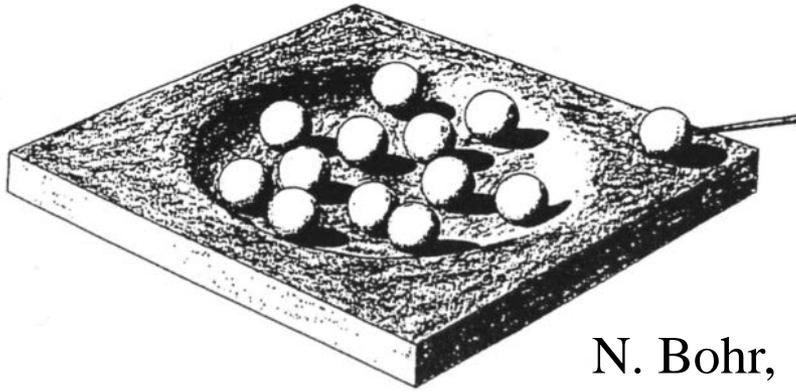
複合核



# 核融合反応： 複合核生成反応

Niels Bohr (1936)

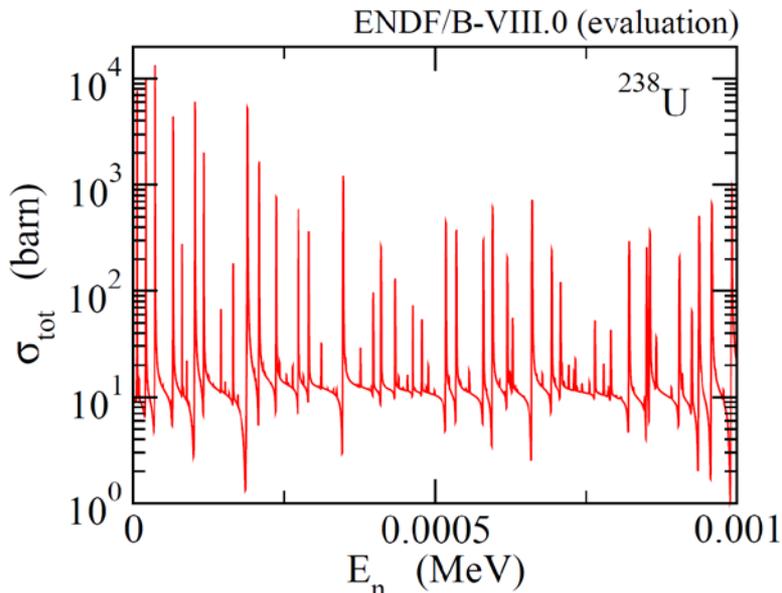
原子核による中性子の吸収 → 複合核



N. Bohr,  
Nature 137 ('36) 351



Wikipedia

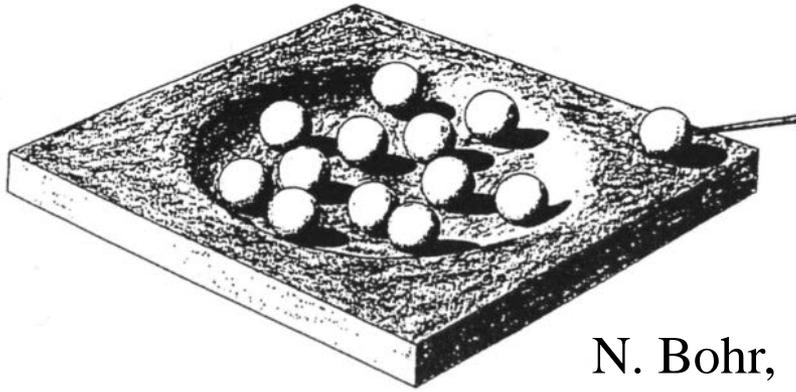


cf. フェルミの実験 (1935)  
MeV スケールの原子核に  
eV スケールの幅の多数の共鳴状態

# 核融合反応： 複合核生成反応

Niels Bohr (1936)

原子核による中性子の吸収 → 複合核

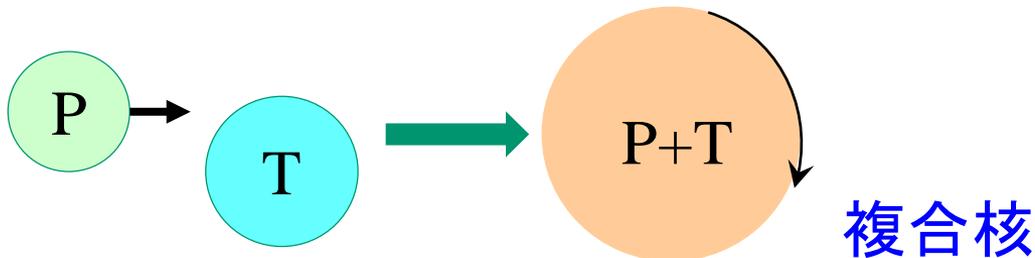


N. Bohr,  
Nature 137 ('36) 351

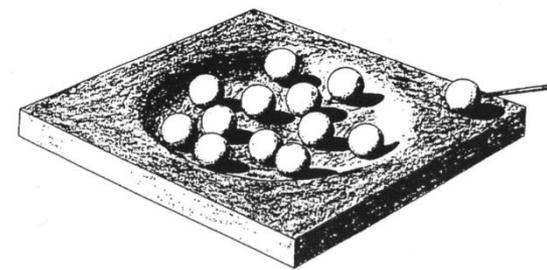
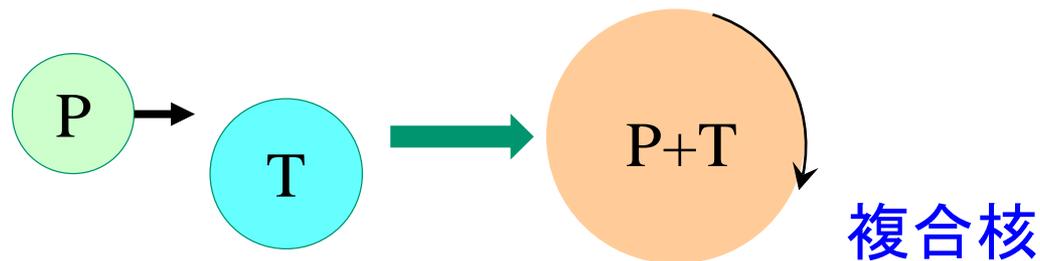


Wikipedia

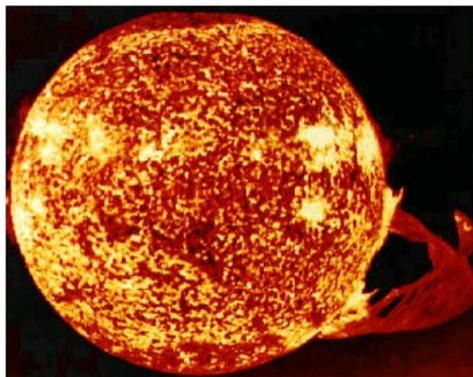
重イオン反応で複合核をつくる = 重イオン核融合反応



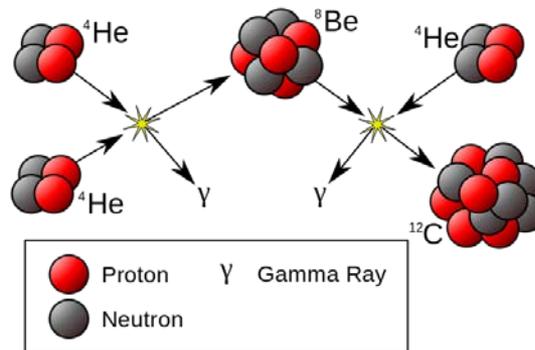
# 核融合反応： 複合核生成反応



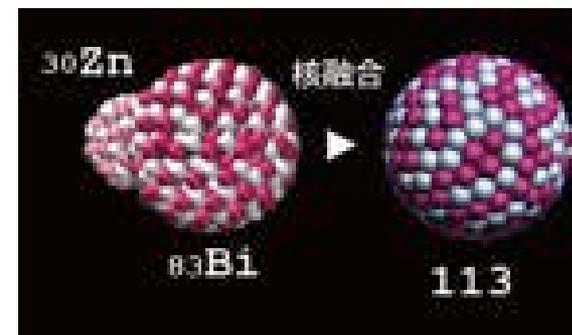
cf. N. Bohr '36



恒星のエネルギー源 (Bethe '39)



元素合成

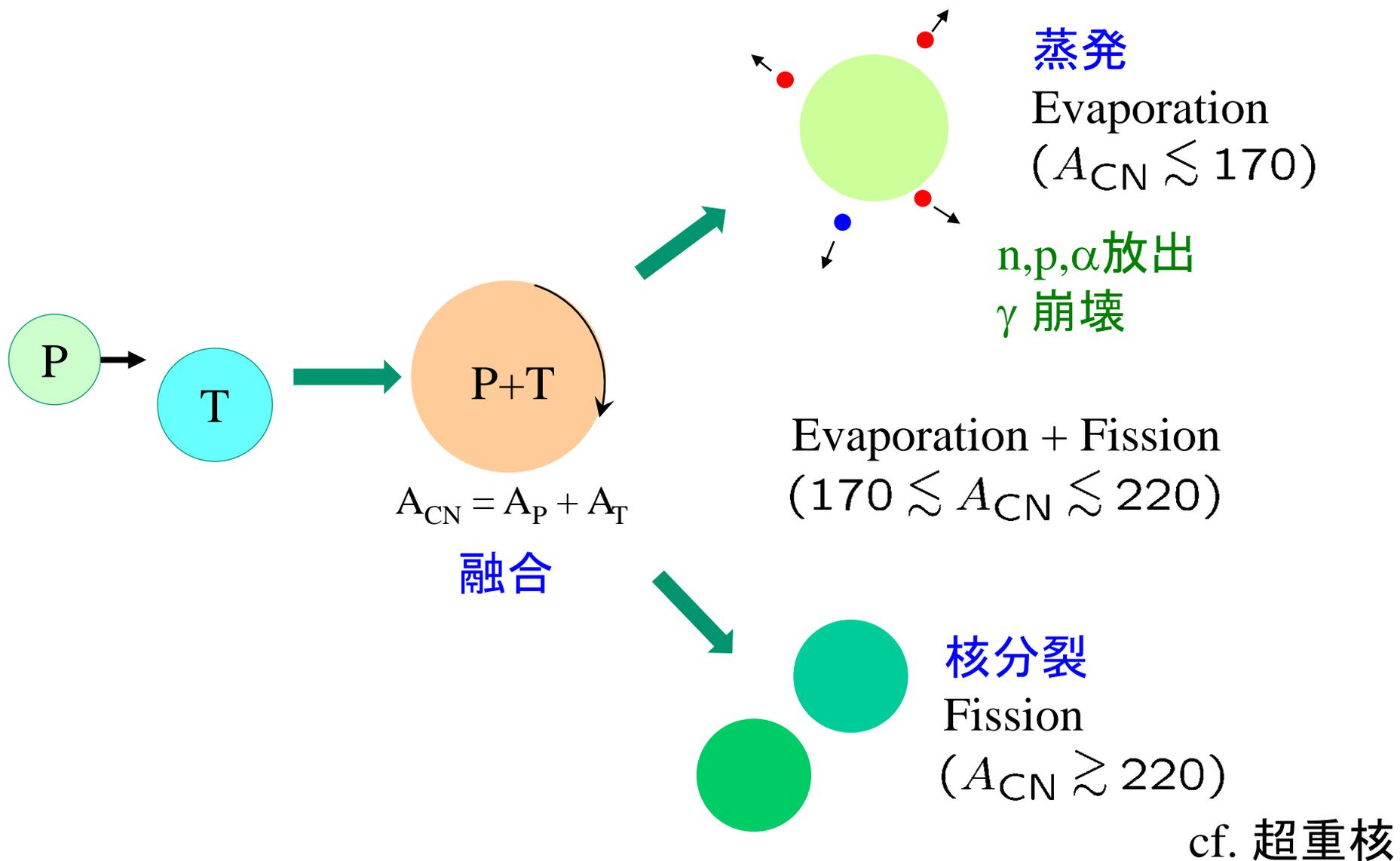


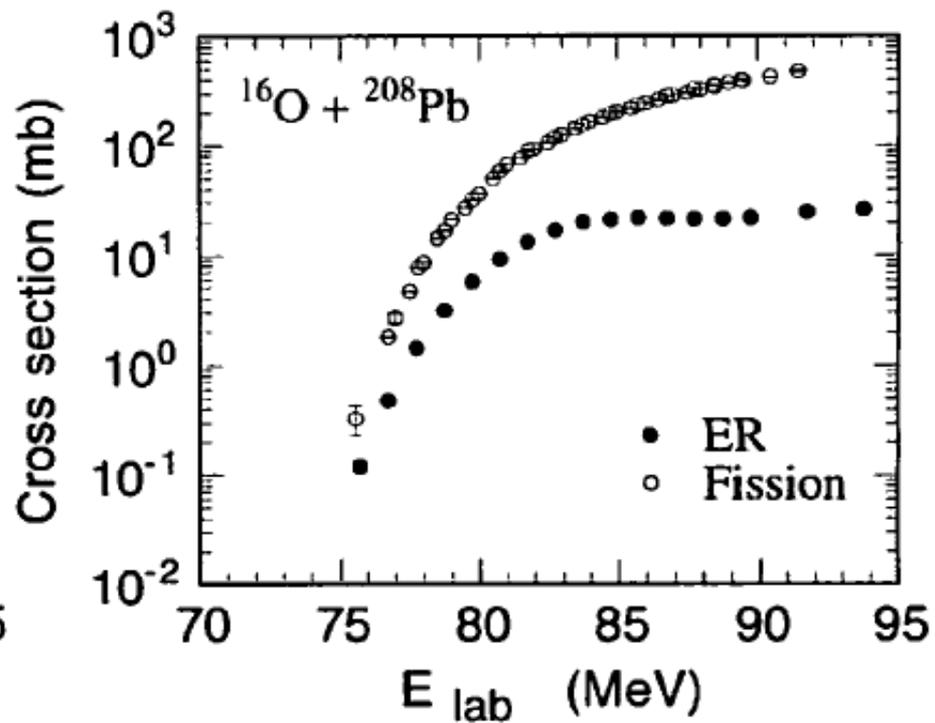
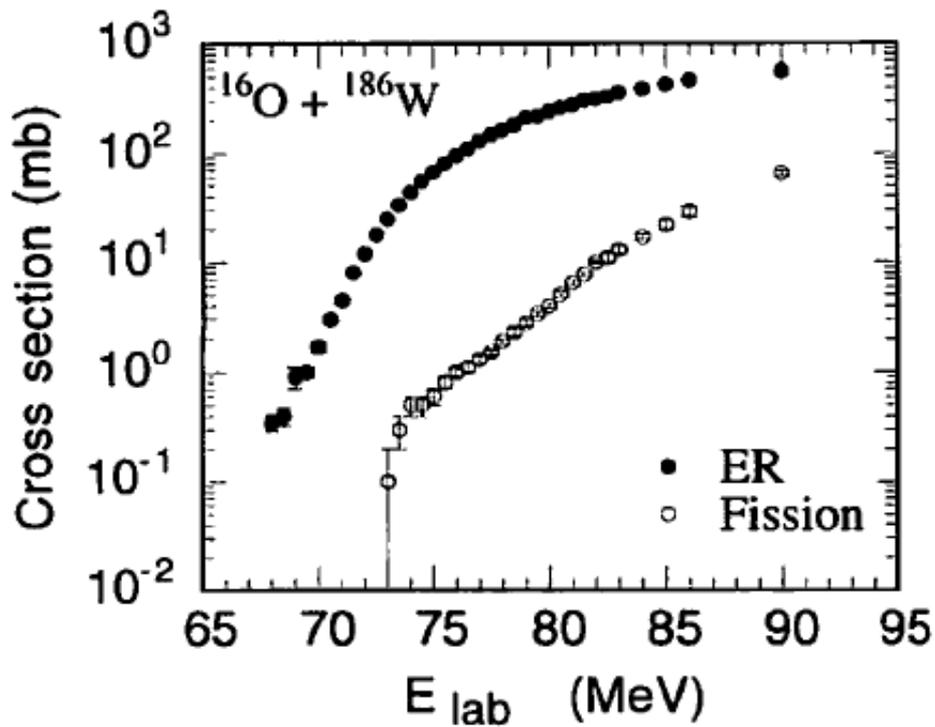
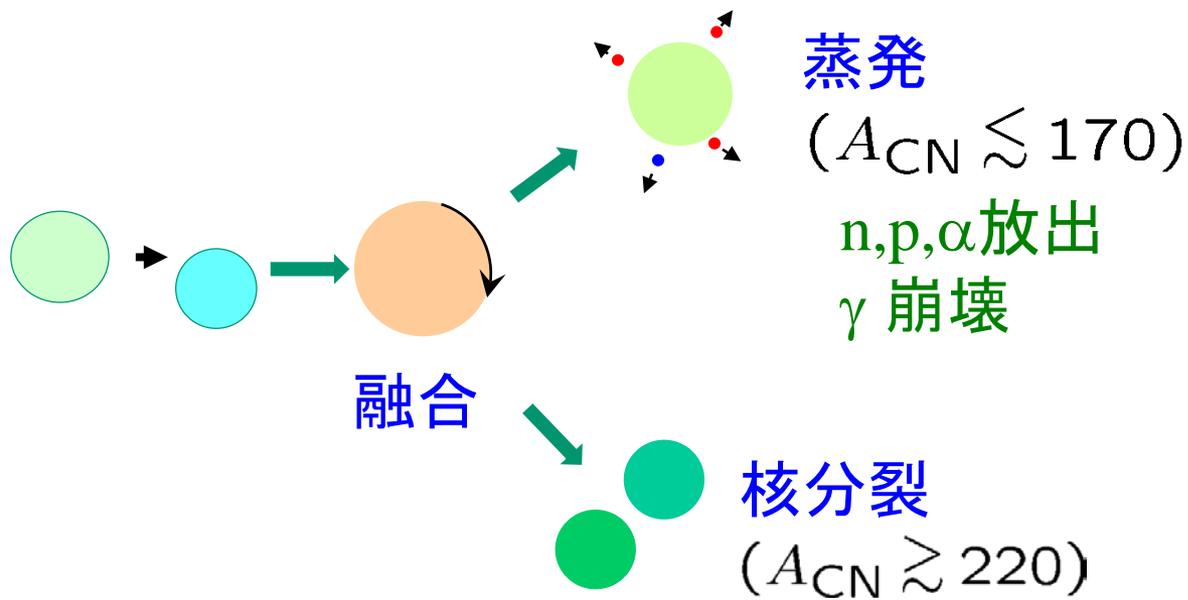
超重元素の合成

核融合・核分裂: 強い相互作用をする量子多体系の大振幅集団運動

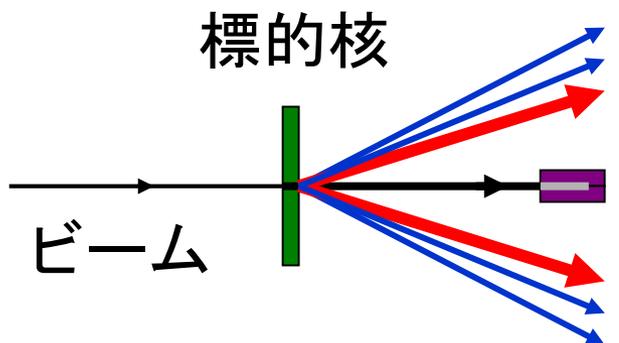
← 微視的理解: 核物理における究極の未解決問題の一つ

# 核融合反応：複合核生成反応

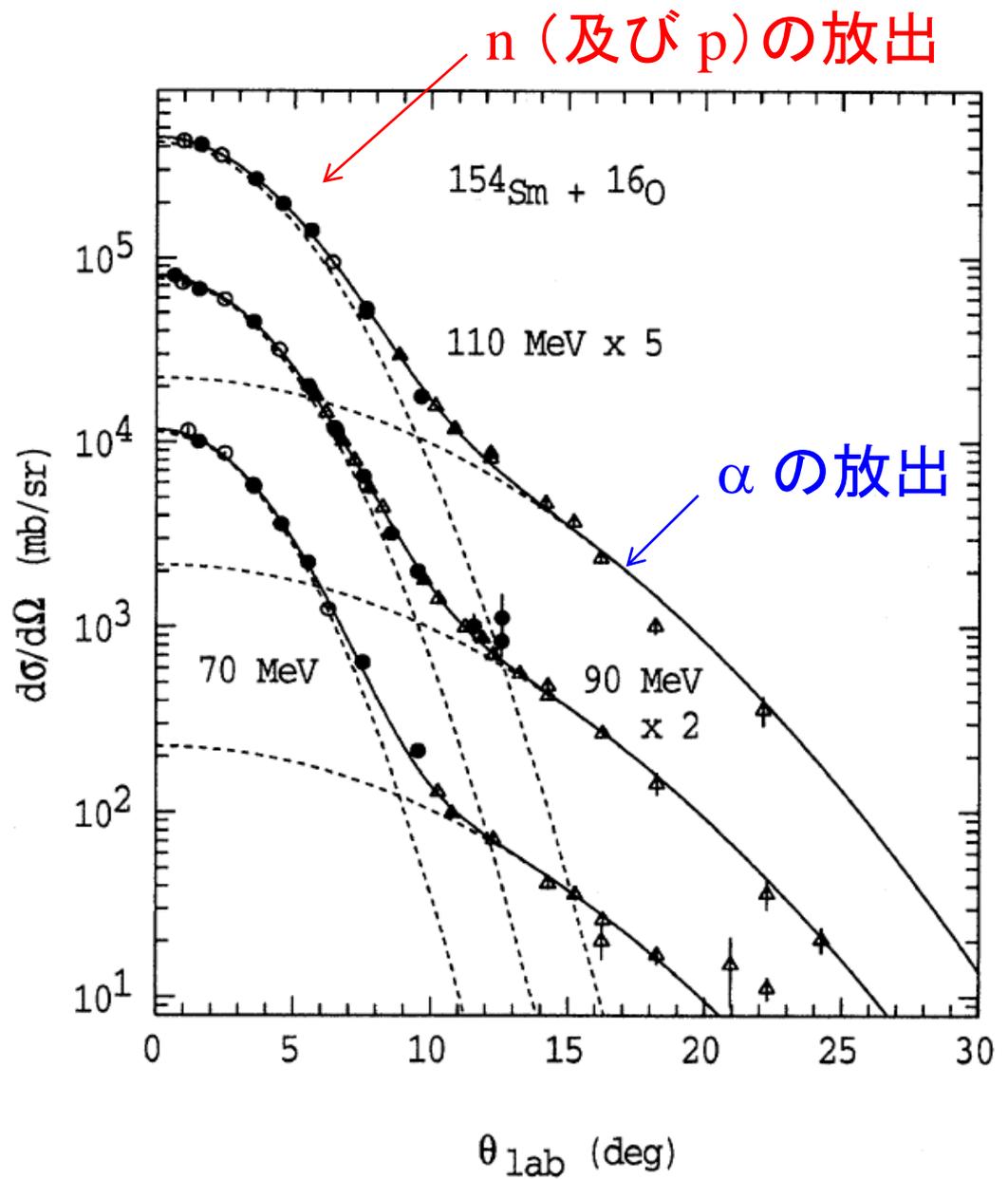




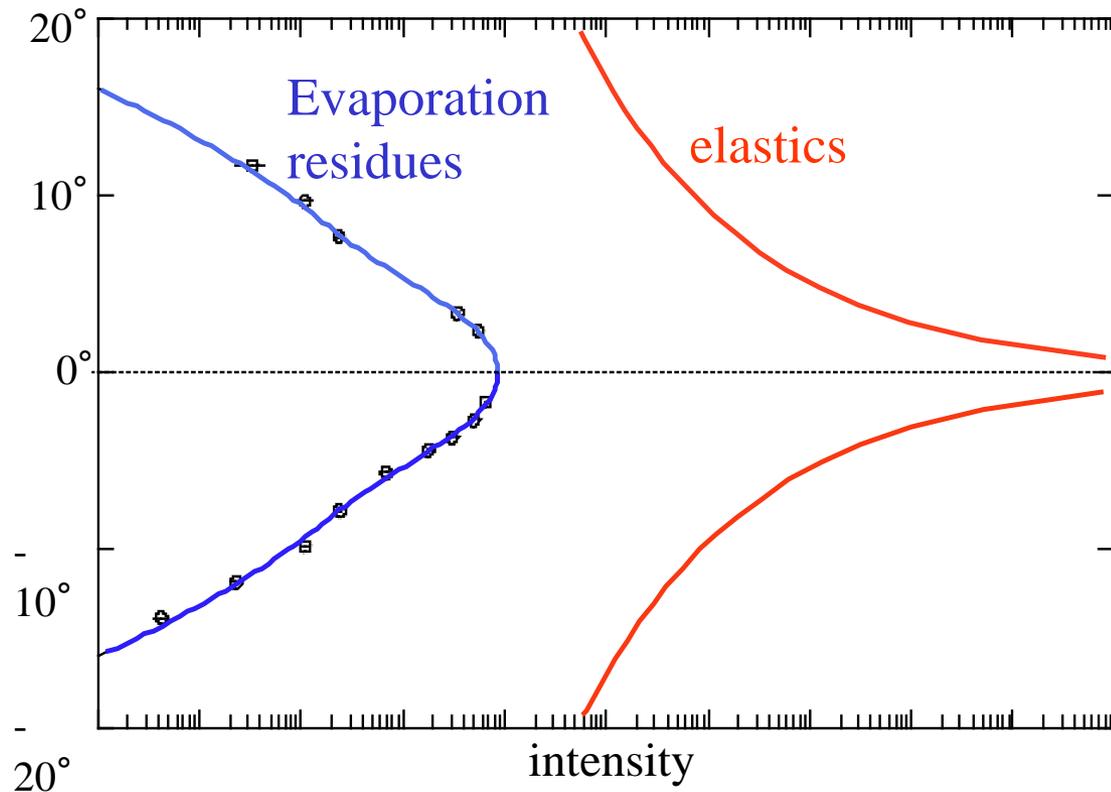
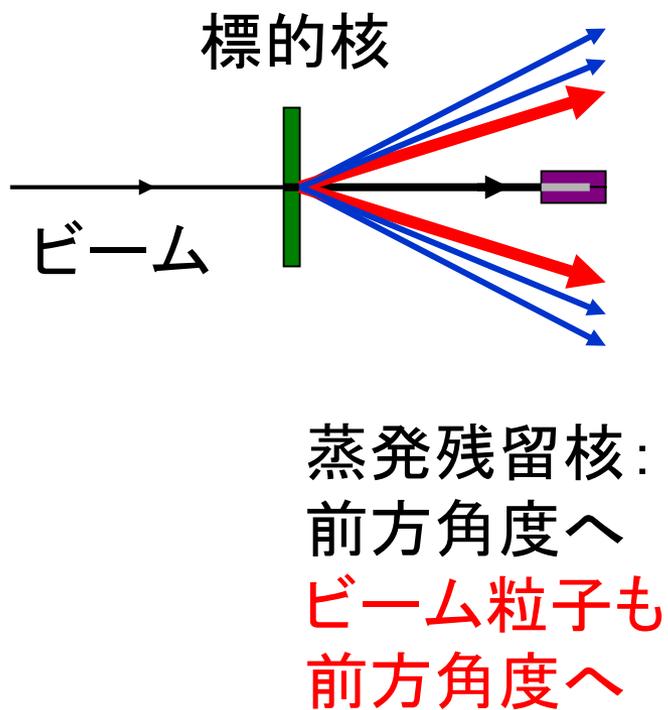
# 蒸発残留核の測定



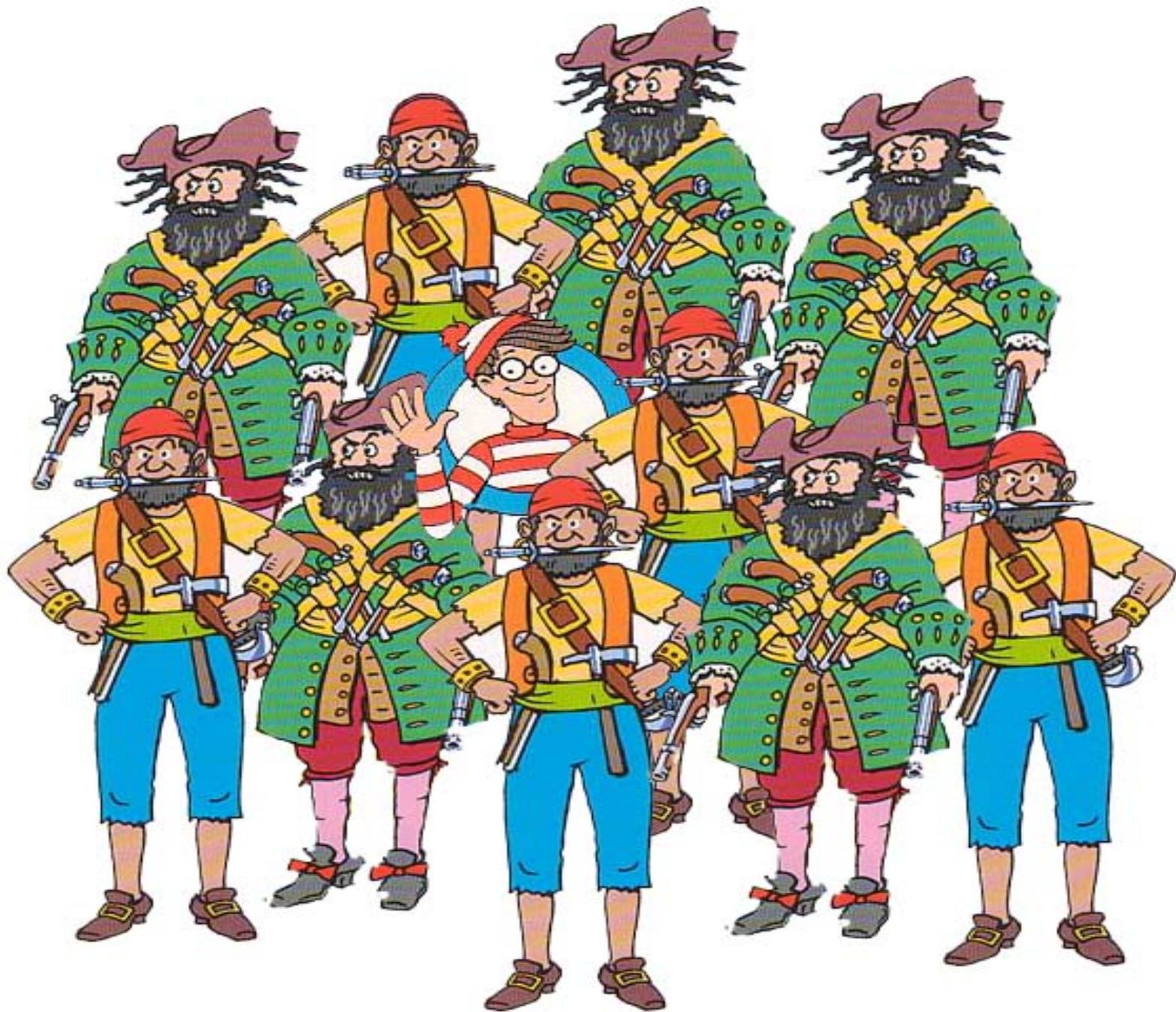
蒸発残留核:  
前方角度へ

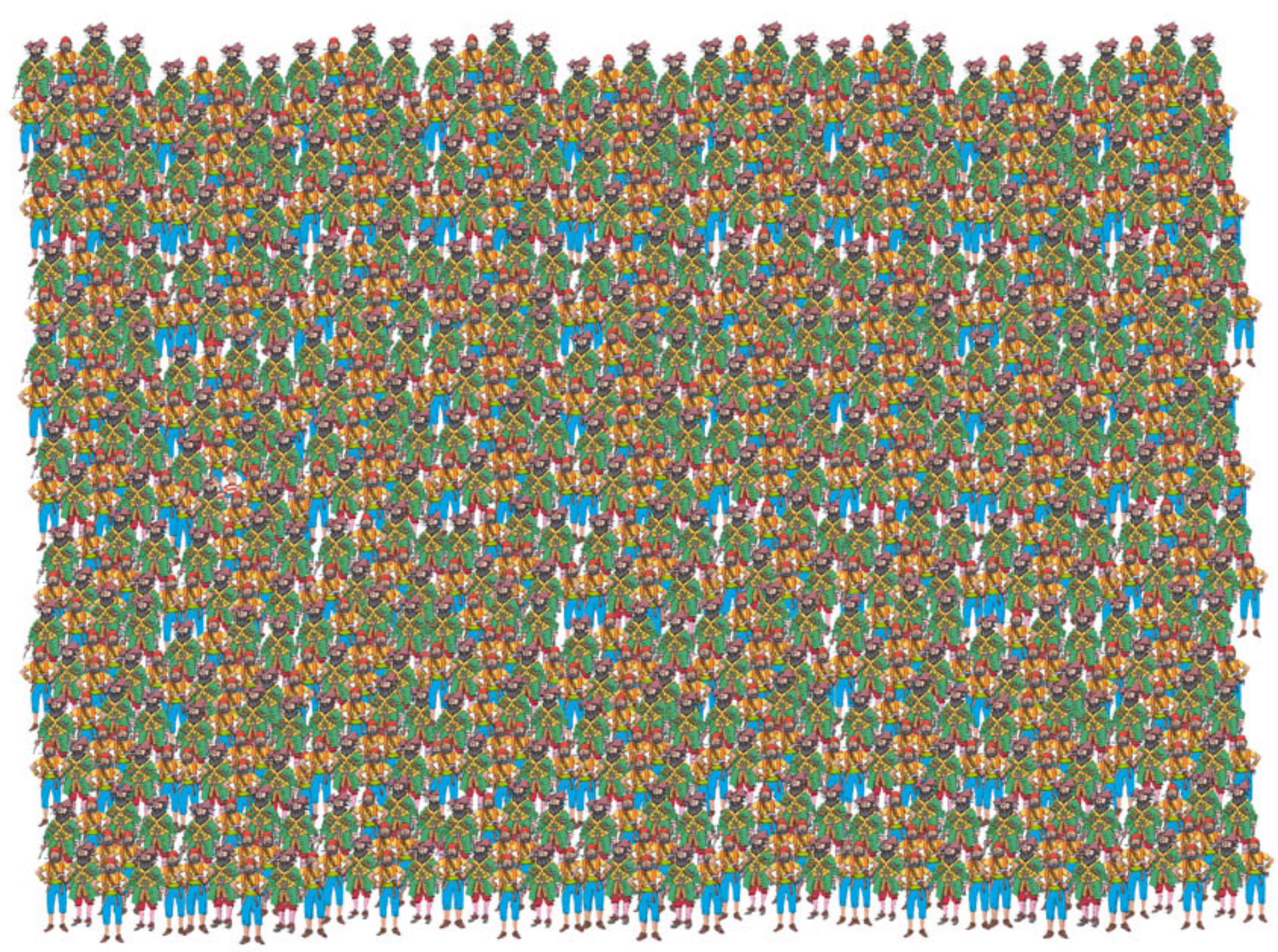


# 蒸発残留核の測定



beam-like 粒子: 蒸発残留核の  $10^4 \sim 10^{12}$  倍の強度

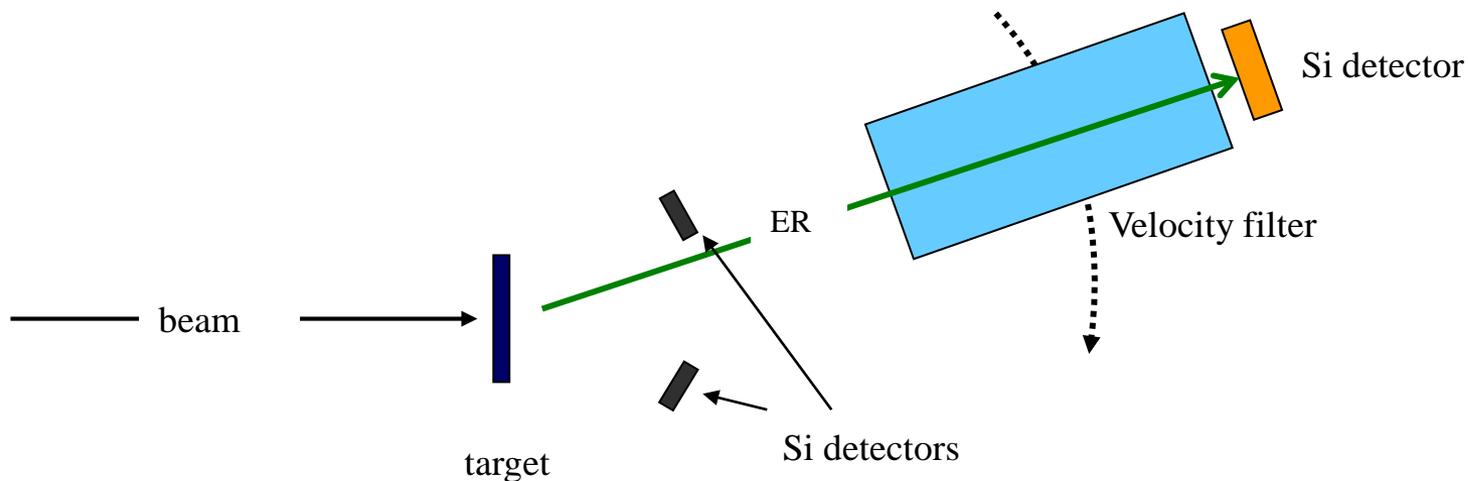




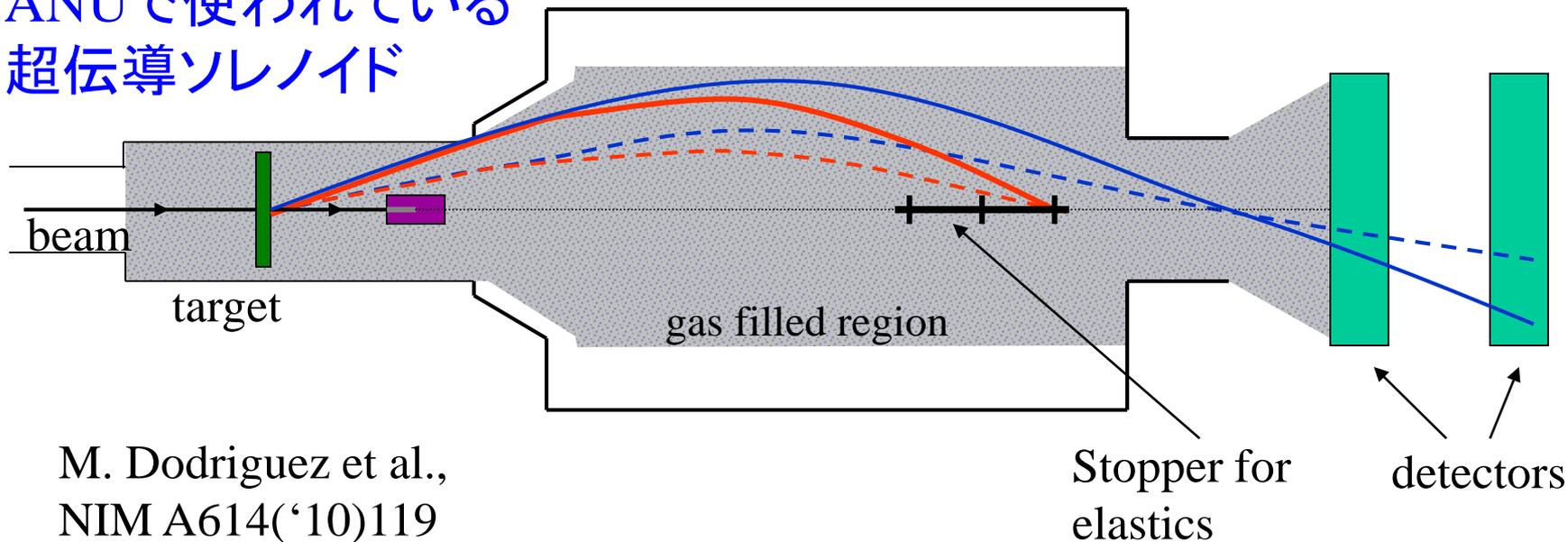


# 蒸発残留核の測定

velocity filter 等を用いてうまく蒸発残留核と beam-like 粒子をわけける



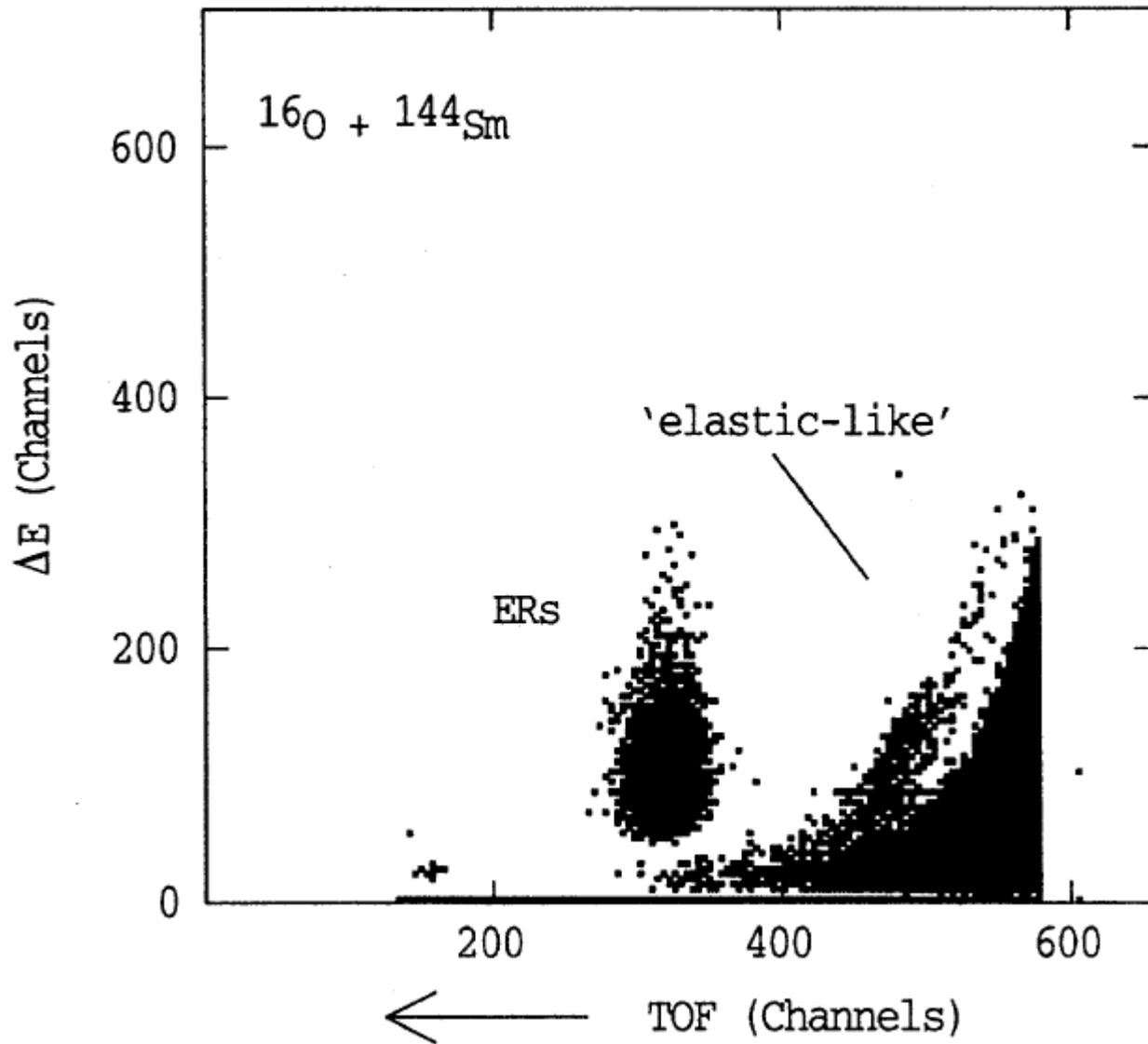
ANUで使われている  
超伝導ソレノイド



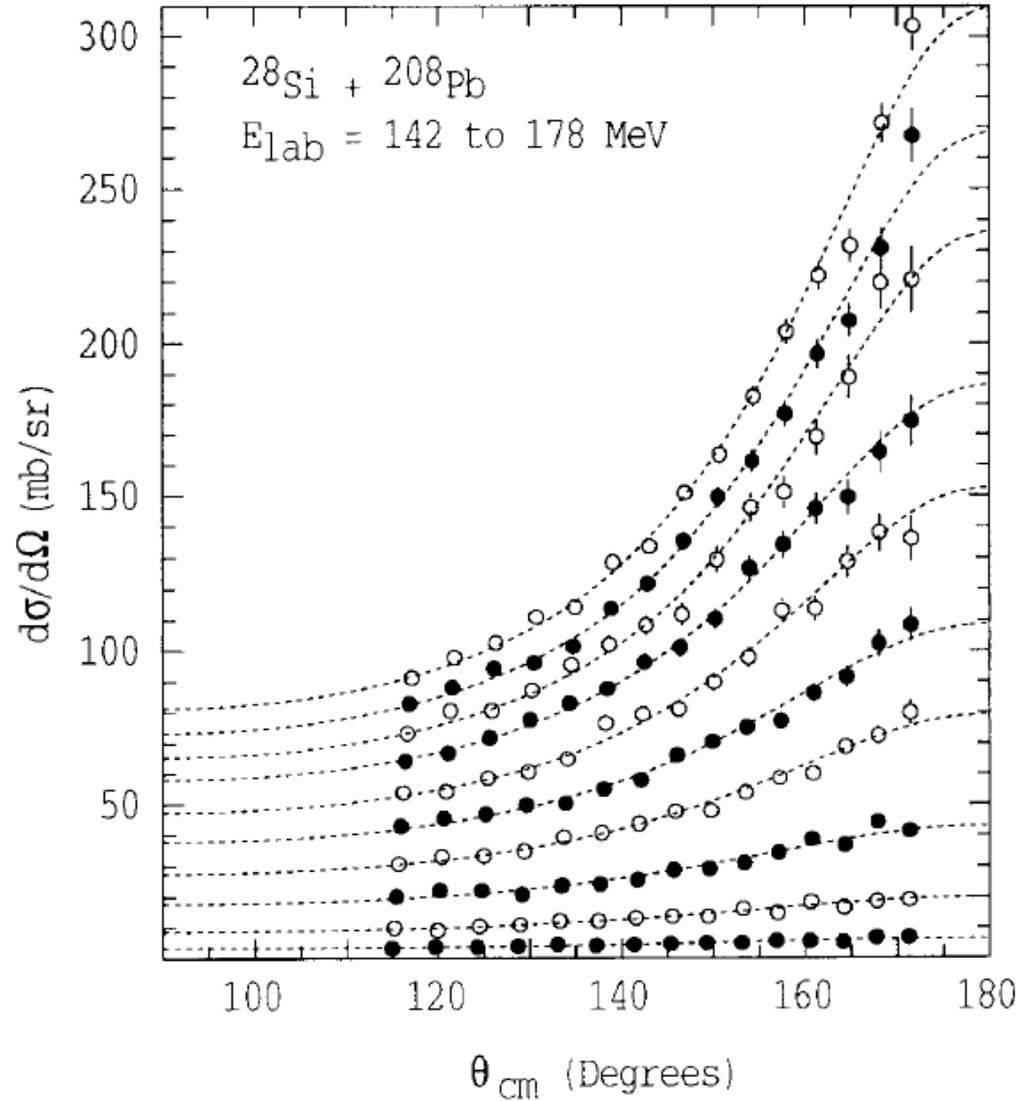
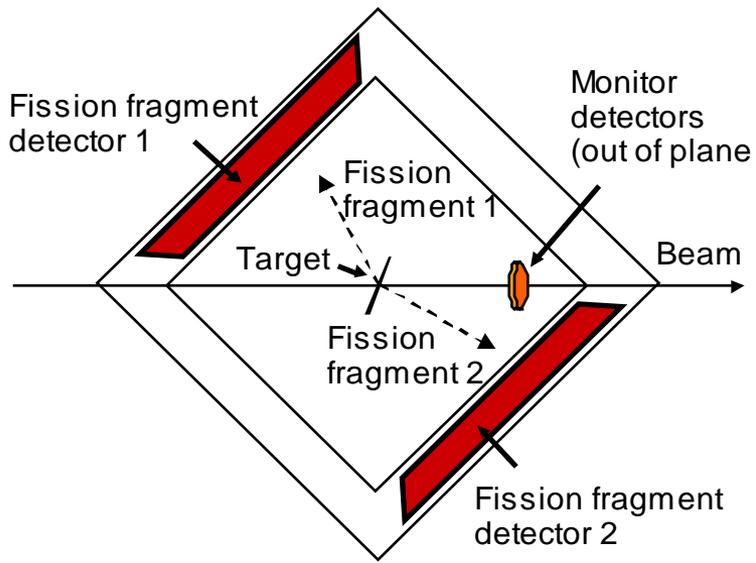
M. Dodríguez et al.,  
NIM A614('10)119

Stopper for  
elastics

detectors

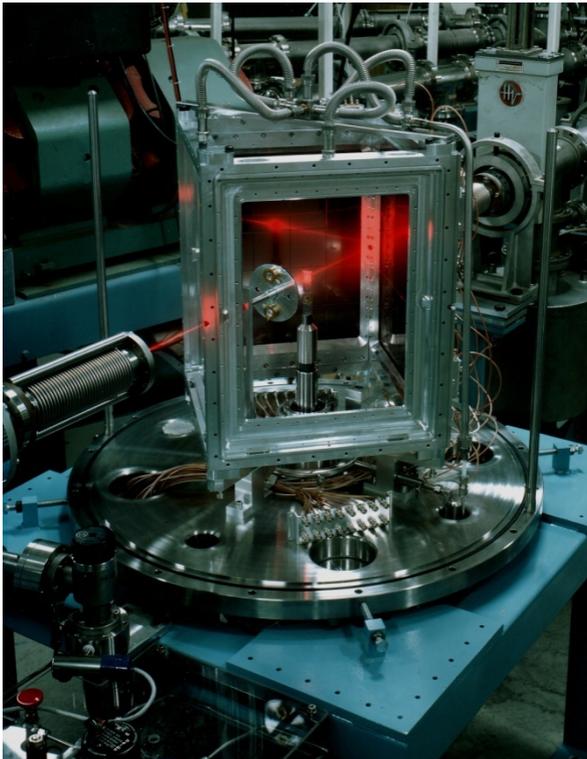


# 核分裂片の測定



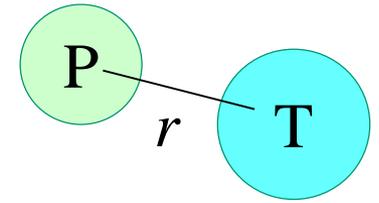
D.J. Hinde et al., NPA592('95)271

← ANUで使われている検出器



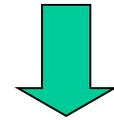
# クーロン障壁

## クーロン障壁



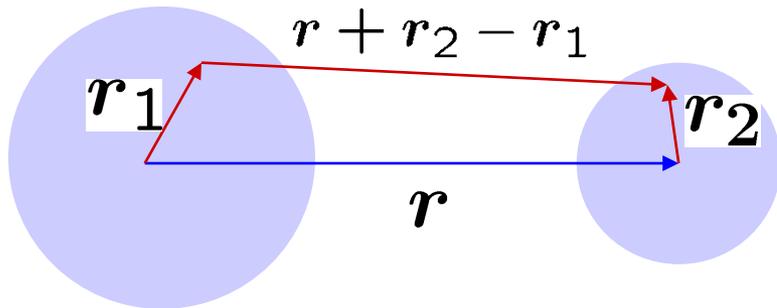
2つの力:

1. **クーロン力**  
長距離斥力
2. **核力**  
短距離引力



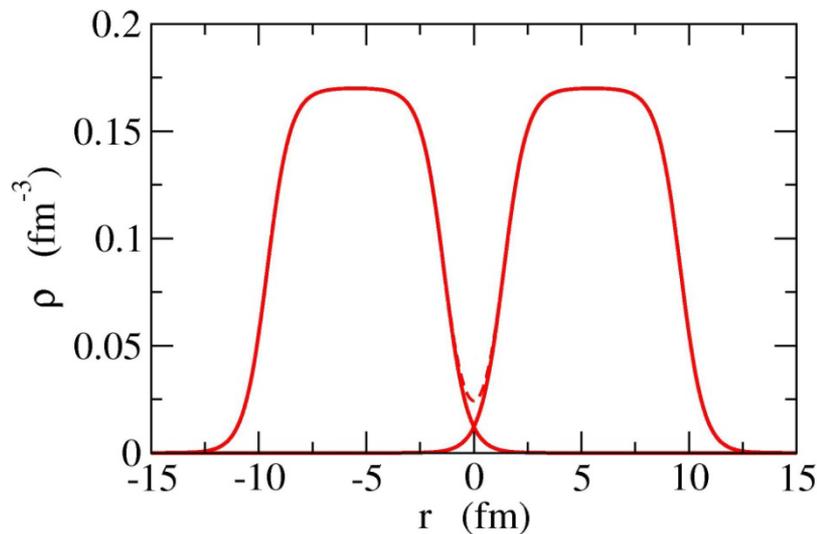
ポテンシャル障壁  
(**クーロン障壁**)

## • Double Folding Potential



$$V_{DF}(r) = \int dr_1 dr_2 \rho_1(r_1) \rho_2(r_2) \times v_{nn}(r + r_2 - r_1)$$

(微視的ポテンシャルの直接項に相当)



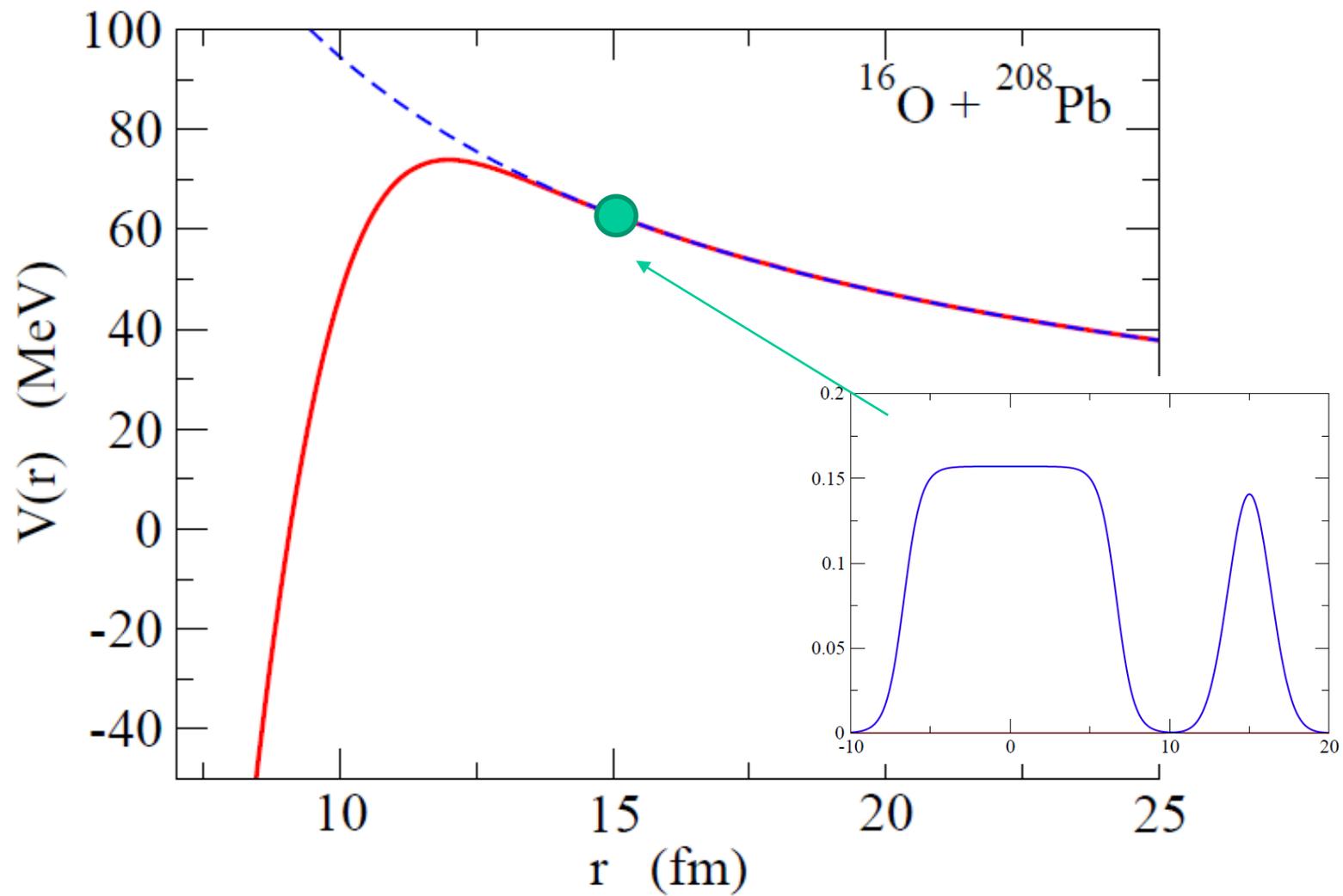
$$\rho(r) \sim \frac{\rho_0}{1 + \exp[(r - R_d)/a_d]}$$

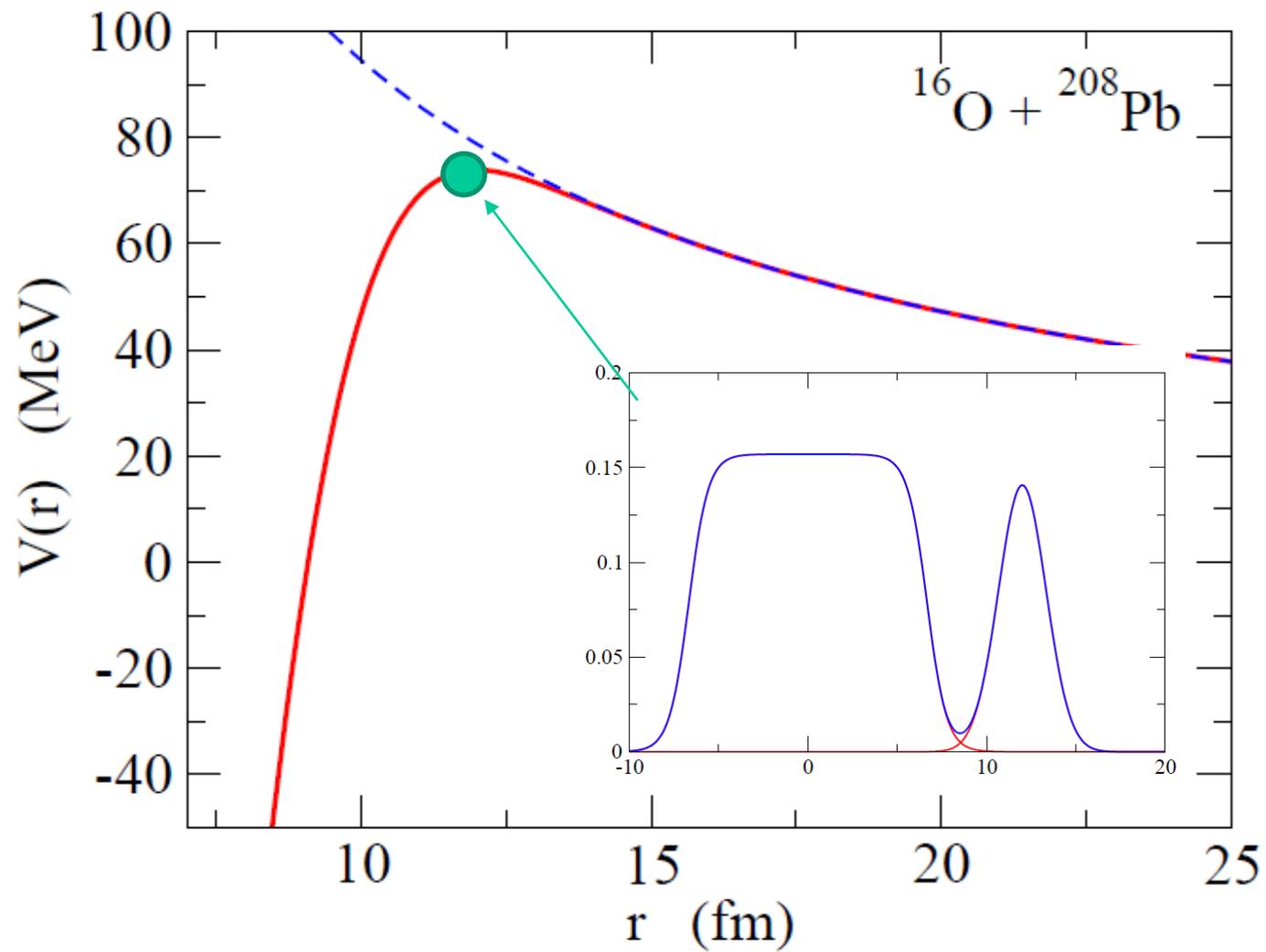
$$a_d \sim 0.54 \text{ (fm)}$$

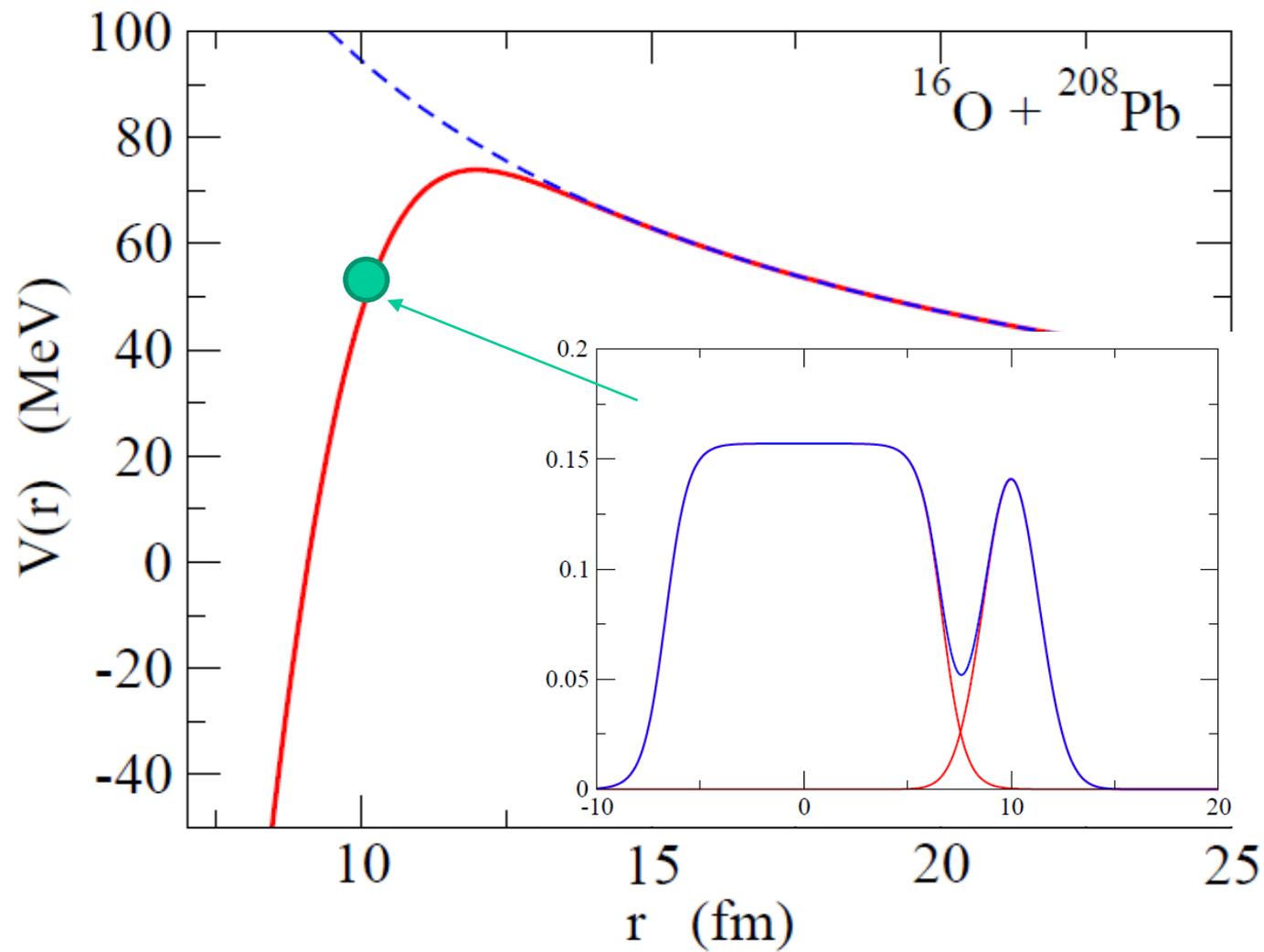
## • Phenomenological potential

$$V_{WS}(r) = -\frac{V_0}{1 + \exp[(r - R_0)/a]}$$

$$a \sim 0.63 \text{ (fm)}$$

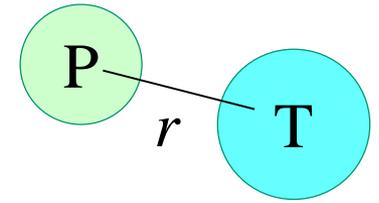






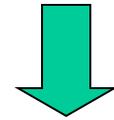
# クーロン障壁

## クーロン障壁



2つの力:

1. クーロン力  
長距離斥力
2. 核力  
短距離引力

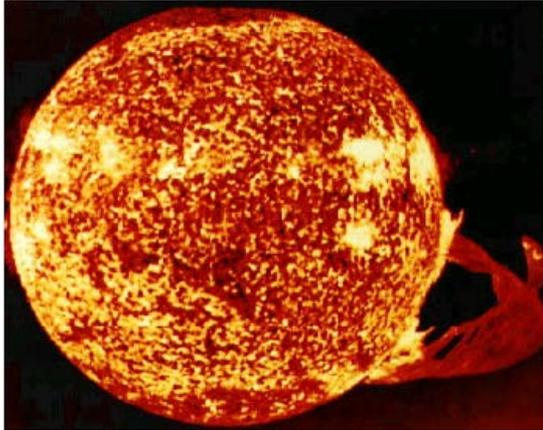


ポテンシャル障壁  
(クーロン障壁)

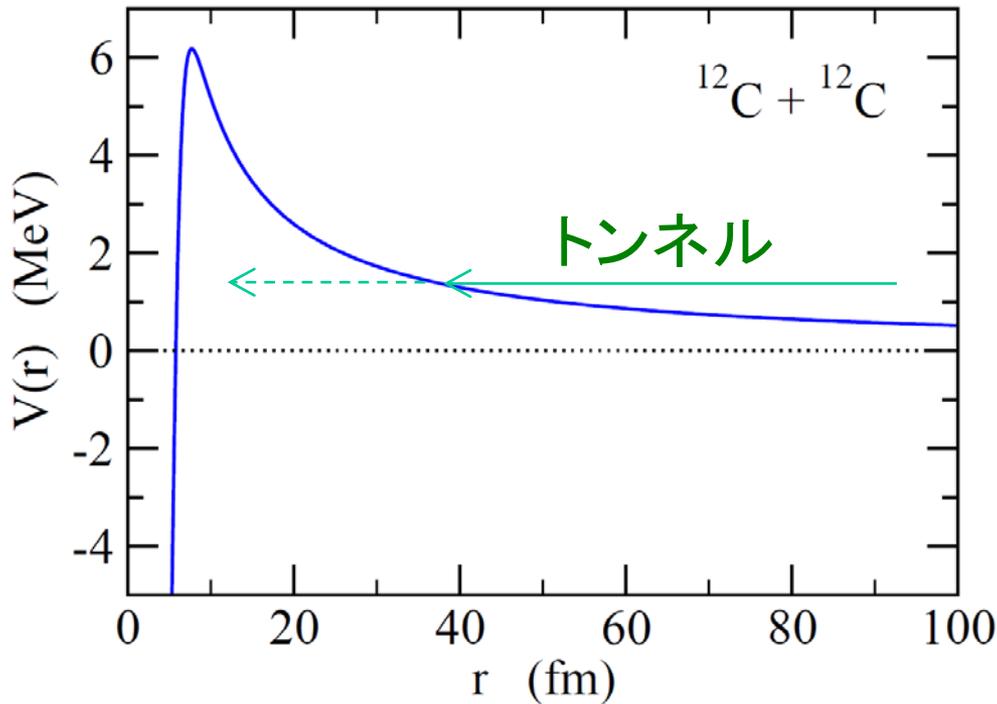
クーロン障壁近傍のエネルギーにおける核融合反応

→ 多粒子系の量子トンネル現象

# nuclear fusion in stars



NASA, Skylab space station December 19, 1973, solar flare reaching 568 000 km off solar surface



$\sigma(E)$   
LOGARITMIC  
SCALE

extrapolation of data

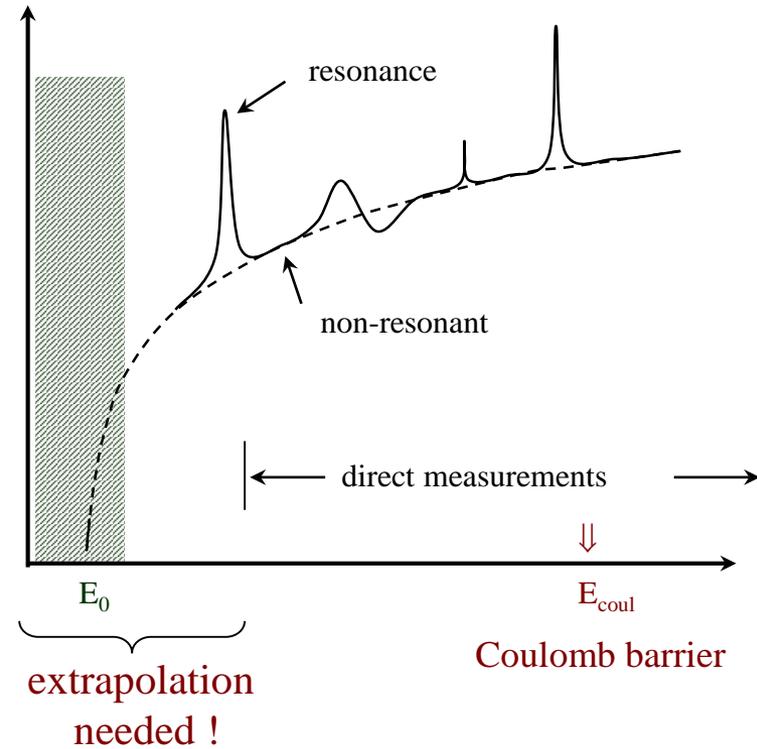
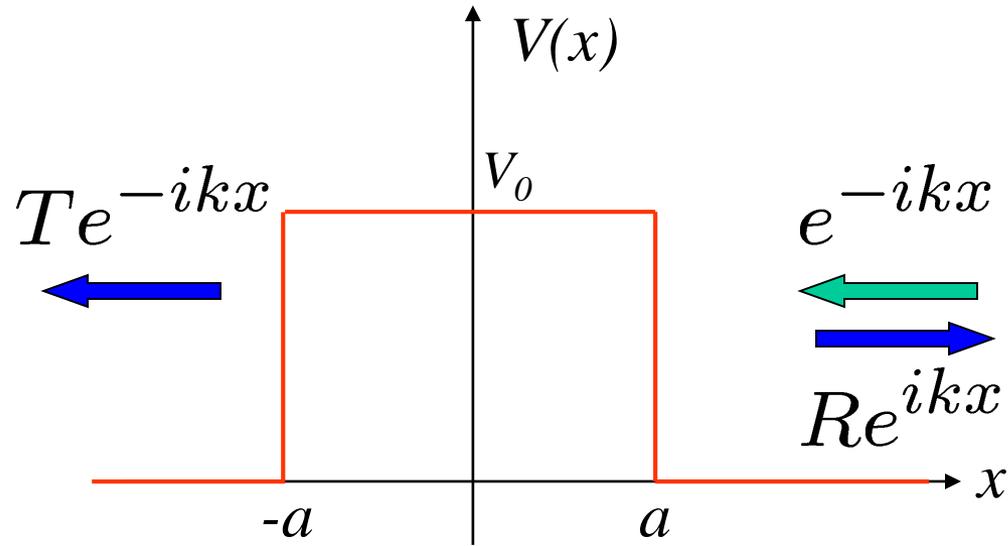


figure: M. Aliotta

# Quantum Tunneling Phenomena

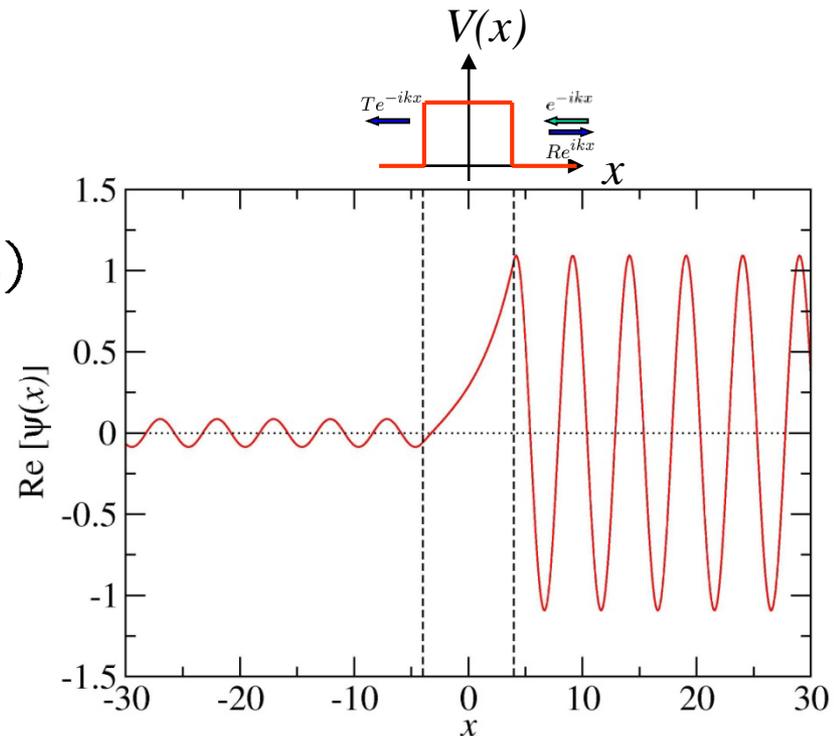


$$\begin{aligned} \psi(x) &= T e^{-ikx} & (x \leq -a) \\ &= A e^{-\kappa x} + B e^{\kappa x} & (-a < x < a) \\ &= e^{-ikx} + R e^{ikx} & (x \geq a) \end{aligned}$$

$$k = \sqrt{2mE/\hbar^2}$$

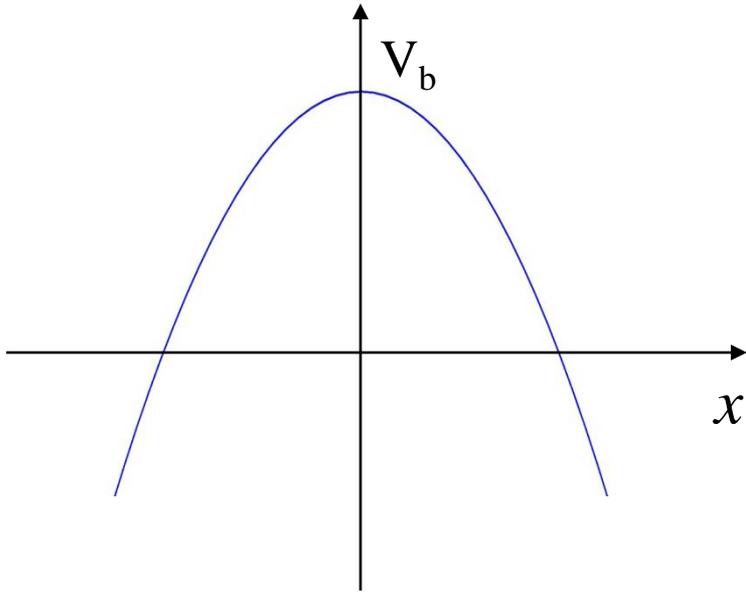
$$\kappa = \sqrt{2m(V_0 - E)/\hbar^2}$$

**Tunnel probability:  $P(E) = |T|^2$**

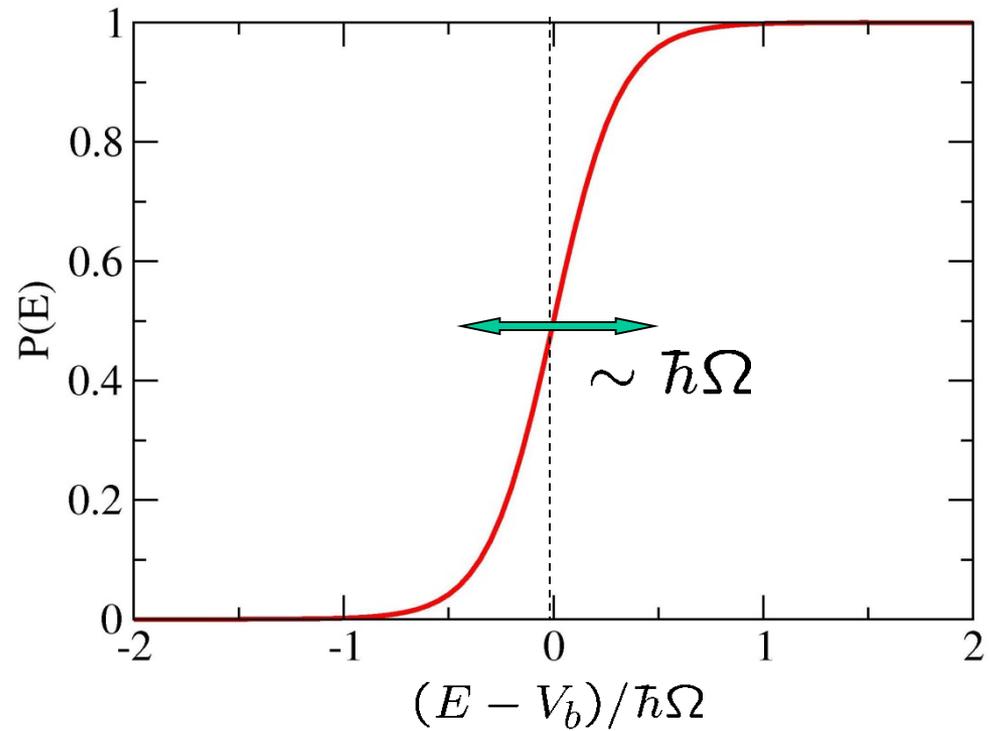


For a parabolic barrier.....

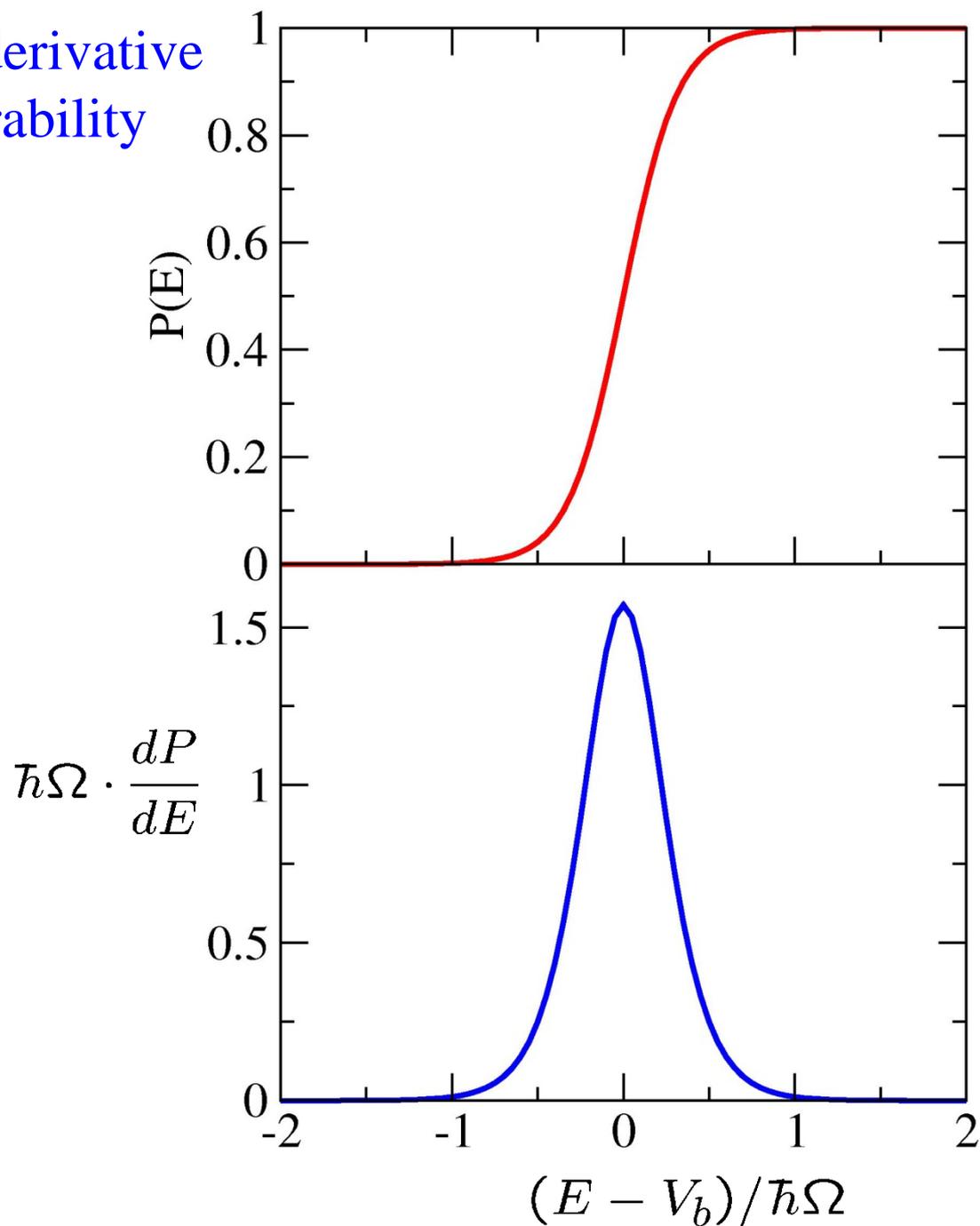
$$V(x) = V_b - \frac{1}{2}m\Omega^2 x^2$$



$$P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$



Energy derivative  
of penetrability



(note) Classical limit

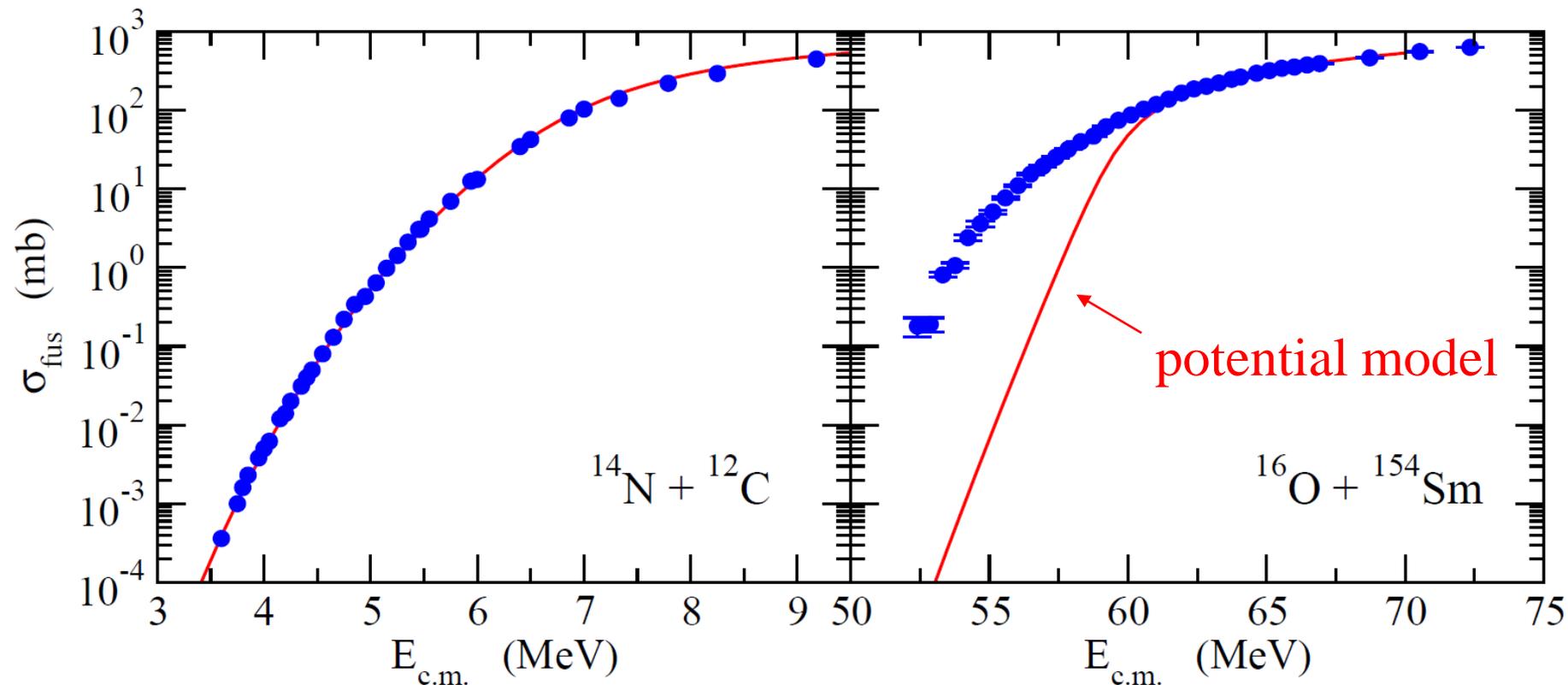
$$P(E) = \theta(E - V_b)$$

$$dP/dE = \delta(E - V_b)$$

# 核融合反応断面積の大きな増大

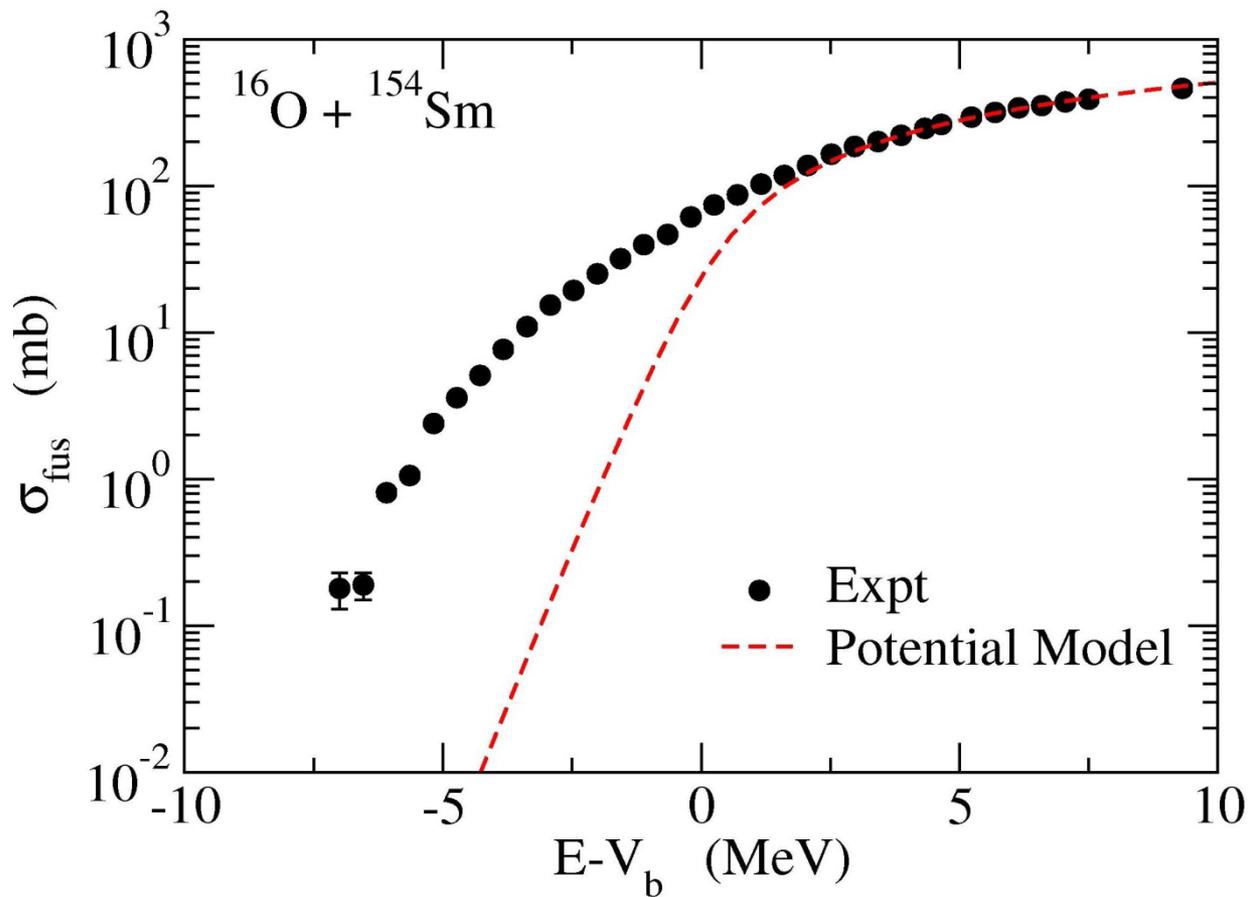
ポテンシャル模型:  $V(r) +$  吸収

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2)$$



cf. 初期の実験:

R.G. Stokstad et al., PRL41('78) 465

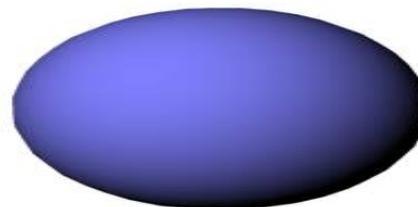


ポテンシャル模型:

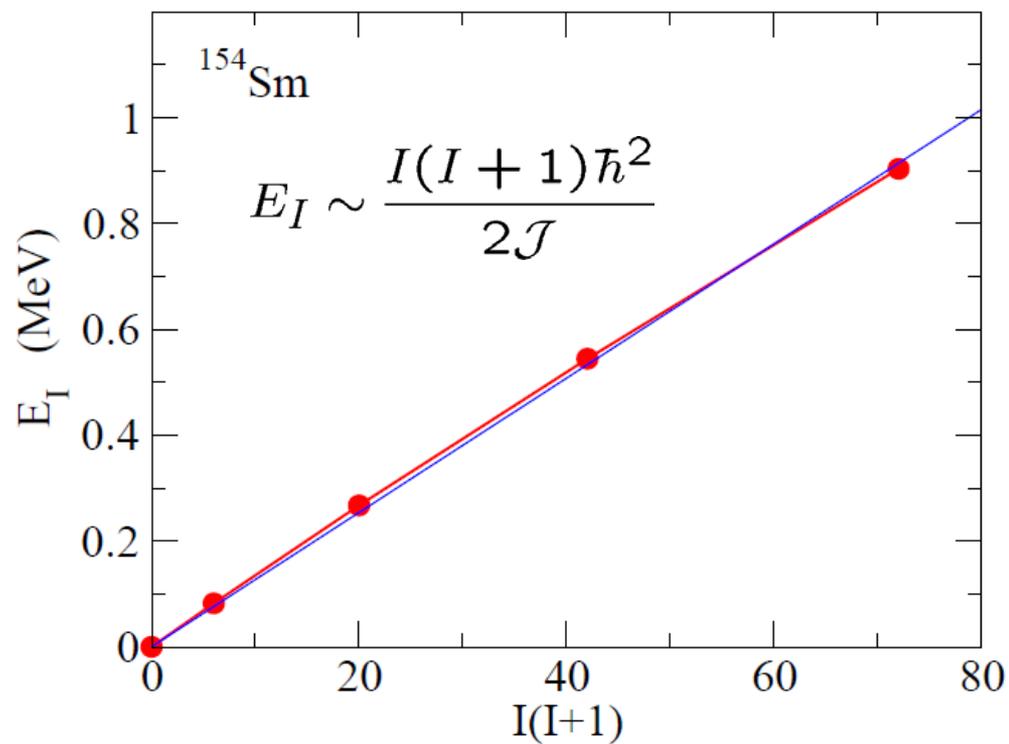
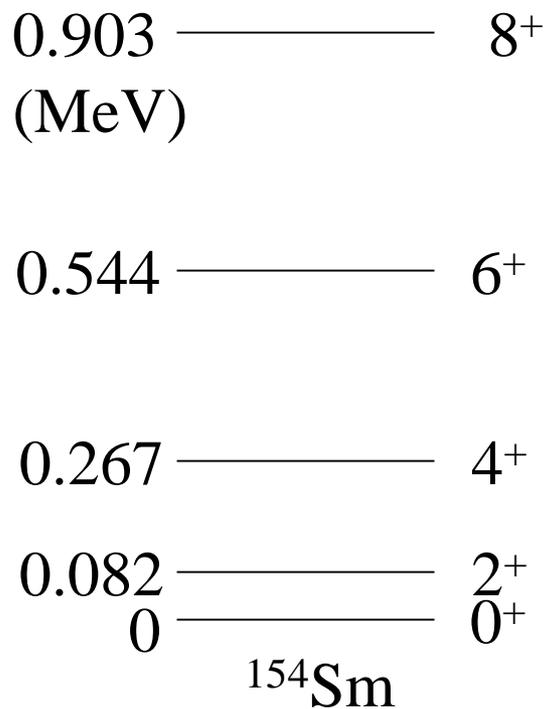
$E > V_b$  では大体データを再現

$E < V_b$  では核融合断面積を過小に評価

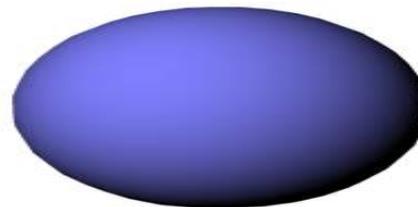
$^{154}\text{Sm}$  : a typical deformed nucleus



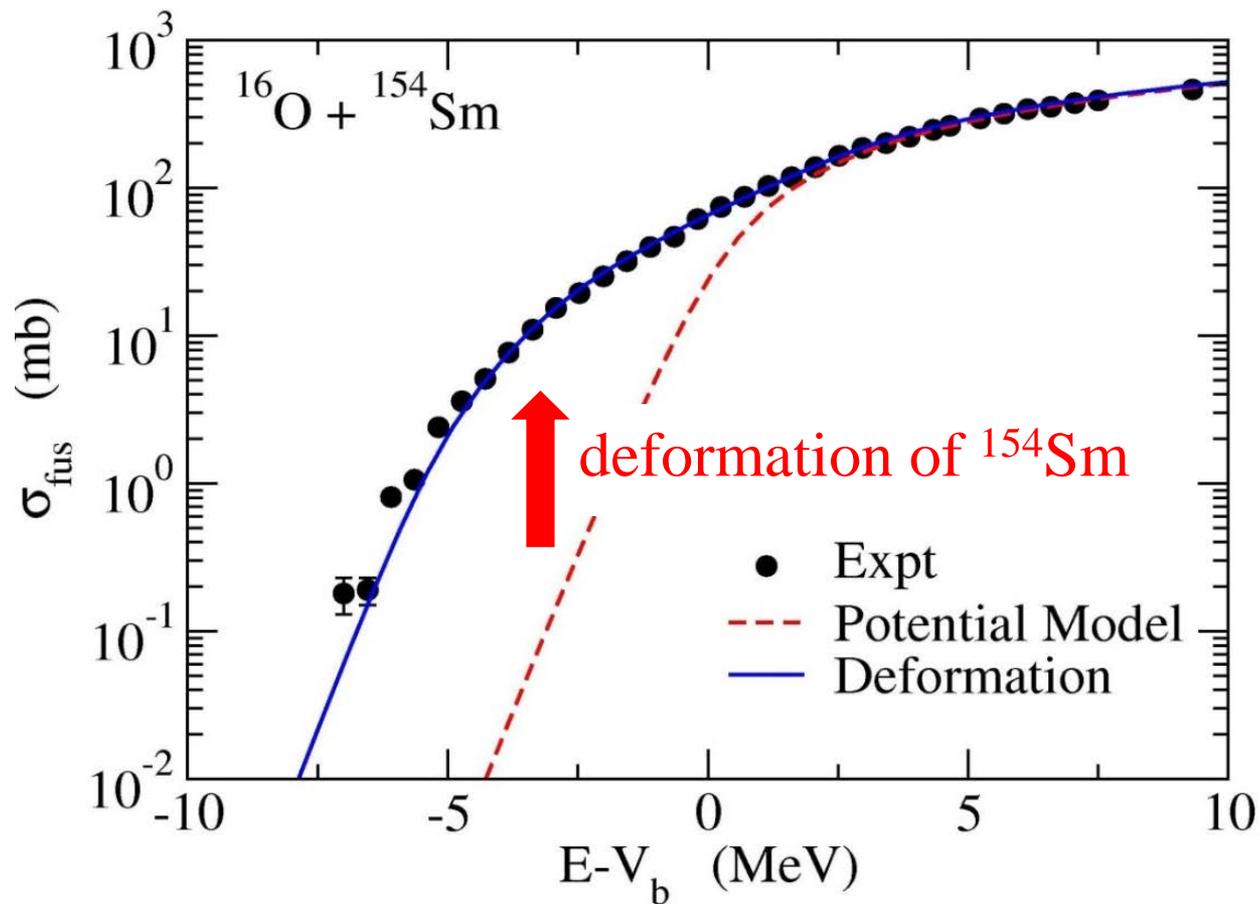
$^{154}\text{Sm}$



$^{154}\text{Sm}$  : a typical deformed nucleus

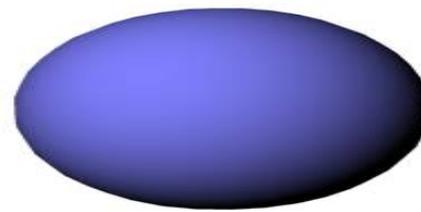


$^{154}\text{Sm}$

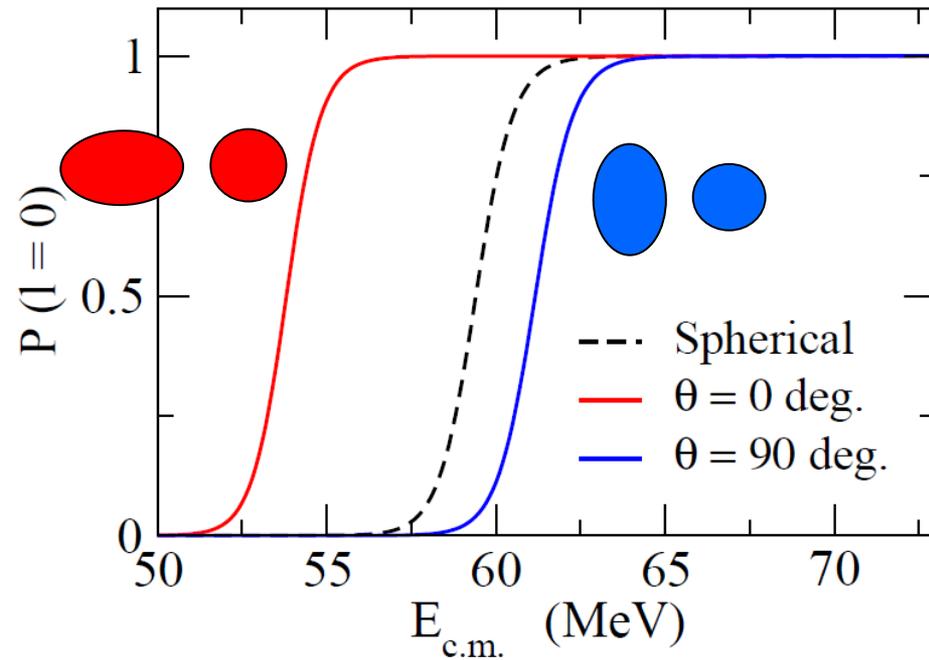
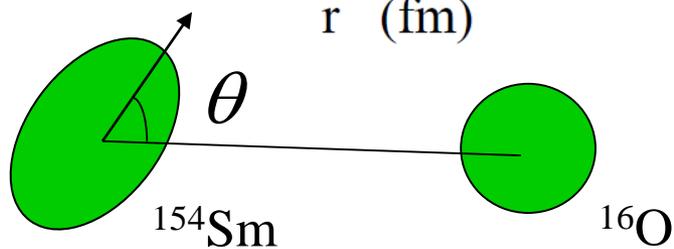
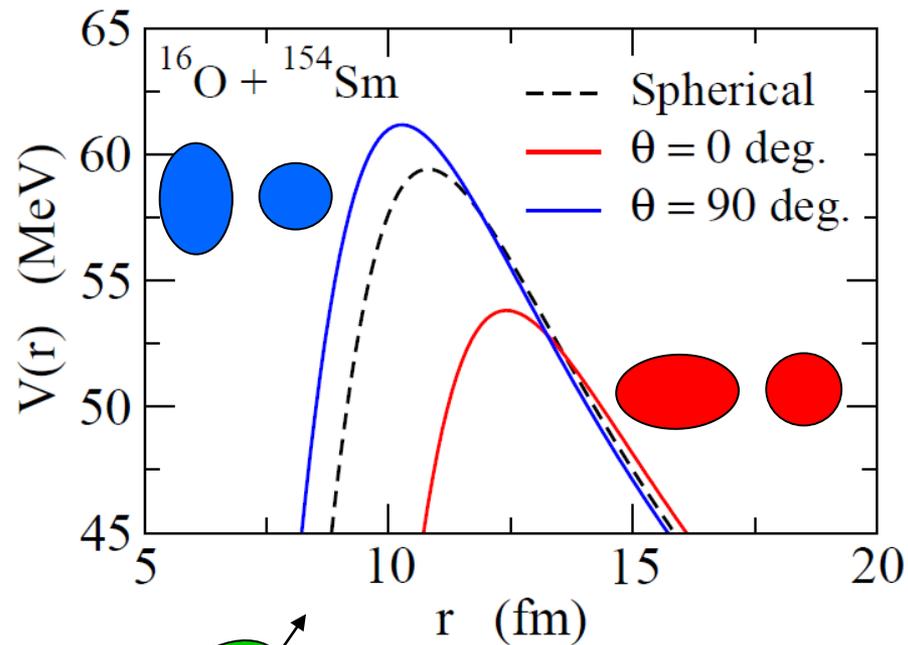


# Effects of nuclear deformation

$^{154}\text{Sm}$  : a typical deformed nucleus

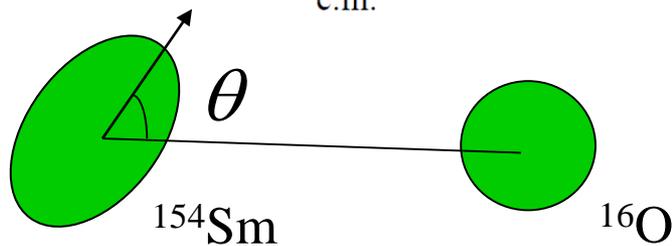
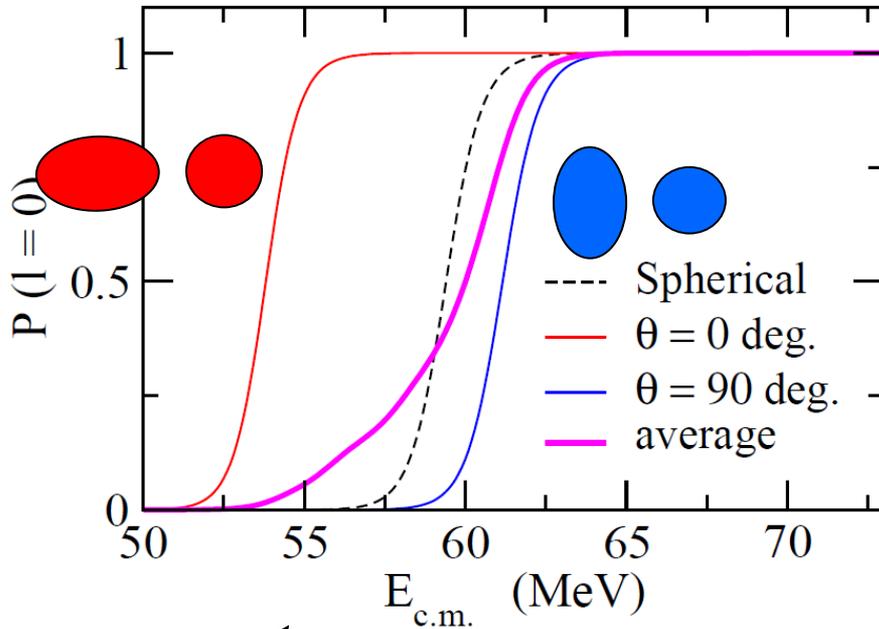


$^{154}\text{Sm}$

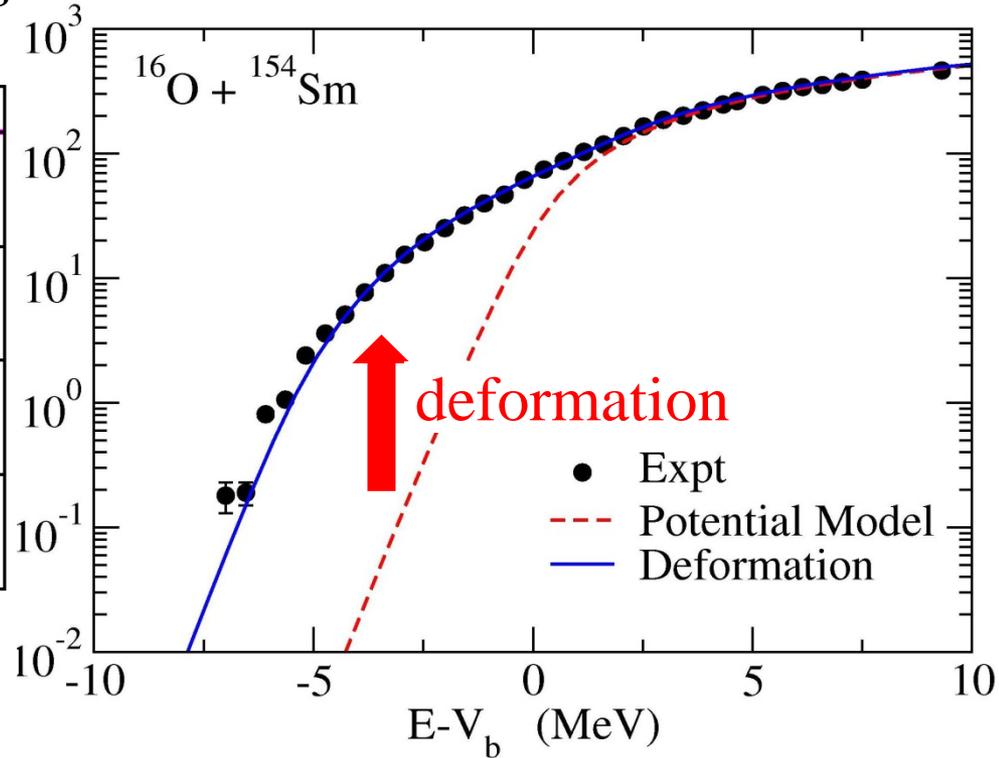


# Effects of nuclear deformation

$^{154}\text{Sm}$  : a typical deformed nucleus



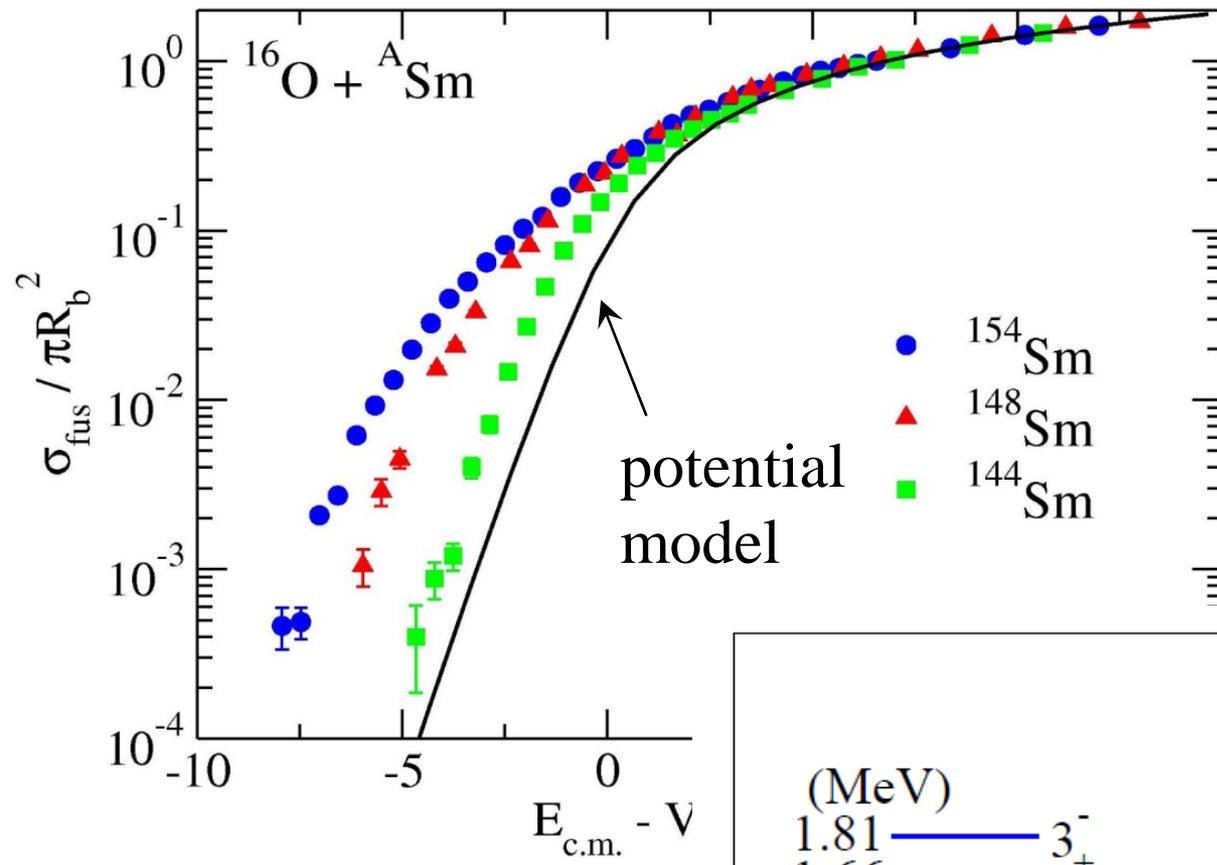
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$



**Fusion: strong interplay between nuclear structure and reaction**

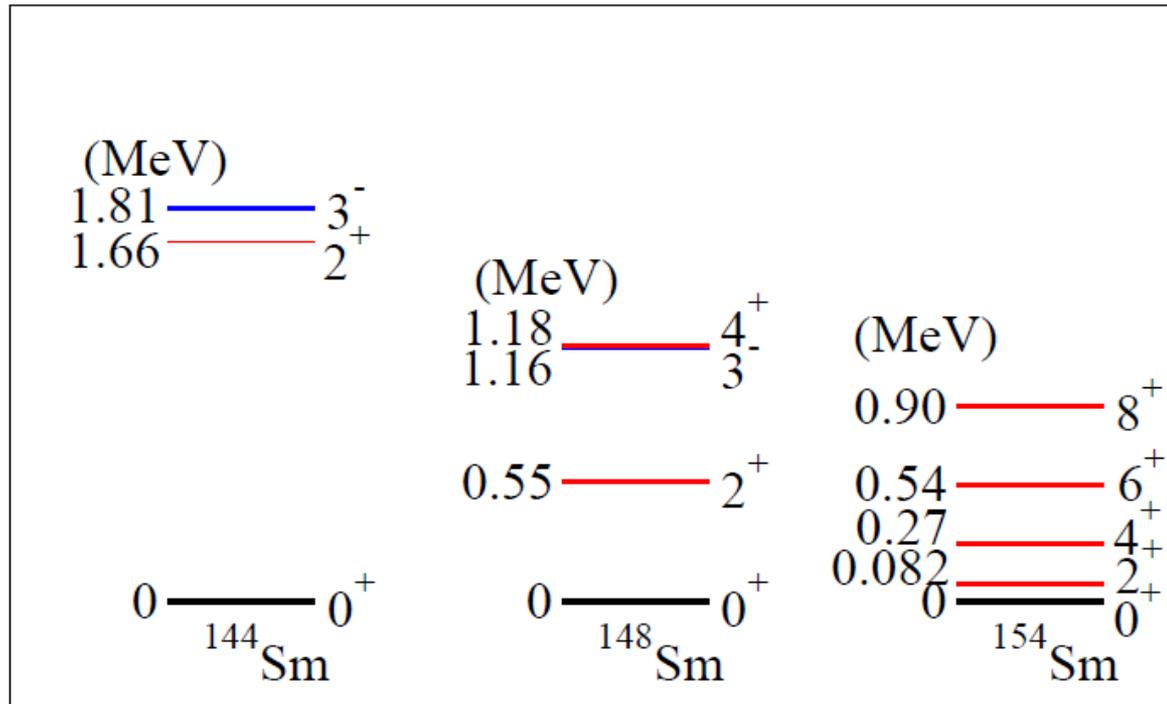
\* Sub-barrier enhancement also for non-deformed targets:

couplings to low-lying collective excitations → coupling assisted tunneling

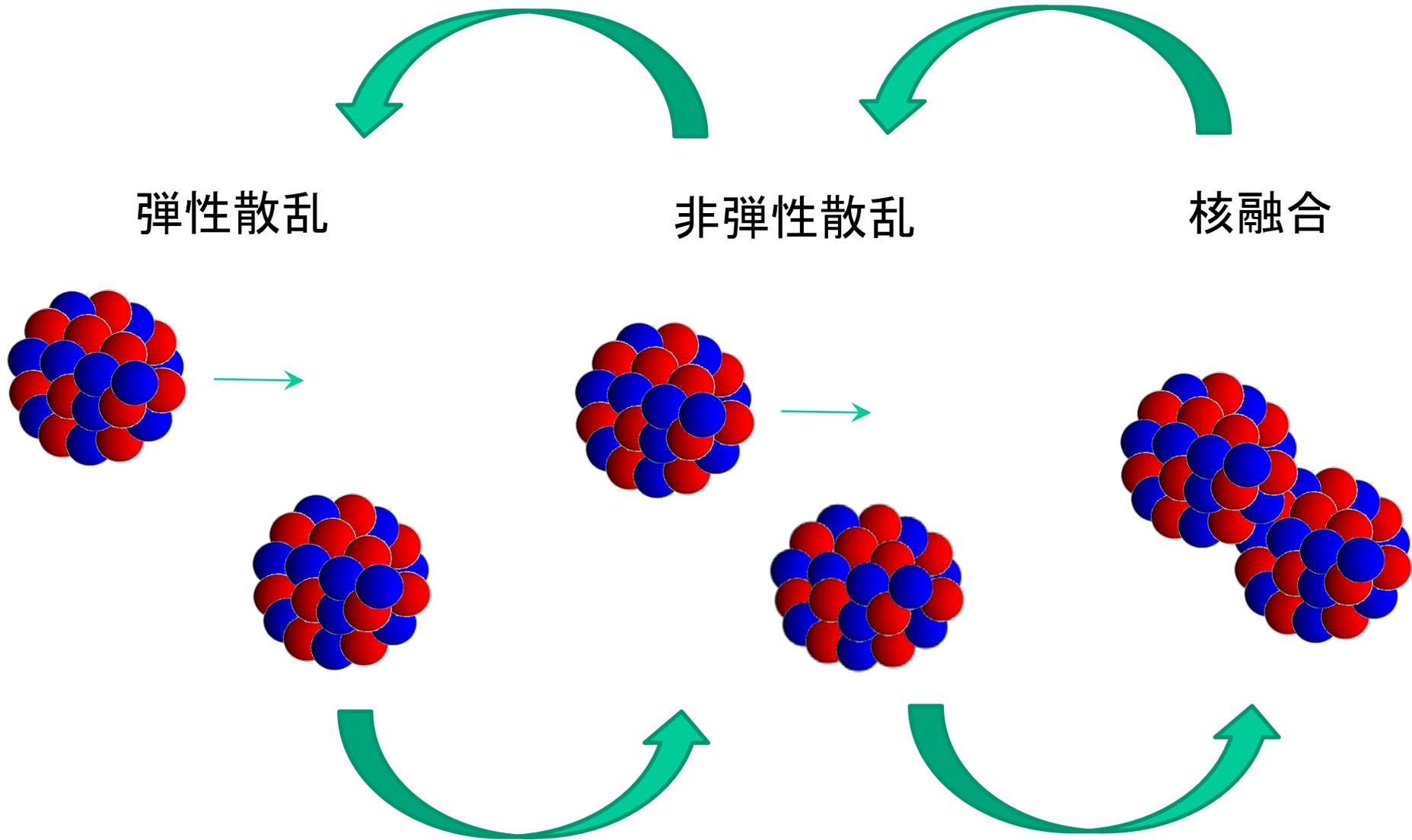


enhancement of fusion cross sections  
: a general phenomenon

strong correlation with nuclear spectrum  
→ coupling assisted tunneling

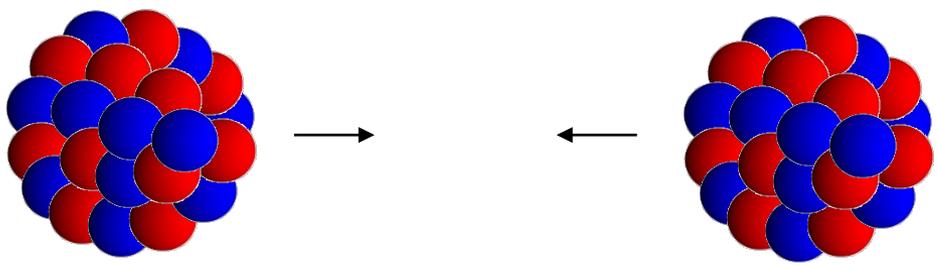


# 量子多体系のダイナミクス(原子核反応)



# Coupled-channels method: a quantal scattering theory with excitations

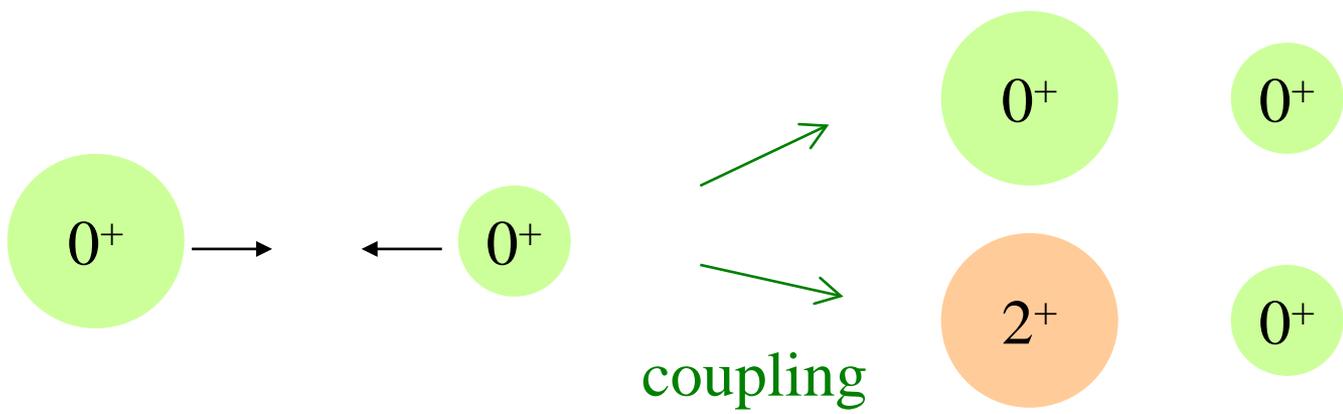
many-body problem



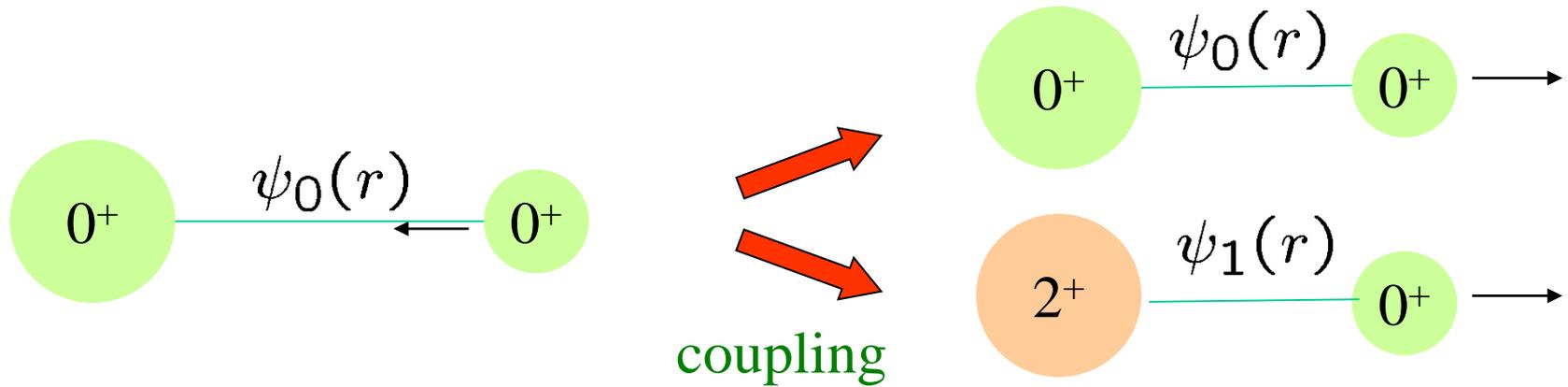
still very challenging



two-body problem, but with excitations  
(coupled-channels approach)



# Coupled-channels method: a quantal scattering theory with excitations



$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + \overleftrightarrow{V}(r) - \overleftrightarrow{E} \right] \overleftrightarrow{\psi}(r) = 0$$

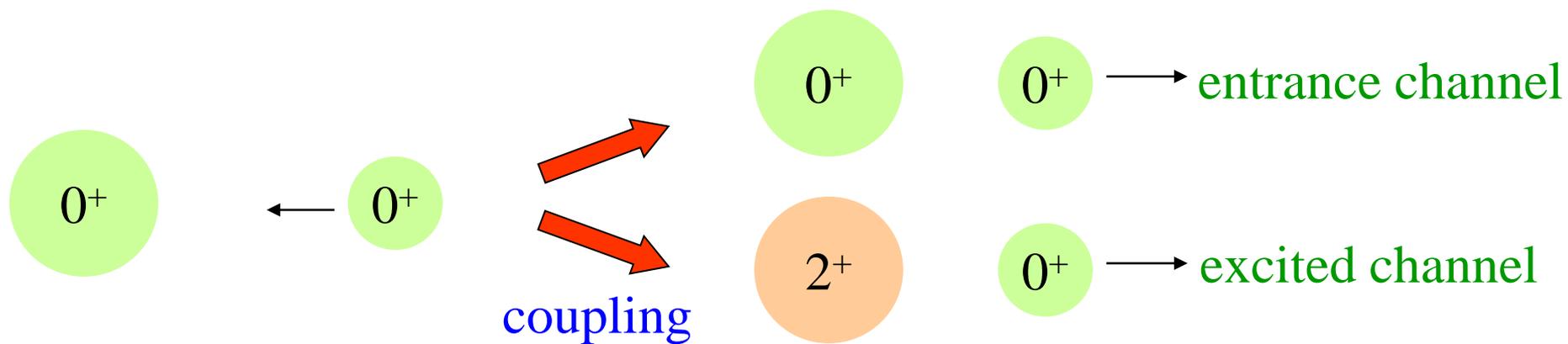
if written down more explicitly:

$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

↑  
excitation energy

↑  
excitation operator

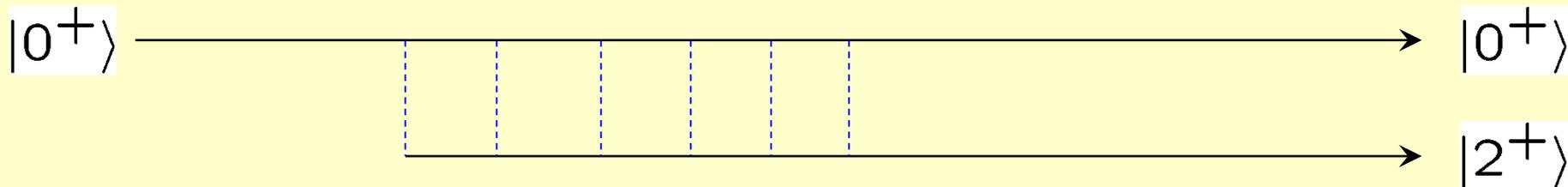
# Coupled-channels method: a quantal scattering theory with excitations



$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

excitation energy

excitation operator



full order treatment of excitation/de-excitation dynamics during reaction

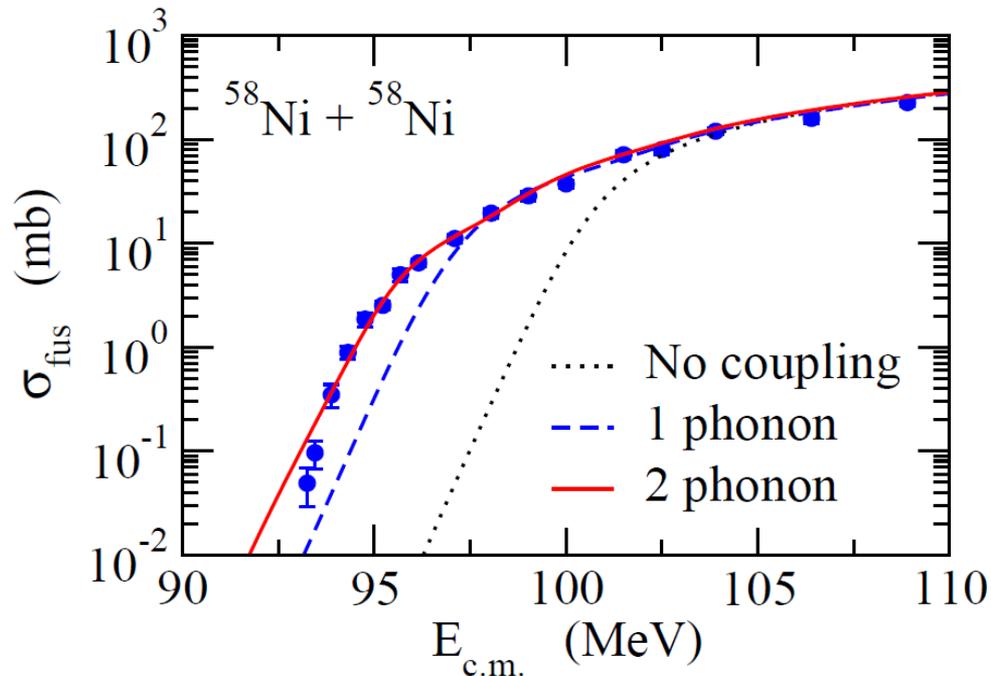
# Inputs for C.C. calculations

## i) Inter-nuclear potential

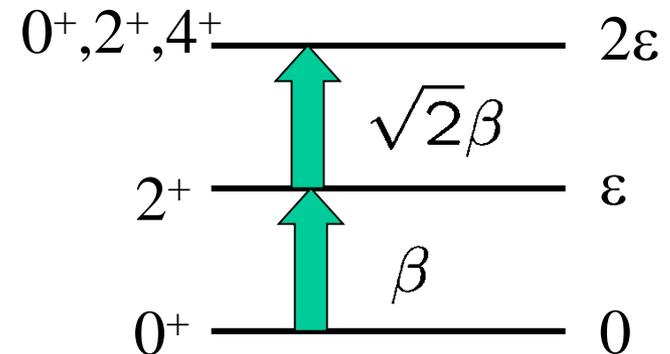
a fit to experimental data at above barrier energies

## ii) Intrinsic degrees of freedom

in most of cases, (macroscopic) collective model  
(rigid rotor / harmonic oscillator)



simple harmonic oscillator



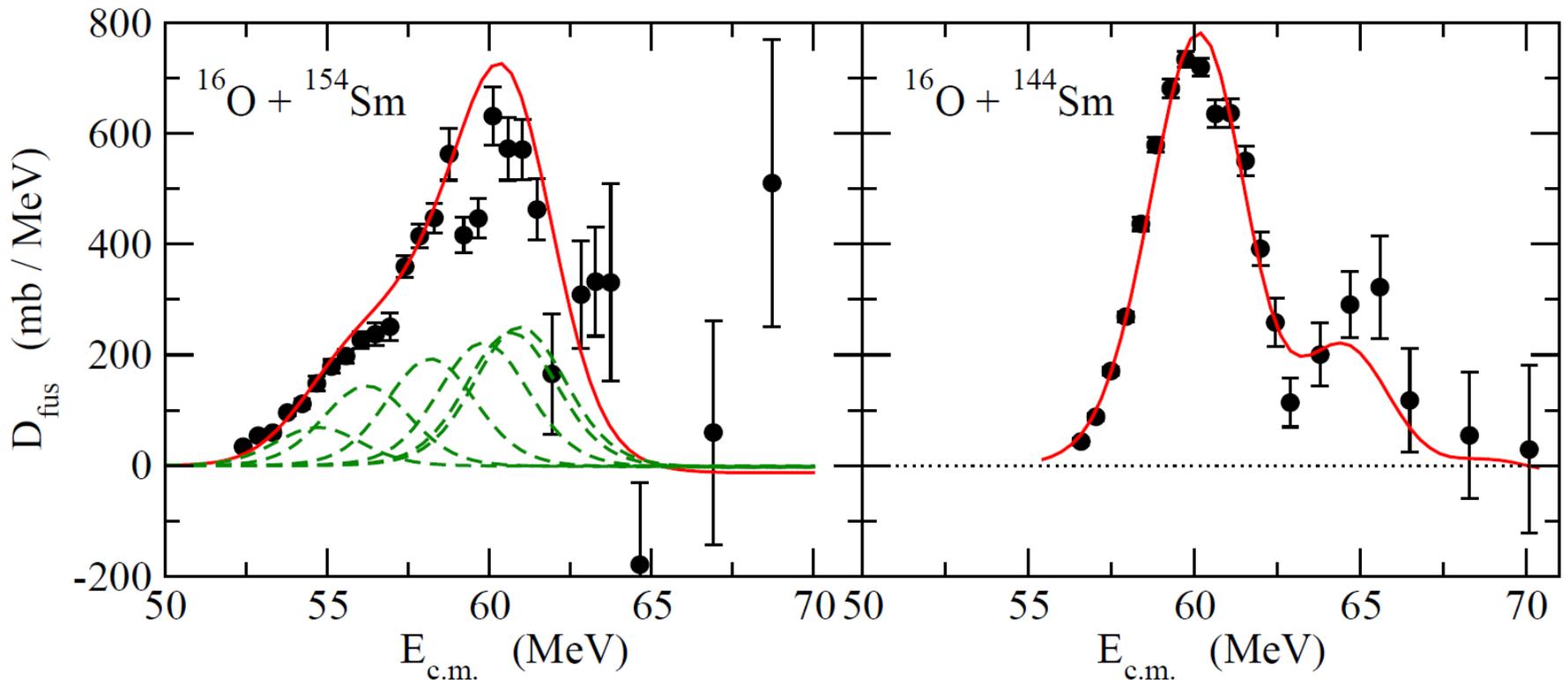
## C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143)

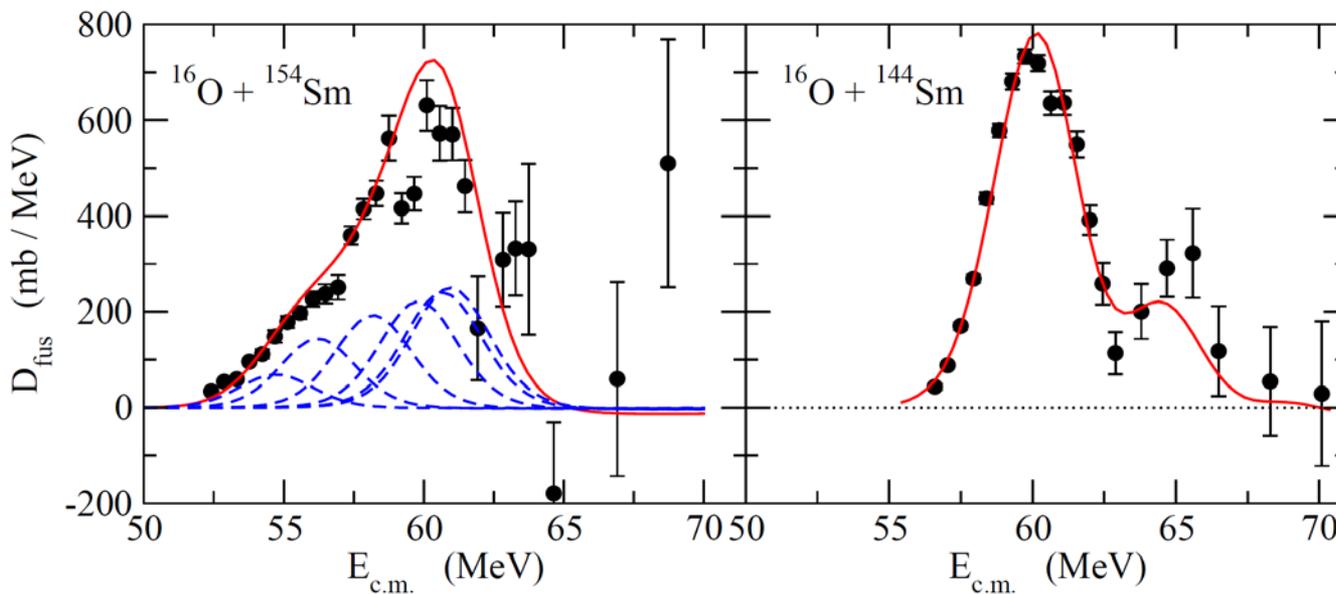
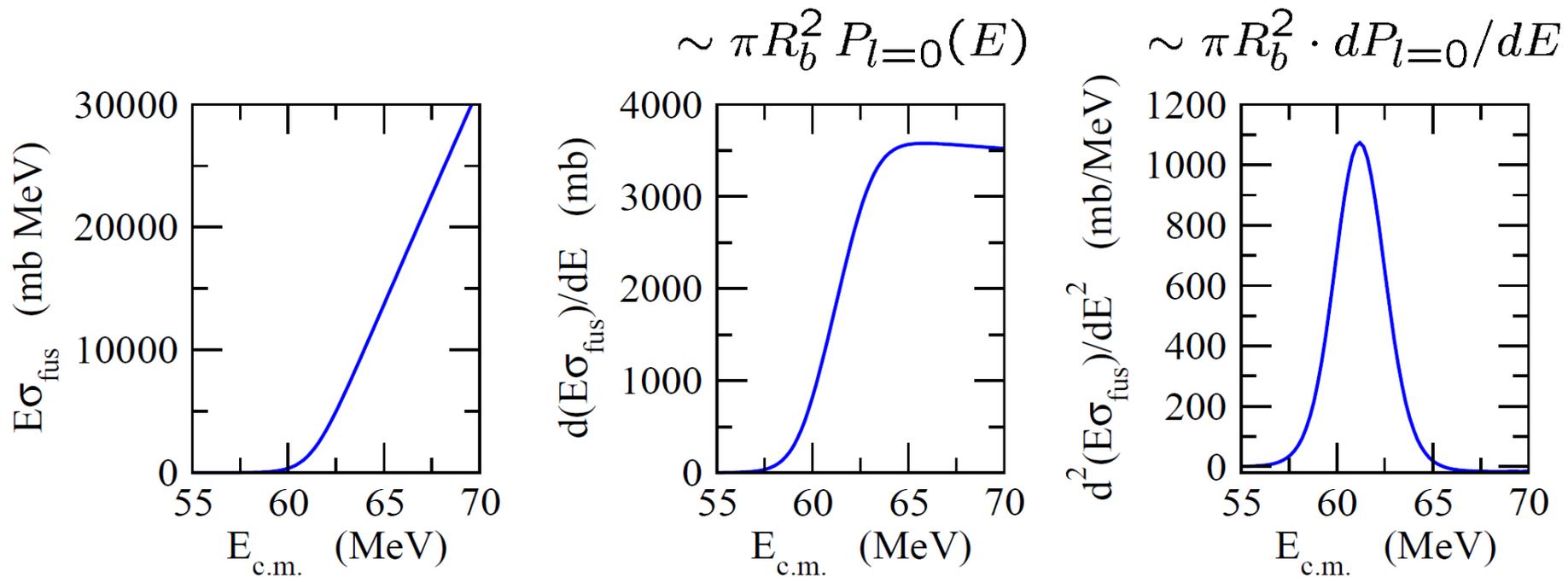
✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

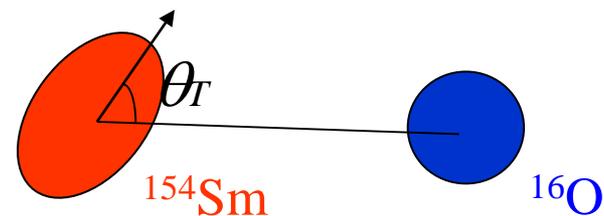
— c.c. calculations



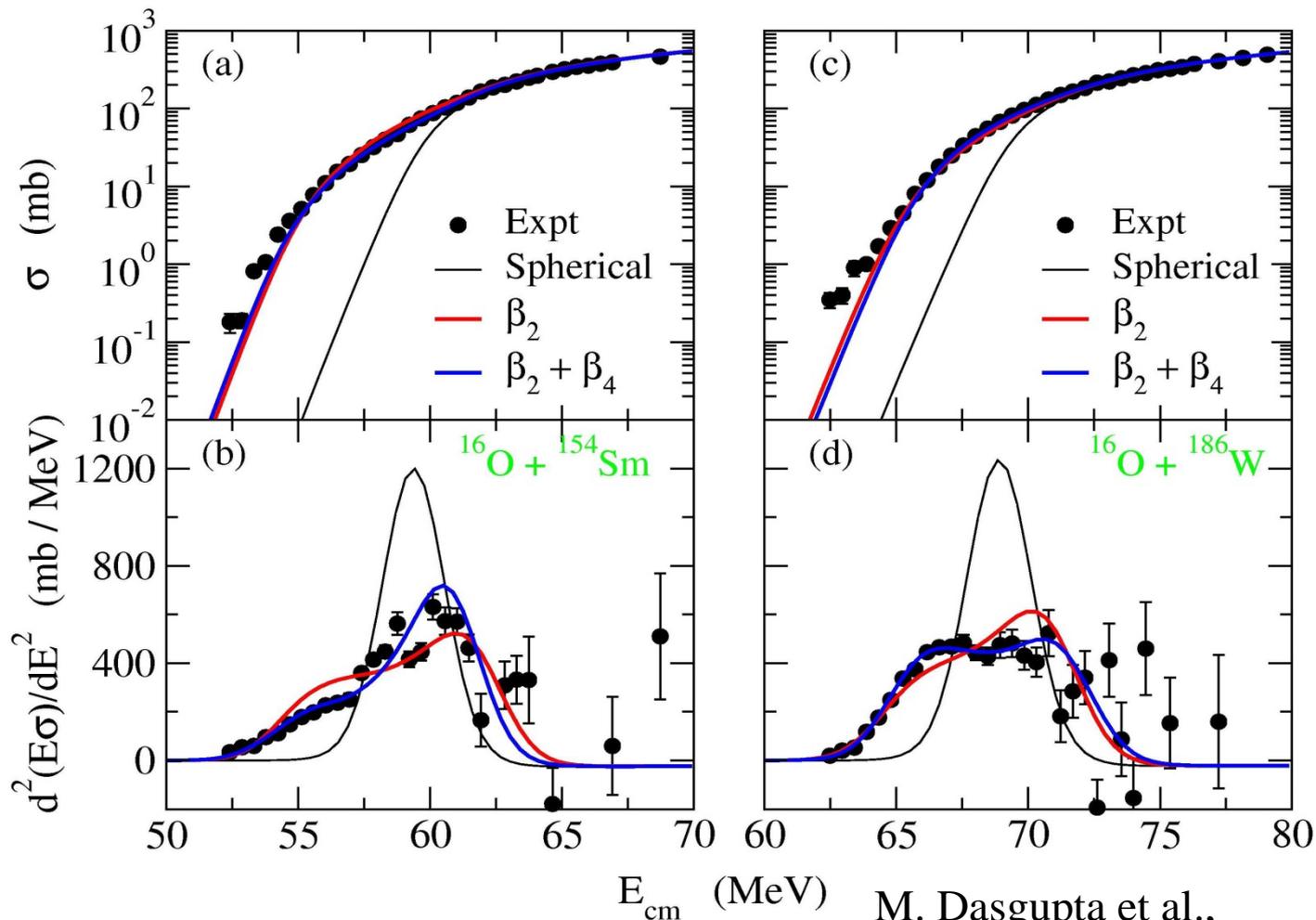
K.H., N. Takigawa, PTP128 ('12) 1061



# 高精度実験データから得られた障壁分布

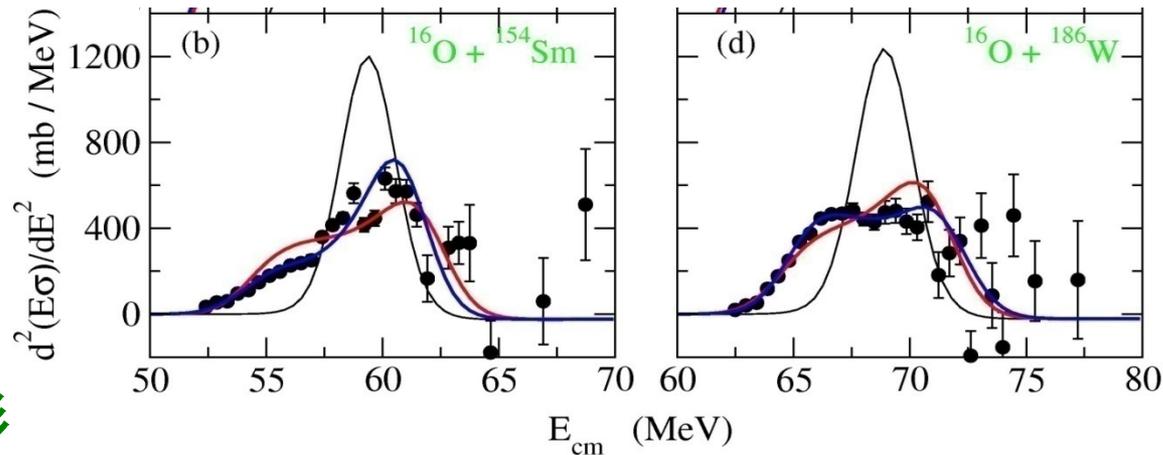


$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta_T) \sigma_{\text{fus}}(E; \theta_T)$$



M. Dasgupta et al.,  
Annu. Rev. Nucl. Part. Sci. 48('98)401

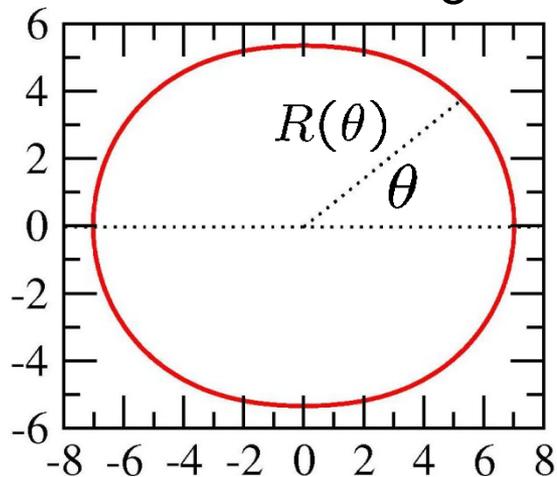
# 障壁分布を通じて原子核の形を見る



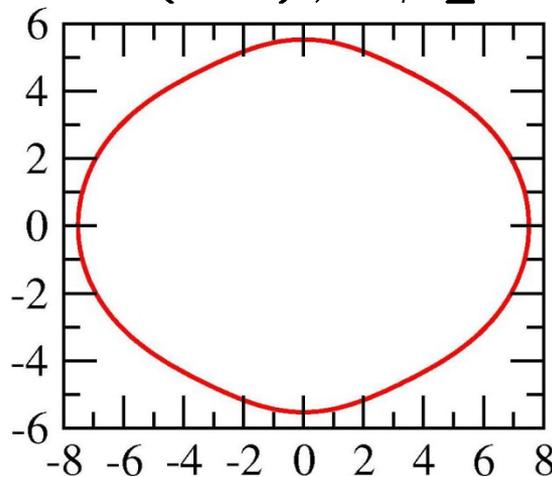
## 原子核の形

$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \dots)$$

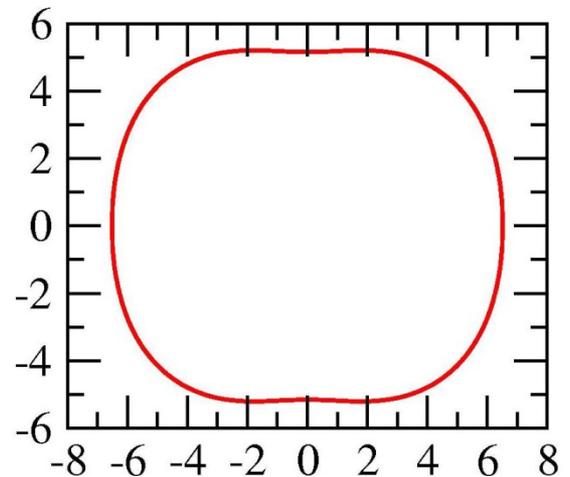
$$R_0 = 5.9 \text{ (fm)}, \quad \beta_2 = 0.3$$



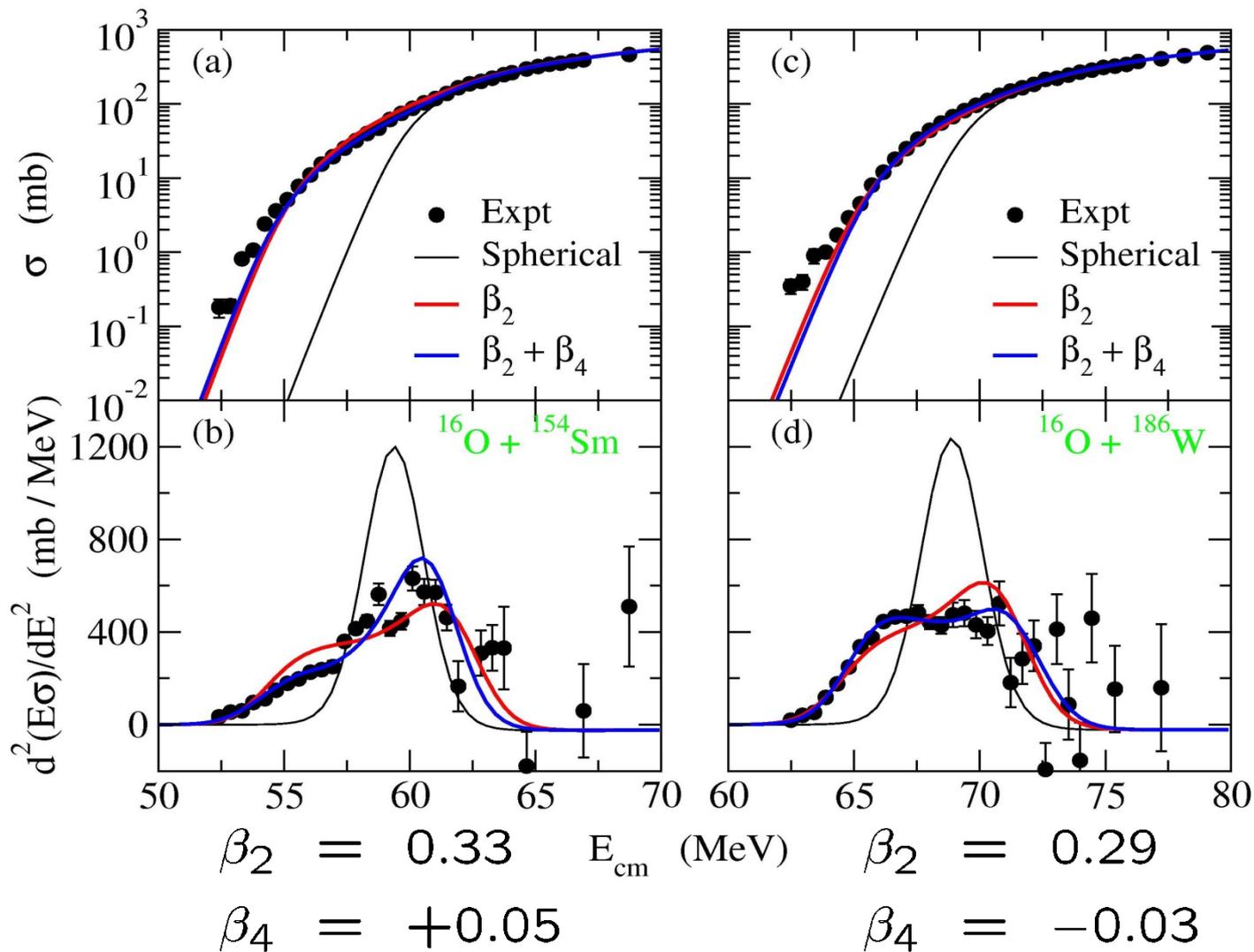
$$\beta_4 = 0$$



$$\beta_4 = 0.1$$



$$\beta_4 = -0.1$$



障壁分布をとることによって、 $\beta_4$ による違いがかなりはっきりと目に見える！

➡ 原子核に対する量子トンネル顕微鏡としての核融合反応