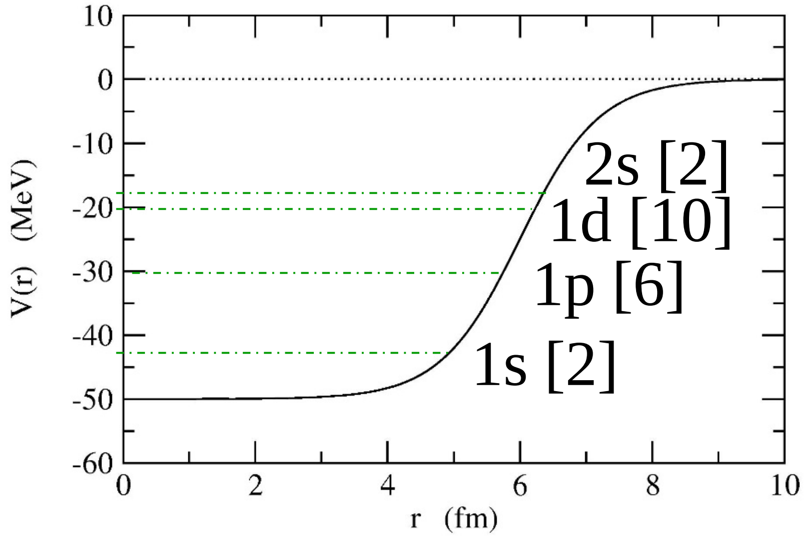
Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

An interpretation: independent particle motion in a potential well

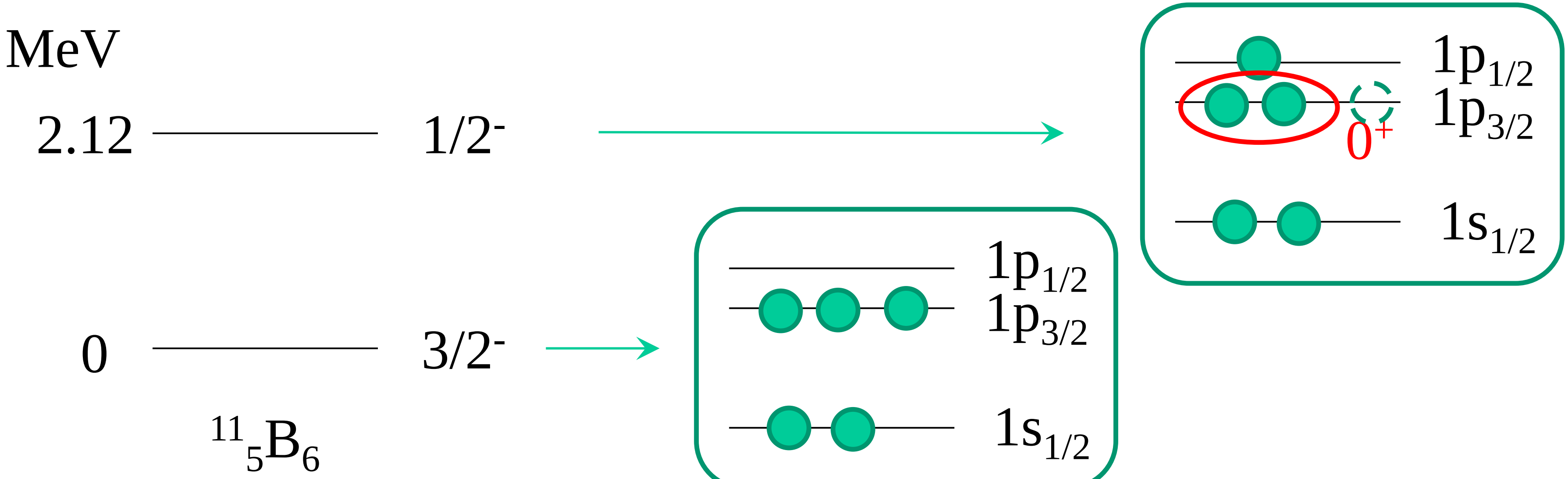


+ spin-orbit interaction

$$f[14] \rightarrow f_{5/2}[6],\quad f_{7/2}[8]$$

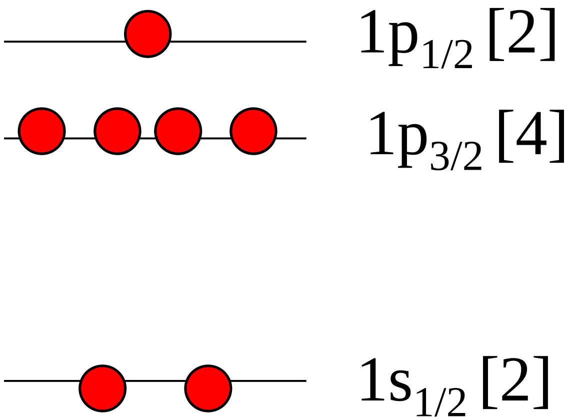
20+8 = 28

example:

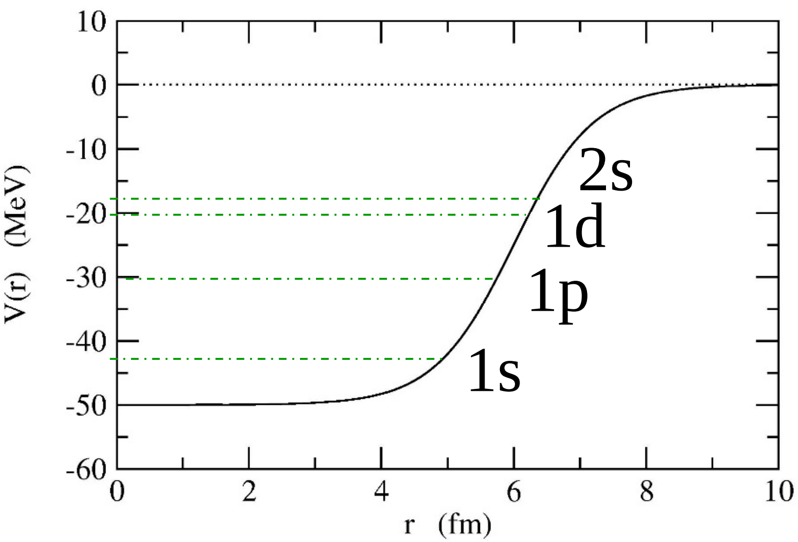


同じように殻模型で$^{11}_{4}\mathrm{Be}_{7}$ のレベルを考えると。。。

殻模型（球形ポテンシャルの準位）で考えた場合：

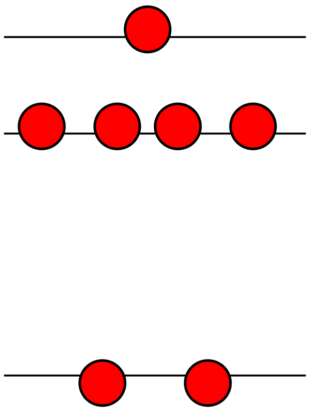


^{11}Be の基底状態は $I^{\pi} = 1/2^{-}$



同じように殻模型で$^{11}_{4}Be_{7}$ のレベルを考えると。。。

殻模型(球形ポテンシャルの準位)で考えた場合:



$1p_{1/2}$ [2]

$1p_{3/2}$ [4]

$1s_{1/2}$ [2]

^{11}Be の基底状態は $I^{\pi} = 1/2^{-}$

実際の ^{11}Be の準位を見てみると:

0.32 MeV

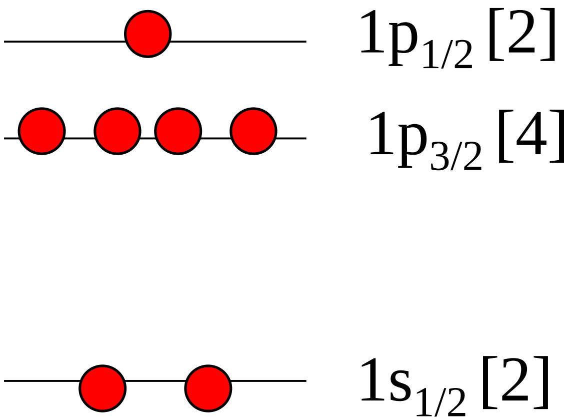
$1/2^{-}$

$1/2^{+}$

^{11}Be

同じように殻模型で$^{11}_{4}Be_{7}$ のレベルを考えると。。。

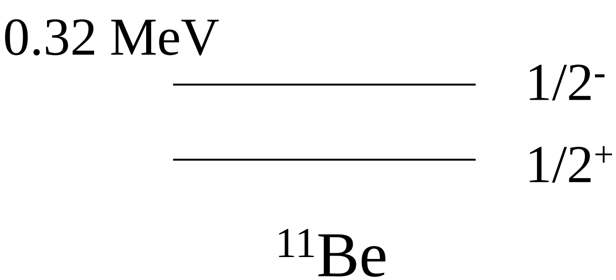
殻模型（球形ポテンシャルの準位）で考えた場合：



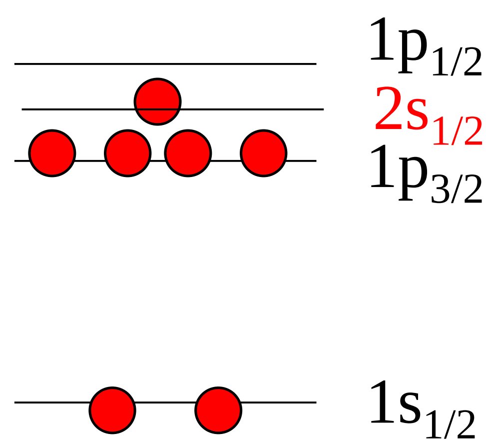
^{11}Be の基底状態は $I^{\pi} = 1/2^{-}$

かなり無理

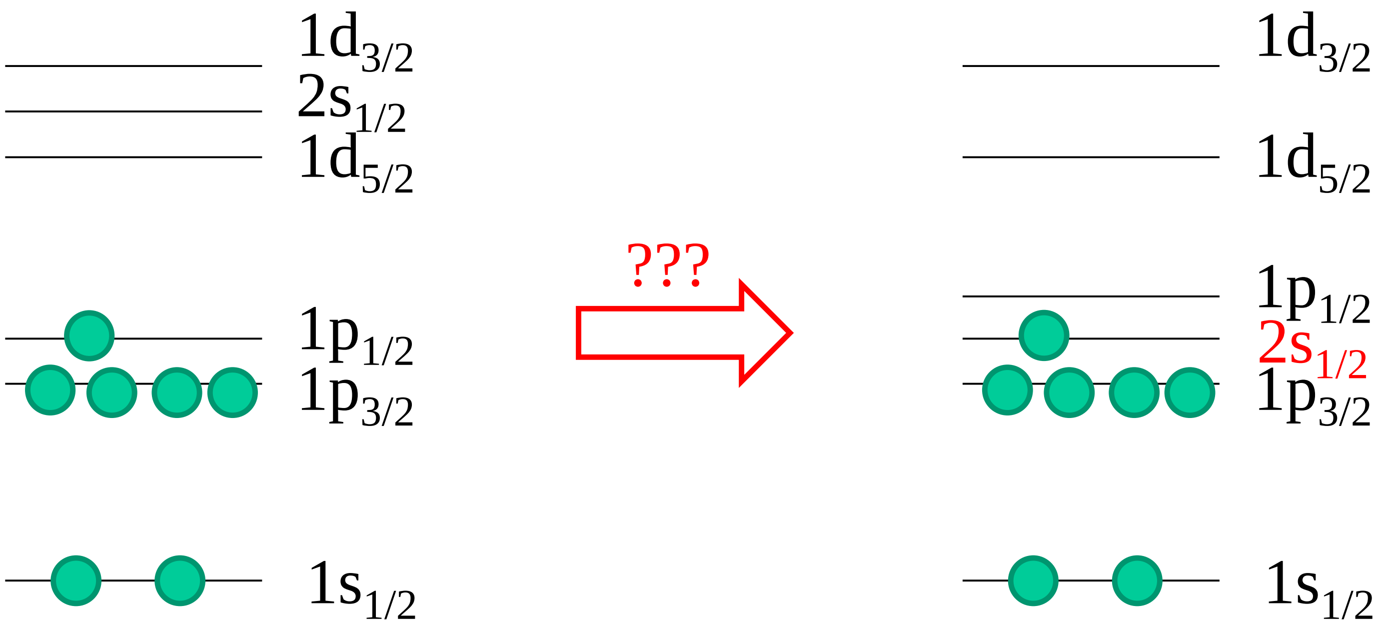
実際の ^{11}Be の準位を見てみると:



?



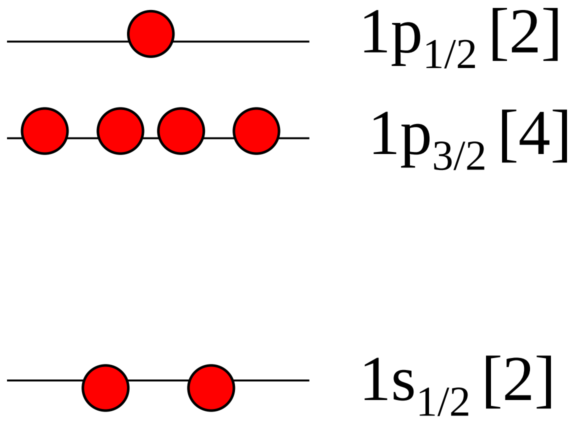
“parity inversion”



1d3/2
2s1/2
1d5/2
1p1/2
1p3/2
1s1/2
???
1d3/2
1d5/2
1p1/2
2s1/2
1p3/2
1s1/2

同じように殻模型で${}^{11}_{4}Be_7$ のレベルを考えると。。。

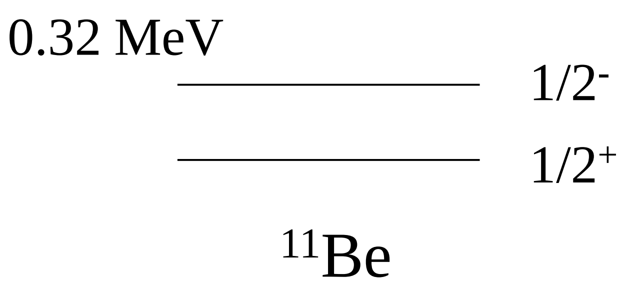
殻模型（球形ポテンシャルの準位）で考えた場合：



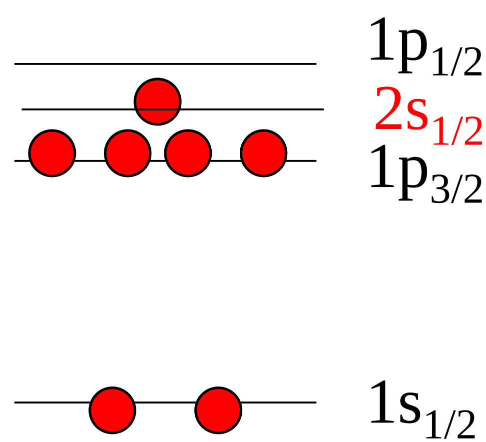
^{11}Be の基底状態は $I^{\pi} = 1/2^{-}$

かなり無理

実際の ^{11}Be の準位を見てみると:



?

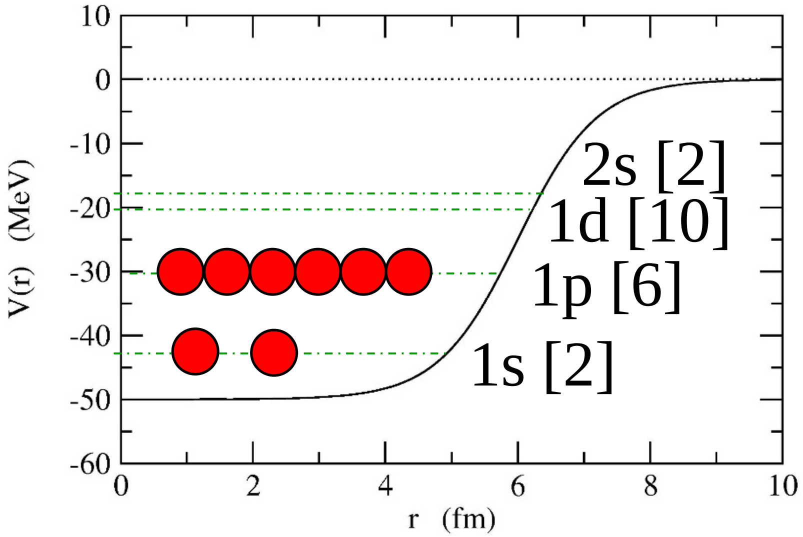


“parity inversion”

球形ポテンシャルに無理があるなら、変形させてみる?

Mean-field approximation and deformation

Mean-field approximation



$$H \sim \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\mathrm{MF}}(\boldsymbol{r}_i)\right)$$

Slater determinant

$$\Psi_{\mathrm{MF}}(1, 2, \cdots, A)$$
$$= \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)]$$

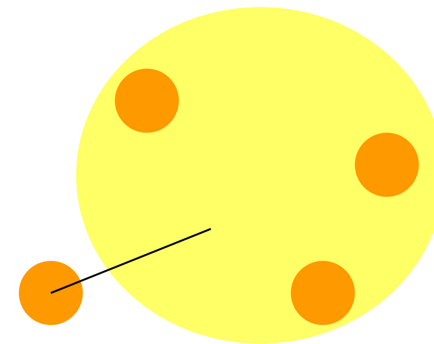
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{MF}}(\boldsymbol{r})\right)\psi_k(\boldsymbol{r}) = \epsilon_k\psi_k(\boldsymbol{r})$$

the original many-body *H*:

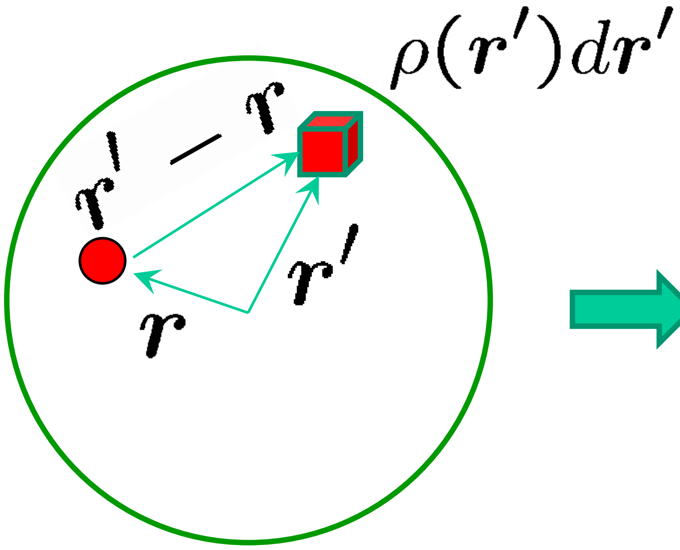
$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j)$$

$$= \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\mathrm{MF}}(\boldsymbol{r}_i)\right) + \cancel{\frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\mathrm{MF}}(\boldsymbol{r}_i)}$$

平均場



interaction for a nucleon inside a nucleus:



$$v(\boldsymbol{r}'-\boldsymbol{r})\cdot\underline{\rho(\boldsymbol{r}')d\boldsymbol{r}'}$$

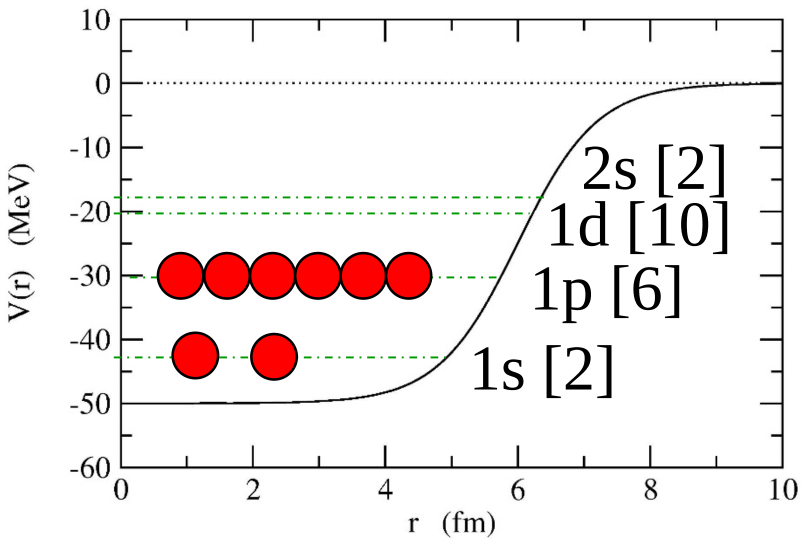
the number of nucleon at *r*’

naively speaking,

$$\boxed{V(\boldsymbol{r})\sim\int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'}$$

Mean-field approximation and deformation

Mean-field approximation



$$H \sim \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\mathrm{MF}}(\boldsymbol{r}_i) \right)$$

Slater determinant

$$\Psi_{\mathrm{MF}}(1,2,\cdots,A)$$
$$= \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)]$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{MF}}(\boldsymbol{r}) \right)\psi_k(\boldsymbol{r}) = \epsilon_k \psi_k(\boldsymbol{r})$$

$$H = \sum_{i=1}^{A} \left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\mathrm{MF}}(\boldsymbol{r}_i) \right) + \cancel{\frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) - \sum_i V_{\mathrm{MF}}(\boldsymbol{r}_i)}$$

Ψ_{MF} : does not necessarily possess the symmetries that H has.

"Symmetry-broken solution"
"Spontaneous Symmetry Broken"

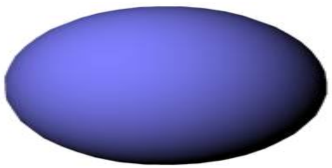
$\Psi_{\rm MF}$: does not necessarily possess the symmetries that H has.

Typical Examples

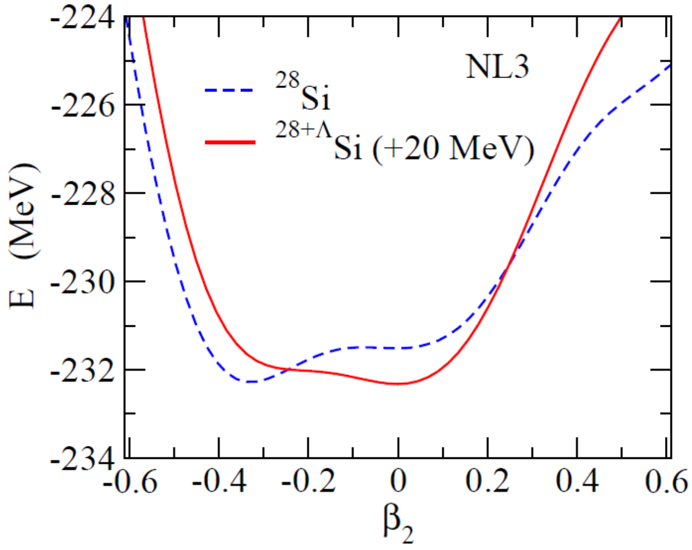
➢ Translational symmetry: always broken in nuclear systems

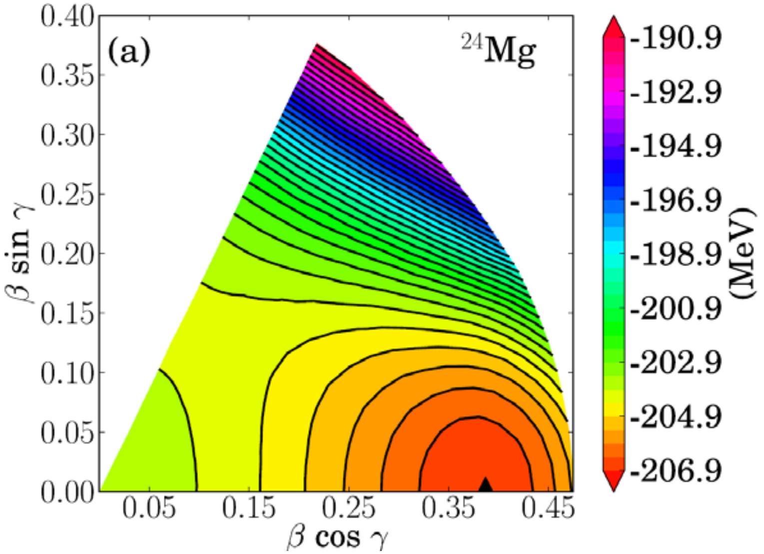
$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\nabla_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i - \boldsymbol{r}_j) \to \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\nabla_i^2 + \underline{\underline{V_{\rm MF}(\boldsymbol{r}_i)}}\right)$$

➢ Rotational symmetry



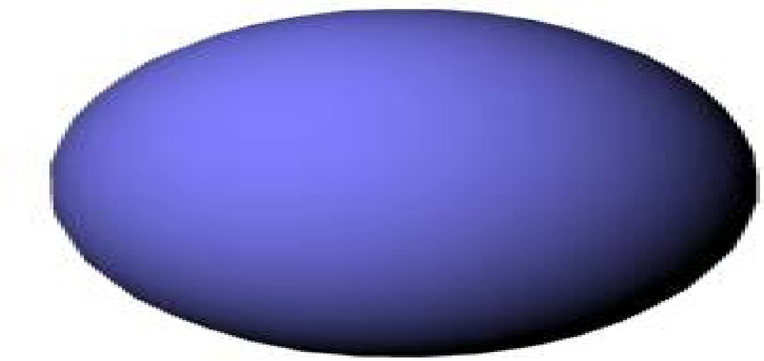
Deformed solution





Nuclear Deformation

実験的な証拠



Nuclear Deformation

Excitation spectra of ^{154}Sm

(MeV)	
0.903	8^+
0.544	6^+
0.267	4^+
0.082	2^+
0	0^+

^{154}Sm

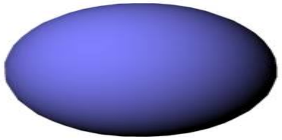
$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

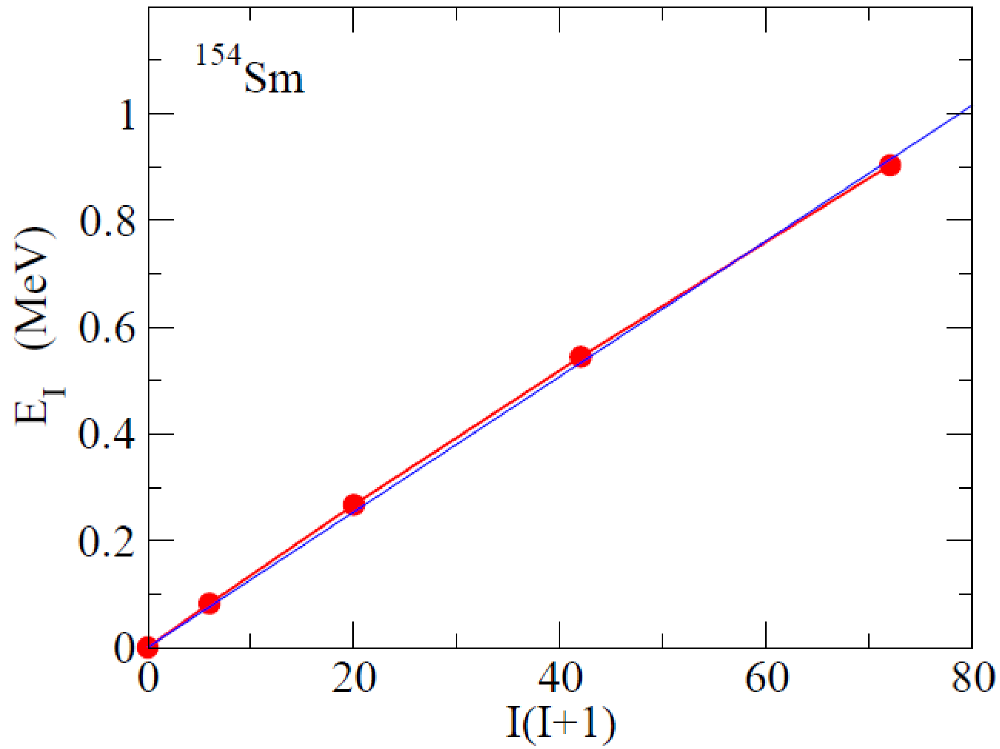
cf. Rotational energy of a rigid body
(Classical mechanics)

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \ \ \omega = \dot{\theta})$$

^{154}Sm is deformed



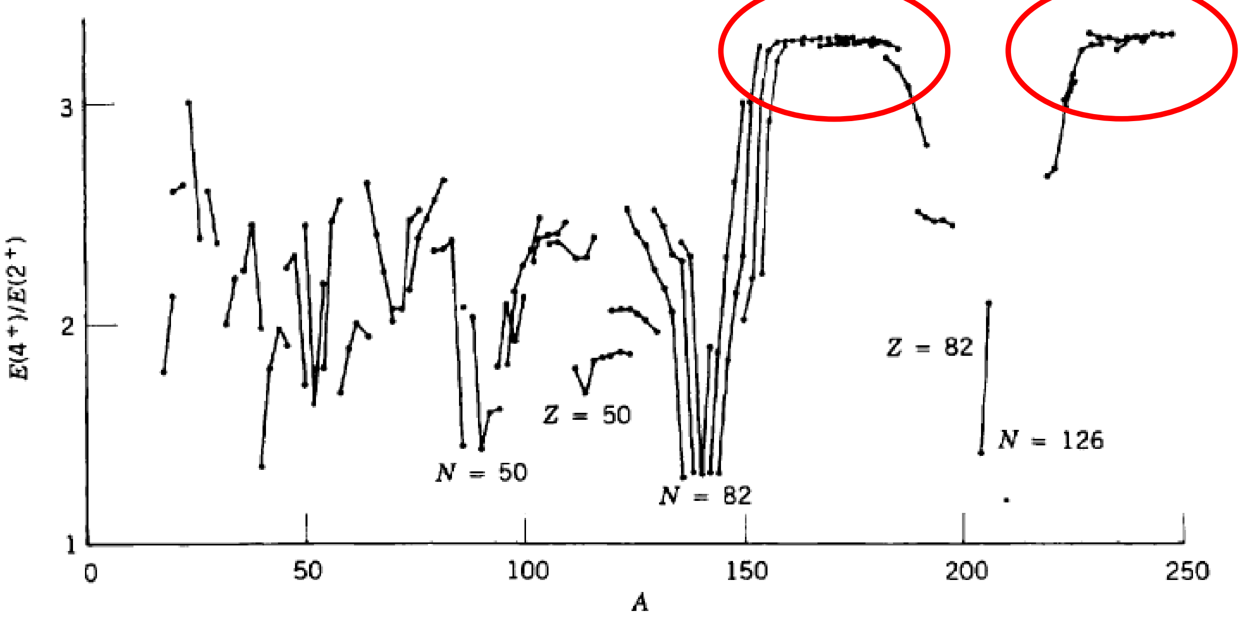


$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

$$\Longrightarrow \quad E_2 \propto 2 \times 3 = 6, \quad E_4 \propto 4 \times 5 = 20$$

$$\Longrightarrow \quad E_4/E_2 = 20/6 = 3.3333\cdots$$

E(4⁺)/E(2⁺)

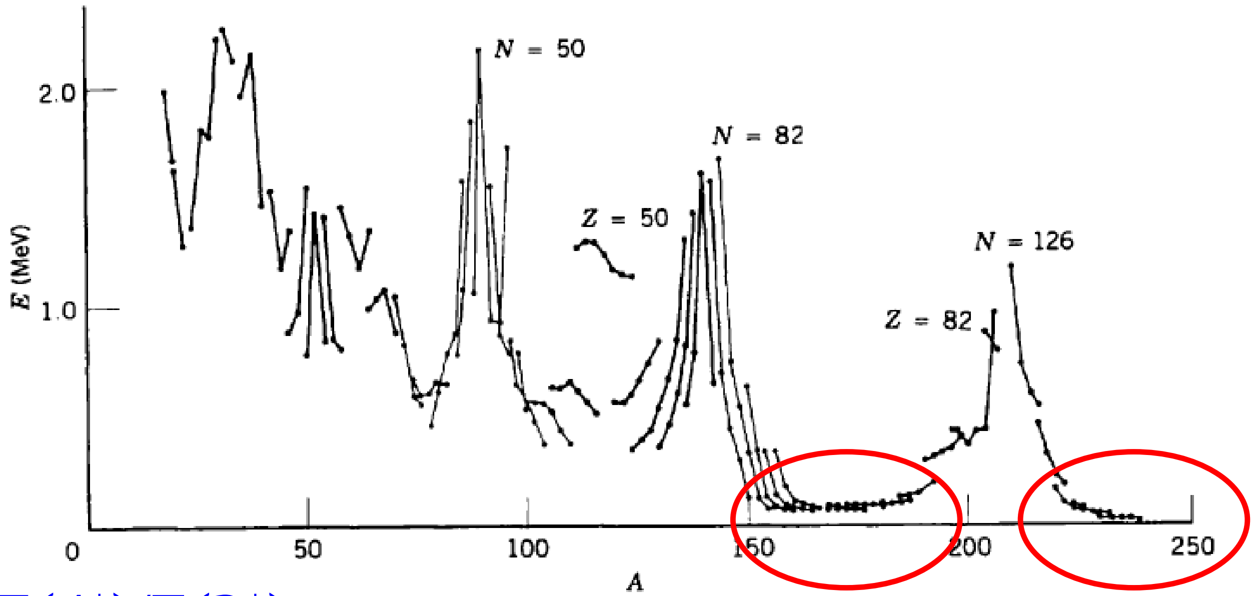


deformed nuclei:
$E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:
$E(4^+)/E(2^+) \sim 2$

K.S. Krane, "Introductory Nuclear Physics"

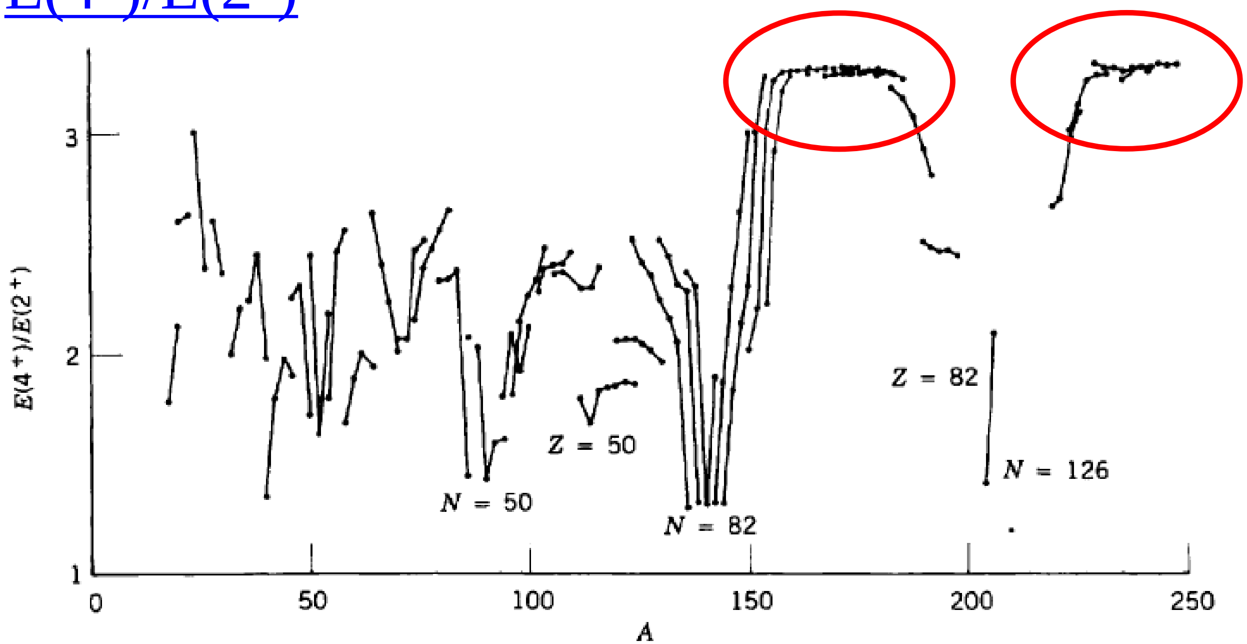
The energy of the first 2^+ state in even-even nuclei



a small energy
→ spontaneously
symm. breaking

deformed nuclei

$E(4^+)/E(2^+)$



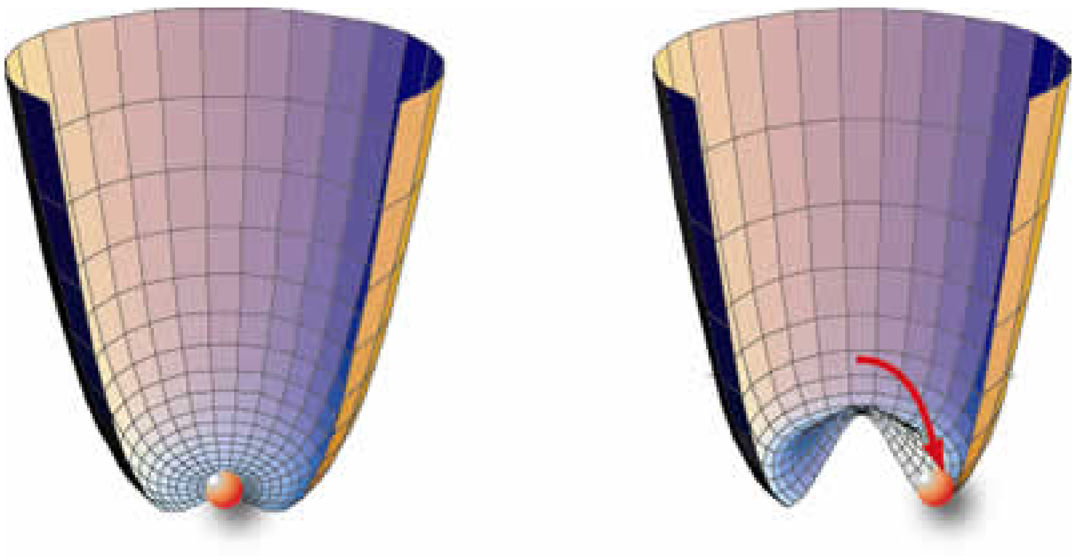
deformed nuclei:
$E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:
$E(4^+)/E(2^+) \sim 2$

K.S. Krane, "Introductory Nuclear Physics"

Spontaneous symmetry breaking

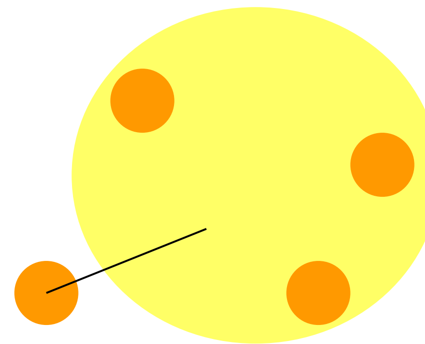
The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.



Nambu-Goldstone mode (zero energy mode) to restore the symmetry

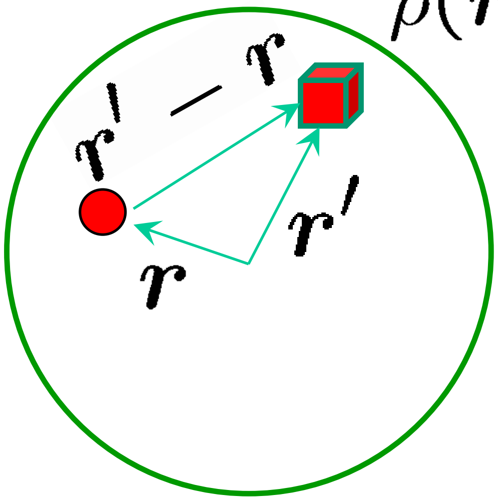
One-particle motion in a deformed potential

平均場



interaction for a nucleon inside a nucleus:

$$\rho(\boldsymbol{r}')d\boldsymbol{r}'$$



$$v(\boldsymbol{r}' - \boldsymbol{r}) \cdot \underline{\rho(\boldsymbol{r}')d\boldsymbol{r}'}$$

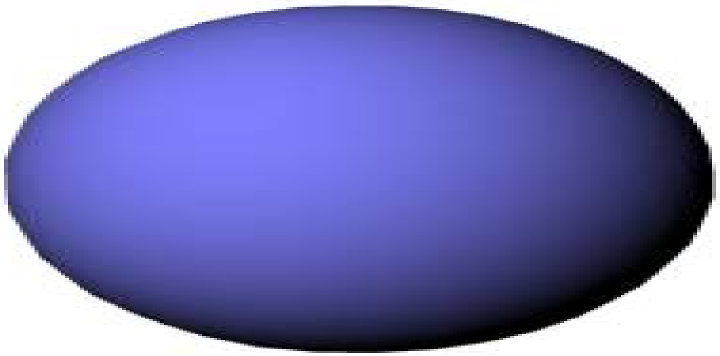
the number of nucleon at *r'*

naively speaking, $$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r} - \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\boxed{V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}, \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \quad \text{if} \quad v(\boldsymbol{r}, \boldsymbol{r}') = -g\delta(\boldsymbol{r} - \boldsymbol{r}')}$$

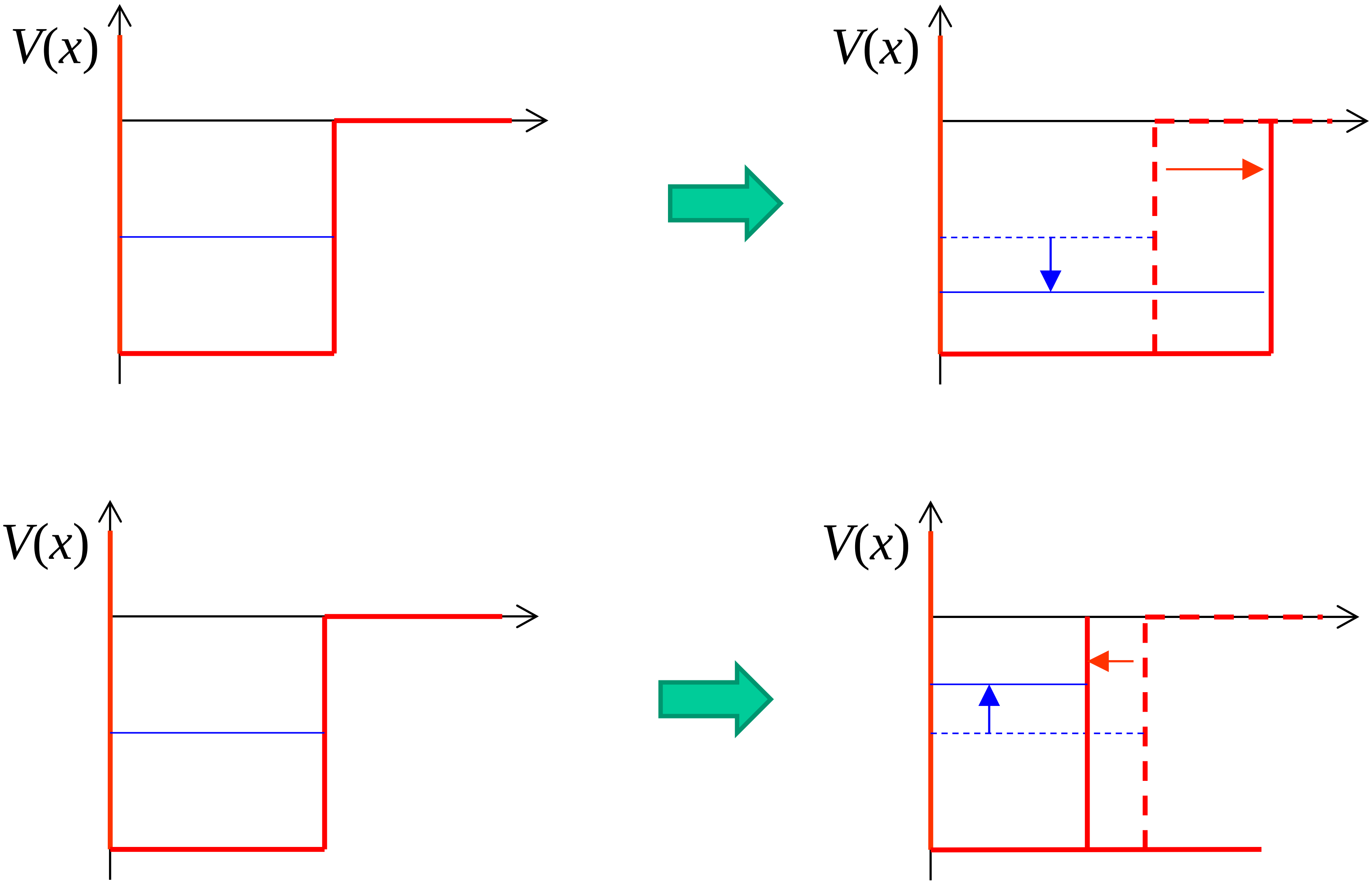
One-particle motion in a deformed potential

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$



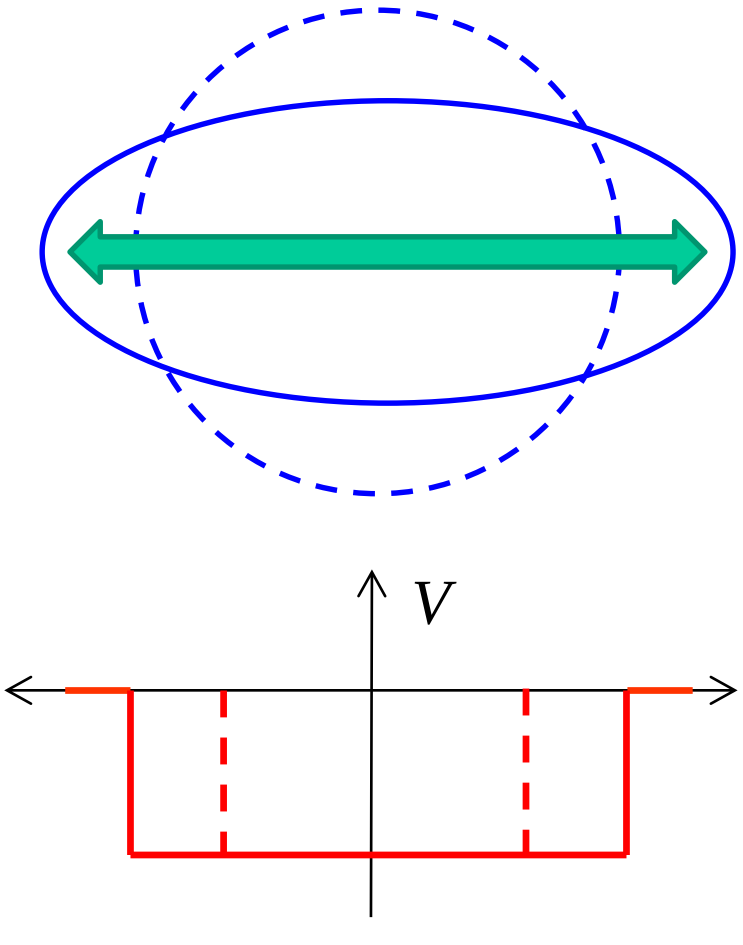
$$V(r) \qquad\qquad V(r,\theta)$$

（準備）1次元井戸型ポテンシャル



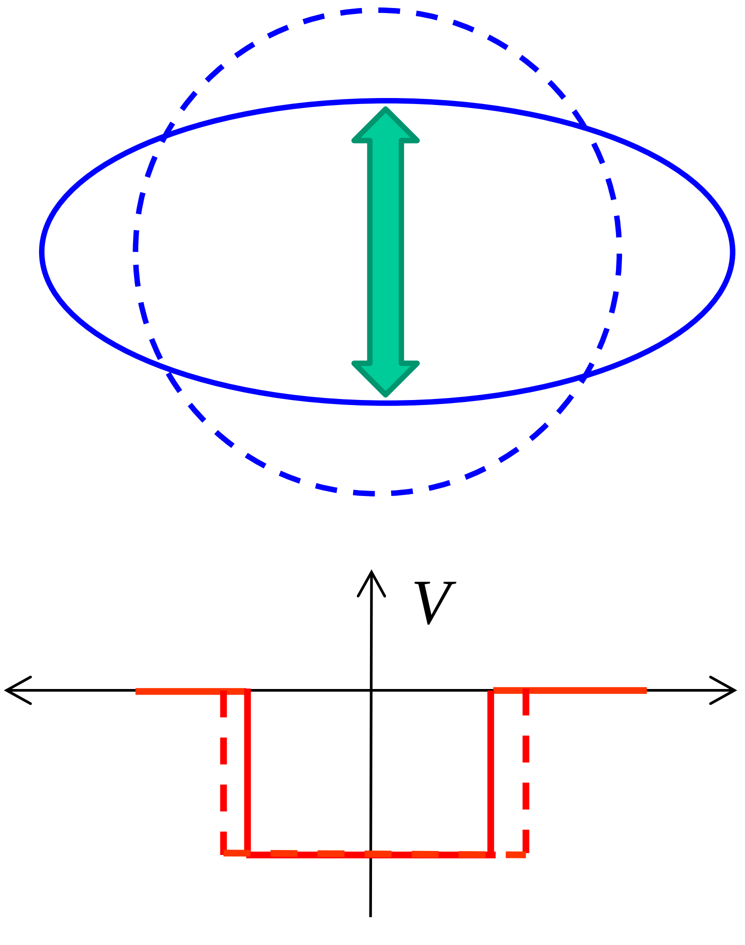
One-particle motion in a deformed potential

長軸に沿った運動



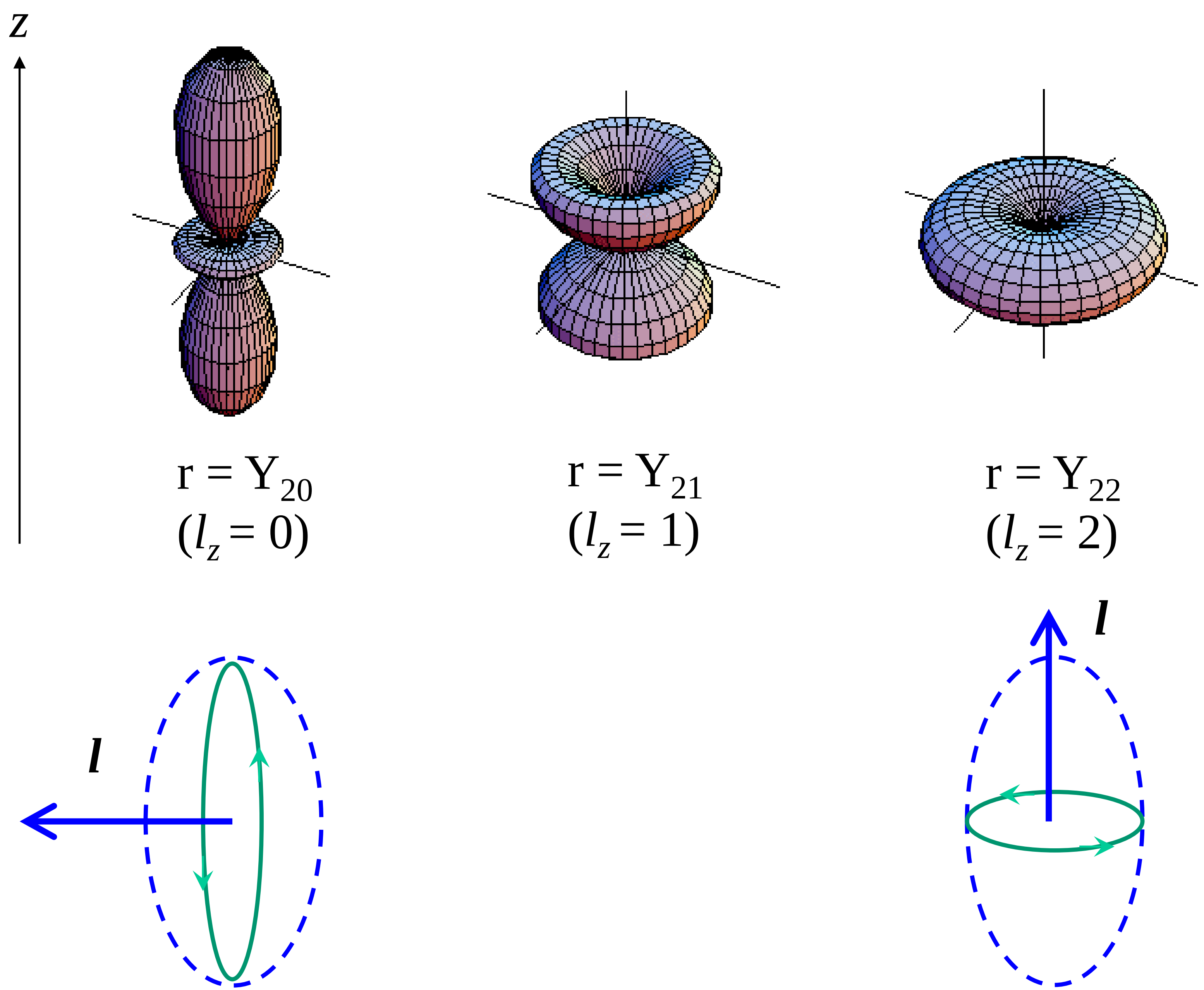
$E \rightarrow$ 小

短軸に沿った運動



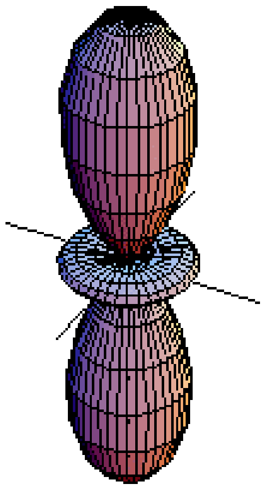
$E \rightarrow$ 大

$l = 2$

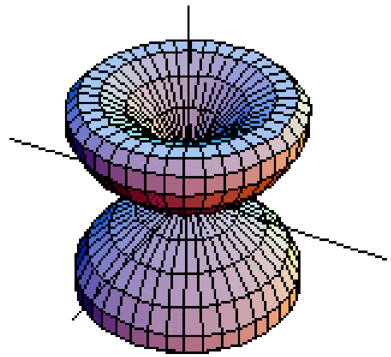


$l = 2$

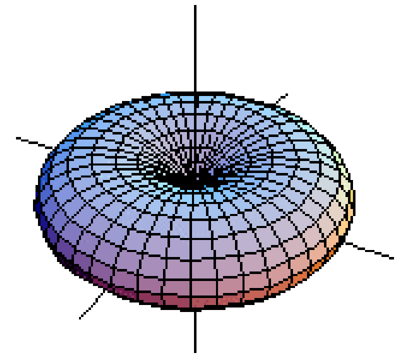
z



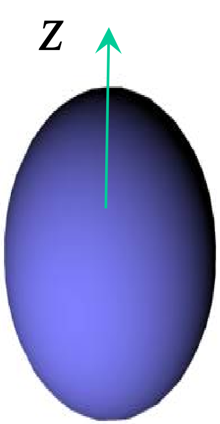
$r = Y_{20}$
$(l_z = 0)$



$r = Y_{21}$
$(l_z = 1)$



$r = Y_{22}$
$(l_z = 2)$

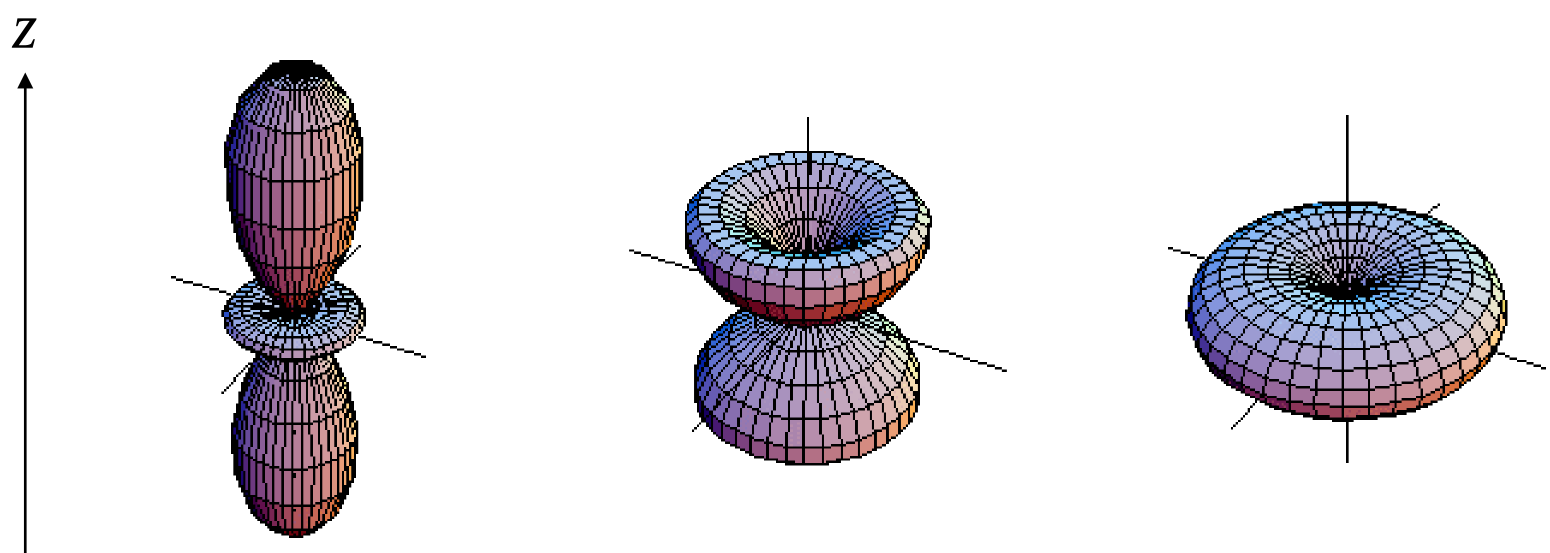


$E \rightarrow$ 小

$E \rightarrow$ 大

なら

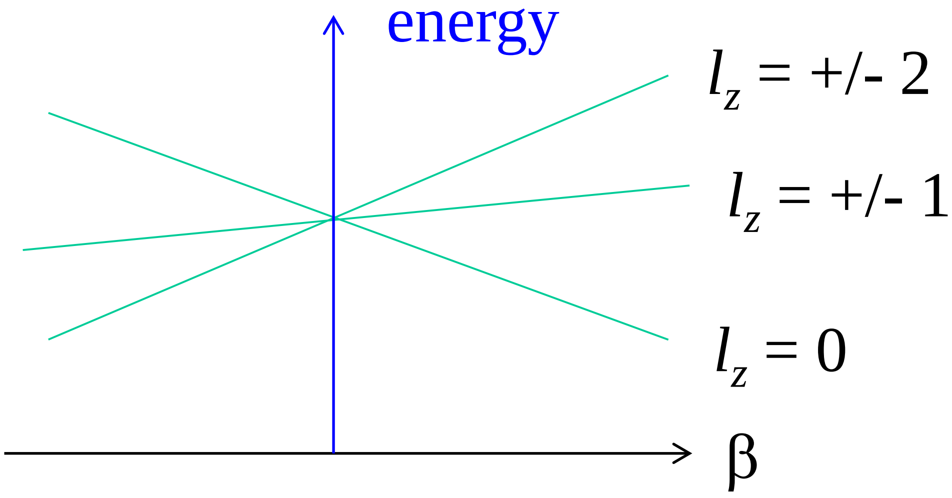
$l = 2$

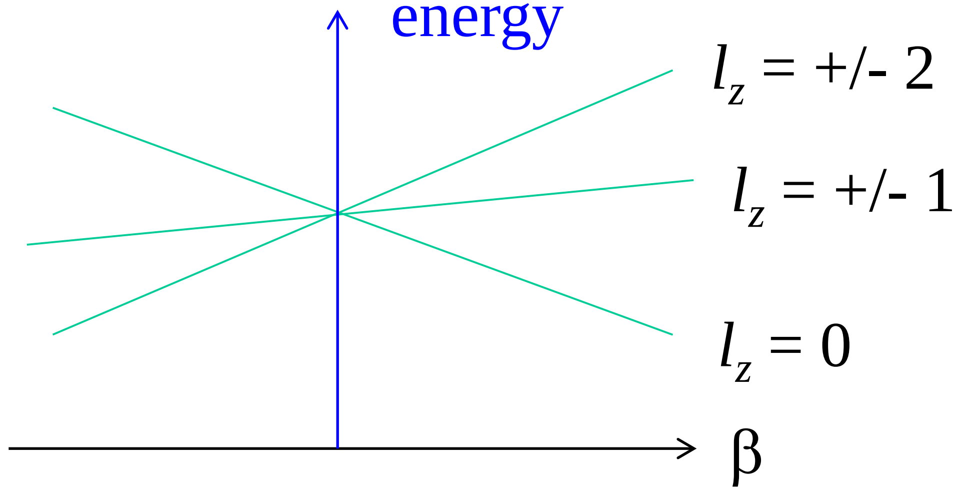


$r = Y_{20}$ ($l_z = 0$)

$r = Y_{21}$ ($l_z = 1$)

$r = Y_{22}$ ($l_z = 2$)





(note)　$V(r,\theta)$　→ 回転対称性を持っていない

→ 角運動量がいい量子数ではない

$$\phi_{nll_z}(r,\theta,\phi) \rightarrow \phi_{nl_z}(r,\theta,\phi) = \sum_l \psi_{nl}(r) Y_{ll_z}(\theta,\phi)$$

いろいろな角運動量成分が混じる

＊軸対称変形であれば l_z は保存

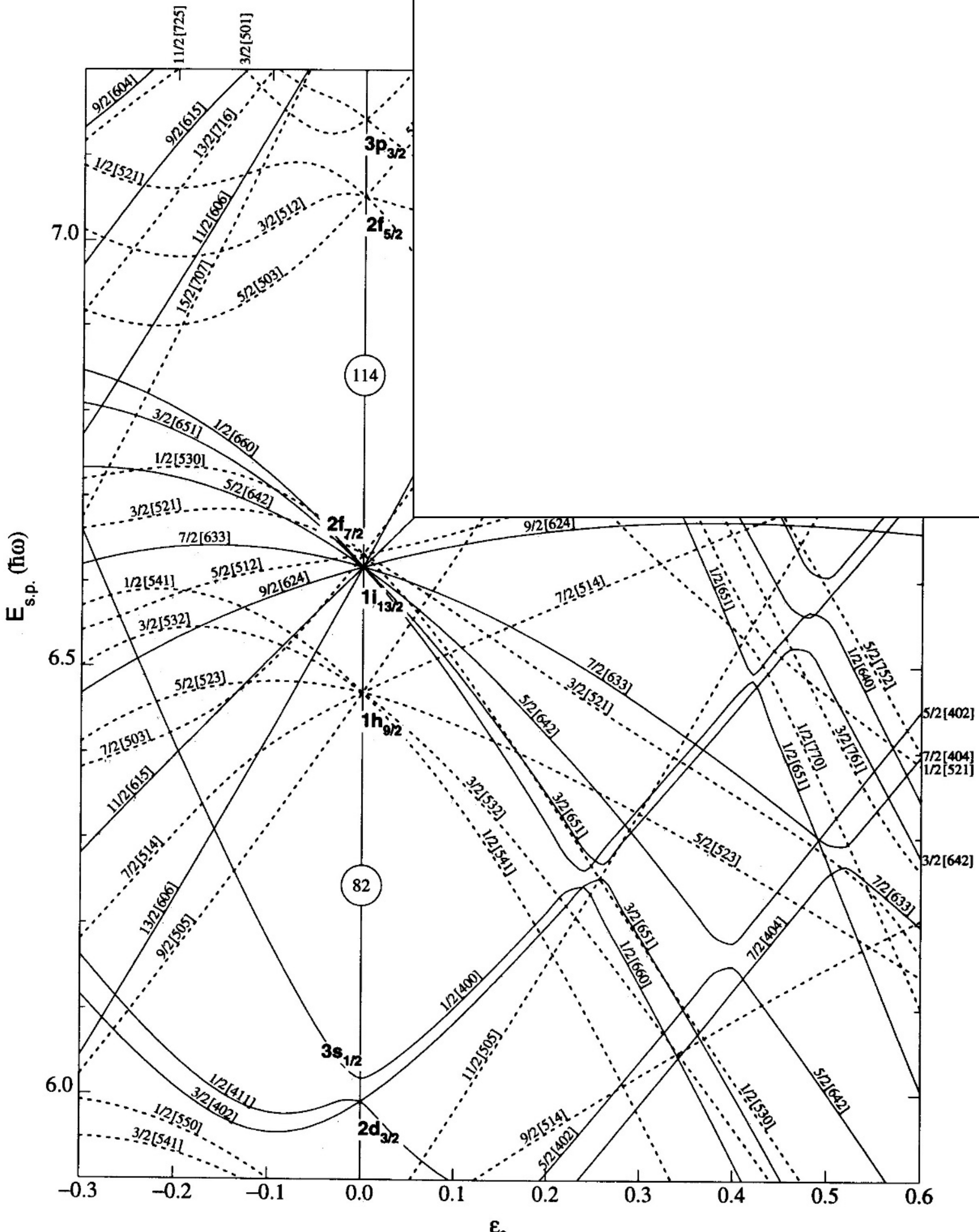


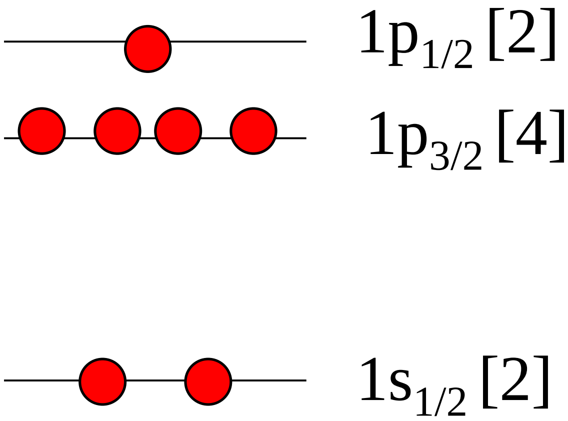
Figure 13. Nilsson diagram for protons, Z ≥ 82 ($\varepsilon_4 = \varepsilon_2^2/6$).

Nilsson diagram

$h = T + V(r,\theta)$
の固有値

Level scheme of $^{11}_{4}\mathrm{Be}_7$

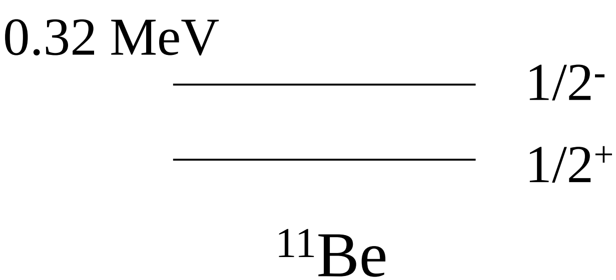
With a spherical potential:



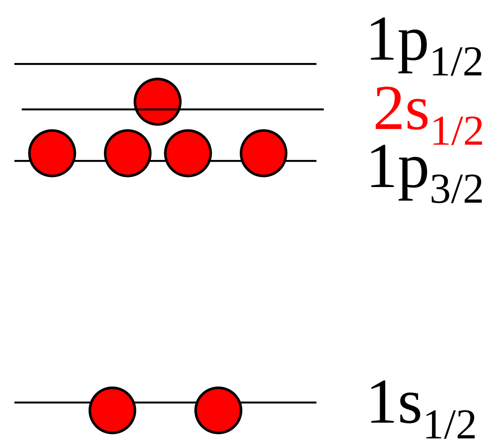
The g.s. of ^{11}Be : $I^\pi = 1/2^-$

very artificial

In reality…..



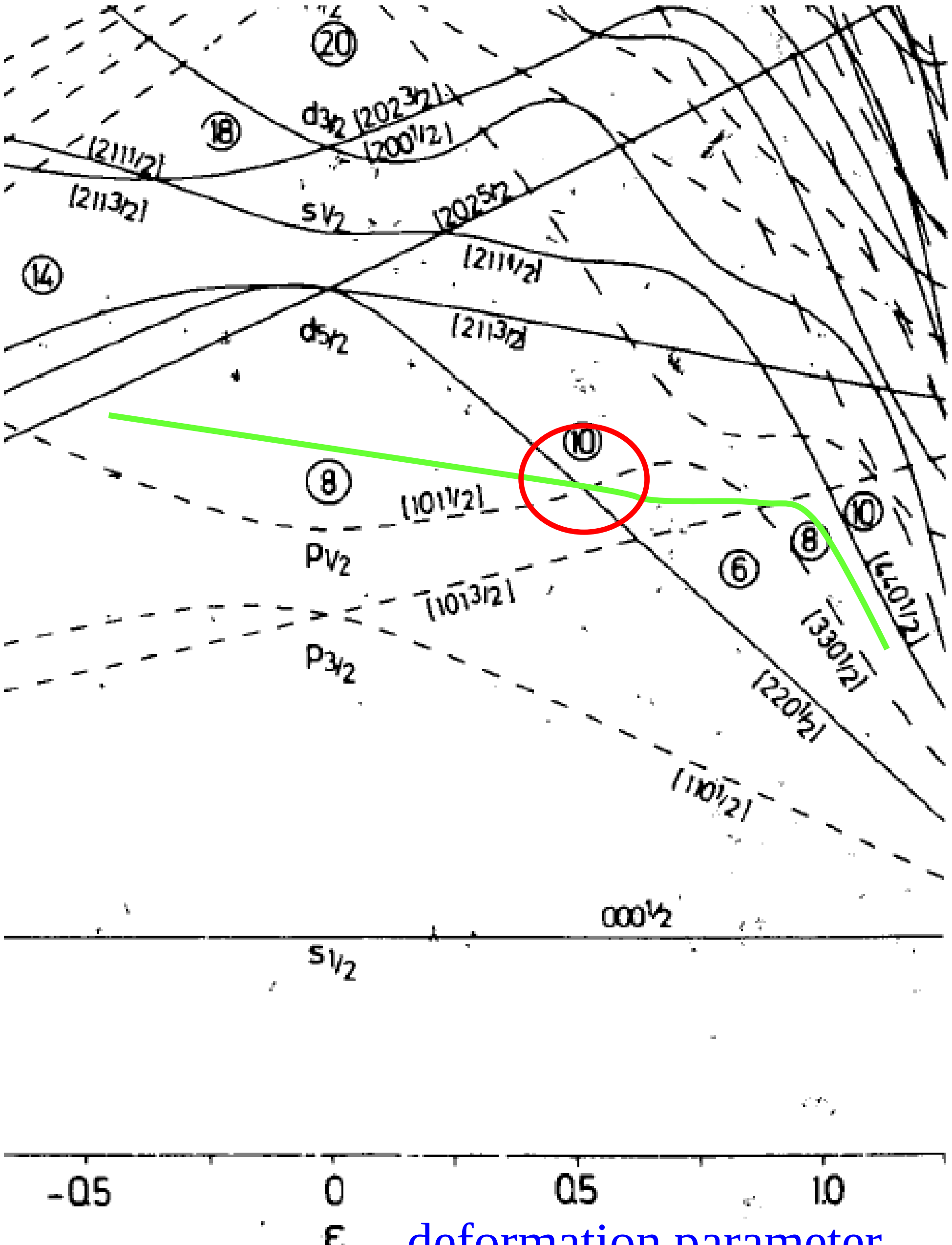
?



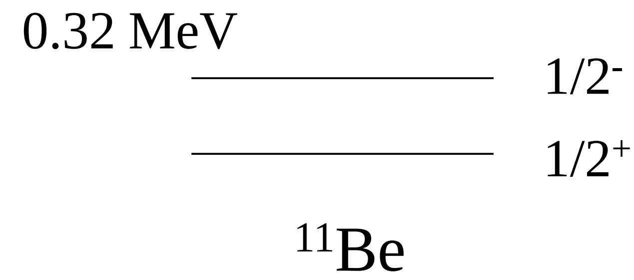
"parity inversion"

What happens if ^{11}Be is deformed?

$^{11}_{4}Be_7$

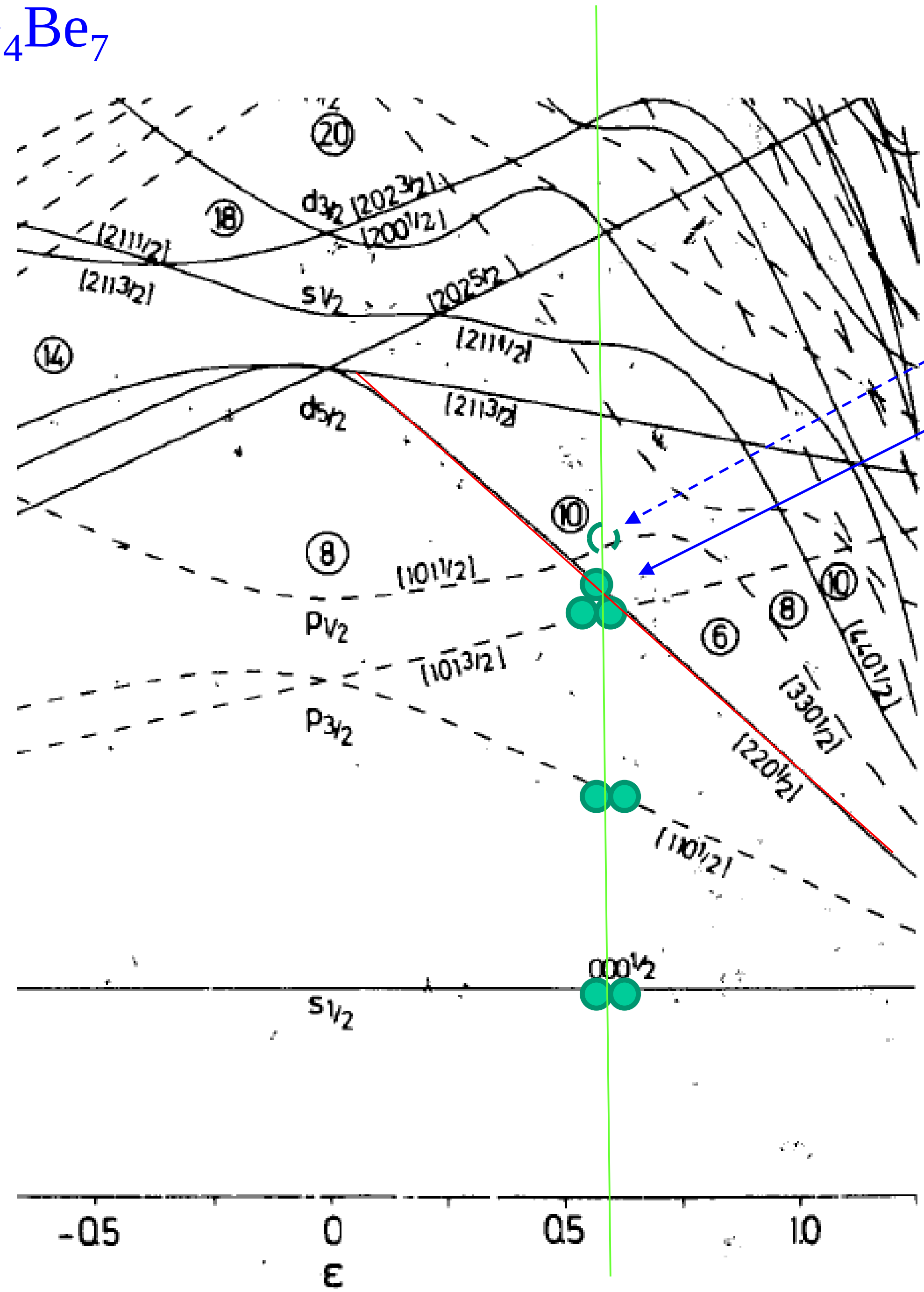


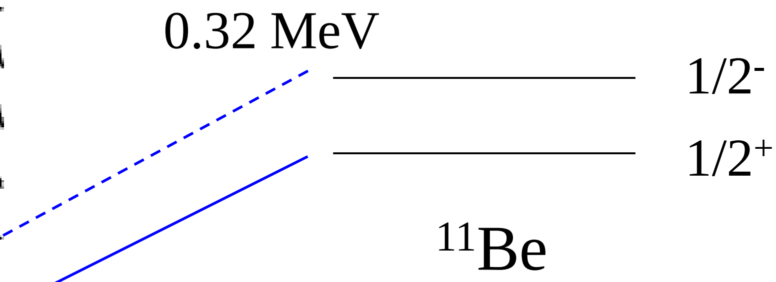
deformation parameter



✓ 7番目の中性子の入る軌道を探す
（それぞれの軌道に2つずつ中性子をつめる）

$^{11}_{4}Be_{7}$

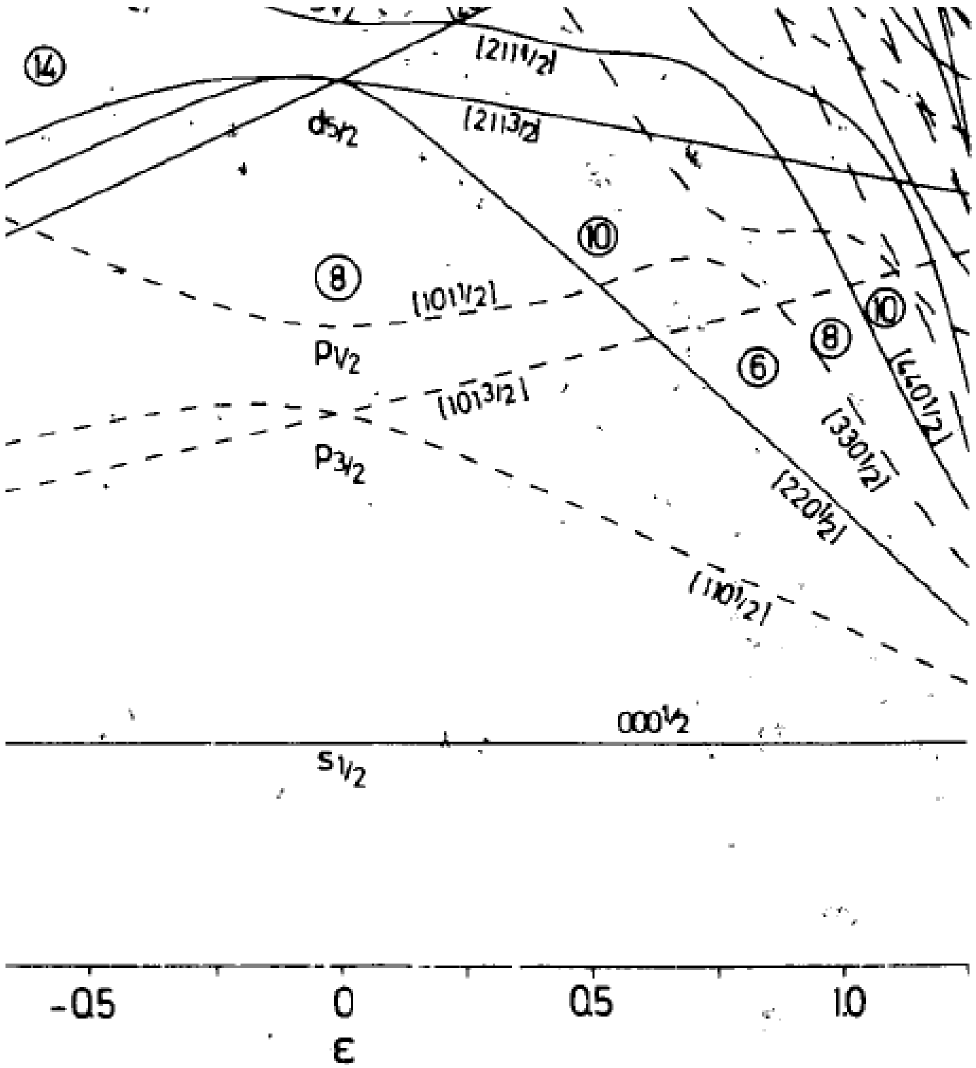


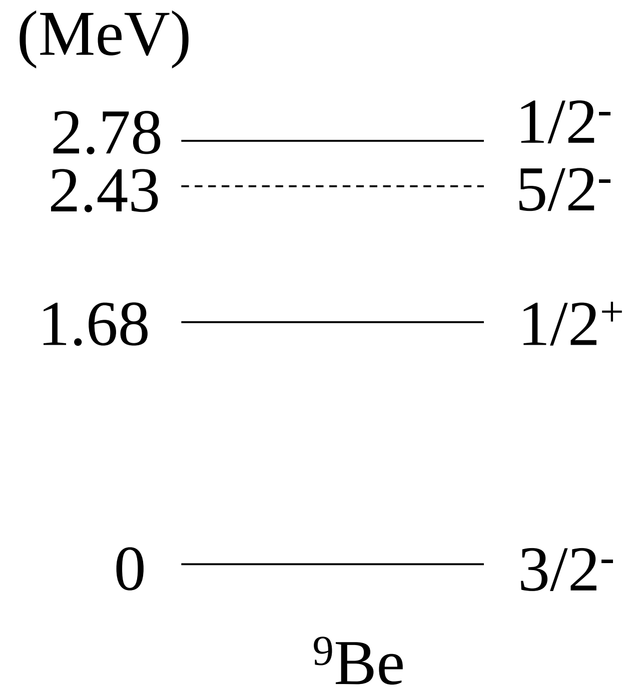


✓ 7番目の中性子の入る軌道を探す
（それぞれの軌道に2つずつ中性子をつめる）

Can the level scheme of $^{9}_{4}\mathrm{Be}_{5}$ be explained in a similar way?

cf. $^{10}\mathrm{B}(e,e'K^{+})^{10}_{\Lambda}\mathrm{Be}$ (= $^{9}\mathrm{Be}+\Lambda$)

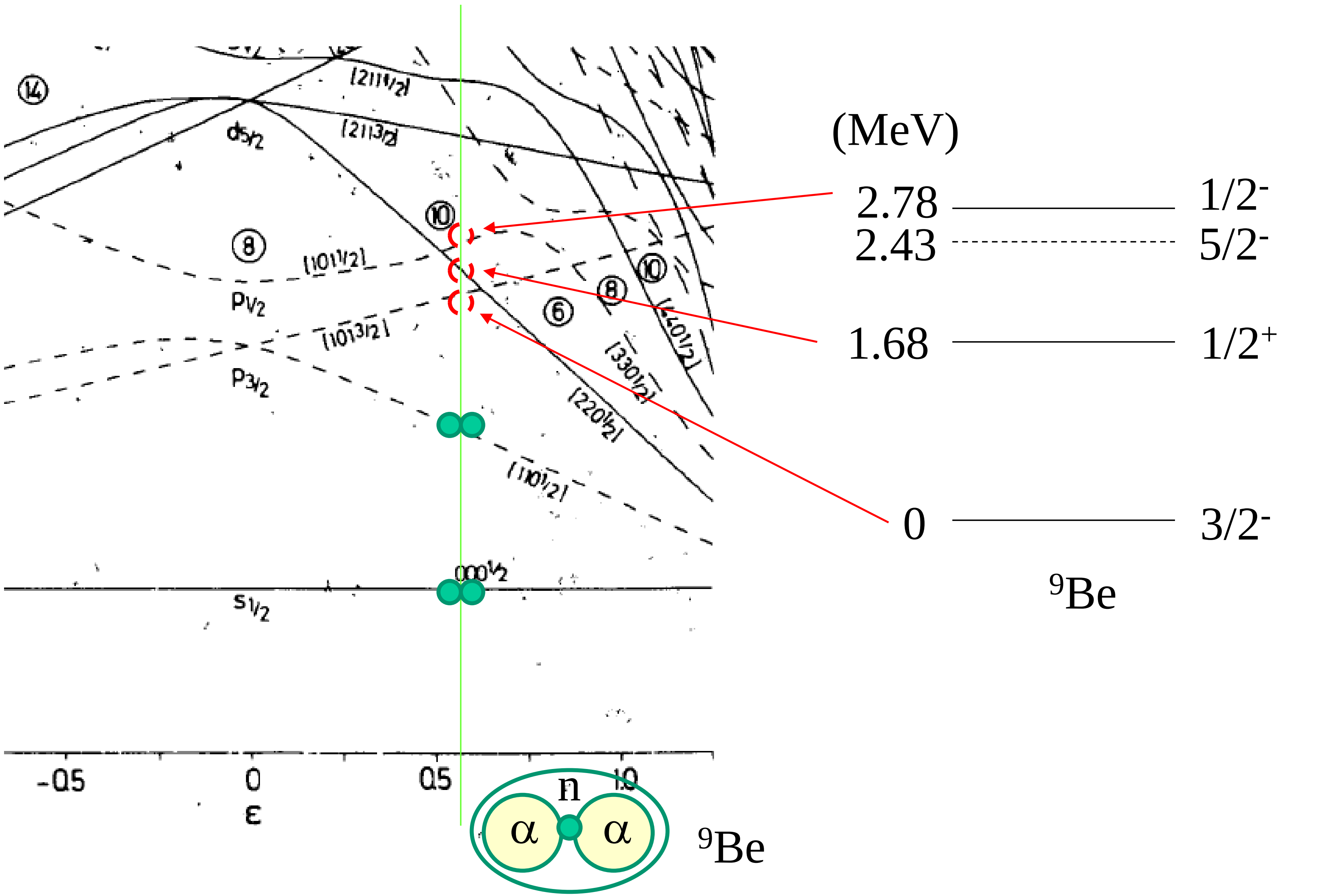




The $5/2^{-}$ state at 2.43 MeV: rotational state with the same configuration as the g.s. state (not considered here)

Can the level scheme of $^{9}_{4}Be_5$ be explained in a similar way?

cf. $^{10}B(e,e'K^+)^{10}_{\Lambda}Be$ (= $^{9}Be+\Lambda$)



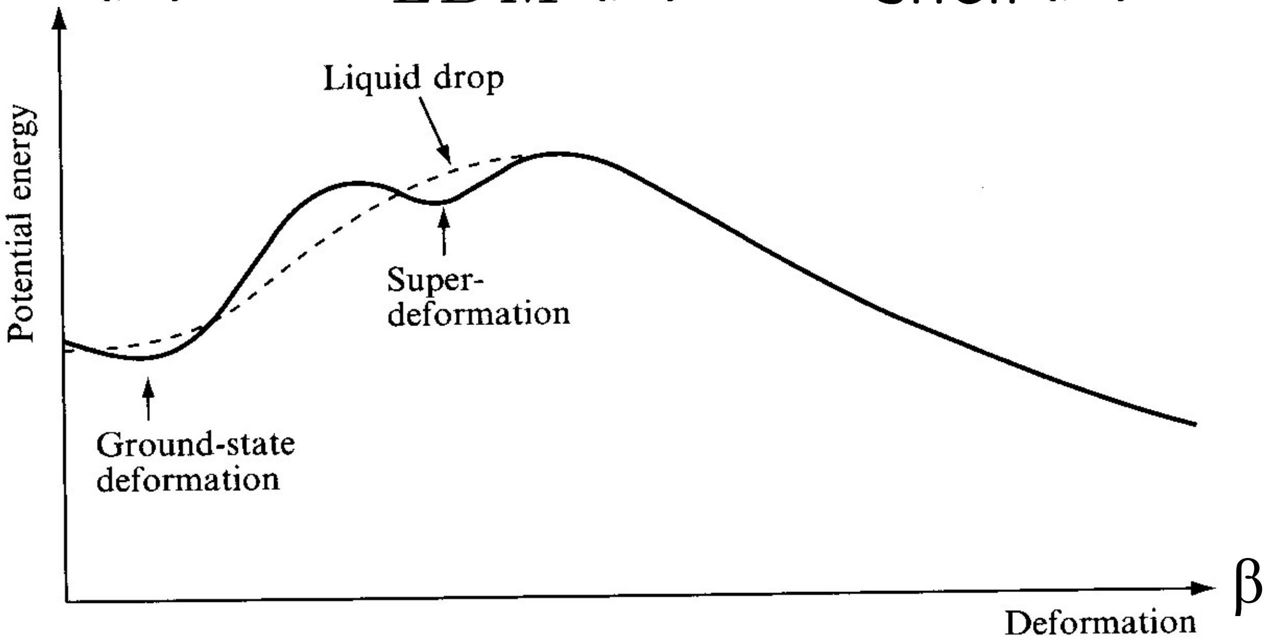
Several topics:

- deformation as a quantum effect
- Neumann-Wigner no-crossing rule
- RMF for deformed hypernuclei
- Quiz: spontaneous symmetry breaking

nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\text{shell}}(\beta)$$



LDM only → always spherical ground state

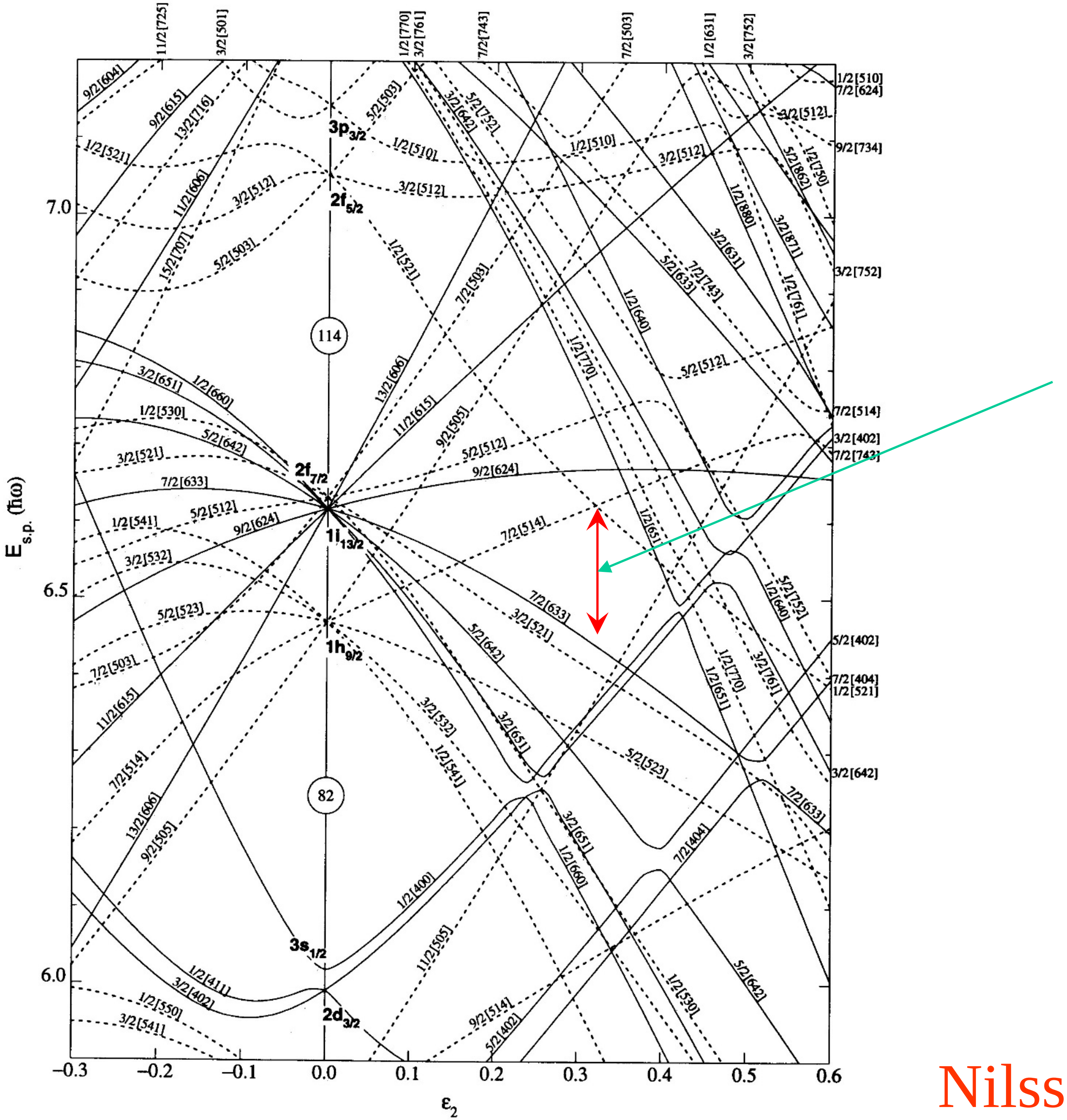


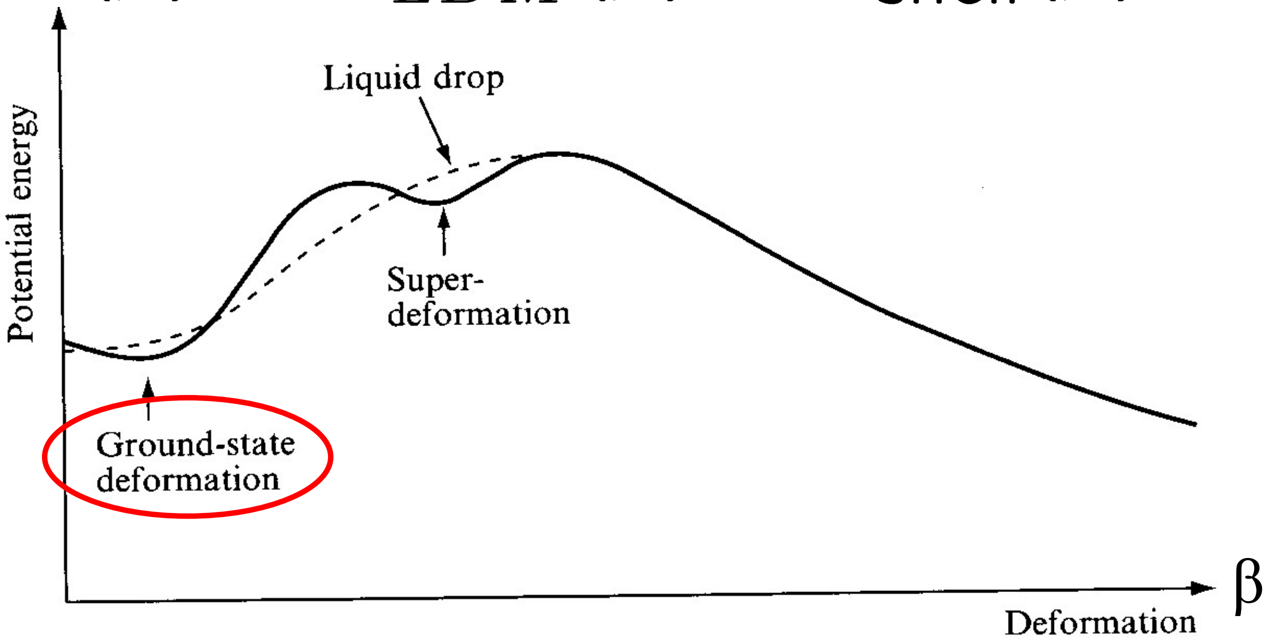
Figure 13. Nilsson diagram for protons, $Z \geq 82$ ($\varepsilon_4 = \varepsilon_2^2/6$).

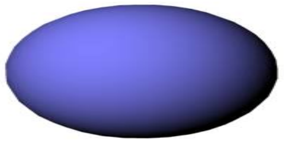
Nilsson diagram

nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$



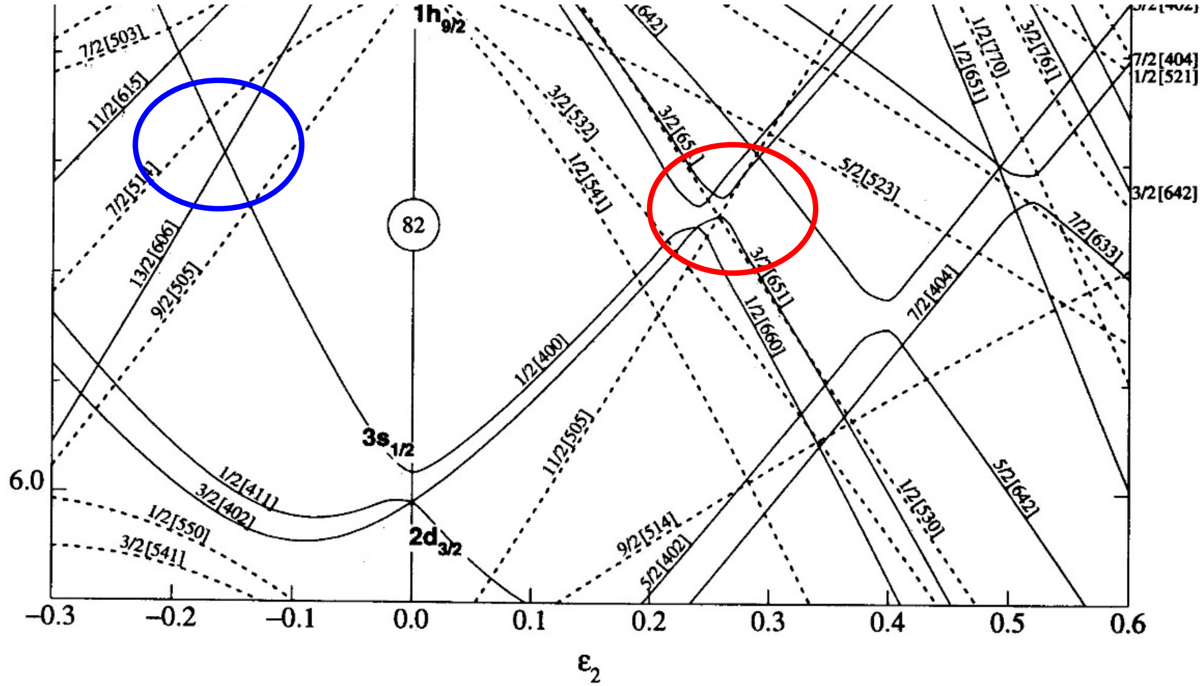


LDM only → always spherical ground state

Shell correction → may lead to a deformed g.s.

* Spontaneous Symmetry Breaking

➢ ニルソン図で準位が反発しているように見えるのは何でですか?



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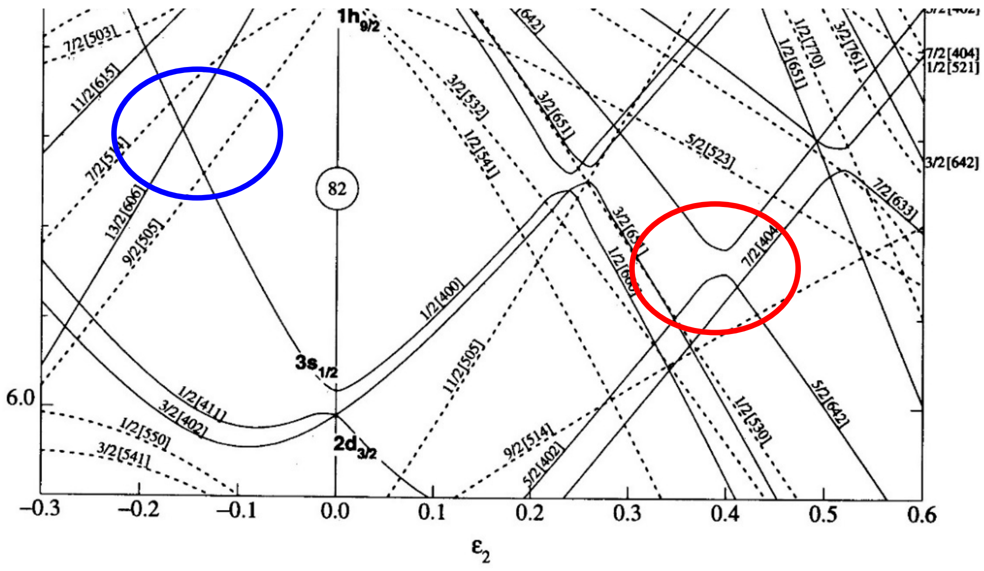
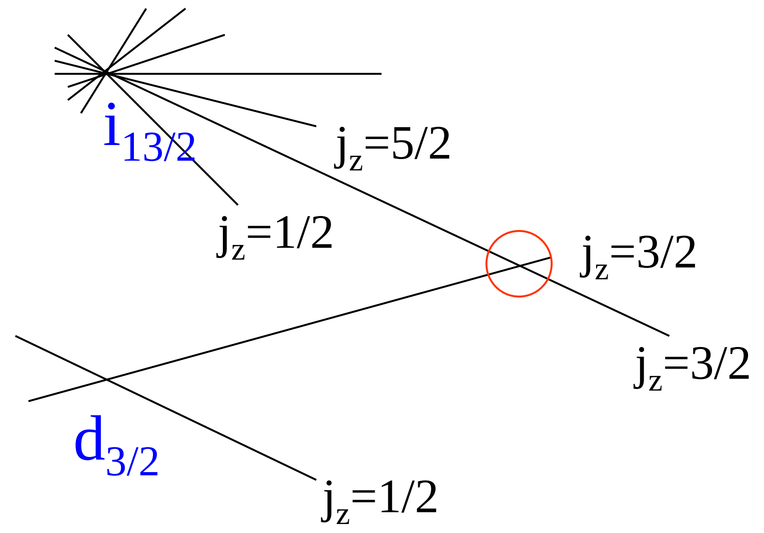
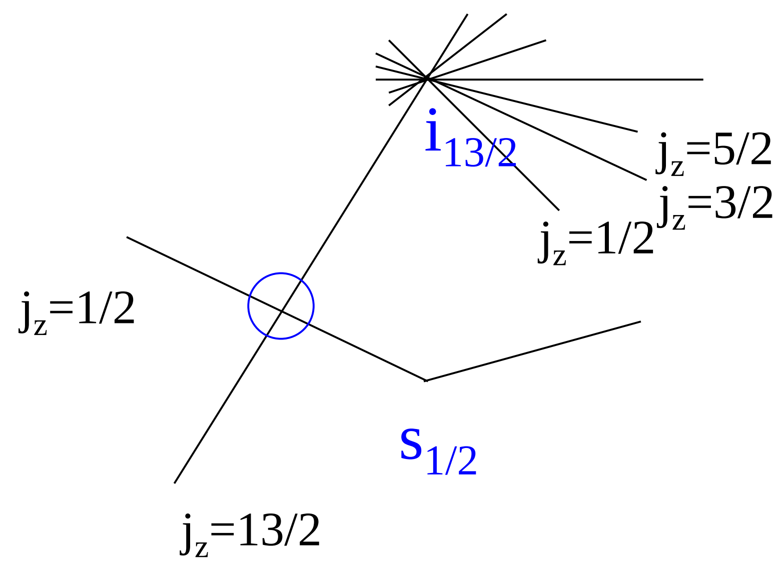




同じ量子数を持つ準位は交わらない

（量子数が違うと交わってもよい）



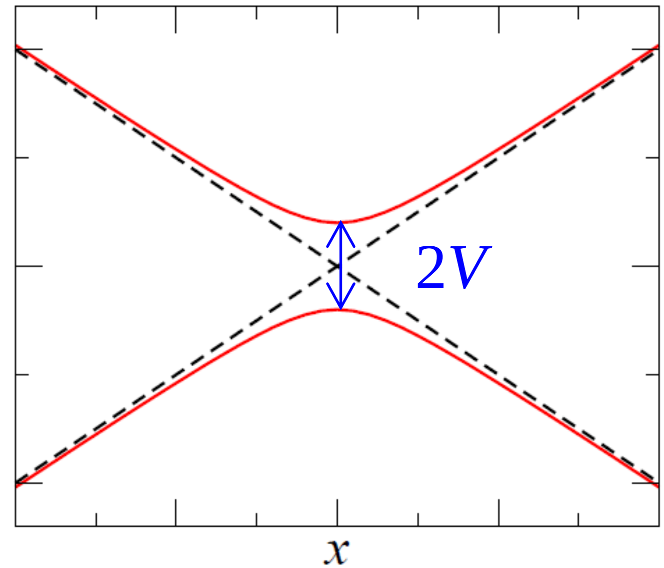
「ノイマン‐ウィグナーの定理」

準位交差の問題：同じ量子数を持つ2つの状態は交差しない

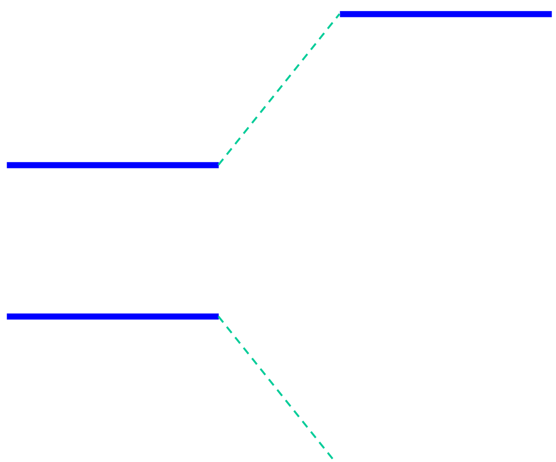
「ノイマン‐ウィグナーの定理」

$$\begin{pmatrix} -\epsilon x & V \\ V & \epsilon x \end{pmatrix}$$

対角化 $\rightarrow \lambda_{\pm}(x) = \pm\sqrt{\epsilon^2 x^2 + V^2}$

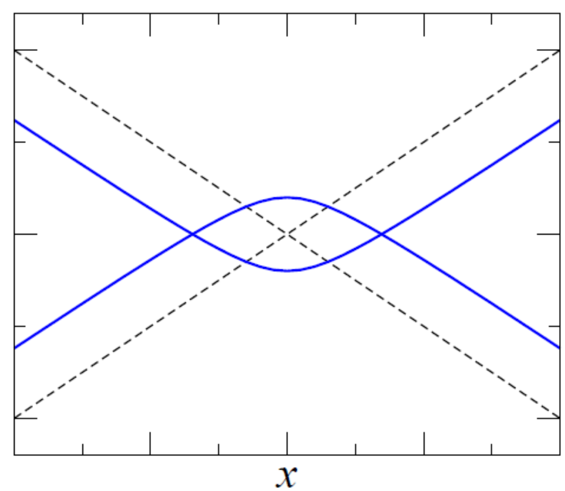


「疑似交差」、「準位反発」



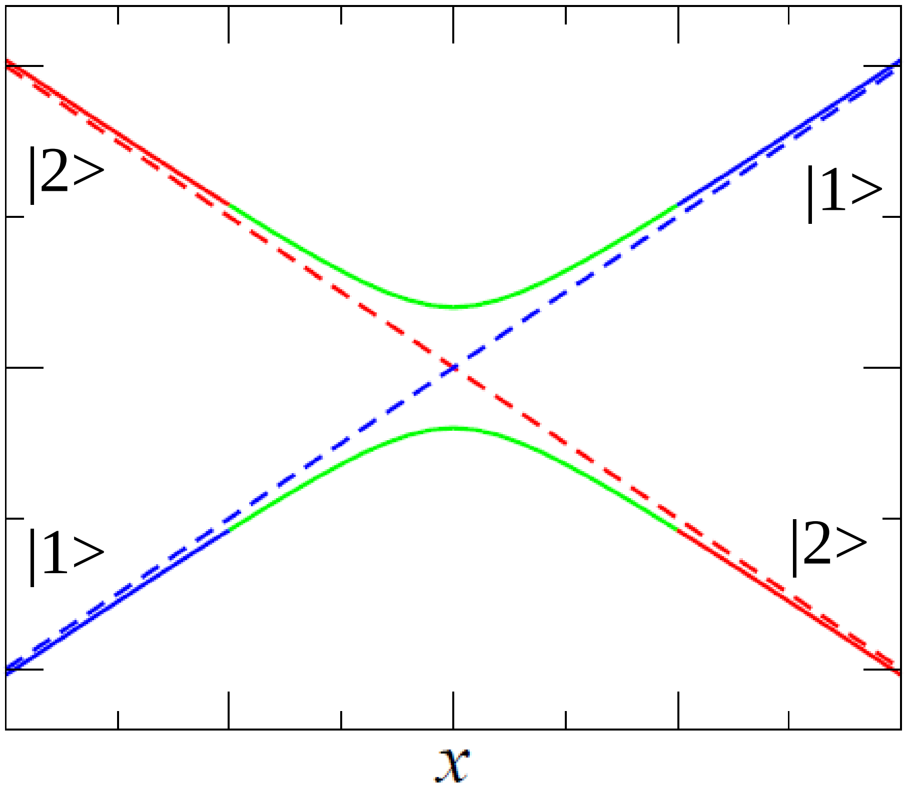
$V = 0$ $\quad$ $V \neq 0$

V の符号によらず必ず反発



このようになることはない

x がゆっくりと変化すると断熱的に状態が |1> から |2> へ変化（断熱遷移）



Landau-Zener の式:

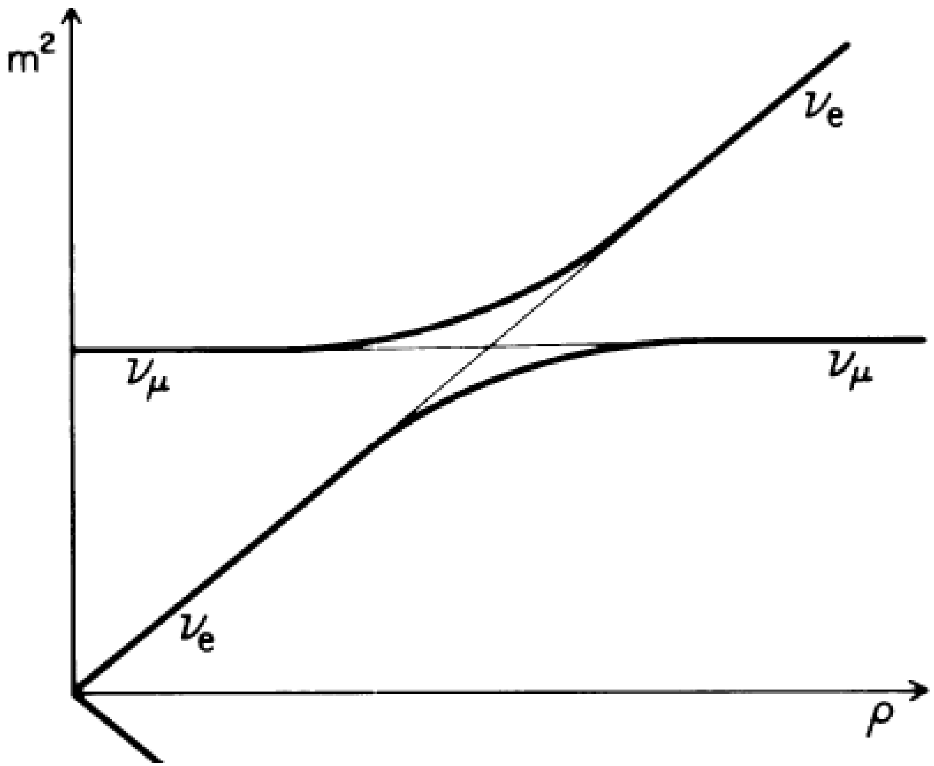
$$P(|1\rangle \to |1\rangle) = \exp\left(-\frac{2\pi V^2}{\hbar|\dot{x}| \cdot 2\epsilon}\right)$$

cf. ニュートリノ振動と準位交差問題

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\Psi_e\\ \Psi_\mu\end{pmatrix}=\left[\begin{pmatrix}E+A(r) & 0\\ 0 & E\end{pmatrix}+a\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}\right]\begin{pmatrix}\Psi_e\\ \Psi_\mu\end{pmatrix}$$

$$E=\frac{1}{2}(m_1^2+m_2^2),\qquad a=\frac{1}{2}(m_2^2-m_1^2)$$

電子ニュートリノと
物質中の電子との相互作用



物質中で共鳴的に
ニュートリノ振動が起こる
= MSW 効果

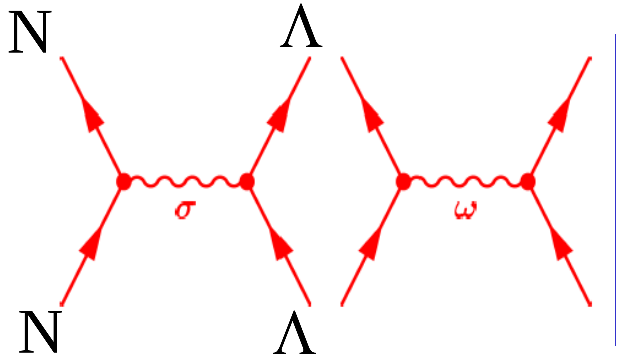
Ref.
H.A. Bethe, PRL56(‘86)1305,
W.C. Haxton, PRL57(‘86)1271

RMF calculations for deformed hypernuclei

Hypernuclei: nucleus + Lambda particle

Effect of a Λ particle on nuclear shapes?

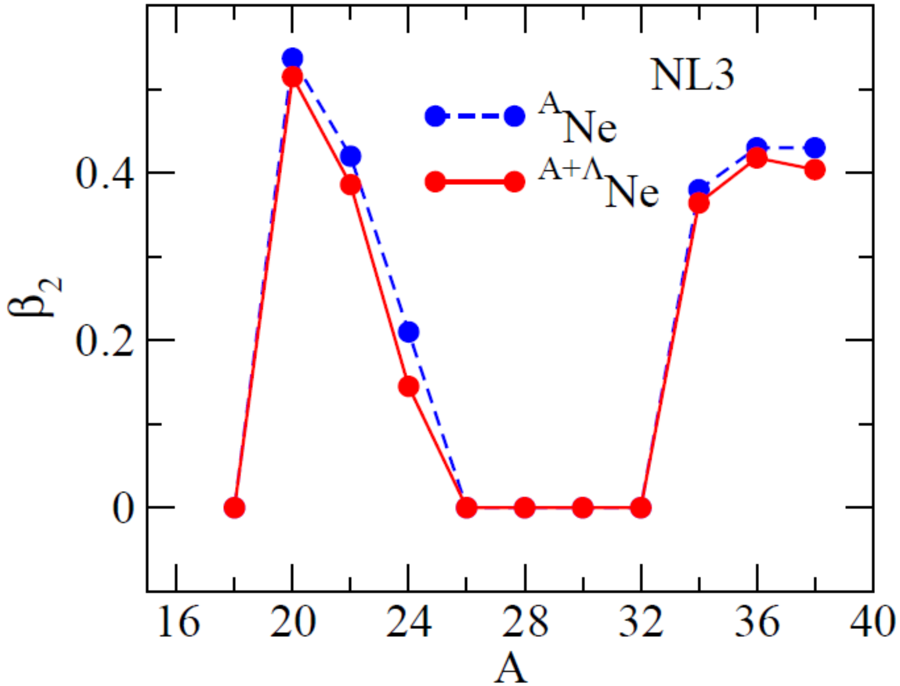
Relativistic Mean-field model



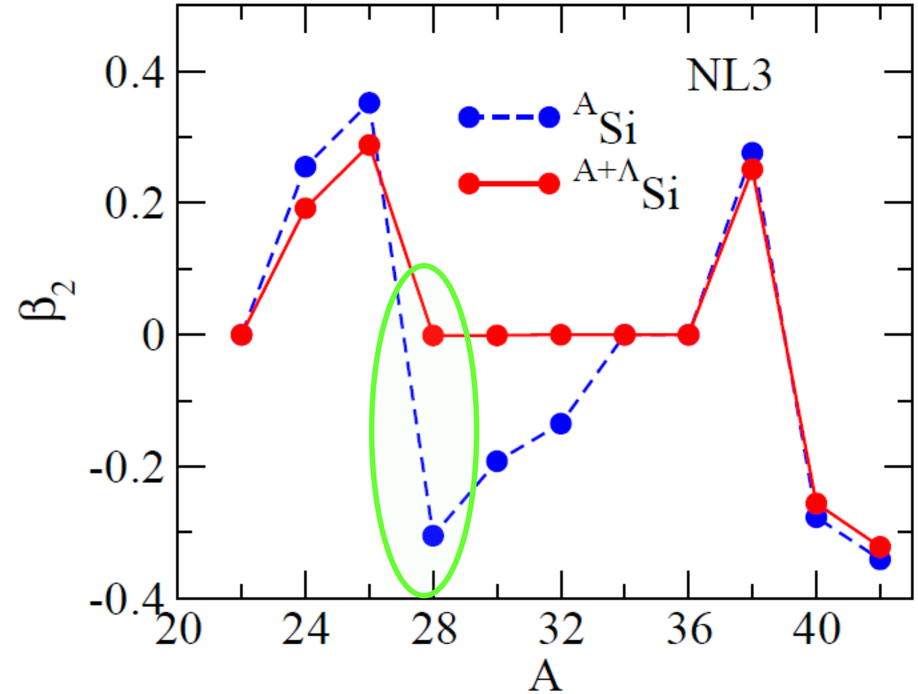
nucleon-nucleon interaction
via meson exchange

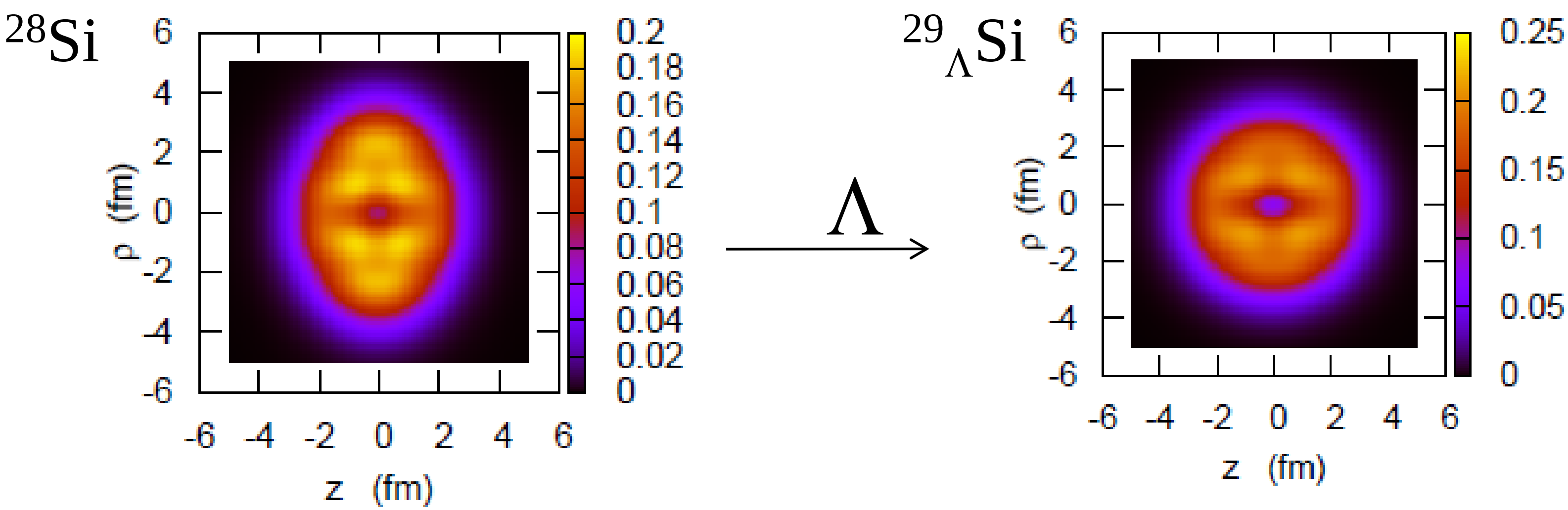
Λσ and Λω couplings

Ne isotopes



Si isotopes





Myaing Thi Win and K.Hagino, PRC78('08)054311

Quiz: spontaneous symmetry breaking

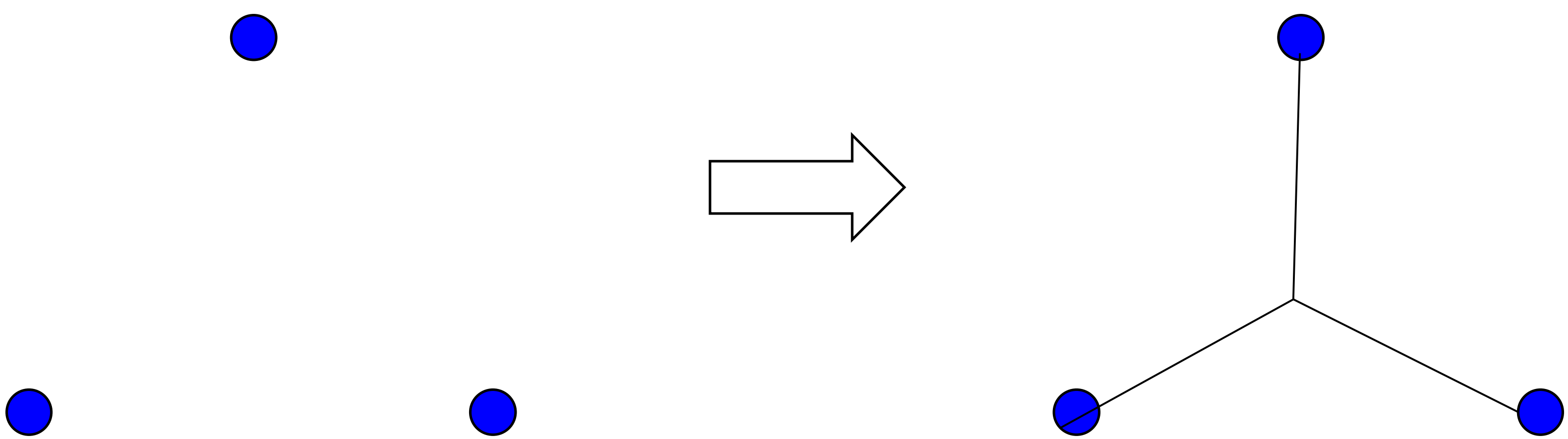
There are a few dots.

- Connect the dots.
- The number of lines is not limited.
- Two lines can cross.
- Connect the dots so that one can go from one dot to all the other dots.

How do you connect the lines if you want to make the total length of lines the shortest?

e.g.) Equilateral triangle

Connect symmetrically



Quiz: spontaneous symmetry breaking

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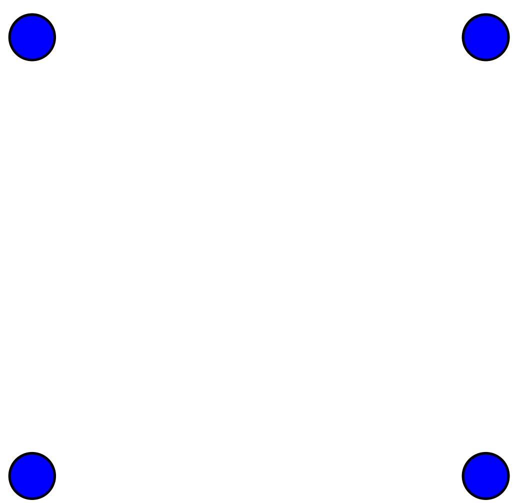
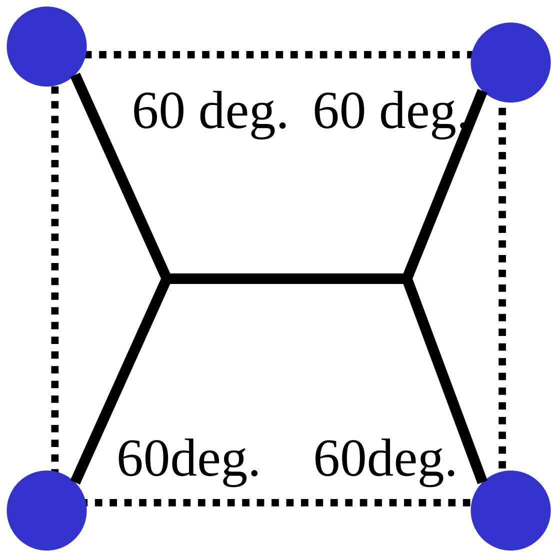
How do you connect the lines if you want to make the total length of lines the shortest?

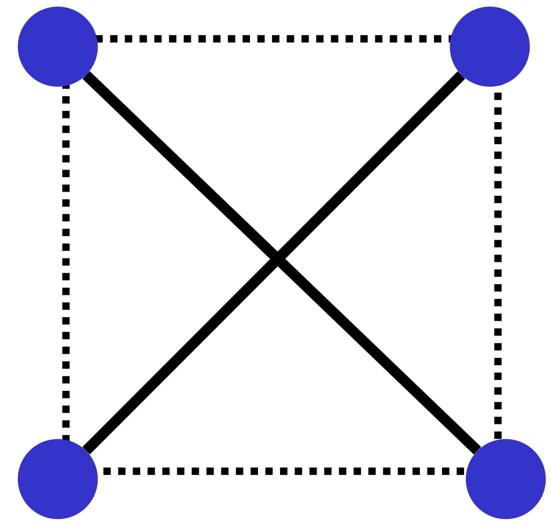
(question) how about the case for a square?



(the answer)



cf.



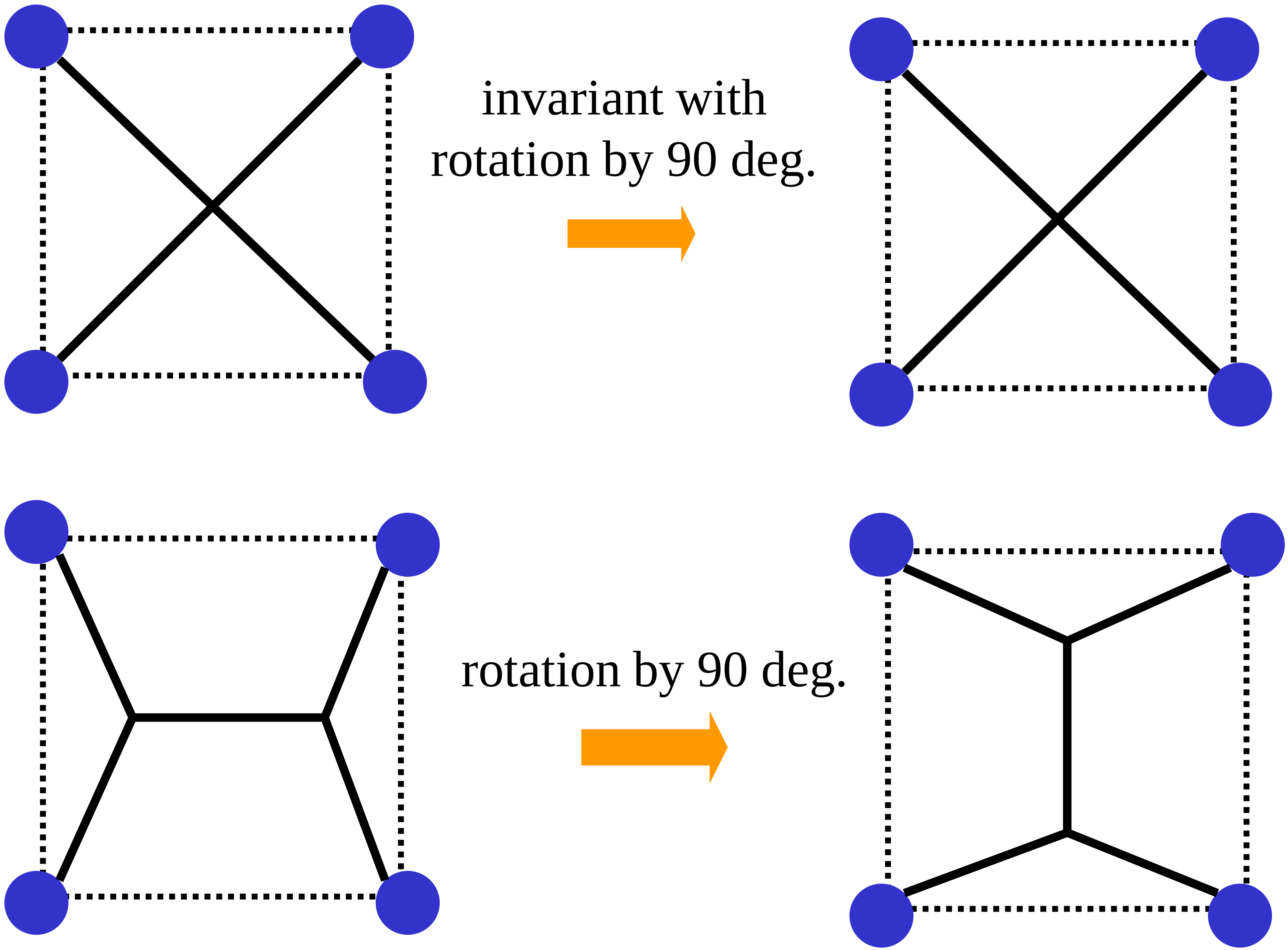
Length

$$4 \times \frac{1}{\sqrt{3}} + \left(1 - 2 \times \frac{1}{2\sqrt{3}}\right)$$
$$= 1 + \sqrt{3}$$
$$= 2.732\cdots$$

Length

$$2 \times \sqrt{2} = 2.828\cdots$$

Ref. Takeshi Koike,
“Genshikaku Kenkyu” Vol. 52 No. 2, p. 14



a good example of spontaneous symm. breaking

Courtesy: Takeshi Koike