

原子核反応

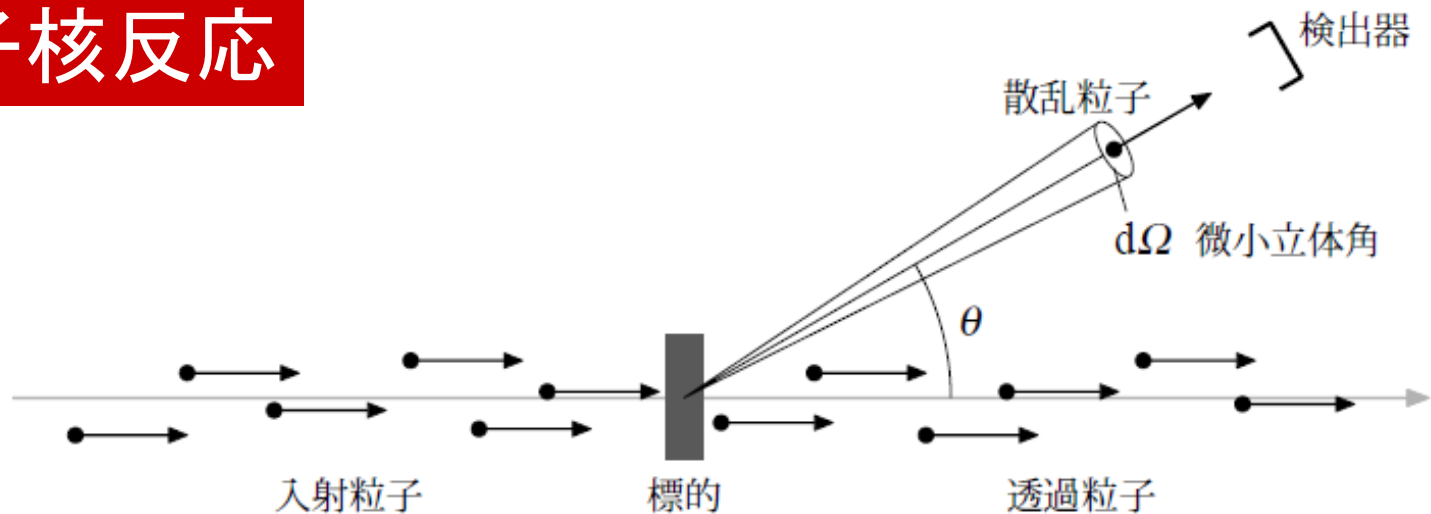


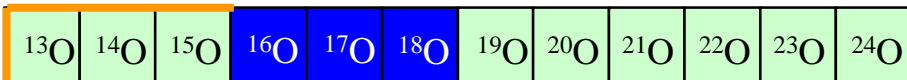
図 21.1: 散乱実験

今日の内容

1. 共鳴反応
2. 核融合反応

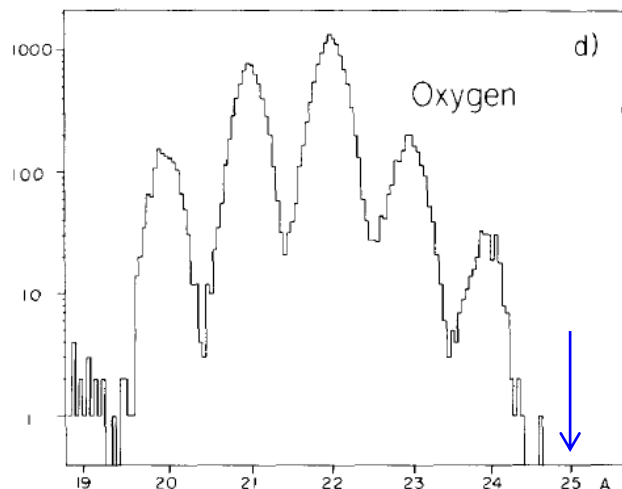
1. 共鳴反応

酸素同位体のドリップ線



酸素原子核 (Z=8)

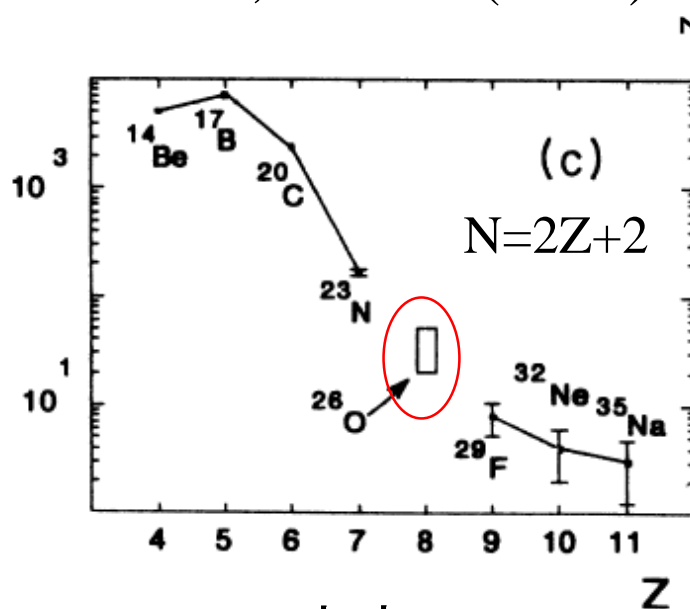
- ✓ 安定同位体: ^{16}O (99.757%), ^{17}O (0.038%), ^{18}O (0.205%)
- ✓ ^{24}O の発見: A.G. Artukh et al., PL32B (1970) 43



^{40}Ar の破碎

M. Langevin et al.,
PL150B ('85) 71

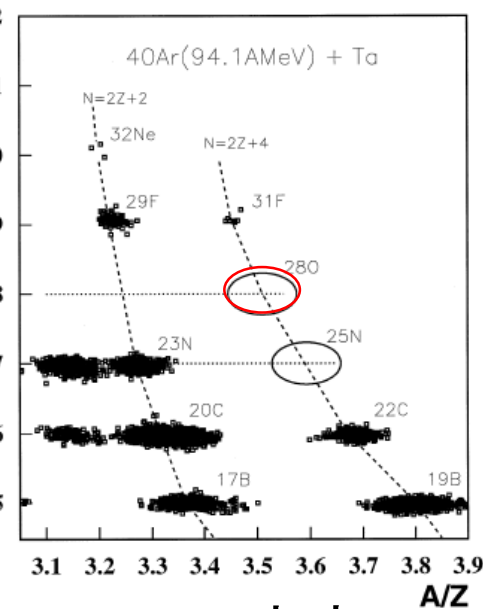
^{25}O は不検出



^{48}Ca の破碎

D. Guillemaud-Mueller et al.,
PRC41 ('90) 937

^{26}O は不検出



^{40}Ar の破碎

H. Sakurai et al.,
PLB448 ('99) 180

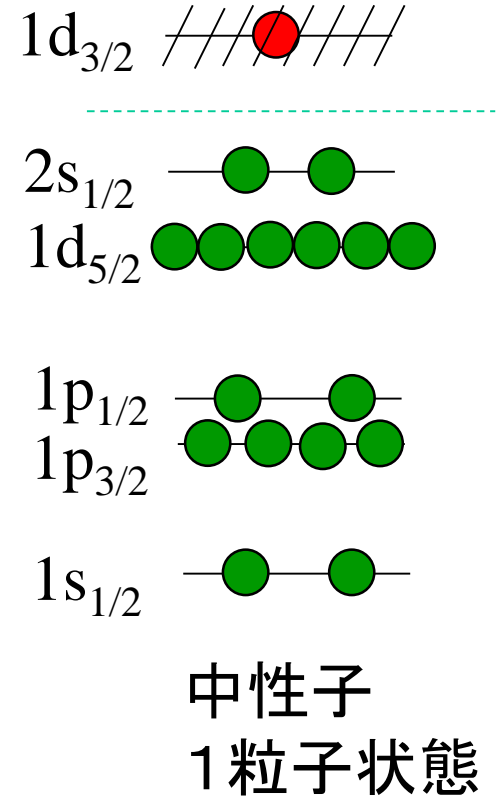
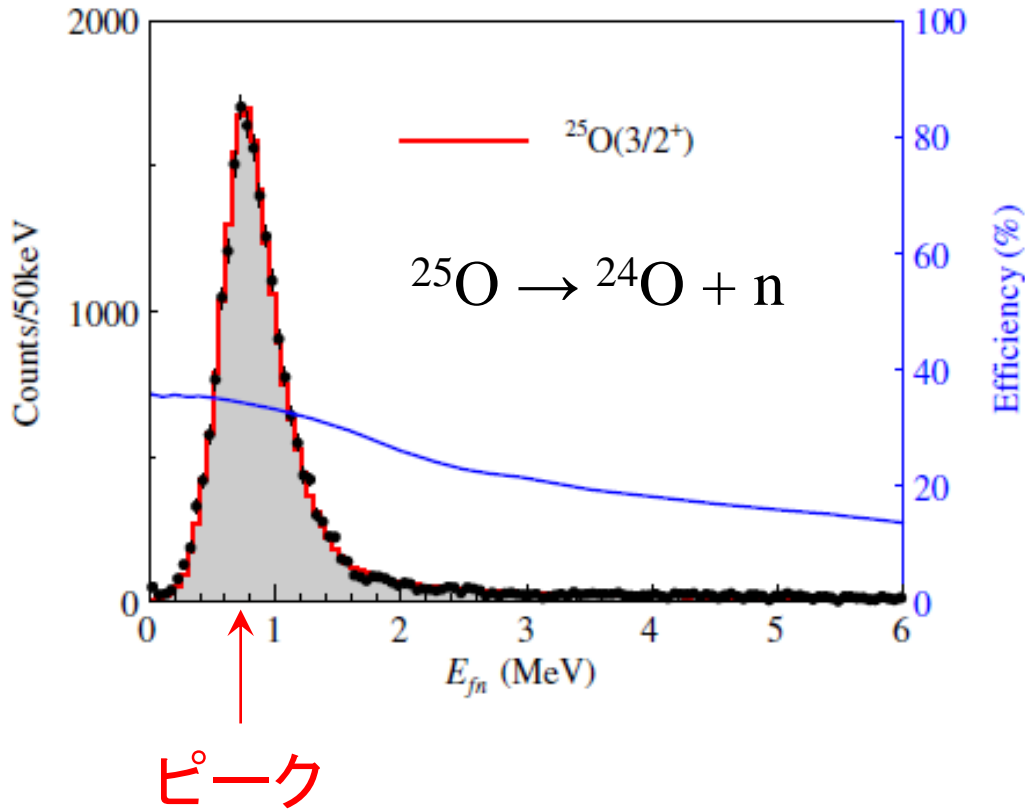
^{28}O は不検出

➡ 酸素の中性子ドリップ線は ^{24}O で確定。 $^{25,26,28}\text{O}$ は非束縛。

^{25}O はどのように見えるのか?

^{22}O	^{23}O	^{24}O	^{25}O	^{26}O
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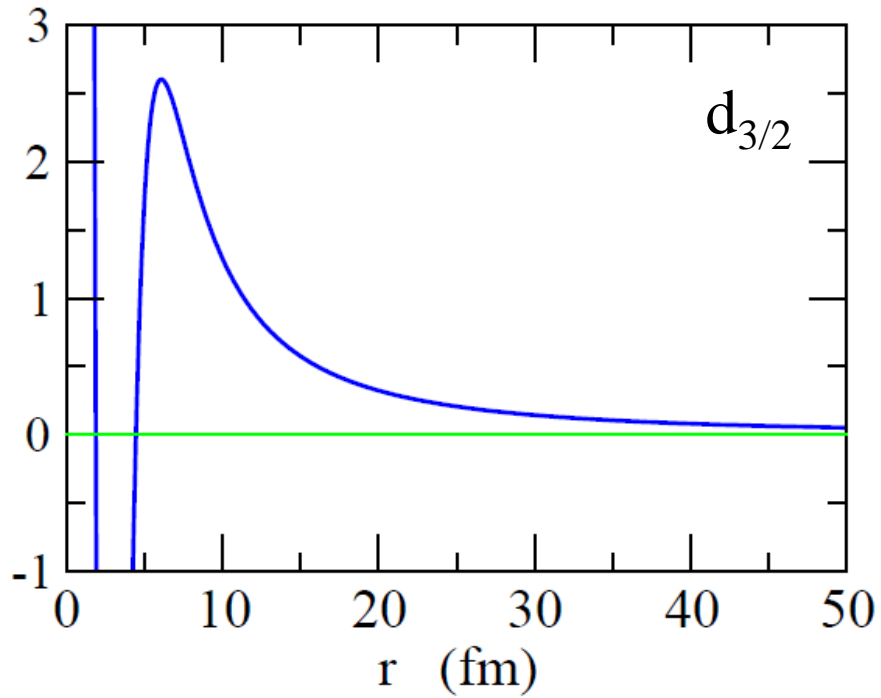
$^{26}_9\text{F}_{17}$ から1つ陽子を抜いて $^{25}_8\text{O}_{17}$ を生成 → 1中性子を放出して崩壊



Y. Kondo et al., PRL116 ('16) 102503

$1d_{3/2}$ の「準束縛」状態と解釈することができる

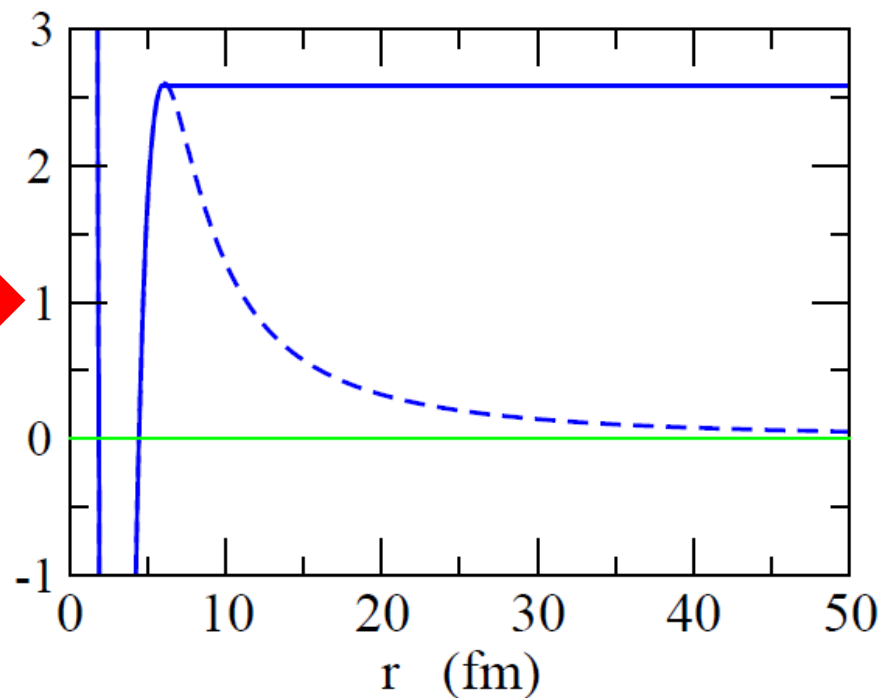
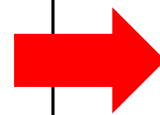
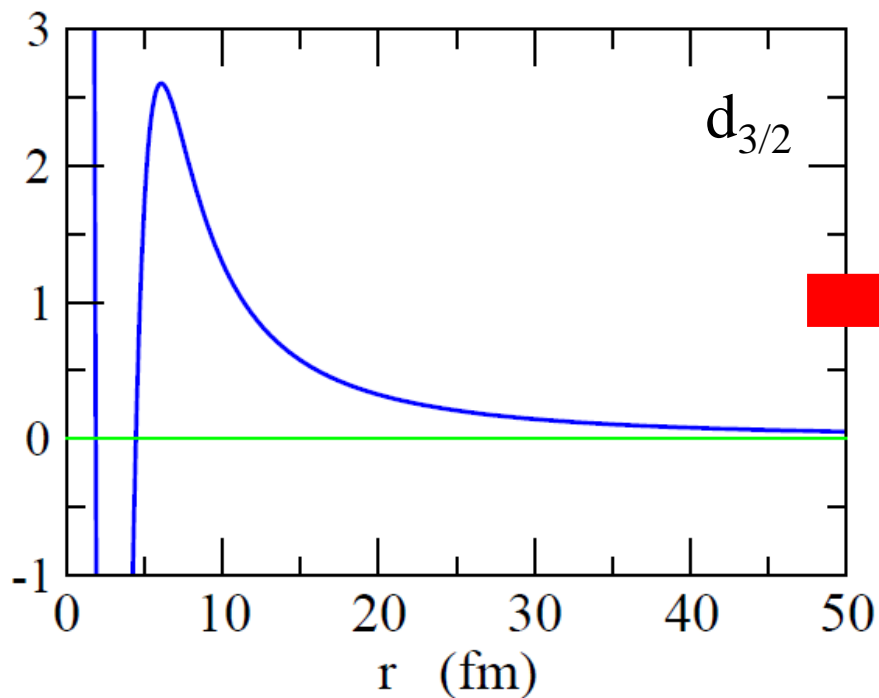
準束縛状態とは?



実際のポテンシャル

束縛状態は $E < 0$ の領域のみ

準束縛状態とは?



実際のポテンシャル

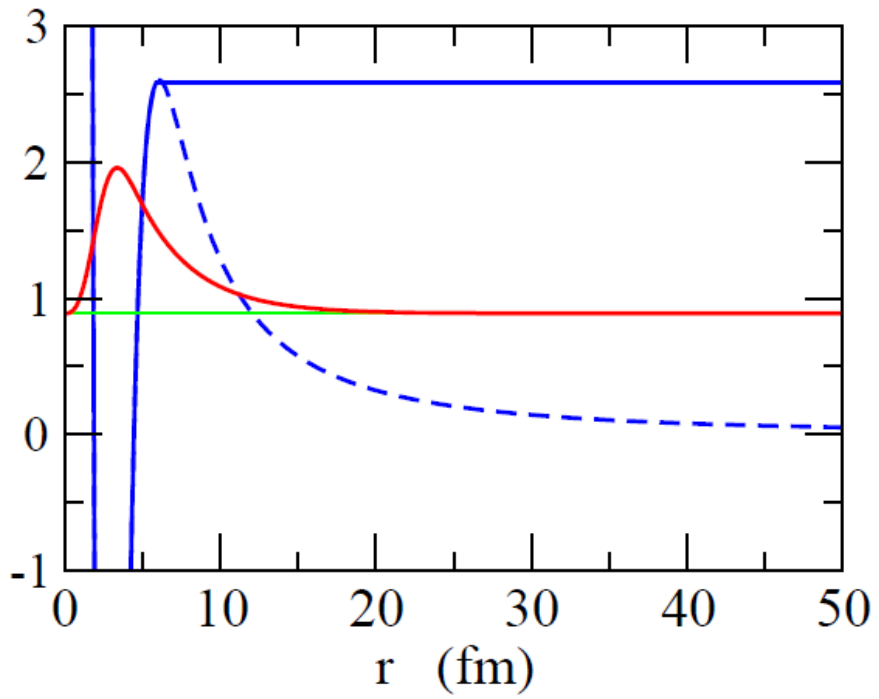
束縛状態は $E < 0$ の領域のみ

このようにポテンシャルを
変更すると

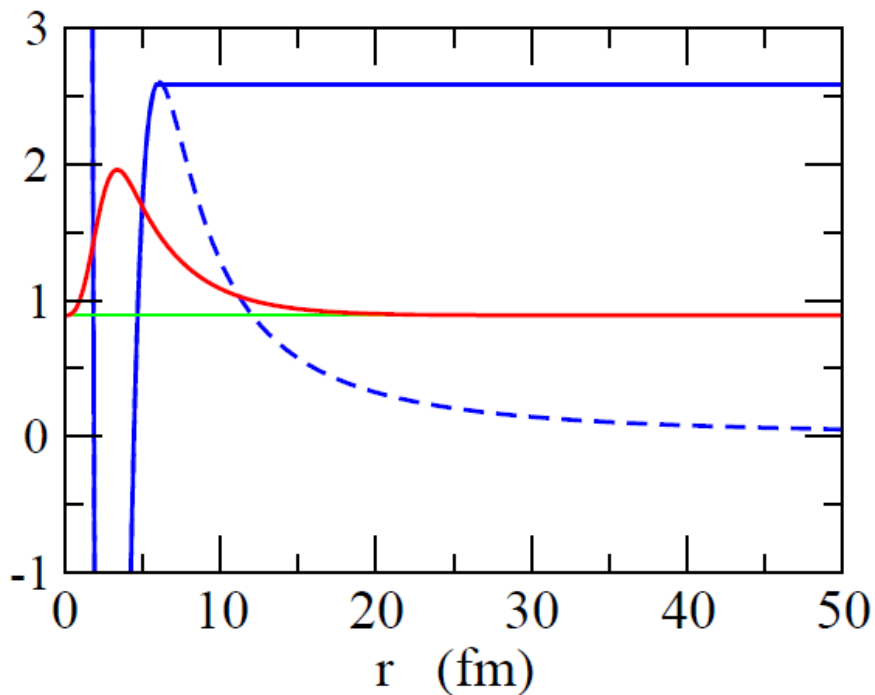
→ $E > 0$ でも束縛状態が
できる

= 準束縛(準安定)状態

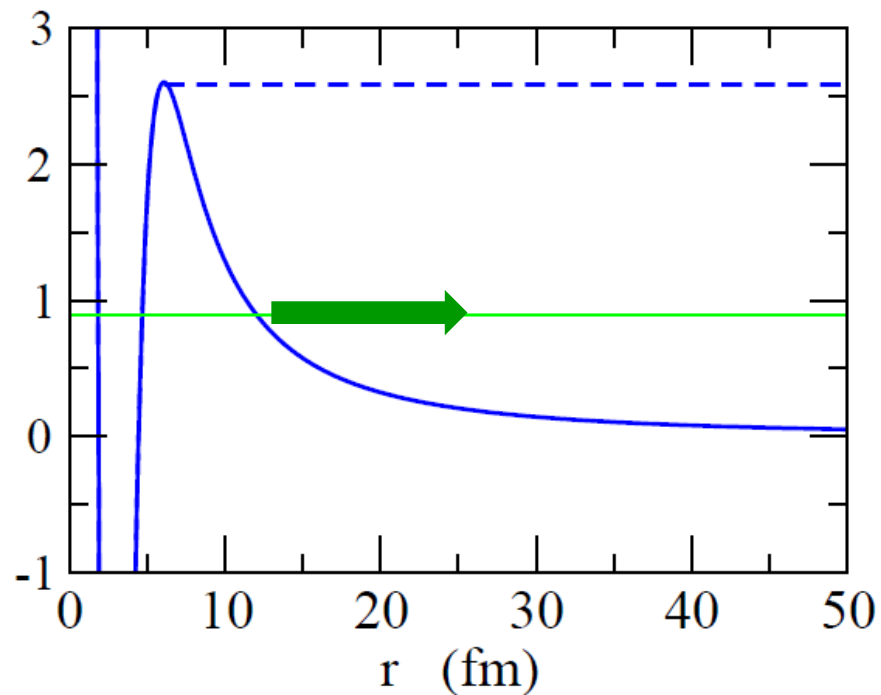
準束縛状態とは?



準束縛状態とは?



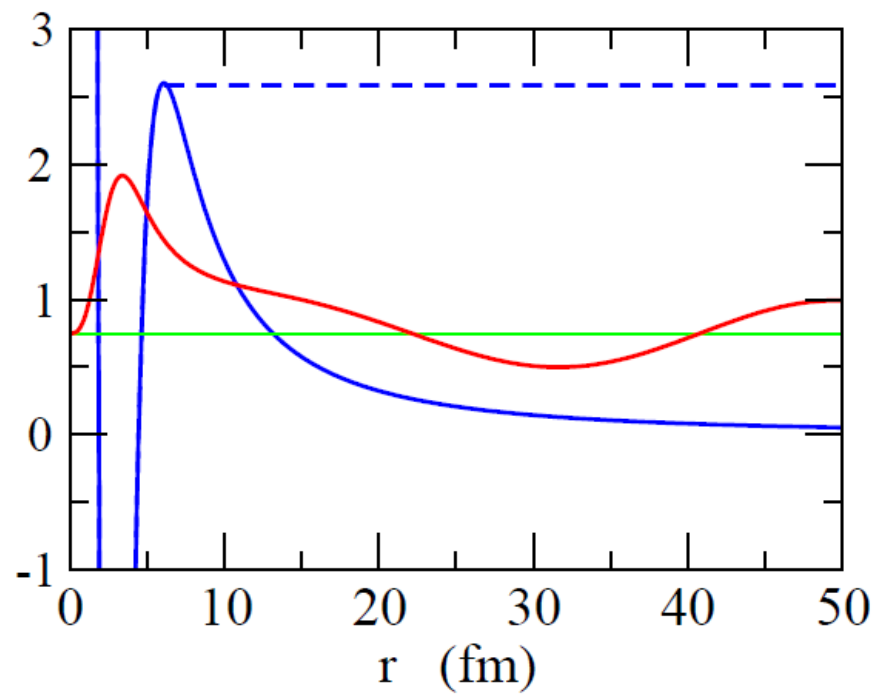
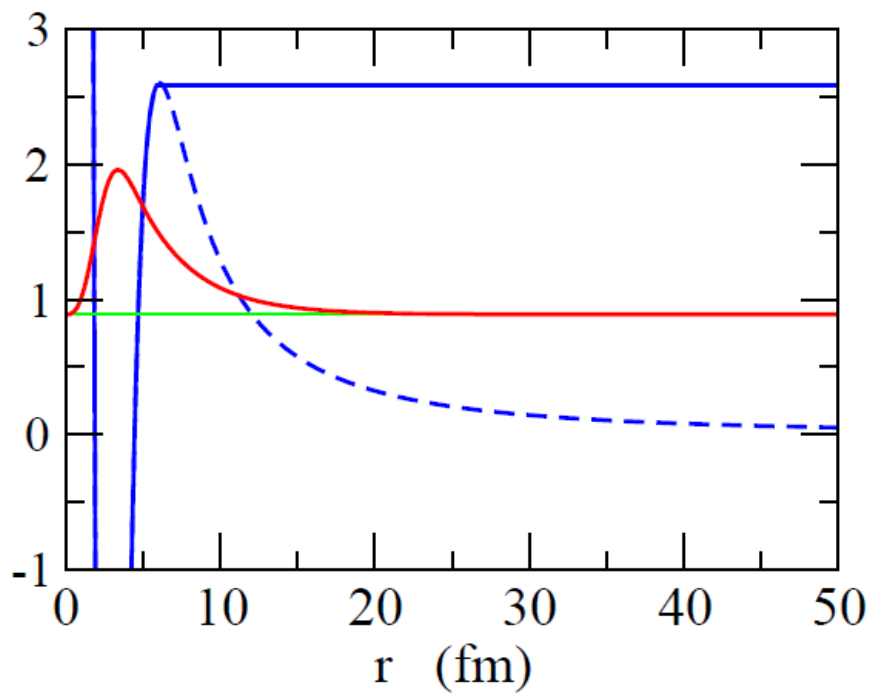
束縛状態 = 無限の寿命



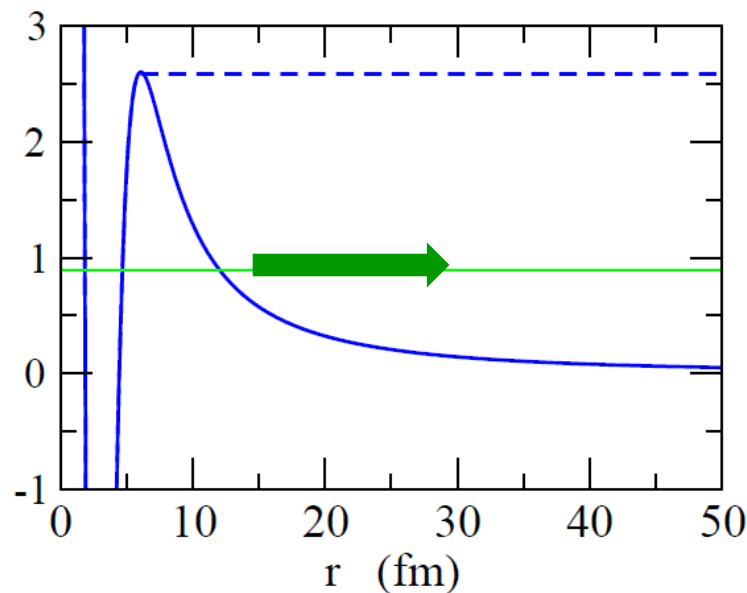
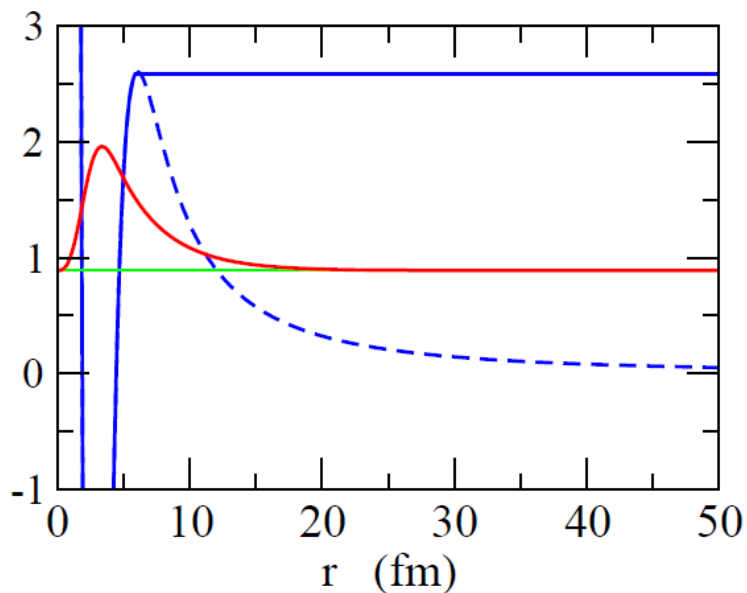
実際には有限の寿命で
障壁をトンネルし崩壊

「準束縛(準安定)状態」

準束縛状態とは?



ガモフ状態



トンネル効果で波動関数が
 沁み出し、**外向きの波として崩壊**

$$\begin{aligned}
 u(r) &\sim r^{l+1} && (r \rightarrow 0) \\
 &\rightarrow \mathcal{N} e^{i(kr - l\pi/2)} && (r \rightarrow \infty)
 \end{aligned}$$

共鳴幅



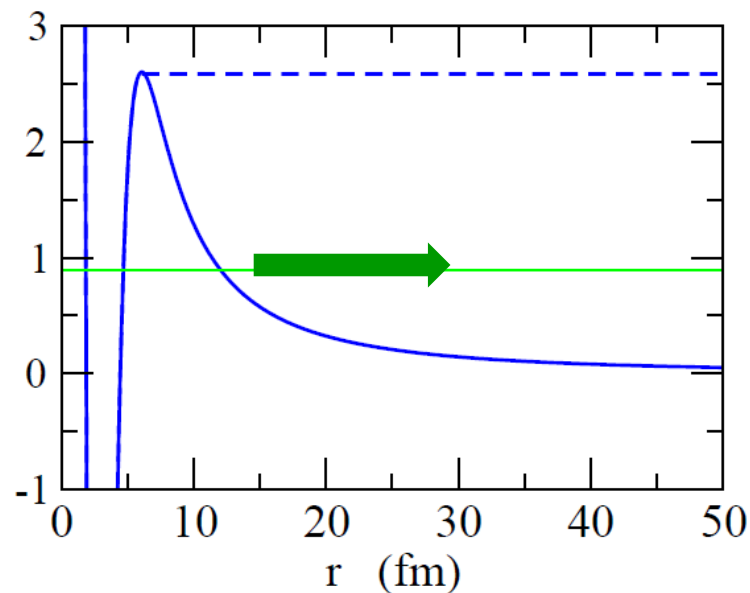
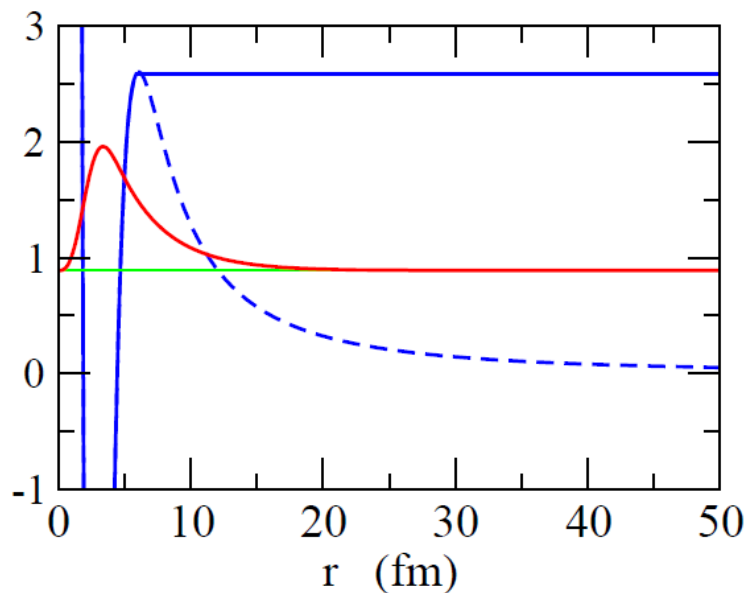
$$E \rightarrow E_R - i \frac{\Gamma}{2}$$



共鳴エネルギー

⇒ エネルギーを複素数にしなければならない:

ガモフ状態



トンネル効果で波動関数が
 沁み出し、**外向きの波として崩壊**

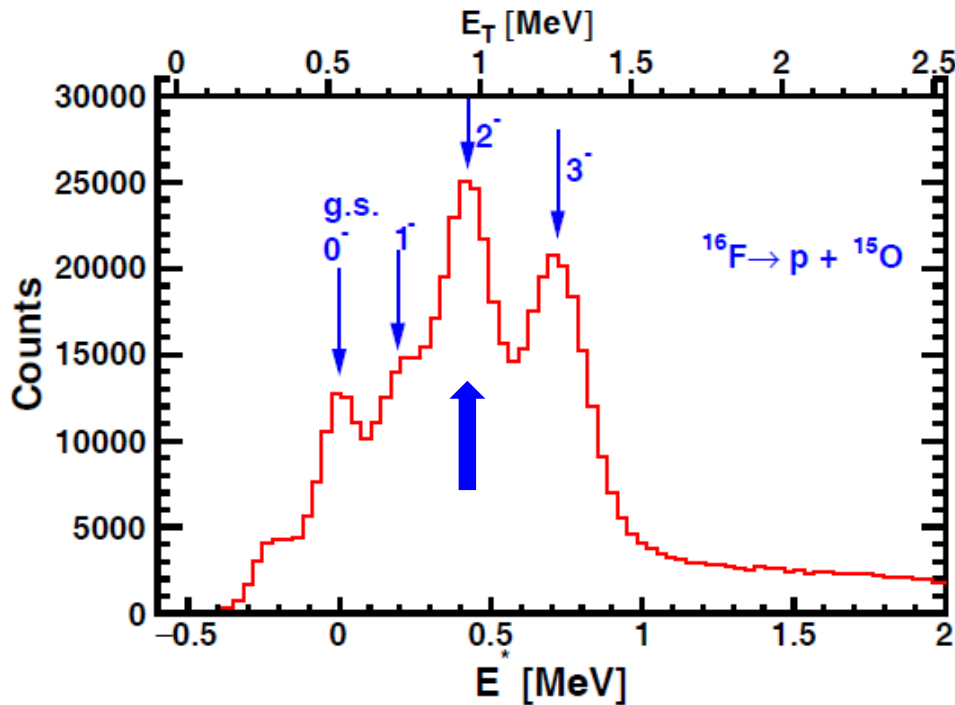
$$E \rightarrow E_R - i\frac{\Gamma}{2}$$

$$\begin{aligned}
 P_{\text{sur}}(t) &\equiv |\langle \psi(0) | \psi(t) \rangle|^2 \\
 &= |\langle \psi(0) | e^{-iHt/\hbar} | \psi(0) \rangle|^2 \\
 &= |\langle \psi(0) | e^{-i(E_R - i\Gamma/2)t/\hbar} | \psi(0) \rangle|^2 \\
 &= e^{-\Gamma t/\hbar} \quad \longrightarrow \quad \boxed{\hbar/\Gamma \text{ が準安定状態の寿命}}
 \end{aligned}$$

共鳴状態と弾性散乱(共鳴散乱)

陽子非束縛核 $^{16}_9\text{F}_7$

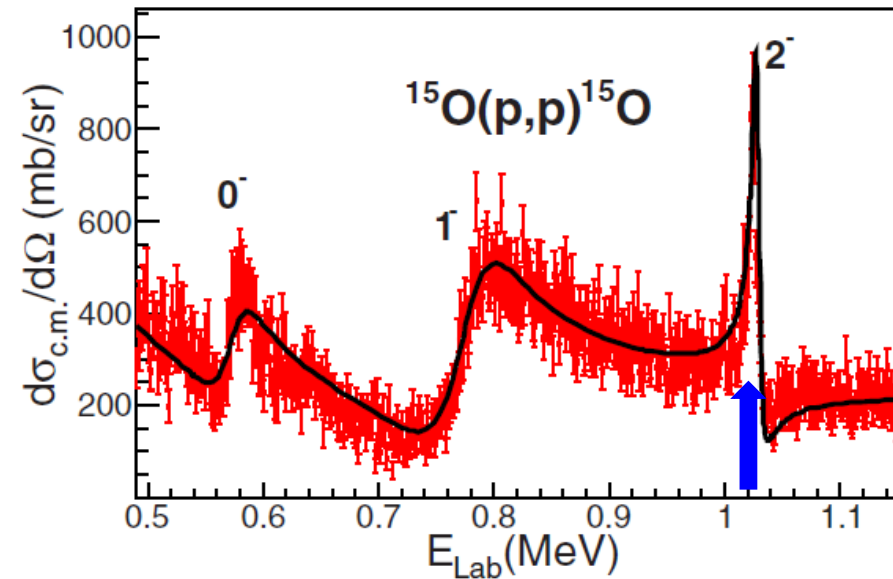
$^{17}_{10}\text{Ne}$ から1つ陽子を抜いて
 $^{16}_9\text{F}$ を生成 → 崩壊スペクトル



R.J. Charity,
Eur. Phys. J. Plus 131 ('16) 63

まず実際の現象から

$^{15}\text{O} + \text{p}$ 弾性散乱の断面積



$\theta_{\text{cm}} = 180$ deg.

I. Stefan et al., PRC90('14) 014307

共鳴エネルギーで弾性散乱の断面積が増大(共鳴散乱)

共鳴状態と弾性散乱(共鳴散乱)

次に散乱理論

自由粒子の運動:

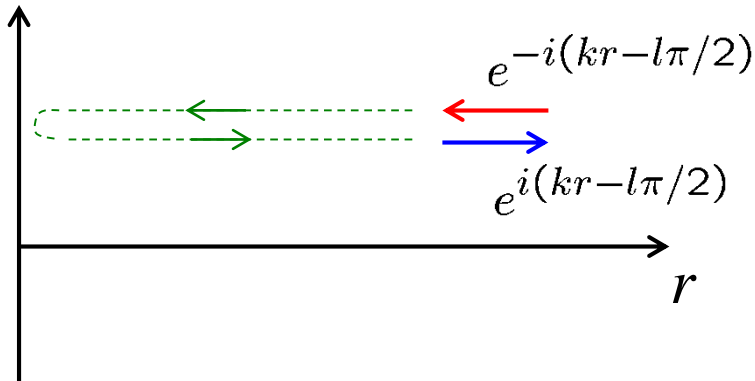
$$\left(-\frac{\hbar^2}{2m}\nabla^2 - E\right)\psi(\mathbf{r}) = 0$$

✓ 解:

$$\psi(\mathbf{r}) \propto j_l(kr)Y_{lm}(\hat{\mathbf{r}})\chi_{m_s}$$

✓ 遠方での振る舞い:

$$\begin{aligned} j_l(kr) &\rightarrow \frac{1}{kr} \sin\left(kr - \frac{l\pi}{2}\right) \\ &= \frac{-1}{2ikr} \left(e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)} \right) \end{aligned}$$



共鳴状態と弾性散乱(共鳴散乱)

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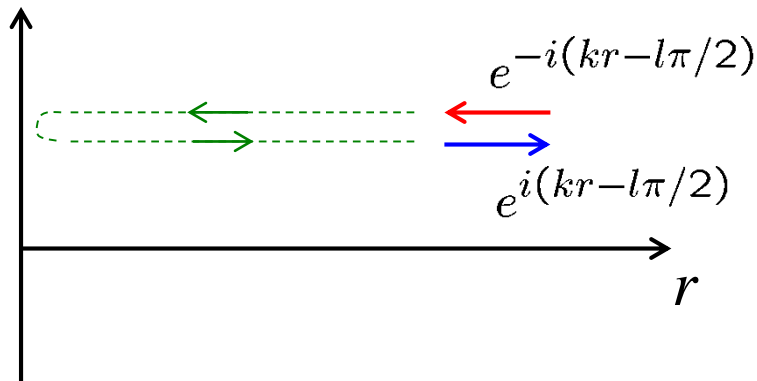
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次に散乱理論

ポテンシャル中の運動:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r) - E\right)\psi(\mathbf{r}) = 0$$

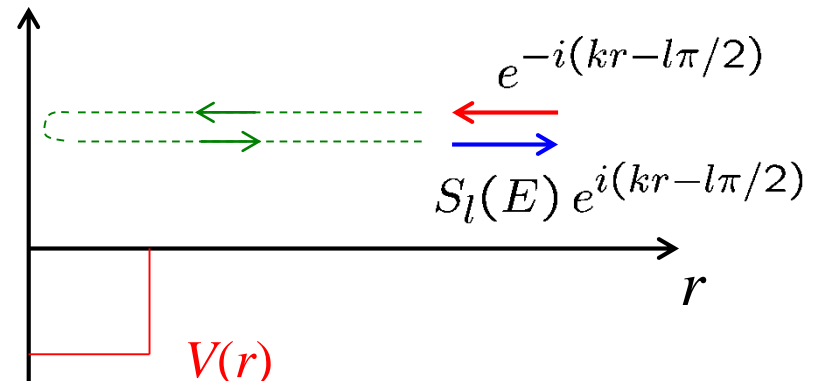
✓解:

$$\psi(\mathbf{r}) \propto R_l(r)Y_{lm}(\hat{\mathbf{r}})\chi_{m_s}$$

✓遠方での振る舞い:

$$R_l(r) \rightarrow \frac{-1}{2ikr} \left(e^{-i(kr-l\pi/2)} - S_l(E) e^{i(kr-l\pi/2)} \right)$$

* 吸収がなければ $|S_l(E)| = 1$



共鳴状態と弾性散乱(共鳴散乱)

自由粒子の運動:

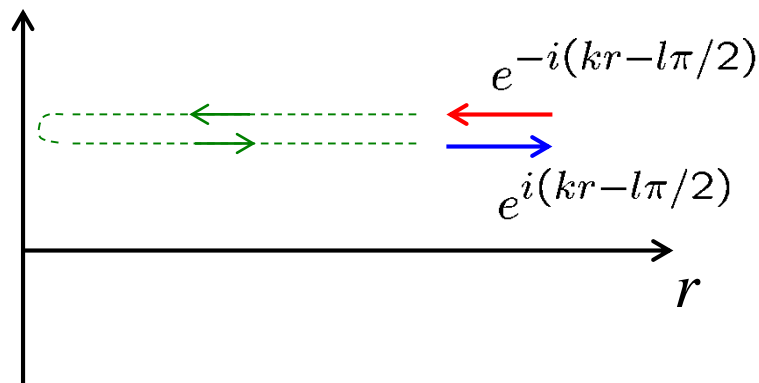
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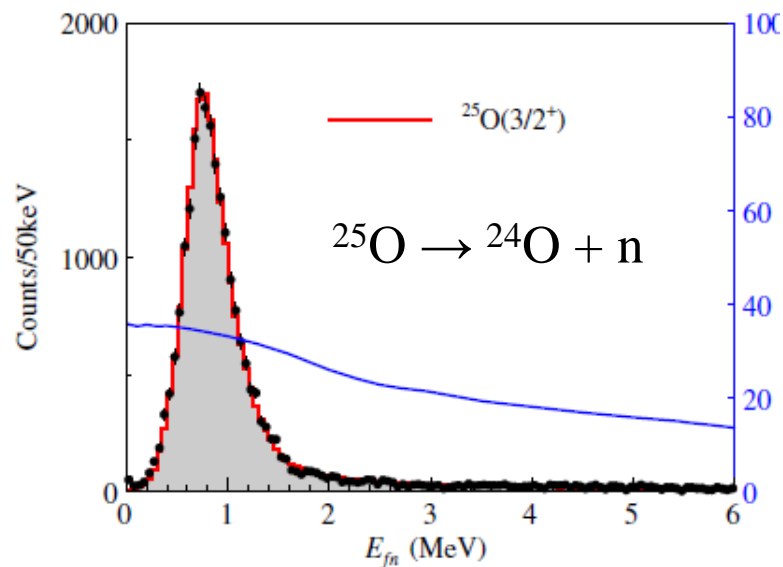
位相のずれ (phase shift)

$$S_l(E) = e^{2i\delta_l(E)}$$

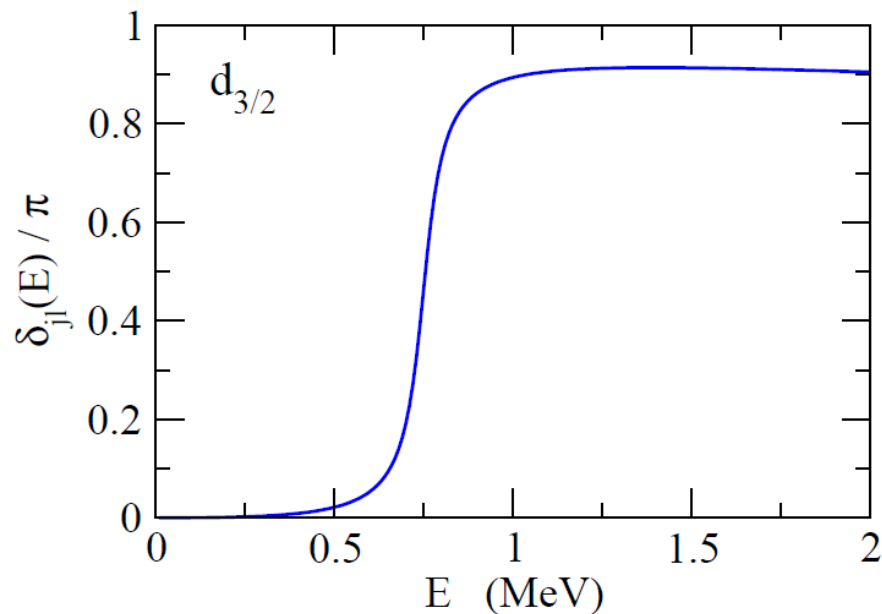
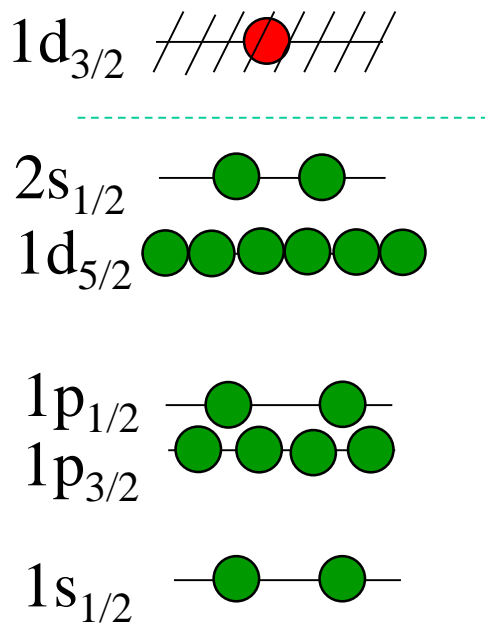
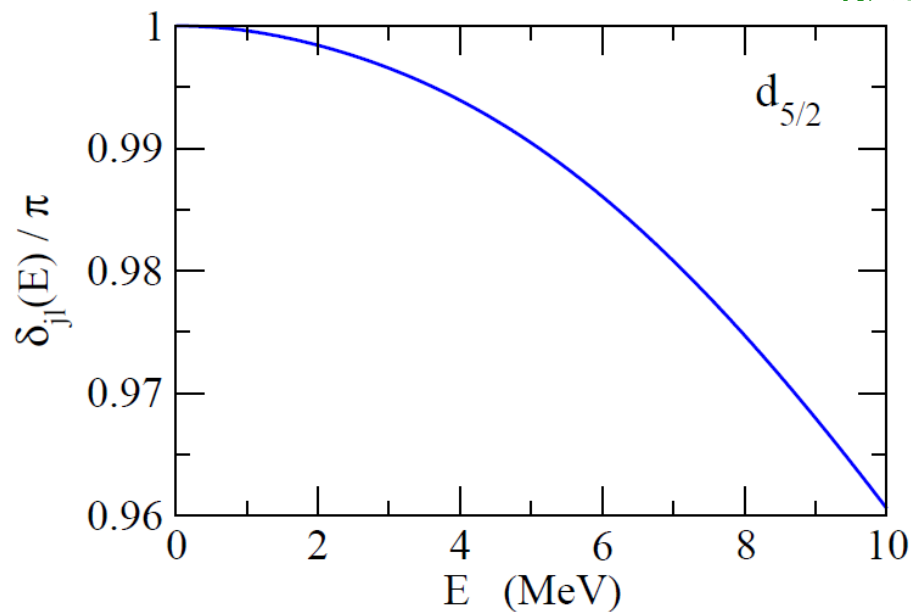


$$R_l(r) \rightarrow -\frac{e^{i\delta_l(E)}}{kr} \sin(kr - l\pi/2 + \delta_l(E))$$

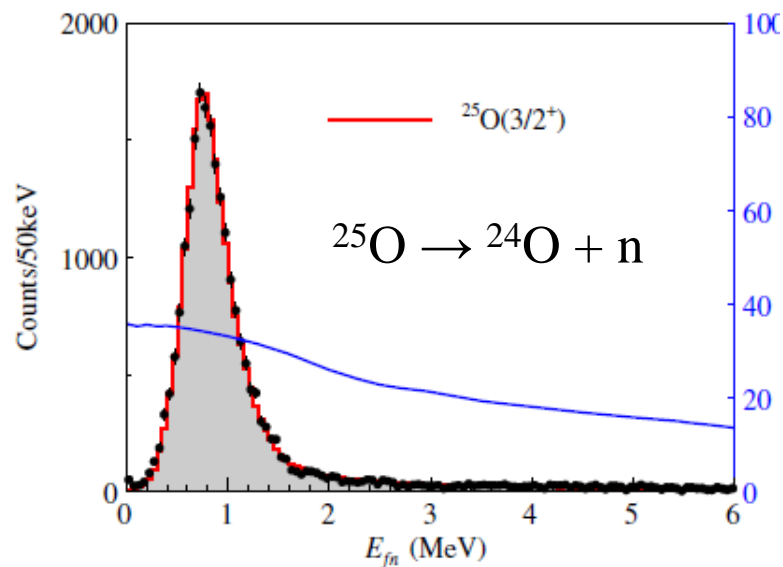
共鳴があると位相のずれはどう振る舞う?



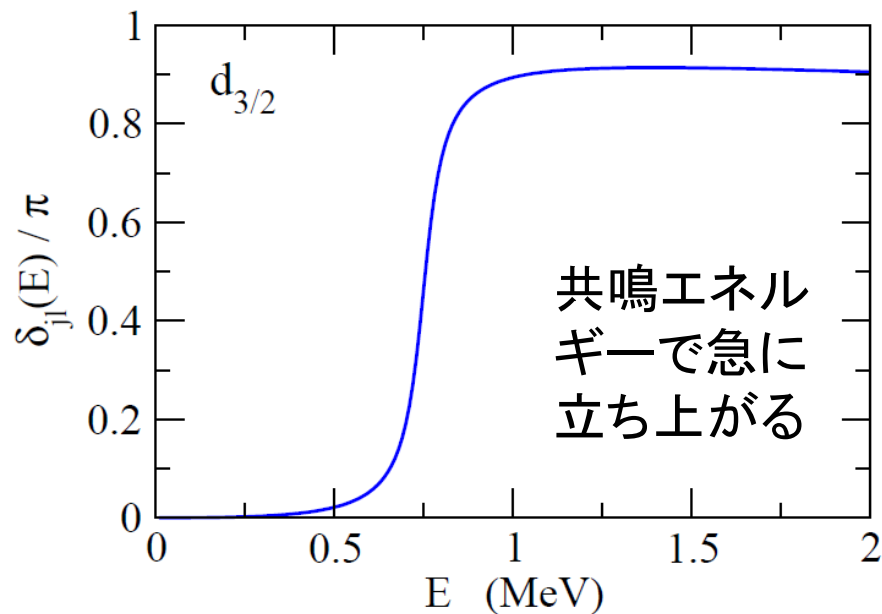
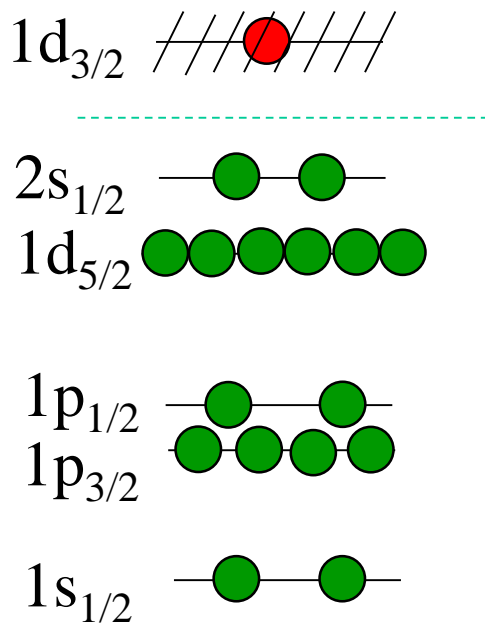
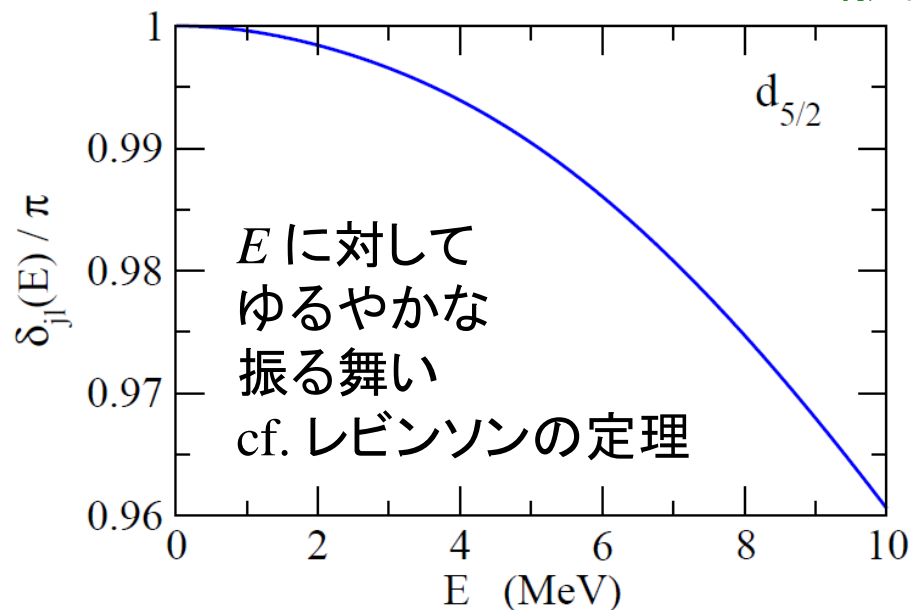
WS ポテンシャルによる n- ^{24}O 散乱



共鳴があると位相のずれはどう振る舞う?

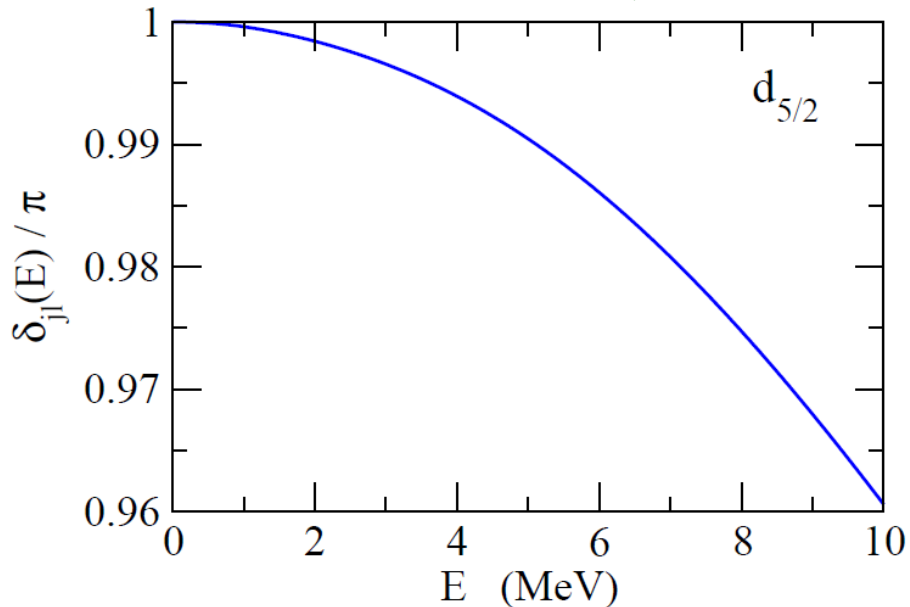


WS ポテンシャルによる n- ^{24}O 散乱



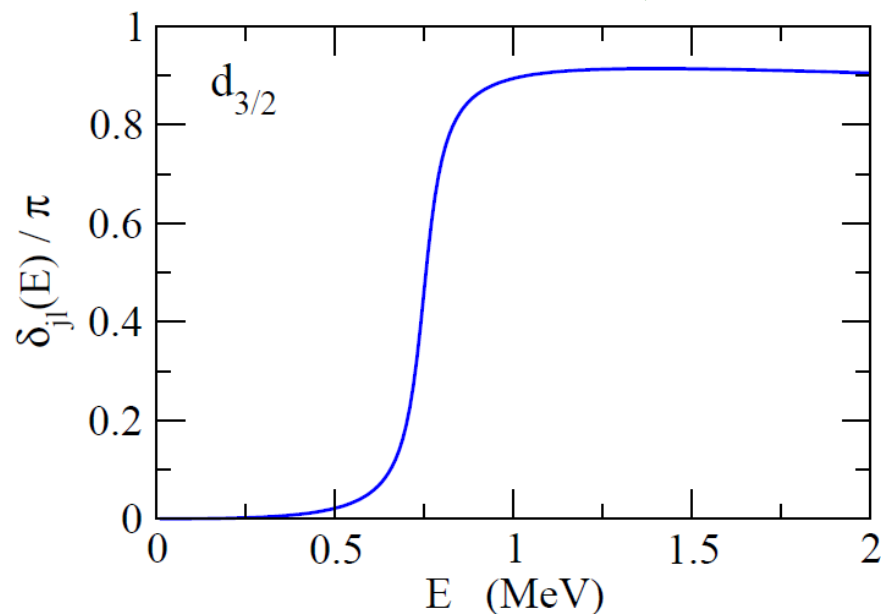
共鳴があると位相のずれはどう振る舞う?

共鳴がない場合



E に対してゆるやかな
振る舞い

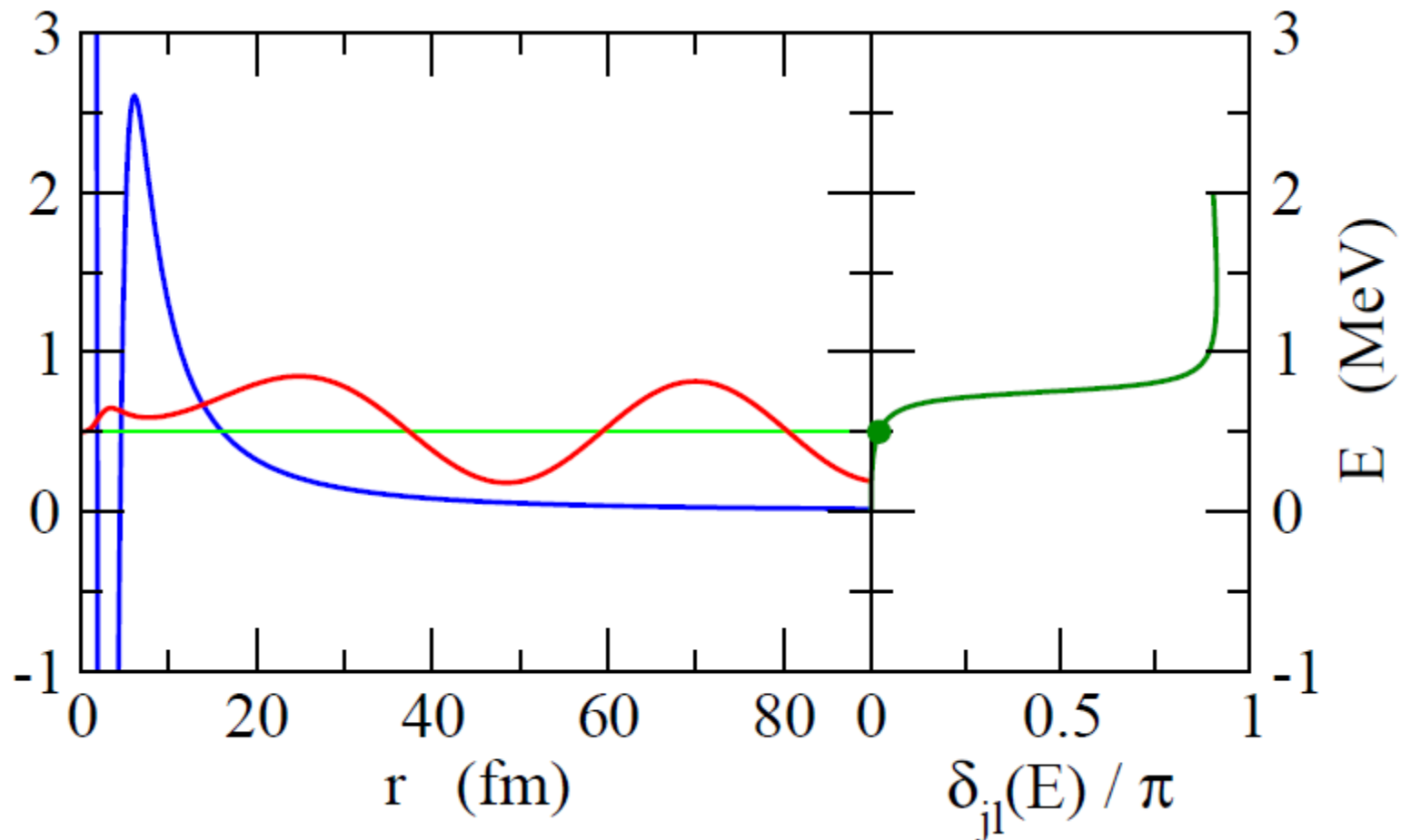
共鳴がある場合



共鳴エネルギーで急に
位相のずれが立ち上がる

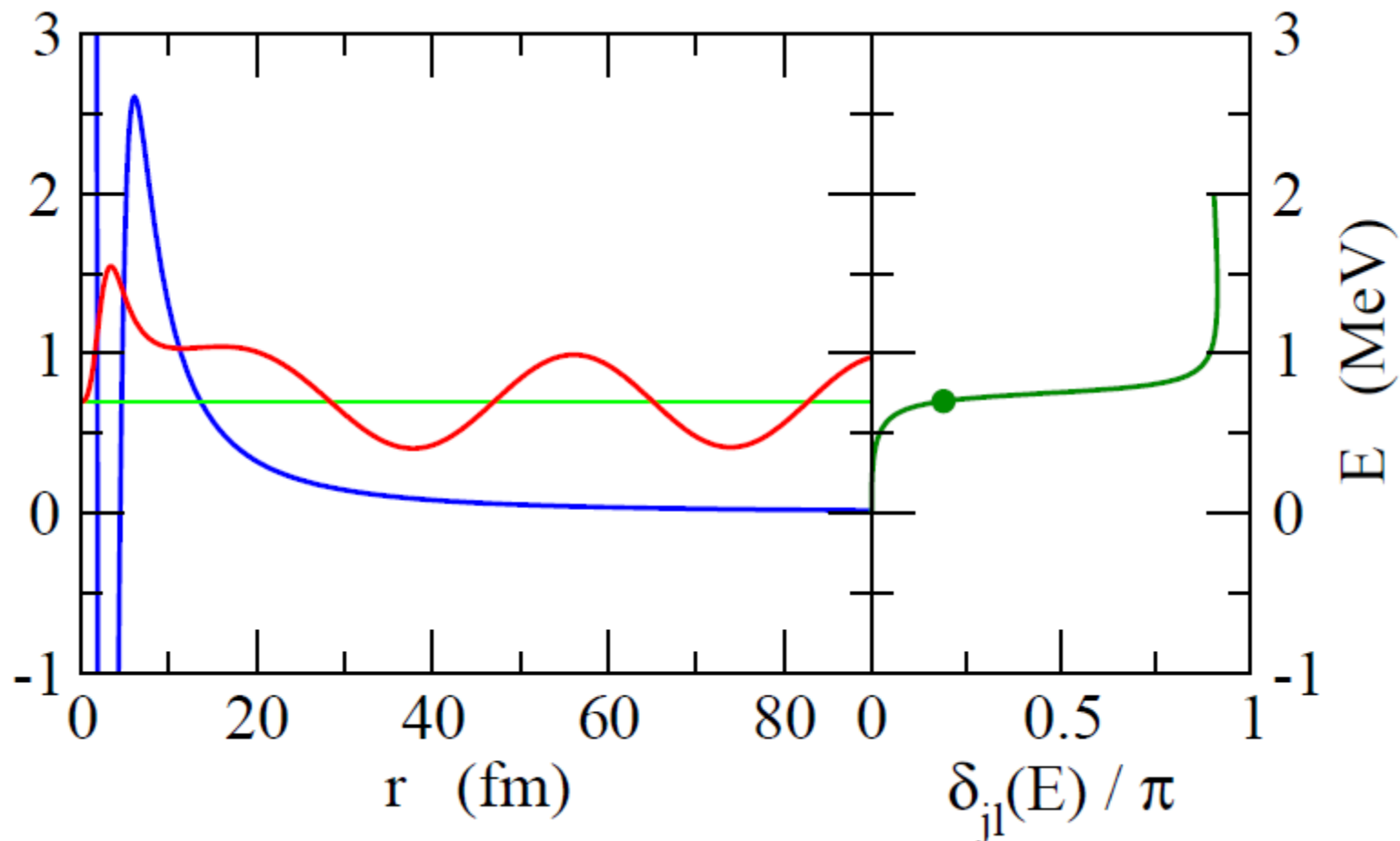
弾性散乱の全断面積: $\sigma_l = \frac{4\pi}{k^2} (2l + 1) \sin^2 \delta_l$ → 共鳴だと大

それでは波動関数は?



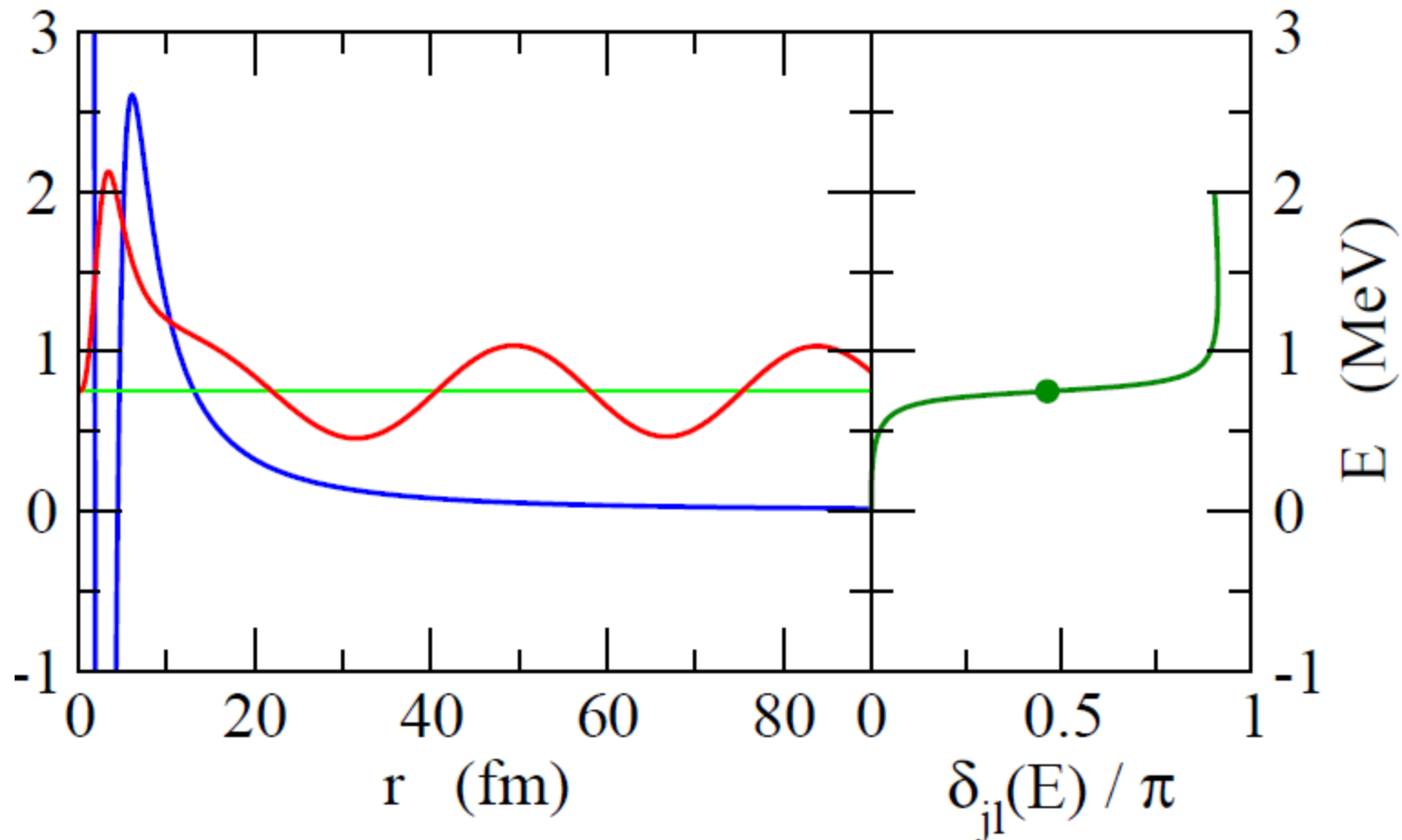
$$rR_{jl}(r) \rightarrow \sqrt{\frac{2m}{\pi k \hbar^2}} \sin\left(kr - \frac{l\pi}{2} + \delta_{jl}(E)\right); \quad \int dr \psi_E(r) \psi_{E'}^*(r) = \delta(E - E')$$

それでは波動関数は?



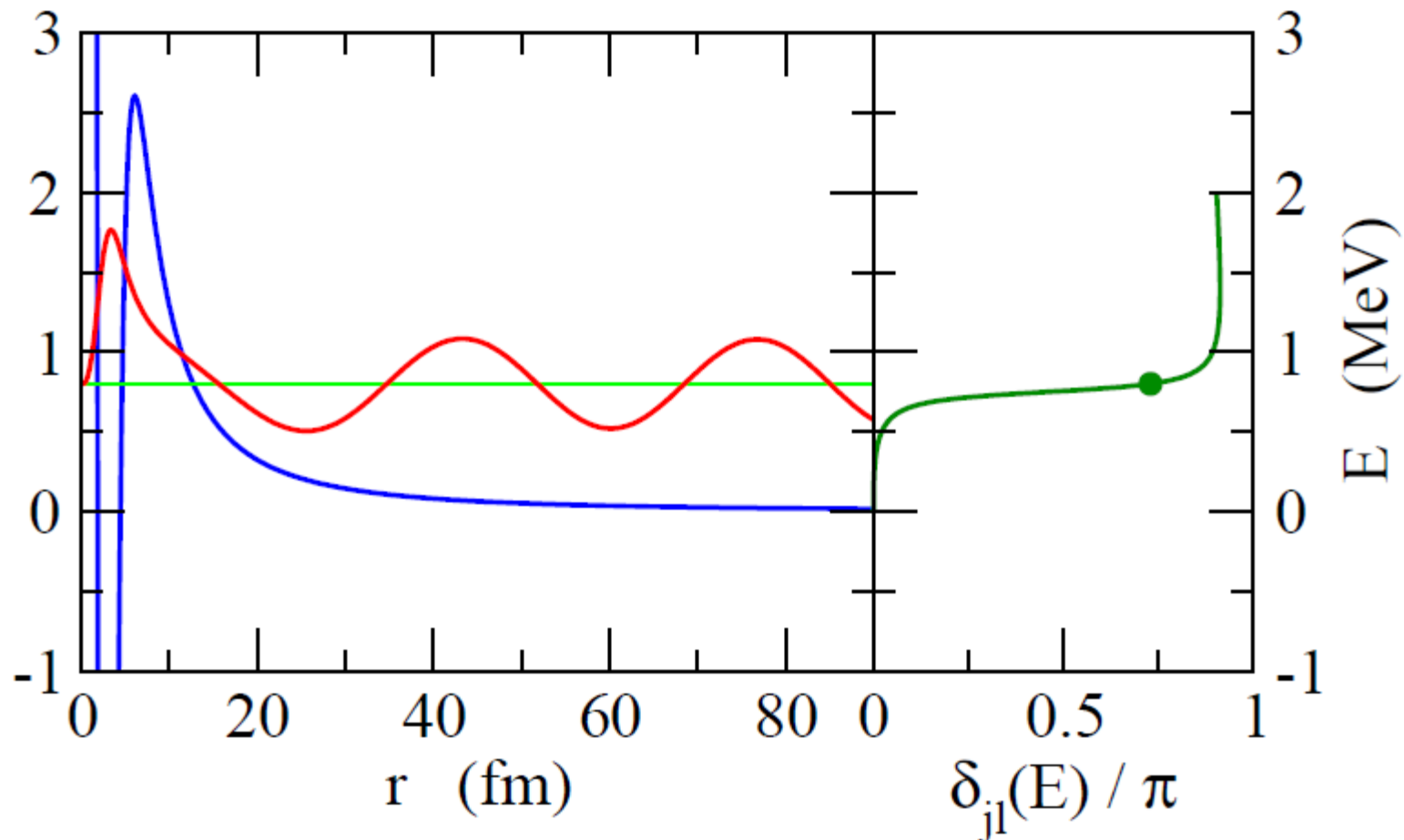
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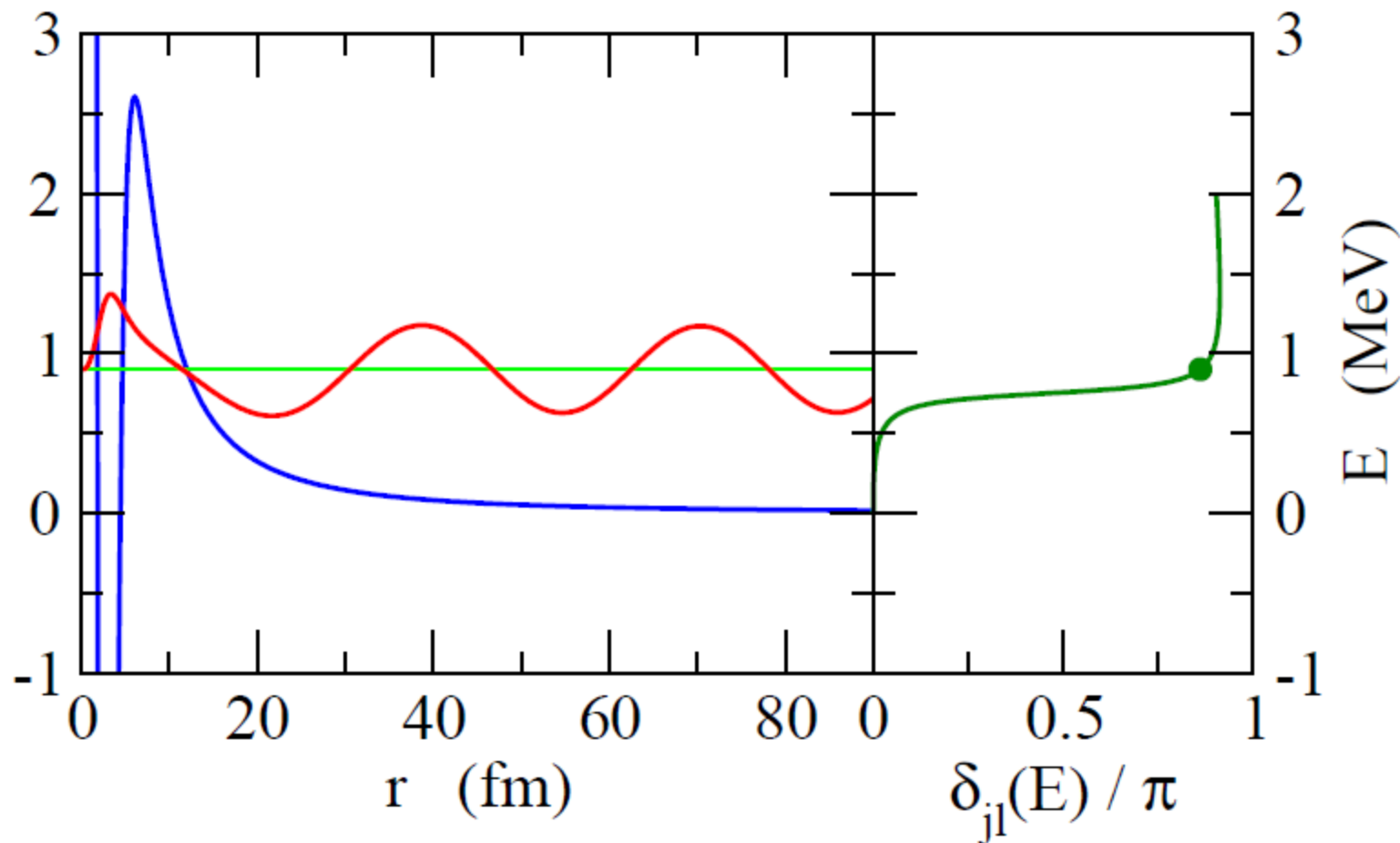
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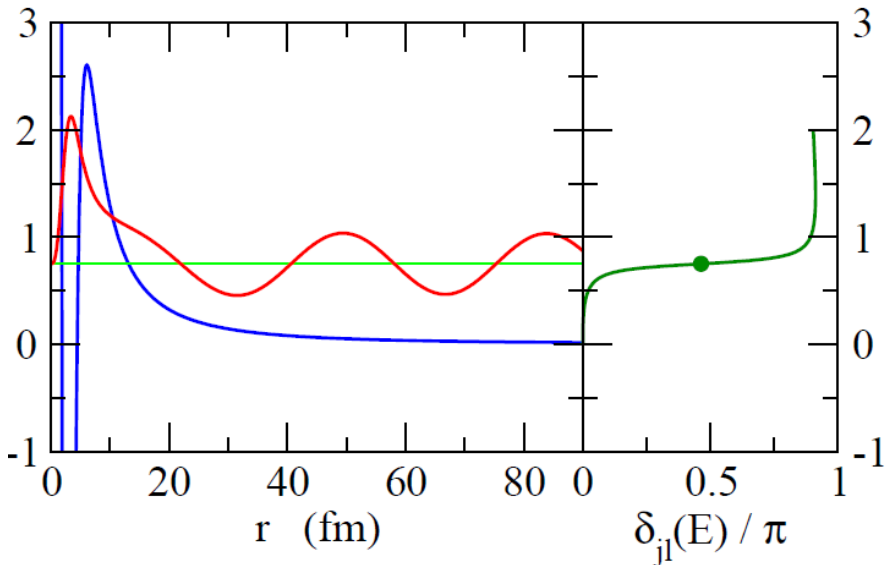
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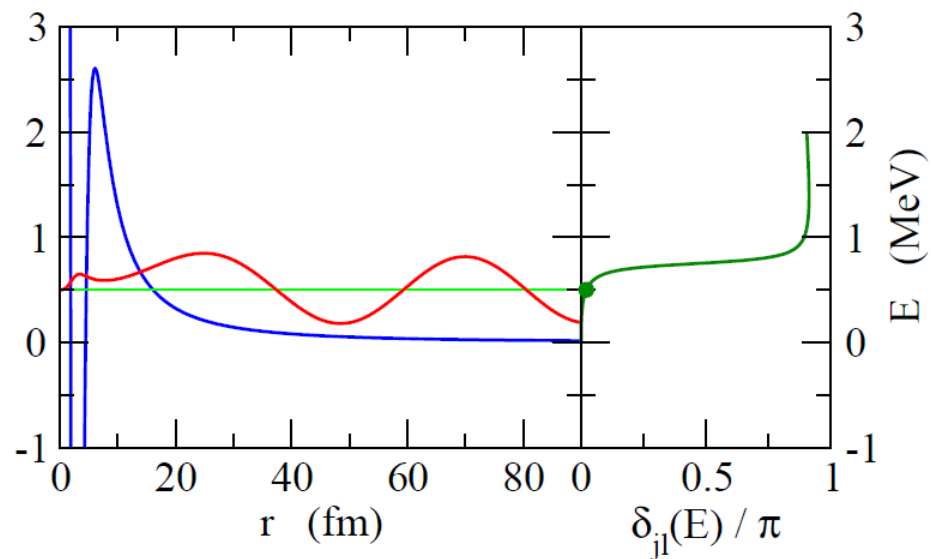
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それでは波動関数は?



on-resonance:

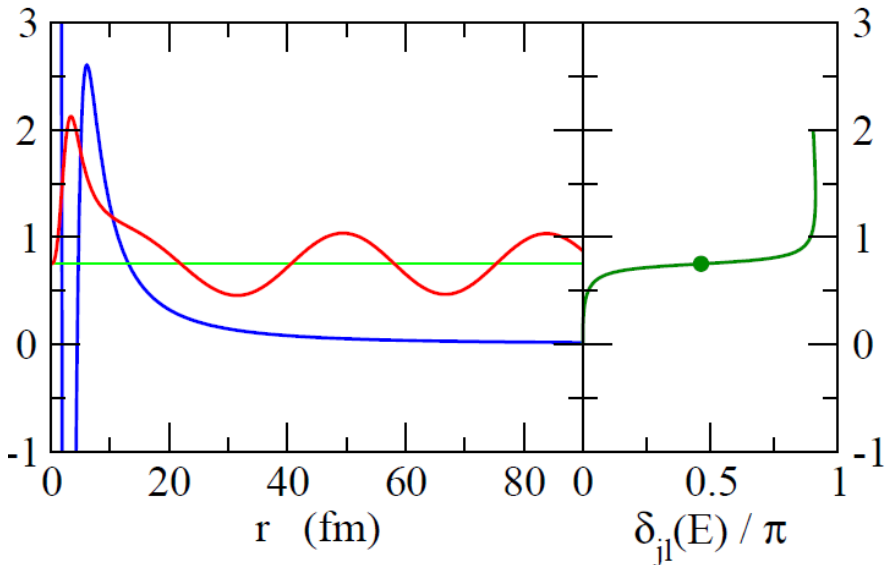
波動関数は障壁の内側で
大きな振幅



off-resonance:

障壁の内側では振幅が
小さい

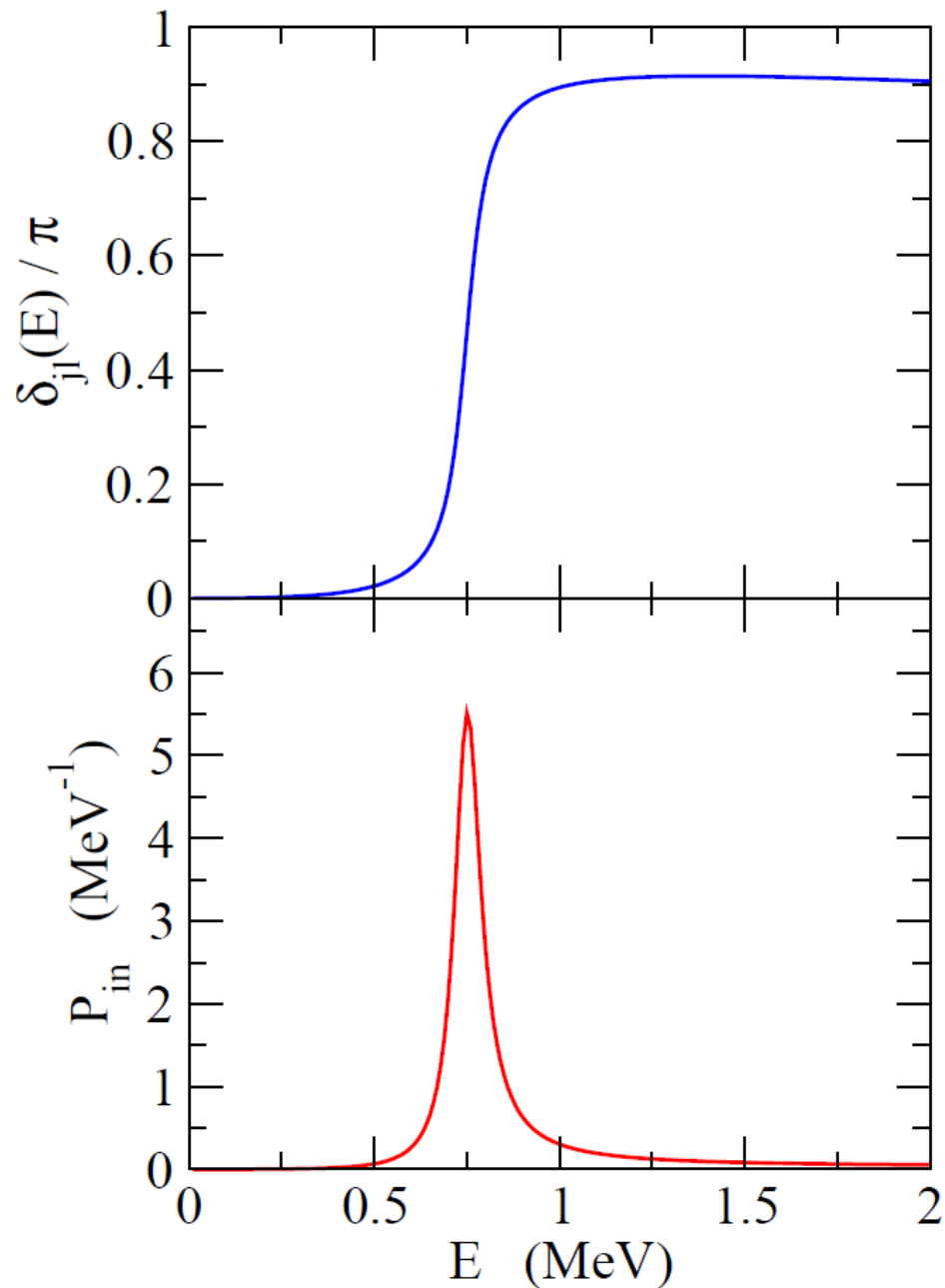
それでは波動関数は?



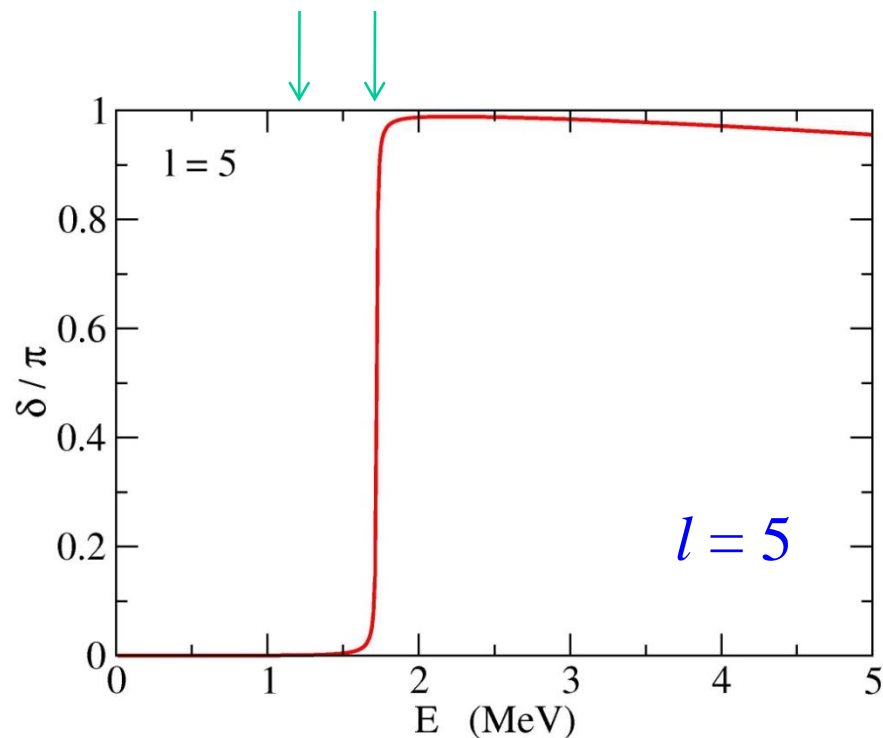
on-resonance:
波動関数は障壁の内側で
大きな振幅

障壁内部の存在確率

$$P_{\text{in}} \equiv \int_0^{r_b} r^2 dr |R_{jl}(r)|^2$$



もう少し極端な例：

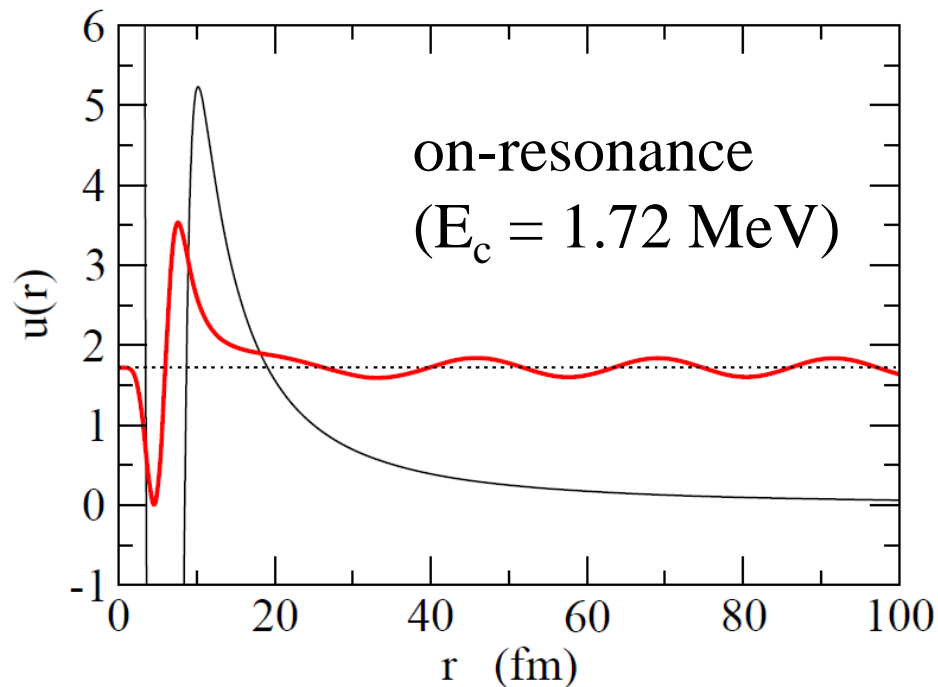
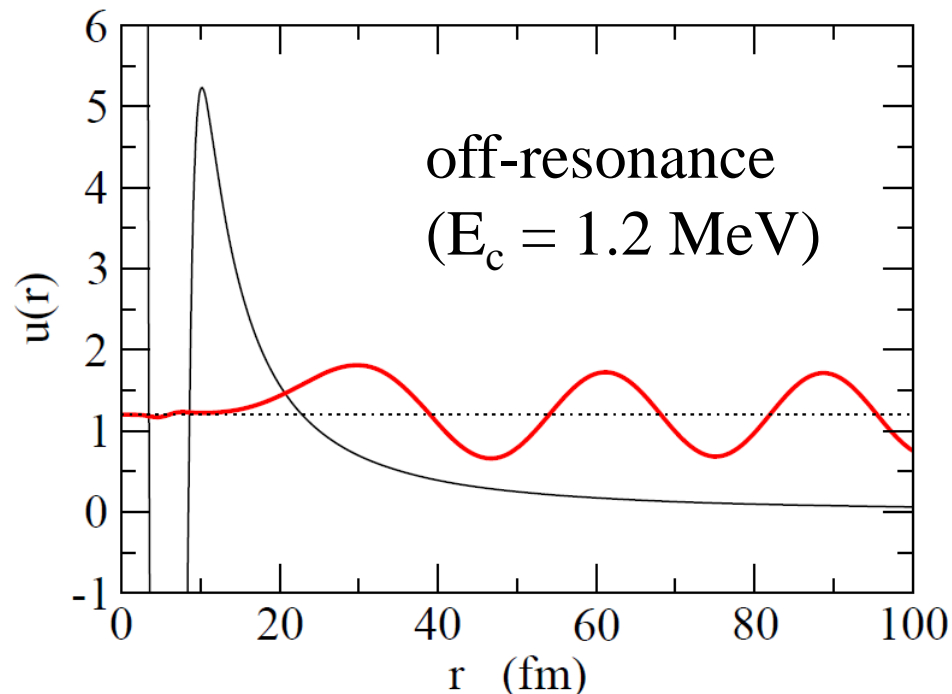


$$V_0 = -50 \text{ MeV}$$

$$R_0 = 1.27 * 200^{1/3} \text{ fm}$$

$$a = 0.67 \text{ fm}$$

$$\mu = 200 m_N / 201$$



➤ off-resonance では

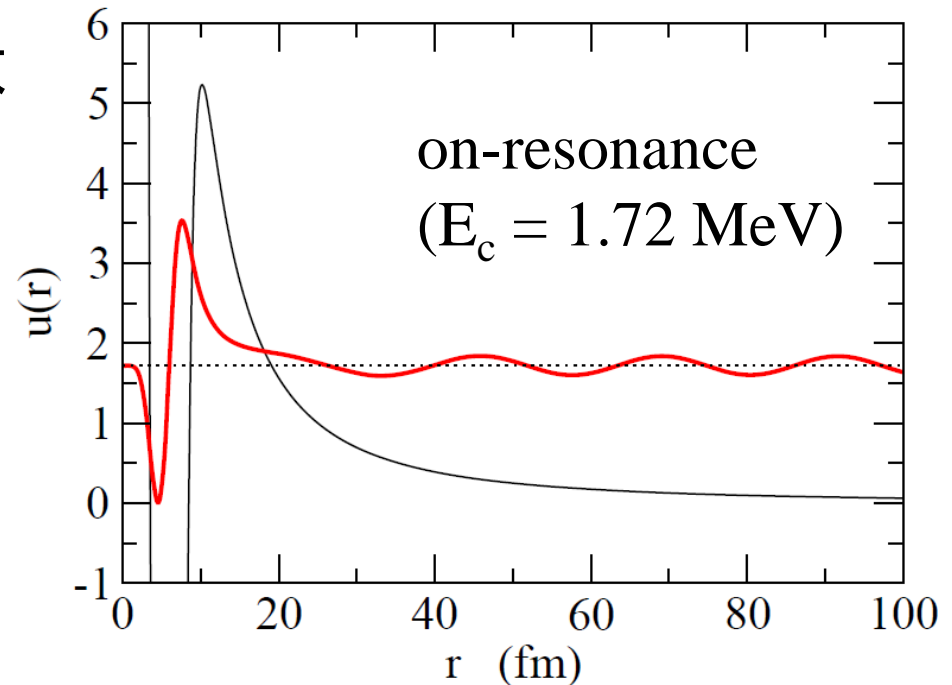
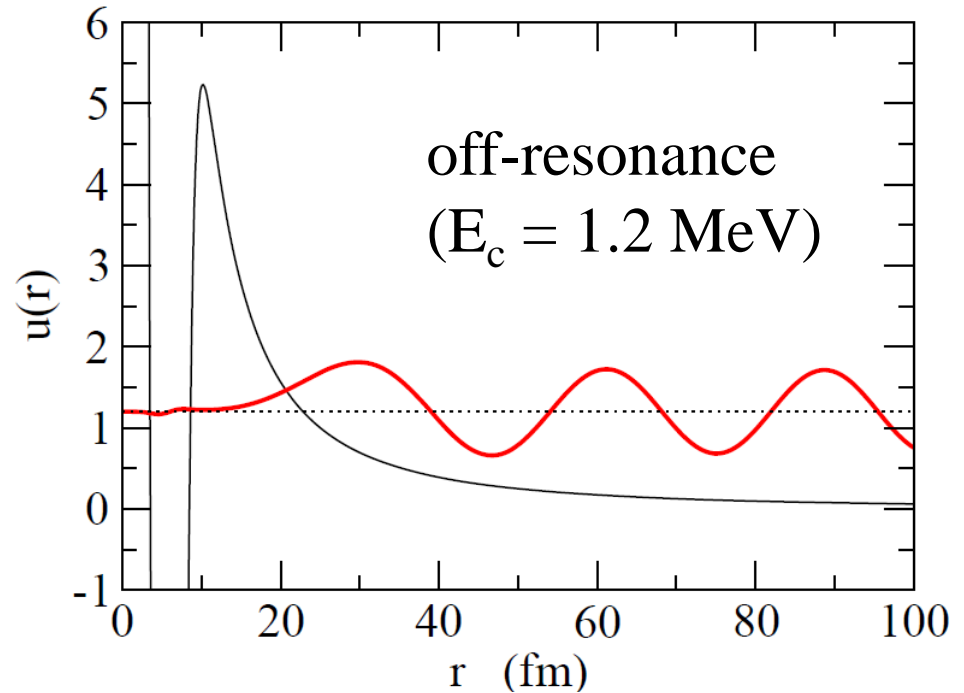
- 波動関数は障壁の外側で大きな振幅
- トンネル効果による障壁内部にしみ込む

➤ on-resonance では

- 波動関数は障壁内部で束縛状態のように振る舞う
- 障壁にトラップされた波動関数がトンネル効果により障壁の外側にしみ出る

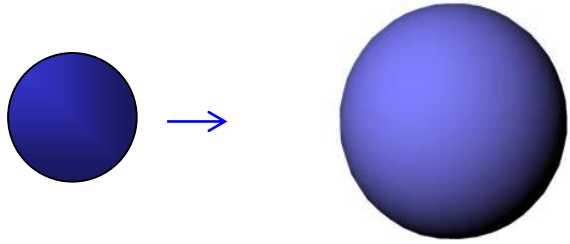


ガモフ状態と同じ振る舞い

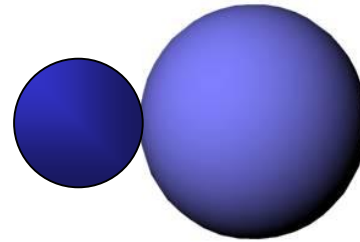


2. 核融合反応

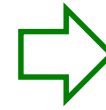
Nuclear fusion reactions



two positive charges
repel each other

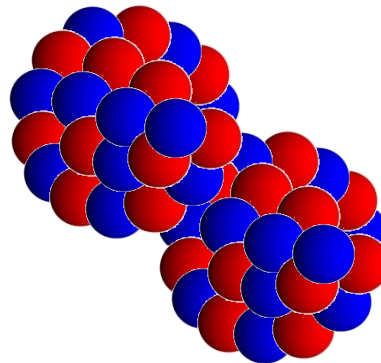


nuclear *attractive*
interaction



compound
nucleus

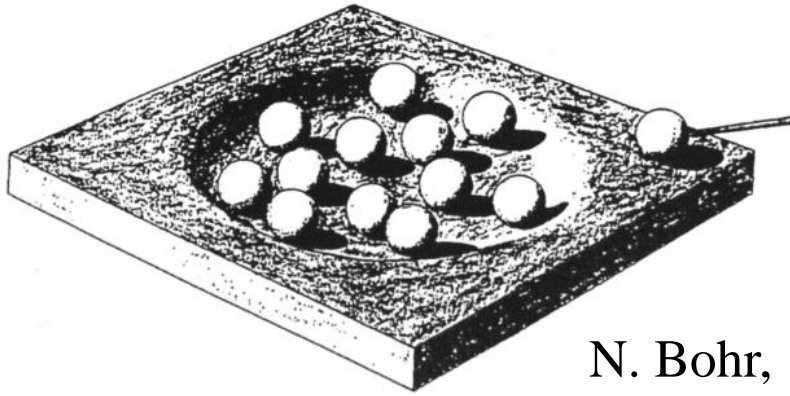
複合核



核融合反応： 複合核生成反応

Niels Bohr (1936)

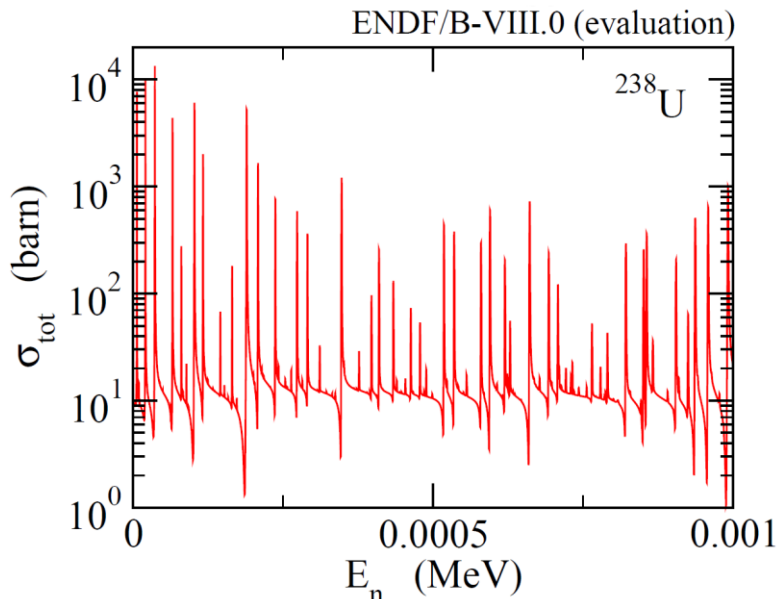
原子核による中性子の吸収 → 複合核



N. Bohr,
Nature 137 ('36) 351



Wikipedia

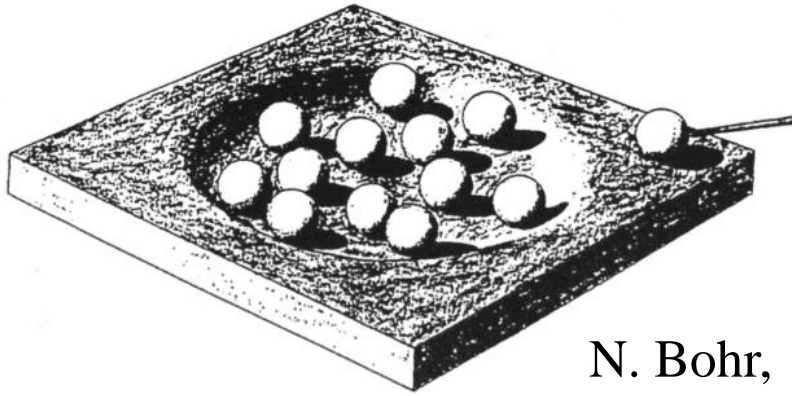


cf. フェルミの実験 (1935)
MeV スケールの原子核に
eV スケールの幅の多数の共鳴状態

核融合反応： 複合核生成反応

Niels Bohr (1936)

原子核による中性子の吸収 → 複合核

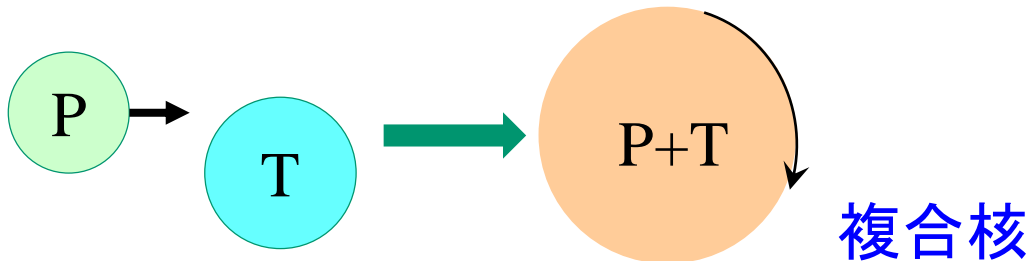


N. Bohr,
Nature 137 ('36) 351

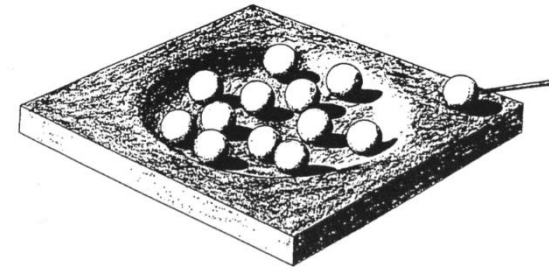
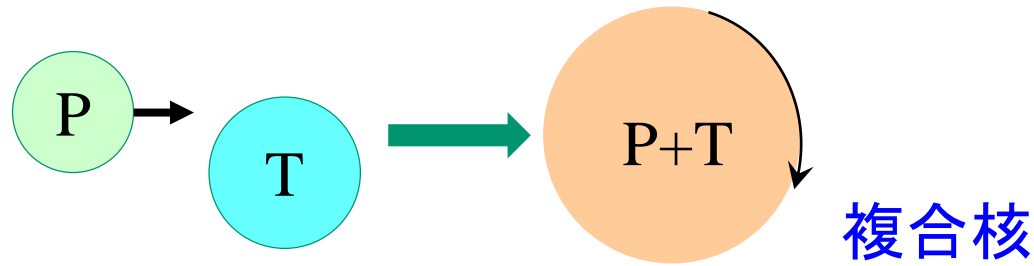


Wikipedia

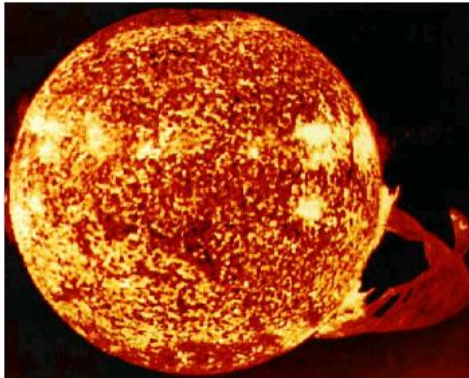
重イオン反応で複合核をつくる = 重イオン核融合反応



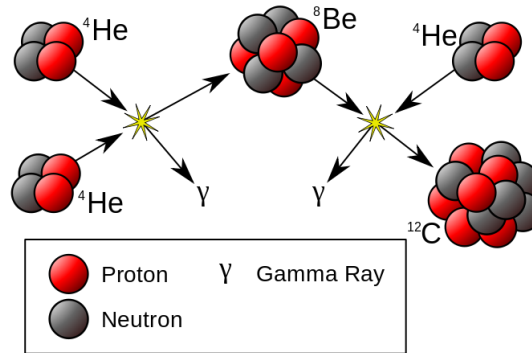
核融合反応： 複合核生成反応



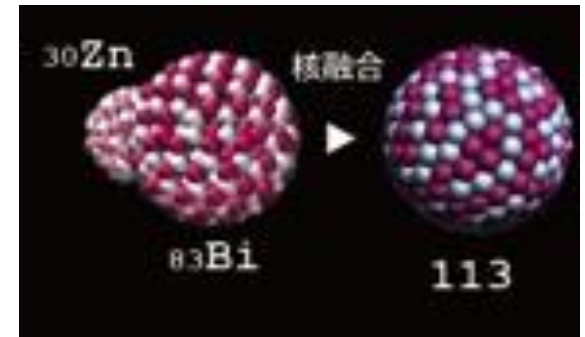
cf. N. Bohr '36



恒星のエネルギー源 (Bethe '39)



元素合成

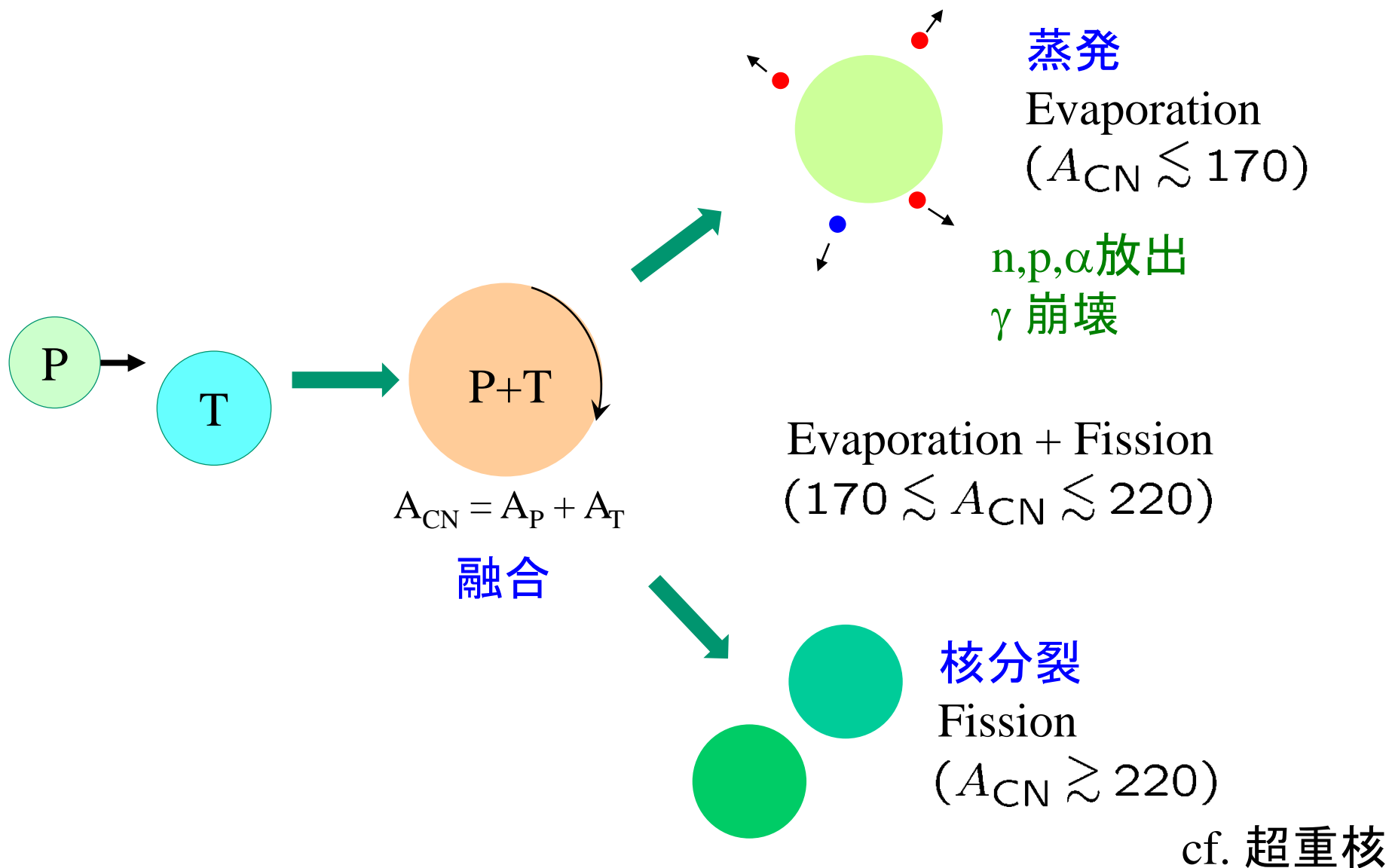


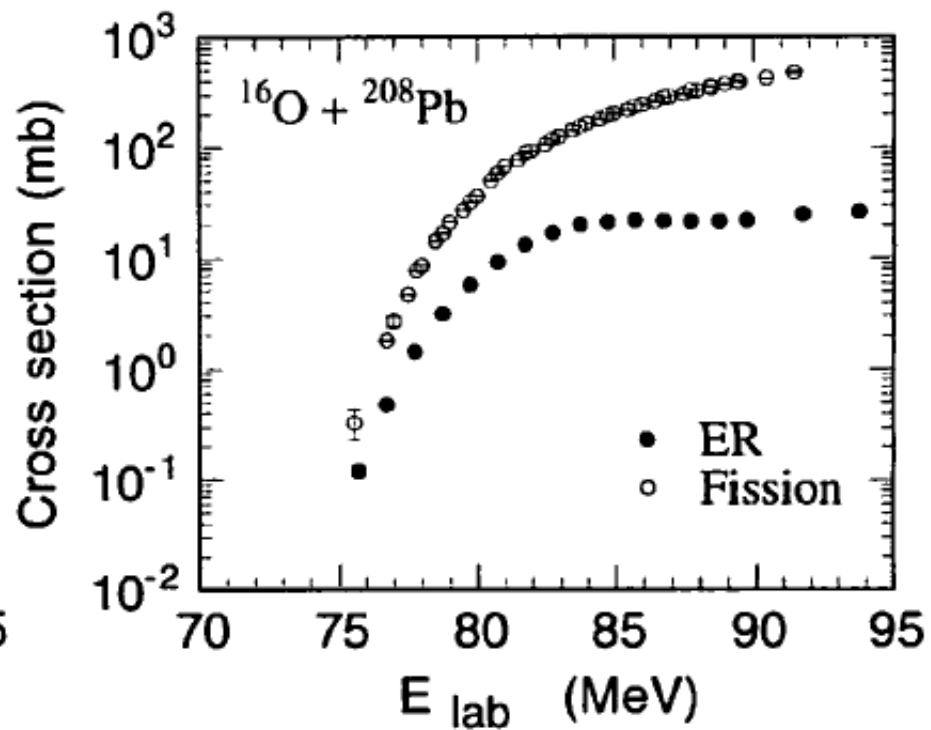
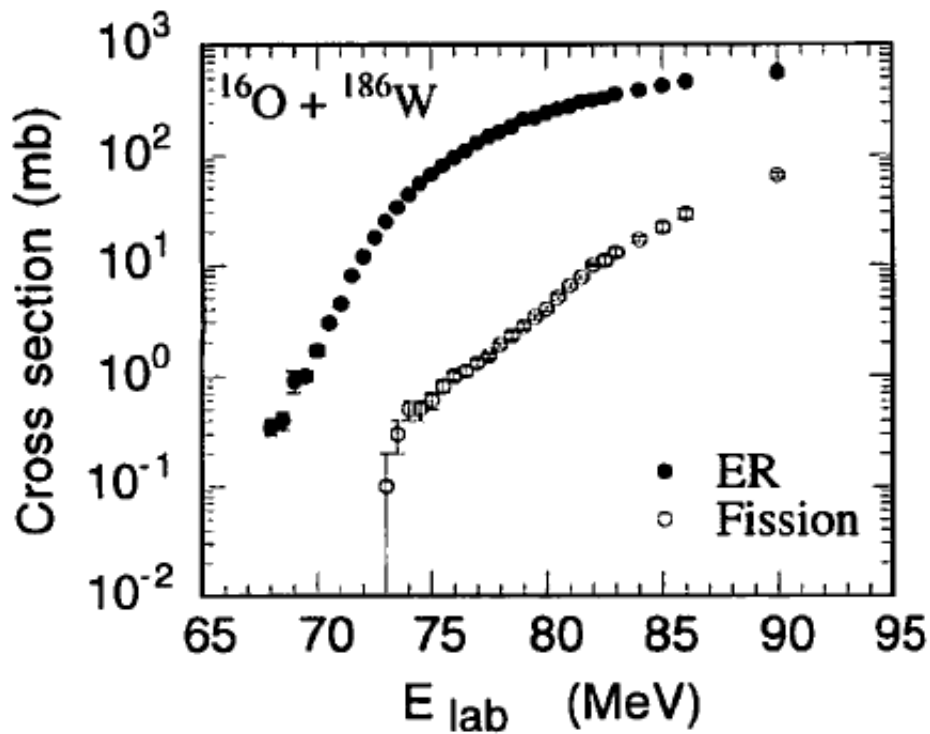
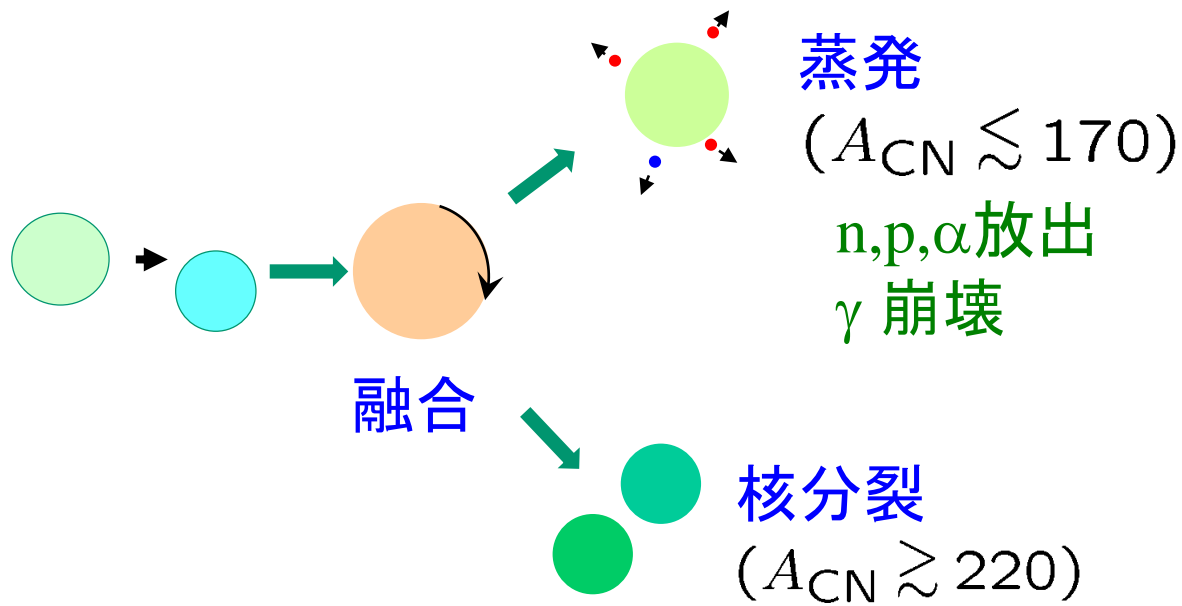
超重元素の合成

核融合・核分裂: 強い相互作用をする量子多体系の大振幅集団運動

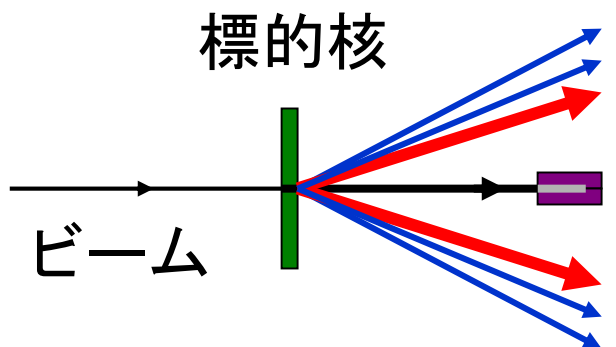
← 微視的理解: 核物理における究極の未解決問題の一つ

核融合反応：複合核生成反応

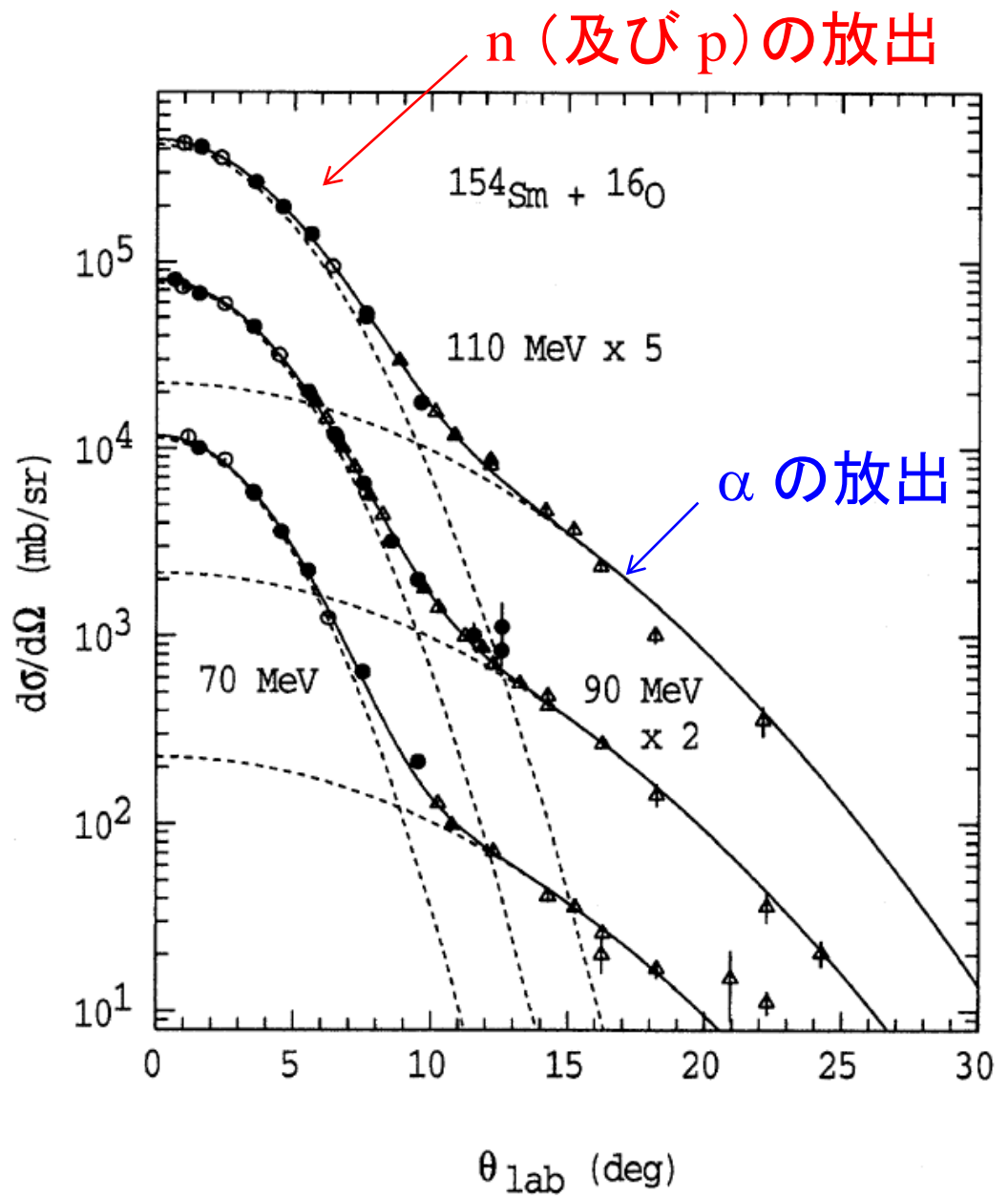




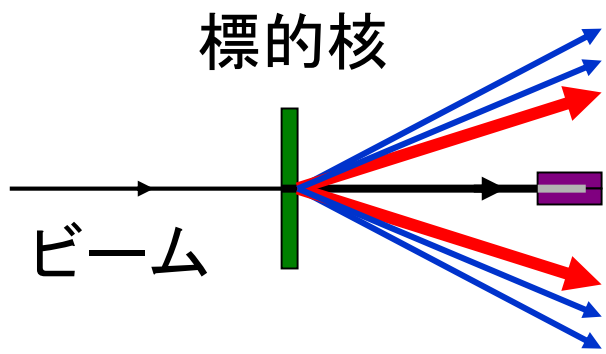
蒸発残留核の測定



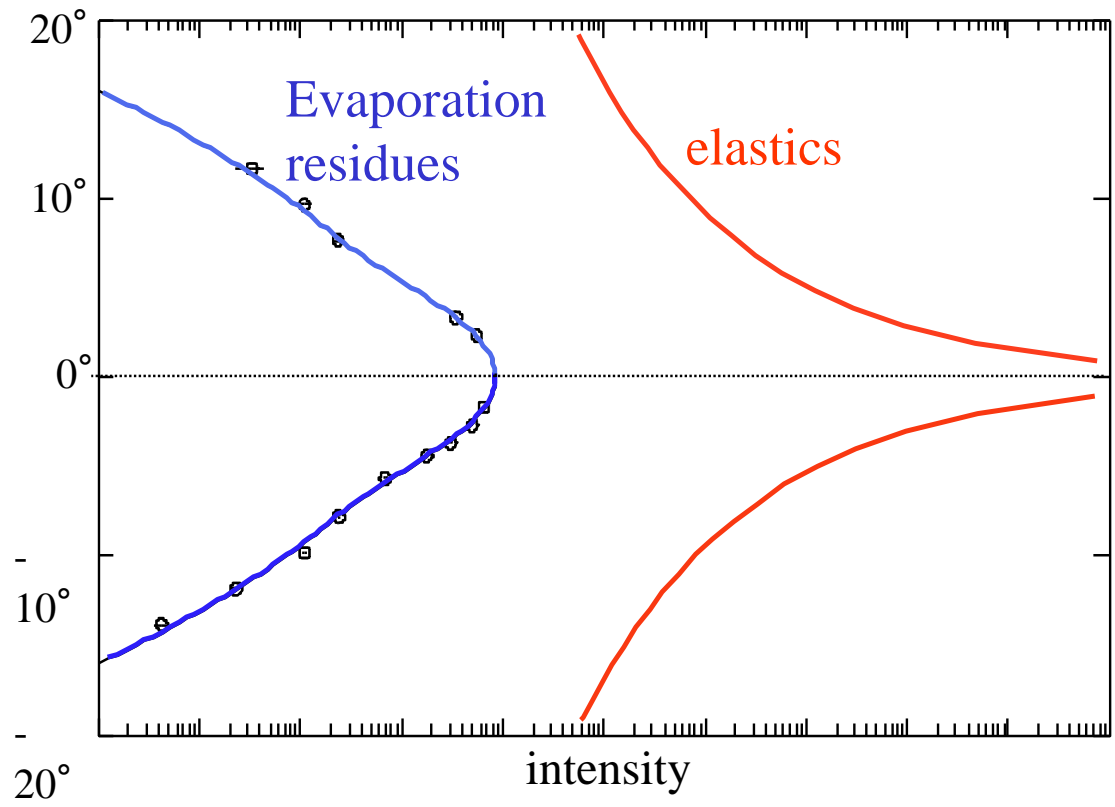
蒸発残留核:
前方角度へ



蒸発残留核の測定

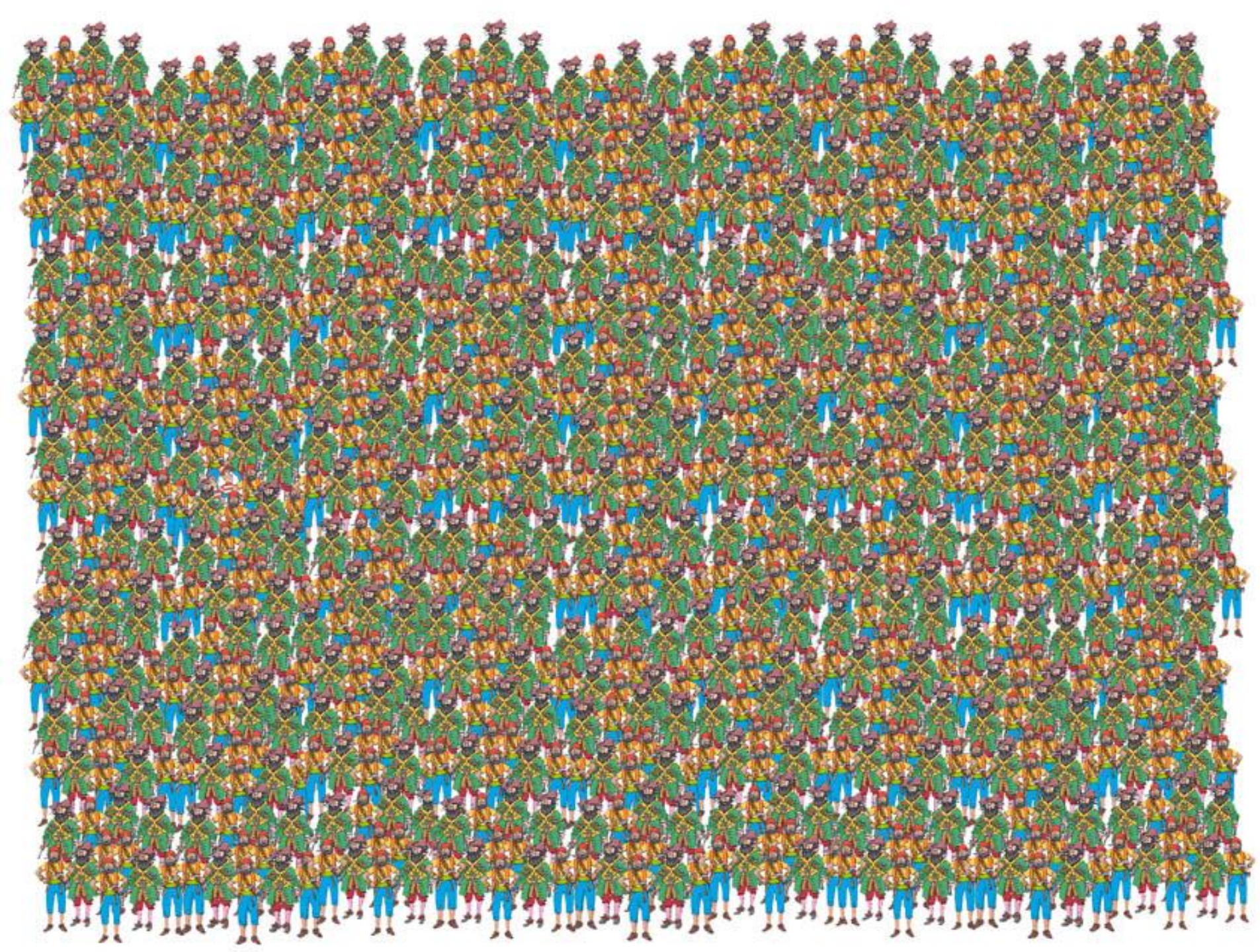


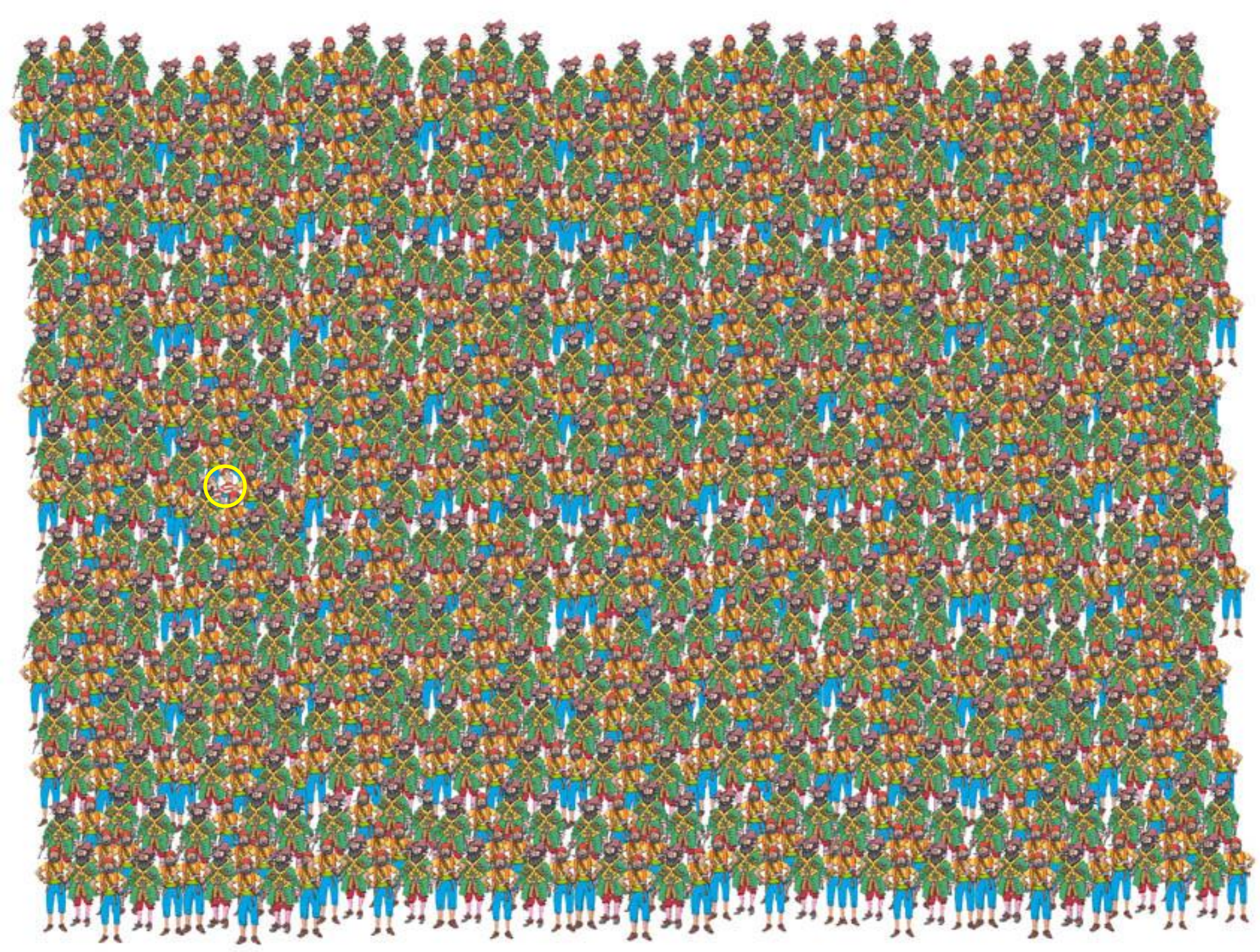
蒸発残留核:
前方角度へ
ビーム粒子も
前方角度へ



beam-like 粒子: 蒸発残留核の $10^4 \sim 10^{12}$ 倍の強度

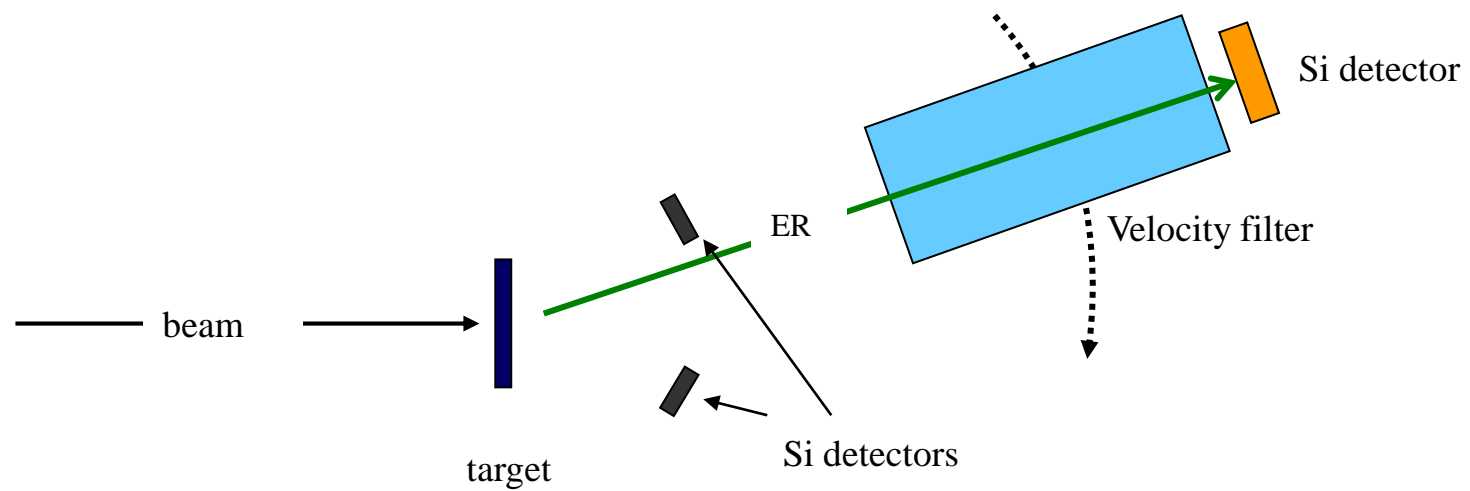




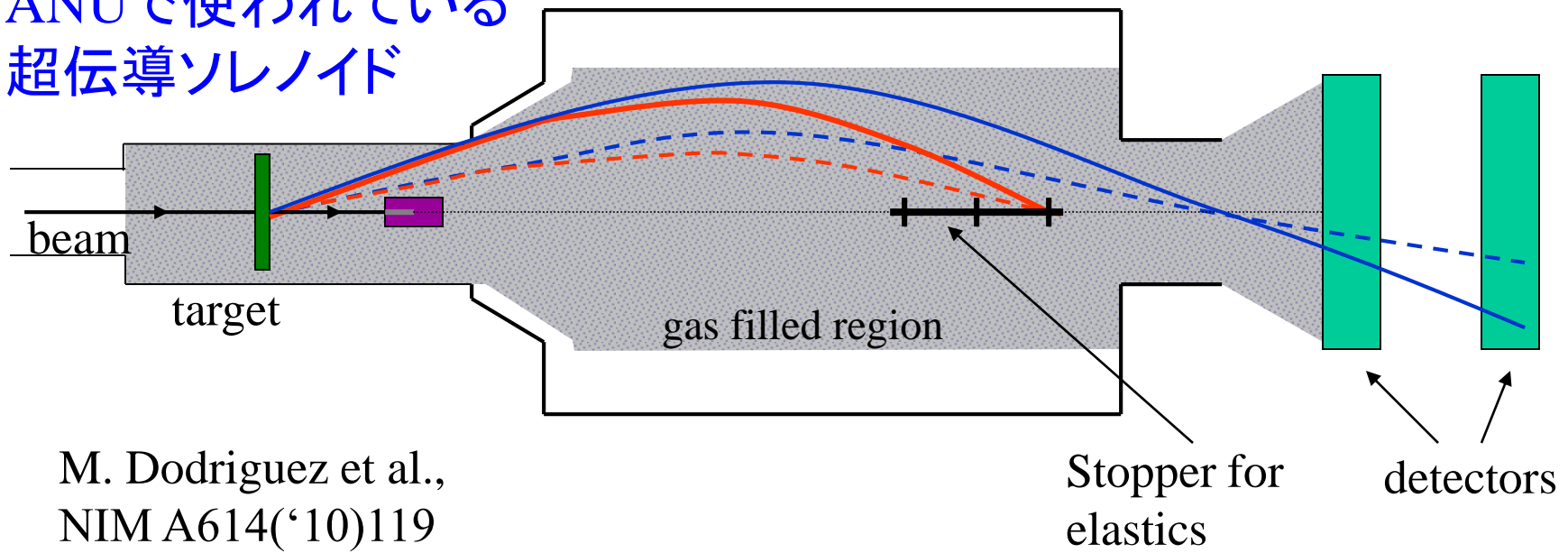


蒸発残留核の測定

velocity filter 等を用いてうまく蒸発残留核と beam-like 粒子をわけける



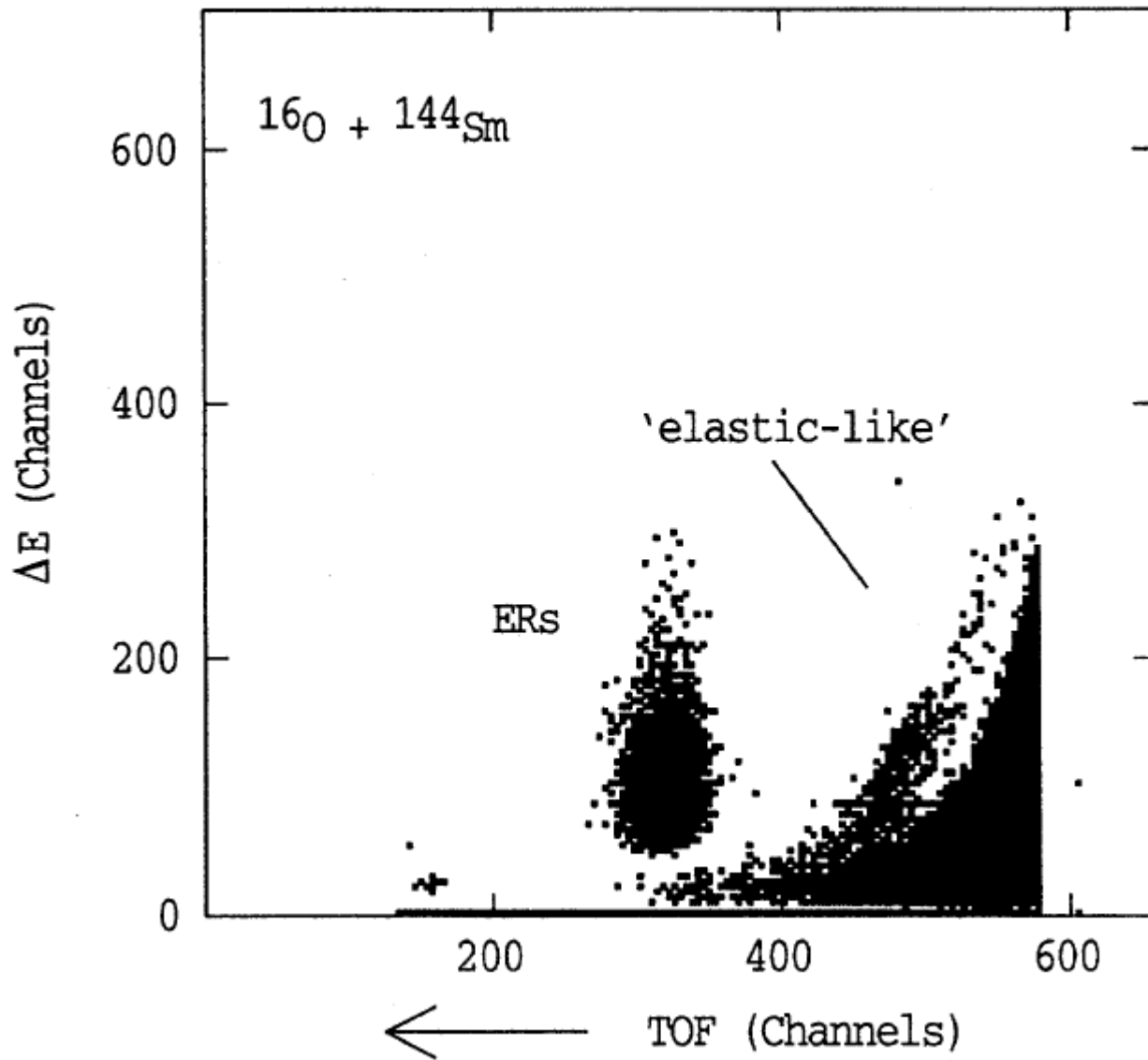
ANUで使われている
超伝導ソレノイド



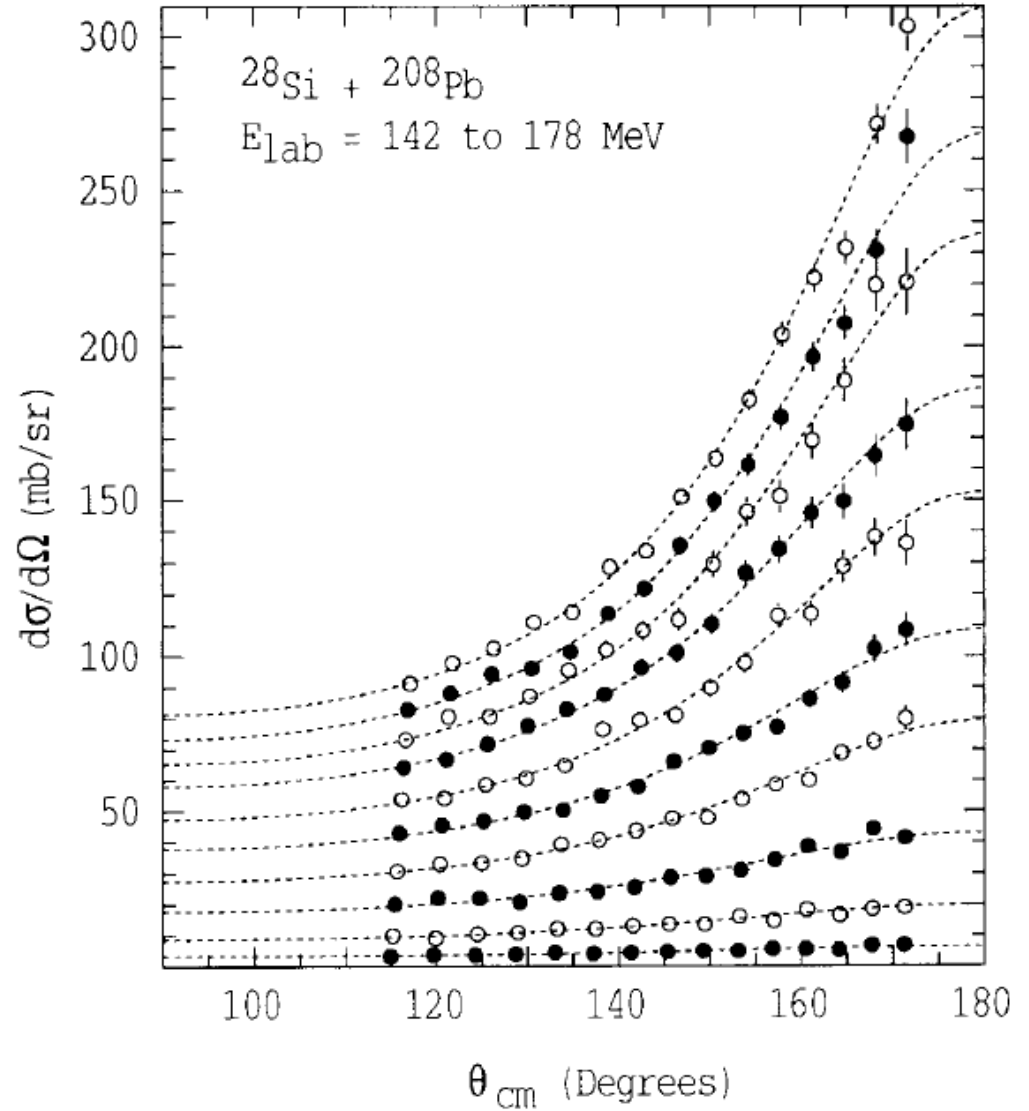
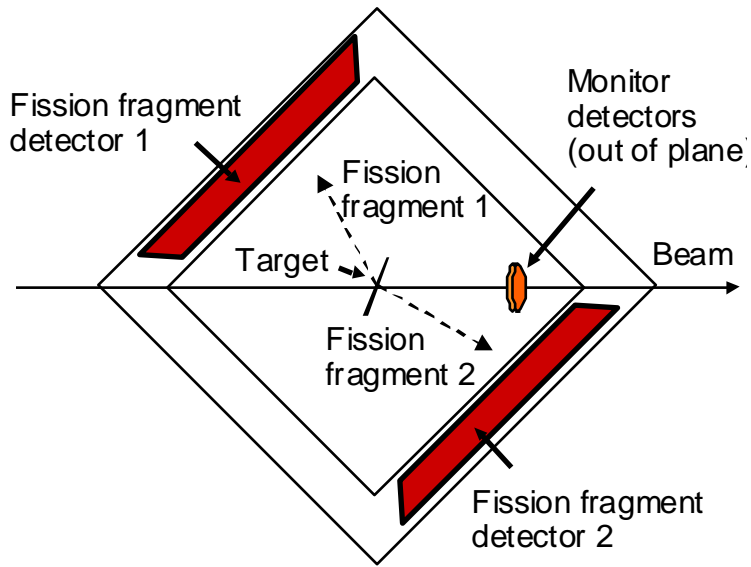
M. Dodríguez et al.,
NIM A614('10)119

Stopper for
elastics

detectors

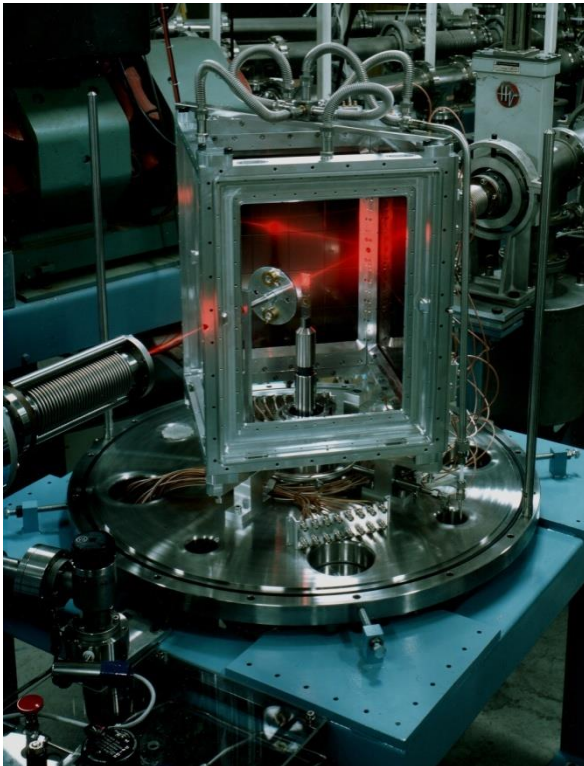


核分裂片の測定

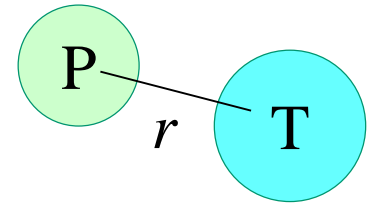


D.J. Hinde et al., NPA592('95)271

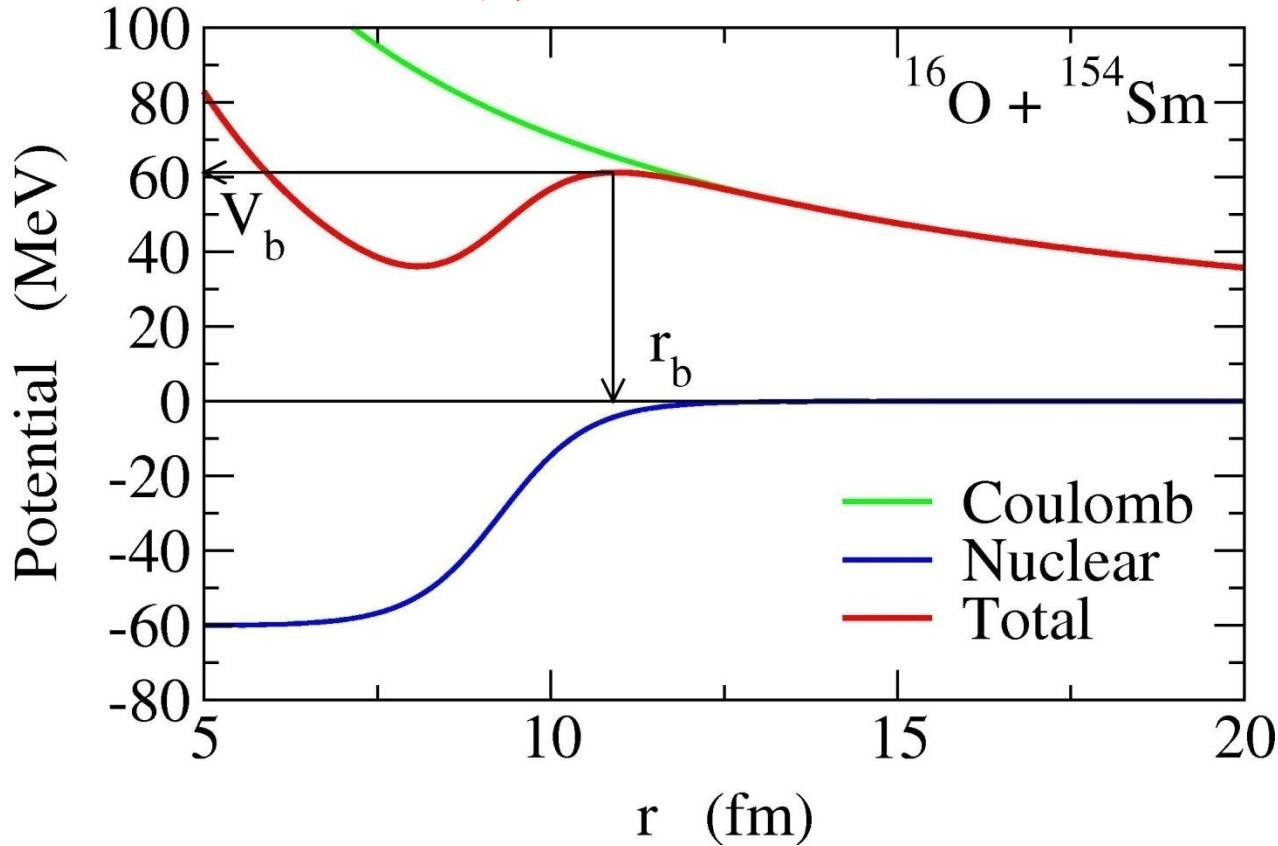
← ANUで使われている検出器



クーロン障壁



クーロン障壁



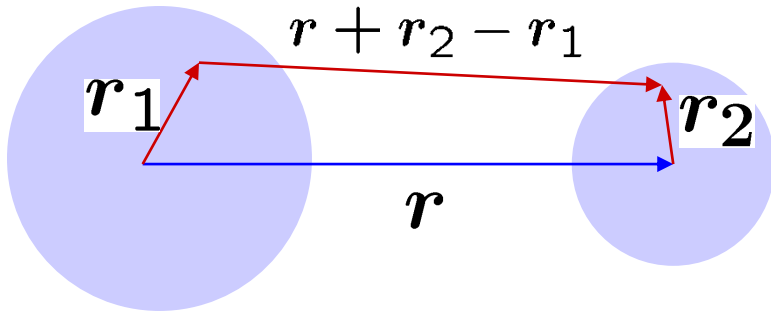
2つの力:

1. クーロン力
長距離斥力
2. 核力
短距離引力



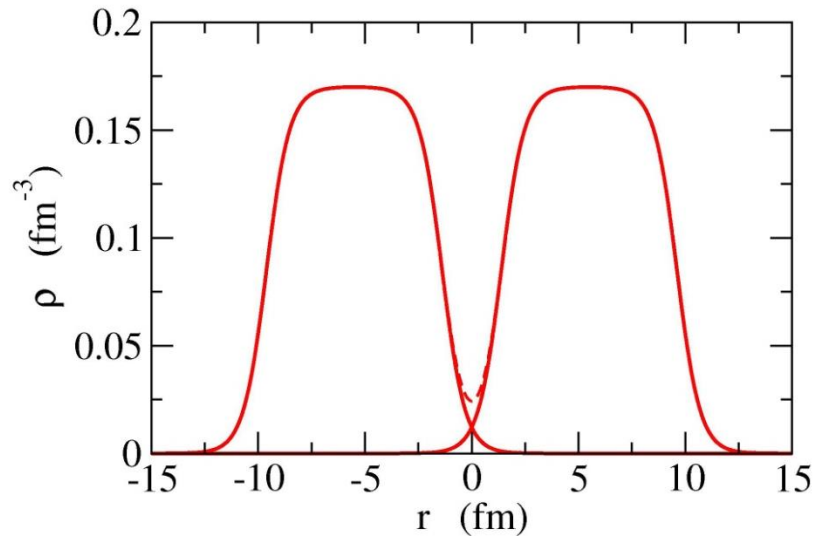
ポテンシャル障壁
(クーロン障壁)

• Double Folding Potential



$$V_{DF}(r) = \int dr_1 dr_2 \rho_1(r_1) \rho_2(r_2) \times v_{nn}(r + r_2 - r_1)$$

(微視的ポテンシャルの直接項に相当)



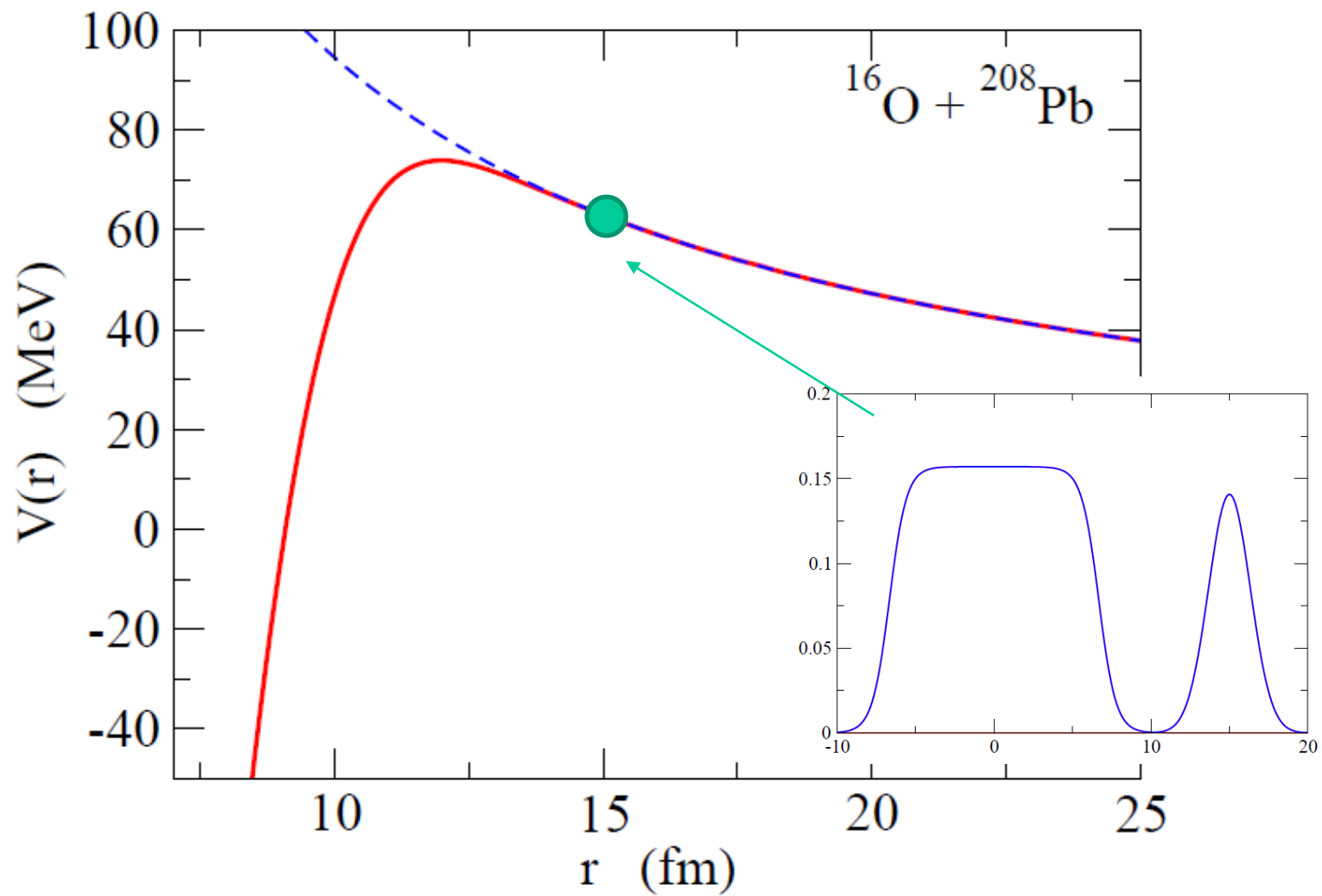
$$\rho(r) \sim \frac{\rho_0}{1 + \exp[(r - R_d)/a_d]}$$

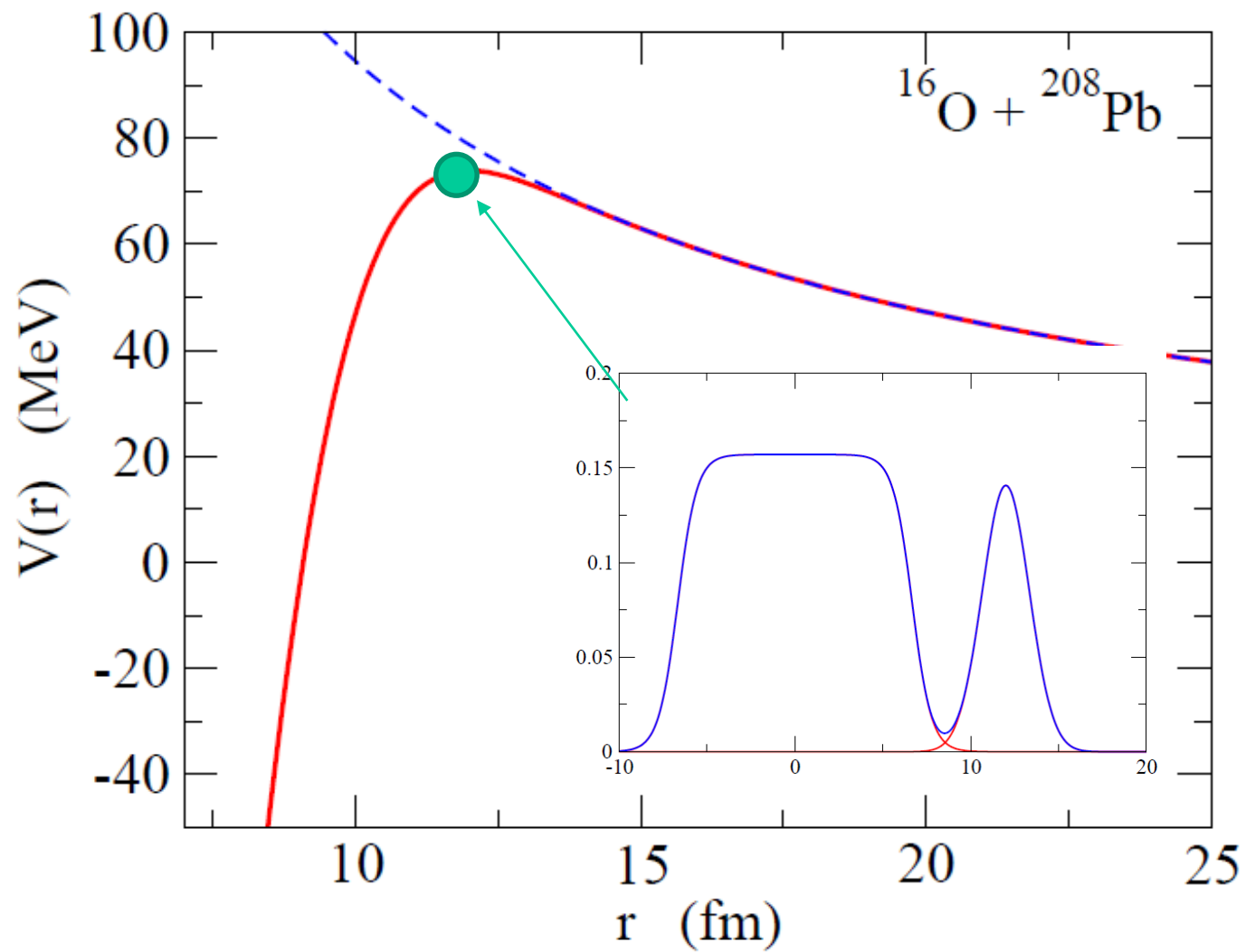
$$a_d \sim 0.54 \text{ (fm)}$$

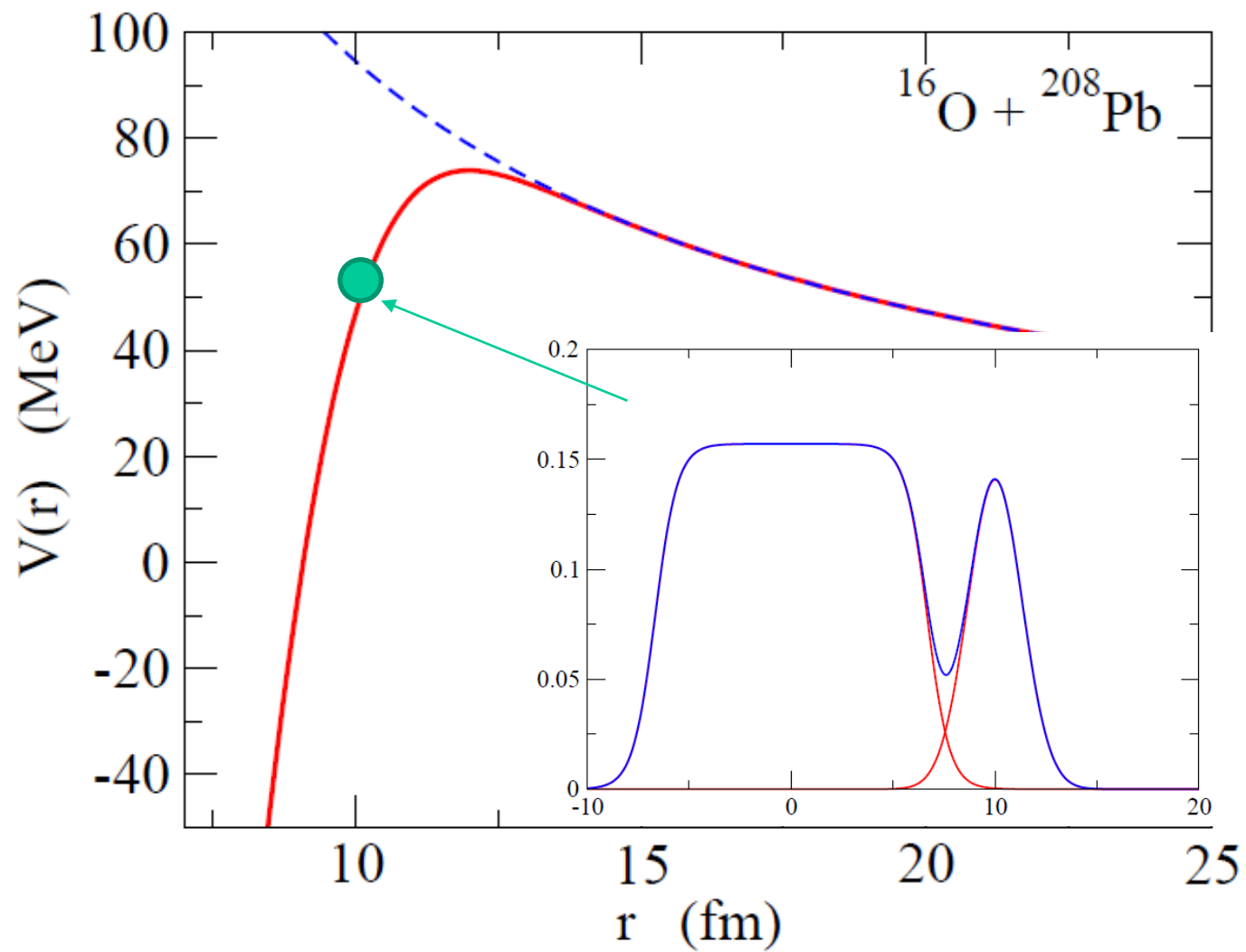
• Phenomenological potential

$$V_{WS}(r) = -\frac{V_0}{1 + \exp[(r - R_0)/a]}$$

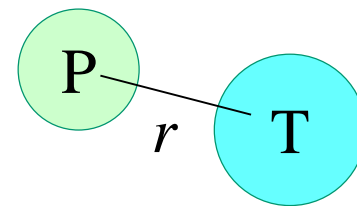
$$a \sim 0.63 \text{ (fm)}$$



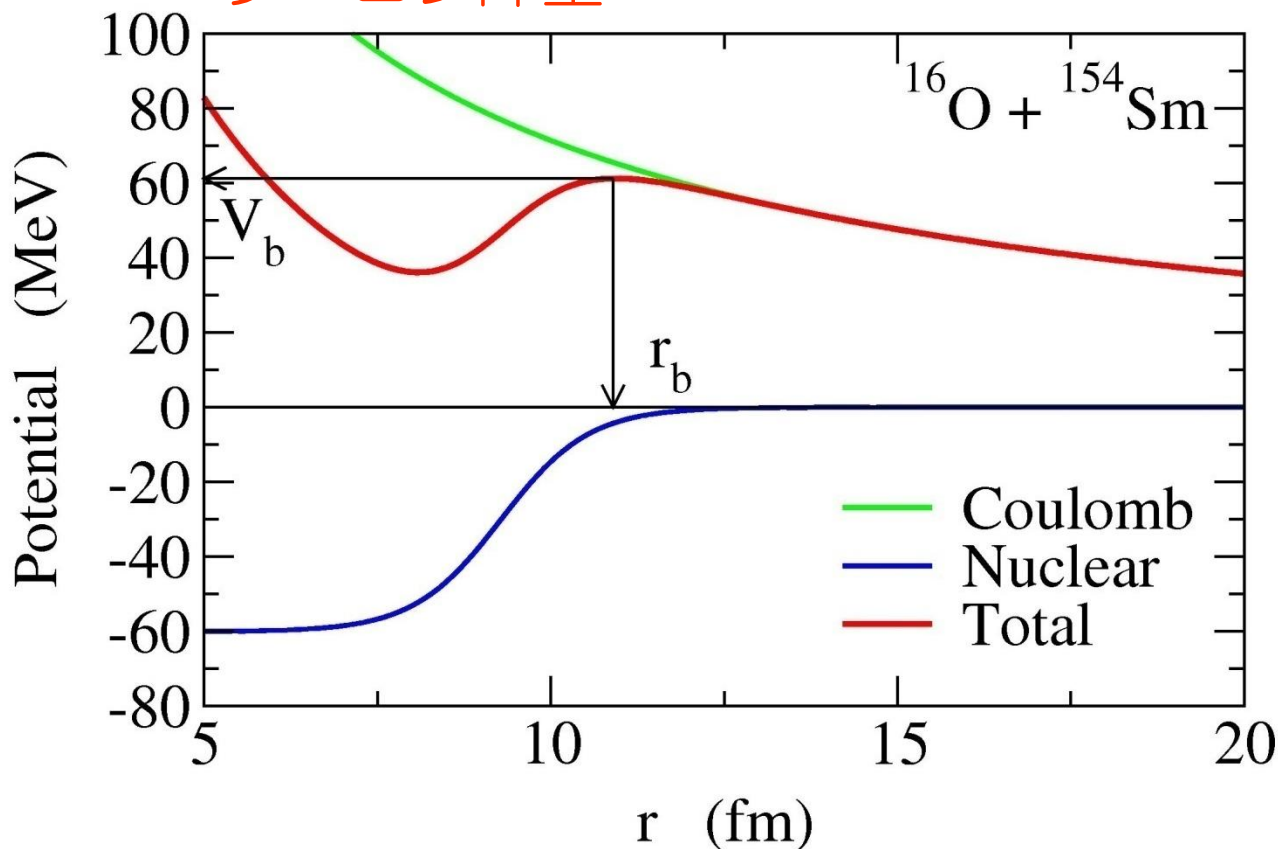




クーロン障壁



クーロン障壁



2つの力:

1. クーロン力
長距離斥力
2. 核力
短距離引力

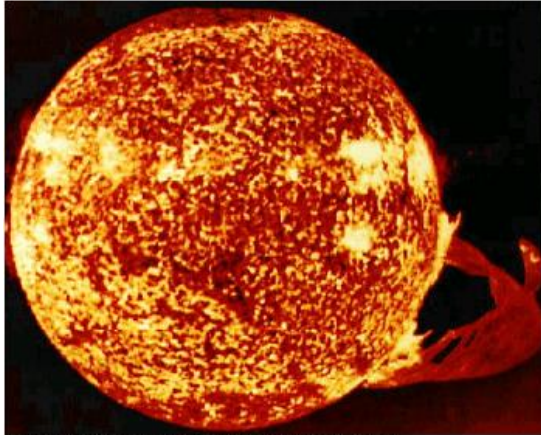


ポテンシャル障壁
(クーロン障壁)

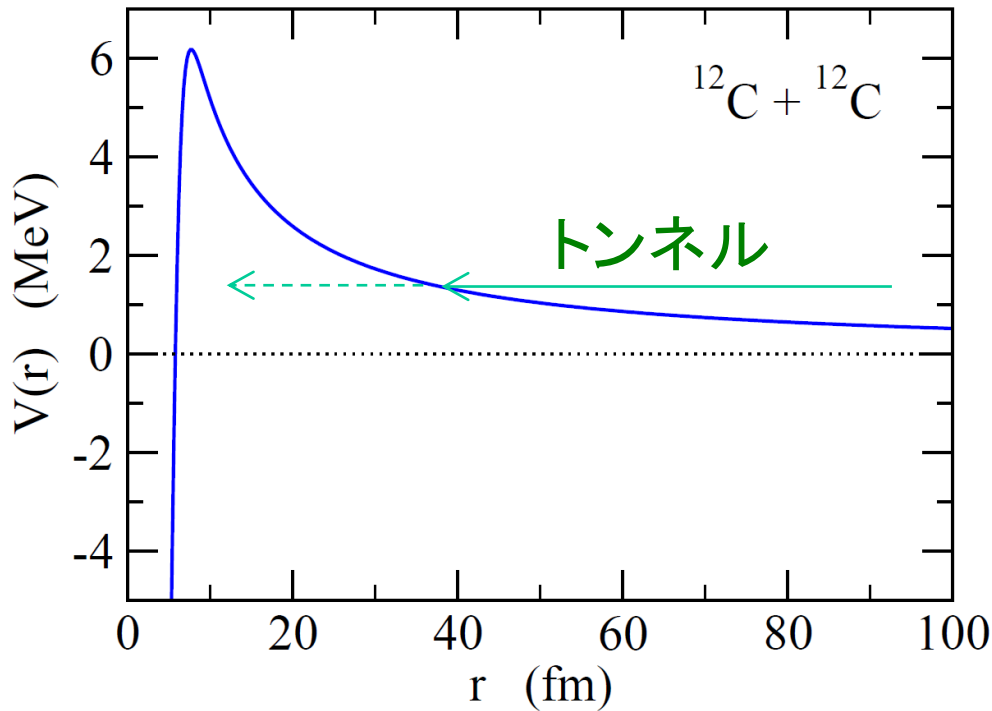
クーロン障壁近傍のエネルギーにおける核融合反応

→ 多粒子系の量子トンネル現象

nuclear fusion in stars



NASA, Skylab space station December 19, 1973, solar flare reaching 568 000 km off solar surface



$\sigma(E)$
LOGARITMIC
SCALE

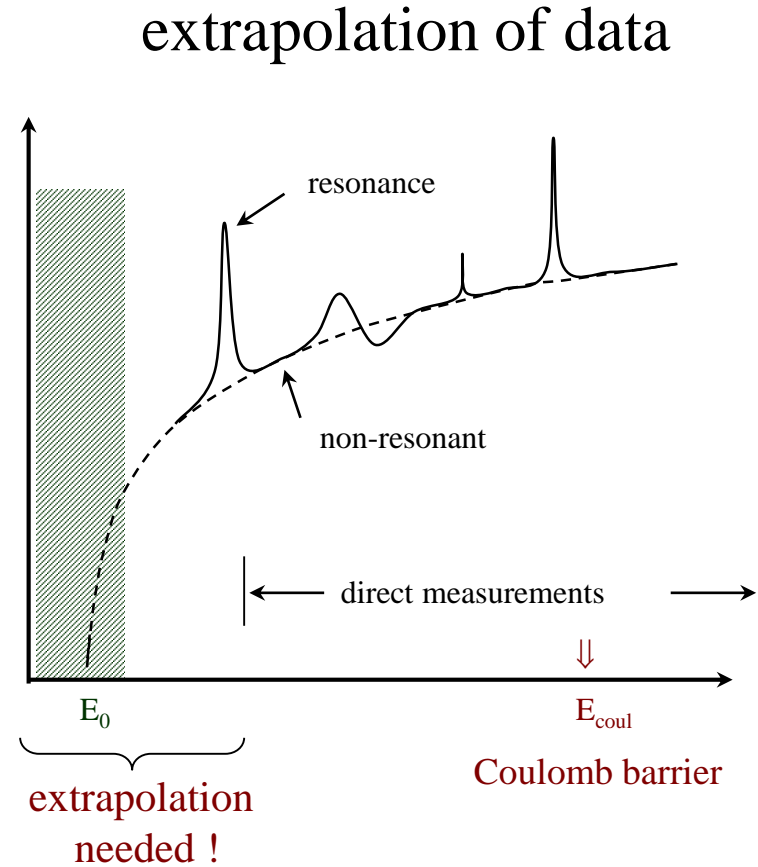
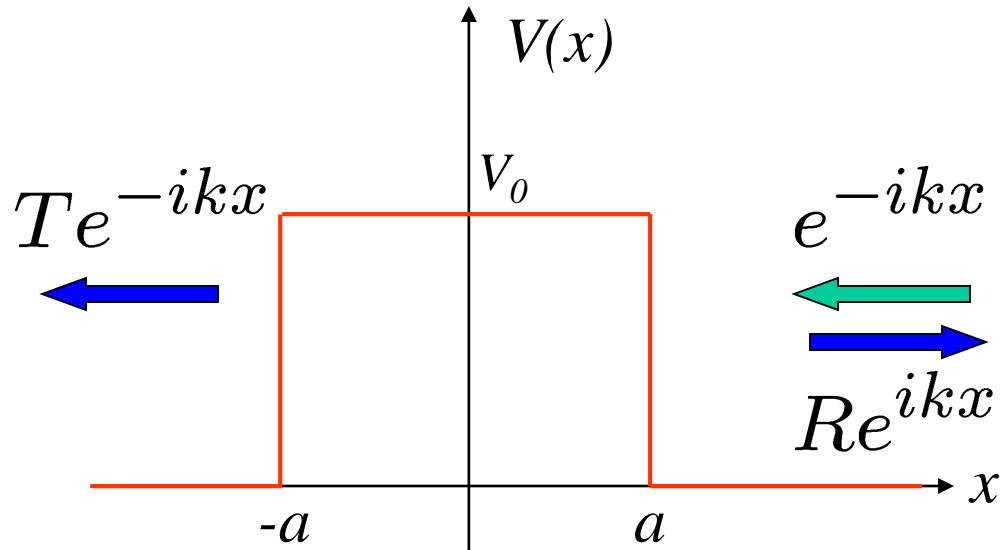


figure: M. Aliotta

Quantum Tunneling Phenomena

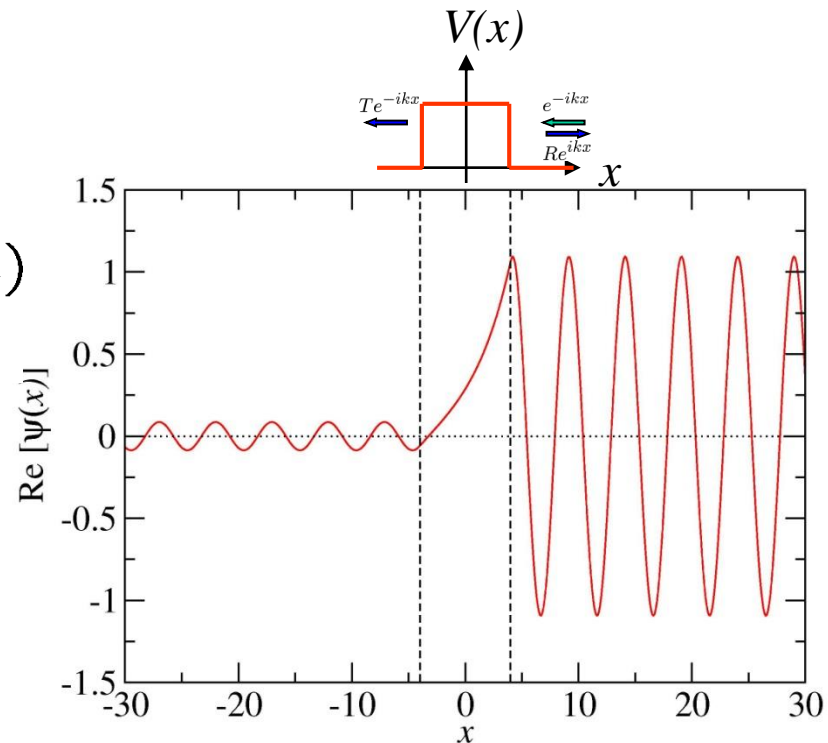


$$\begin{aligned} \psi(x) &= T e^{-ikx} & (x \leq -a) \\ &= A e^{-\kappa x} + B e^{\kappa x} & (-a < x < a) \\ &= e^{-ikx} + R e^{ikx} & (x \geq a) \end{aligned}$$

$$k = \sqrt{2mE/\hbar^2}$$

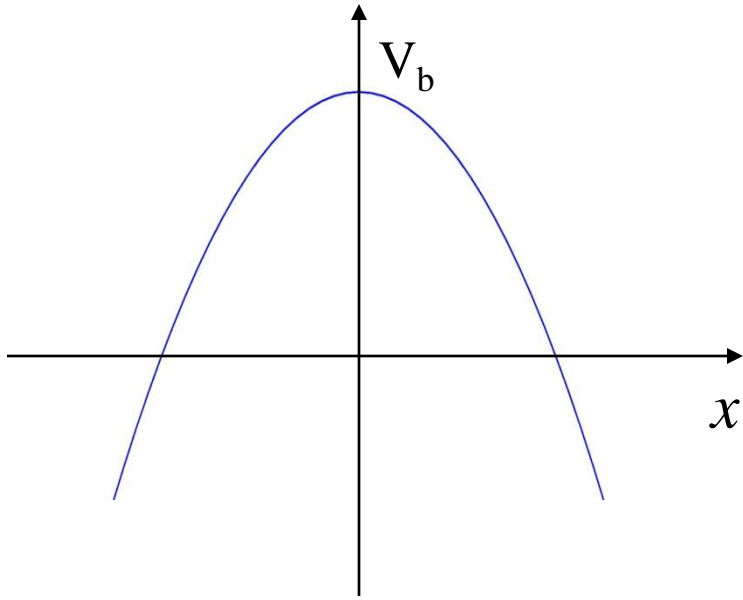
$$\kappa = \sqrt{2m(V_0 - E)/\hbar^2}$$

Tunnel probability: $P(E) = |T|^2$

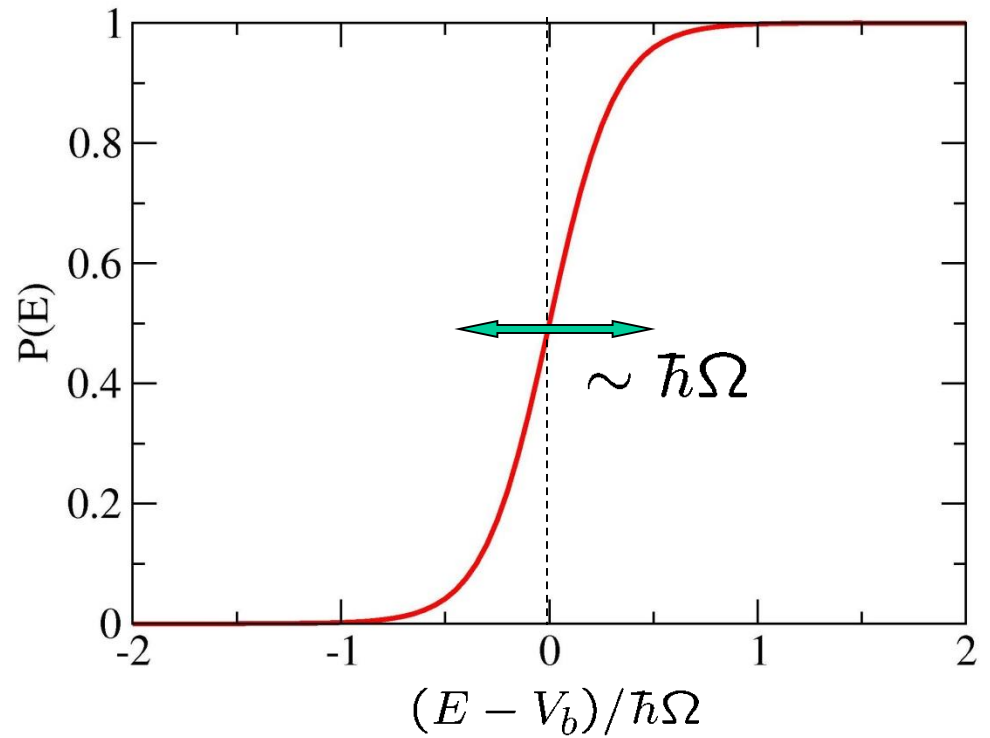


For a parabolic barrier.....

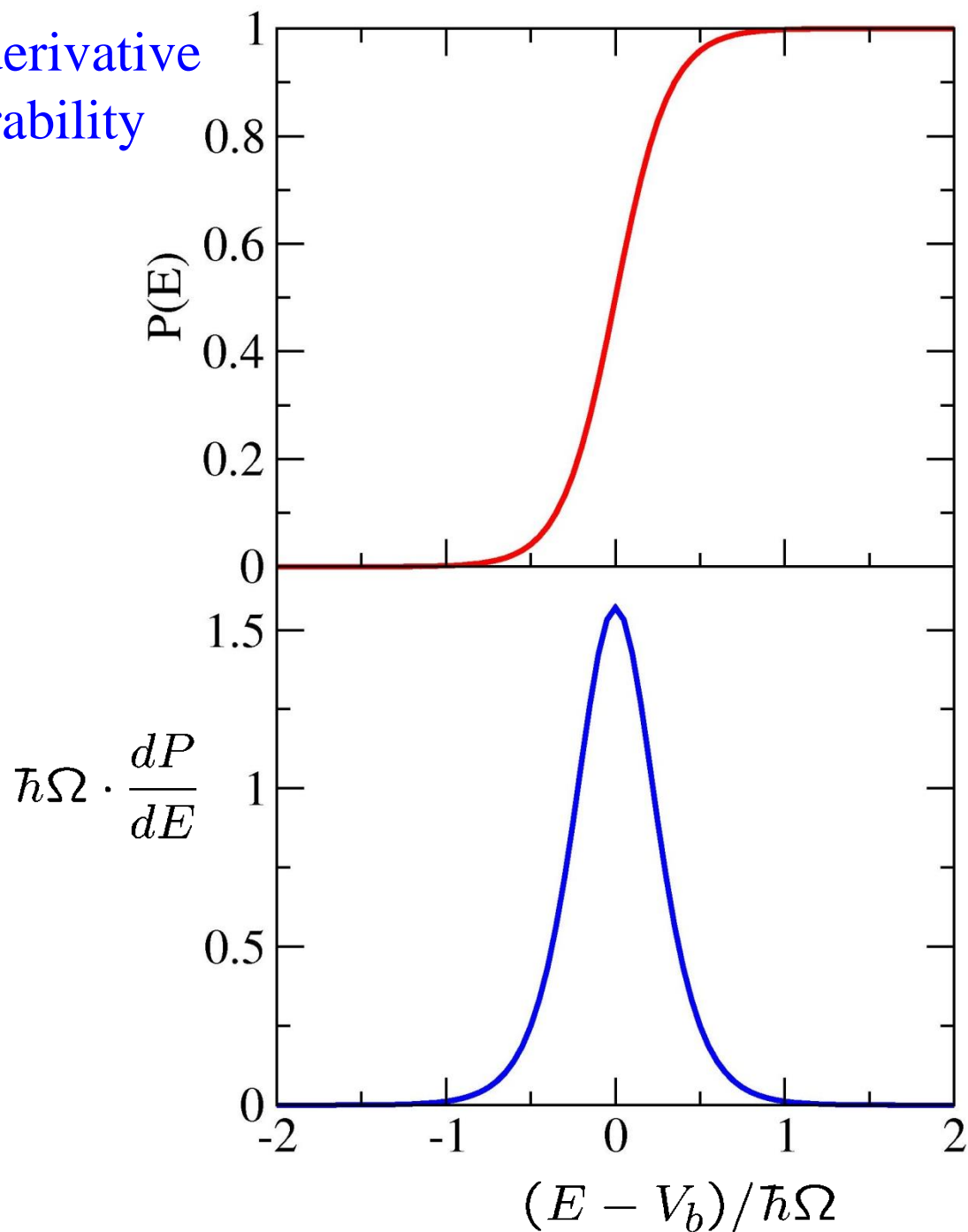
$$V(x) = V_b - \frac{1}{2}m\Omega^2 x^2$$



$$P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$



Energy derivative
of penetrability



(note) Classical limit

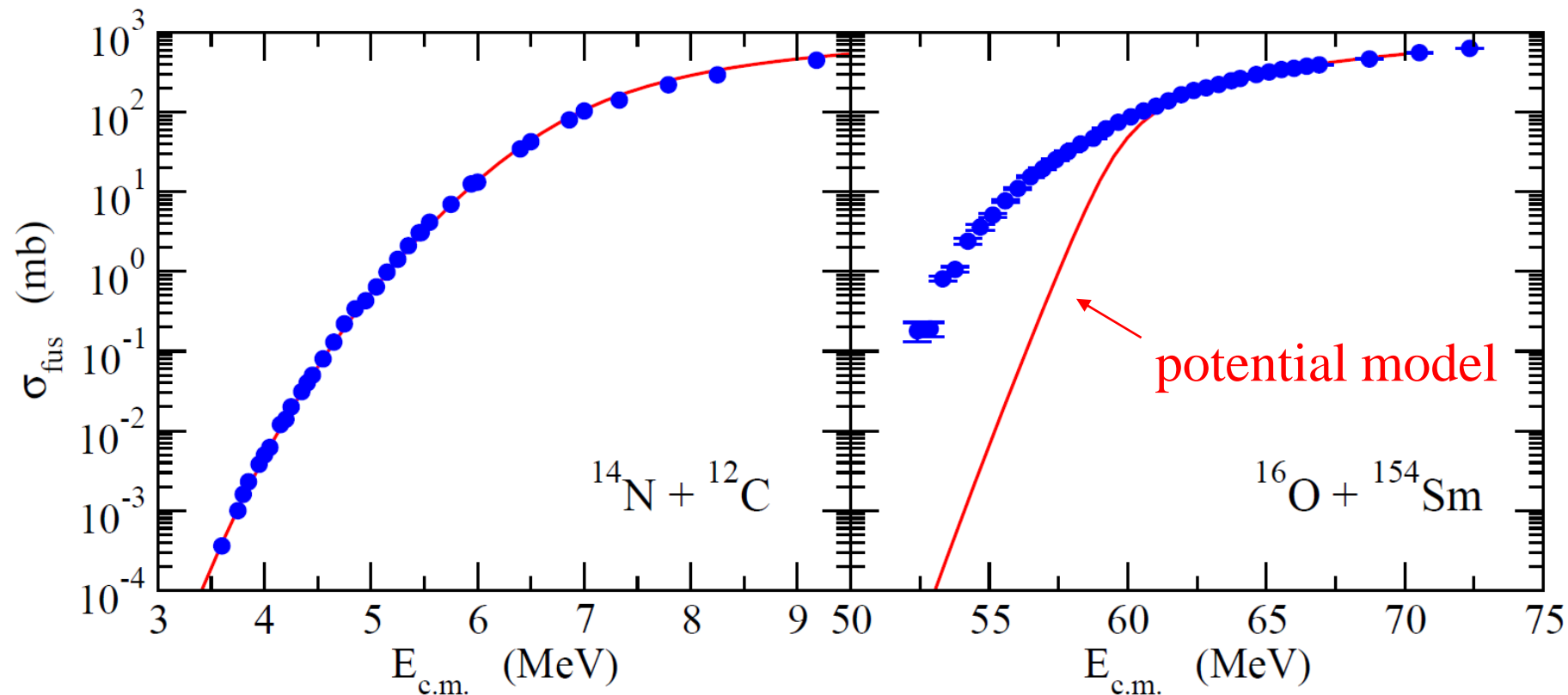
$$P(E) = \theta(E - V_b)$$

$$dP/dE = \delta(E - V_b)$$

核融合反応断面積の大きな増大

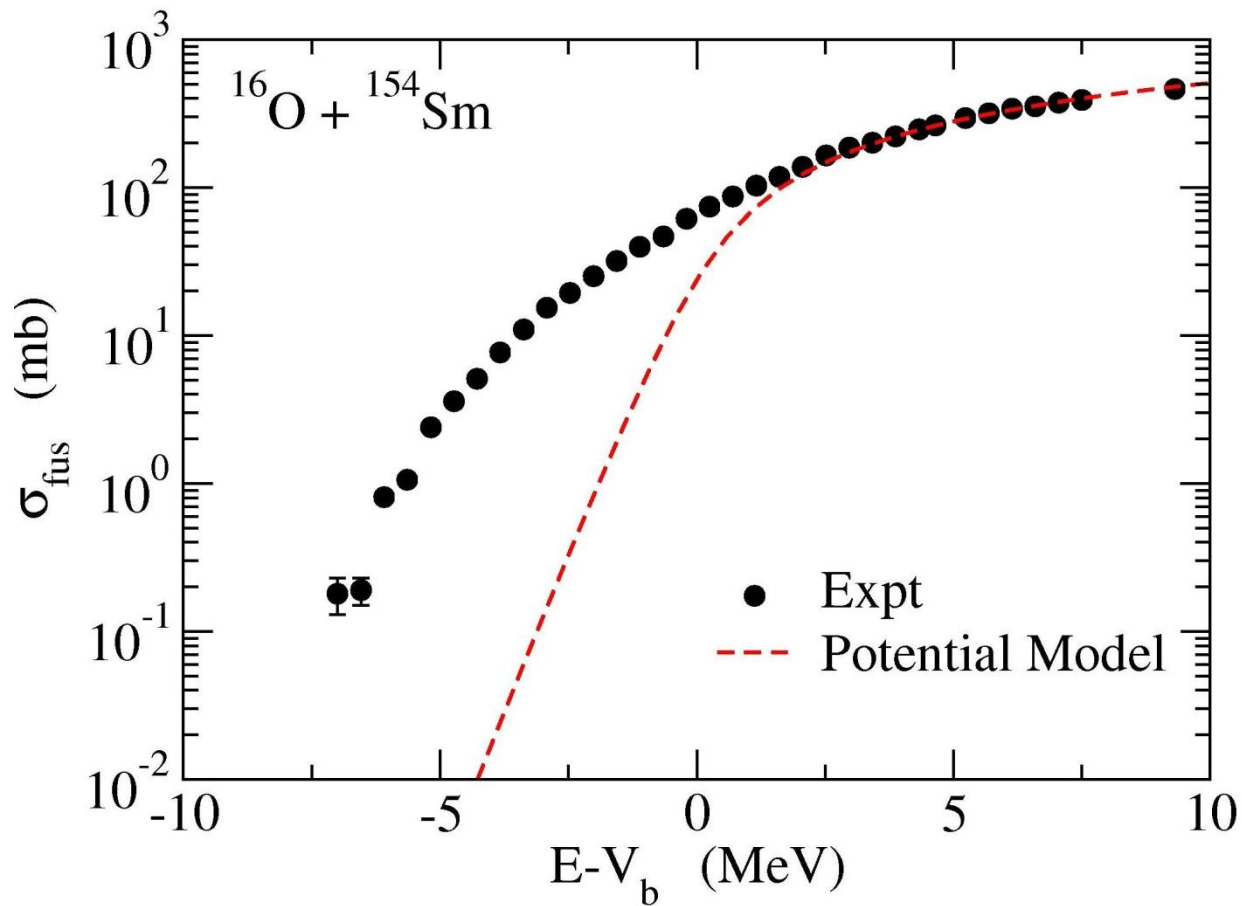
ポテンシャル模型: $V(r) +$ 吸収

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2)$$



cf. 初期の実験:

R.G. Stokstad et al., PRL41('78) 465

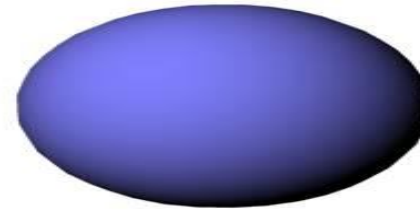


ポテンシャル模型:

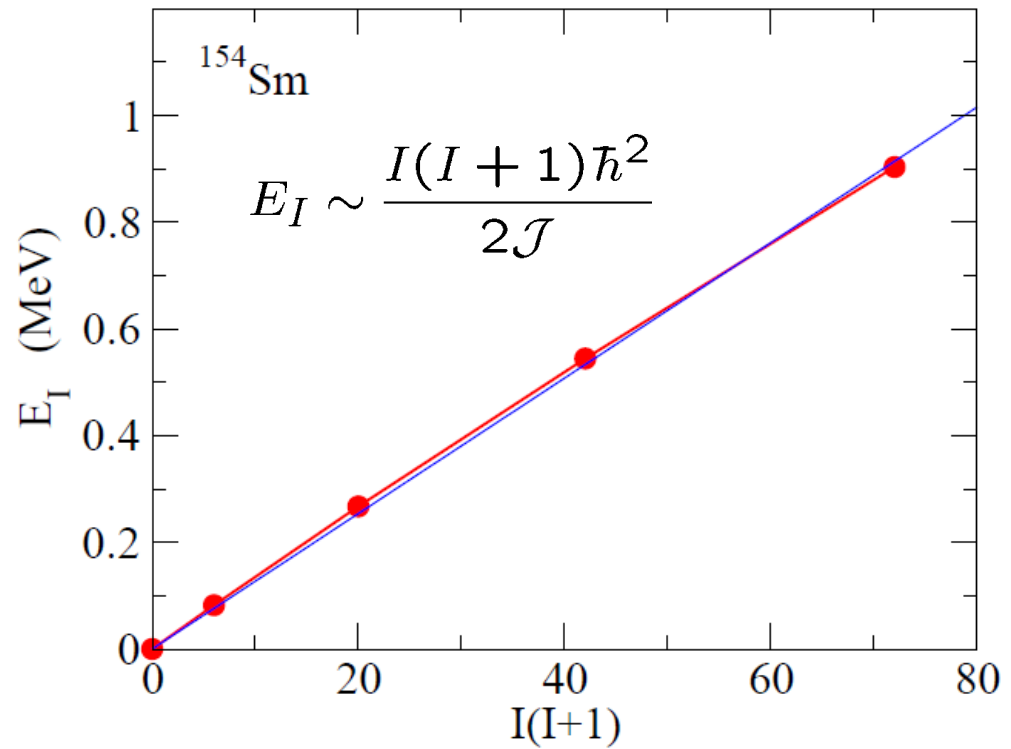
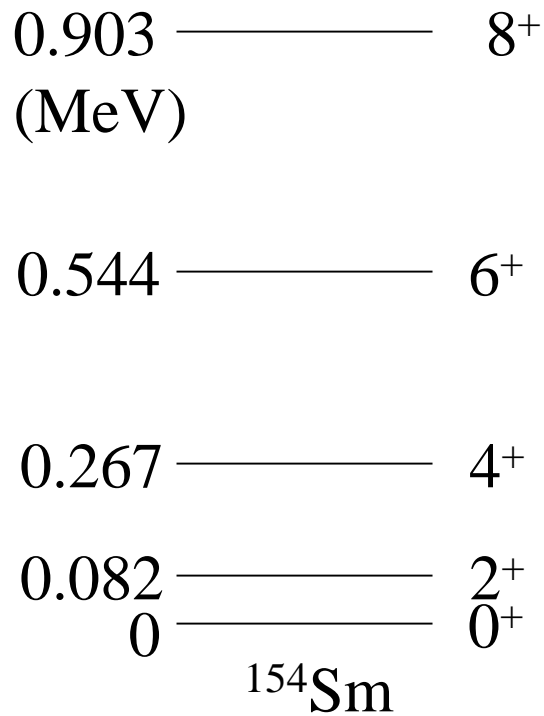
$E > V_b$ では大体データを再現

$E < V_b$ では核融合断面積を過小に評価

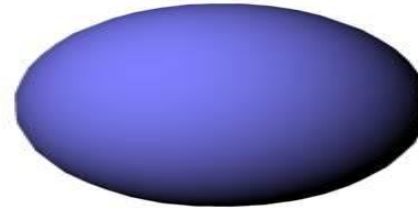
^{154}Sm : a typical deformed nucleus



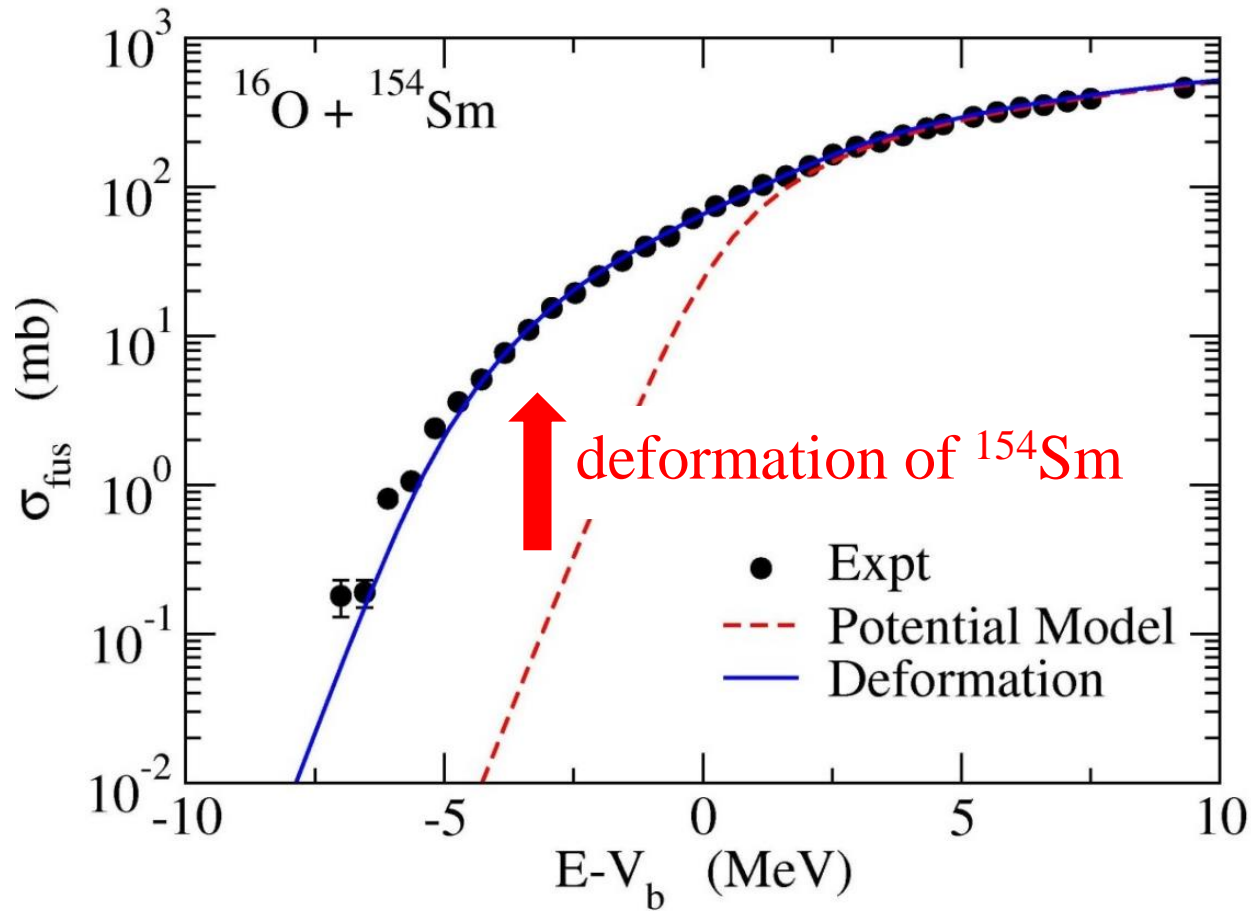
^{154}Sm



^{154}Sm : a typical deformed nucleus

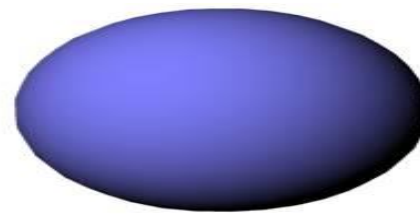


^{154}Sm

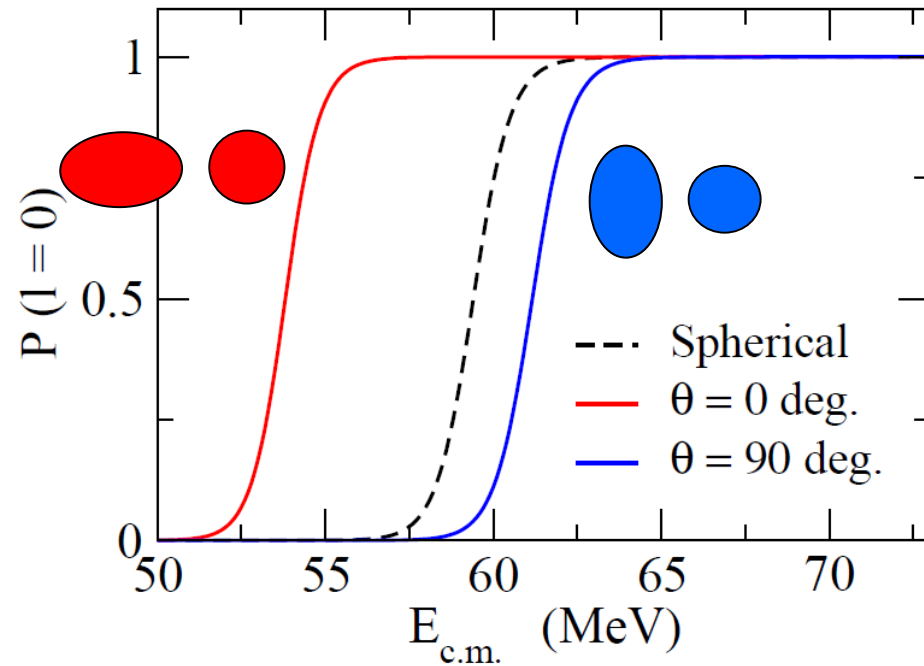
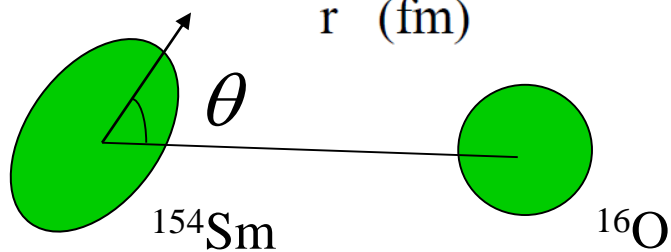
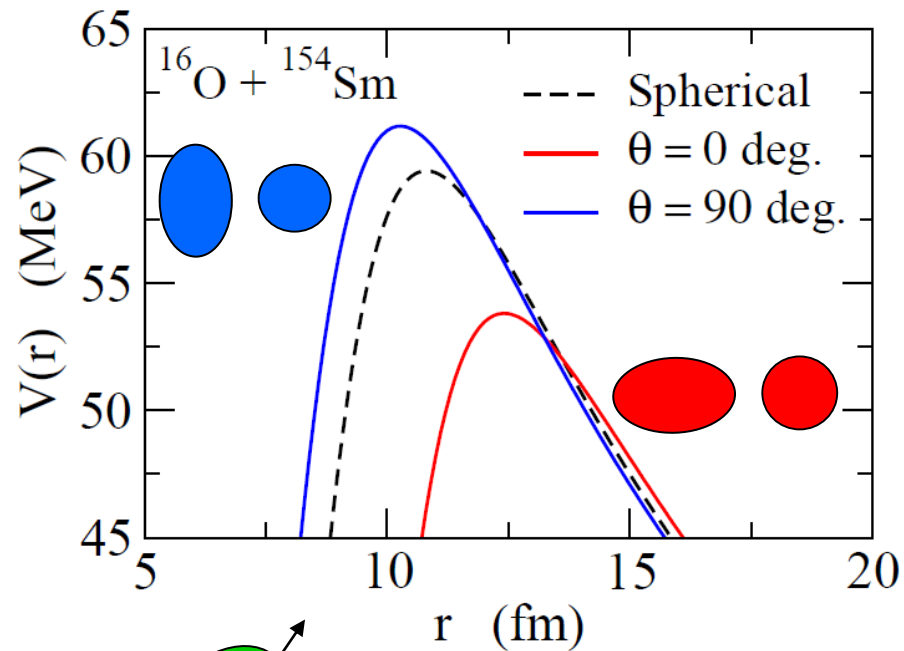


Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus

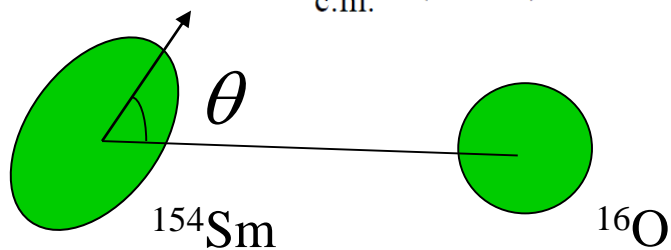
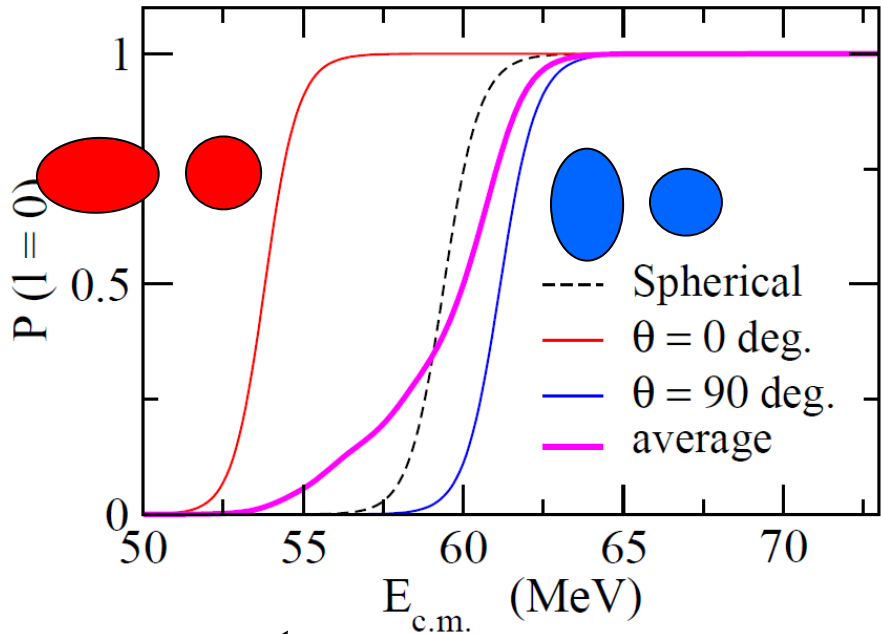


^{154}Sm

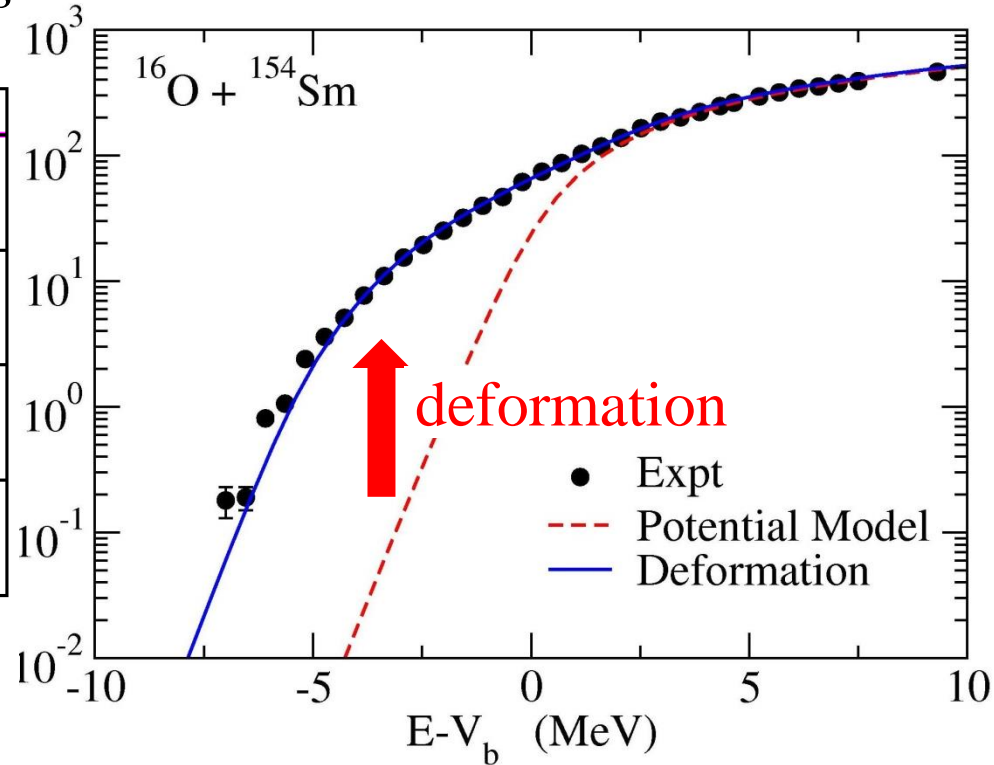


Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus



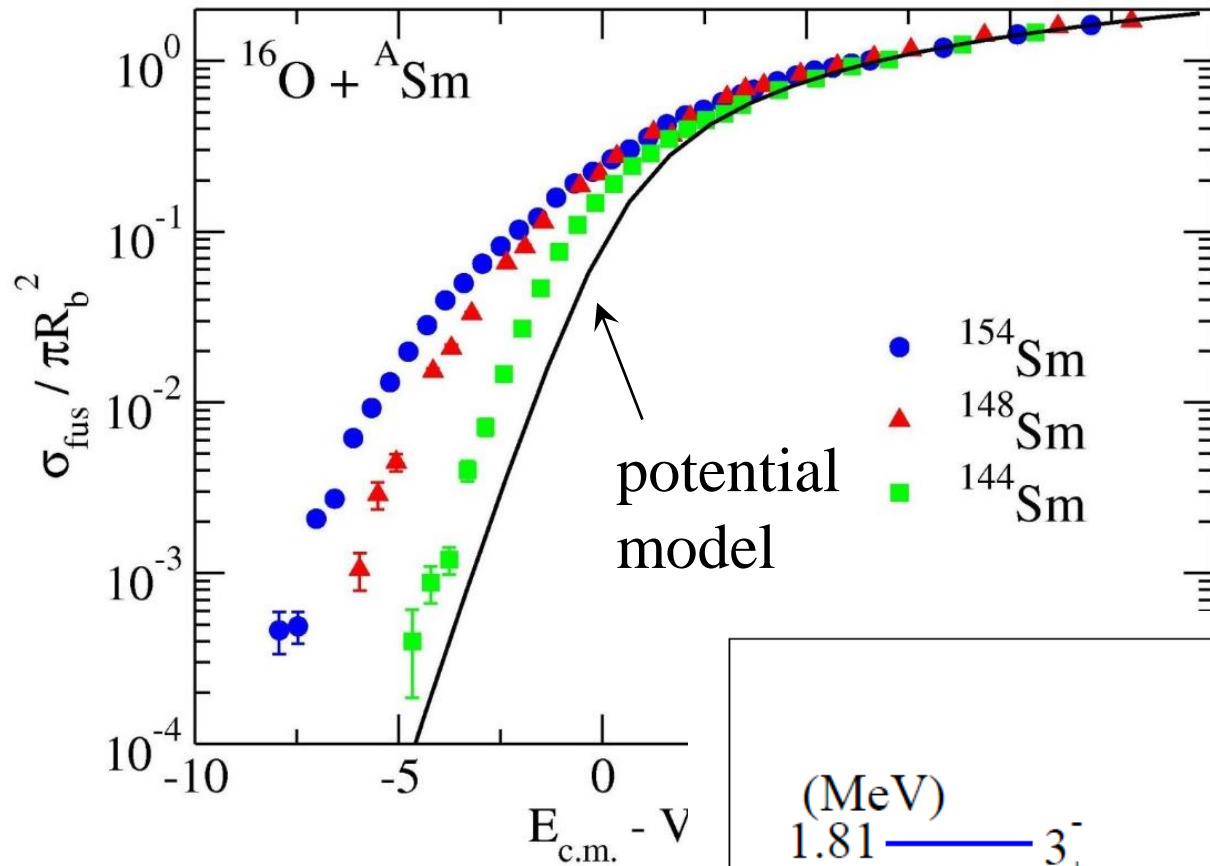
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$



Fusion: strong interplay between nuclear structure and reaction

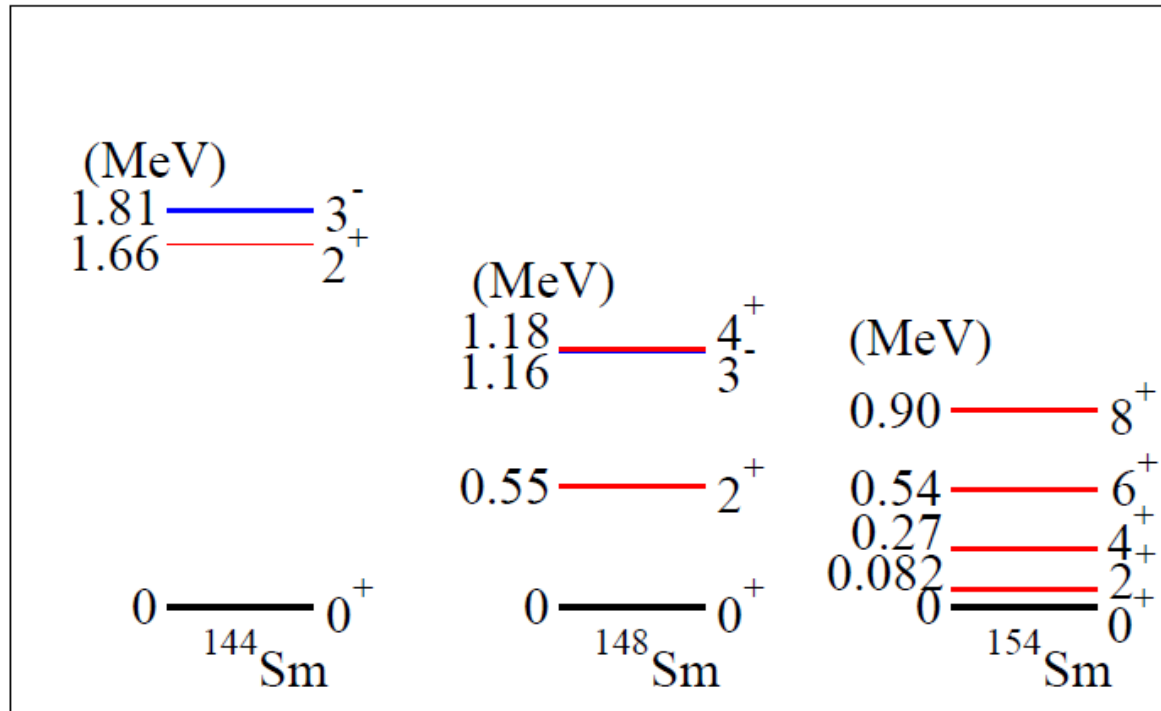
* Sub-barrier enhancement also for non-deformed targets:

couplings to low-lying collective excitations \rightarrow coupling assisted tunneling

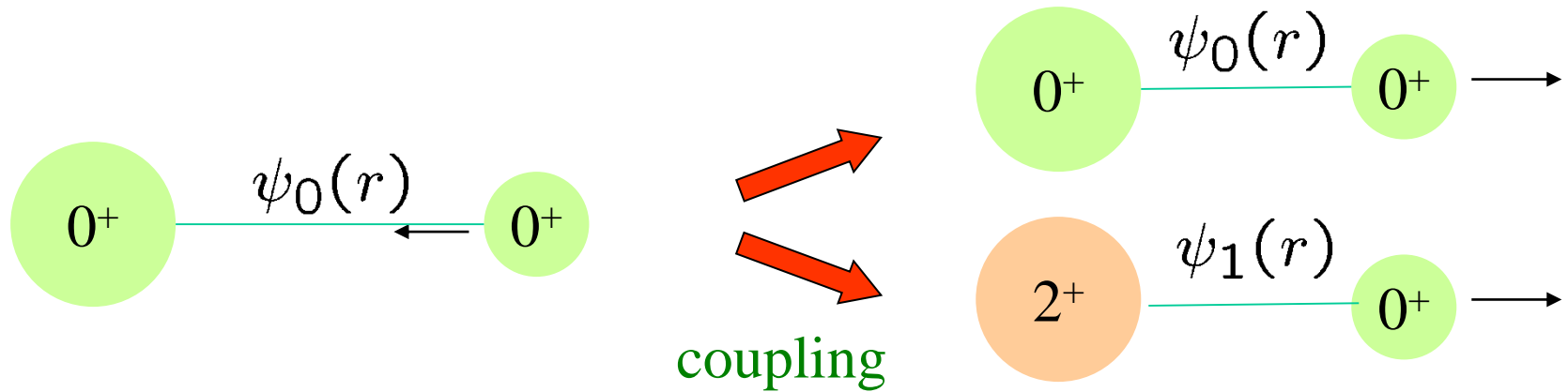


enhancement of fusion cross sections
: a general phenomenon

strong correlation with nuclear spectrum
→ coupling assisted tunneling



Coupled-channels method: a quantal scattering theory with excitations



$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + \overleftarrow{V}(r) - \overleftarrow{E} \right] \overrightarrow{\psi}(r) = 0$$

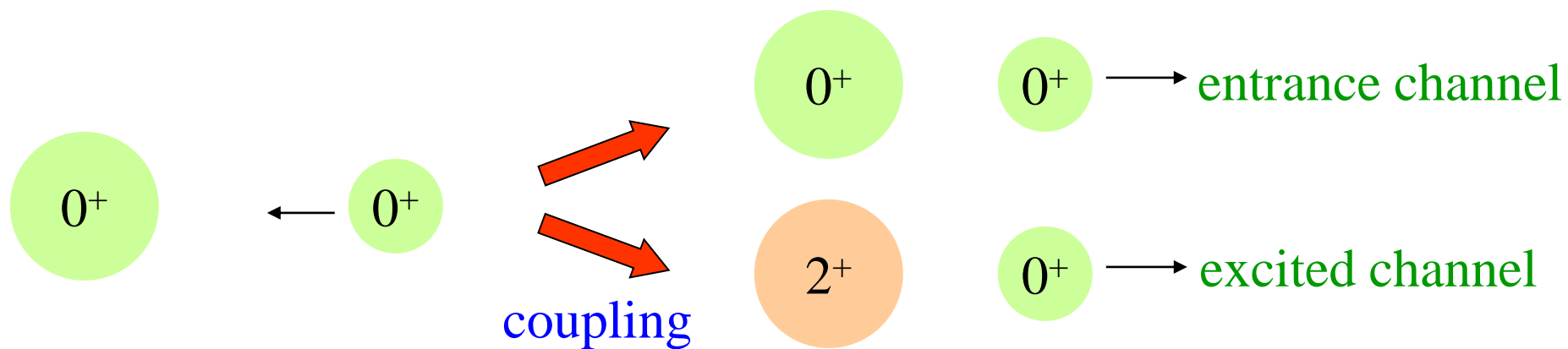
if written down more explicitly:

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

↑
excitation energy

↑
excitation operator

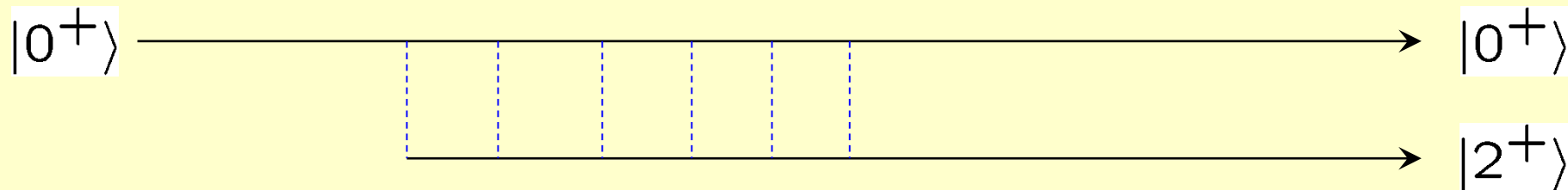
Coupled-channels method: a quantal scattering theory with excitations



$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

excitation energy

excitation operator



full order treatment of excitation/de-excitation dynamics during reaction

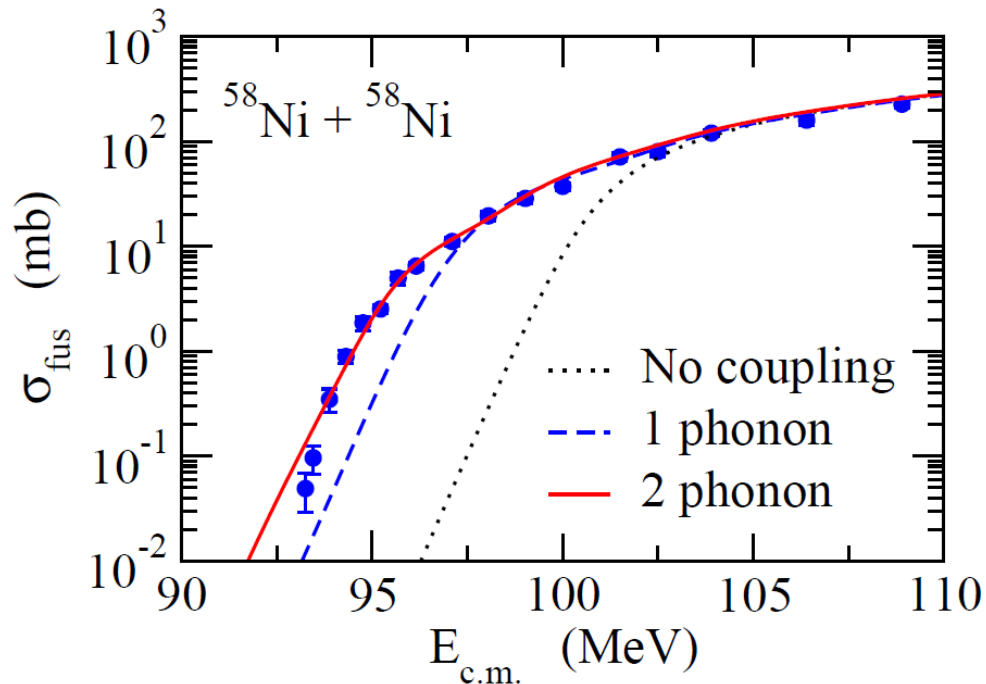
Inputs for C.C. calculations

i) Inter-nuclear potential

a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom

in most of cases, (macroscopic) collective model
(rigid rotor / harmonic oscillator)



simple harmonic oscillator

