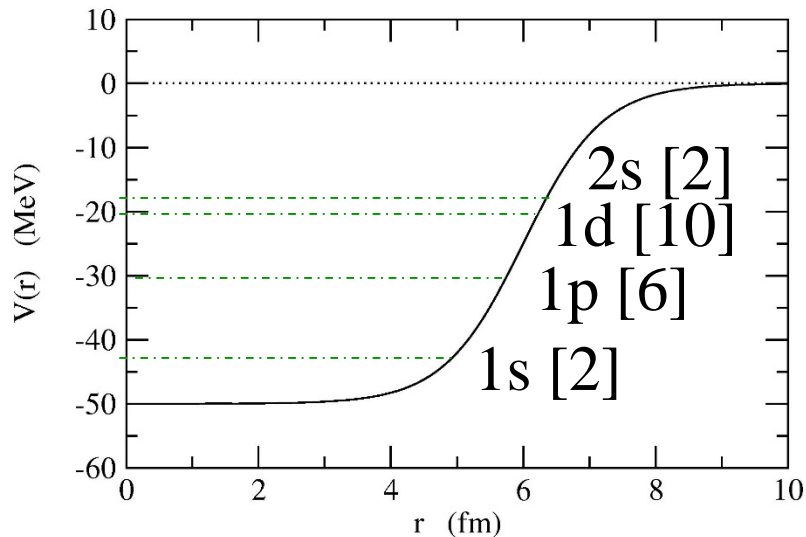


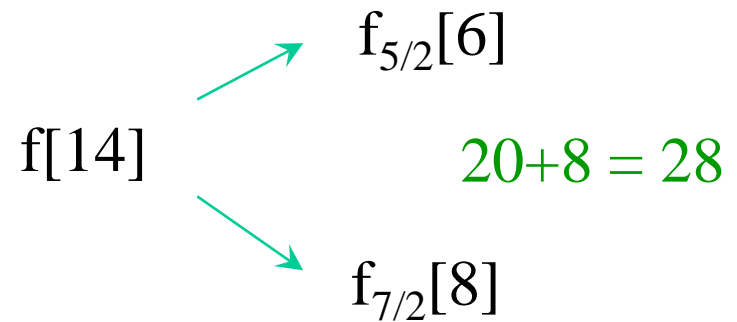
# Mean-field approximation and deformation

Extra binding for  $N$  or  $Z = 2, 8, 20, 28, 50, 82, 126$  (magic numbers)

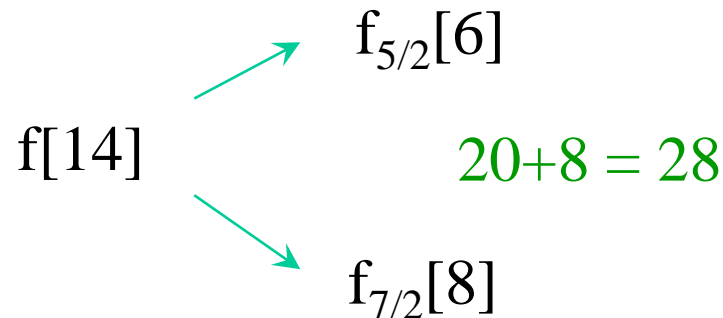
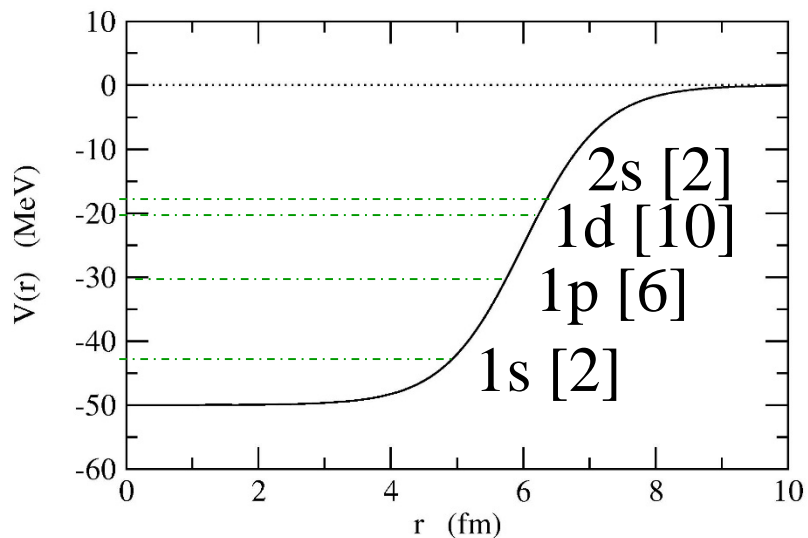
An interpretation: independent particle motion in a potential well



+ spin-orbit interaction



# Mean-field approximation and deformation



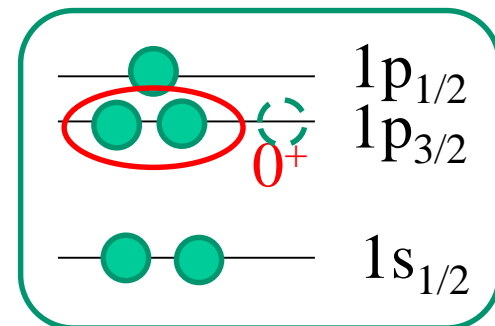
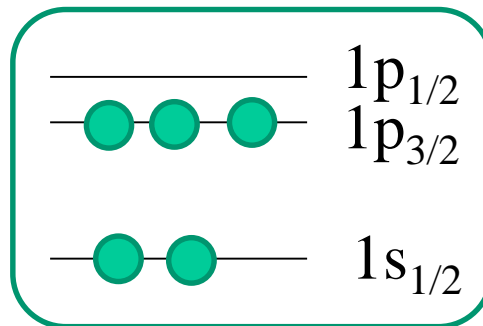
example:

MeV

2.12 —————  $1/2^-$

0 —————  $3/2^-$

$^{11}_5\text{B}_6$

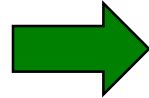


同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型(球形ポテンシャルの準位)で考えた場合:

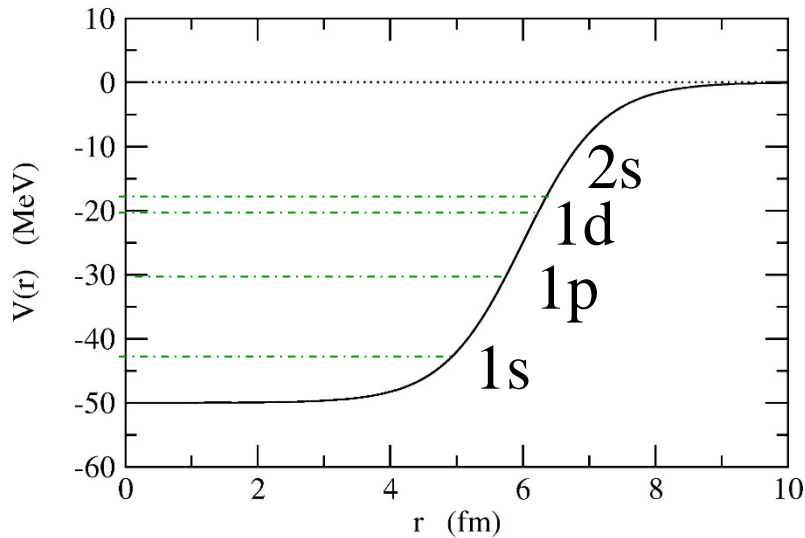
—●—  $1p_{1/2}$  [2]

●●●●  $1p_{3/2}$  [4]



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

—●—●—  $1s_{1/2}$  [2]

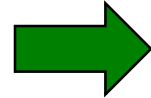


同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型 (球形ポテンシャルの準位) で考えた場合:

—●—  $1p_{1/2}$  [2]

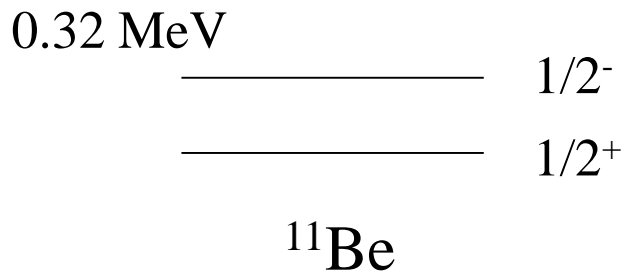
—●●●●—  $1p_{3/2}$  [4]



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

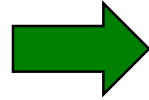
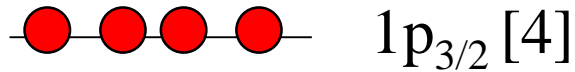
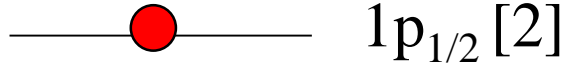
—●●—  $1s_{1/2}$  [2]

実際の  $^{11}\text{Be}$  の準位を見てみると:



同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型 (球形ポテンシャルの準位) で考えた場合:



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

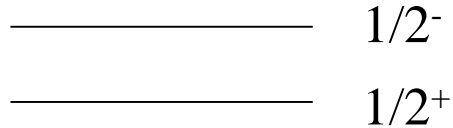


かなり無理

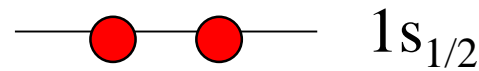
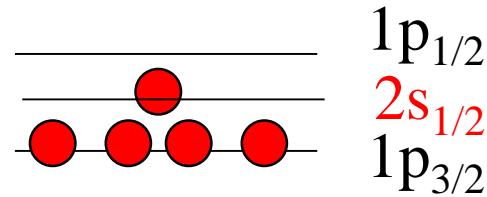
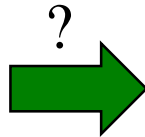


実際の  $^{11}\text{Be}$  の準位を見てみると:

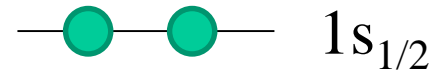
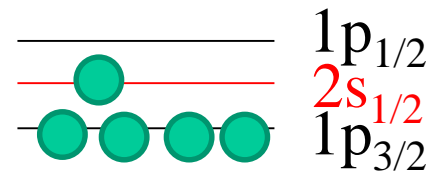
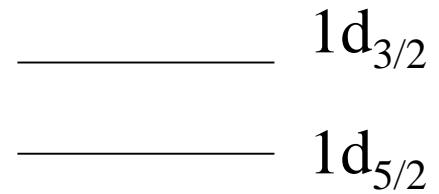
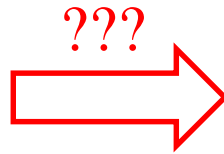
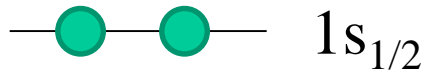
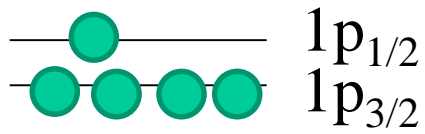
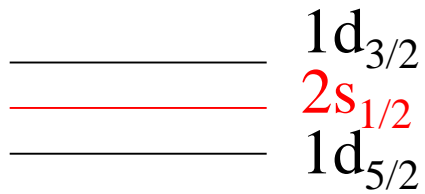
0.32 MeV



$^{11}\text{Be}$

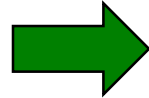
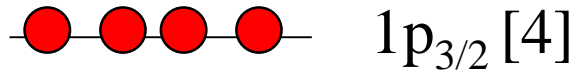
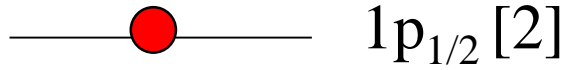


“parity inversion”

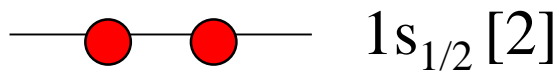


同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型 (球形ポテンシャルの準位) で考えた場合:



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

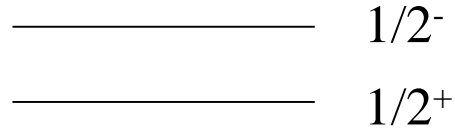


かなり無理

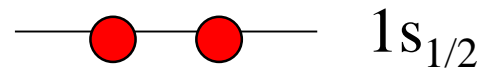
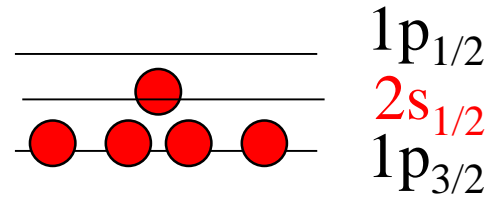
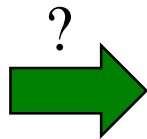


実際の  $^{11}\text{Be}$  の準位を見てみると:

0.32 MeV



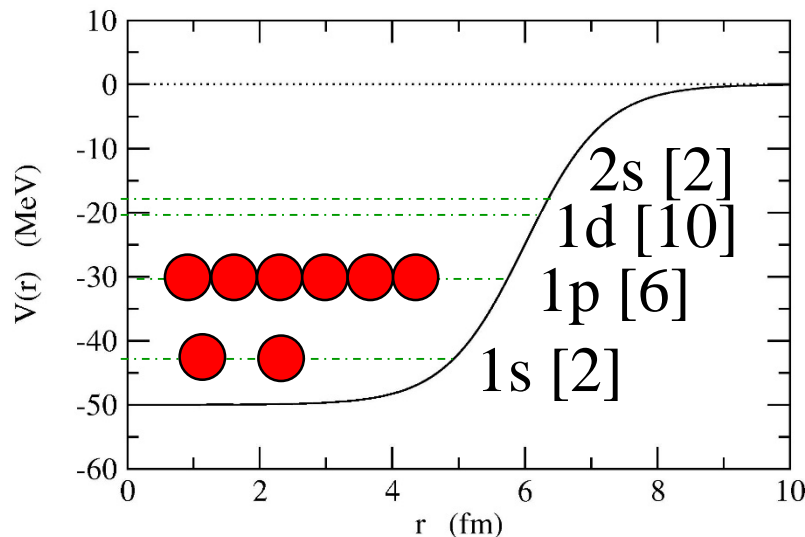
$^{11}\text{Be}$



球形ポテンシャルに無理があるなら、変形させてみる?

# Mean-field approximation and deformation

## Mean-field approximation



$$H \sim \sum_i \left( -\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{MF}}(\mathbf{r}_i) \right)$$

## Slater determinant

$$\begin{aligned} \Psi_{\text{MF}}(1, 2, \dots, A) \\ = \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)] \end{aligned}$$

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{MF}}(\mathbf{r}) \right) \psi_k(\mathbf{r}) = \epsilon_k \psi_k(\mathbf{r})$$

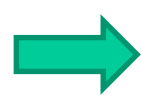
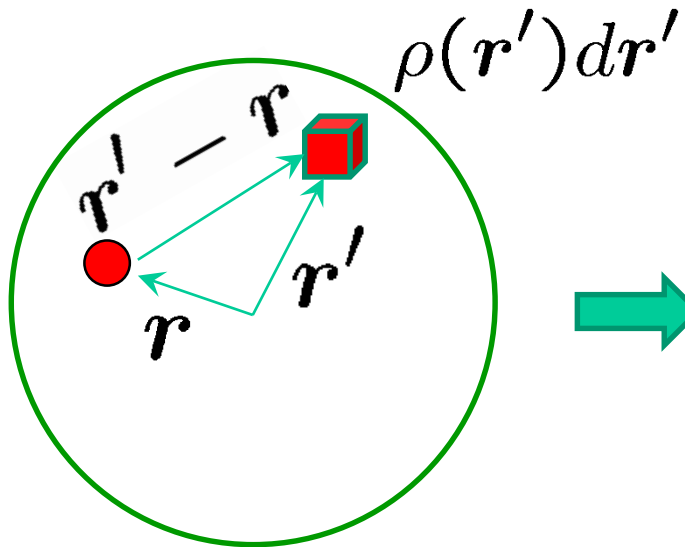
the original many-body  $H$ :

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} \sum_{i,j}^A v(\mathbf{r}_i, \mathbf{r}_j)$$

$$= \sum_{i=1}^A \left( -\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{MF}}(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i,j}^A v(\mathbf{r}_i, \mathbf{r}_j) - \sum_i V_{\text{MF}}(\mathbf{r}_i)$$



interaction for a nucleon inside a nucleus:



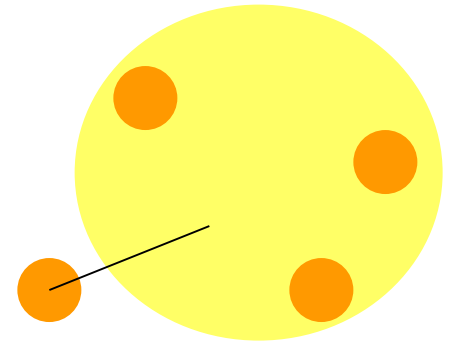
$$v(\mathbf{r}' - \mathbf{r}) \cdot \rho(\mathbf{r}') d\mathbf{r}'$$

the number of nucleon  
at  $\mathbf{r}'$

naively speaking,

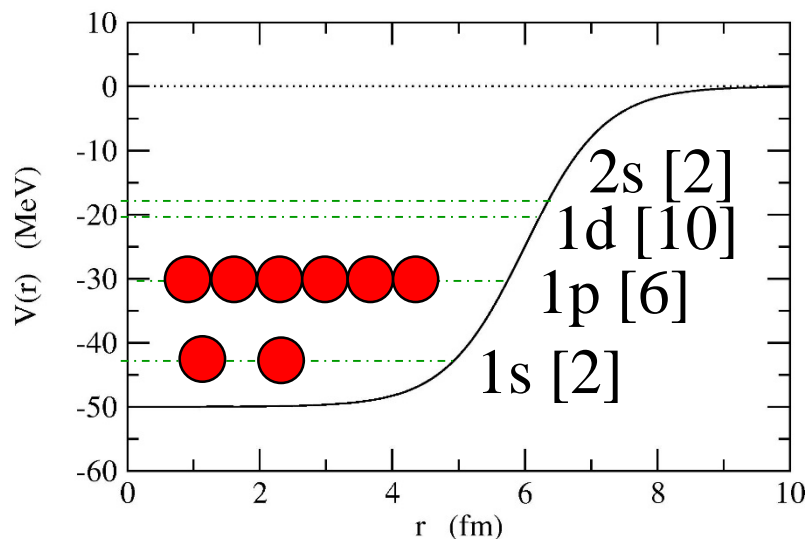
$$V(\mathbf{r}) \sim \int v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}'$$

平均場



# Mean-field approximation and deformation

## Mean-field approximation



$$H \sim \sum_i \left( -\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{MF}}(\mathbf{r}_i) \right)$$

## Slater determinant

$$\begin{aligned} \Psi_{\text{MF}}(1, 2, \dots, A) \\ = \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)] \end{aligned}$$

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{MF}}(\mathbf{r}) \right) \psi_k(\mathbf{r}) = \epsilon_k \psi_k(\mathbf{r})$$

$$H = \sum_{i=1}^A \left( -\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{MF}}(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i,j} v(\mathbf{r}_i, \mathbf{r}_j) - \sum_i V_{\text{MF}}(\mathbf{r}_i)$$

➔  $\Psi_{\text{MF}}$  : does not necessarily possess the symmetries that  $H$  has.

“Symmetry-broken solution”

“Spontaneous Symmetry Broken”

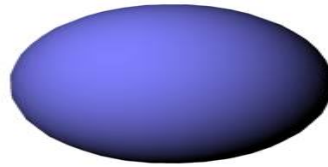
$\Psi_{\text{MF}}$  : does not necessarily possess the symmetries that  $H$  has.

## Typical Examples

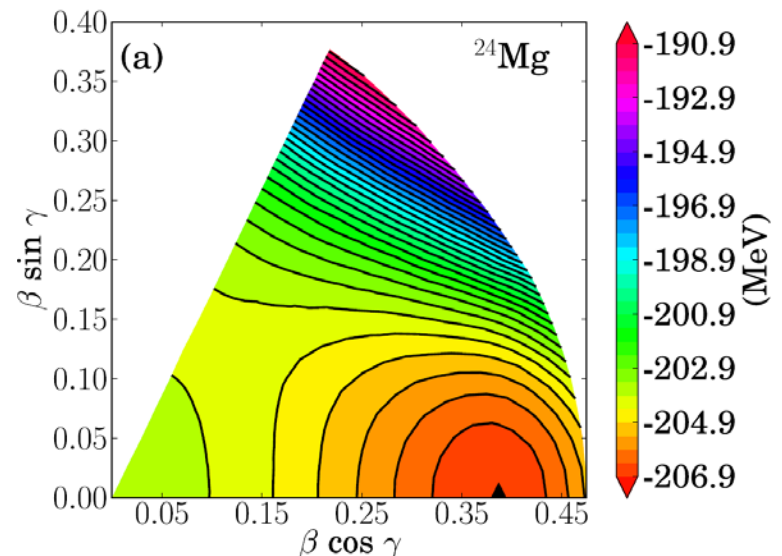
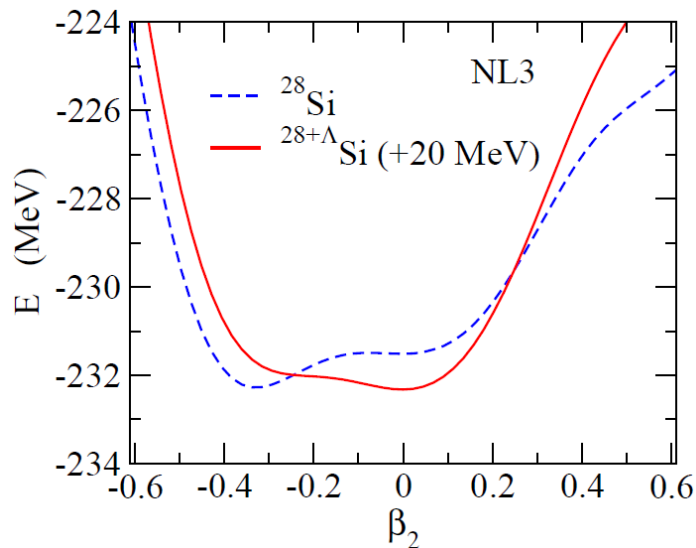
➤ Translational symmetry: always broken in nuclear systems

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} \sum_{i,j} v(r_i - r_j) \rightarrow \sum_{i=1}^A \left( -\frac{\hbar^2}{2m} \nabla_i^2 + \underline{V_{\text{MF}}(r_i)} \right)$$

➤ Rotational symmetry

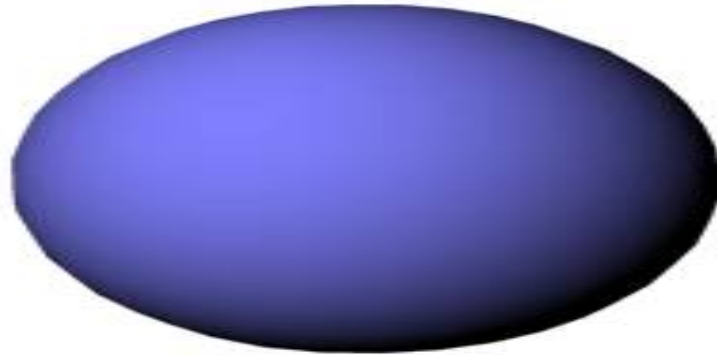


*Deformed solution*



# Nuclear Deformation

実験的な証拠



# Nuclear Deformation

## Excitation spectra of $^{154}\text{Sm}$

(MeV)

0.903 —————  $8^+$

0.544 —————  $6^+$

0.267 —————  $4^+$

0.082 —————  $2^+$   
0 —————  $0^+$

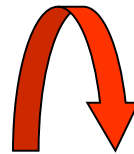
$^{154}\text{Sm}$

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

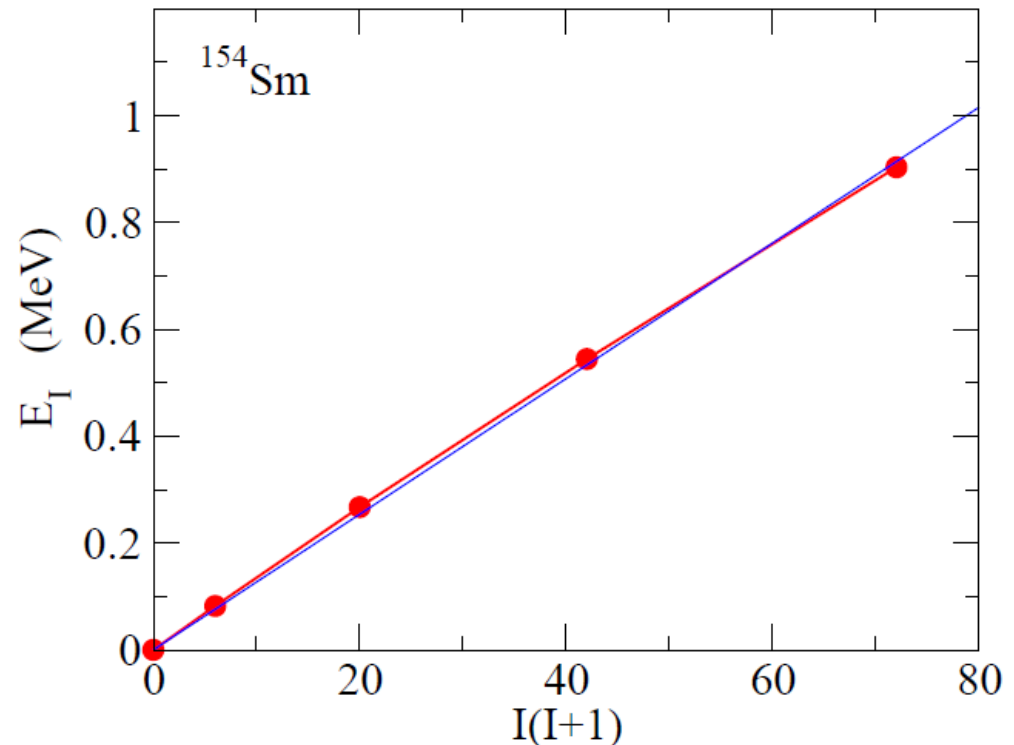
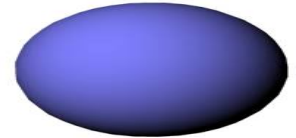
cf. Rotational energy of a rigid body  
(Classical mechanics)

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \omega = \dot{\theta})$$



$^{154}\text{Sm}$  is deformed

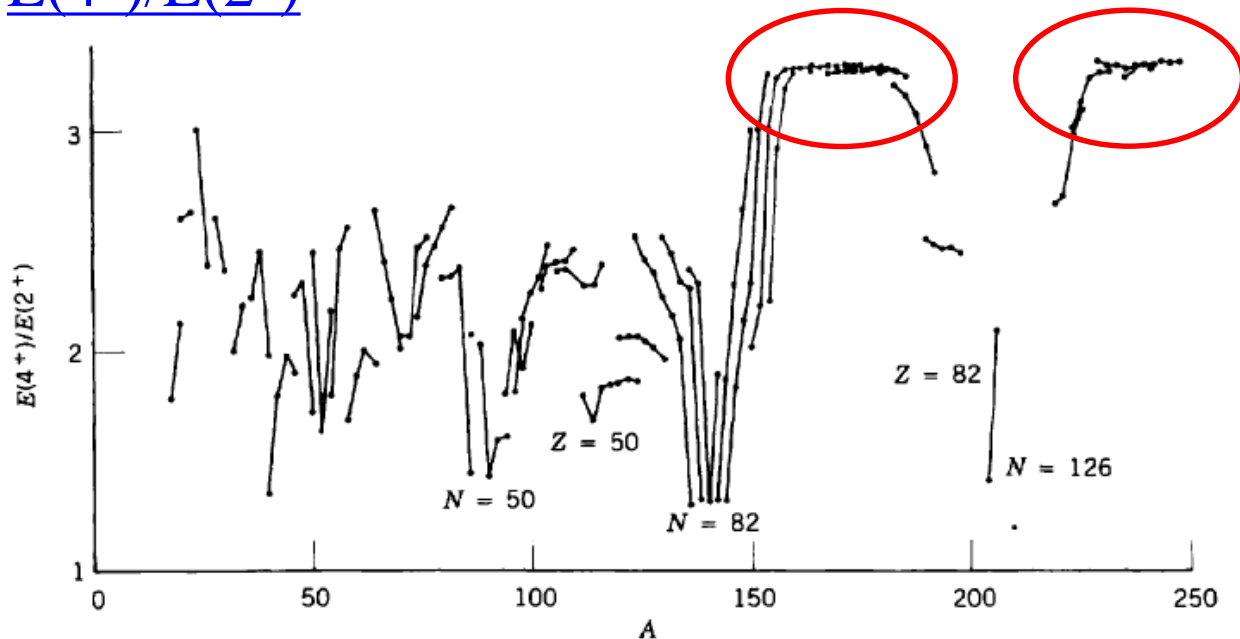


$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→  $E_2 \propto 2 \times 3 = 6, \quad E_4 \propto 4 \times 5 = 20$

→  $E_4/E_2 = 20/6 = 3.3333 \dots$

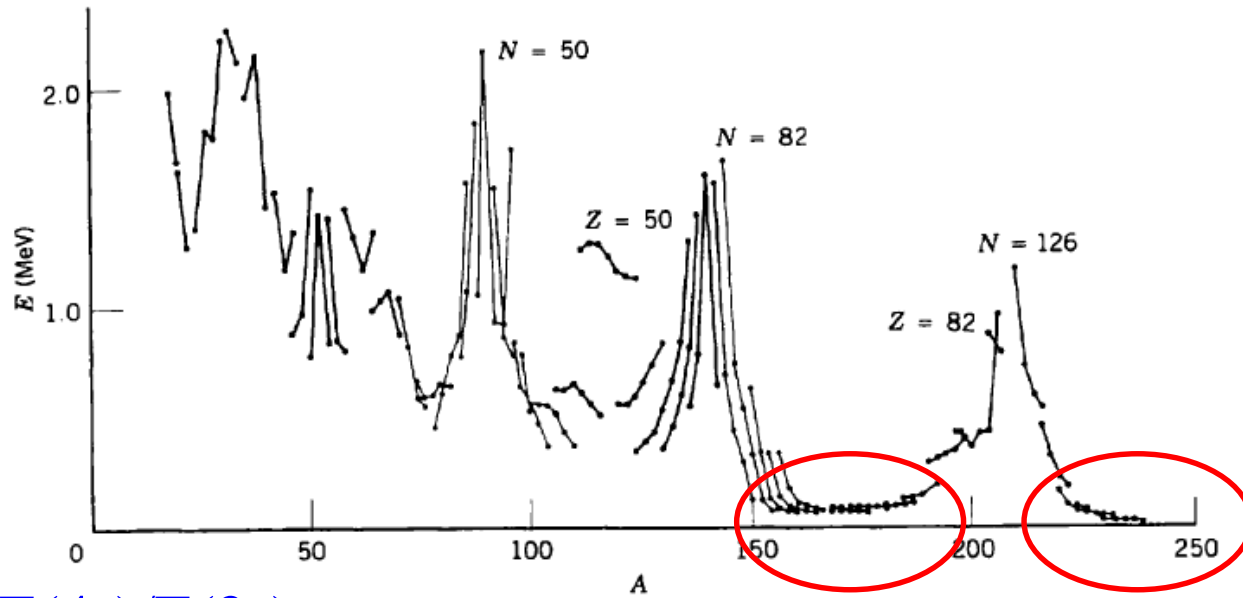
$E(4^+)/E(2^+)$



deformed nuclei:  
 $E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:  
 $E(4^+)/E(2^+) \sim 2$

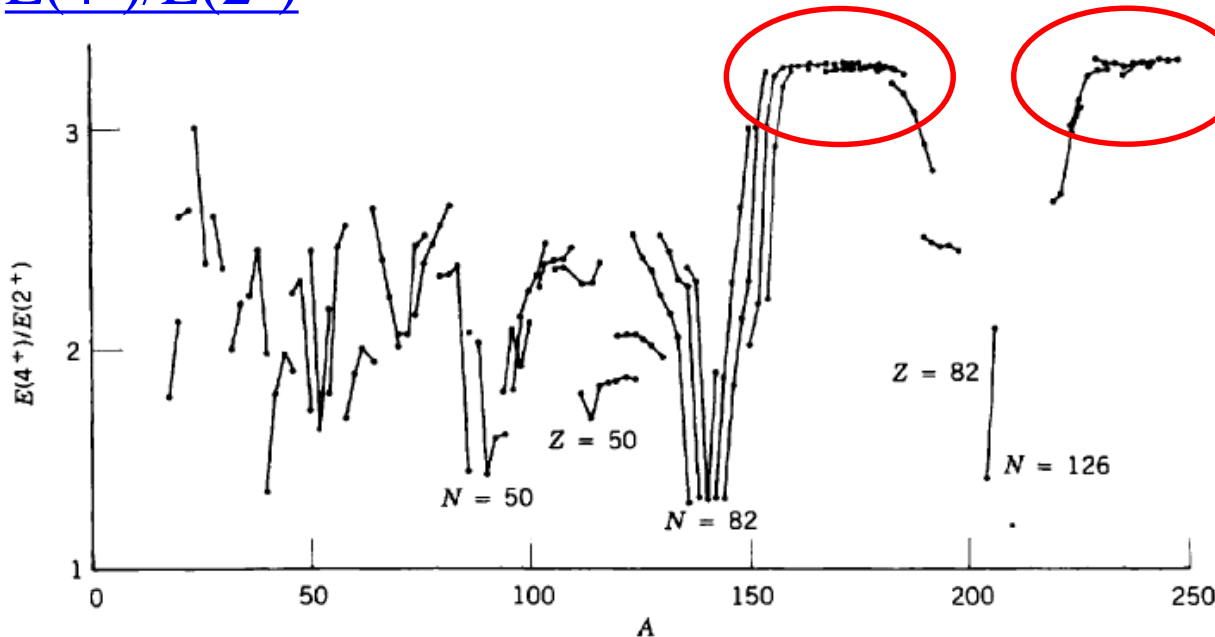
# The energy of the first $2^+$ state in even-even nuclei



a small energy  
→ spontaneously  
symm. breaking

deformed nuclei

## $E(4^+)/E(2^+)$

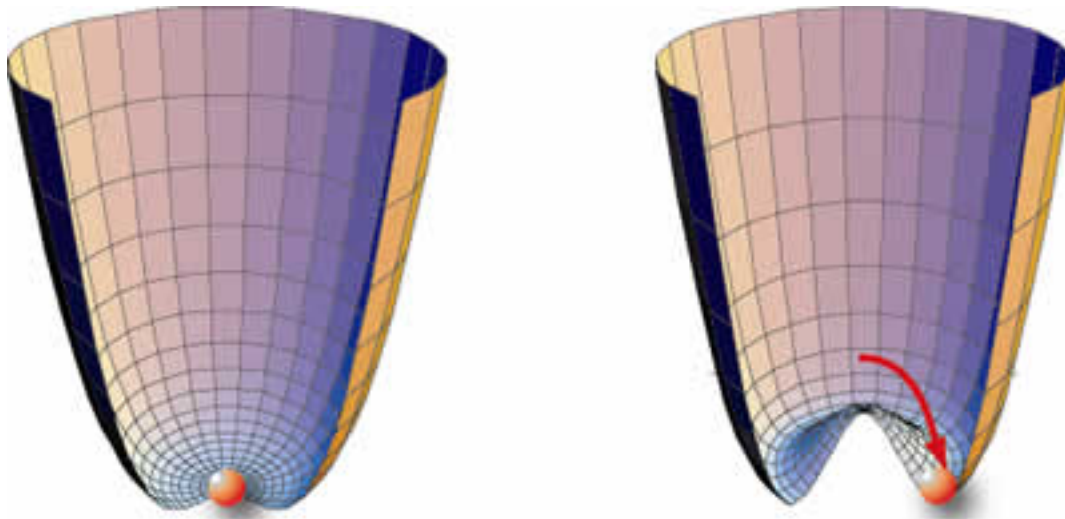


deformed nuclei:  
 $E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:  
 $E(4^+)/E(2^+) \sim 2$

## Spontaneous symmetry breaking

The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.



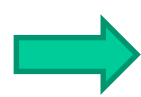
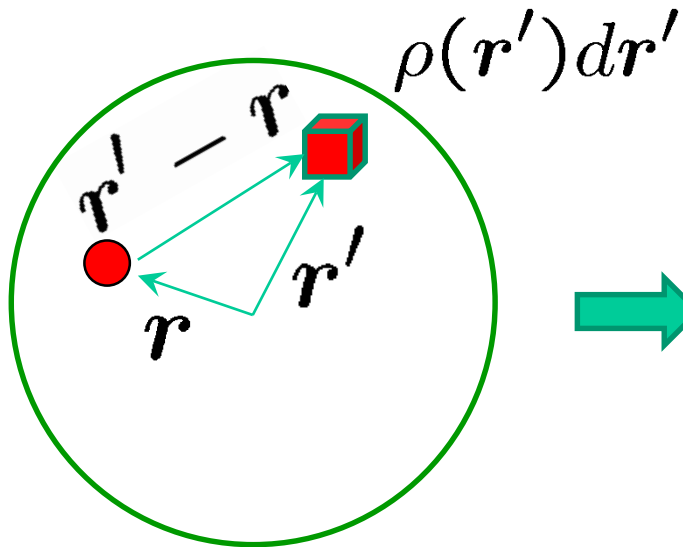
Nambu-Goldstone mode (zero energy mode)  
to restore the symmetry



# One-particle motion in a deformed potential

平均場

interaction for a nucleon inside a nucleus:



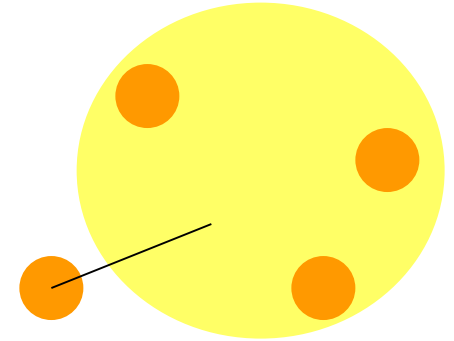
$$v(\mathbf{r}' - \mathbf{r}) \cdot \underline{\rho(\mathbf{r}') d\mathbf{r}'}$$

the number of nucleon  
at  $r'$

naively speaking,

$$V(\mathbf{r}) \sim \int v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}'$$

$$V(\mathbf{r}) \sim \int v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}' \sim -g\rho(\mathbf{r}) \quad \text{if } v(\mathbf{r}, \mathbf{r}') = -g\delta(\mathbf{r} - \mathbf{r}')$$

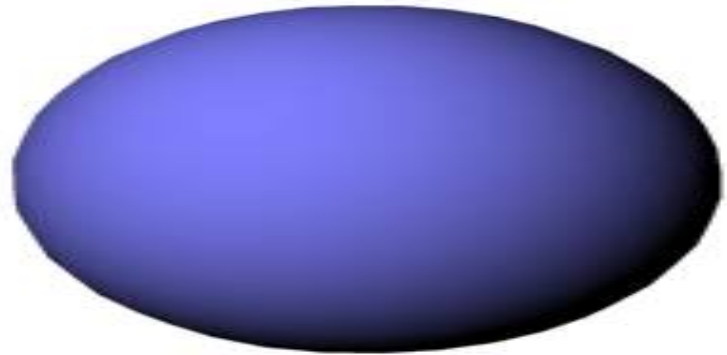


# One-particle motion in a deformed potential

$$V(\mathbf{r}) \sim \int v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}' \sim -g\rho(\mathbf{r}) \quad \text{if } v(\mathbf{r}, \mathbf{r}') = -g\delta(\mathbf{r} - \mathbf{r}')$$

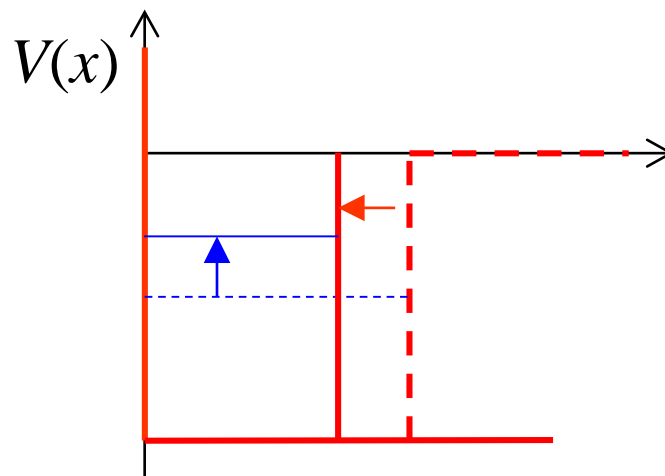
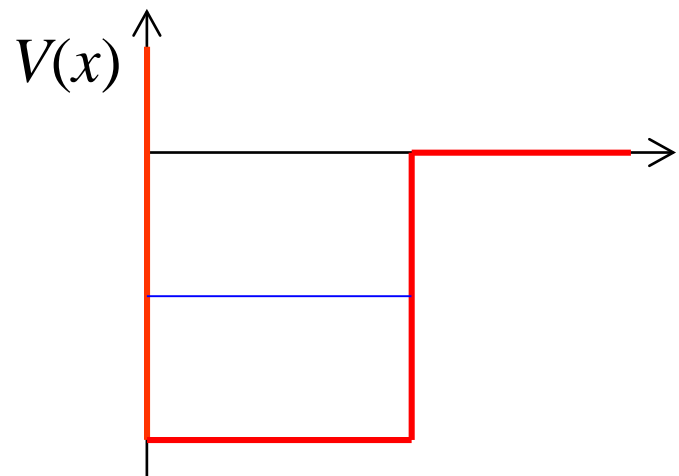
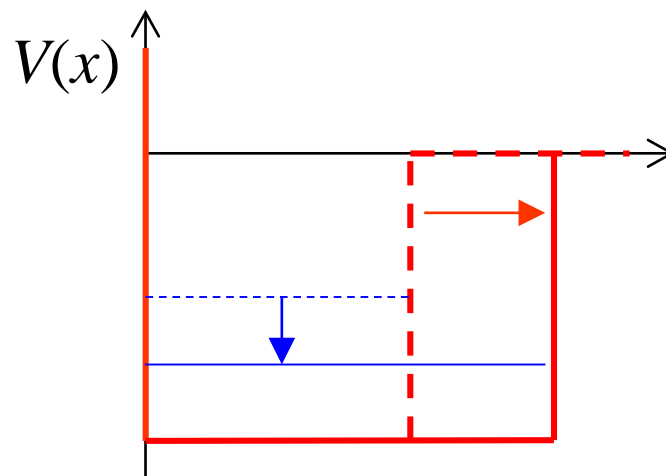
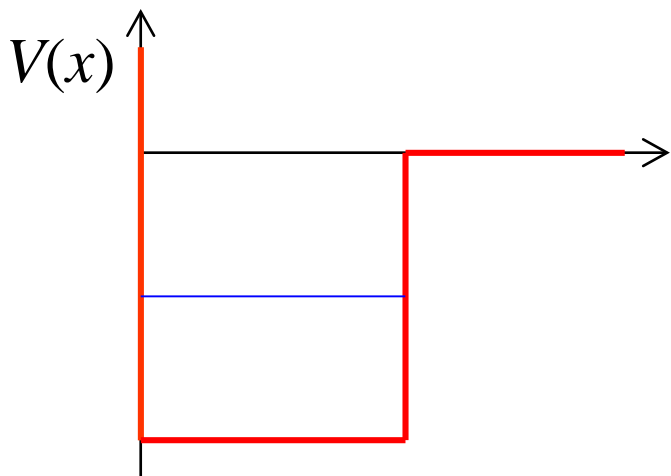


$V(r)$



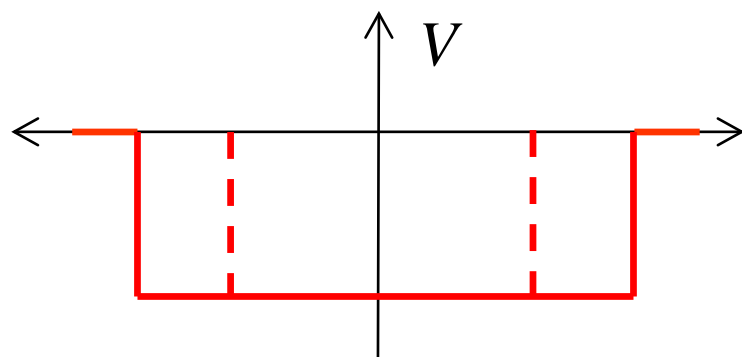
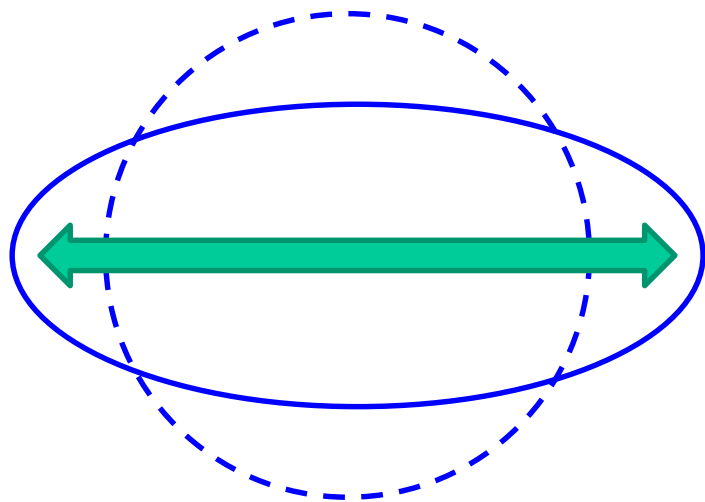
$V(r, \theta)$

# (準備) 1次元井戸型ポテンシャル



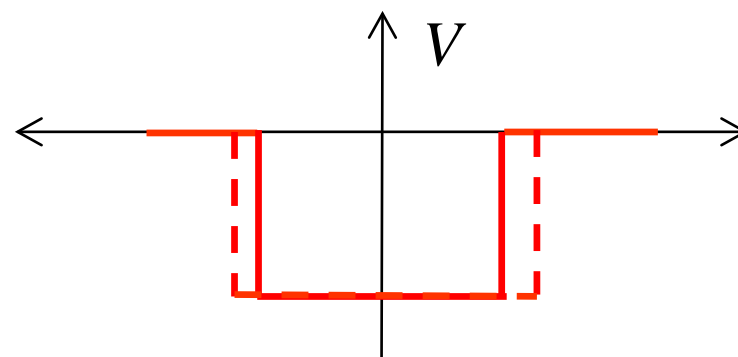
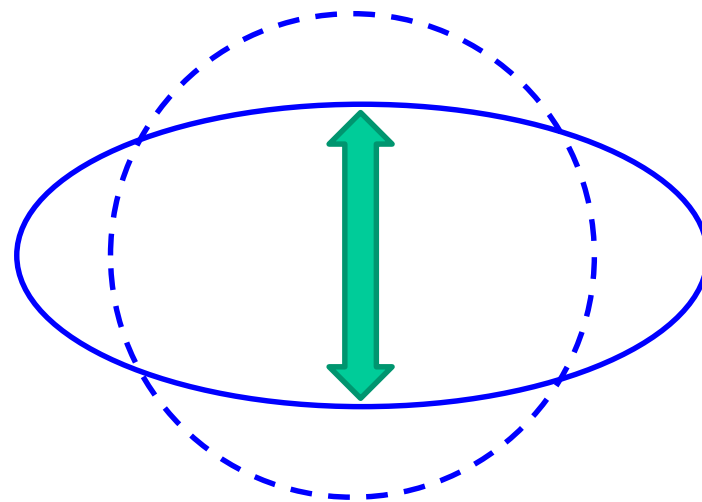
# One-particle motion in a deformed potential

長軸に沿った運動



$E \rightarrow$  小

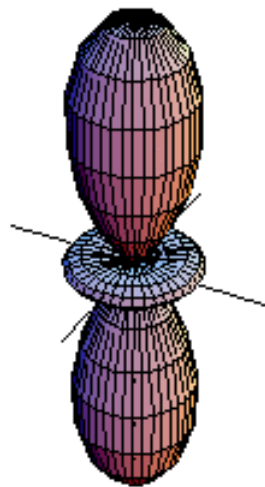
短軸に沿った運動



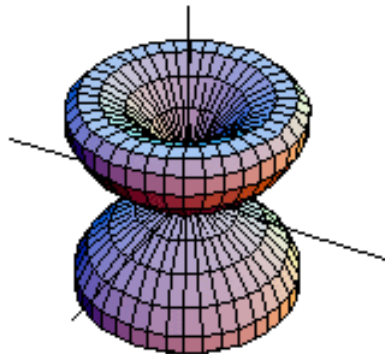
$E \rightarrow$  大

$l = 2$

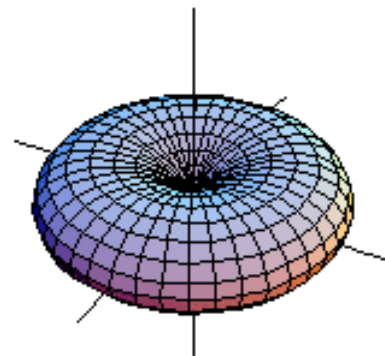
$z$



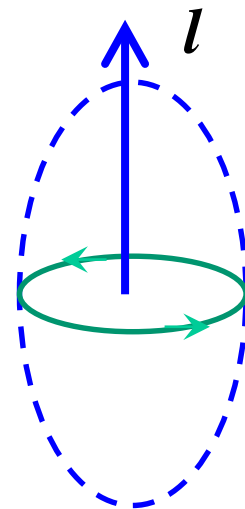
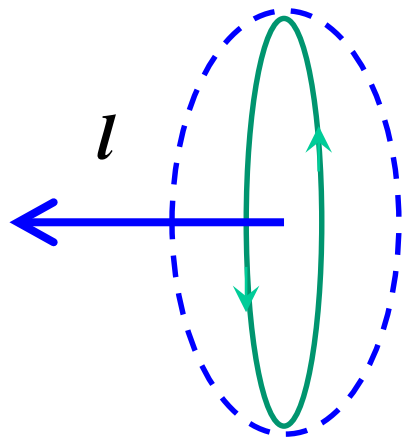
$r = Y_{20}$   
( $l_z = 0$ )



$r = Y_{21}$   
( $l_z = 1$ )

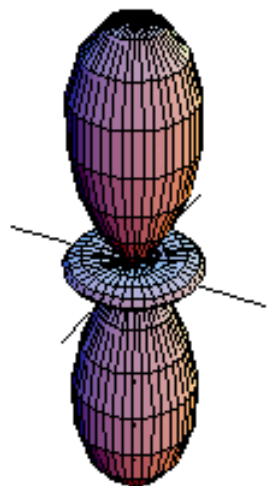


$r = Y_{22}$   
( $l_z = 2$ )

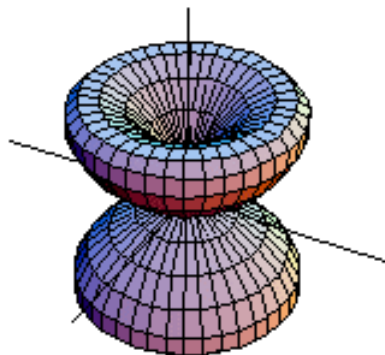


$l = 2$

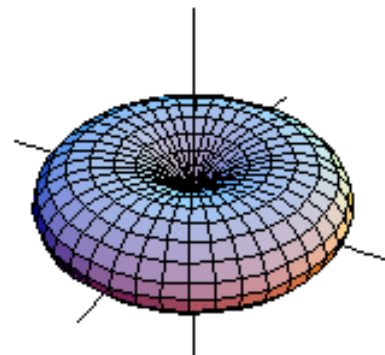
$z$



$r = Y_{20}$   
 $(l_z = 0)$

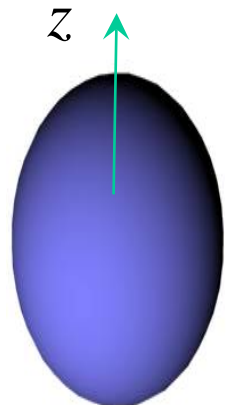


$r = Y_{21}$   
 $(l_z = 1)$



$r = Y_{22}$   
 $(l_z = 2)$

$z$



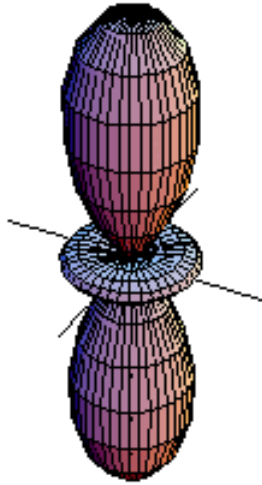
$E \rightarrow$  小

なら

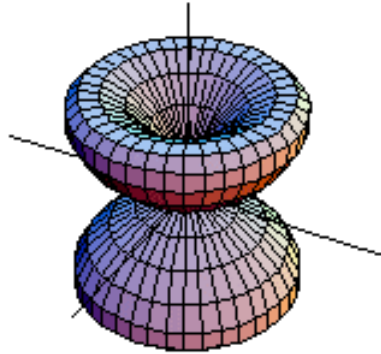
$E \rightarrow$  大

$l = 2$

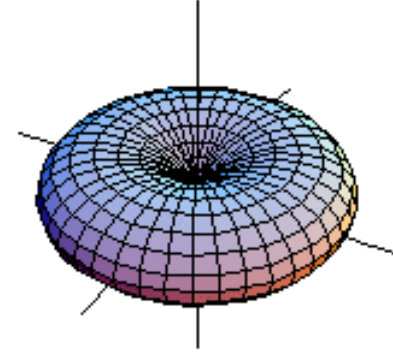
$z$



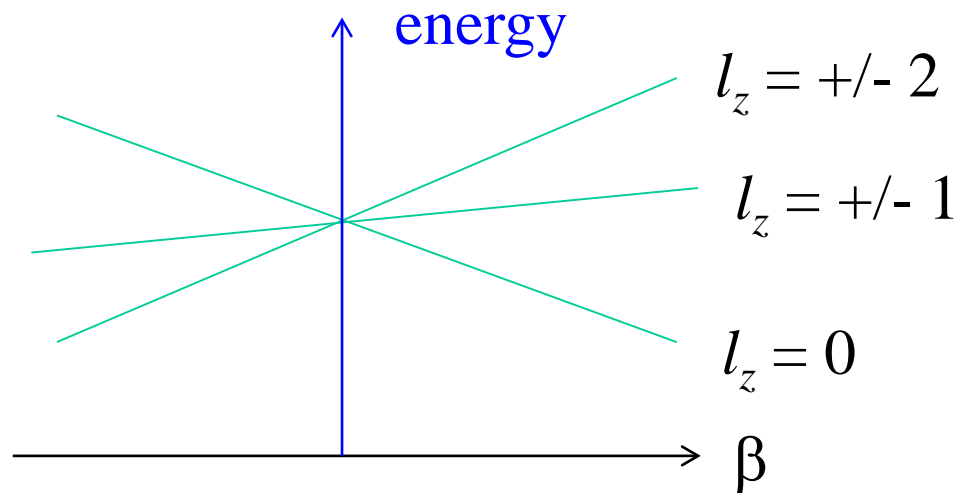
$r = Y_{20}$   
( $l_z = 0$ )

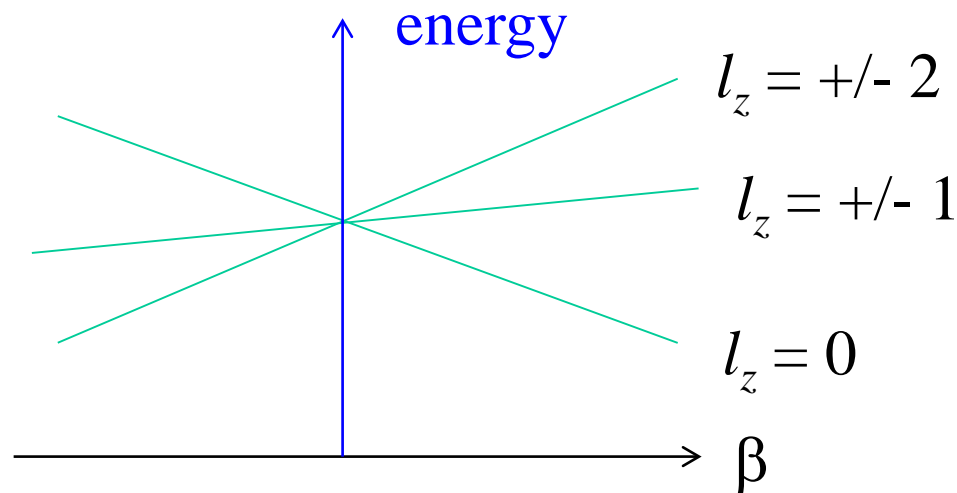


$r = Y_{21}$   
( $l_z = 1$ )



$r = Y_{22}$   
( $l_z = 2$ )





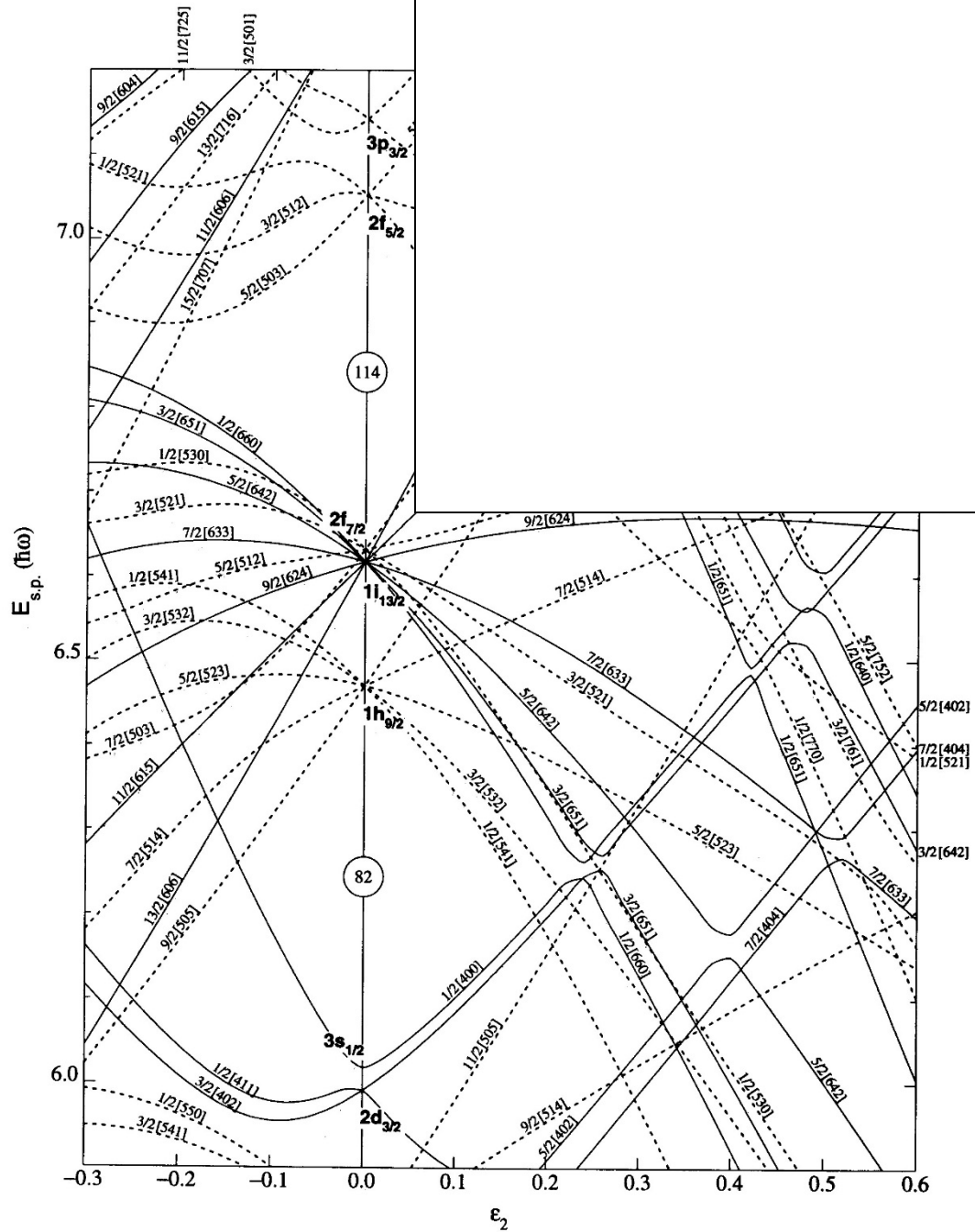
- (note)  $V(r, \theta) \rightarrow$  回転対称性を持っていない  
 $\rightarrow$  角運動量がいい量子数ではない

$$\phi_{nll_z}(r, \theta, \phi) \rightarrow \phi_{nl_z}(r, \theta, \phi) = \sum_l \psi_{nl}(r) Y_{ll_z}(\theta, \phi)$$

いろいろな角運動量成分  
 が混じる

\* 軸対称変形であれば  $l_z$  は保存





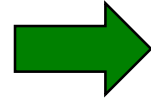
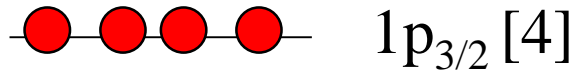
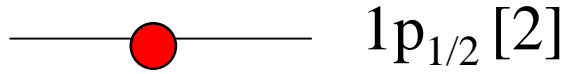
## Nilsson diagram

$h = T + V(r, \theta)$   
の固有値

Figure 13. Nilsson diagram for protons,  $Z \geq 82$  ( $\epsilon_4 = \epsilon_2^2/6$ ).

# Level scheme of $^{11}_4\text{Be}_7$

With a spherical potential:

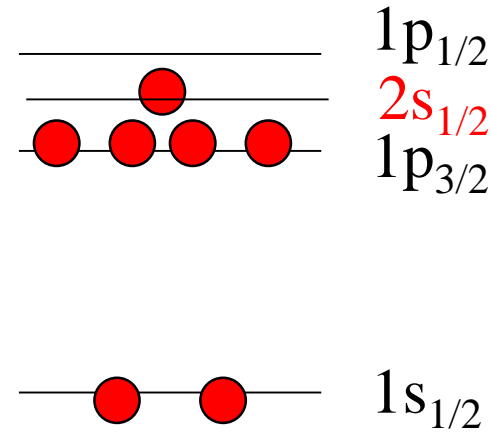
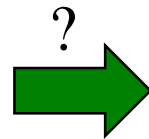
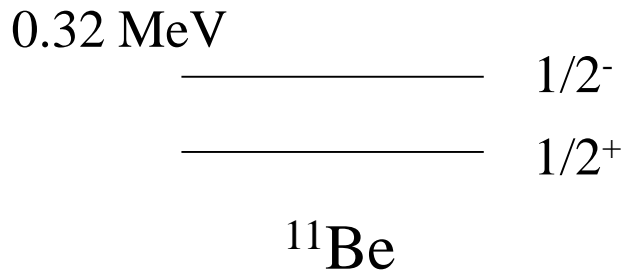


The g.s. of  $^{11}\text{Be}$  :  $I^\pi = 1/2^-$

very artificial



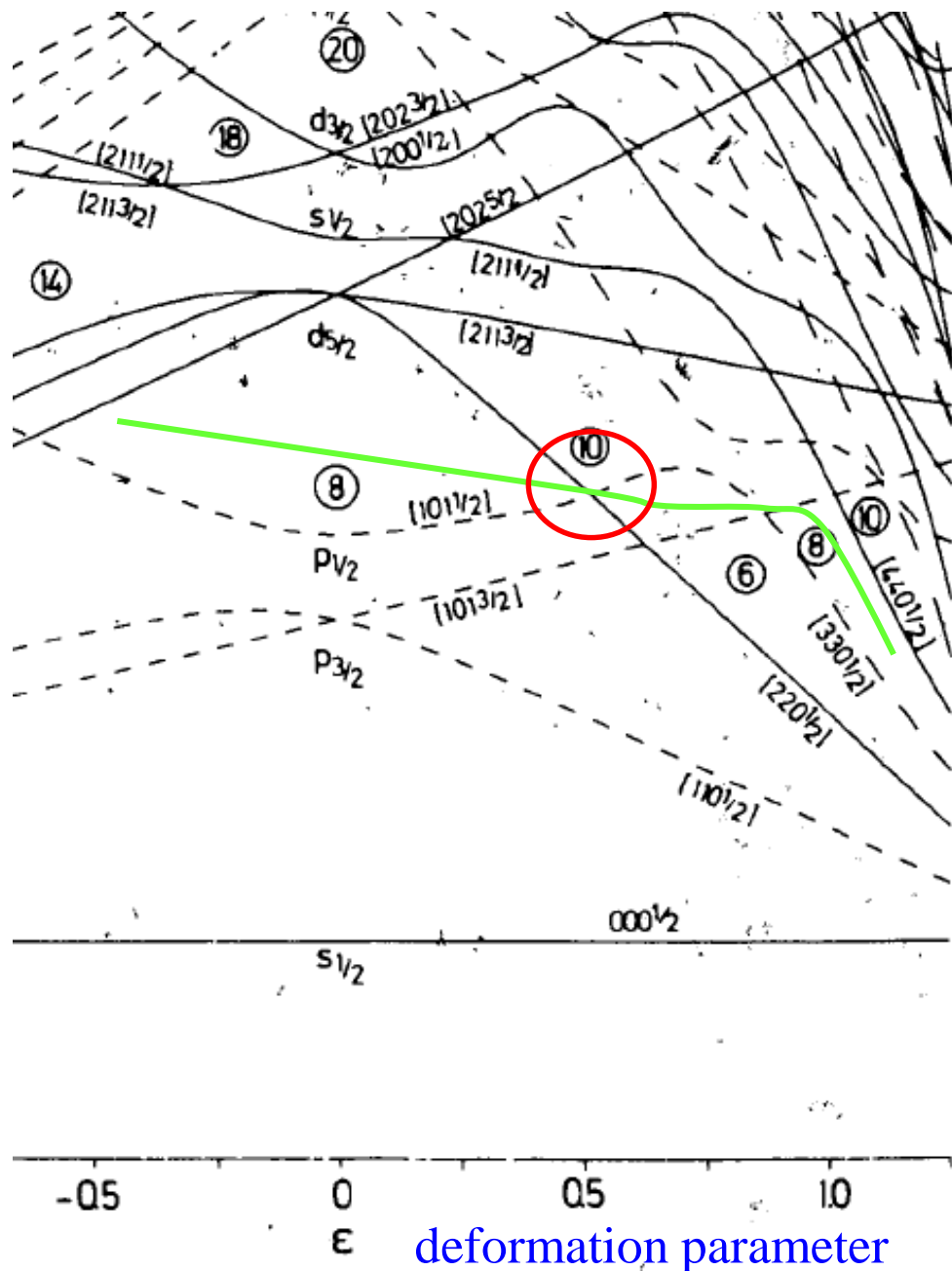
In reality.....



“parity inversion”

What happens if  $^{11}\text{Be}$  is deformed?

$^{11}_4\text{Be}_7$



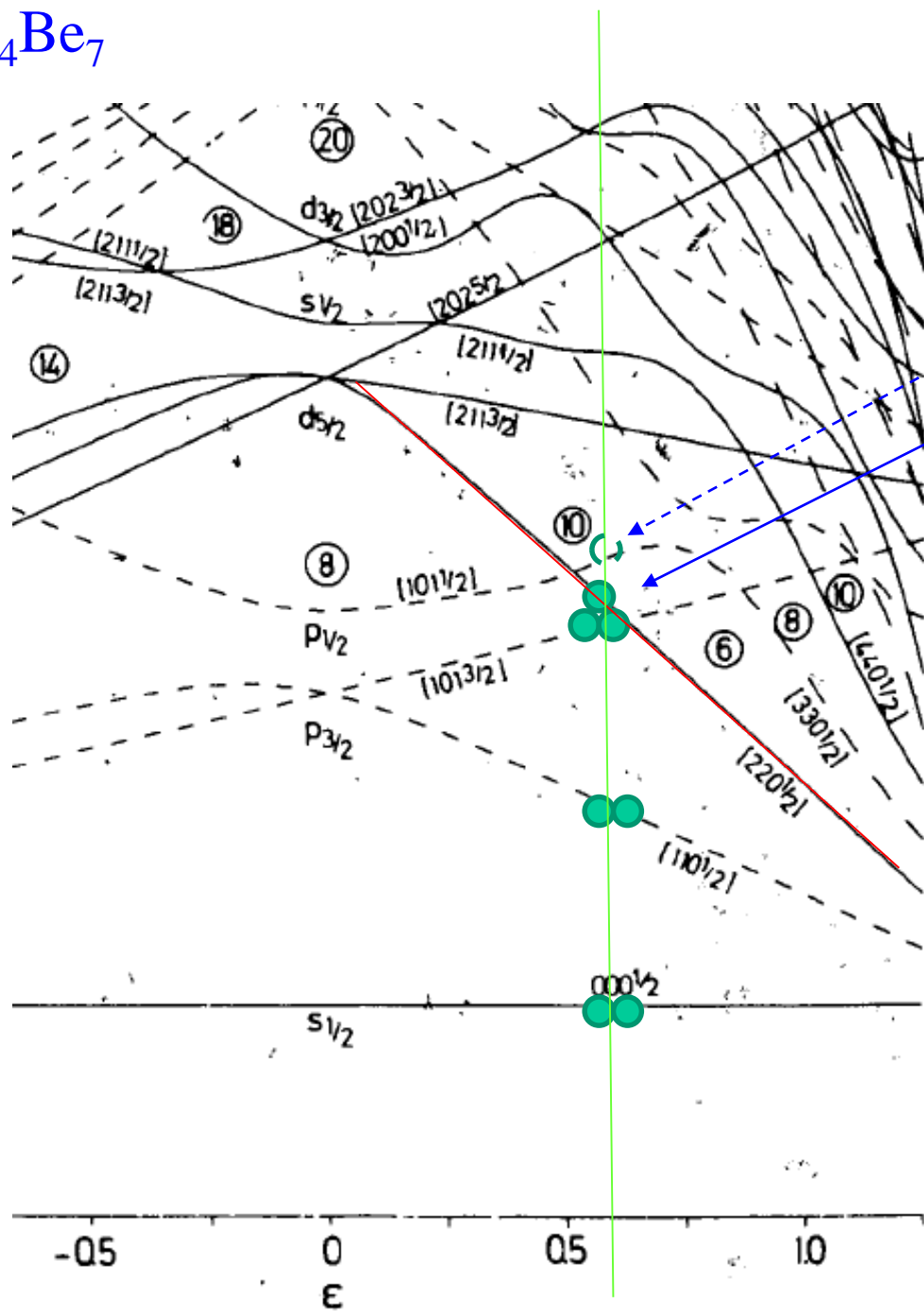
0.32 MeV

—————  $1/2^-$   
—————  $1/2^+$

$^{11}\text{Be}$

- ✓ 7番目の中性子の入る軌道を探す  
(それぞれの軌道に2つずつ中性子をつめる)

$^{11}_4\text{Be}_7$



0.32 MeV

$1/2^-$

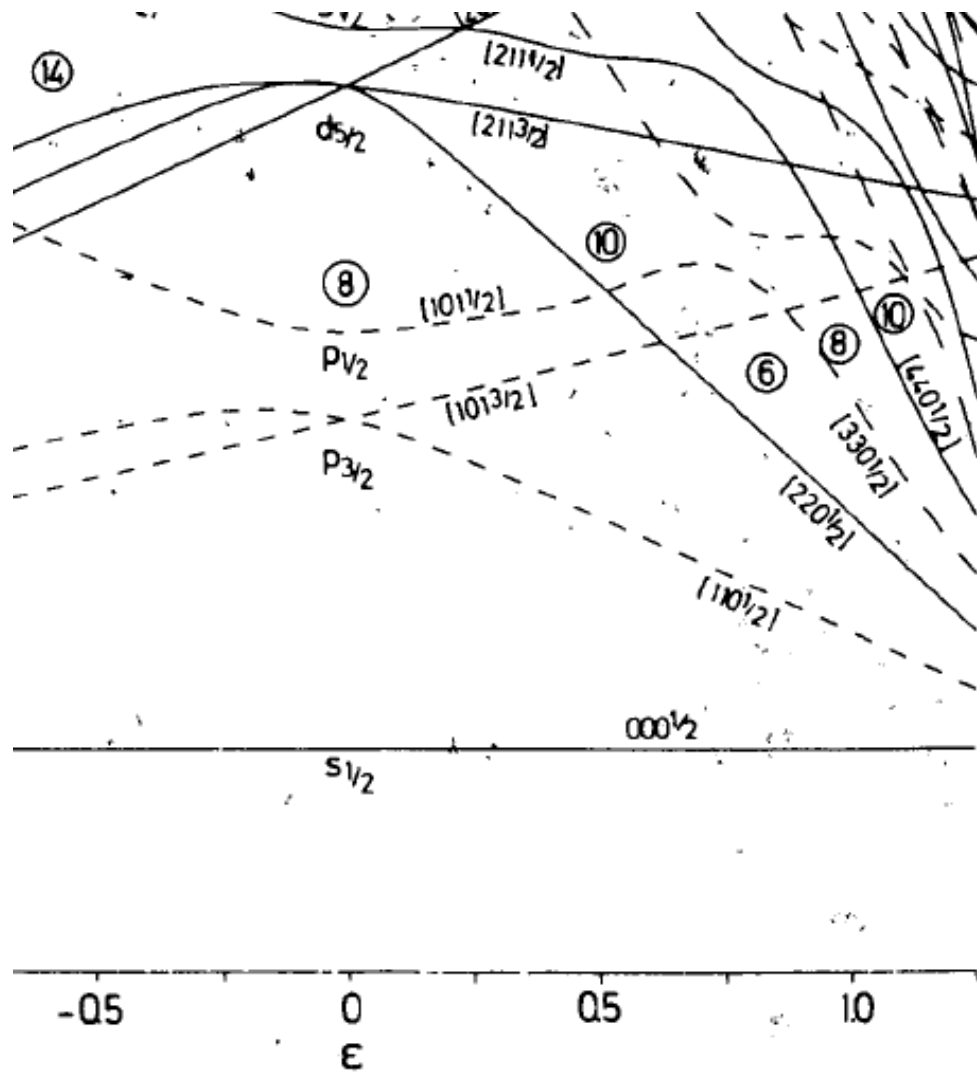
$1/2^+$

$^{11}\text{Be}$

✓ 7番目の中性子の入る軌道を探す  
(それぞれの軌道に2つずつ中性子をつめる)

Can the level scheme of  ${}^9_4\text{Be}_5$  be explained in a similar way?

cf.  ${}^{10}\text{B}(e,e'\text{K}^){}^{10}_\Lambda\text{Be} (= {}^9\text{Be}+\Lambda)$



(MeV)

2.78 ————— 1/2<sup>-</sup>

2.43 ·········· 5/2<sup>-</sup>

1.68 ————— 1/2<sup>+</sup>

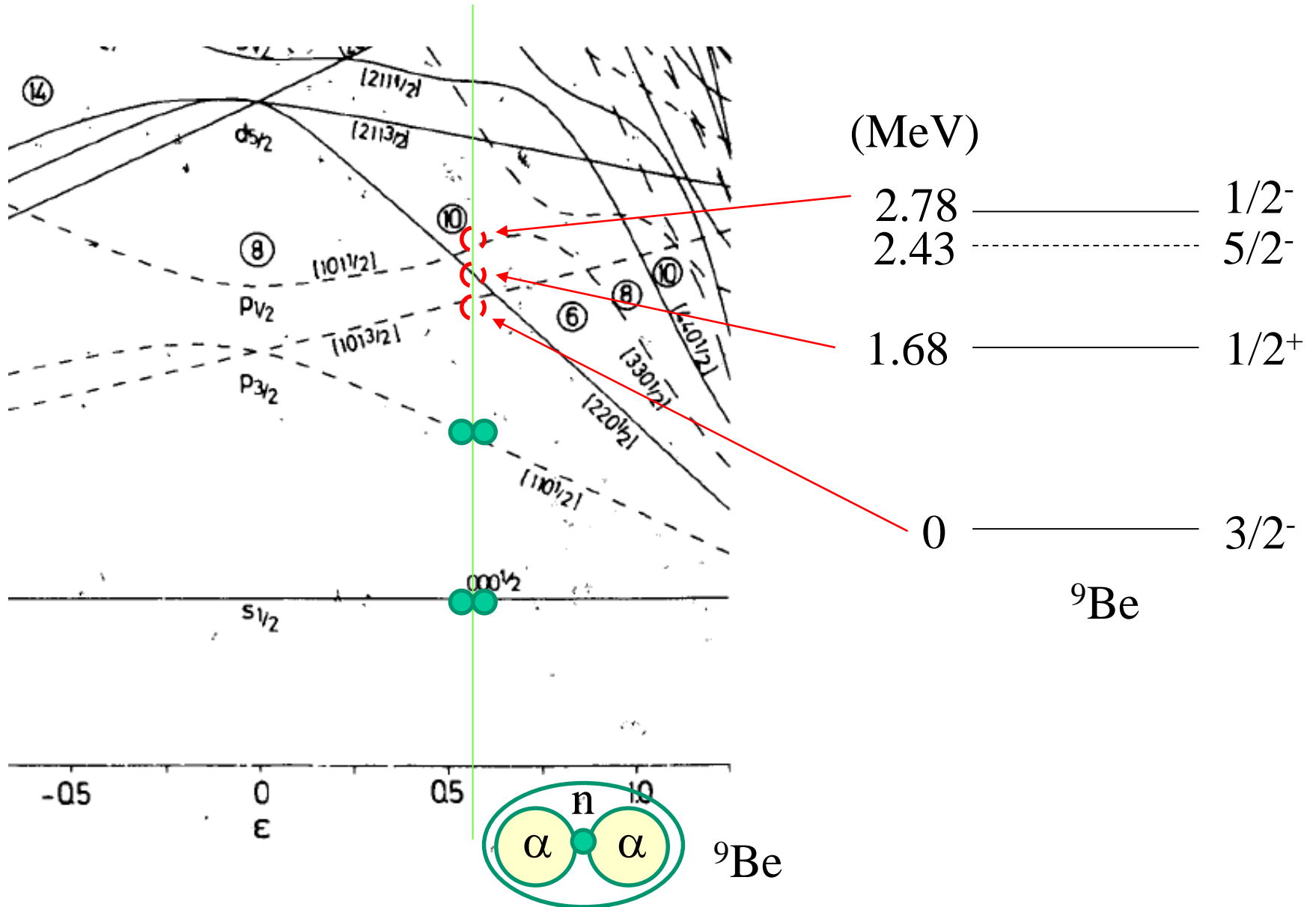
0 ————— 3/2<sup>-</sup>

${}^9\text{Be}$

The 5/2<sup>-</sup> state at 2.43 MeV:  
rotational state with the same  
configuration as the g.s. state  
(not considered here)

Can the level scheme of  ${}^9_4\text{Be}_5$  be explained in a similar way?

cf.  ${}^{10}\text{B}(e,e'\text{K}^+){}^{10}_\Lambda\text{Be} (= {}^9\text{Be} + \Lambda)$



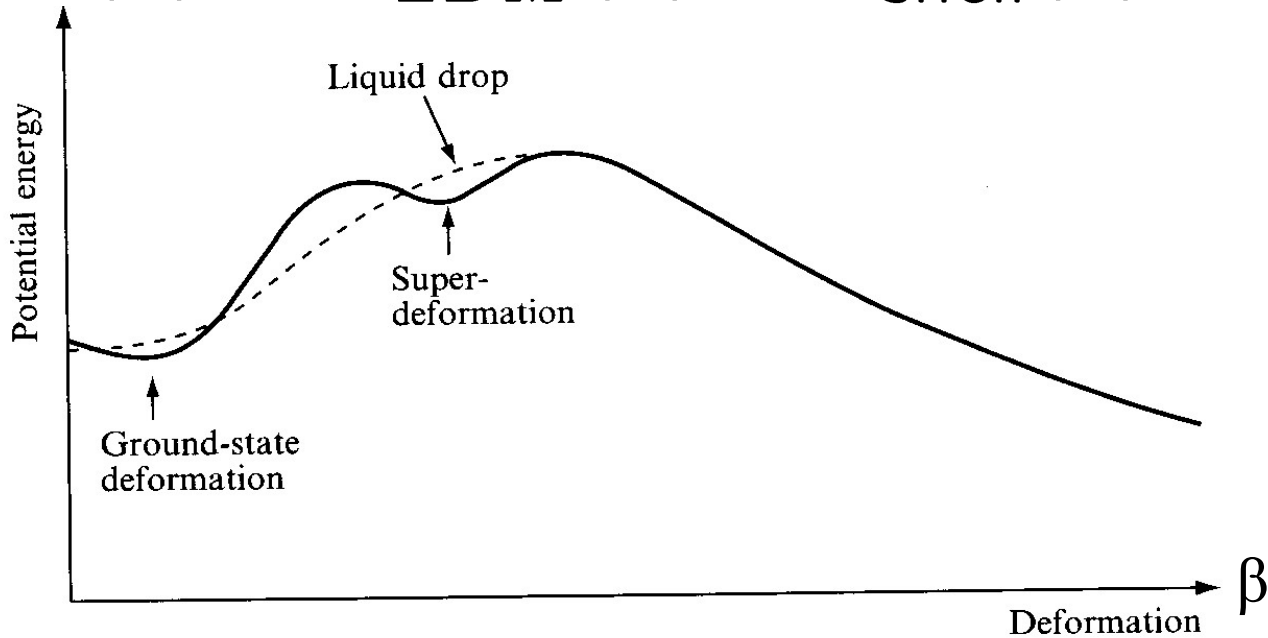
## Several topics:

- deformation as a quantum effect
- Neumann-Wigner no-crossing rule
- RMF for deformed hypernuclei
- Quiz: spontaneous symmetry breaking

## nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{shell}(\beta)$$

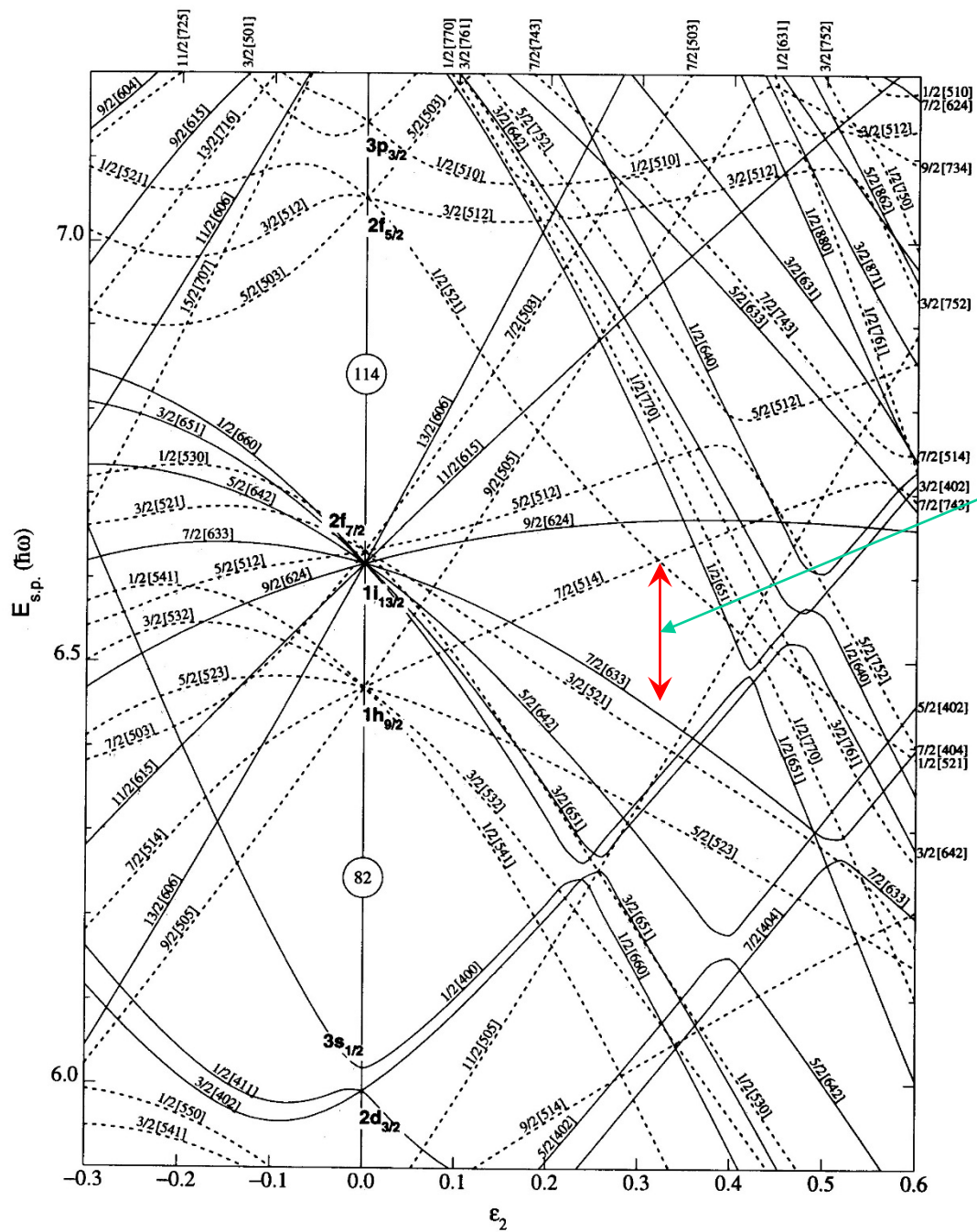


LDM only



always spherical ground state





energy gap if deformed

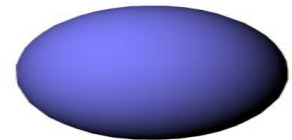
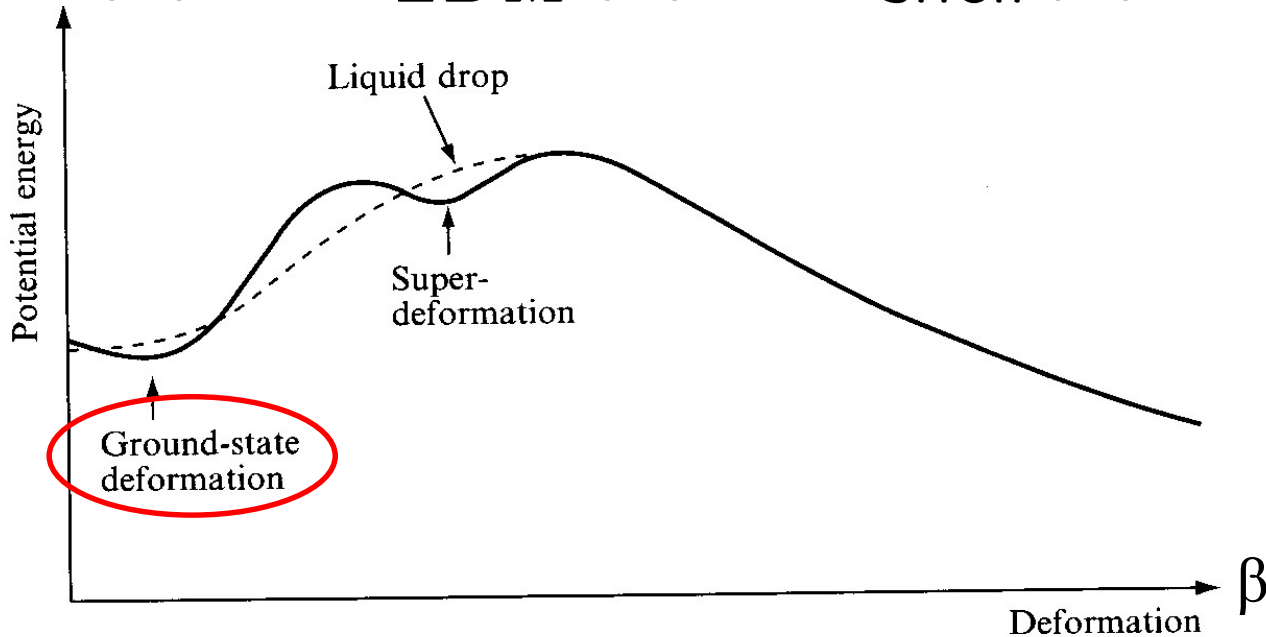
# Nilsson diagram

Figure 13. Nilsson diagram for protons,  $Z \geq 82$  ( $\epsilon_4 = \epsilon_2^2/6$ ).

## nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{shell}(\beta)$$

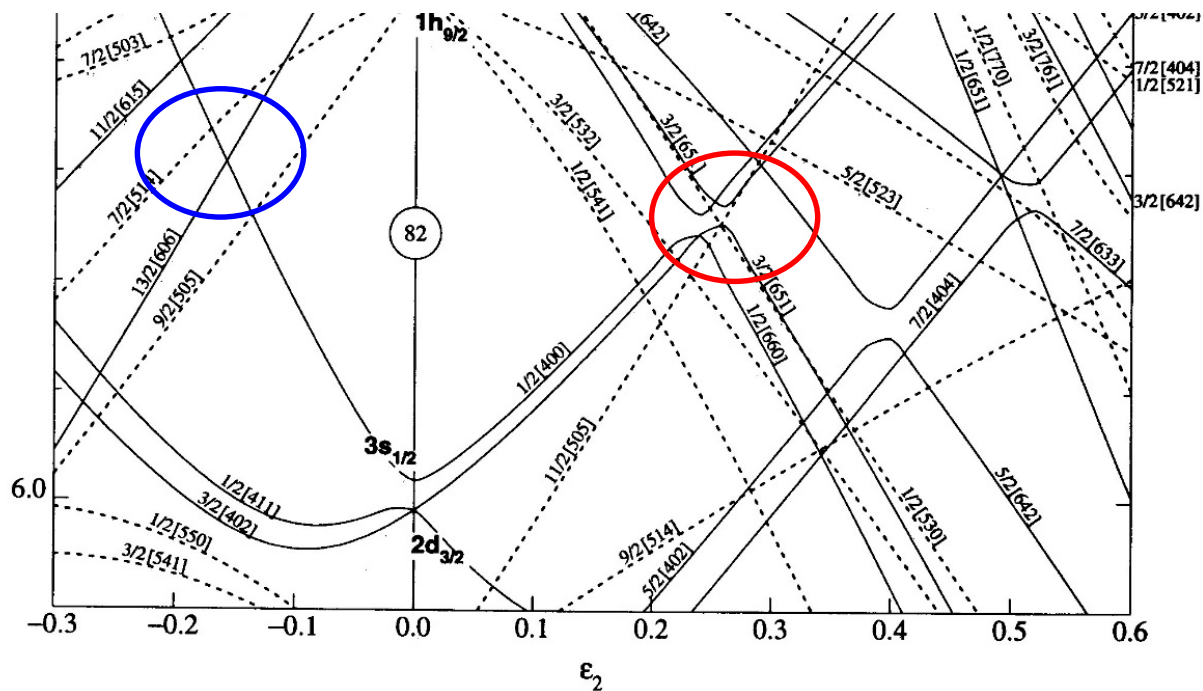


LDM only  $\longrightarrow$  always spherical ground state

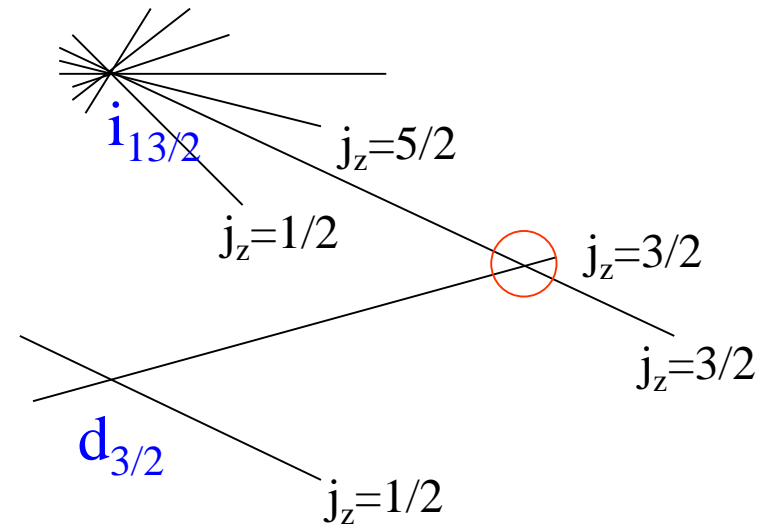
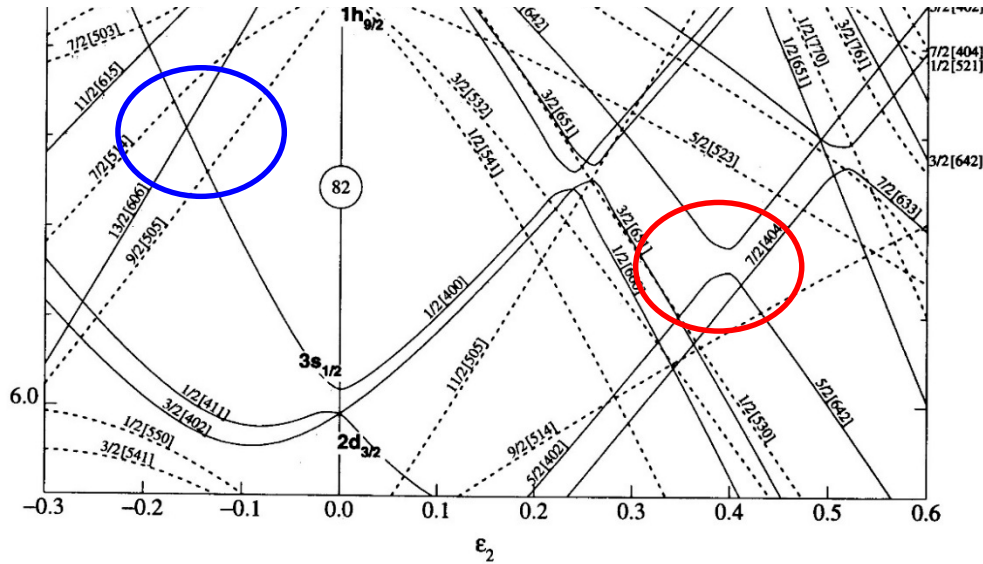
Shell correction  $\longrightarrow$  may lead to a **deformed g.s.**

\* Spontaneous Symmetry Breaking

➤ ニルソン図で準位が反発しているように見えるのは何ですか？

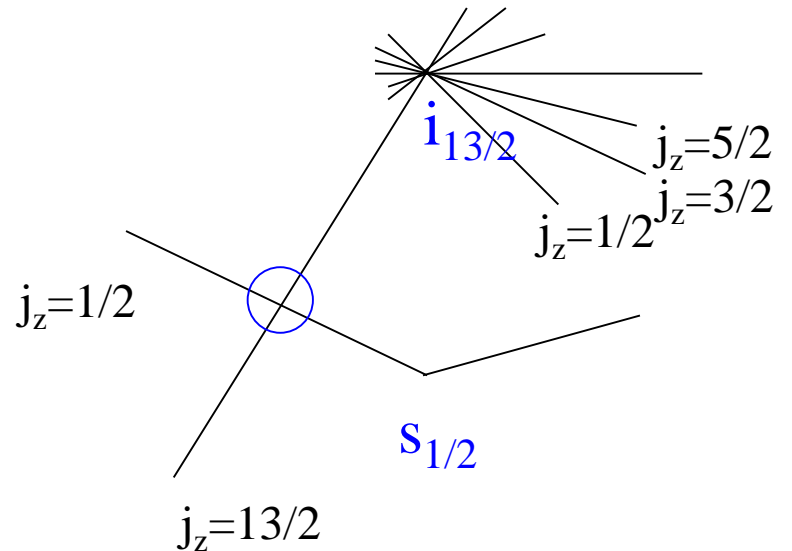


➤ ニルソン図で準位が反発しているように見えるのは何ですか？



同じ量子数を持つ準位は交わらない  
(量子数が違うと交わってもよい)

「ノイマン-ウィグナーの定理」

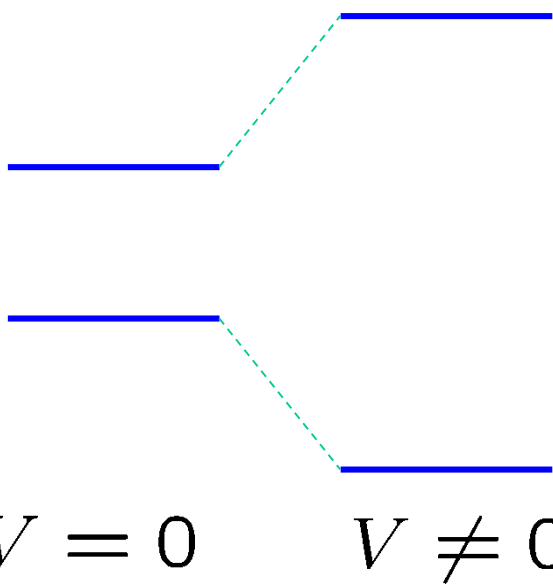


# 準位交差の問題：同じ量子数を持つ2つの状態は交差しない

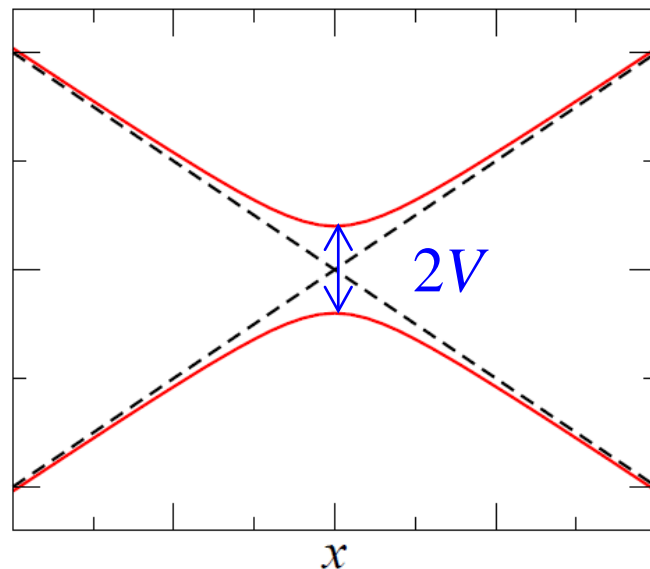
## 「ノイマン-ウィグナーの定理」

$$\begin{pmatrix} -\epsilon x & V \\ V & \epsilon x \end{pmatrix}$$

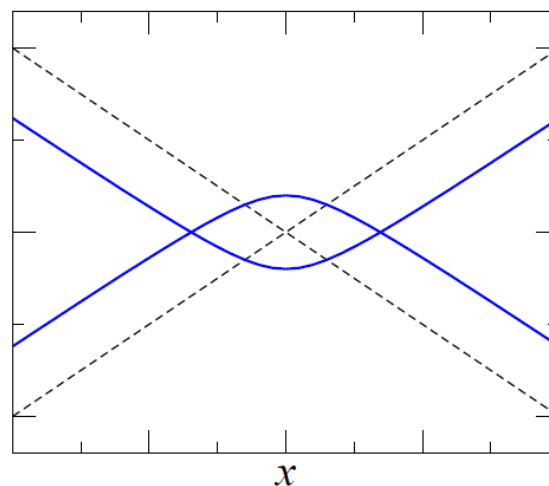
対角化  $\rightarrow \lambda_{\pm}(x) = \pm \sqrt{\epsilon^2 x^2 + V^2}$



Vの符号によらず必ず反発

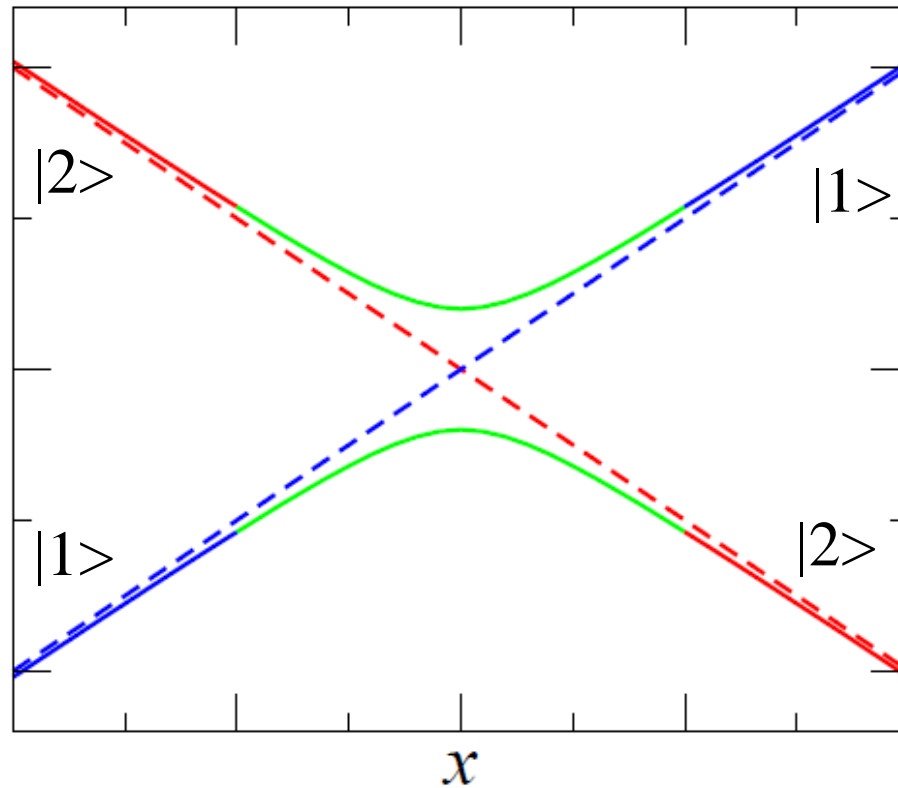


「疑似交差」、「準位反発」



このようになることはない

$x$  がゆっくりと変化すると断熱的に状態が  $|1\rangle$  から  $|2\rangle$  へ変化  
(断熱遷移)



Landau-Zener の式:

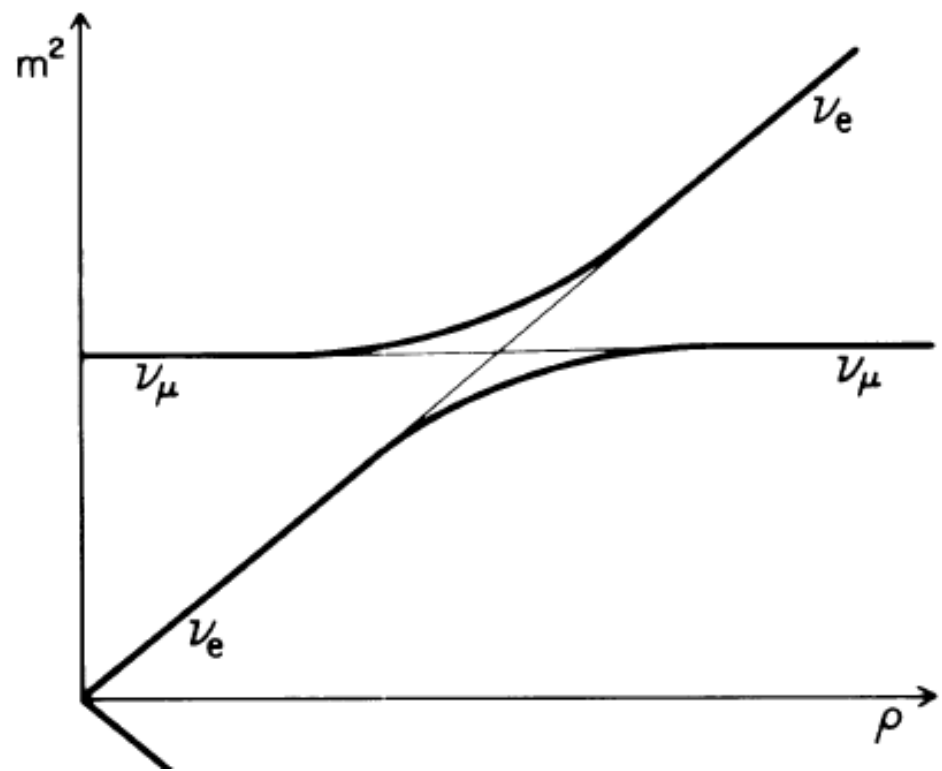
$$P(|1\rangle \rightarrow |1\rangle) = \exp\left(-\frac{2\pi V^2}{\hbar|\dot{x}| \cdot 2\epsilon}\right)$$

# cf. ニュートリノ振動と準位交差問題

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \Psi_e \\ \Psi_\mu \end{pmatrix} = \left[ \begin{pmatrix} E + A(r) & 0 \\ 0 & E \end{pmatrix} + a \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \right] \begin{pmatrix} \Psi_e \\ \Psi_\mu \end{pmatrix}$$

$$E = \frac{1}{2}(m_1^2 + m_2^2), \quad a = \frac{1}{2}(m_2^2 - m_1^2)$$

電子ニュートリノと物質中の電子との相互作用



物質中で共鳴的にニュートリノ振動が起こる = MSW 効果

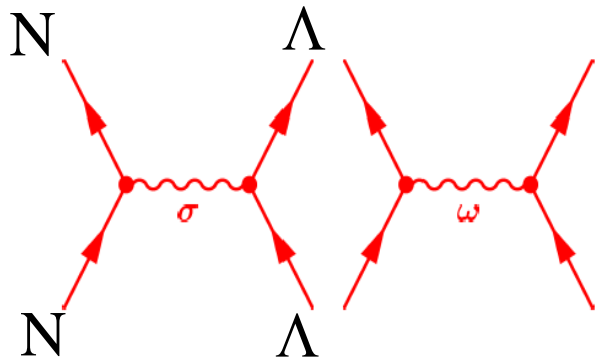
Ref.  
H.A. Bethe, PRL56('86)1305,  
W.C. Haxton, PRL57('86)1271

# RMF calculations for deformed hypernuclei

Hypernuclei: nucleus + Lambda particle

Effect of a  $\Lambda$  particle on nuclear shapes?

Relativistic Mean-field model

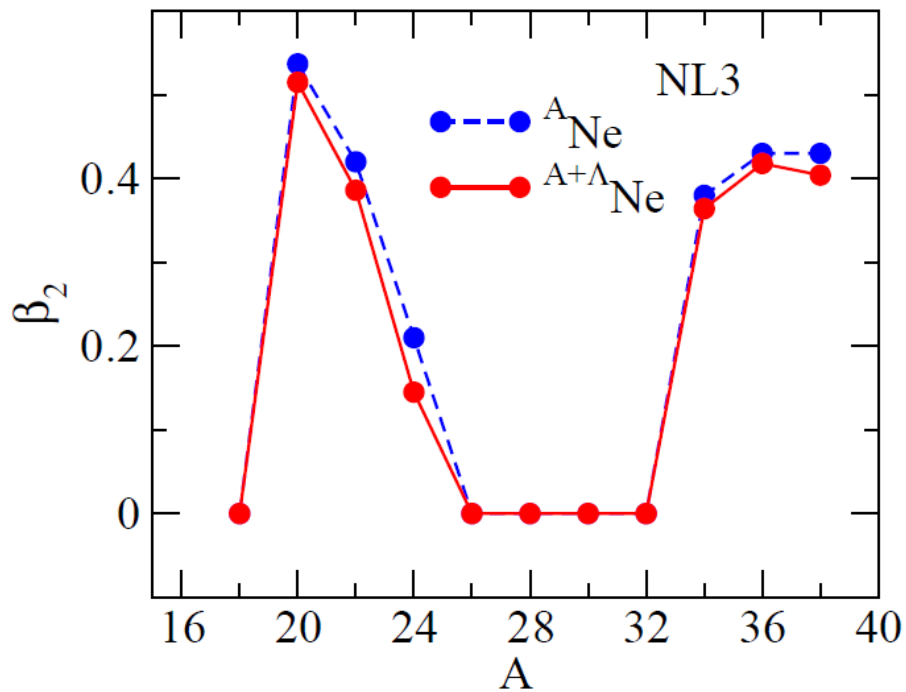


nucleon-nucleon interaction  
via meson exchange

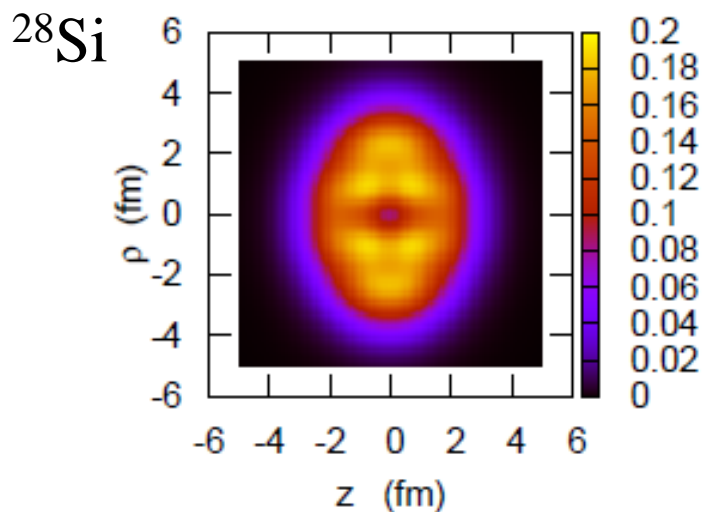
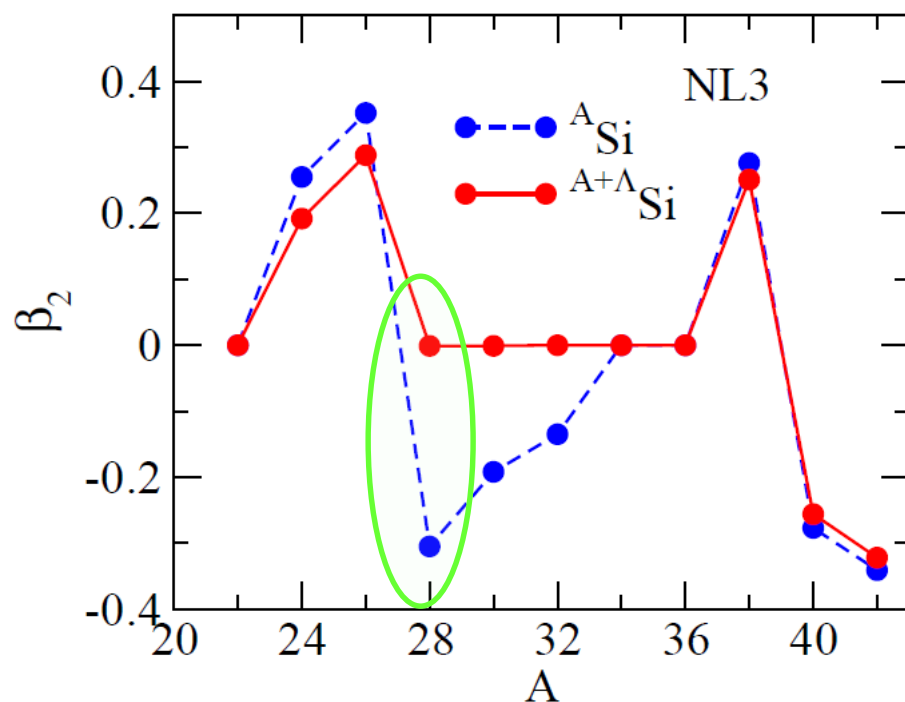
$\Lambda\sigma$  and  $\Lambda\omega$  couplings



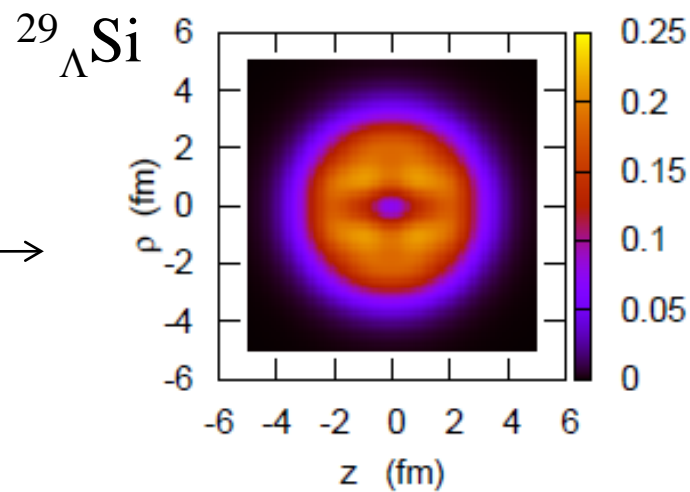
# Ne isotopes



# Si isotopes



$\Lambda \rightarrow$



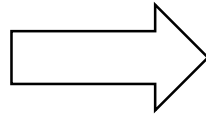
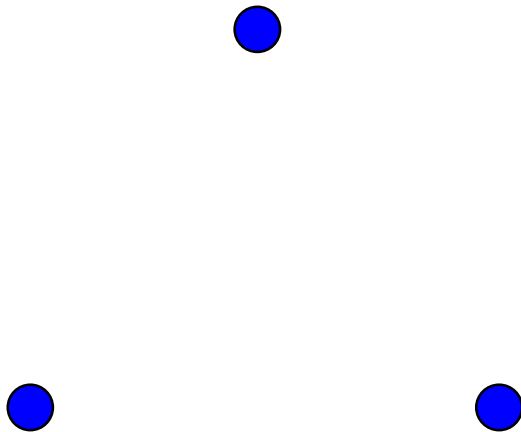
## Quiz: spontaneous symmetry breaking

There are a few dots.

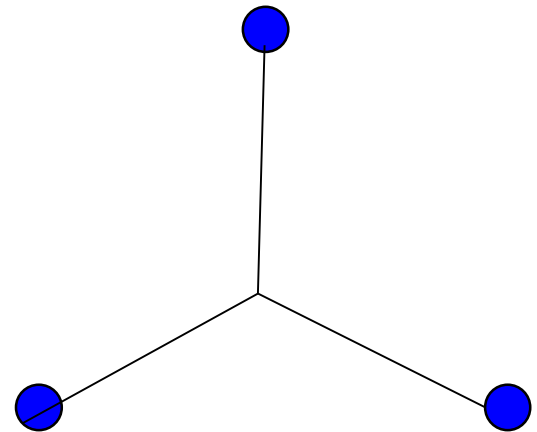
- Connect the dots.
- The number of lines is not limited.
- Two lines can cross.
- Connect the dots so that one can go from one dot to all the other dots.

How do you connect the lines if you want to make the total length of lines the shortest?

e.g.) Equilateral triangle



Connect symmetrically



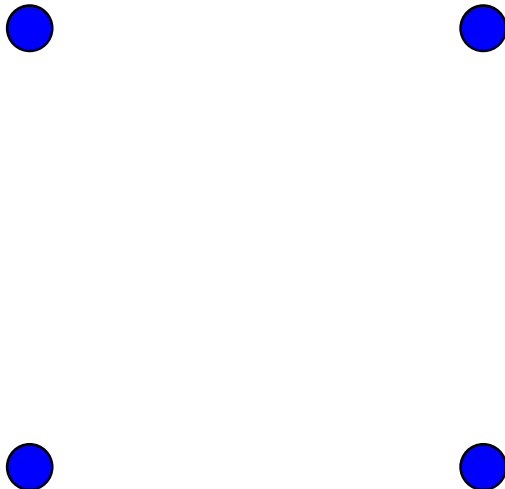
## Quiz: spontaneous symmetry breaking

There are a few dots.

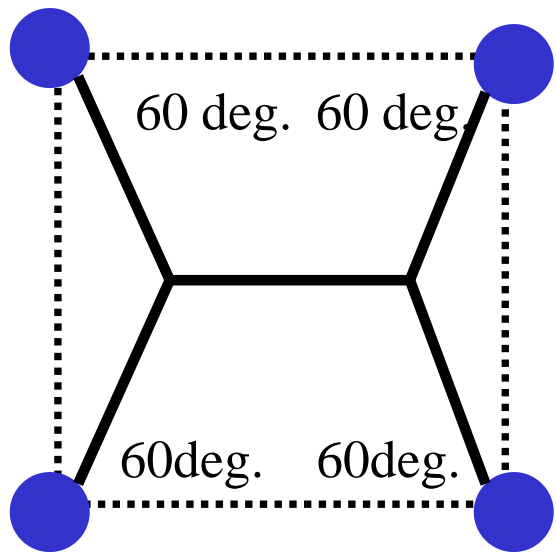
- Connect the dots.
- The number of lines is not limited.
- Two lines can cross.
- Connect the dots so that one can go from one dot to all the other dots.

How do you connect the lines if you want to make the total length of lines the shortest?

(question) how about the case for a square?



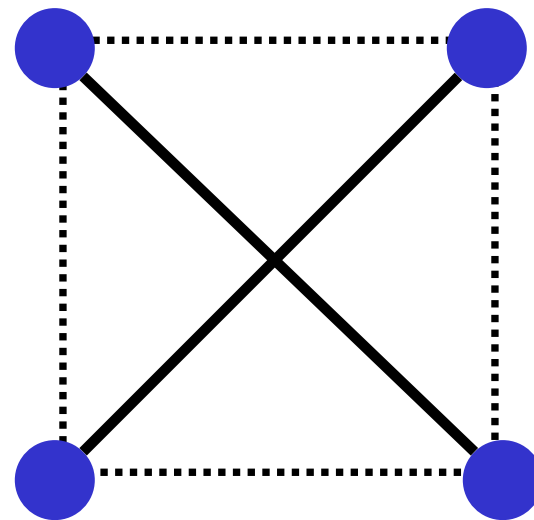
(the answer)



Length

$$\begin{aligned} & 4 \times \frac{1}{\sqrt{3}} + \left( 1 - 2 \times \frac{1}{2\sqrt{3}} \right) \\ & = 1 + \sqrt{3} \\ & = 2.732 \dots \end{aligned}$$

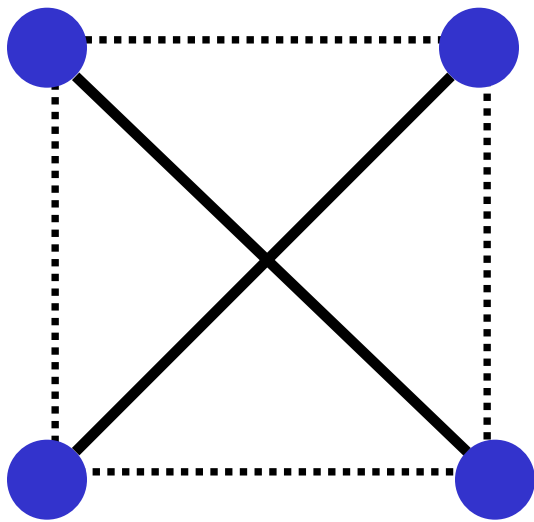
cf.



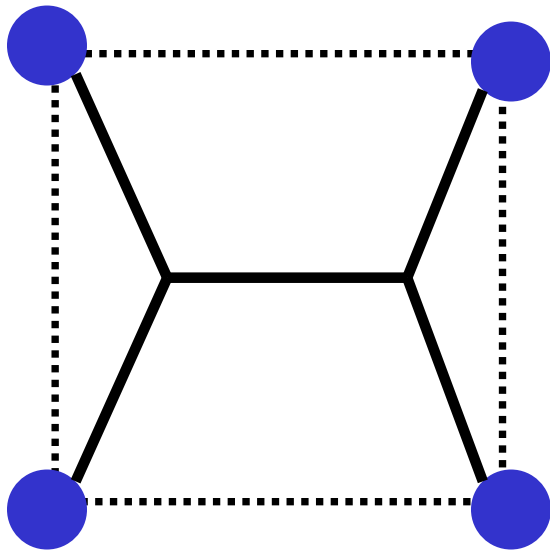
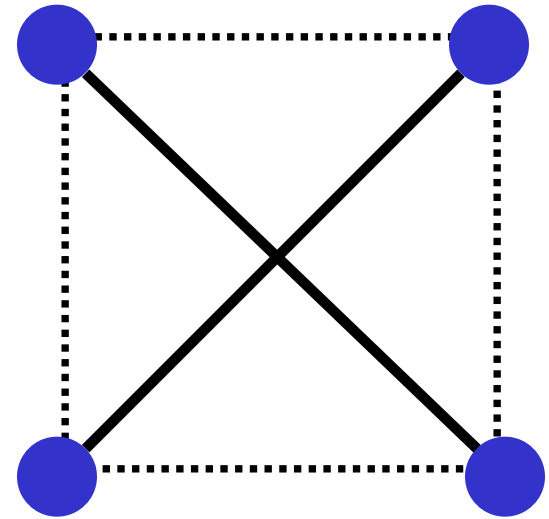
Length

$$2 \times \sqrt{2} = 2.828 \dots$$

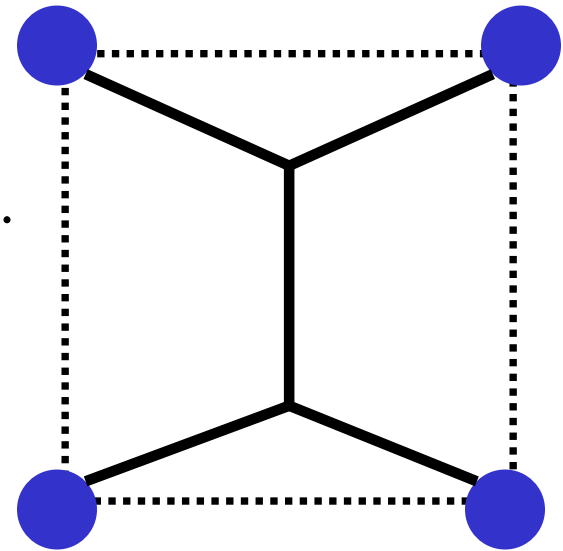
Ref. Takeshi Koike,  
“Genshikaku Kenkyu” Vol. 52 No. 2, p. 14



invariant with  
rotation by 90 deg.



rotation by 90 deg.



a good example of spontaneous symm. breaking

Courtesy: Takeshi Koike