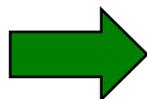
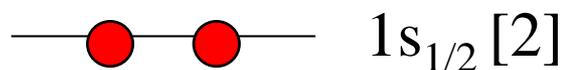
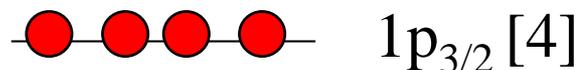
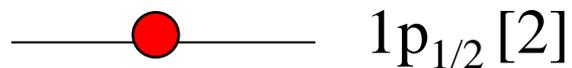


同じように殻模型で $^{11}_4\text{Be}_7$ のレベルを考えると。。。

殻模型 (球形ポテンシャルの準位) で考えた場合:



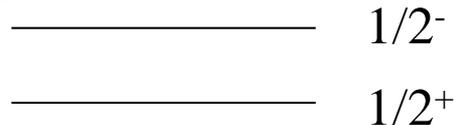
^{11}Be の基底状態は $I^\pi = 1/2^-$

かなり無理

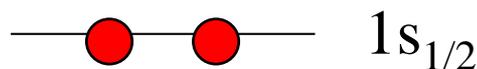
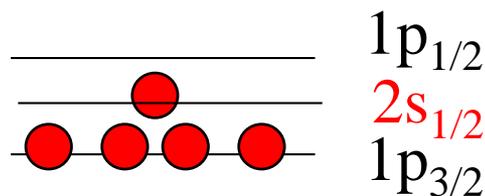


実際の ^{11}Be の準位を見てみると:

0.32 MeV



^{11}Be



球形ポテンシャルに無理があるなら、変形させてみる?

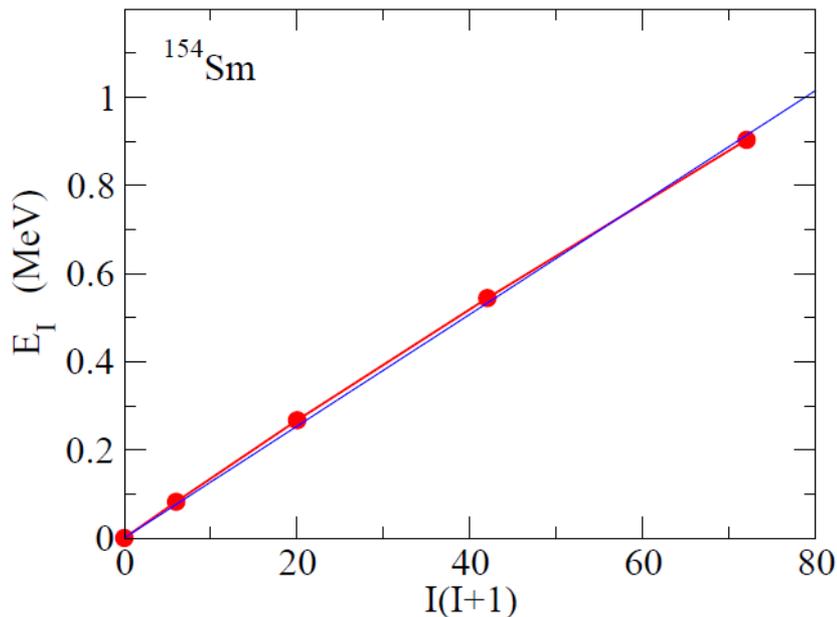
Mean-field approximation and deformation

$$H = \sum_{i=1}^A \left(-\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{MF}}(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i,j}^A v(\mathbf{r}_i, \mathbf{r}_j) - \sum_i V_{\text{MF}}(\mathbf{r}_i)$$

➔ Ψ_{MF} : does not necessarily possess the symmetries that H has.

“Symmetry-broken solution”

“Spontaneous Symmetry Broken”

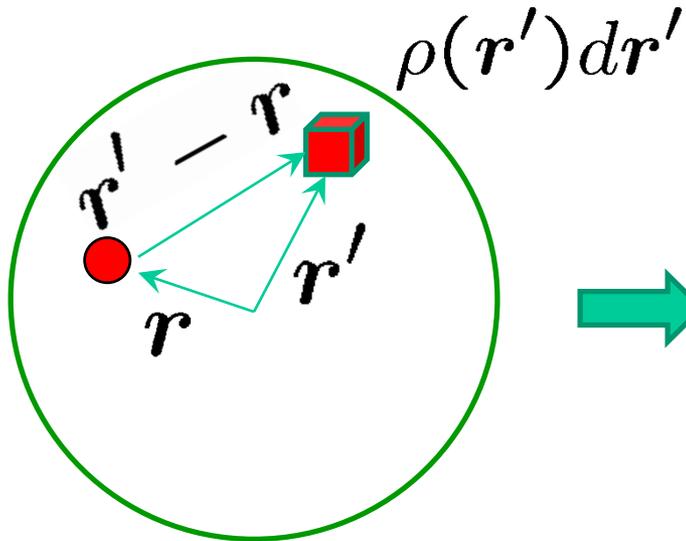


$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

One-particle motion in a deformed potential

平均場

interaction for a nucleon inside a nucleus:



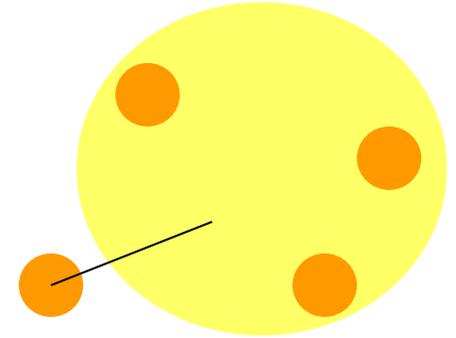
$$v(\mathbf{r}' - \mathbf{r}) \cdot \underline{\rho(\mathbf{r}') d\mathbf{r}'}$$

the number of nucleon
at r'

naively speaking,

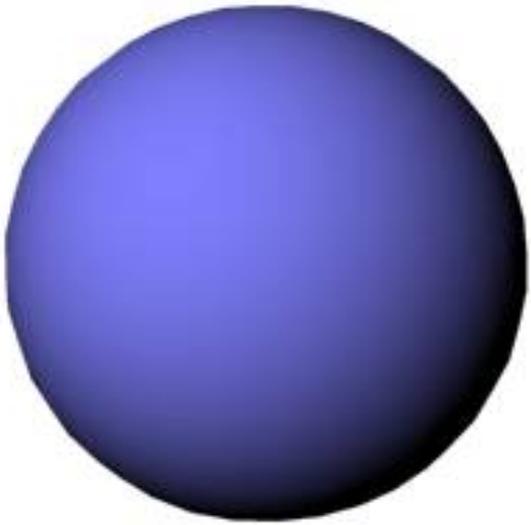
$$V(\mathbf{r}) \sim \int v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}'$$

$$V(\mathbf{r}) \sim \int v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}' \sim -g\rho(\mathbf{r}) \quad \text{if} \quad v(\mathbf{r}, \mathbf{r}') = -g\delta(\mathbf{r} - \mathbf{r}')$$

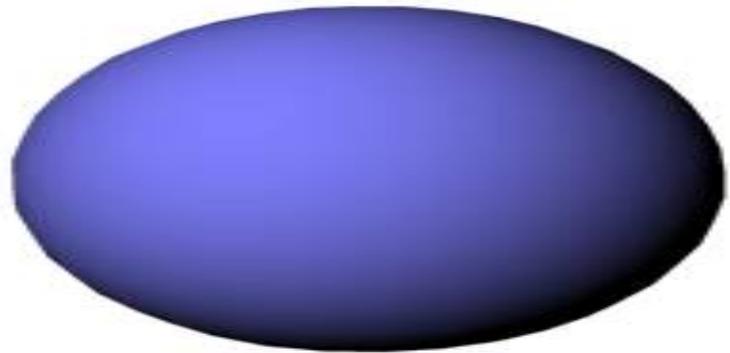


One-particle motion in a deformed potential

$$V(\mathbf{r}) \sim \int v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') d\mathbf{r}' \sim -g\rho(\mathbf{r}) \quad \text{if } v(\mathbf{r}, \mathbf{r}') = -g\delta(\mathbf{r} - \mathbf{r}')$$

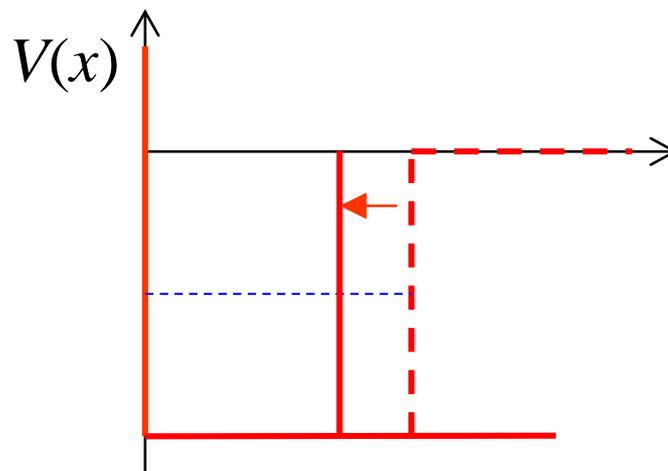
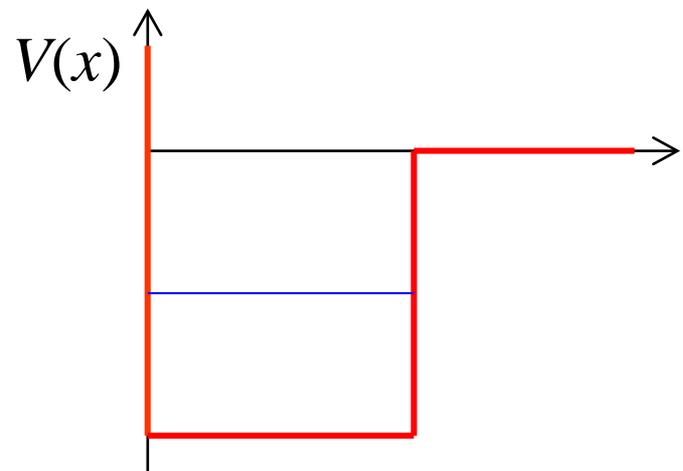
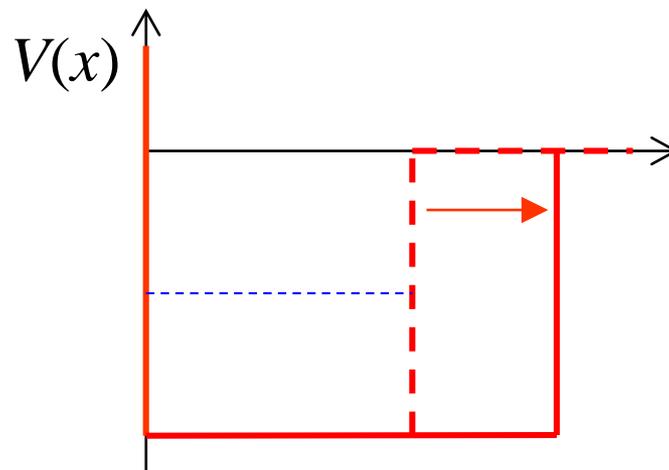
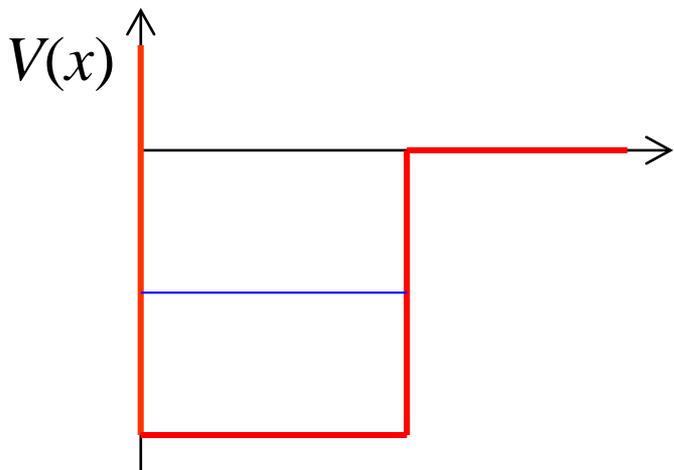


$V(r)$

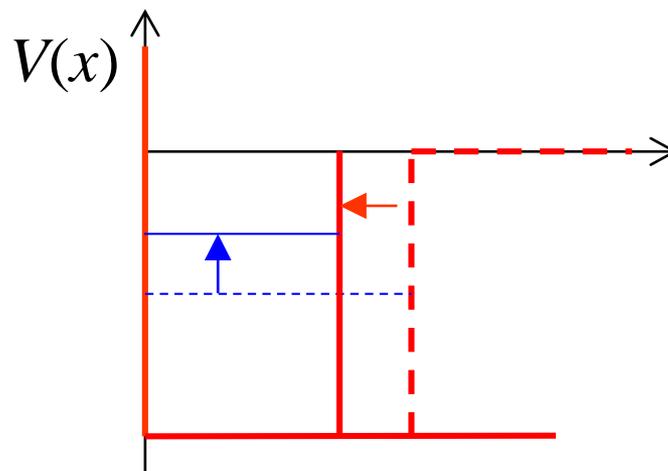
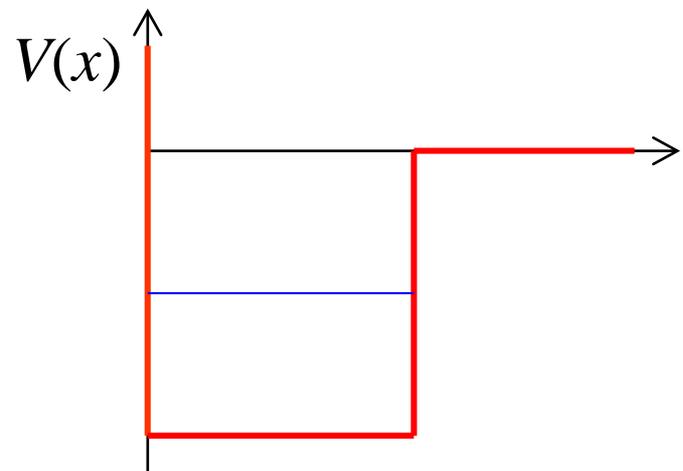
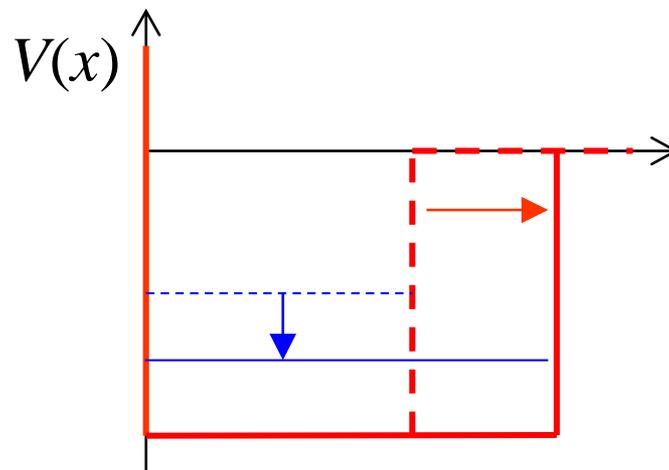
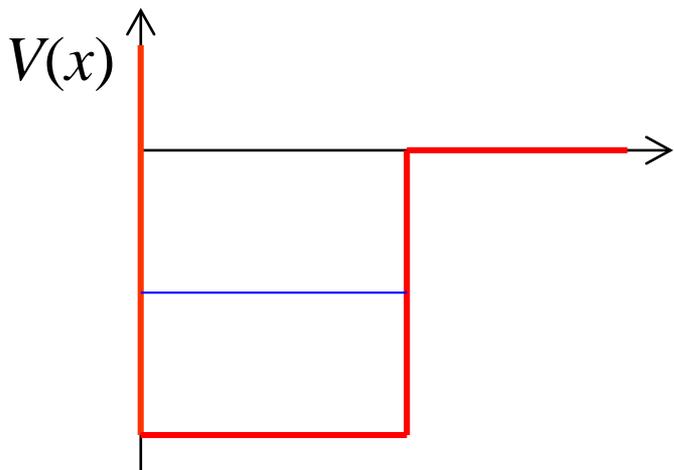


$V(r, \theta)$

(準備) 1次元井戸型ポテンシャル

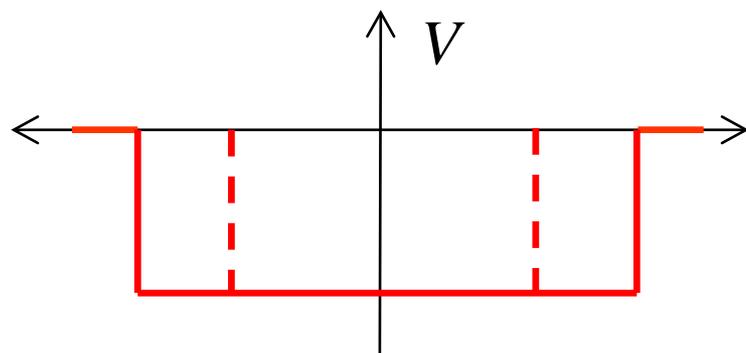
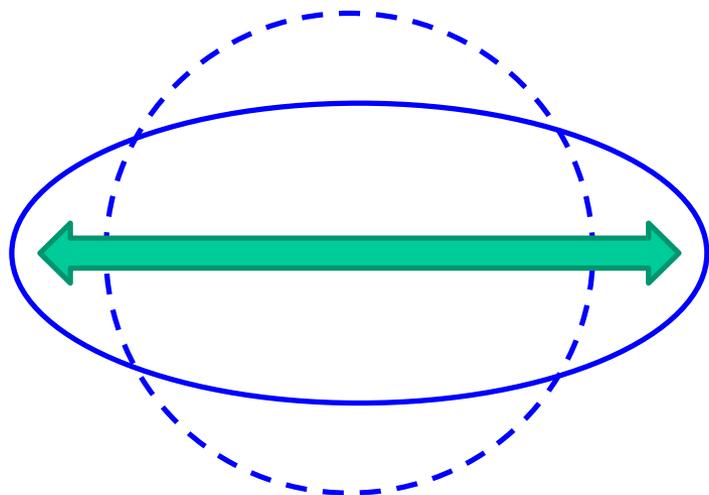


(準備) 1次元井戸型ポテンシャル



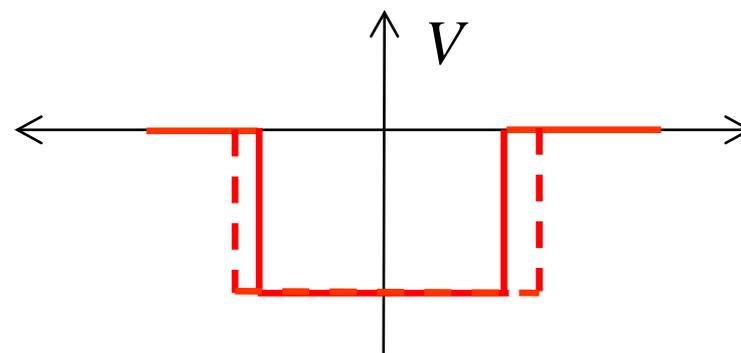
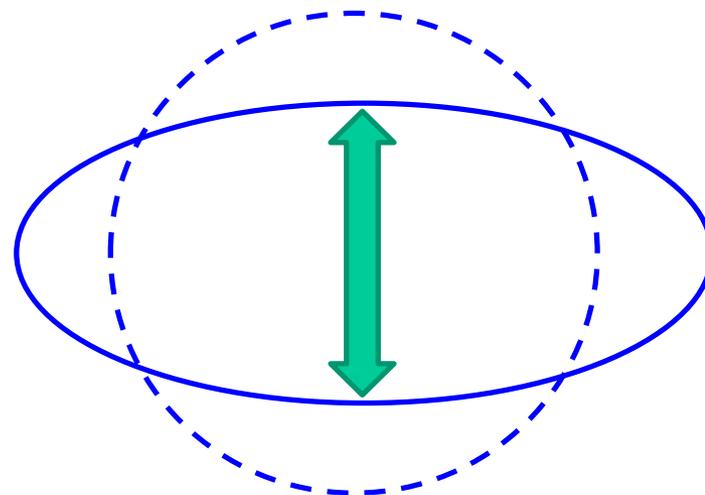
One-particle motion in a deformed potential

長軸に沿った運動



$E \rightarrow$ 小

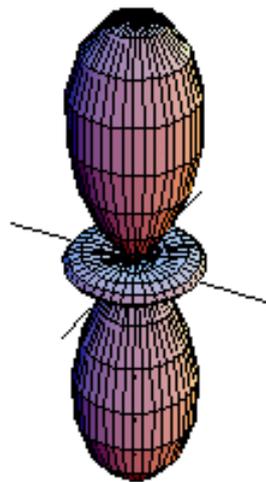
短軸に沿った運動



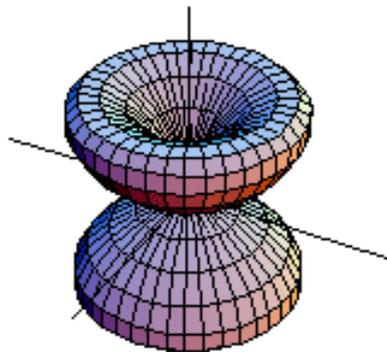
$E \rightarrow$ 大

$l = 2$

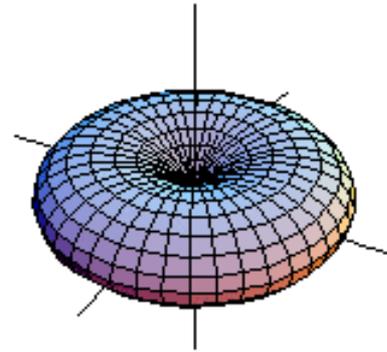
z



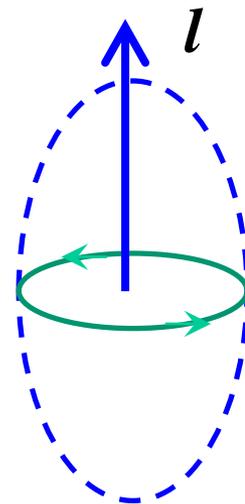
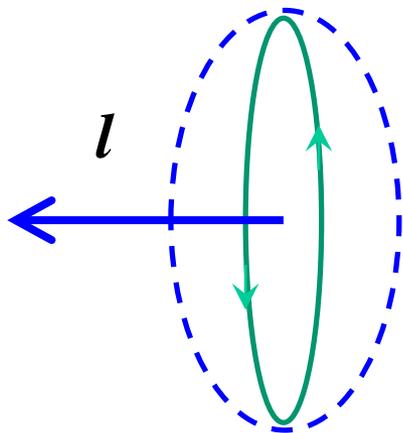
$r = Y_{20}$
($l_z = 0$)



$r = Y_{21}$
($l_z = 1$)

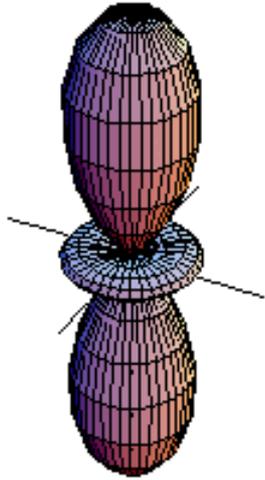


$r = Y_{22}$
($l_z = 2$)

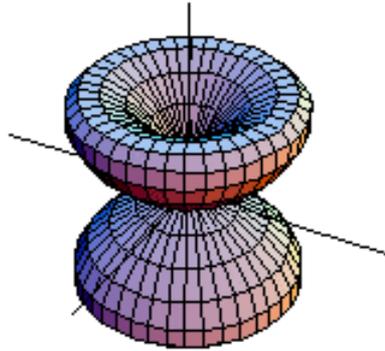


$l = 2$

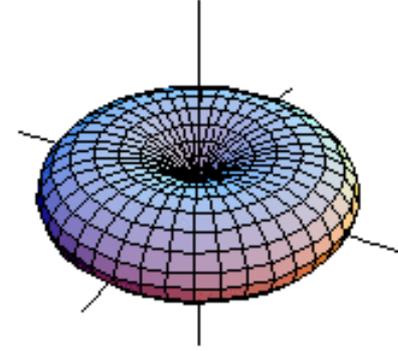
z



$r = Y_{20}$
($l_z = 0$)

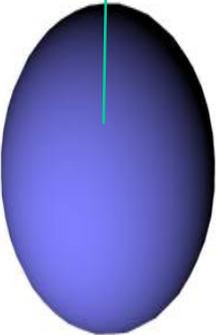


$r = Y_{21}$
($l_z = 1$)



$r = Y_{22}$
($l_z = 2$)

z



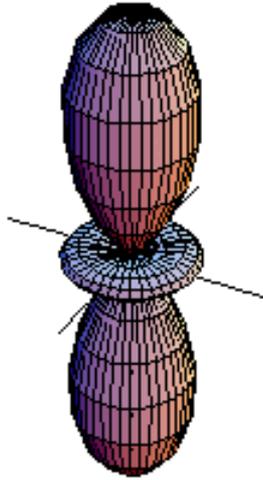
$E \rightarrow$ 小

なら

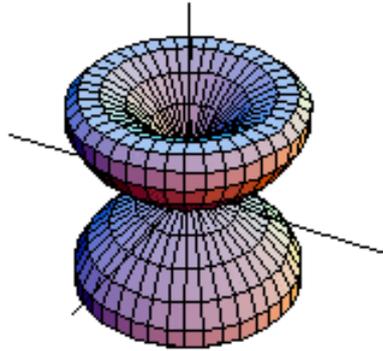
$E \rightarrow$ 大

$l = 2$

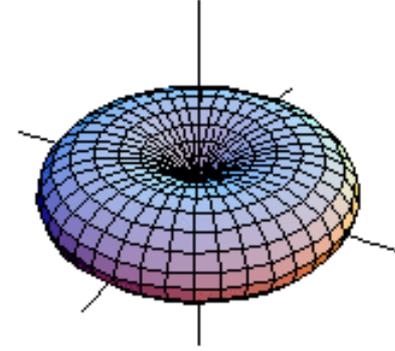
z



$r = Y_{20}$
($l_z = 0$)

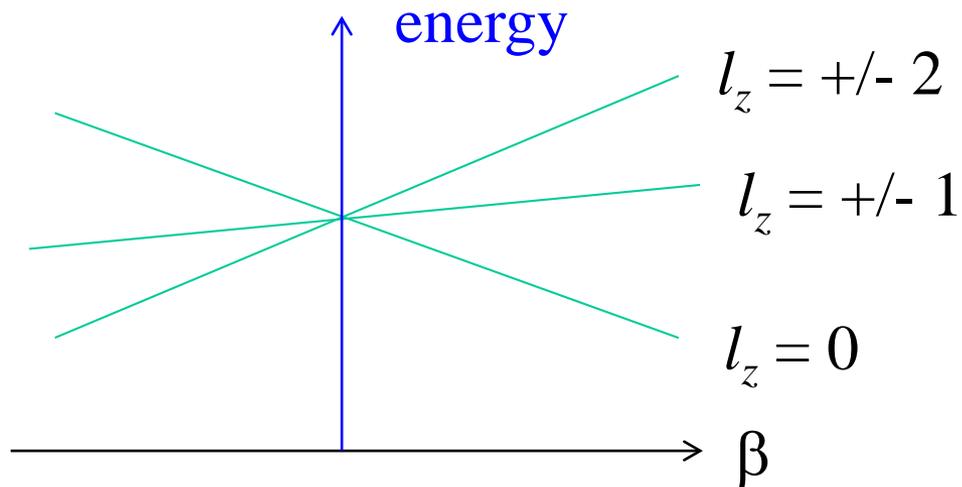


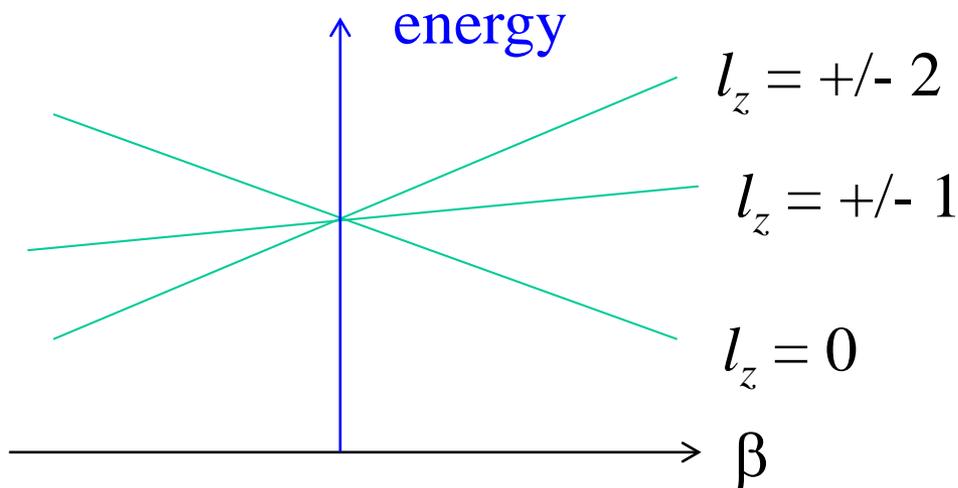
$r = Y_{21}$
($l_z = 1$)



$r = Y_{22}$
($l_z = 2$)

軌道が
スプリット





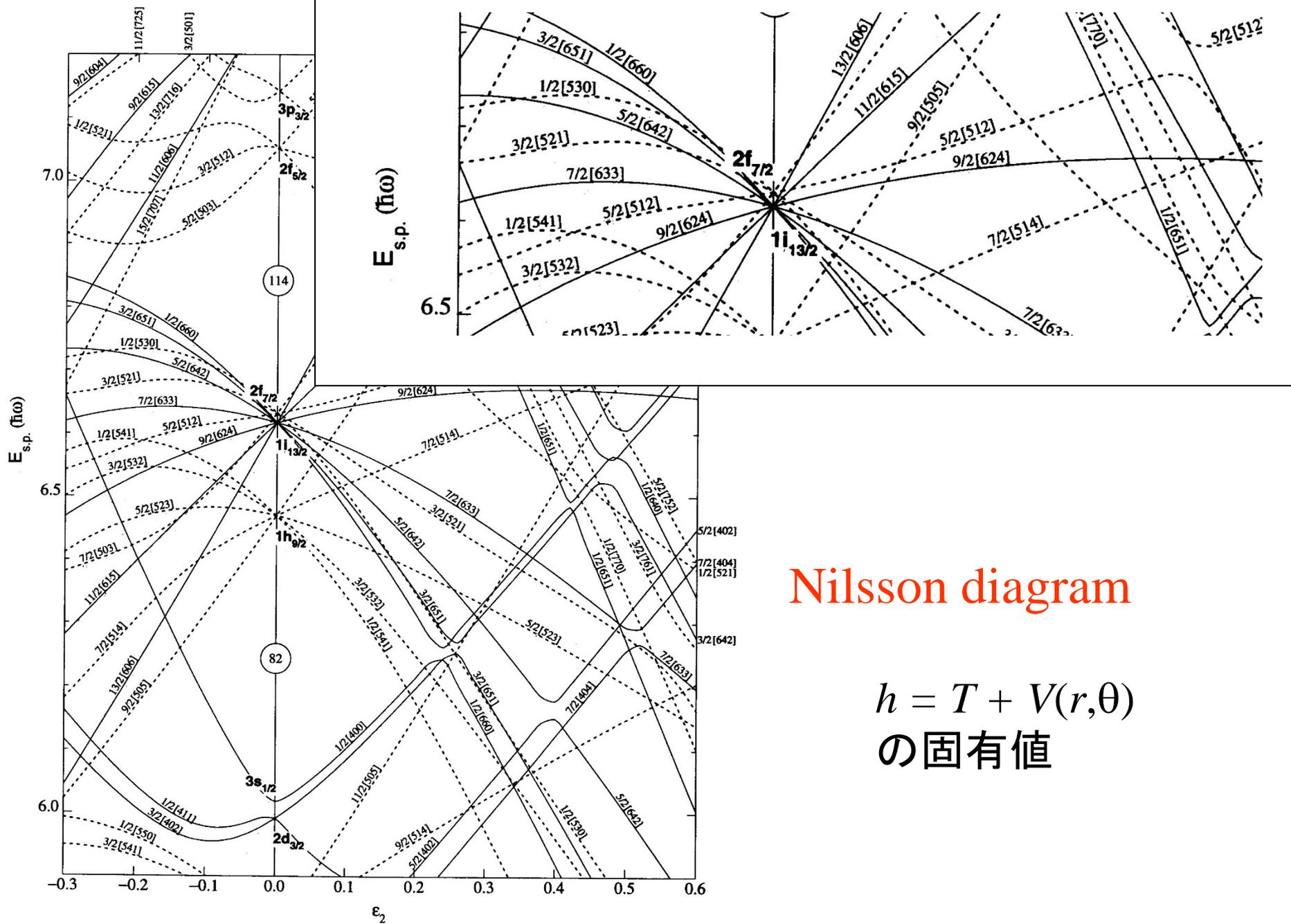
波動関数

- (note) $V(r, \theta) \rightarrow$ 回転対称性を持っていない
 \rightarrow 角運動量がいい量子数ではない

$$\phi_{nll_z}(r, \theta, \phi) \rightarrow \phi_{nl_z}(r, \theta, \phi) = \sum_l \psi_{nl}(r) Y_{ll_z}(\theta, \phi)$$

いろいろな角運動量成分
 が混じる

* 軸対称変形であれば l_z は保存



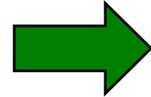
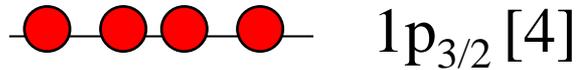
Nilsson diagram

$h = T + V(r, \theta)$
 の固有値

Figure 13. Nilsson diagram for protons, $Z \geq 82$ ($\epsilon_4 = \epsilon_2^2/6$).

Level scheme of $^{11}_4\text{Be}_7$

With a spherical potential:

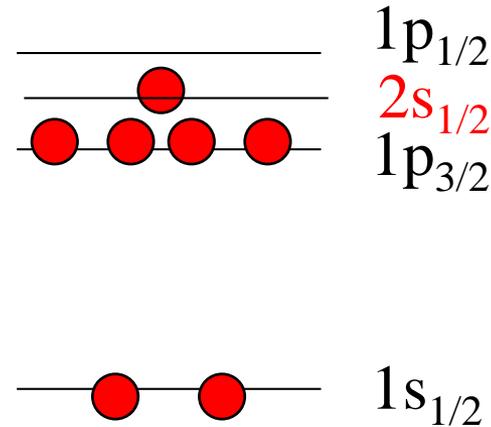
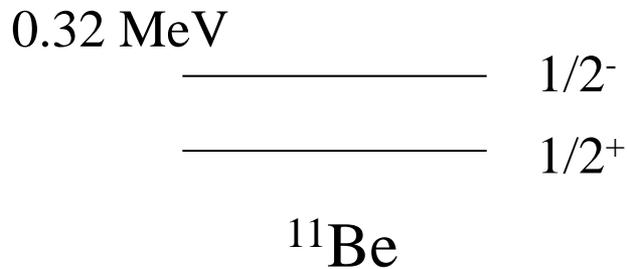


The g.s. of ^{11}Be : $I^\pi = 1/2^-$

very artificial



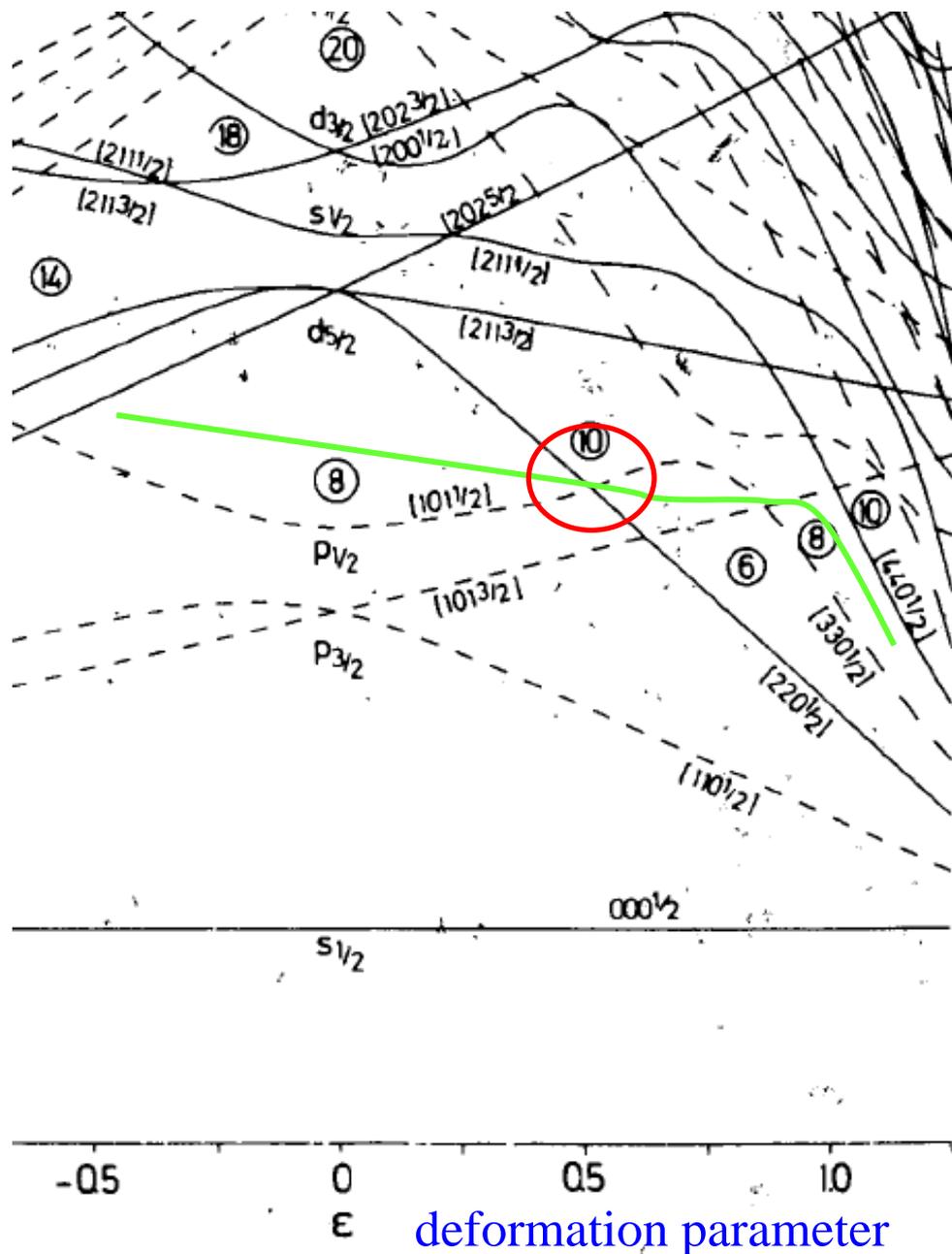
In reality.....



“parity inversion”

What happens if ^{11}Be is deformed?

$^{11}_4\text{Be}_7$



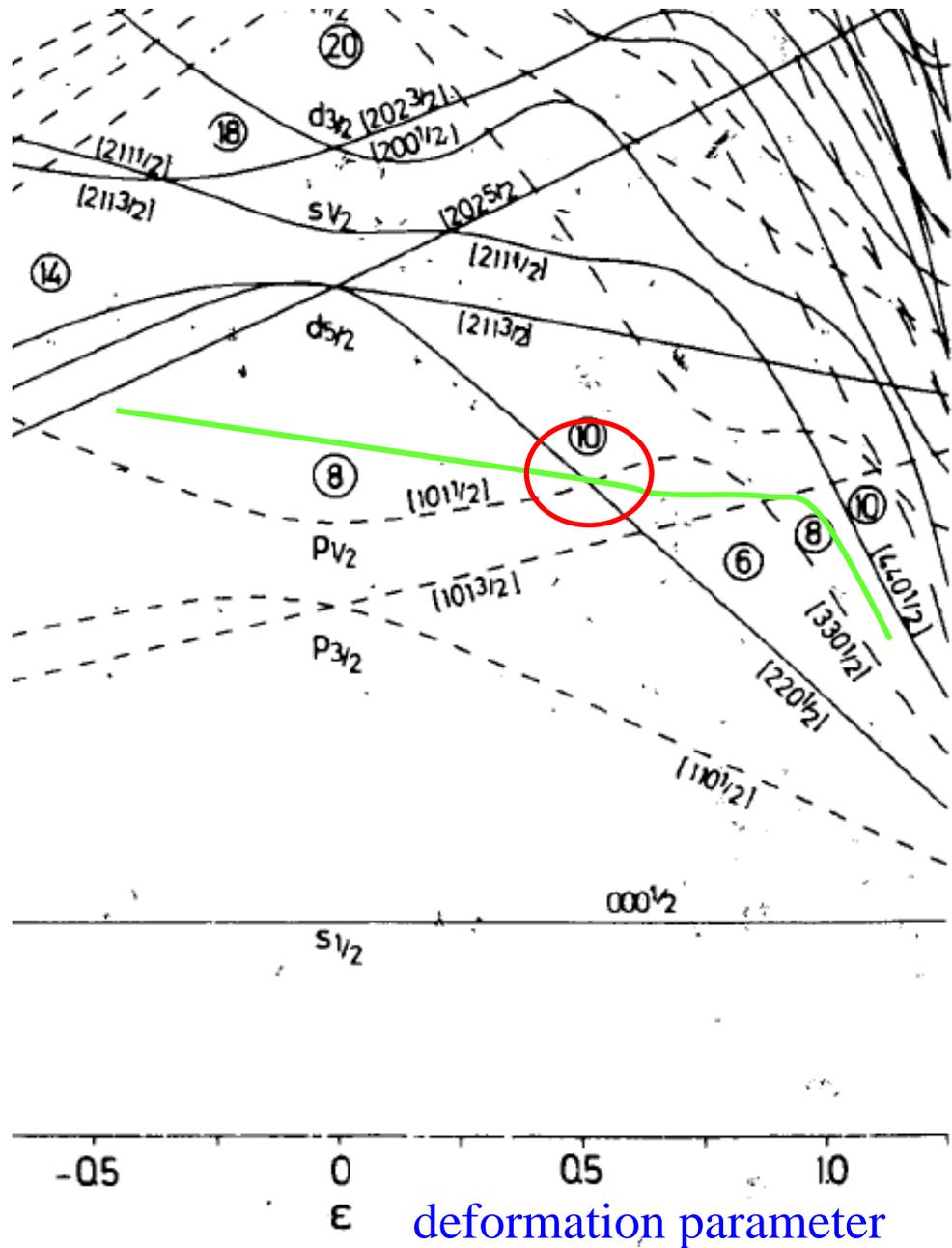
0.32 MeV

————— $1/2^-$
————— $1/2^+$

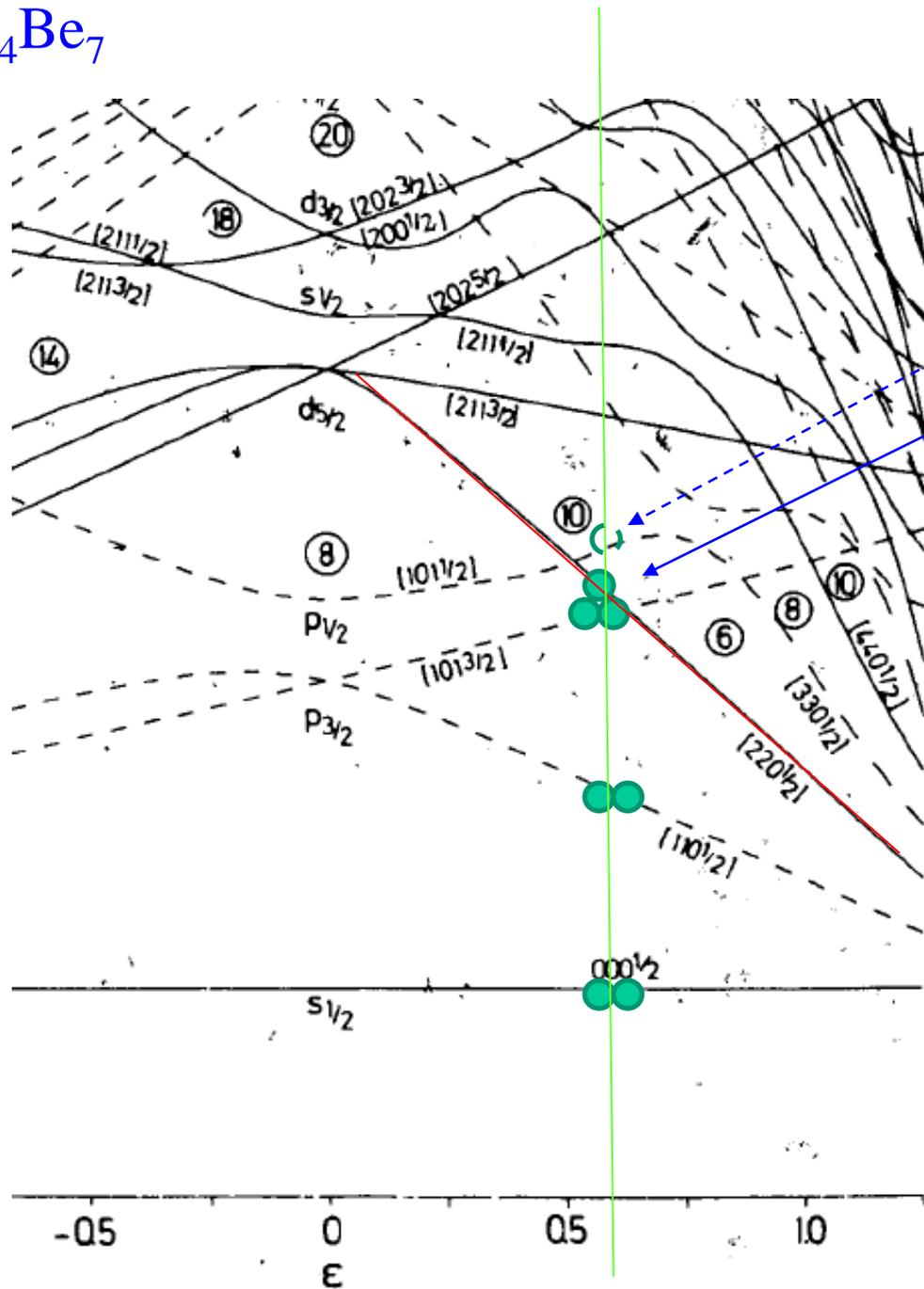
^{11}Be

- ✓ 7番目の中性子の入る軌道を探す
(それぞれの軌道に2つずつ中性子をつめる)

$^{11}_4\text{Be}_7$



$^{11}_4\text{Be}_7$



0.32 MeV

$1/2^-$
 $1/2^+$

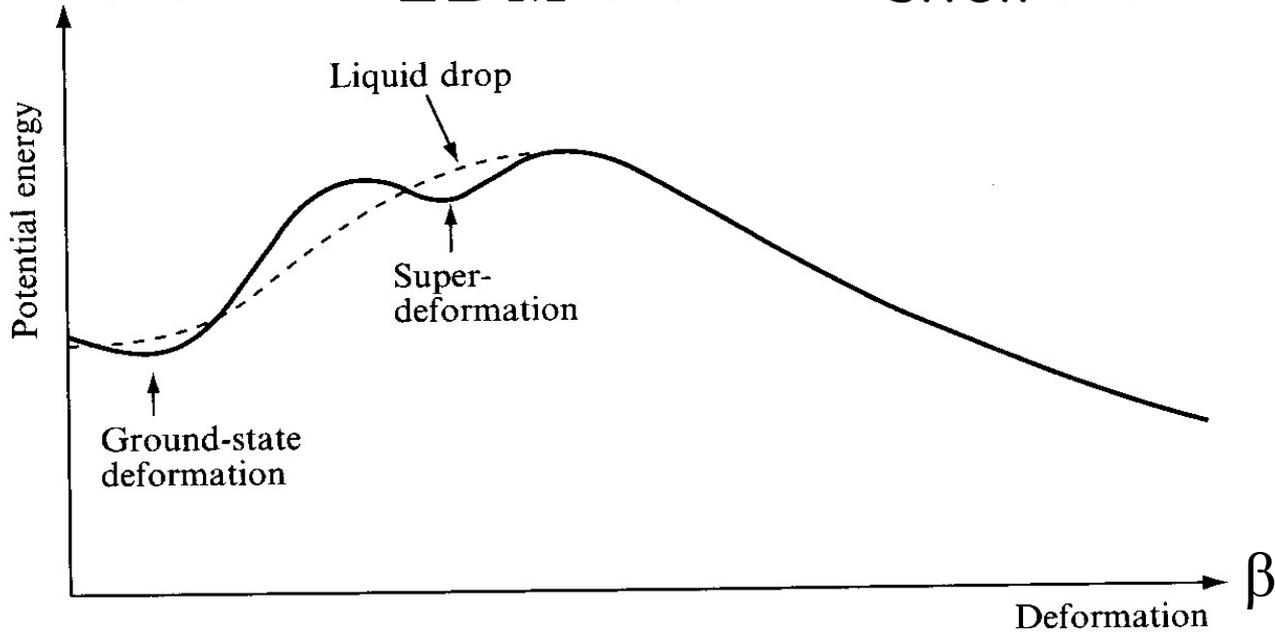
^{11}Be

✓ 7番目の中性子の入る軌道を探す
(それぞれの軌道に2つずつ中性子をつめる)

nuclear deformation

Deformed energy surface for a given nucleus

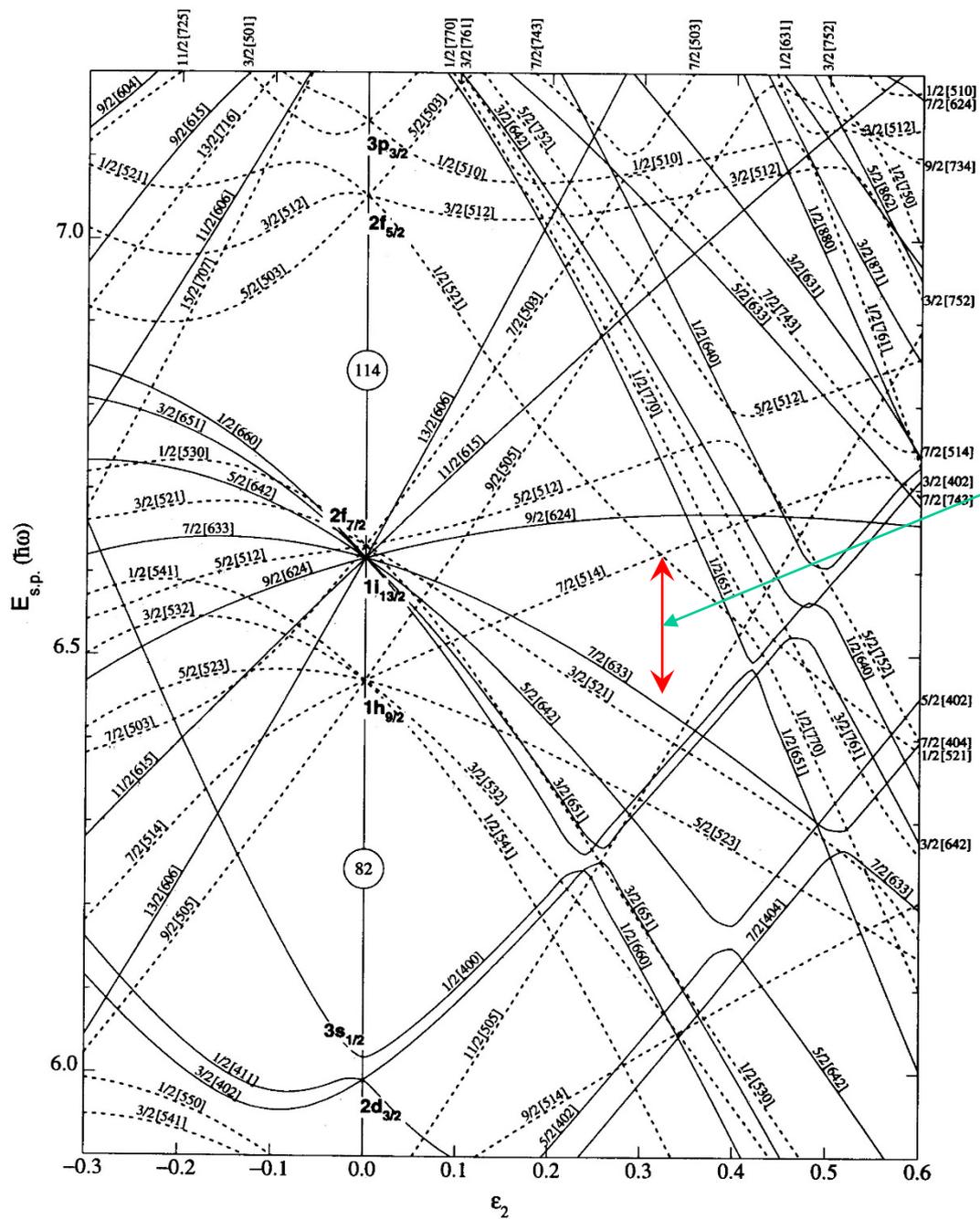
$$E(\beta) = E_{LDM}(\beta) + E_{\text{shell}}(\beta)$$



LDM only



always spherical ground state



energy gap if deformed

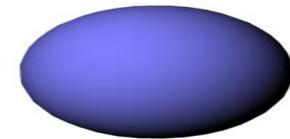
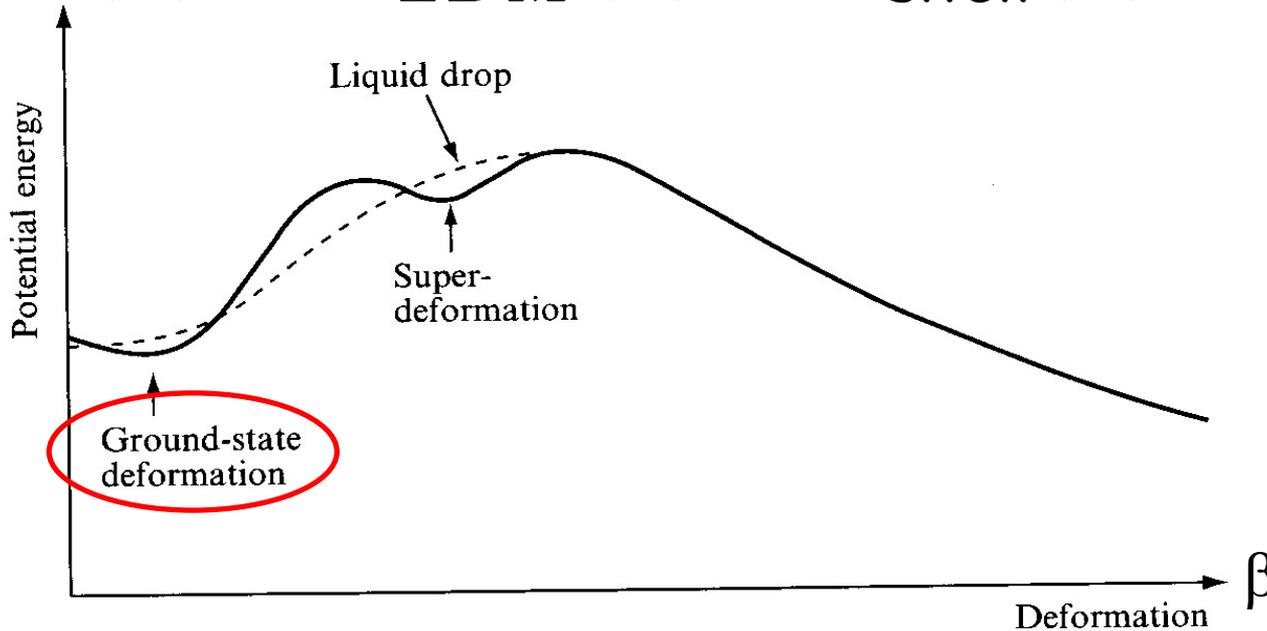
Nilsson diagram

Figure 13. Nilsson diagram for protons, $Z \geq 82$ ($\epsilon_4 = \epsilon_2^2/6$).

nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\text{shell}}(\beta)$$



LDM only \longrightarrow always spherical ground state

Shell correction \longrightarrow may lead to a **deformed g.s.**

* Spontaneous Symmetry Breaking

レポート問題3

3次元非等方調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_z^2 z^2 + \frac{1}{2}m\omega_{\perp}^2 (x^2 + y^2)$$

$$\omega_{\perp} = \omega_0 \left(1 + \frac{\epsilon}{3}\right)$$

$$\omega_z = \omega_0 \left(1 - \frac{2}{3}\epsilon\right)$$

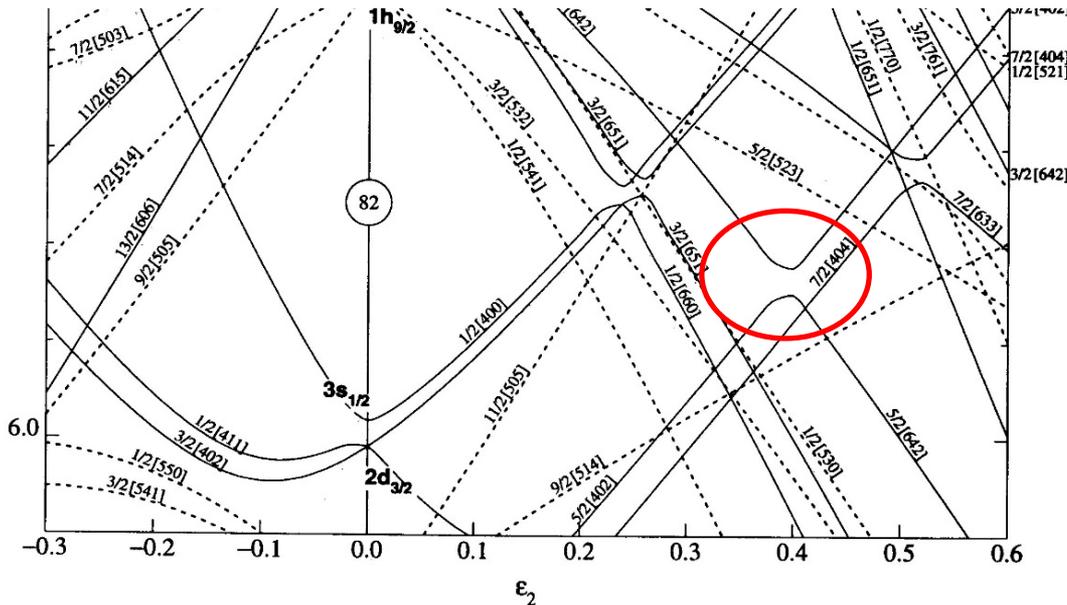
を考える。 ϵ を0から1まで変化させるとき、 $\epsilon = 0$ の時の基底状態、第一励起状態、第二励起状態のエネルギーはどのように変化するか図を使って議論せよ。

レポート問題4

$E_0 = -\epsilon, E_1 = \epsilon$ のエネルギーをもつ2つの状態が強さ V で相互作用しているとする。このときの固有状態は 2×2 行列

$$\begin{pmatrix} -\epsilon & V \\ V & \epsilon \end{pmatrix}$$

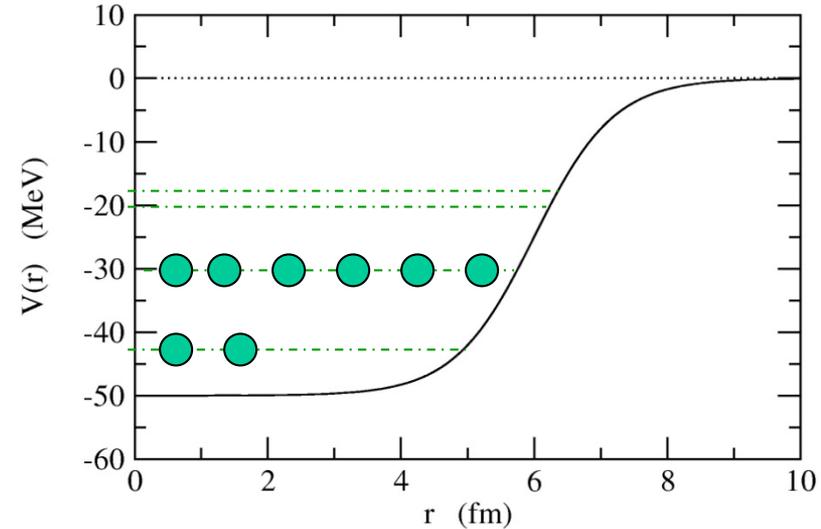
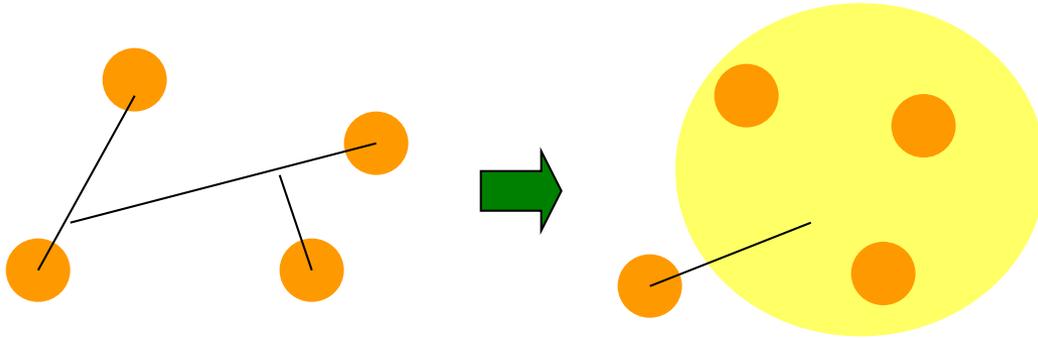
を対角化して得られる。2つの固有エネルギーの差が必ず 2ϵ ($V=0$ のときのエネルギー差) より大きくなることを示せ。



*これを「ノイマン-ウィグナーの定理」といい、ニルソンレベルで準位反発が見られる理由である。

Pairing Correlation (対相関)

平均場近似

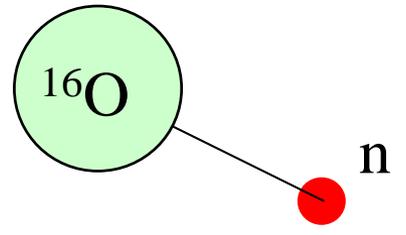
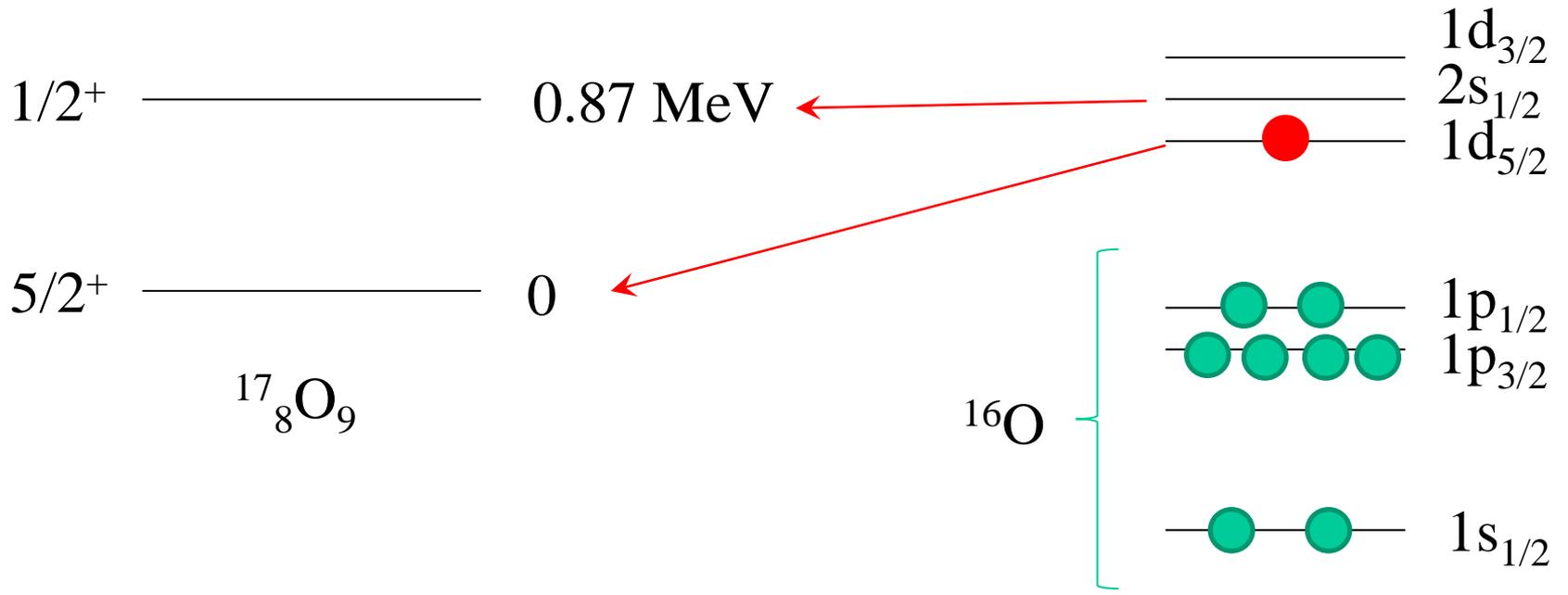


ポテンシャルの中での
独立粒子描像

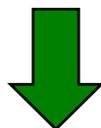
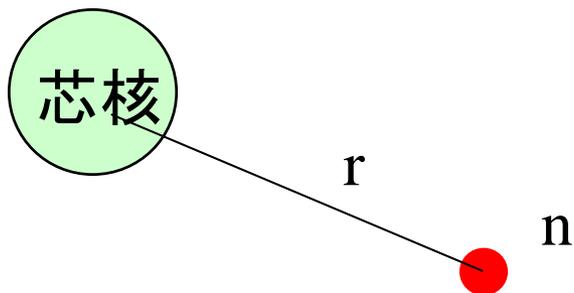
核子間の相互作用:
核子の感じるポテンシャル
としてののみ



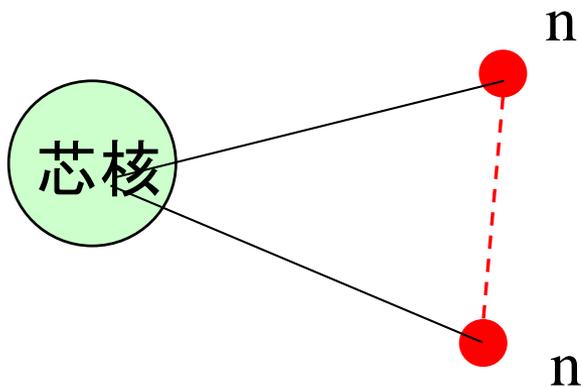
魔法数の説明



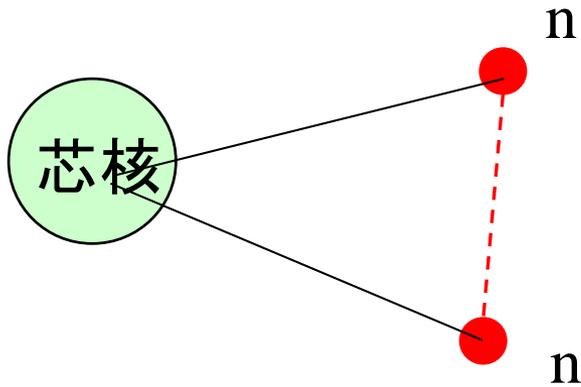
対相関



芯核のまわりに中性子が2個あるとどうなる？

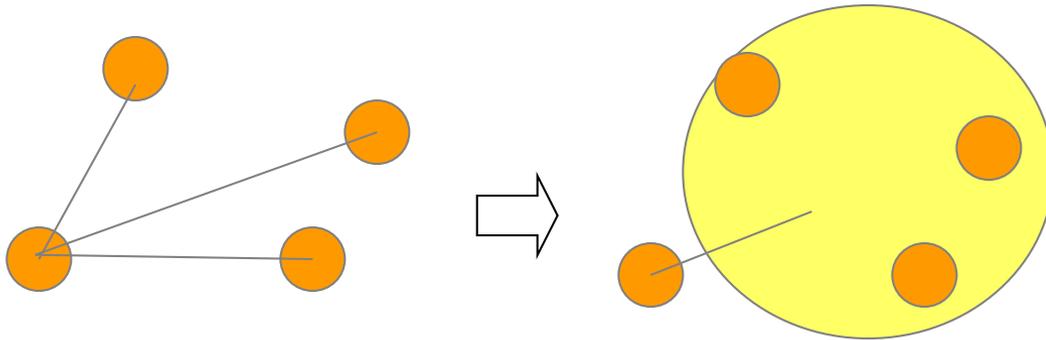


2中性子間に働く相互作用の影響は？



2中性子間に働く相互作用の影響は?

平均場理論



他の核子との相互作用を平均的に取り扱う

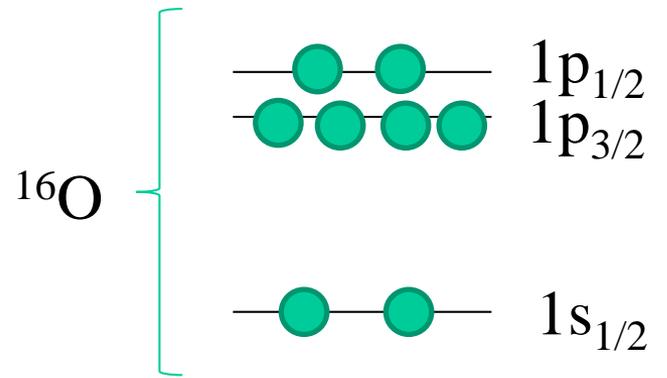
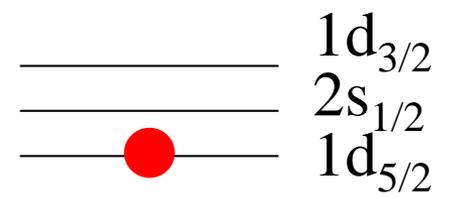
単純な平均場近似 → 2中性子が独立に運動
(2中性子間の相互作用は平均ポテンシャル
にのみ反映される)

$1/2^+$ ————— 0.87 MeV

$5/2^+$ ————— 0
 $^{17}_8\text{O}_9$

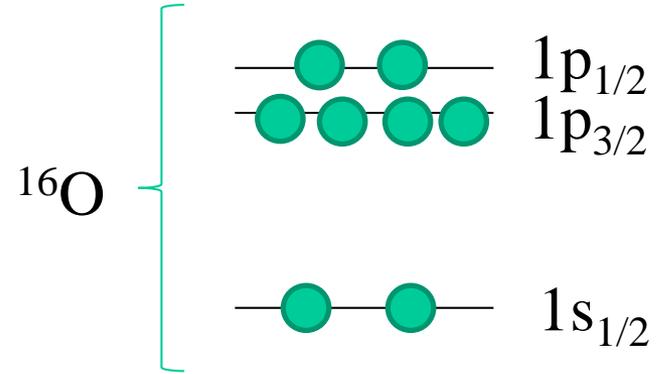
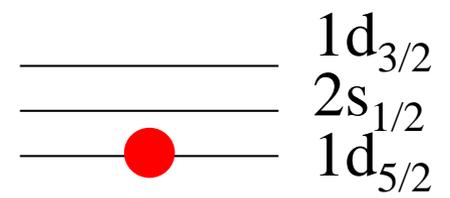


^{18}O の状態を予想すると。。。

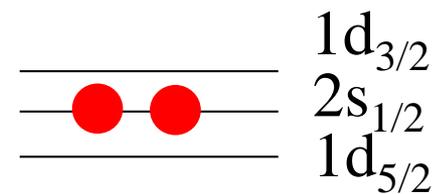
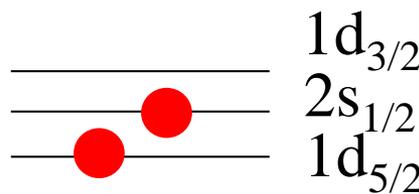
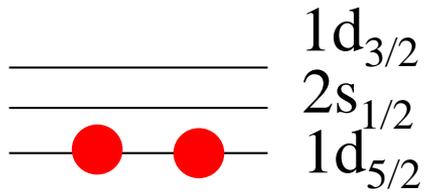


$1/2^+$ ————— 0.87 MeV

$5/2^+$ ————— 0
 $^{17}_8\text{O}_9$



→ ^{18}O の状態を予想すると。。。



$E = 0$

$I^\pi = d_{5/2} \times d_{5/2}$
 $= 0^+, 2^+, 4^+$

$E = 0.87 \text{ MeV}$

$I^\pi = d_{5/2} \times s_{1/2}$
 $= 2^+, 3^+$

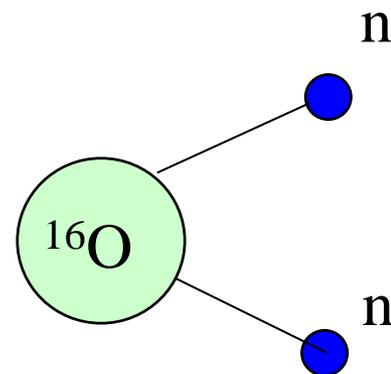
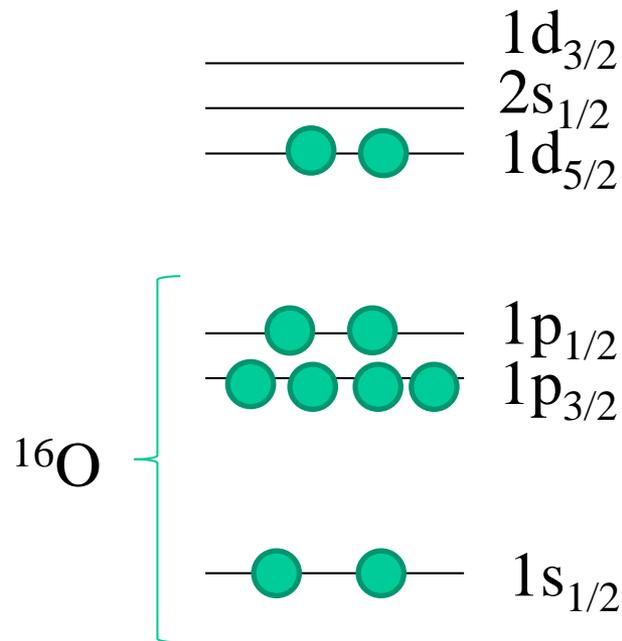
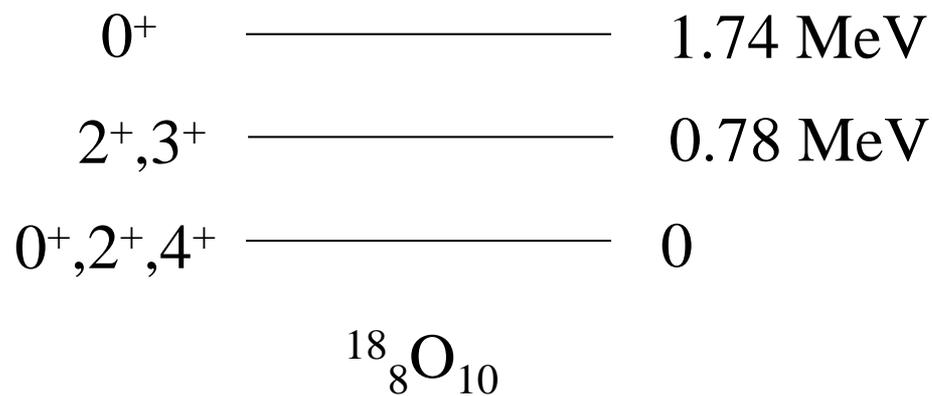
$E = 0.87 \times 2 = 1.74 \text{ MeV}$

$I^\pi = s_{1/2} \times s_{1/2} = 0^+$

2 MeV 以下に少なくとも6本の状態(?)

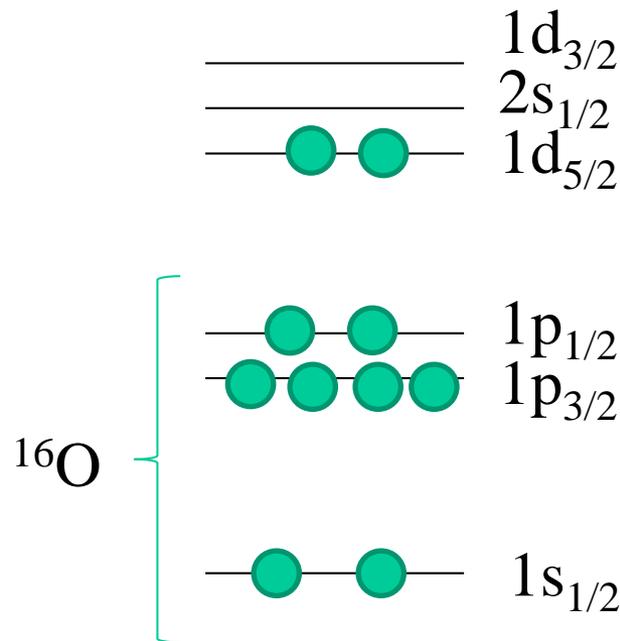
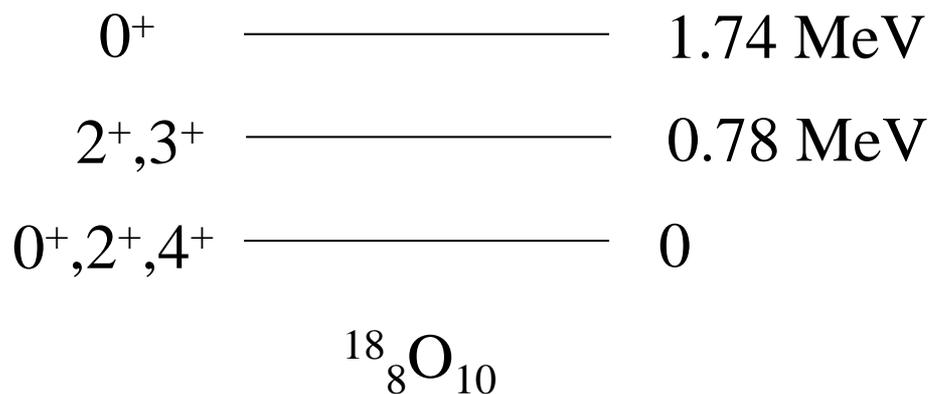
単純な平均場近似:

6 levels

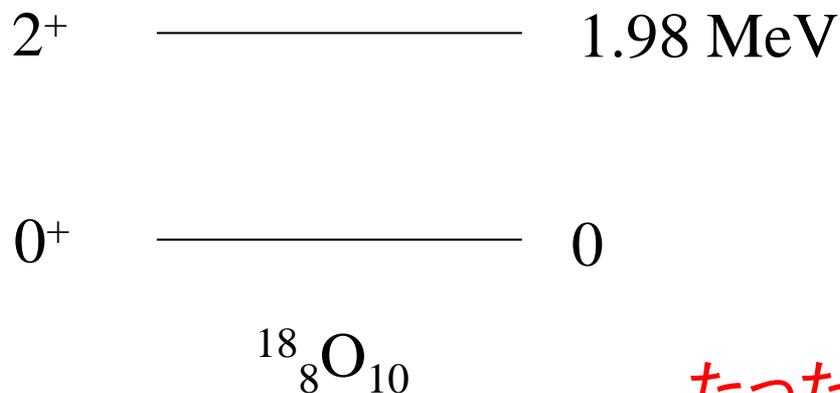


単純な平均場近似:

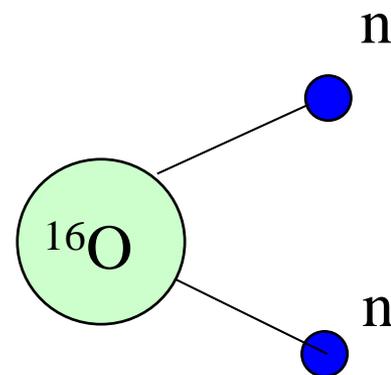
6 levels



実際には:



たったの2本!



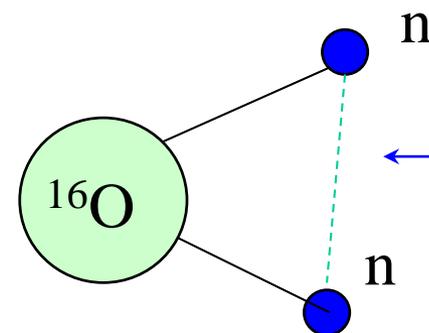
$$H = \sum_i T_i + \sum_{i < j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i < j} v_{ij} - \sum_i V_i}_{\text{平均からのずれ (残留相互作用)}}$$

平均からのずれ
(残留相互作用)

残留相互作用は完全に無視してもよいのか?

答え: no

開殻原子核では重要な役割を果たす
ことが知られている(ペアリング)



(note) 摂動論がよい条件

$$H = H_0 + \Delta V$$

$$H_0 |\phi_n^{(0)}\rangle = E_n^{(0)} |\phi_n^{(0)}\rangle$$

$$\rightarrow |\phi_n\rangle = |\phi_n^{(0)}\rangle + \sum_{m \neq n} \frac{\langle \phi_m^{(0)} | \Delta V | \phi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} |\phi_m^{(0)}\rangle$$



$$|\langle \phi_m^{(0)} | \Delta V | \phi_n^{(0)} \rangle| \ll |E_n^{(0)} - E_m^{(0)}|$$

なら ΔV を無視できる

$$H = H_0 + \Delta V$$

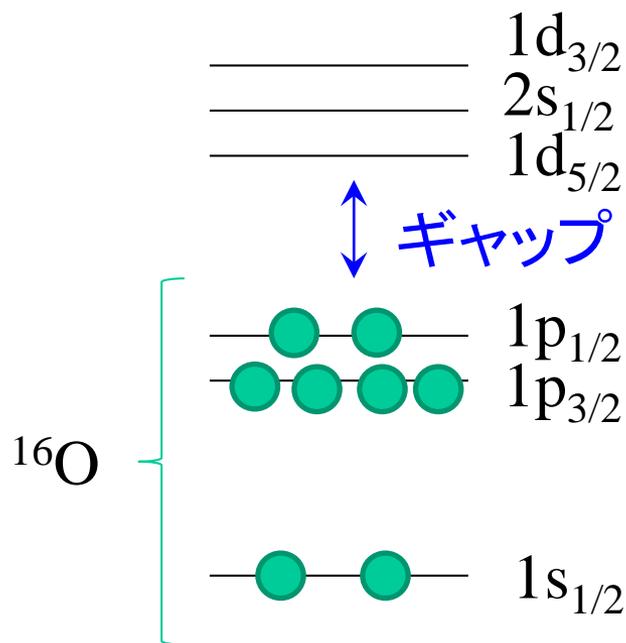


$$|\langle \phi_m^{(0)} | \Delta V | \phi_n^{(0)} \rangle| \ll |E_n^{(0)} - E_m^{(0)}|$$

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$$H = \sum_i T_i + \sum_{i < j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i < j} v_{ij} - \sum_i V_i}_{\text{平均からのずれ (残留相互作用)}}$$

平均からのずれ
(残留相互作用)



閉殻核:

ギャップのために ΔE が大きい

→ 残留相互作用を無視できる

$$H = H_0 + \Delta V$$



$$|\langle \phi_m^{(0)} | \Delta V | \phi_n^{(0)} \rangle| \ll |E_n^{(0)} - E_m^{(0)}|$$

なら ΔV を無視できる

$$H = \sum_i T_i + \sum_{i < j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i < j} v_{ij} - \sum_i V_i}_{\text{平均からのずれ (残留相互作用)}}$$

0^+ ————— 1.74 MeV

$2^+, 3^+$ ————— 0.78 MeV

$0^+, 2^+, 4^+$ ————— 0

$^{18}_8\text{O}_{10}$

6 levels

平均からのずれ
(残留相互作用)

開殻核:

ΔE が小さい

→ 残留相互作用を無視
できない

対相関(ペアリング)

$$H = \sum_{i=1}^A \left(-\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{HF}}(i) \right) + \frac{1}{2} \sum_{i,j} v(\mathbf{r}_i, \mathbf{r}_j) - \sum_i V_{\text{HF}}(i)$$

簡単のために、残留相互作用としてデルタ関数を仮定してみる
(超短距離力)

$$v_{\text{res}}(\mathbf{r}, \mathbf{r}') \sim -g \delta(\mathbf{r} - \mathbf{r}')$$

摂動論で残留相互作用の効果を見積もってみる:

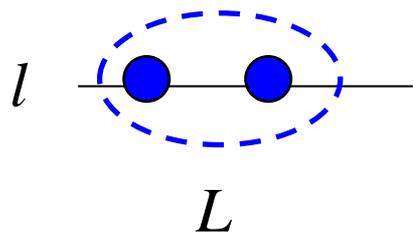
対相関(ペアリング)

$$H = \sum_{i=1}^A \left(-\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{HF}}(i) \right) + \frac{1}{2} \sum_{i,j} v(\mathbf{r}_i, \mathbf{r}_j) - \sum_i V_{\text{HF}}(i)$$

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摂動論で残留相互作用の効果を見積もってみる:



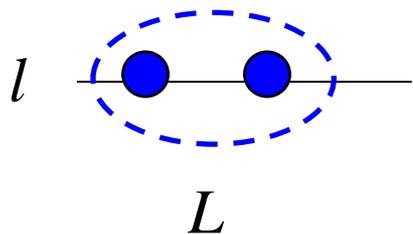
非摂動な波動関数:

角運動量 l の状態に中性子2個、それが
全角運動量 L を組んでいる

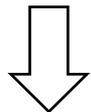
$$|(ll)LM\rangle = \sum_{m,m'} \langle lmlm' | LM \rangle \psi_{lm}(\mathbf{r}) \psi_{lm'}(\mathbf{r}')$$

対相関(ペアリング)

$$v_{\text{res}}(\mathbf{r}, \mathbf{r}') \sim -g \delta(\mathbf{r} - \mathbf{r}')$$



$$|(ll)LM\rangle = \sum_{m,m'} \langle lmlm'|LM\rangle \psi_{lm}(\mathbf{r}) \psi_{lm'}(\mathbf{r}')$$



残留相互作用によるエネルギー変化:

$$\begin{aligned} \Delta E_L &= \langle (ll)LM | v_{\text{res}} | (ll)LM \rangle \\ &= -g I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \end{aligned}$$

$$I_r^{(l)} = \int_0^\infty r^2 dr (R_l(r))^4$$

$$\Delta E_L = -g I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \equiv -g I_r^{(l)} \frac{A(l; L)}{4\pi}$$

$A(l; L)$	$L=0$	$L=2$	$L=4$	$L=6$	$L=8$
$l=2$	5.00	1.43	1.43	---	---
$l=3$	7.00	1.87	1.27	1.63	---
$l=4$	9.00	2.34	1.46	1.26	1.81

$$\Delta E_L = -g I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \equiv -g I_r^{(l)} \frac{A(l; L)}{4\pi}$$

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$$\Delta E_L = -g I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \equiv -g I_r^{(l)} \frac{A(l; L)}{4\pi}$$

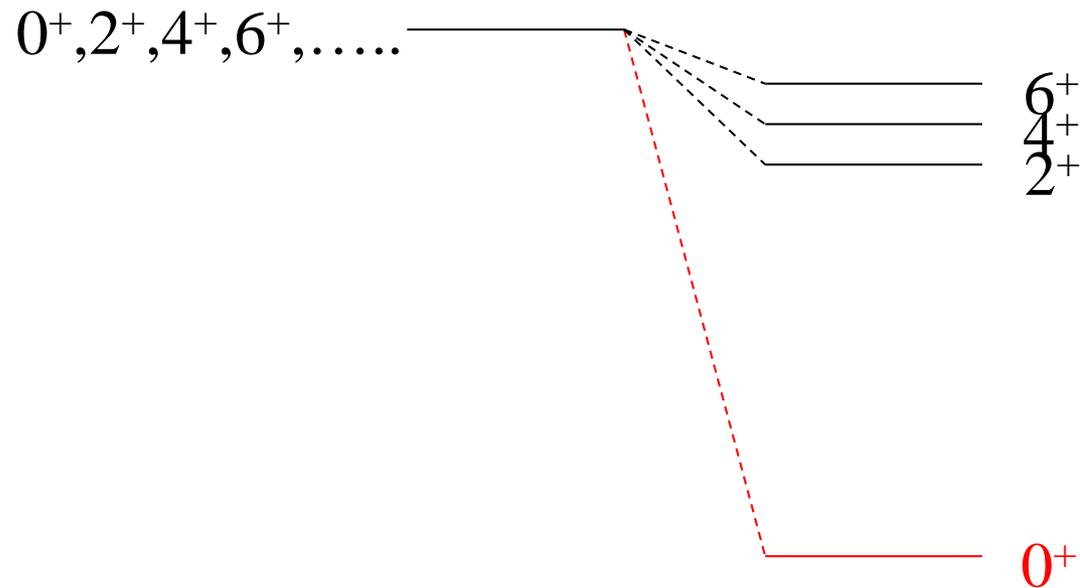
$A(l; L)$	$L=0$	$L=2$	$L=4$	$L=6$	$L=8$
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$l=3$	7.00	1.87	1.27	1.63	---
$l=4$	9.00	2.34	1.46	1.26	1.81

$0^+, 2^+, 4^+, 6^+, \dots$ —————

残留相互
作用なし

$$\Delta E_L = -g I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \equiv -g I_r^{(l)} \frac{A(l; L)}{4\pi}$$

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残留相互
作用なし

残留相互
作用あり