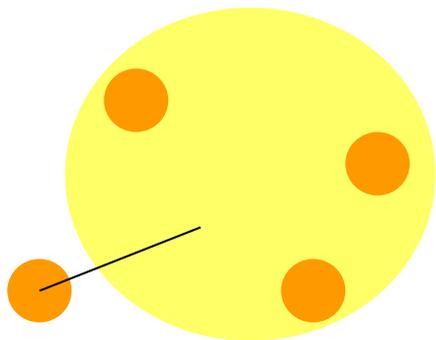
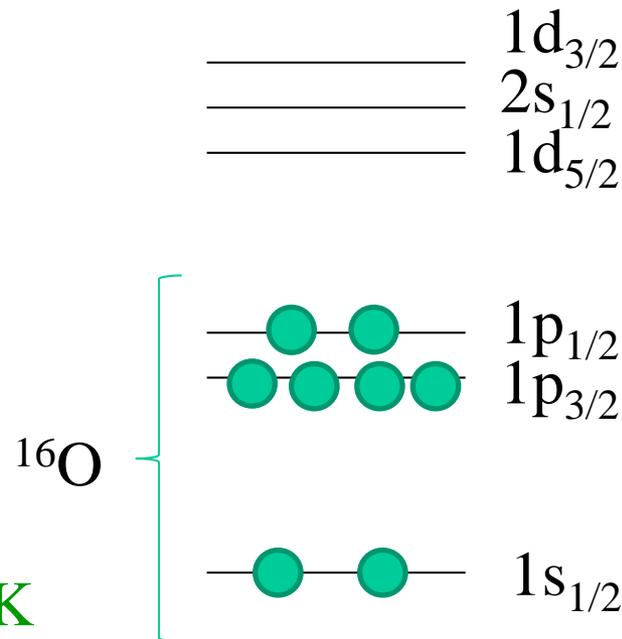


# 対相関

単純な平均場近似(独立粒子描像):



閉殻の場合はOK

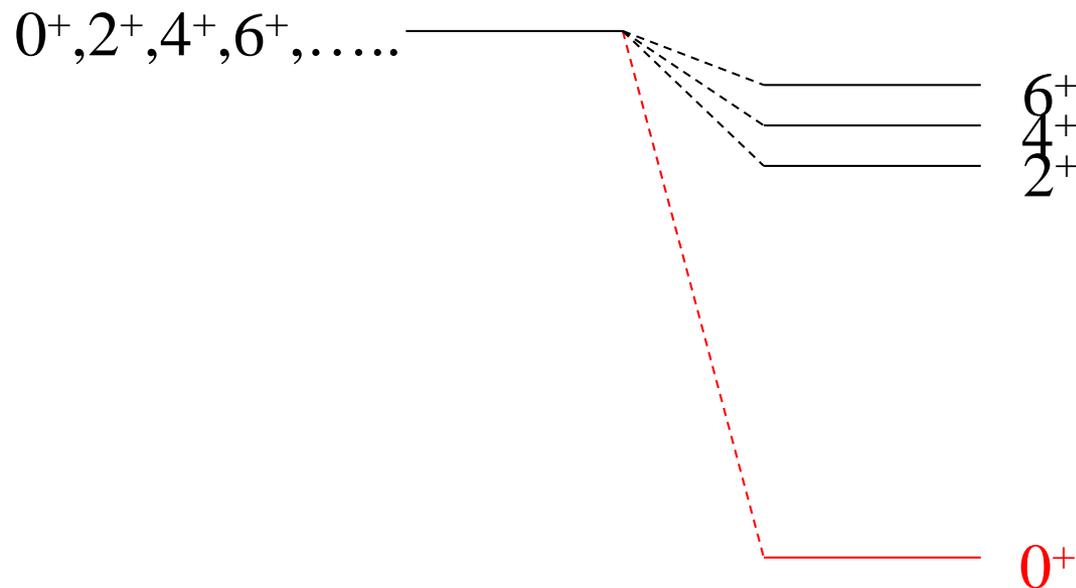
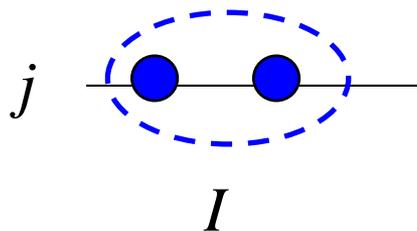


開殻原子核では「対相関」が重要

$$H = \sum_i T_i + \sum_{i < j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \sum_{i < j} v_{ij} - \sum_i V_i$$

平均からのずれ  
(残留相互作用)

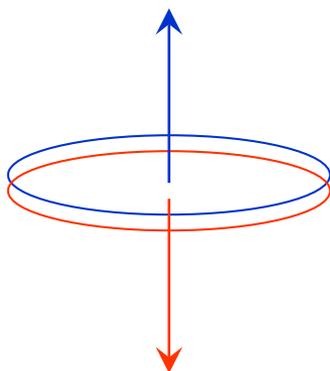
$$v_{\text{res}}(\mathbf{r}, \mathbf{r}') \sim -g \delta(\mathbf{r} - \mathbf{r}')$$



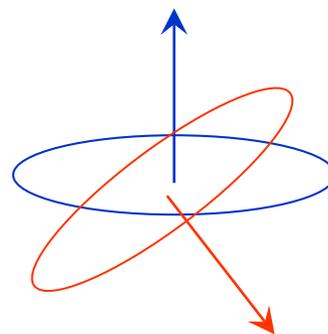
残留相互  
作用なし

残留相互  
作用あり

## 簡単な解釈:



$L=0$  対

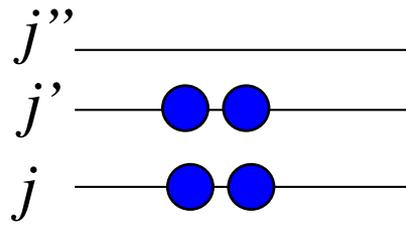


$L \neq 0$  対

$L=0$  対に対して空間的重なりが最大(エネルギー的に得)

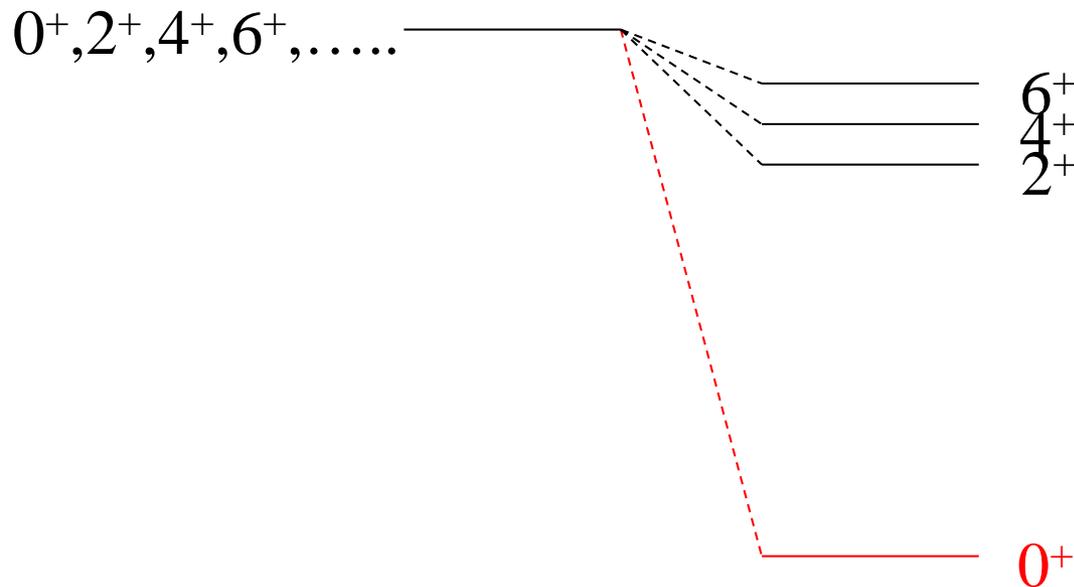
“対相関”

# BCS 理論



複数個のレベルに  
複数個のペアがある問題

$$v_{\text{res}}(r, r') \sim -g \delta(r - r')$$



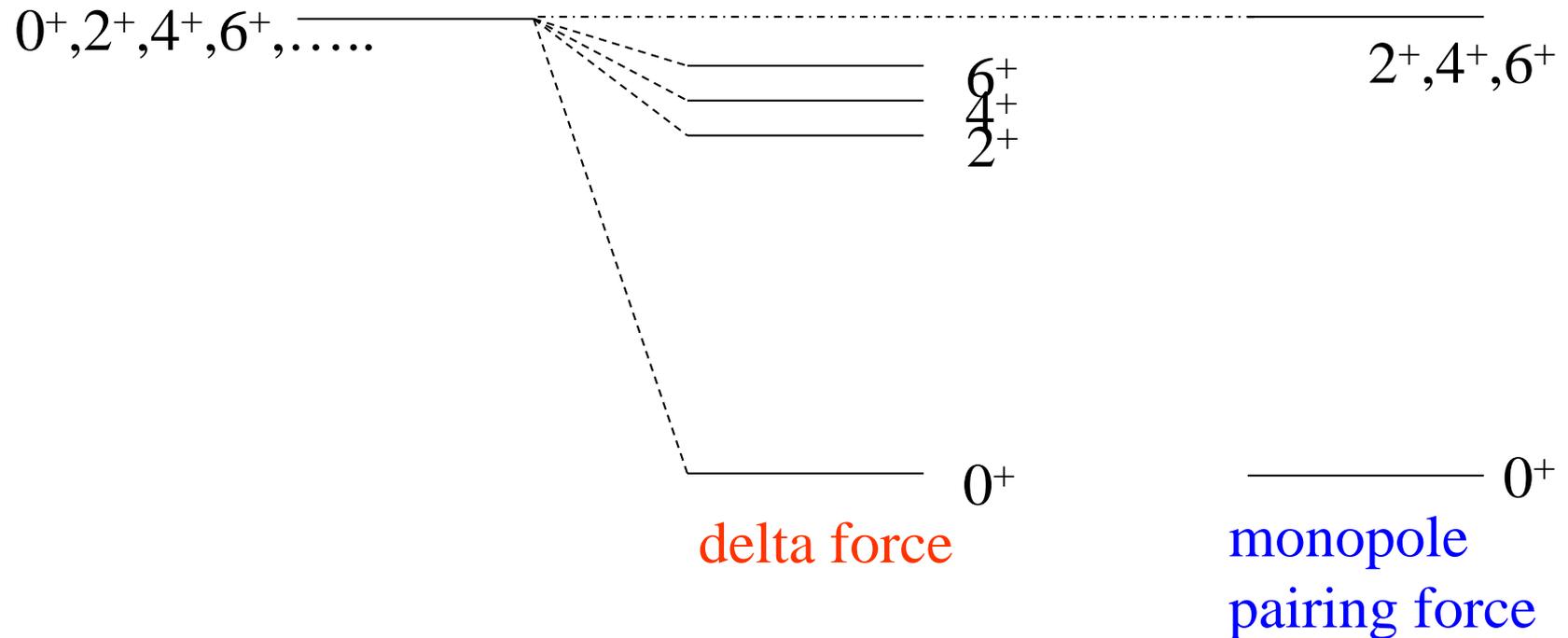
デルタ関数のままでもいいが、説明を簡単にするためにもう少し簡単にした相互作用を導入する。

## Simplified pairing interaction

$$V = -G P^\dagger P; \quad P^\dagger = \sum_{\nu > 0} a_\nu^\dagger a_{\bar{\nu}}^\dagger$$

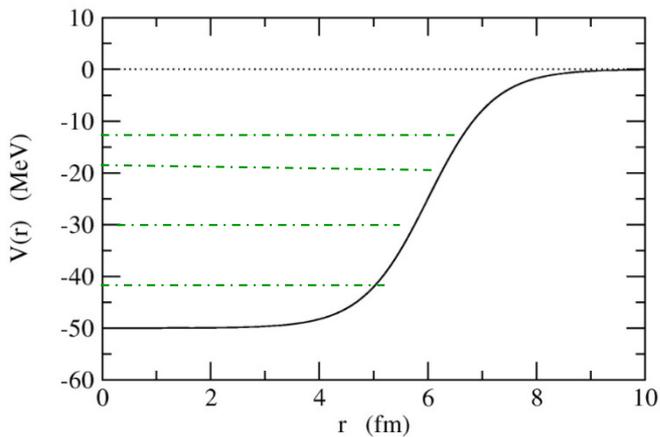
$\bar{\nu}$  : the time reversed state  
of  $\nu$

e.g.,  $|\nu\rangle = |n j l m\rangle, \quad |\bar{\nu}\rangle = |n j l - m\rangle$



# HF+BCS theory

- ① 平均場近似をして核子の感じるポテンシャルを求める  
(平均的な振る舞いをまず決める)



$$H = \sum_{\nu} \epsilon_k (a_k^{\dagger} a_k + a_{\bar{k}}^{\dagger} a_{\bar{k}}) - G \left( \sum_{k>0} a_k^{\dagger} a_{\bar{k}}^{\dagger} \right) \left( \sum_{k>0} a_{\bar{k}} a_k \right)$$

- ② 各準位の占有確率を決める。

決め方は、残留相互作用も含めてエネルギーが最小になるようにする。

$$H = \sum_{\nu} \epsilon_{\nu} (a_{\nu}^{\dagger} a_{\nu} + a_{\bar{\nu}}^{\dagger} a_{\bar{\nu}}) - G \underbrace{\left( \sum_{\nu > 0} a_{\nu}^{\dagger} a_{\bar{\nu}}^{\dagger} \right) \left( \sum_{\nu > 0} a_{\bar{\nu}} a_{\nu} \right)}$$

2体の相互作用

→ 1体近似をする

cf. HF potential

$$V_H(\mathbf{r}) = \int v(\mathbf{r}, \mathbf{r}') \rho_{\text{HF}}(\mathbf{r}') d\mathbf{r}'$$

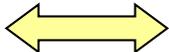
Solve the pairing Hamiltonian

$$H = \sum_{\nu} \epsilon_{\nu} (a_{\nu}^{\dagger} a_{\nu} + a_{\bar{\nu}}^{\dagger} a_{\bar{\nu}}) - G \left( \sum_{\nu > 0} a_{\nu}^{\dagger} a_{\bar{\nu}}^{\dagger} \right) \left( \sum_{\nu > 0} a_{\bar{\nu}} a_{\nu} \right)$$

in the mean-field approximation

- Mean-field approximation:

$$V = -G P^{\dagger} P \rightarrow -G \left( \langle P^{\dagger} \rangle P + P^{\dagger} \langle P \rangle \right) = -\Delta (P^{\dagger} + P)$$

 particle number violation



we consider  $H' = H - \lambda \hat{N}$  instead of  $H$ :

$$\begin{aligned} H' &= \sum_{k>0} (\epsilon_k - \lambda) (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \hat{P}^\dagger \hat{P} \\ &\rightarrow \sum_{k>0} (\epsilon_k - \lambda) (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta (\hat{P}^\dagger + \hat{P}) \\ &= \sum_{k>0} (\epsilon_k - \lambda) (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \end{aligned}$$

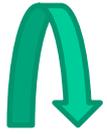


we consider  $H' = H - \lambda \hat{N}$  instead of  $H$  :

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● Transform  $H'$  in a form of

$$H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$



we consider  $H' = H - \lambda \hat{N}$  instead of  $H$ :

$$\begin{aligned} H' &= \sum_{k>0} (\epsilon_k - \lambda) (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \hat{P}^\dagger \hat{P} \\ &\rightarrow \sum_{k>0} (\epsilon_k - \lambda) (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta (\hat{P}^\dagger + \hat{P}) \\ &= \sum_{k>0} (\epsilon_k - \lambda) (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \end{aligned}$$

● Transform  $H'$  in a form of

$$H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$



g.s.:  $\alpha_k |BCS\rangle = 0$

1<sup>st</sup> excited state:  $|1_k\rangle = \alpha_k^\dagger |BCS\rangle$  at  $E_k$

.... and so on.

## Bogoliubov transformation

$$\alpha_{\nu}^{\dagger} = u_{\nu} a_{\nu}^{\dagger} - v_{\nu} a_{\bar{\nu}}, \quad \alpha_{\bar{\nu}}^{\dagger} = u_{\nu} a_{\bar{\nu}}^{\dagger} + v_{\nu} a_{\nu}$$

(Quasi-particle operator)

or  $a_{\nu}^{\dagger} = u_{\nu} \alpha_{\nu}^{\dagger} + v_{\nu} \alpha_{\bar{\nu}}, \quad a_{\bar{\nu}}^{\dagger} = u_{\nu} \alpha_{\bar{\nu}}^{\dagger} + -v_{\nu} \alpha_{\nu}$

(note)

$$\{\alpha_{\nu}, \alpha_{\nu'}\} = 0, \quad \{\alpha_{\nu}, \alpha_{\nu'}^{\dagger}\} = \delta_{\nu, \nu'}$$

$$\longrightarrow u_{\nu}^2 + v_{\nu}^2 = 1$$

## Bogoliubov transformation

$$\alpha_{\nu}^{\dagger} = u_{\nu} a_{\nu}^{\dagger} - v_{\nu} a_{\bar{\nu}}, \quad \alpha_{\bar{\nu}}^{\dagger} = u_{\nu} a_{\bar{\nu}}^{\dagger} + v_{\nu} a_{\nu}$$

(Quasi-particle operator)

or  $a_{\nu}^{\dagger} = u_{\nu} \alpha_{\nu}^{\dagger} + v_{\nu} \alpha_{\bar{\nu}}, \quad a_{\bar{\nu}}^{\dagger} = u_{\nu} \alpha_{\bar{\nu}}^{\dagger} + -v_{\nu} \alpha_{\nu}$

$$H' = \sum_{k>0} (\epsilon_k - \lambda) (a_k^{\dagger} a_k + a_{\bar{k}}^{\dagger} a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^{\dagger} a_{\bar{k}}^{\dagger} + a_{\bar{k}} a_k)$$

→

using the quasi-particle operators:

$$\begin{aligned} H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\ &= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\ &\quad + \sum_{k>0} [2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k) \end{aligned}$$

using the quasi-particle operators:

$$\begin{aligned} H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\ &= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\ &\quad + \sum_{k>0} [2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k) \end{aligned}$$



$$\text{if } 2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2) = 0$$

$$\text{then } H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

$$\text{with } E_k = (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k$$

$$\begin{aligned}
 H' &= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k] (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
 &+ \sum_{k>0} [2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2)] (\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
 \end{aligned}$$

$$\left\{ \begin{array}{l} 0 = 2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2) \\ 1 = u_k^2 + v_k^2 \end{array} \right.$$



$$\begin{aligned}
 u_\nu^2 &= \frac{1}{2} \left( 1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}} \right) \\
 v_\nu^2 &= \frac{1}{2} \left( 1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 H' &= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
 &+ \sum_{k>0} [2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
 \end{aligned}$$

$$\begin{aligned}
 u_\nu^2 &= \frac{1}{2} \left( 1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}} \right) \\
 v_\nu^2 &= \frac{1}{2} \left( 1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}} \right)
 \end{aligned}$$



$$\begin{aligned}
 E_k &= (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k \\
 &= \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}
 \end{aligned}$$

$$H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

Ground state wave function:

$$\alpha_k |BCS\rangle = 0$$



$$\begin{aligned} |BCS\rangle &\propto \prod_{\nu>0} \alpha_\nu \alpha_{\bar{\nu}} |0\rangle \\ &= \prod_{\nu>0} v_\nu (u_\nu + v_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger) |0\rangle \end{aligned}$$



$$|BCS\rangle = \prod_{\nu>0} (u_\nu + v_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger) |0\rangle$$

Ground state wave function:

$$\alpha_k |BCS\rangle = 0$$



$$|BCS\rangle = \prod_{\nu>0} (u_\nu + v_\nu a_\nu^\dagger a_\nu^\dagger) |0\rangle$$

(note)  $\langle BCS | a_\nu^\dagger a_\nu | BCS \rangle = |v_\nu|^2$  : occupation probability

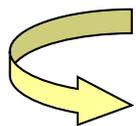
(note)

$$E'_{BCS} = \langle BCS | H' | BCS \rangle \sim 2 \sum_{\nu>0} (\epsilon_\nu - \lambda) v_\nu^2 - \frac{\Delta^2}{G}$$

## Gap equation

$$\begin{cases} u_\nu^2 &= \frac{1}{2} \left( 1 + \frac{\epsilon_\nu - \lambda}{E_\nu} \right) \\ v_\nu^2 &= \frac{1}{2} \left( 1 - \frac{\epsilon_\nu - \lambda}{E_\nu} \right) \end{cases}$$

$$E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}$$



$$\begin{aligned} \Delta &= G \langle BCS | \hat{P} | BCS \rangle = G \sum_{\nu > 0} u_\nu v_\nu \\ &= \frac{G}{2} \sum_{\nu > 0} \frac{\Delta}{E_\nu} \end{aligned}$$

(Gap equation)

$$N = 2 \sum_{\nu > 0} v_\nu^2 \quad \leftarrow \lambda$$

$$\Delta = \frac{G}{2} \sum_{\nu > 0} \frac{\Delta}{E_\nu}$$

i) Trivial solution: always exists

$$\Delta = 0$$

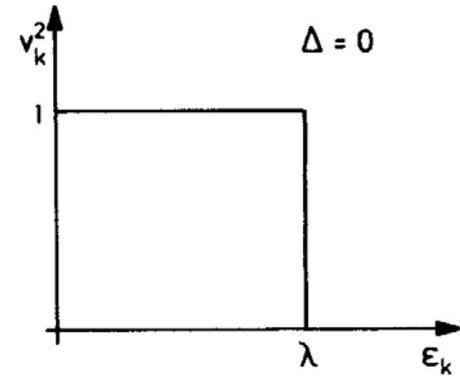
$$\Delta = G \sum_{\nu > 0} u_\nu v_\nu$$

$$v_\nu^2 = 1 \quad (\epsilon_\nu \leq \lambda)$$

$$= 0 \quad (\epsilon_\nu > \lambda)$$

$$|\Psi\rangle = \prod_{\nu > 0} a_\nu^\dagger a_\nu^\dagger |0\rangle$$

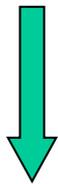
Occupation probability



$$\Delta = \frac{G}{2} \sum_{\nu > 0} \frac{\Delta}{E_{\nu}}$$

i) Trivial solution: always exists

$$\Delta = 0$$



$G \text{ a/o } N \longrightarrow \text{large}$

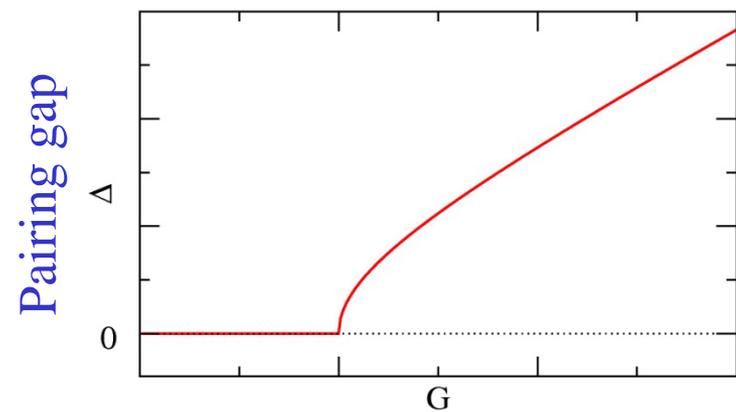
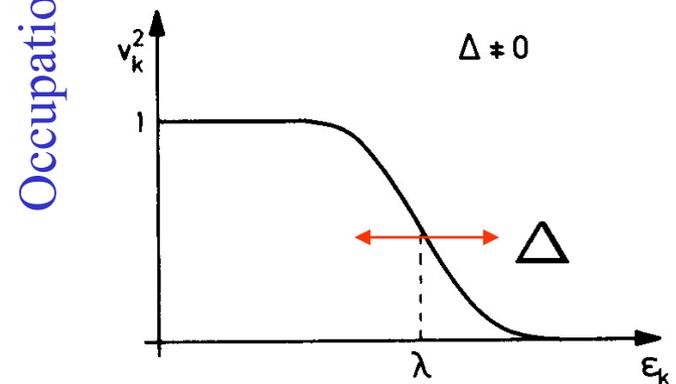
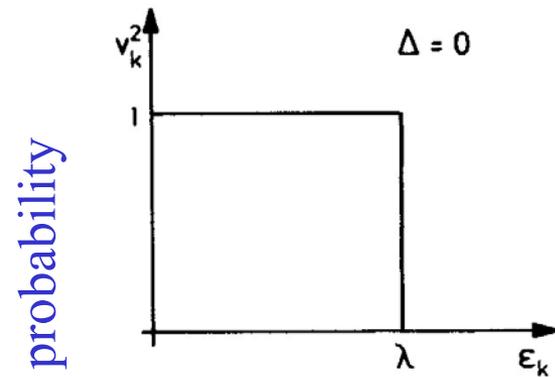
ii) Superfluid solution

$$\Delta \neq 0$$

$$v_{\nu}^2 < 1$$

$$|BCS\rangle = \prod_{\nu > 0} (u_{\nu} + v_{\nu} a_{\nu}^{\dagger} a_{\bar{\nu}}^{\dagger}) |0\rangle$$

Number fluctuation



Normal-Superfluid phase transition

## Quasi-particle excitations

$$H = \sum_{\nu} \epsilon_k (a_k^{\dagger} a_k + a_{\bar{k}}^{\dagger} a_{\bar{k}}) - G \left( \sum_{k>0} a_k^{\dagger} a_{\bar{k}}^{\dagger} \right) \left( \sum_{k>0} a_{\bar{k}} a_k \right)$$

ハミルトニアンを書き直すと:

$$H \sim E_{BCS} + \sum_k E_k \alpha_k^{\dagger} \alpha_k$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

$$\alpha_{\nu}^{\dagger} = u_{\nu} a_{\nu}^{\dagger} - v_{\nu} a_{\bar{\nu}}, \quad \alpha_{\bar{\nu}}^{\dagger} = u_{\nu} a_{\bar{\nu}}^{\dagger} + v_{\nu} a_{\nu}$$

(ボゴリューボフ変換)

## Quasi-particle excitations

$$H \sim E_{BCS} + \sum_k E_k \alpha_k^\dagger \alpha_k$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \quad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

(ボゴリューボフ変換)

基底状態:  $|\text{BCS}\rangle$

1準粒子状態:  $\alpha_k^\dagger |\text{BCS}\rangle$

2準粒子状態:  $\alpha_k^\dagger \alpha_{k'}^\dagger |\text{BCS}\rangle$

奇核に対応

- ・  $N \pm 2$  の原子核
- ・ 同じ原子核の励起状態に対応

## Quasi-particle excitations

$$H \sim E_{BCS} + \sum_k E_k \alpha_k^\dagger \alpha_k$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}},$$

$$\alpha_{\bar{\nu}}^\dagger = u_\nu a_\nu^\dagger + v_\nu a_\nu$$

(ボゴリューボフ変換)

基底状態:  $|\text{BCS}\rangle$

1準粒子状態:  $\alpha_k^\dagger |\text{BCS}\rangle$

奇核に対応

2準粒子状態:  $\alpha_k^\dagger \alpha_{k'}^\dagger |\text{BCS}\rangle$

・N +/- 2 の原子核  
・同じ原子核の励起状態  
に対応

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2} \geq \Delta$$

(エネルギー・ギャップ)

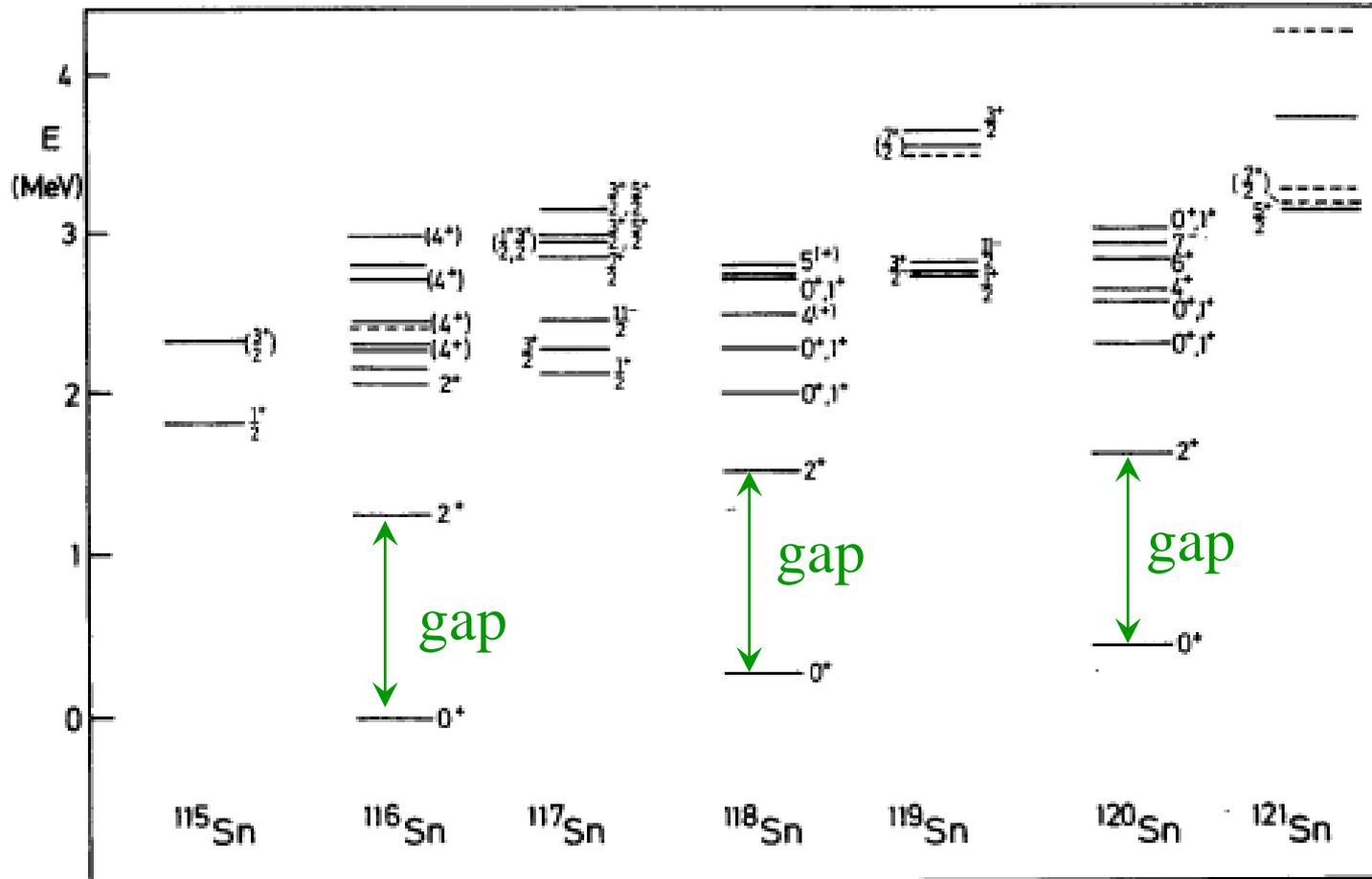
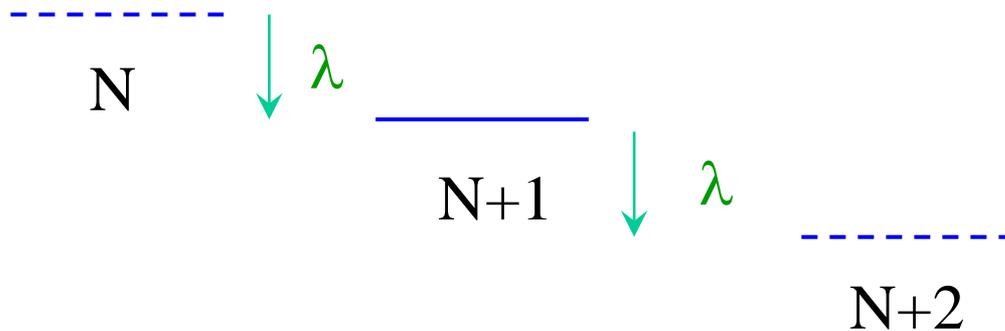


Figure 6.1. Excitation spectra of the  ${}_{50}\text{Sn}$  isotopes.

# Even-odd mass difference and pairing gap

$$E(N + 2, Z) = E(N, Z) + 2\lambda$$

$$E(N + 1, Z) = E(N, Z) + \lambda + \Delta$$



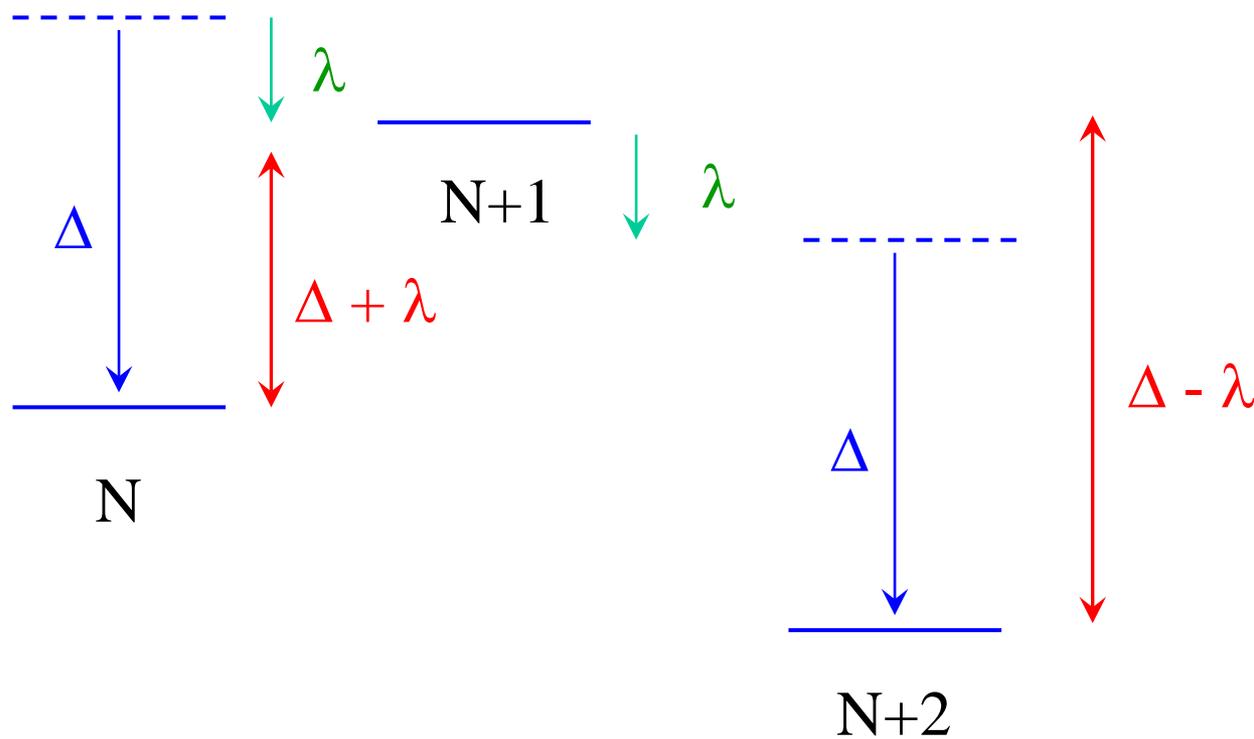
(note)  $\lambda < 0$

$$-\Delta_n \sim [E(N + 2, Z) - 2E(N + 1, Z) + E(N, Z)]/2$$

# Even-odd mass difference and pairing gap

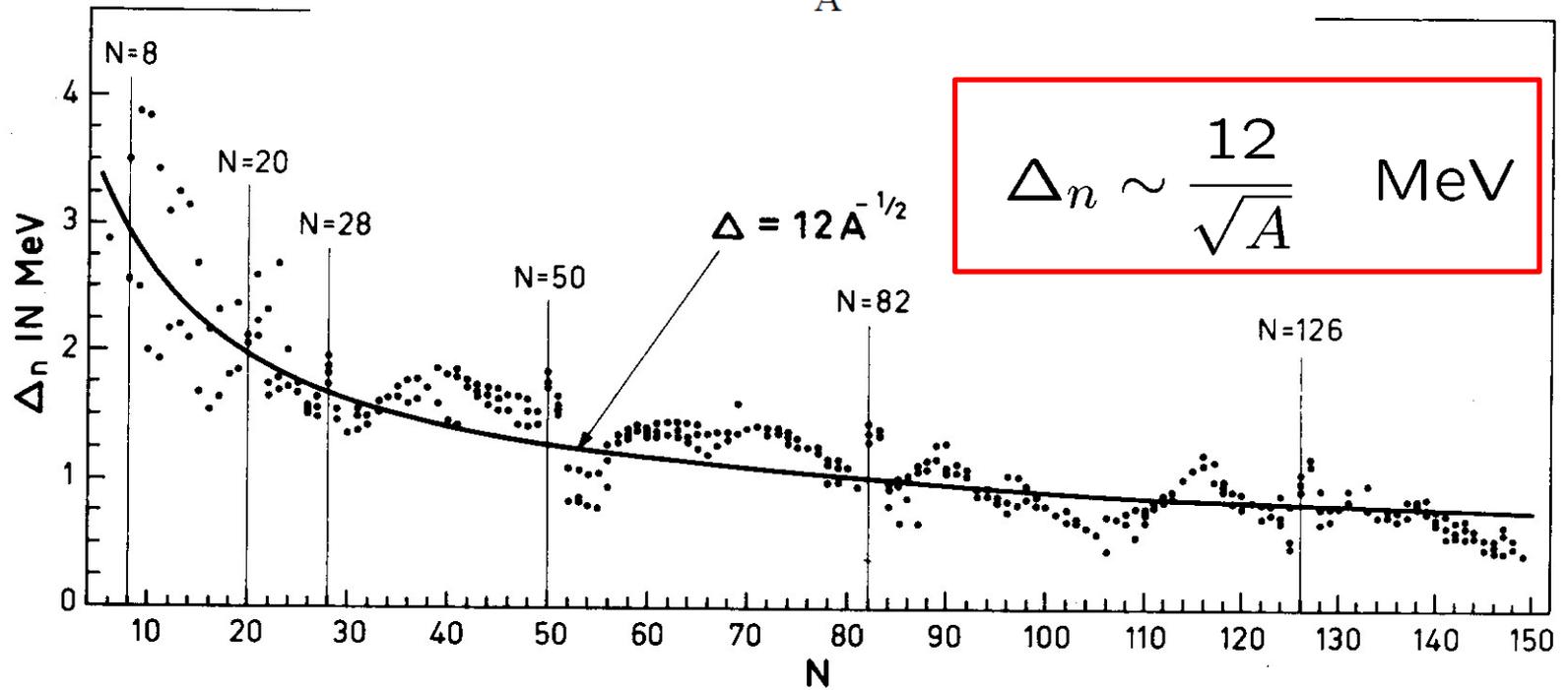
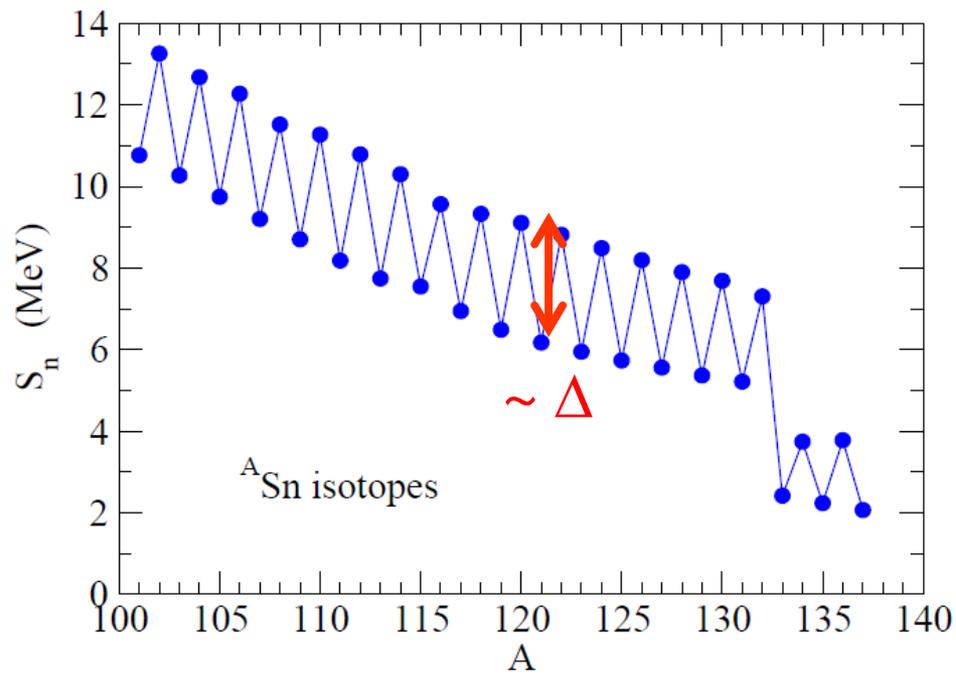
$$E(N + 2, Z) = E(N, Z) + 2\lambda$$

$$E(N + 1, Z) = E(N, Z) + \lambda + \Delta$$



(note)  $\lambda < 0$

$$-\Delta_n \sim [E(N + 2, Z) - 2E(N + 1, Z) + E(N, Z)]/2$$



# Hartree-Fock-Bogoliubov (HFB) Theory

HF+BCS 法: 2ステップ

(まず平均場を求め、次に占有確率)

$$\psi_k(\mathbf{r}), u_k, v_k$$



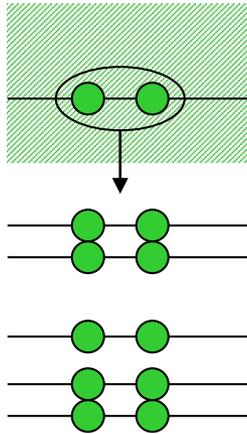
改良: 両方同時に行う

Hartree-Fock-Bogoliubov (HFB) theory:

波動関数と占有確率を同時に求める

$$U_k(\mathbf{r}), V_k(\mathbf{r})$$

cf. weakly bound systems  
(ガスの問題)



束縛

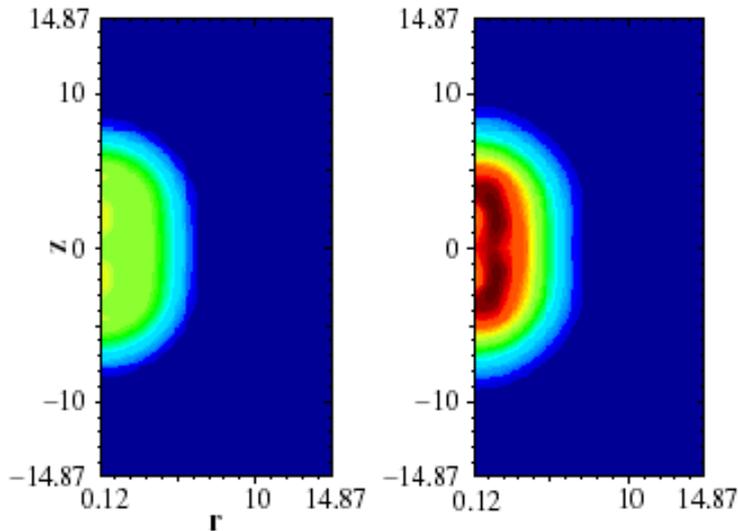


BCS法だと散乱状態をそのまま占有させるので、中性子が抜けていく（束縛核のまわりに中性子のガスができる）。

HFB法だと全体で束縛するということがもともと取り入れられているので中性子ガスは発生しない。

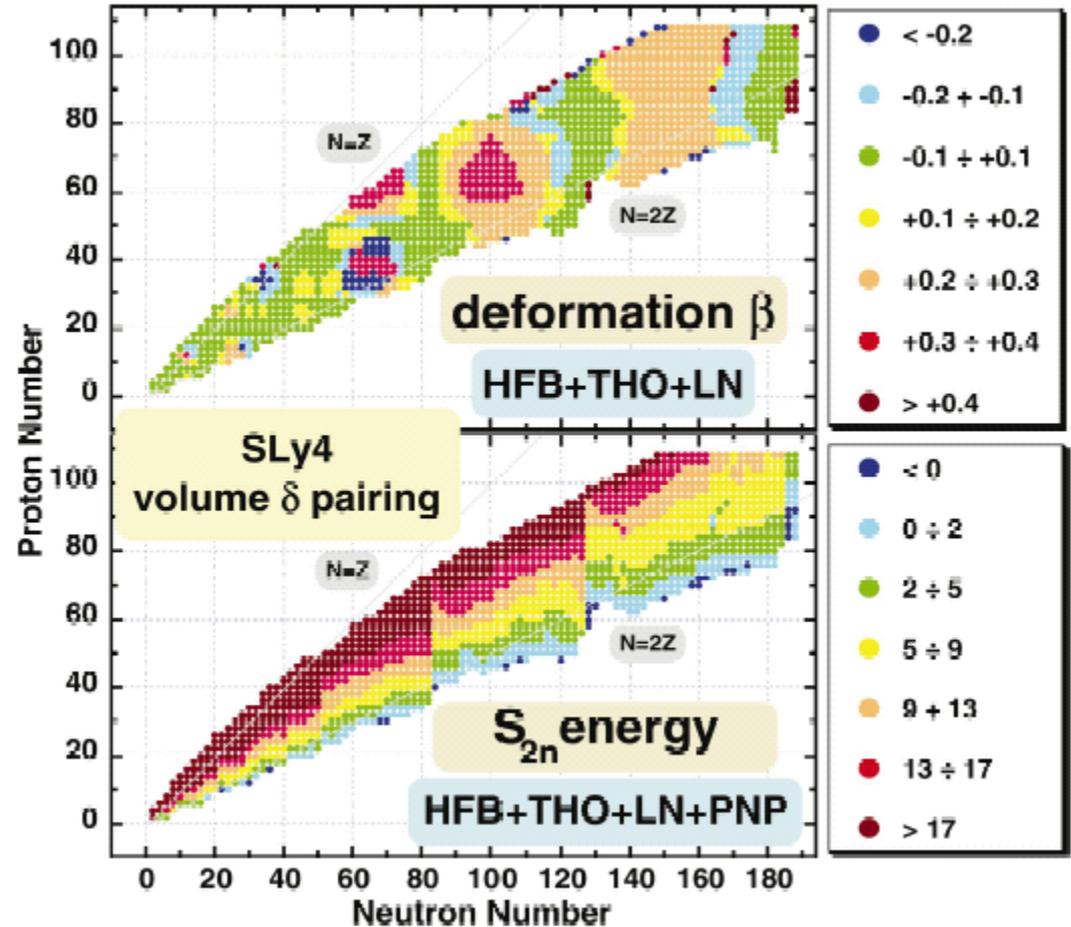
# Application of the HFB method

Density of  $^{110}\text{Zr}$  (SHFB-SLy4)



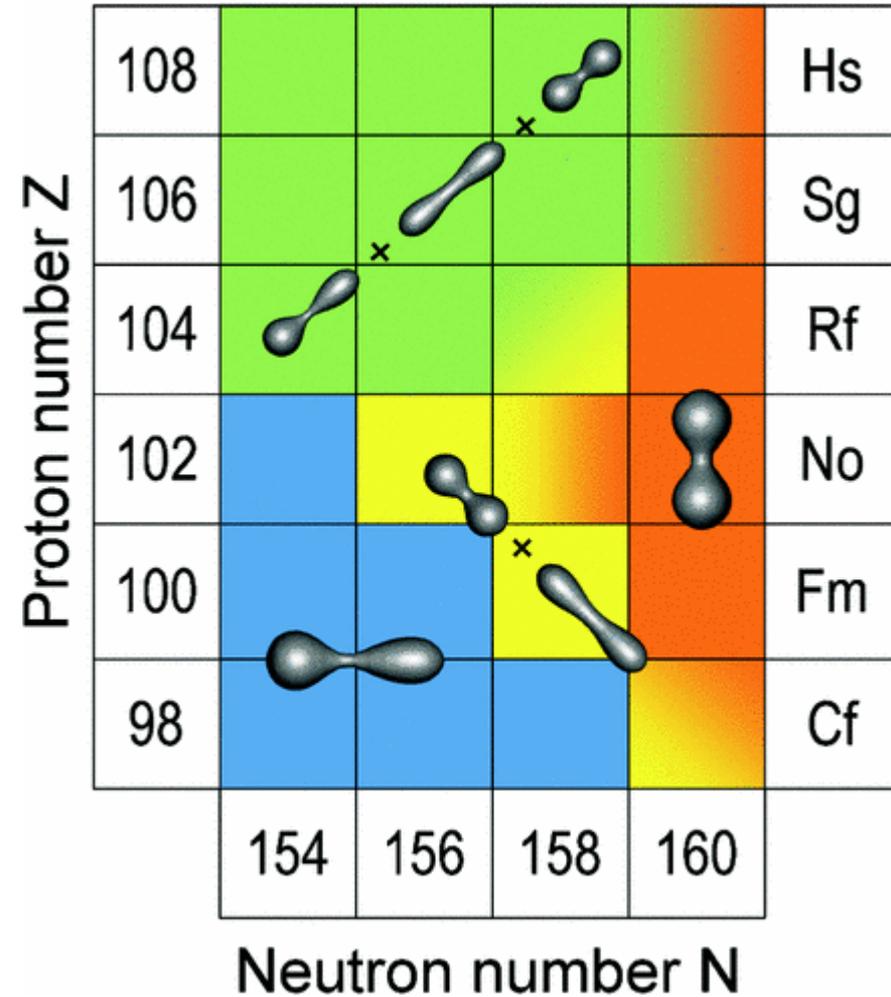
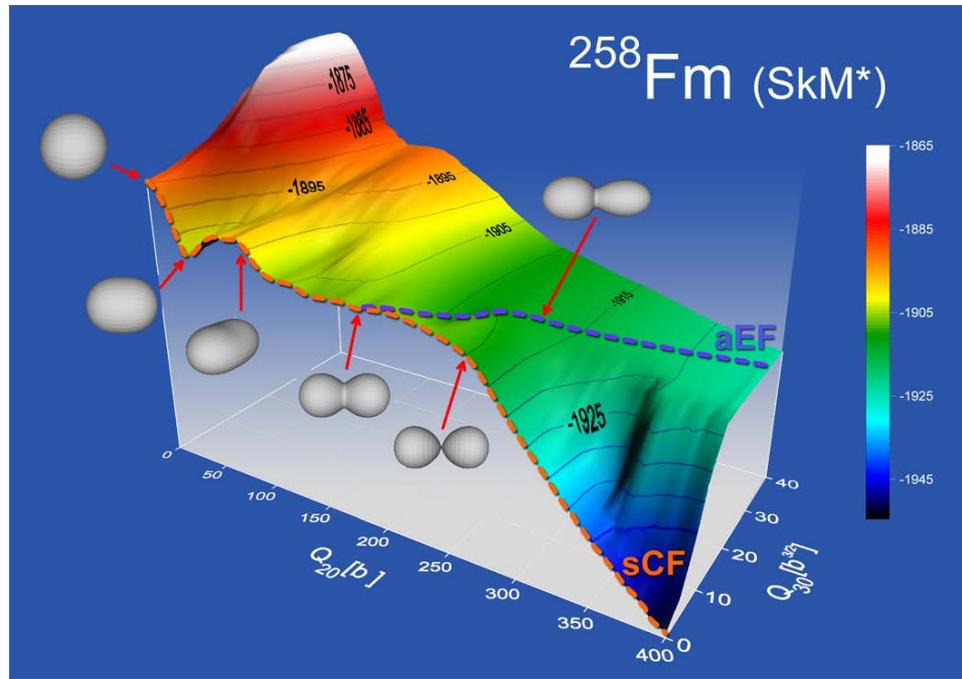
A. Blazkiewicz et al.,  
PRC71('05)054231

Systematics of  $\beta_2$  and  $S_{2n}$



M.V. Stoitsov et al., PRC68('03)054312

# potential energy surface for fission process

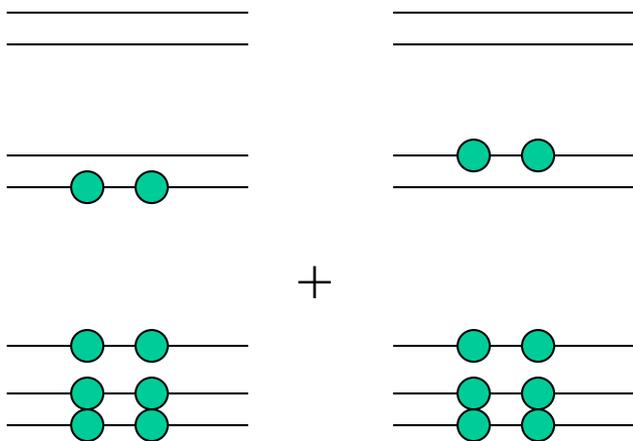


A. Staszczak, A. Baran, J. Dobaczewski,  
and W. Nazarewicz, PRC80 ('09) 014309

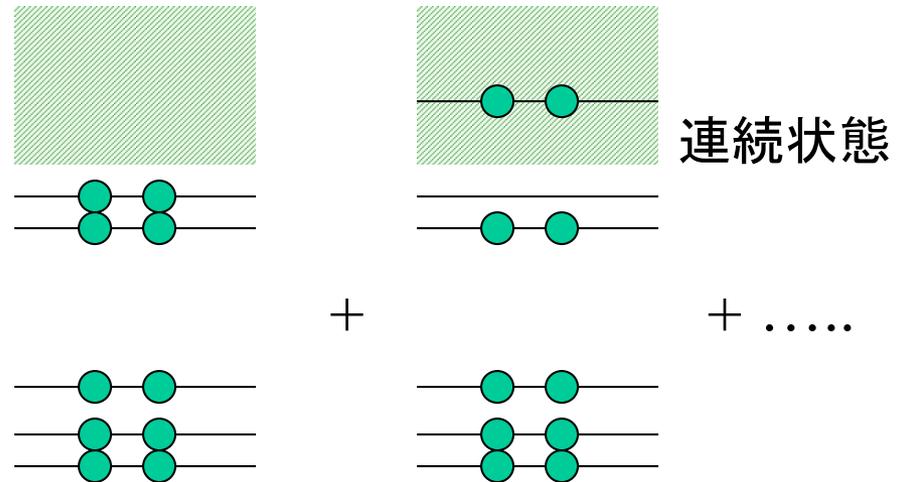
# 弱束縛核における対相関

$$H = \sum_i T_i + \sum_{i < j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i < j} v_{ij} - \sum_i V_i}_{\text{平均からのずれ (残留相互作用)}}$$

平均からのずれ  
(残留相互作用)



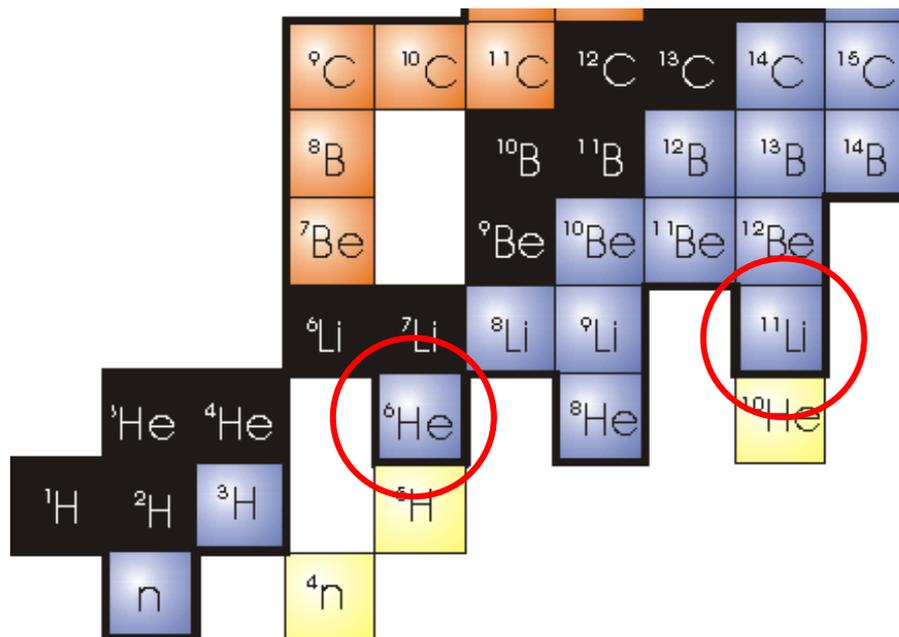
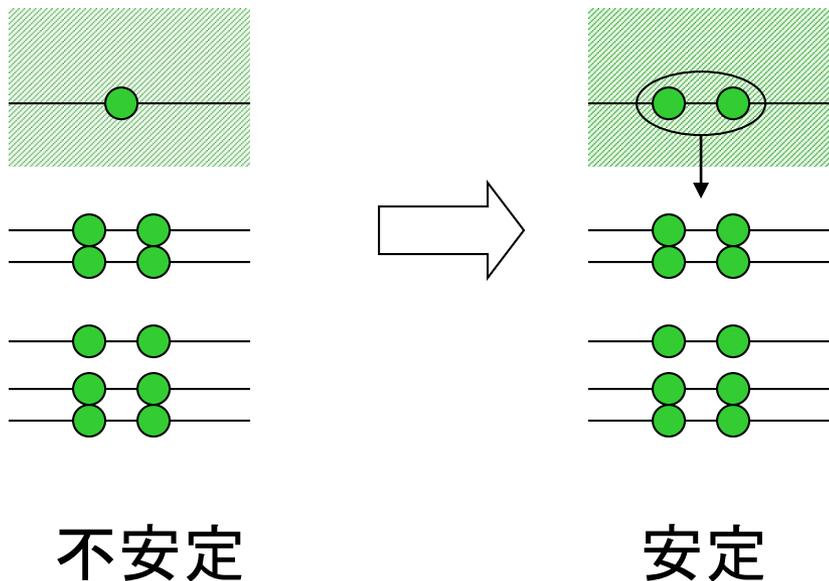
安定な原子核  
→ 超流動状態



弱く束縛された系

# ボロミアン原子核

残留相互作用 → 引力



“ボロミアン核”



10 Google - 地図データ ©2010 Basarsoft, Europa Technologies, Goo

10 Google - 地図データ ©2010 Basarsoft, Europa Technologies, Goo



ボロミオ諸島



ボッロメオ諸島  
(北イタリア、マッジョー  
レ湖)

ミラノの近く



ボッロメオ家(13世紀)の紋章



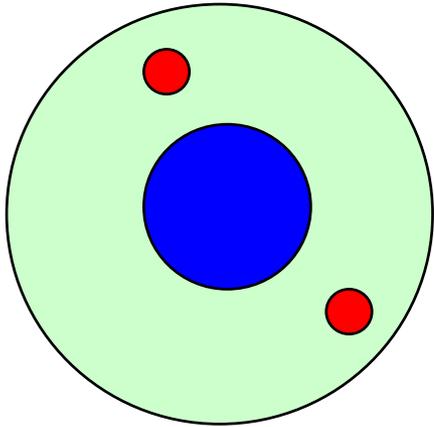
法金剛院(京都)

3つの輪はつながっているけど、どれか1つをはずすとバラバラになる

「ボロミアン・リング」

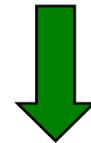


## 双中性子 (di-neutron) 相関



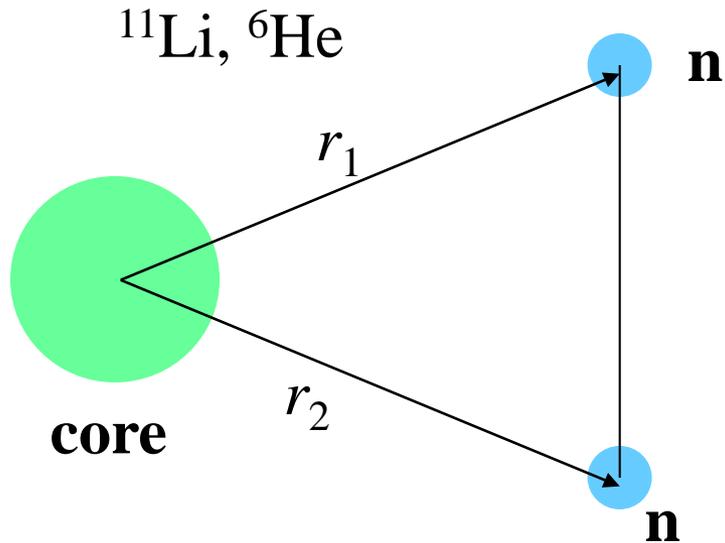
原子核中で2つの中性子は空間的にどのように配置されているのか？

2つの中性子が独立に運動しているとすると、片方の中性子がどこにいてももう片方は関知しない



対相関が働くとどうなるか？

## 3体模型計算: di-neutron 相関の微視的理解



$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn}(r_1, r_2) + \frac{(p_1 + p_2)^2}{2A_c m}$$

(最後の項は3体系の静止系で考えた芯原子核の運動エネルギー一項。)

⇒ この3体ハミルトニアン基底状態を求め、密度分布を調べる:

(例えば)  $V_{nn}$  が無いときの状態で展開し、展開係数を求める

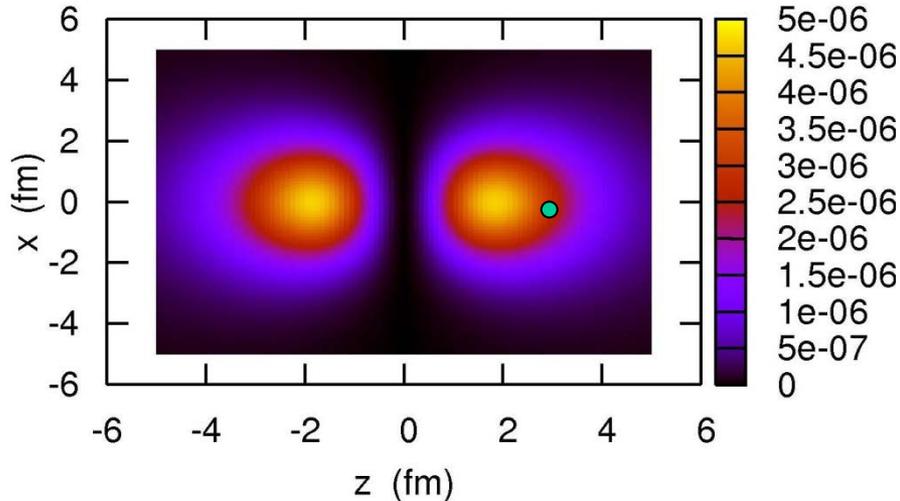
$$\Psi_{gs}(r_1, r_2) = A \sum_{nn'lj} \alpha_{nn'lj} \Psi_{nn'lj}^{(2)}(r_1, r_2)$$

$$\Psi_{nn'lj}^{(2)}(r_1, r_2) = \sum_m \langle jmj - m | 00 \rangle \psi_{nljm}(r_1) \psi_{n'lj-m}(r_2)$$

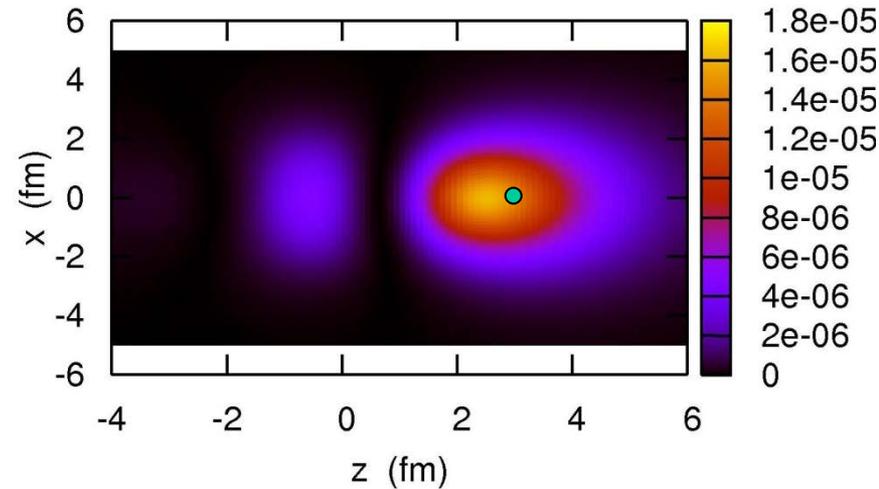
# 対相関力がある場合とない場合の比較

$^{11}\text{Li}$  1つの中性子を  $(z_1, x_1)=(3.4 \text{ fm}, 0)$  に置いたときのもう一つの中性子の分布

対相関がない場合  $[1p_{1/2}]^2$



対相関がある場合



- 対相関がないと、 $z$  と  $-z$  で対称的な分布。片方の中性子がどこにいても分布は変わらない。
- 対相関があると、2つの中性子は近くにいる。1つの中性子の場所が変わると、もう1つも変わる。

# What is Di-neutron correlation?

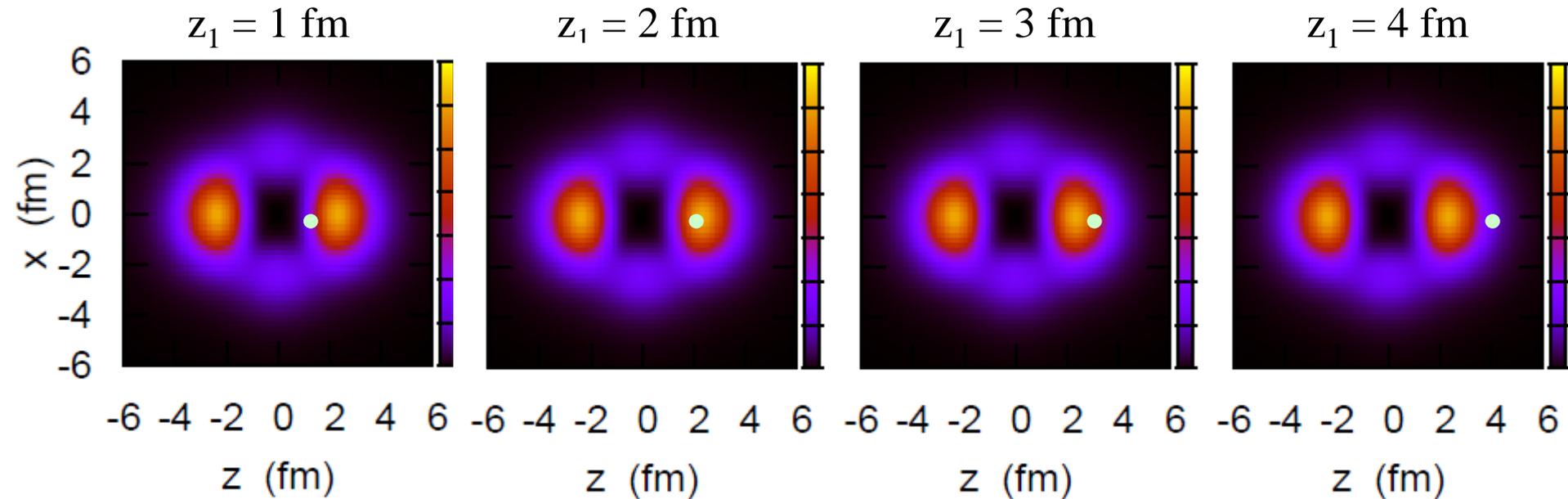
$$\text{Correlation: } \langle AB \rangle \neq \langle A \rangle \langle B \rangle$$

Example:  $^{18}\text{O} = ^{16}\text{O} + n + n$

cf.  $^{16}\text{O} + n$  : 3 bound states ( $1d_{5/2}$ ,  $2s_{1/2}$ ,  $1d_{3/2}$ )

i) Without nn interaction:  $|nn\rangle = |(1d_{5/2})^2\rangle$

Distribution of the 2<sup>nd</sup> neutron when the 1<sup>st</sup> neutron is at  $z_1$  :



✓ Two neutrons move independently

✓ No influence of the 2<sup>nd</sup> neutron from the 1<sup>st</sup> neutron

$$\langle AB \rangle = \langle A \rangle \langle B \rangle$$

# What is Di-neutron correlation?

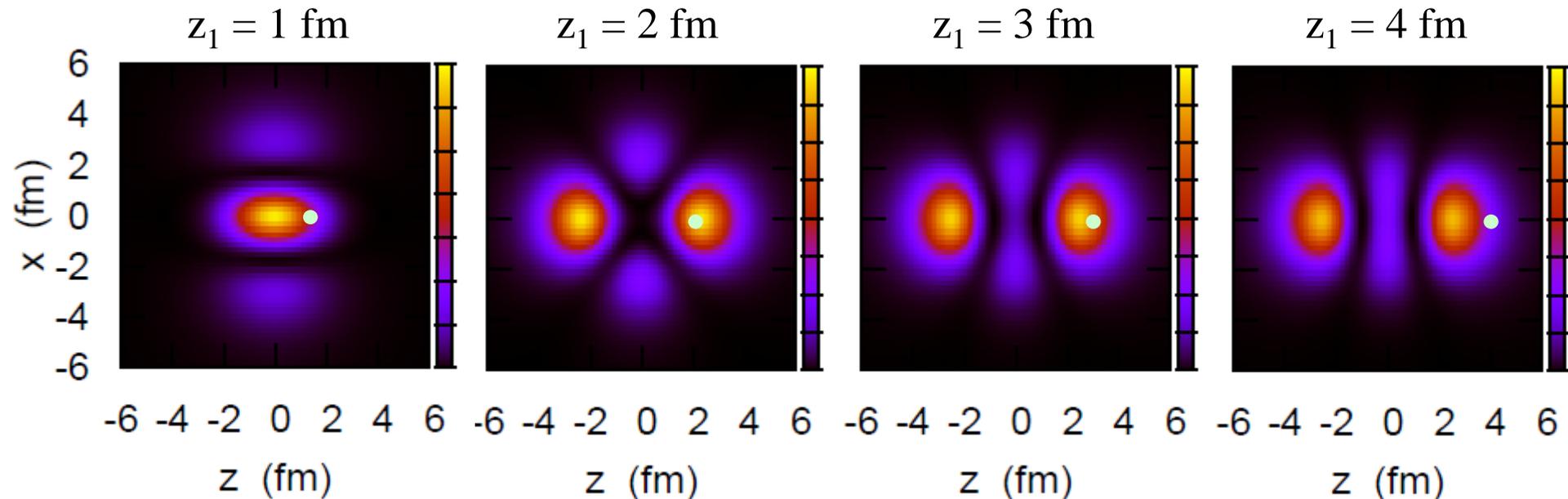
Correlation:  $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example:  $^{18}\text{O} = ^{16}\text{O} + n + n$

cf.  $^{16}\text{O} + n$  : 3 bound states ( $1d_{5/2}$ ,  $2s_{1/2}$ ,  $1d_{3/2}$ )

ii) nn interaction: works only on the positive parity (bound) states

$$|nn\rangle = \alpha|(1d_{5/2})^2\rangle + \beta|(2s_{1/2})^2\rangle + \gamma|(1d_{3/2})^2\rangle$$



✓ distribution changes according to the 1<sup>st</sup> neutron (nn correlation)

✓ but, the distribution of the 2<sup>nd</sup> neutron has peaks both at  $z_1$  and  $-z_1$

→ this is NOT called the di-neutron correlation

# What is Di-neutron correlation?

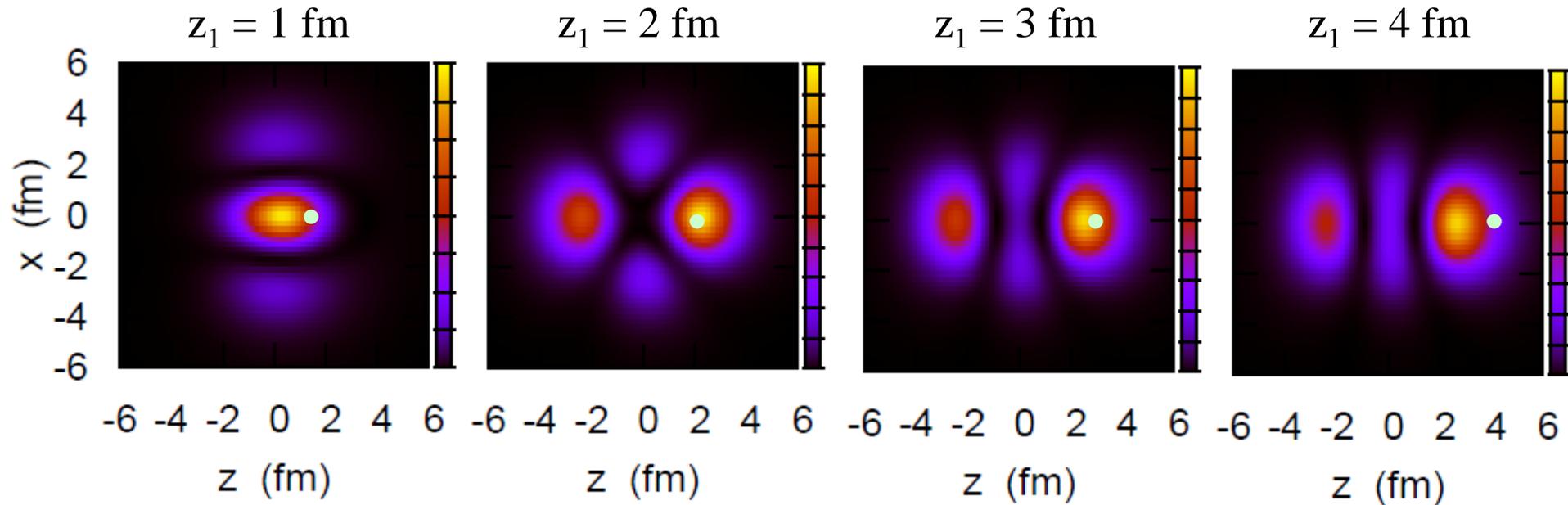
Correlation:  $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example:  $^{18}\text{O} = ^{16}\text{O} + n + n$

cf.  $^{16}\text{O} + n$  : 3 bound states ( $1d_{5/2}$ ,  $2s_{1/2}$ ,  $1d_{3/2}$ )

iii) nn interaction: works also on the continuum states

$$|nn\rangle = \sum_{n,n',j,l} C_{nn'jl} |(nn'jl)^2\rangle$$

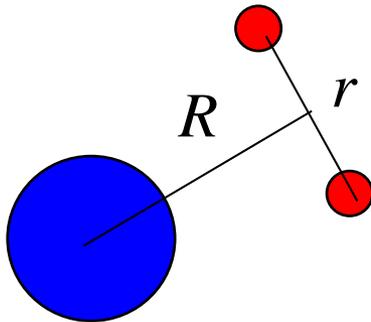
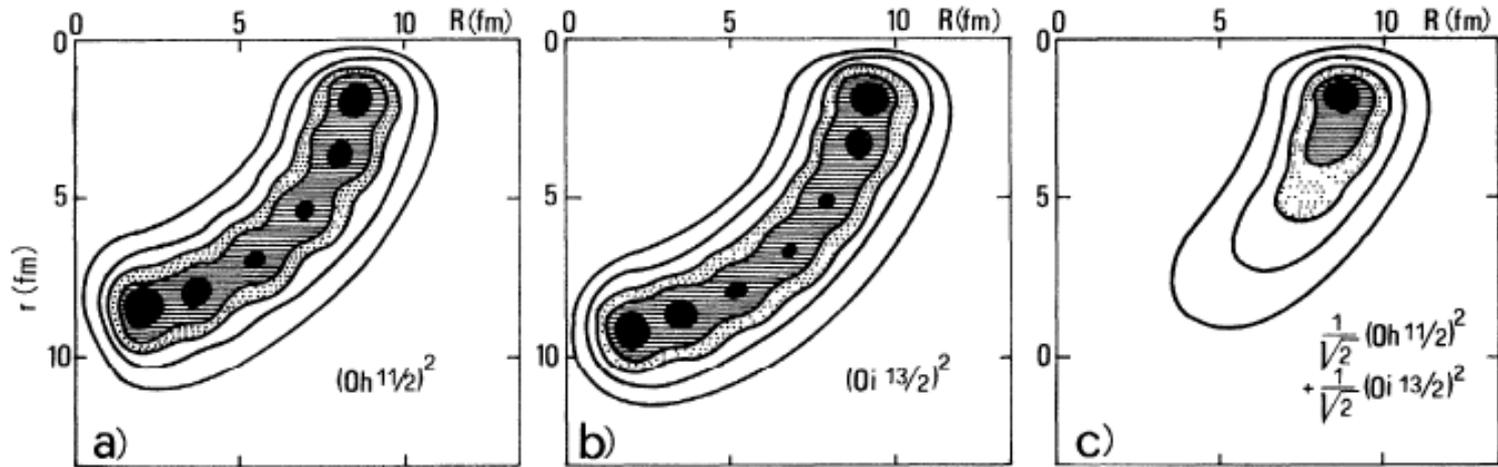


✓ spatial correlation: the density of the 2<sup>nd</sup> neutron localized close to the 1<sup>st</sup> neutron (dineutron correlation)

✓ parity mixing: essential role

cf. F. Catara et al., PRC29('84)1091

dineutron correlation: caused by the admixture of different parity states

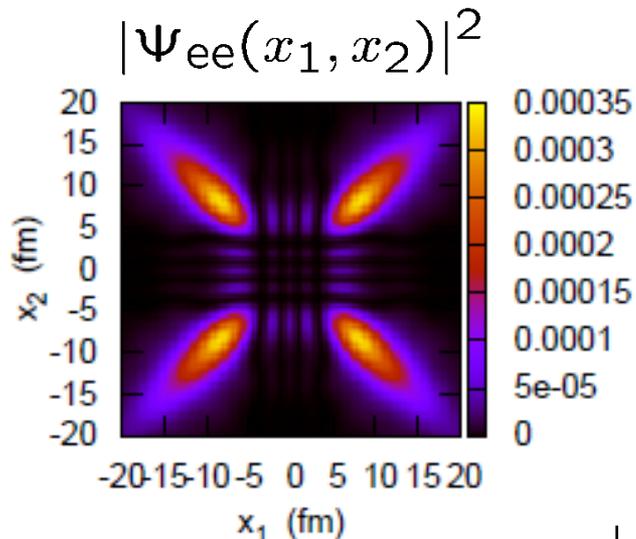


F. Catara, A. Insolia, E. Maglione,  
and A. Vitturi, PRC29('84)1091

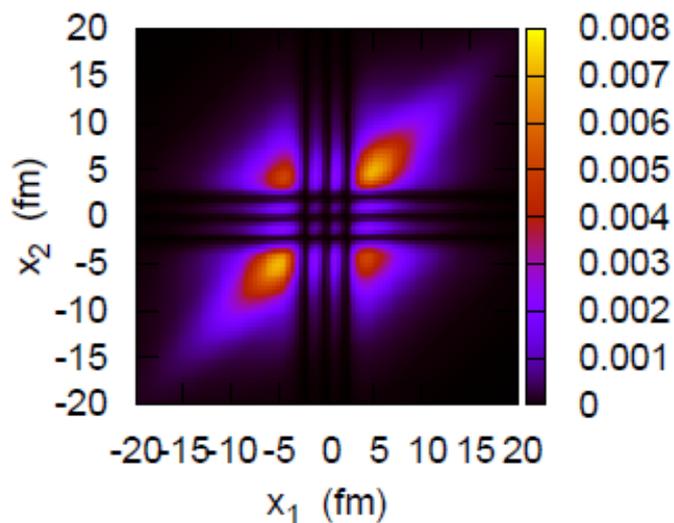
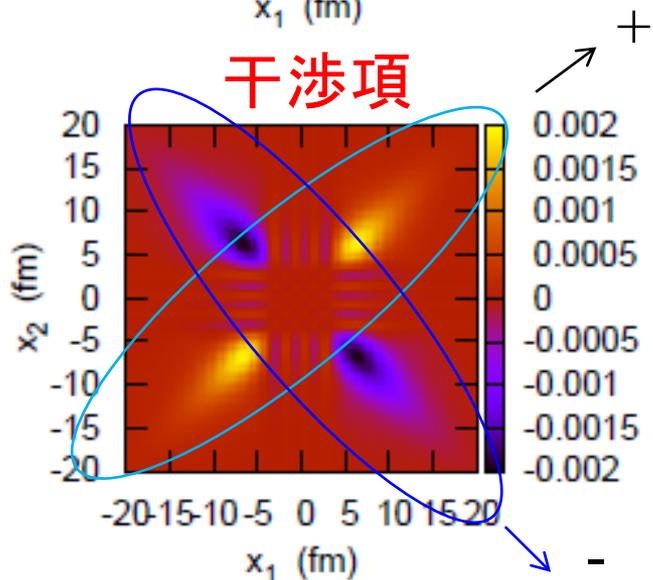
# 何故、異なるパリティが混ざると dineutron 相関が生じるのか?

$$\Psi_{gs}(x_1, x_2) = \Psi_{ee}(x_1, x_2) + \Psi_{oo}(x_1, x_2)$$

$$\longrightarrow \rho_2(x_1, x_2) = |\Psi_{ee}(x_1, x_2)|^2 + |\Psi_{oo}(x_1, x_2)|^2 + 2\Psi_{ee}(x_1, x_2)\Psi_{oo}(x_1, x_2)$$



$$\begin{aligned} \Psi_{ee}(-x_1, x_2) &= \Psi_{ee}(x_1, x_2) \\ \Psi_{oo}(-x_1, x_2) &= -\Psi_{oo}(x_1, x_2) \\ \rho_2(-x_1, x_2) &= |\Psi_{ee}(x_1, x_2)|^2 + |\Psi_{oo}(x_1, x_2)|^2 - 2\Psi_{ee}(x_1, x_2)\Psi_{oo}(x_1, x_2) \end{aligned}$$

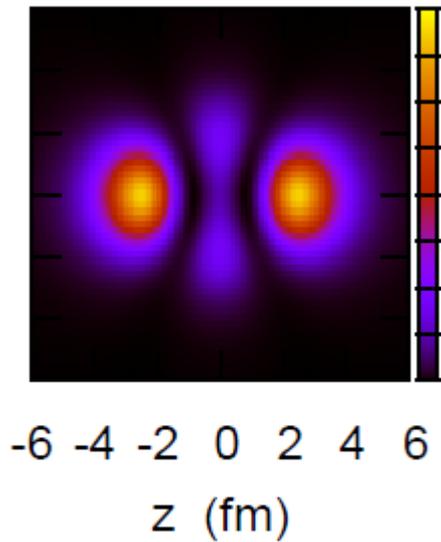


# spatial localization of two neutrons (dineutron correlation)

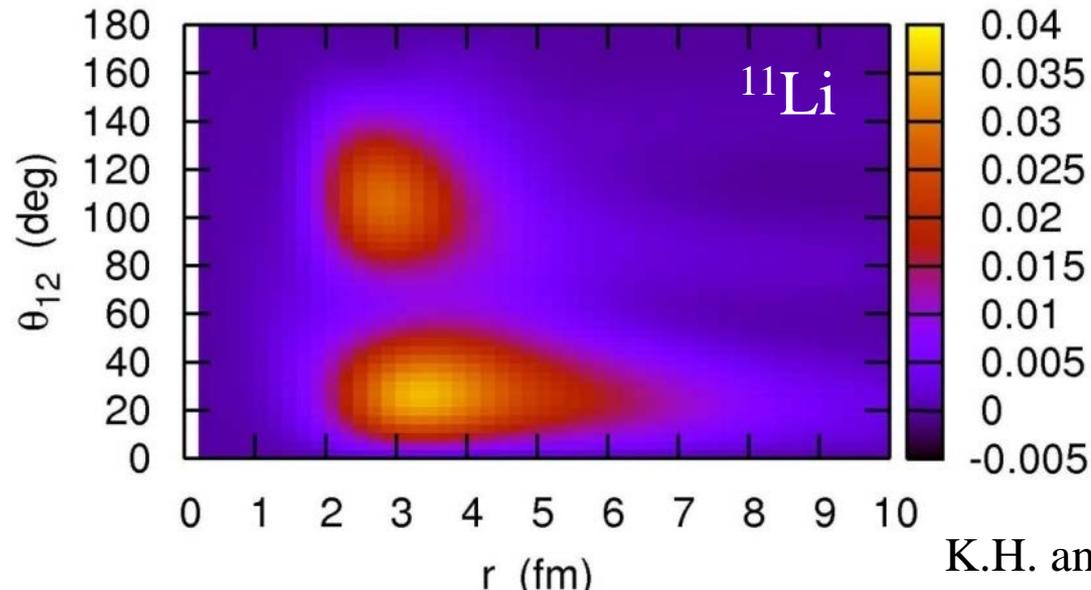
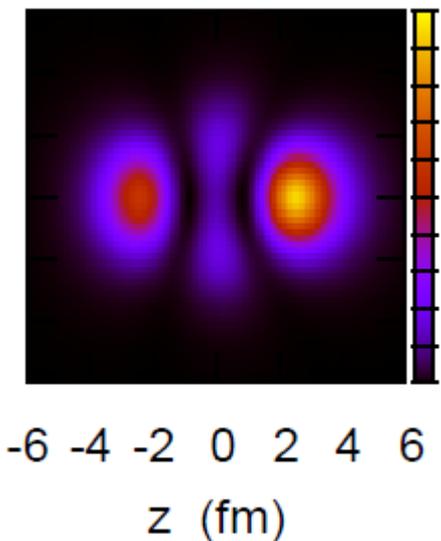
cf. Migdal, Soviet J. of Nucl. Phys. 16 ('73) 238  
Bertsch, Broglia, Riedel, NPA91('67)123

## weakly bound systems

- easy to mix different parity states due to the continuum couplings
- + enhancement of pairing on the surface

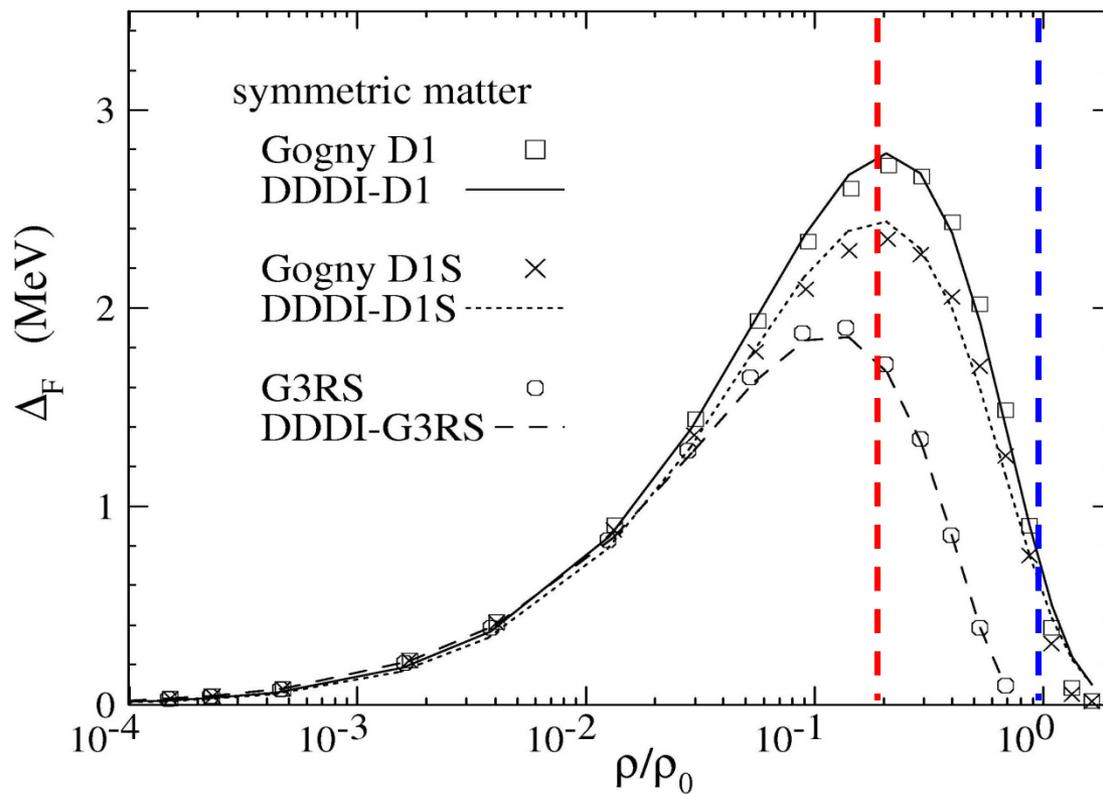


parity mixing



K.H. and H. Sagawa,  
PRC72('05)044321

## pairing gap in infinite nuclear matter



M. Matsuo, PRC73('06)044309

# spatial localization of two neutrons (dineutron correlation)

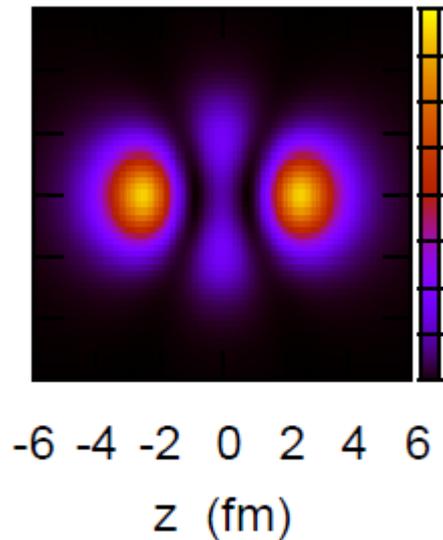
cf. Migdal, Soviet J. of Nucl. Phys. 16 ('73) 238  
Bertsch, Broglia, Riedel, NPA91('67)123

## weakly bound systems

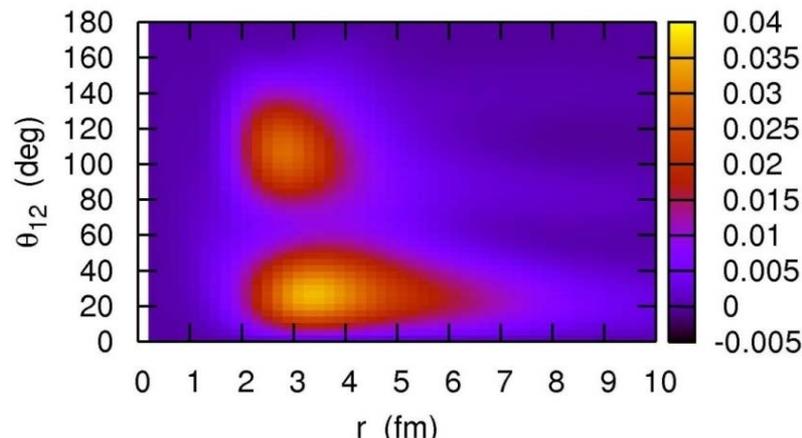
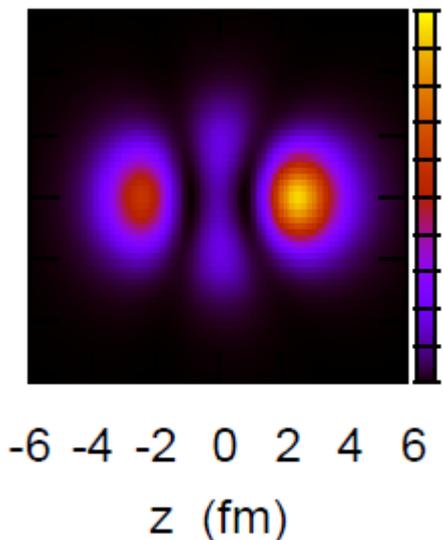
→ easy to mix different parity states due to the continuum couplings  
+ enhancement of pairing on the surface

→ dineutron correlation: enhanced

cf. - Bertsch, Esbensen, Ann. of Phys. 209('91)327  
- M. Matsuo, K. Mizuyama, Y. Serizawa, PRC71('05)064326

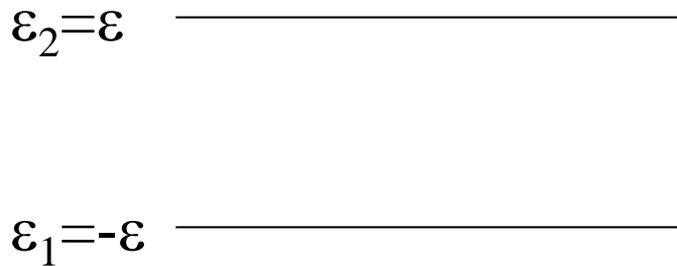


parity mixing



K.H. and H. Sagawa,  
PRC72('05)044321

## レポート問題5



左の図のように、 $\epsilon$  と  $-\epsilon$  のエネルギーを持つ2つの準位があるとする。それぞれの準位はともに角運動量  $j$  を持ち、 $2j+1$  重に縮退しているとする。粒子数が

$$N = 2j + 1 \equiv 2\Omega$$

で与えられているときにハミルトニアン

$$H = \sum_{\nu} \epsilon_{\nu} (a_{\nu}^{\dagger} a_{\nu} + a_{\bar{\nu}}^{\dagger} a_{\bar{\nu}}) - G \left( \sum_{\nu > 0} a_{\nu}^{\dagger} a_{\bar{\nu}}^{\dagger} \right) \left( \sum_{\nu > 0} a_{\bar{\nu}} a_{\nu} \right)$$

をBCS近似で解き、それぞれの準位に対する占有確率、およびペアリングギャップ  $\Delta$  を求めよ。ただし、BCS近似の式は既知のものとして使ってもよい。 $G$  の大小によって、場合分けをして答えよ。

(注) 対称性から、化学ポテンシャル  $\lambda$  は  $\lambda=0$  となる。

## レポート問題6

$\varepsilon=0$

レポート問題5で、準位が1つしかない場合を考える。このときにも、この準位は角運動量  $j$  を持ち、 $2j+1$  重に縮退しているとする。この準位のエネルギーを 0 にとると、ハミルトニアンは

$$H = -G\Omega A^\dagger A; \quad \Omega \equiv \frac{2j+1}{2}, \quad A^\dagger \equiv \frac{1}{\sqrt{\Omega}} \sum_{\nu>0} a_\nu^\dagger a_\nu^\dagger$$

で与えられる。

- (i) 粒子数を  $N$  としたときに、この準位の占有確率  $v^2$  をBCS近似で見積もれ。(化学ポテンシャル  $\lambda$  を 0 にし、粒子数と  $v^2$  の関係を使う。)
- (ii) BCS近似による基底状態のエネルギー  $E_{\text{BCS}} = -\Delta^2/G$  を計算せよ。

(iii)

$$[H, A^\dagger] = -G\Omega A^\dagger \left(1 - \frac{\hat{N}}{\Omega}\right); \quad \hat{N} = \sum_{\nu} a_\nu^\dagger a_\nu$$

となることを示すことができる。 $A|0\rangle = 0$ ,  $\hat{N}|0\rangle = 0$  という状態を定義するとき、状態  $A^\dagger|0\rangle$  が  $H$  の固有状態になっていることを示し、その固有値を求めよ。

(次ページへつづく)

(iv)  $H(A^\dagger)^k|0\rangle = E_k(A^\dagger)^k|0\rangle$

が成り立っているとするとき、状態  $(A^\dagger)^{k+1}|0\rangle$  が  $H$  の固有状態になっていることを示し、その固有値を求めよ。

(v) (iii) と (iv) の結果を用いて、粒子数が  $N$  である状態

$(A^\dagger)^{N/2}|0\rangle$  のエネルギー固有値を求めよ。

このようにして求めたエネルギー固有値は、ハミルトニアン  
の厳密解である。これと、(ii) でBCS近似を用いて求めたエネルギー  
を比較して、BCS近似がどのような場合により近似になるか議論  
せよ。