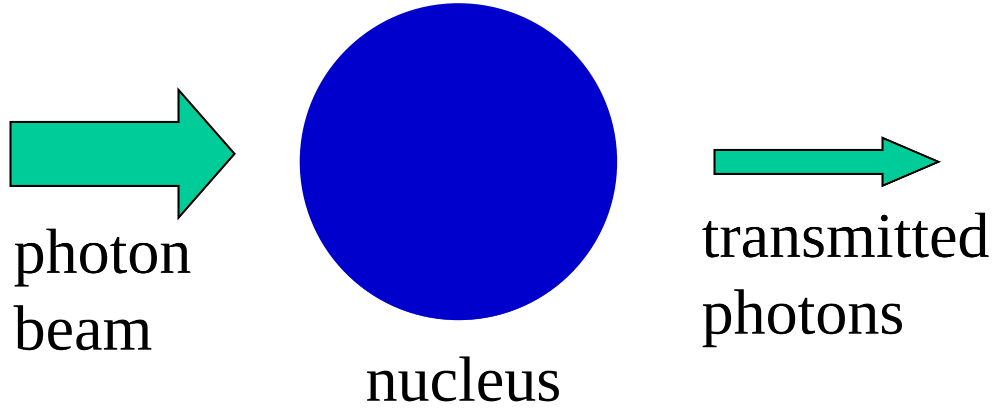
Collective Vibrations



光吸収断面積

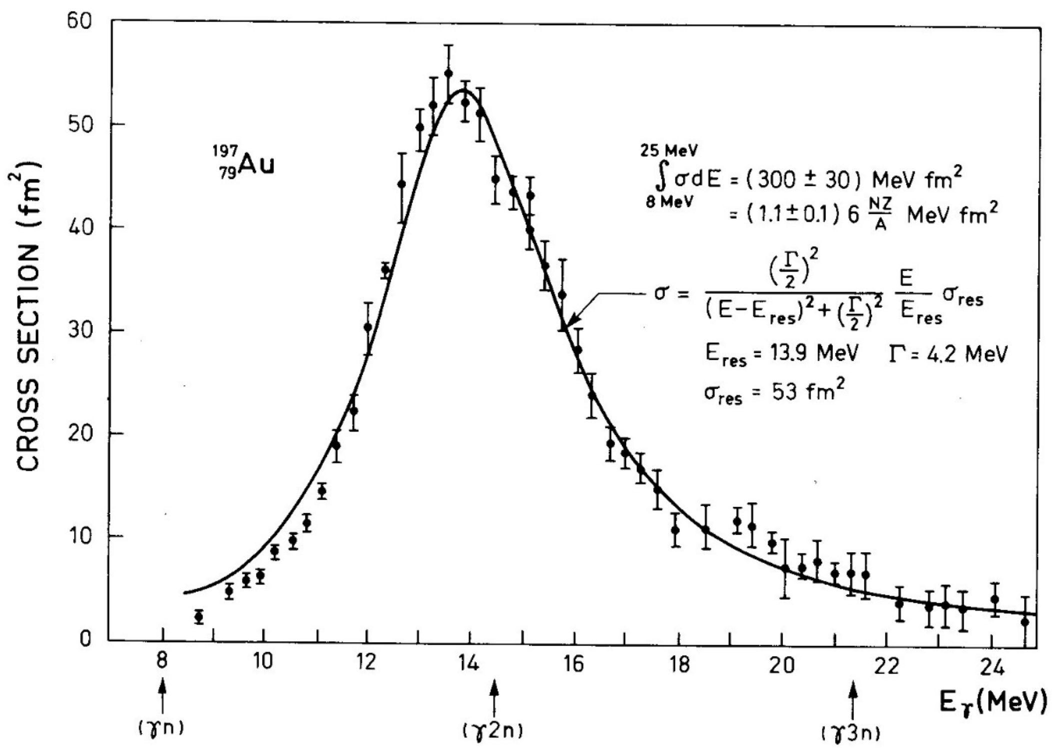
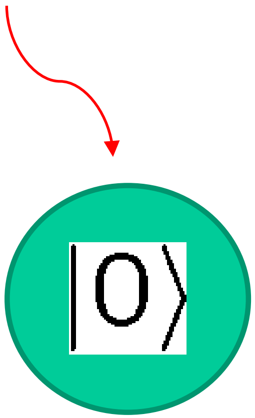


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

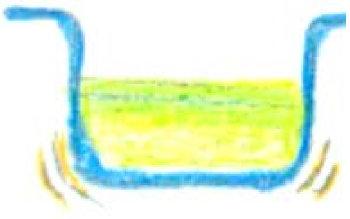
Sum Rule

$$|\psi\rangle = F|0\rangle$$

F (external field)



$|0\rangle$



図：松柳研一氏

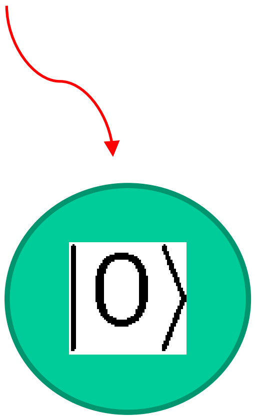
外場をかけて原子核をゆすってみる

Sum Rule

$$|\psi\rangle = F|0\rangle$$

$$= \sum_n |n\rangle\langle n|F|0\rangle$$

F (external field)



Sum Rule

$$|\psi\rangle = F|0\rangle$$

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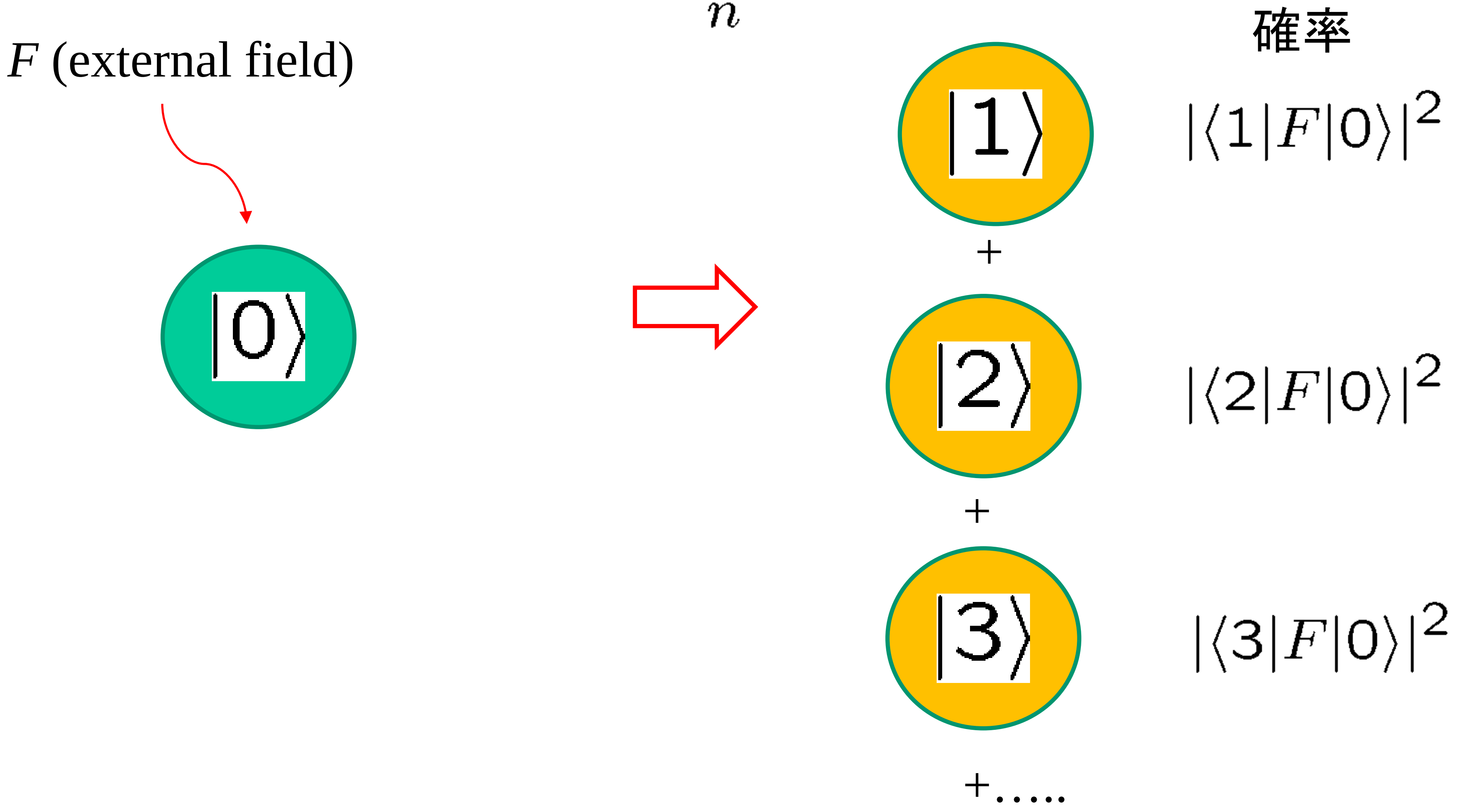
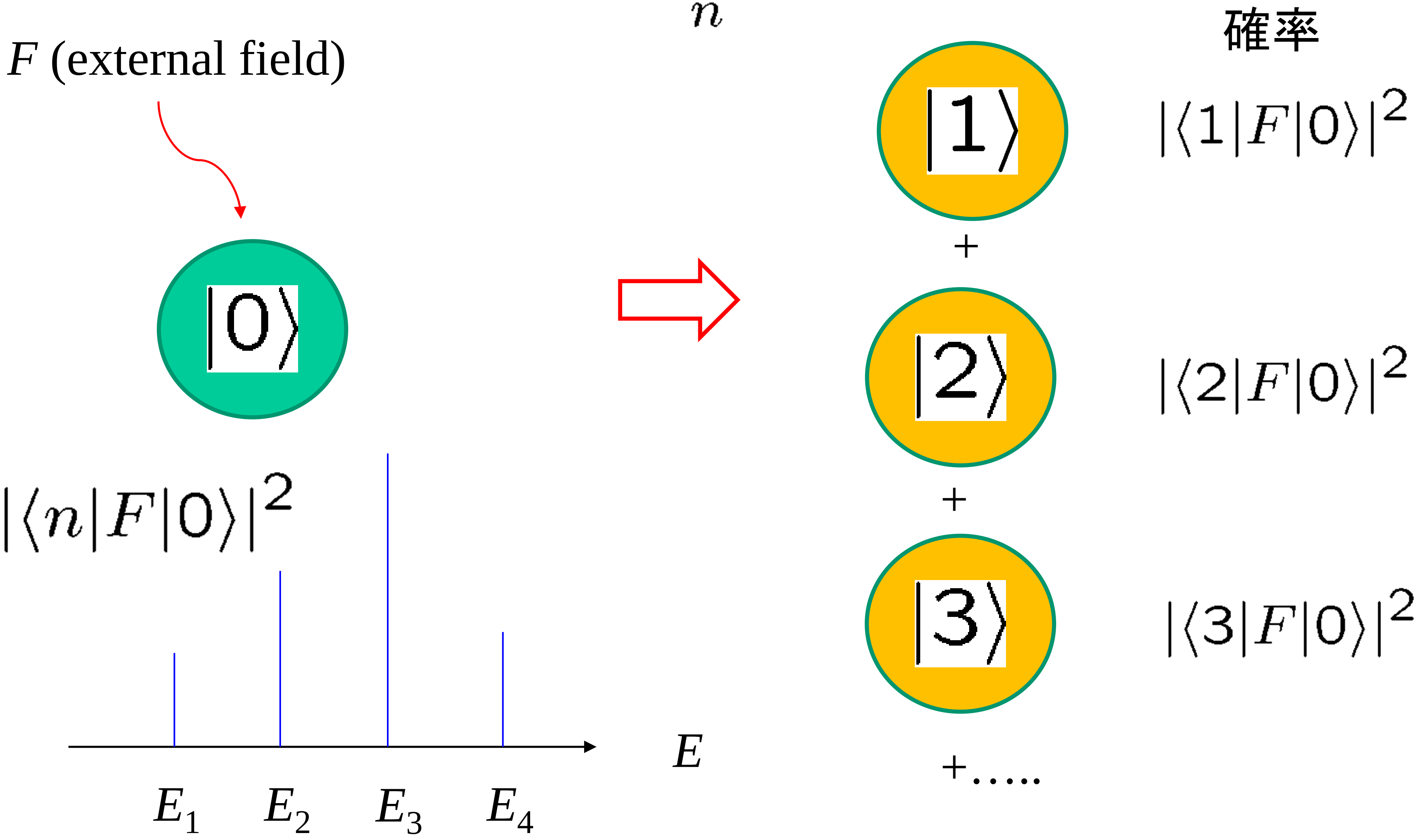


Sum Rule

$$|\psi\rangle = F|0\rangle$$

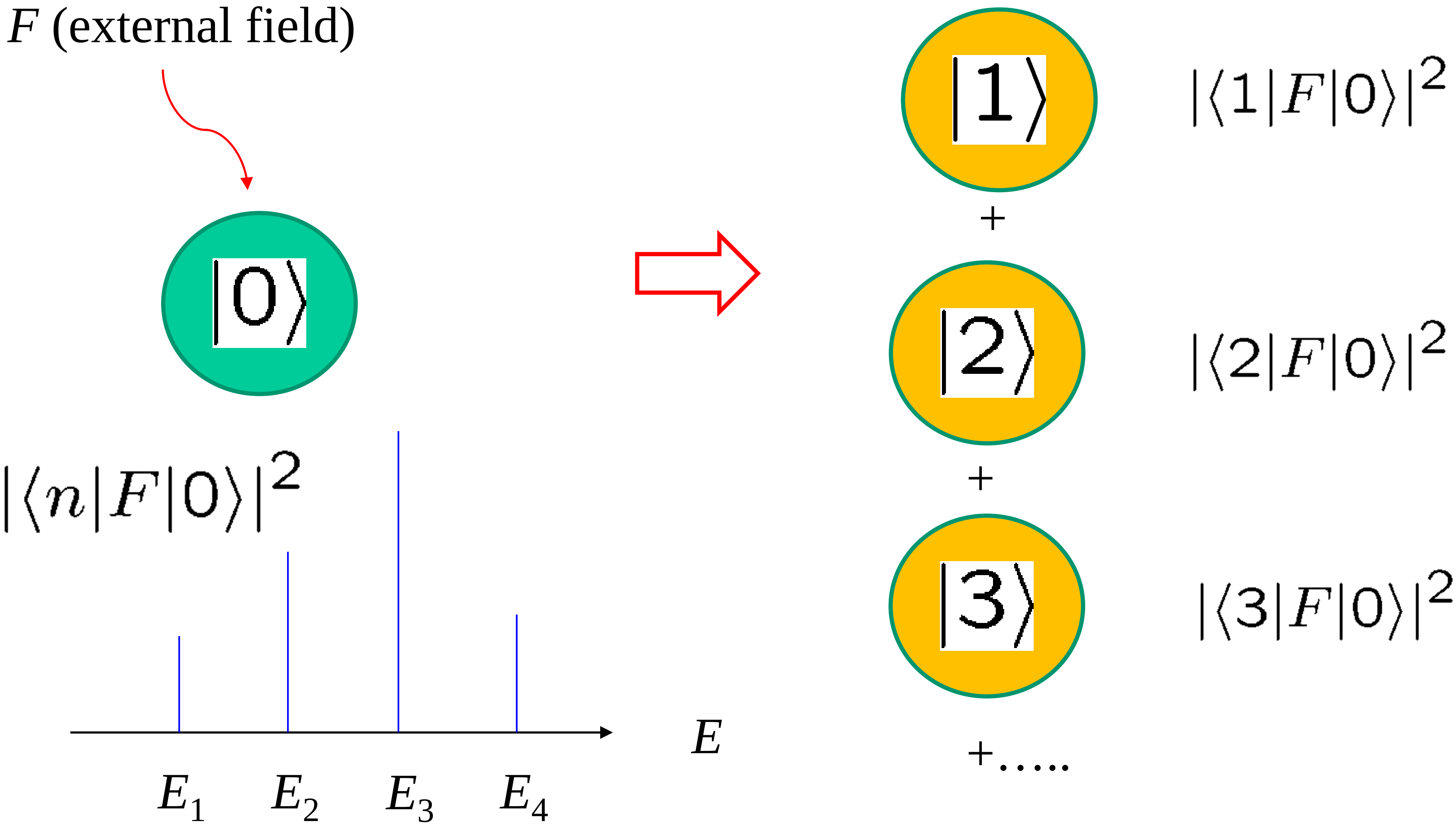
$$= \sum_n |n\rangle\langle n|F|0\rangle$$



Sum Rule

Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \, \delta(E_n - E_0 - E)$$



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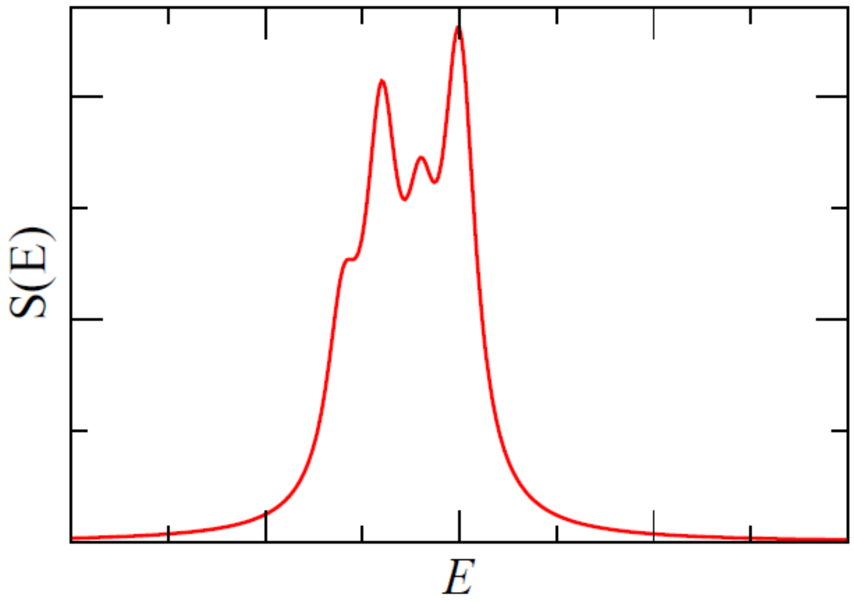
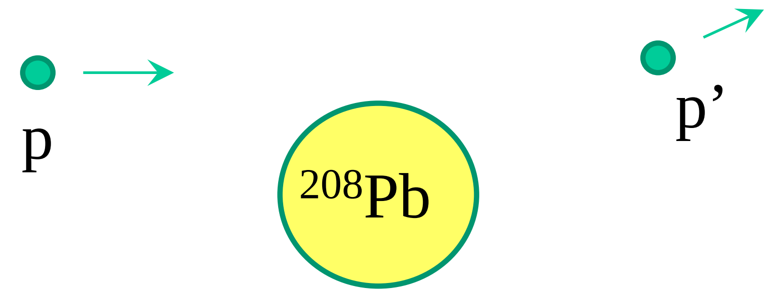
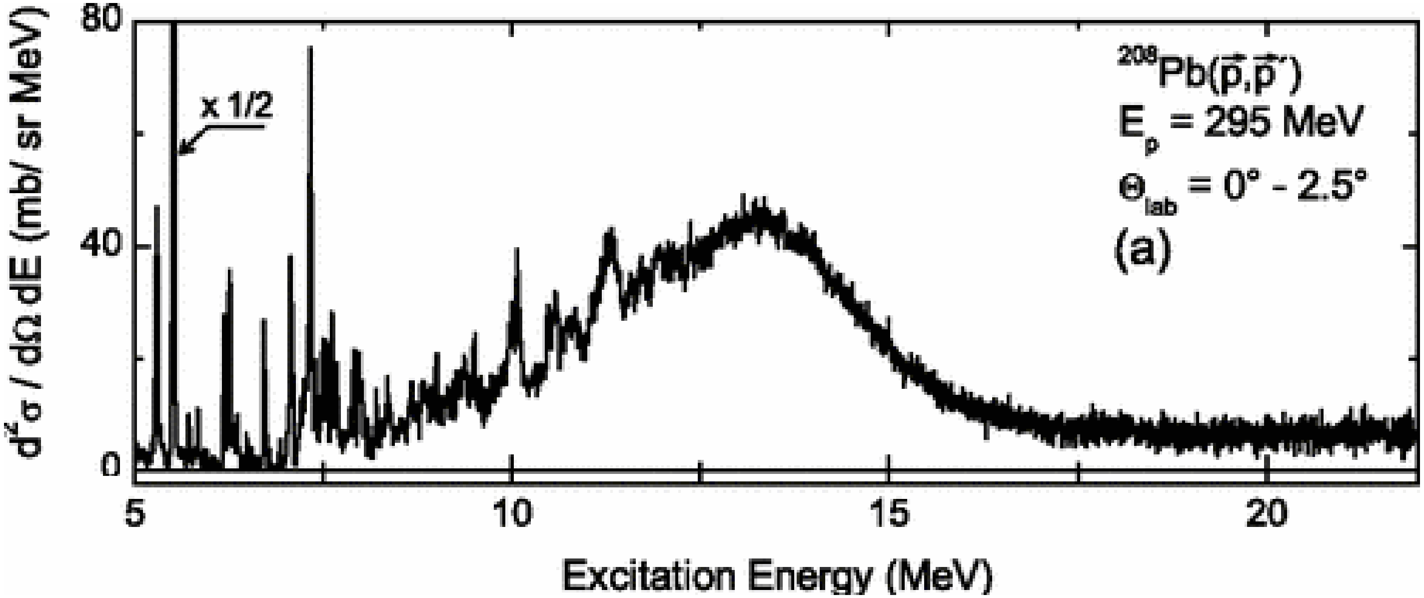


Sum Rule

例えば:



非弾性散乱(^{208}Pb の励起)のスペクトル

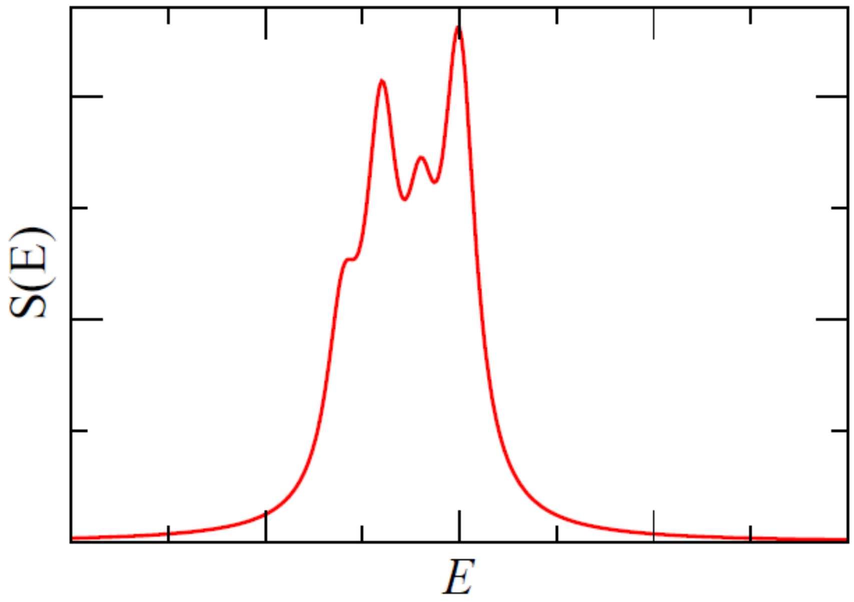


A. Tamii et al., PRL107, 062502 (2011)

Sum Rule

Strength function:

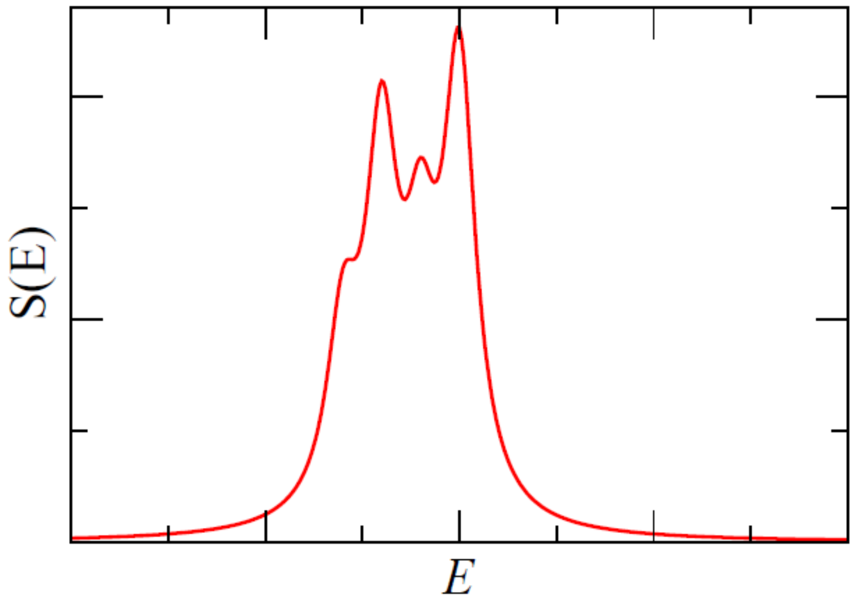
$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times \delta(E_n - E_0 - E)$$



Sum Rule

Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times \delta(E_n - E_0 - E)$$



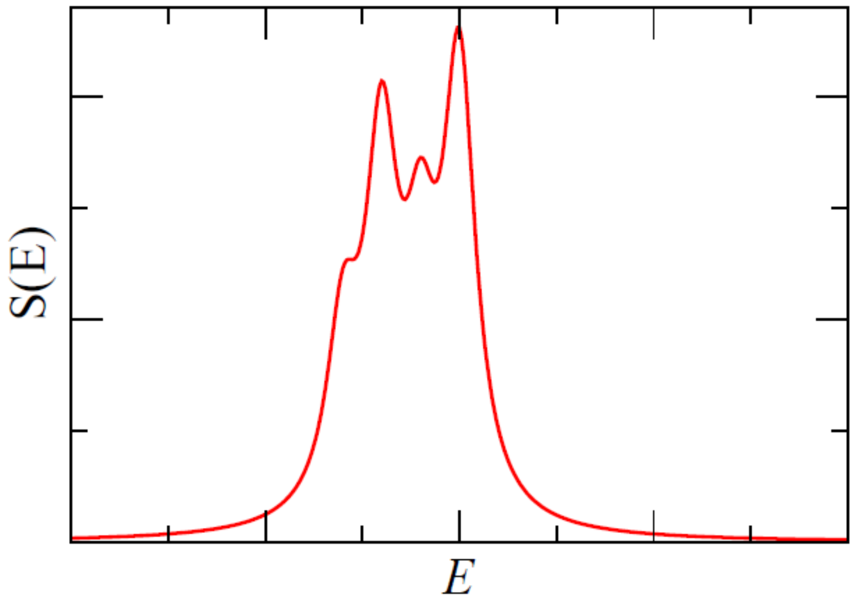
✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$

Sum Rule

Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times \delta(E_n - E_0 - E)$$



✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$

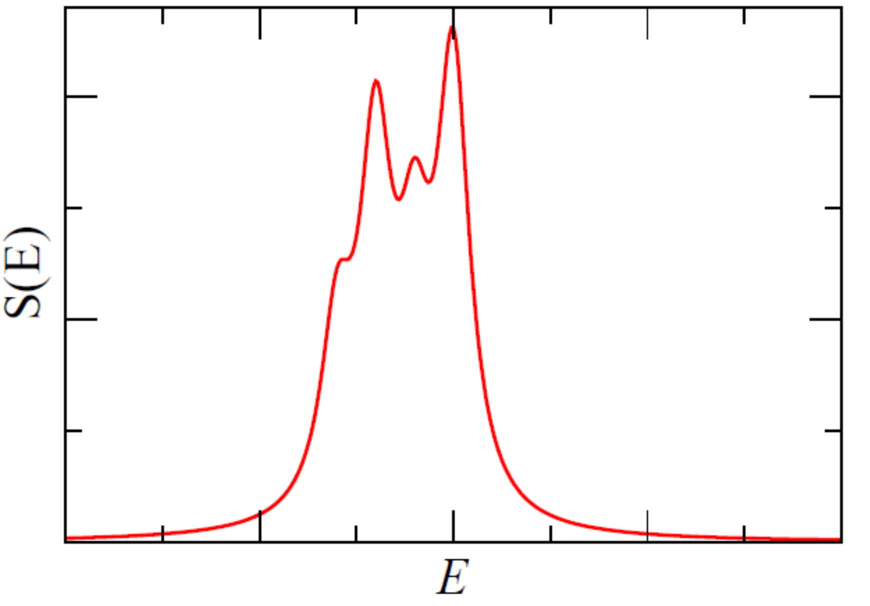
✓ energy weighted sum rule

$$S_1 \equiv \int ES(E)dE = \sum_n (E_n - E_0)|\langle n|F|0\rangle|^2$$

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

F^2の基底状態期待値



$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times \delta(E_n - E_0 - E)$$

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
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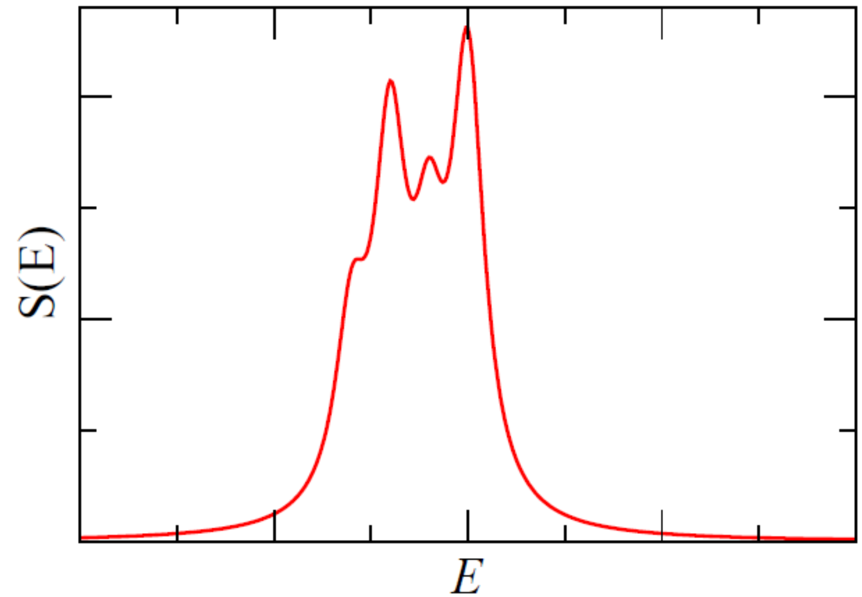
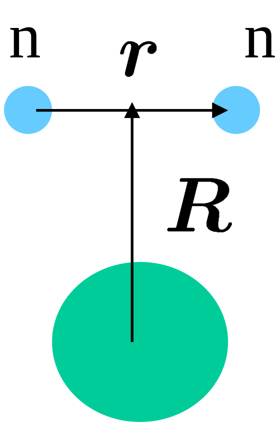
F^2の基底状態期待値



$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

cf. geometry of Borromean nuclei



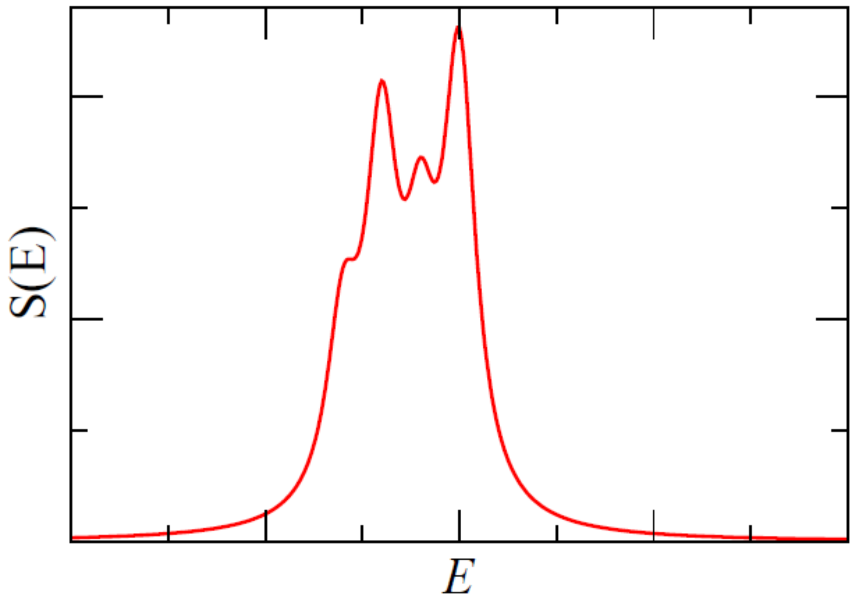
$$B(E1) = \sum_i B(E1; gs \to i)$$
$$= \frac{3}{\pi}\left(\frac{Ze}{A}\right)^2 \langle R^2\rangle$$

$$\Rightarrow \langle\theta_{nn}\rangle = 65.2^{+11.4}_{-13.0} \quad (^{11}\mathrm{Li})$$
$$= 74.5^{+11.2}_{-13.1} \quad (^{6}\mathrm{He})$$

K.H. and H. Sagawa,
PRC76('07)047302

✓ energy weighted sum rule

$$
\begin{aligned}
S_1 &\equiv \int E S(E) dE \\
&= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \\
&= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle
\end{aligned}
$$



$$
\begin{aligned}
S(E) &= \sum_\nu |\langle \nu | F | 0 \rangle|^2 \\
&\times \delta(E_\nu - E_0 - E)
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle &= \frac{1}{2} \langle F(HF - FH) - (HF - FH)F \rangle \\
&= \langle FHF - E_0 F^2 \rangle \\
&= \sum_\nu E_\nu |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \\
&= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2
\end{aligned}
$$

Energy weighted sum rule:

$$S_1 = \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 = \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle$$

For $F = F(\boldsymbol{r})$ (local operator)

$$[H, F] = \left[-\frac{\hbar^2}{2m} \nabla^2, F \right] = -\frac{\hbar^2}{2m} (\nabla^2 F + 2 \nabla F \cdot \nabla)$$

$$[F, [H, F]] = \frac{\hbar^2}{m} (\nabla F)^2$$

$$S_1 = \frac{\hbar^2}{2m} \int d\boldsymbol{r}\, \rho(\boldsymbol{r}) \cdot (\nabla F)^2$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle \nu | F | 0 \rangle|^2 = \frac{\hbar^2}{2m} \int d\boldsymbol{r}\, \rho(\boldsymbol{r}) \cdot (\nabla F)^2$$

For *F=z*

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle \nu | z | 0 \rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

⬆

Model independent

For $F = r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$

$$S_1 = \frac{\lambda(2\lambda+1)\hbar^2}{8\pi m} A \langle r^{2\lambda-2} \rangle$$

Photo absorption cross section:

$$\sigma_{\mathsf{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\begin{aligned}\tilde{z} = \sum_p (z_p - Z_{cm}) &= \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n\right)\right\} \\ &= \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n\right)\end{aligned}$$

$$\begin{aligned}\int \sigma_{\mathsf{abs}}(E_\gamma)dE_\gamma &= \frac{4\pi^2 e^2}{\hbar c}\cdot\frac{\hbar^2}{2m}\cdot\frac{NZ}{A} \\ &= \frac{2\pi^2 e^2 \hbar}{mc}\cdot\frac{NZ}{A}\end{aligned}$$

Giant Dipole Resonance (GDR)

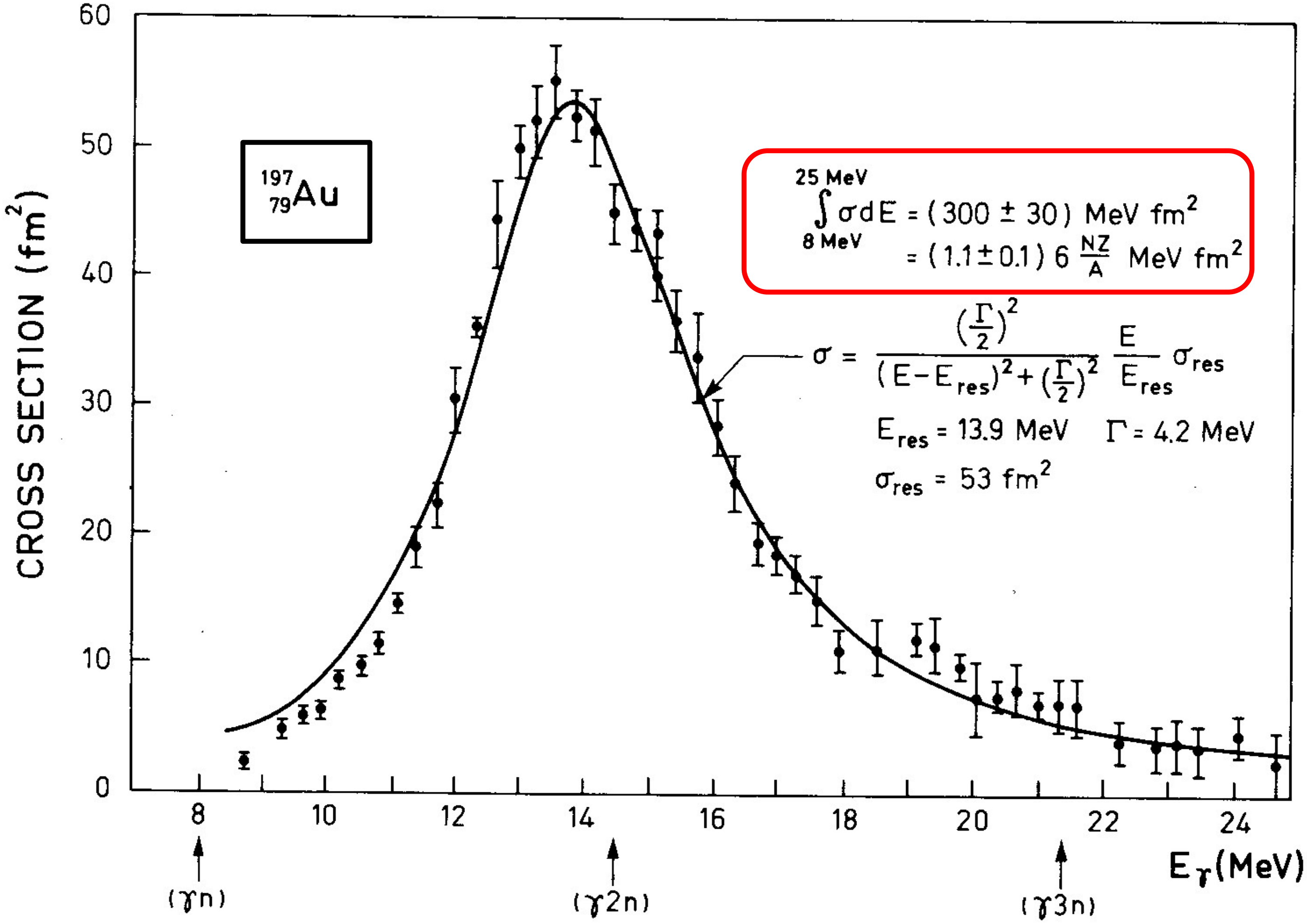


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

cf. $41 \times 197^{-1/3} = 7.05$ MeV

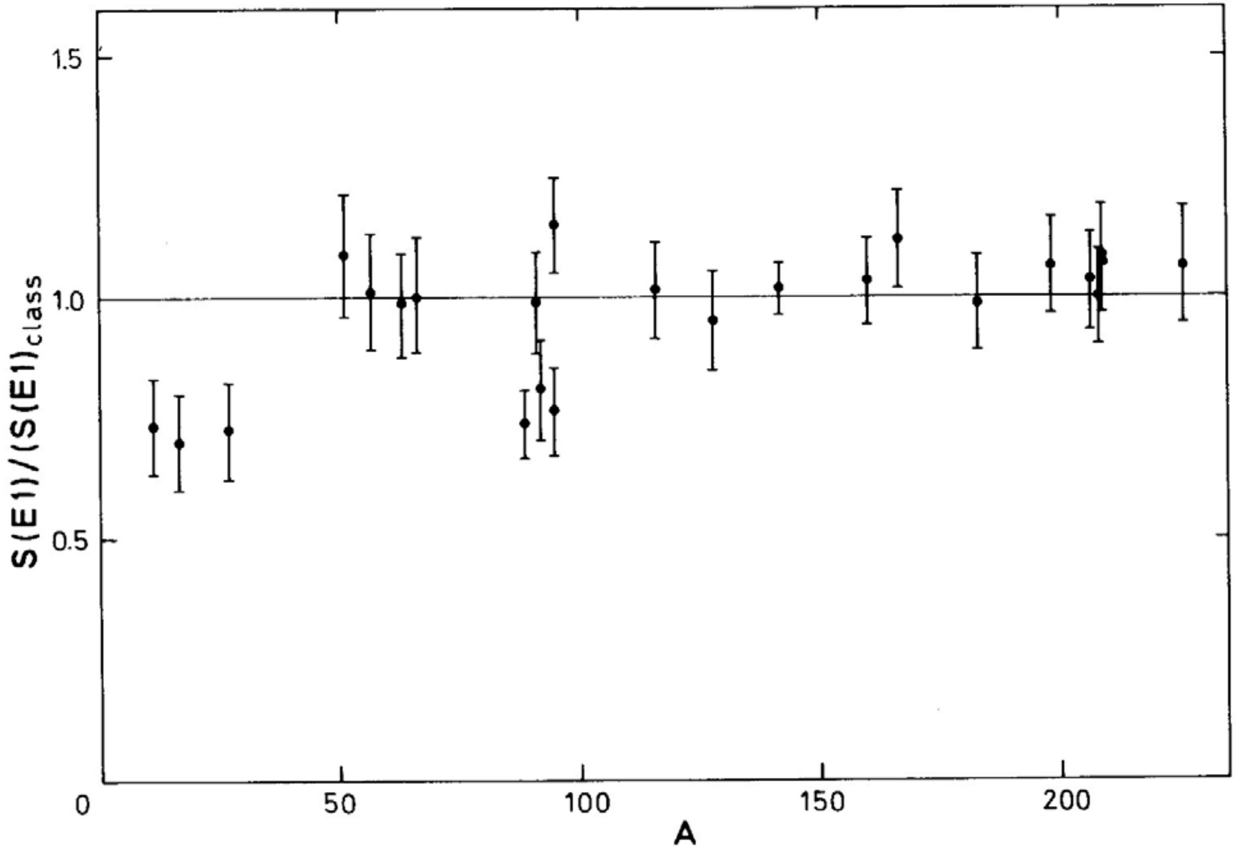
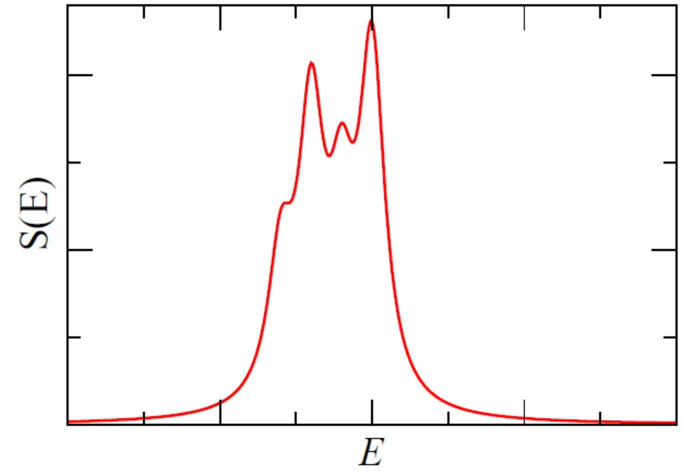


Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

和則の利点

$$S_0 = \langle 0|F^2|0\rangle$$
$$S_1 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$



和則：
励起状態の（ある種の）情報が基底状態の
性質のみによって表わされる
（励起状態の情報を知っている必要がない）。

- 実験で強度分布が測られた時、測られた範囲外にも強度があるかどうか (missing strength) 判断できる。
- 強度分布を測ることによって原子核の半径などの情報を得られる。
- 実験データや数値計算のチェックになる。
（和則の値よりとても大きくなると何かがおかしい）。

Giant Quadrupole Resonance (GQR)

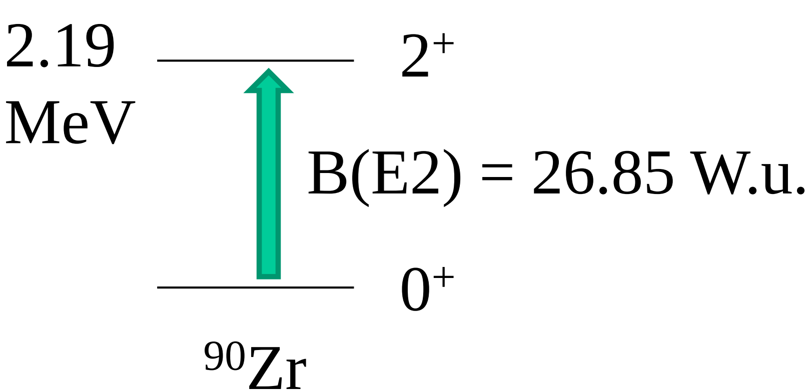
VOLUME 29, NUMBER 16 PHYSICAL REVIEW LETTERS 16 OCTOBER 1972

Giant Multipole Resonances in ^{90}Zr Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in ^{90}Zr was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

GQRの発見以前は、低エネルギー 2^+ 状態のみが知られていた

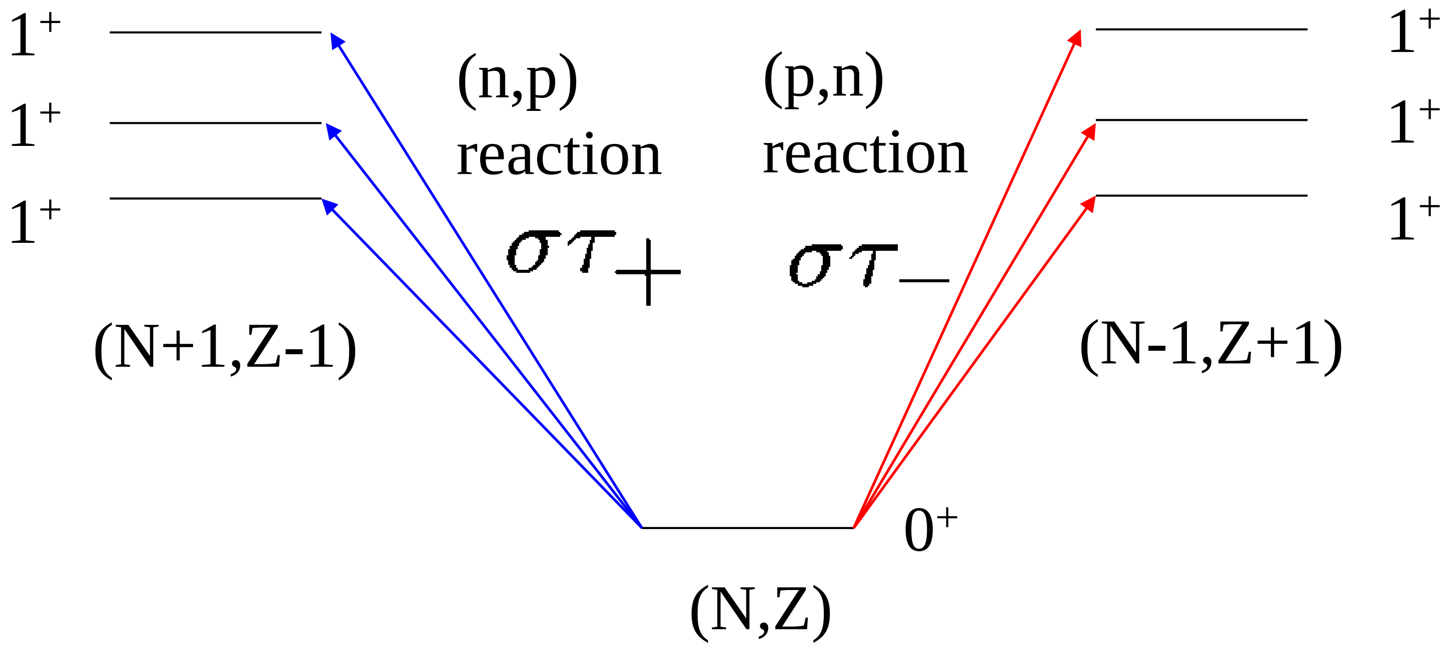


→ EWSR の 2.5% 程度
残りはどこに?

→ GQR (~14 MeV) が
発見され解決

Ikeda sum rule

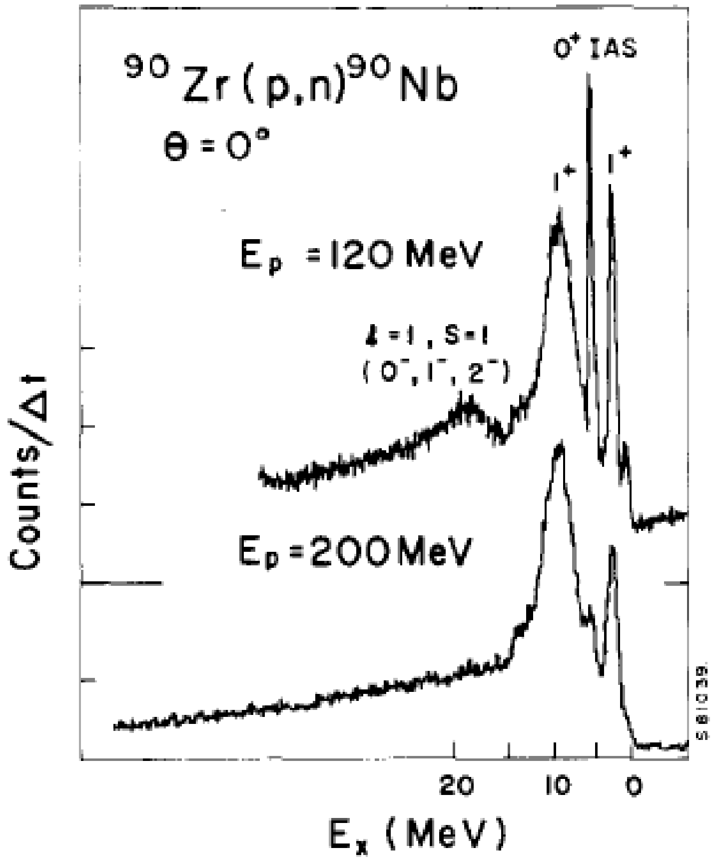
charge exchange reactions: Gamow-Teller transitions

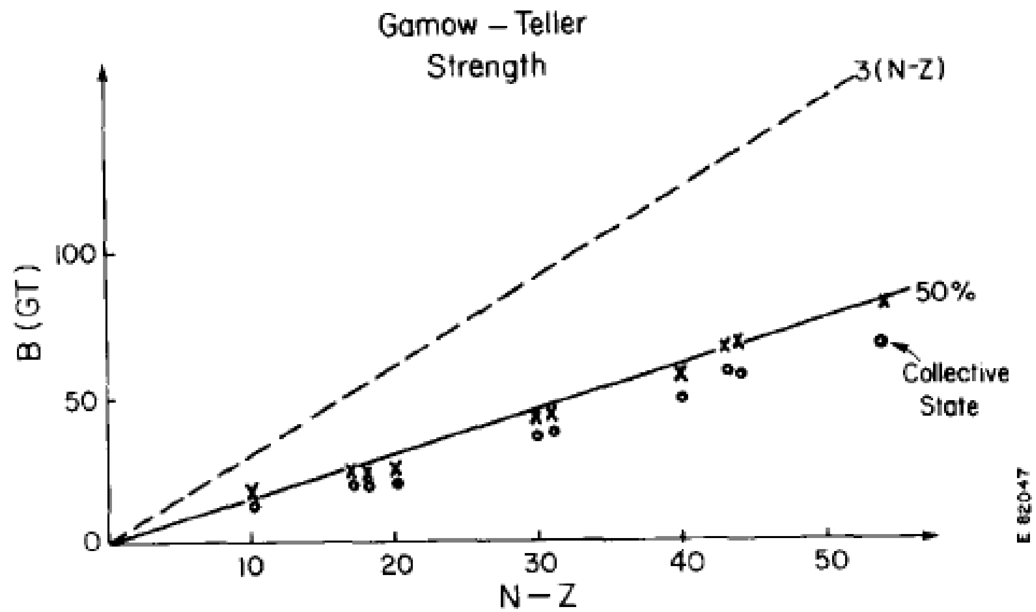


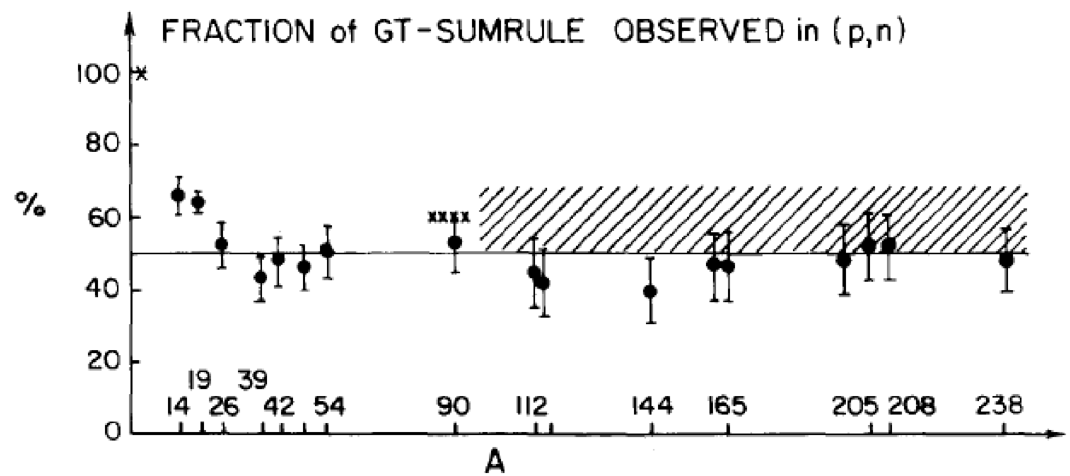
Ikeda sum rule

$$S_0(\sigma\tau_-) - S_0(\sigma\tau_+) = 3(N-Z)$$

the situation before 1997





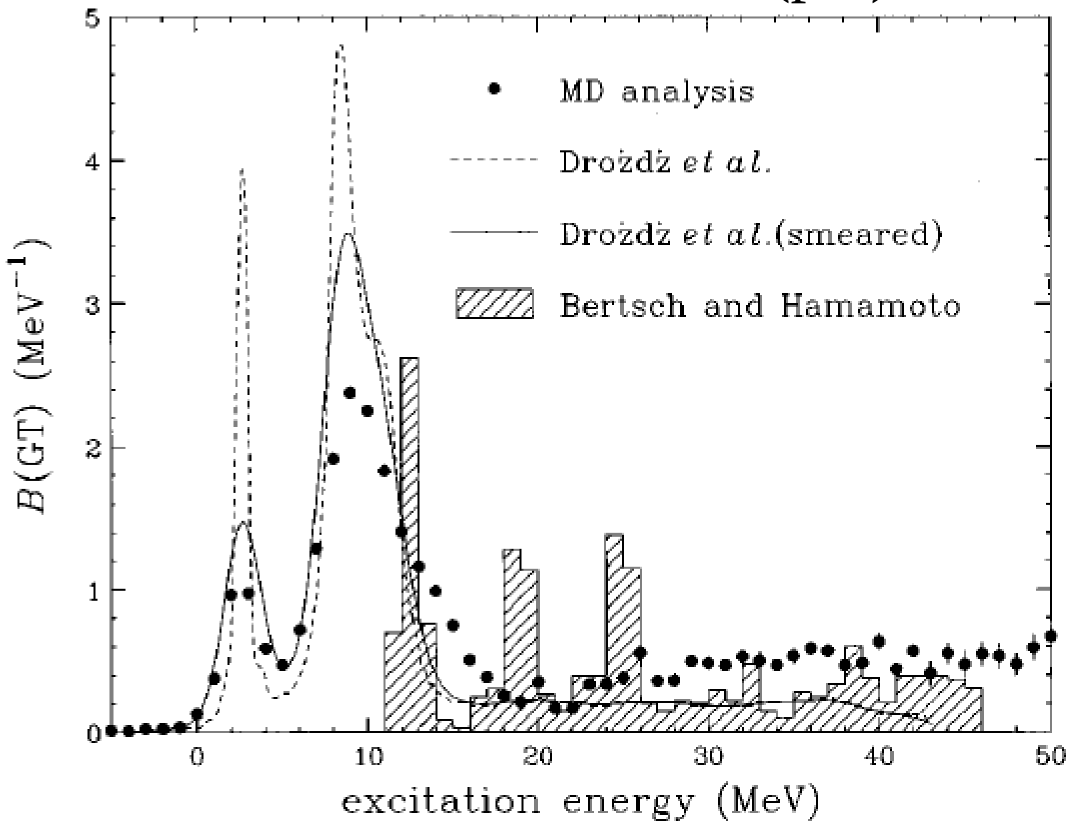


the "quenching problem"
of GT strength

quark (Δ resonance)?

C. Gaarde, Nucl. Phys. A396('83)127c

^{90}Zr (p,n) ^{90}Nb



T. Wakasa et al., PRC55 (‘97) 2909

$S_- - S_+$ = 27.0 +/- 1.6 = (90 +/-5)% of Ikeda sum rule

→ quark contribution: small

レポート問題2（〆切：12月5日（土））

1次元調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

の n 番目の固有状態 $|n\rangle$ を考える (n=0 が基底状態)。

1) 演算子 x で遷移できる状態 $|k\rangle$ を全て書き出し（状態 k も調和振動子の固有状態）、遷移確率

$$P_{n\to k} = |\langle k|x|n\rangle|^2$$

を求めよ。

2) energy weighted sum rule

$$S_1 = \sum_k (E_k - E_n) P_{n\to k}$$

を計算し、TRK和則が成り立っていることを示せ。

レポート問題3（〆切：12月5日（土））

以下の Ikeda sum rule を証明せよ。

$$(Y_\pm)_\mu \equiv \sum_{i=1}^{A} \tau_\pm(i)\sigma_\mu(i) \quad ; \quad [(Y_\pm)_\mu]^\dagger = (-)^\mu (Y_\mp)_{-\mu}$$

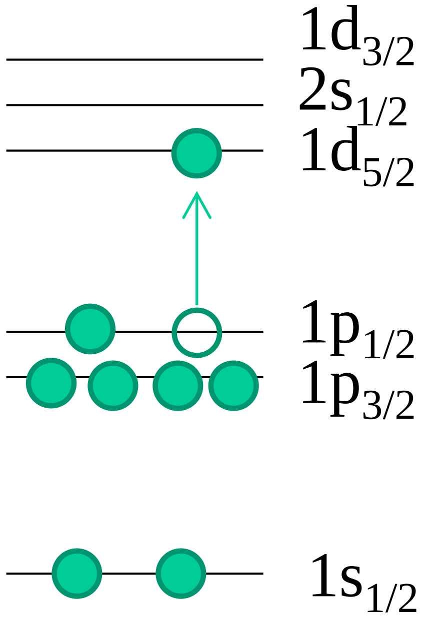
$$\begin{aligned} S_- - S_+ &= \langle 0|Y_-^\dagger Y_-|0\rangle - \langle 0|Y_+^\dagger Y_+|0\rangle \\ &= \sum_\mu \langle 0|(Y_-)_\mu^\dagger (Y_-)_\mu|0\rangle - \langle 0|(Y_+)_\mu^\dagger (Y_+)_\mu|0\rangle \\ &= 3(N-Z) \end{aligned}$$

ここで、|0> は陽子数 Z、中性子数 N, 質量数 A=Z+N を持つ原子核の基底状態である。

集団励起の微視的理論

原子核の励起状態

✓ 一粒子励起（一つの核子が励起に関与）

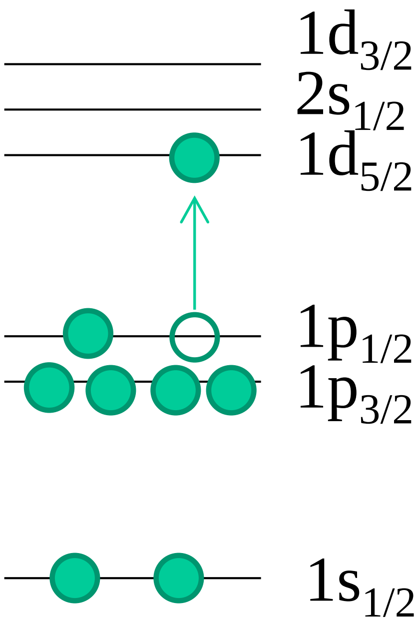


一粒子励起の例

集団励起の微視的理論

原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）



一粒子励起の例

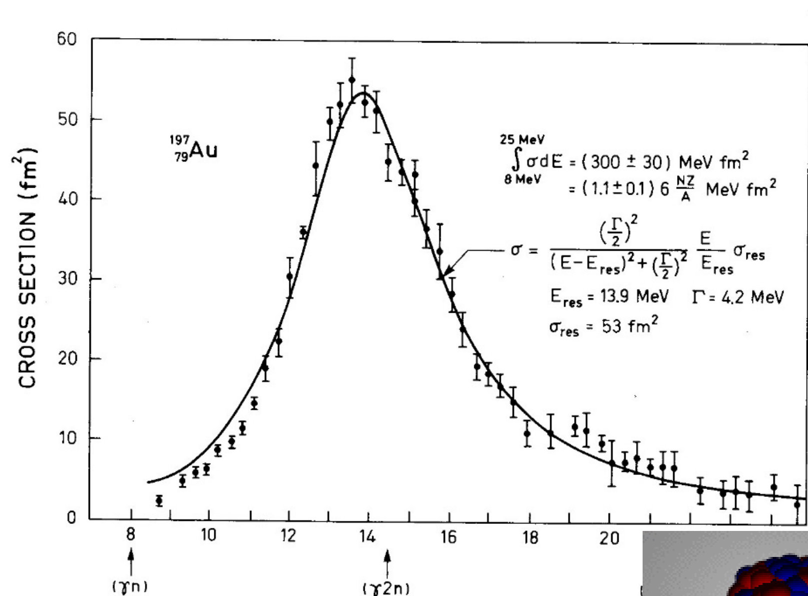
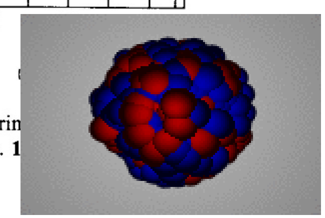
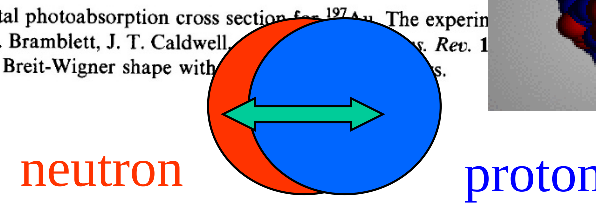


Figure 6-18 Total photoabsorption cross section for ¹⁹⁷Au. The experim… S. C. Fultz, R. L. Bramblett, J. T. Caldwell, … s. *Rev.* 1… solid curve is of Breit-Wigner shape with … s.





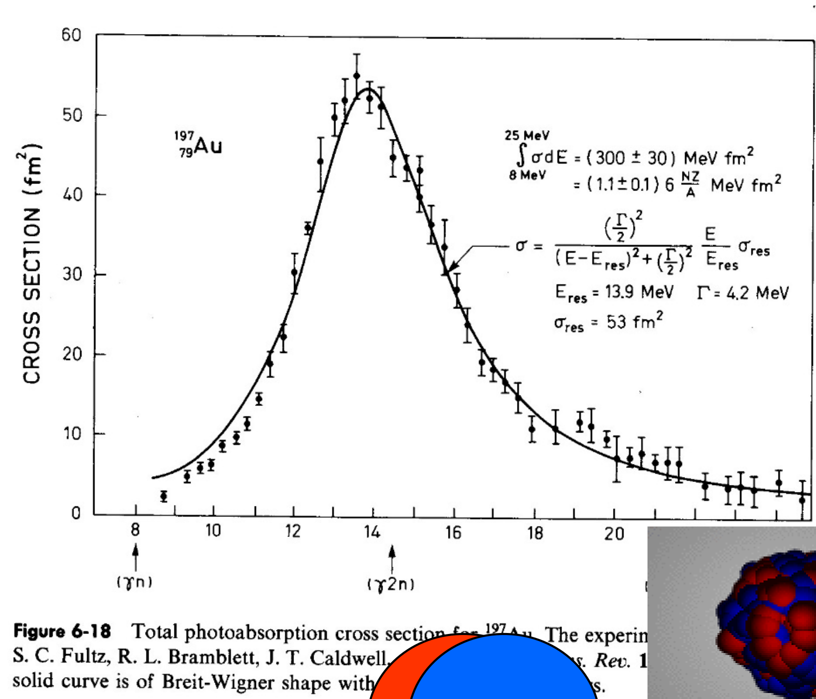
集団励起の例：巨大双極子共鳴

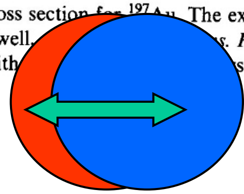
集団励起の微視的理論

原子核の励起状態

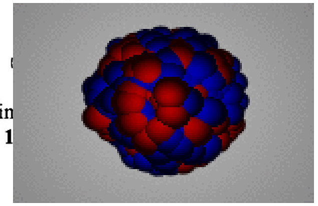
- ✓ 一粒子励起(一つの核子が励起に関与)
- ✓ 集団励起(多くの核子が集団として励起に関与)

集団励起を微視的に理解してみる
(集団励起をミクロに見てみるとどうなっているのか?)





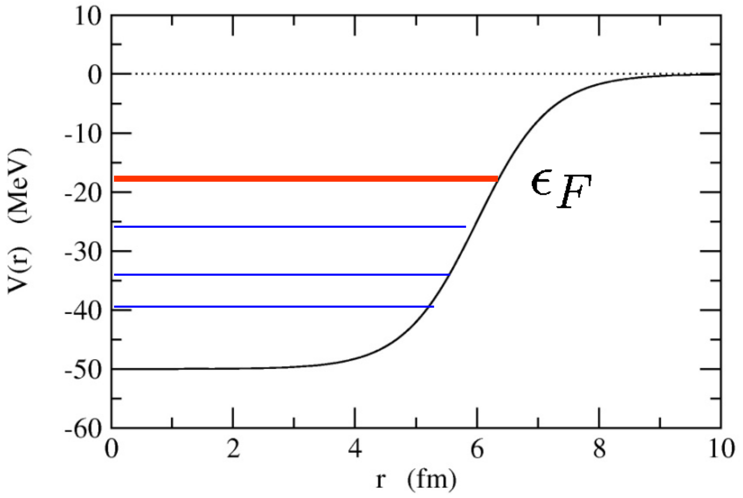
neutron
proton



集団励起の例:巨大双極子共鳴

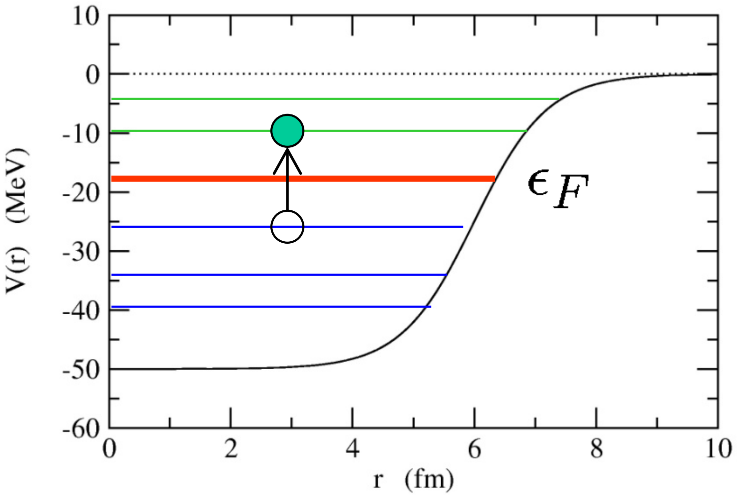
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle = \prod_h a_h^\dagger |0\rangle$$

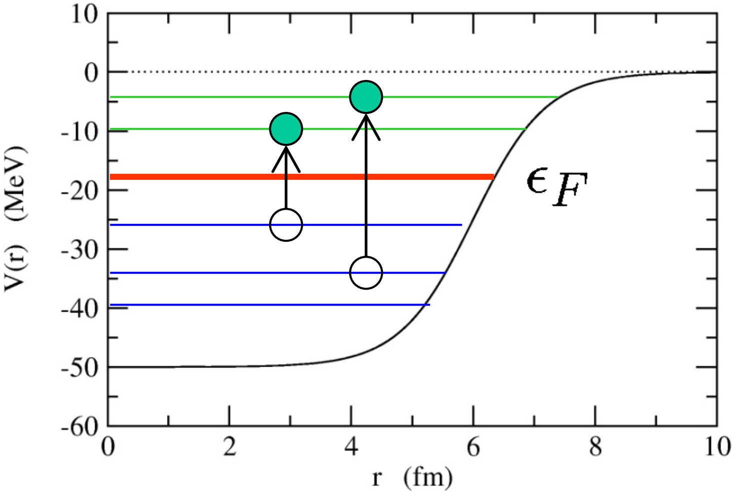
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$

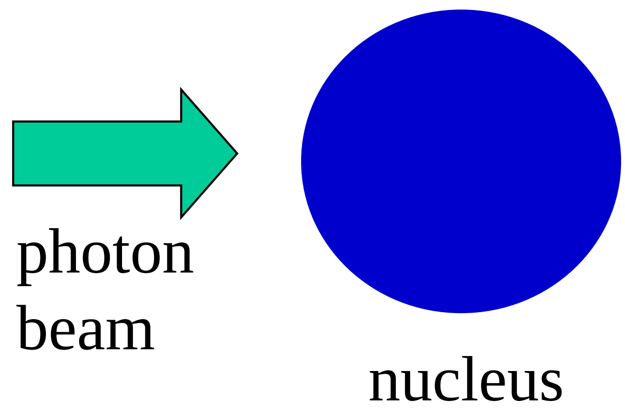


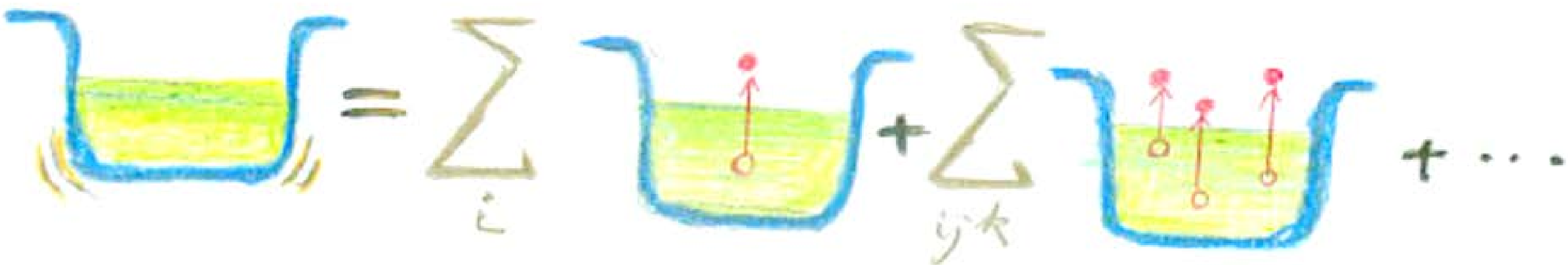
Tamm-Dancoff Approximation

Assume: $$|\nu\rangle = Q_\nu^\dagger |HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle$$
$$\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

(superposition of 1p1h states)

原子核を外場により揺らしてみると何が起こるのか?





スライド：松柳研一氏

Tamm-Dancoff Approximation

Assume: $$|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle \equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

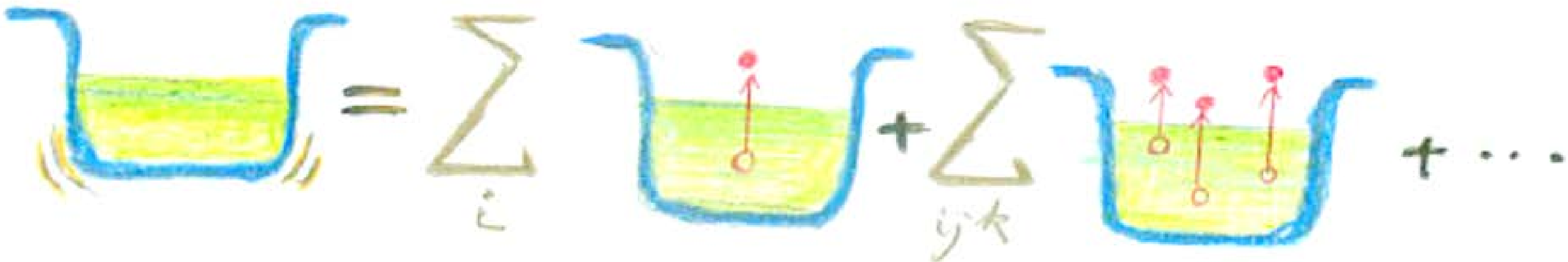
$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

residual interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

Tamm-Dancoff equation

残留相互作用の意味



スライド：松柳研一氏

$$V(\boldsymbol{r}) \sim \int d\boldsymbol{r}'\, v(\boldsymbol{r}, \boldsymbol{r}')\rho(\boldsymbol{r}')$$

vibration: $\rho = \rho_0(\boldsymbol{r}) \rightarrow \rho_0(\boldsymbol{r}) + \delta\rho(\boldsymbol{r}, t)$

↓ ($\delta\rho(\boldsymbol{r}, t)$)

residual
interaction