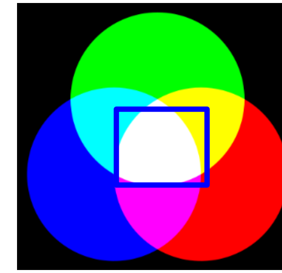
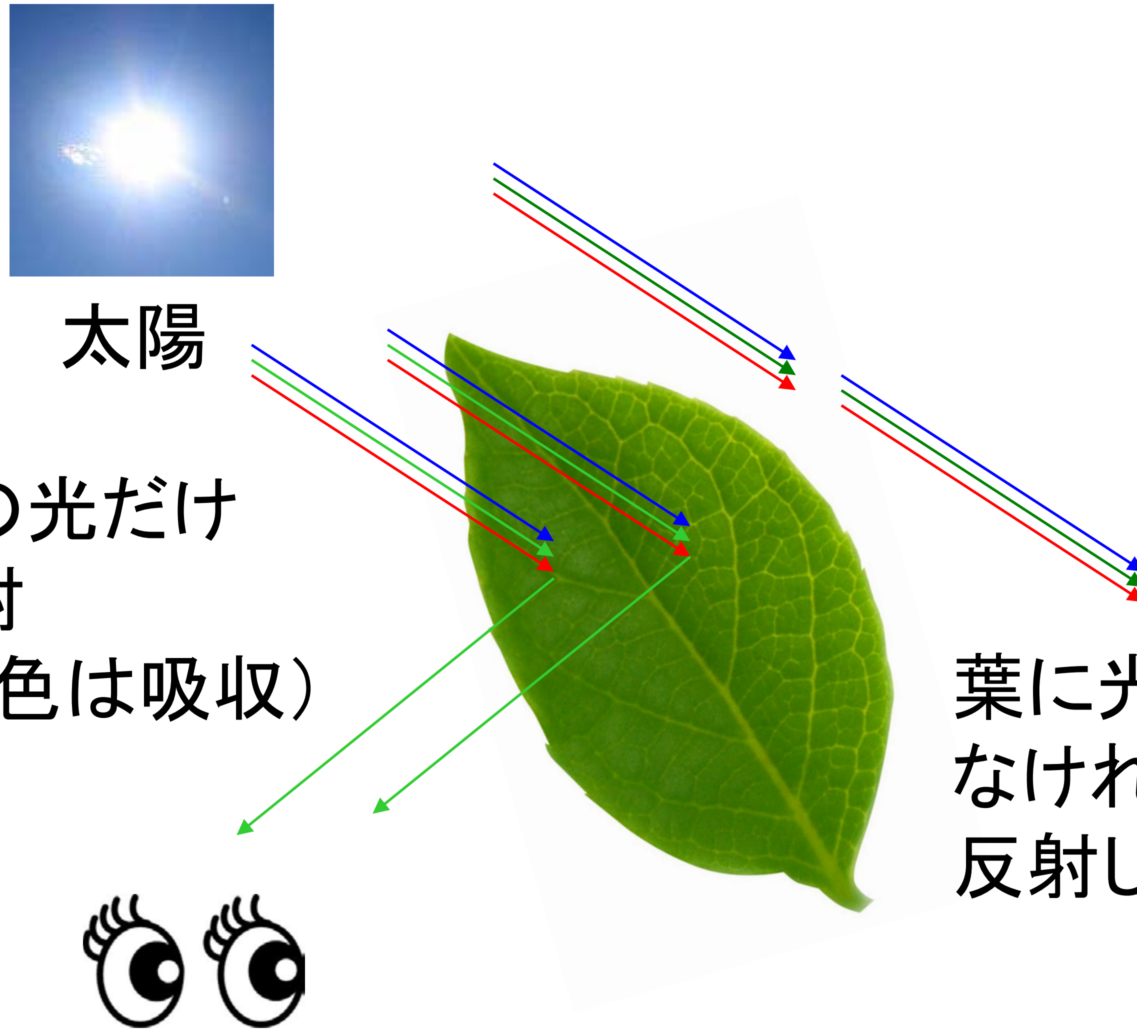
Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

そもそも、ものが見えるとはどういうことか？





太陽

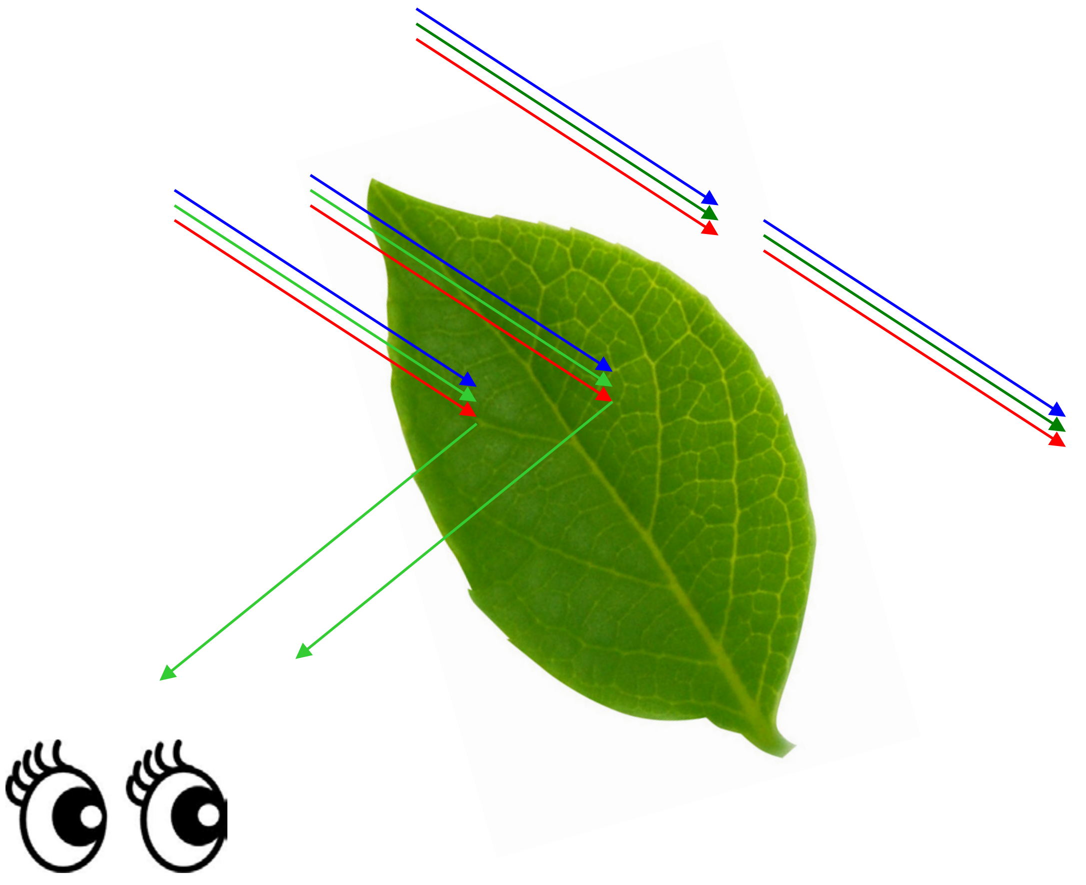
緑色の光だけ
が反射
（他の色は吸収）

葉に光が当たら
なければ緑は
反射しない

⇩

葉の形

そもそも、ものが見えるとはどういうことか？



原子核のようなミクロなものの大きさを測るのも基本的には同じ

何かをぶつけて、どのように散乱されるか観測する

Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

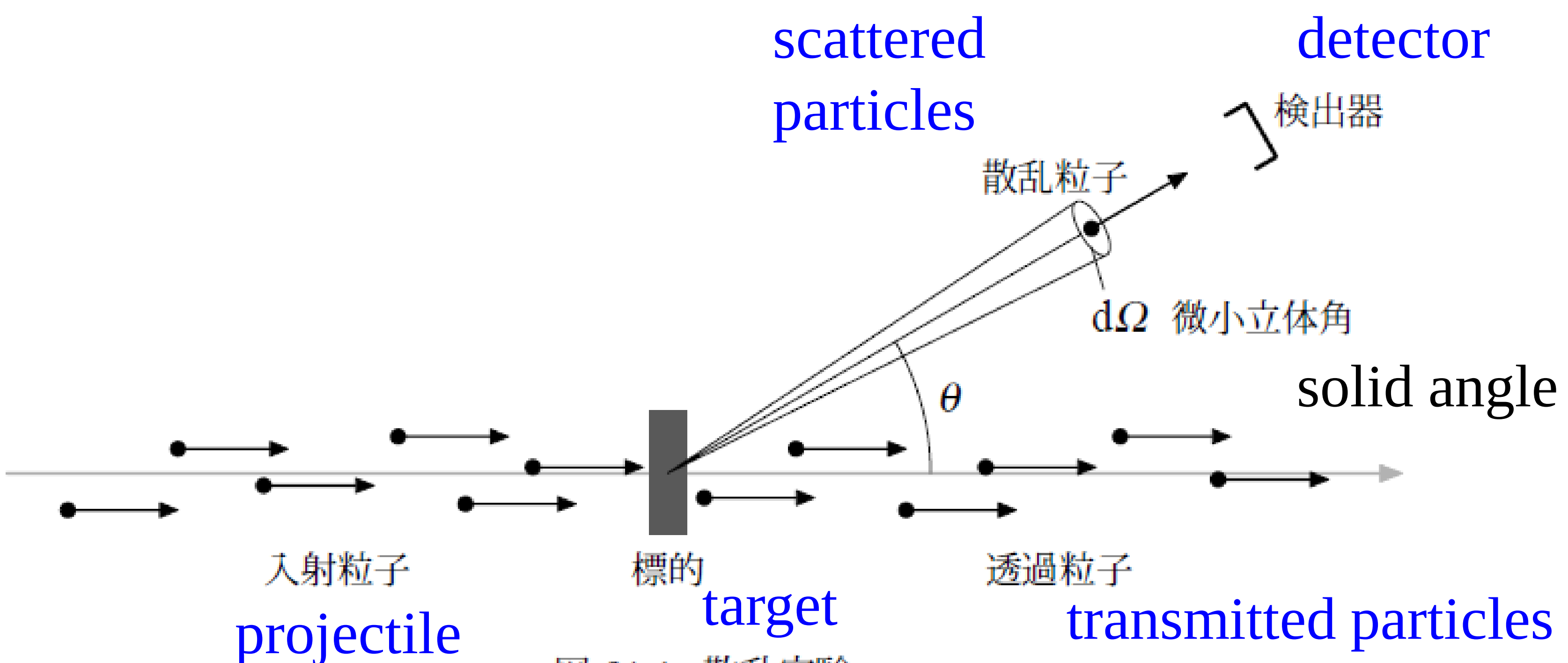


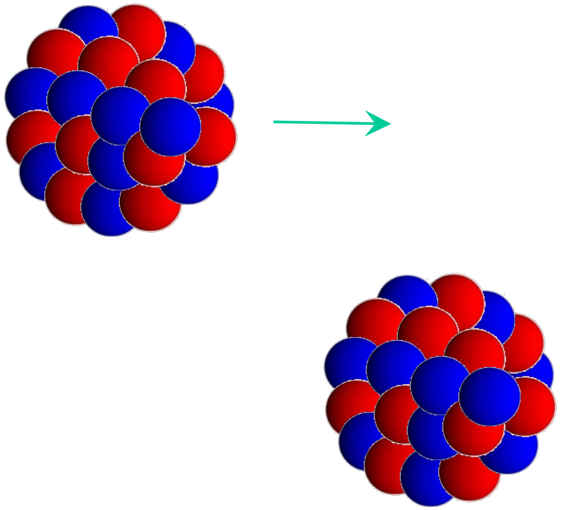
図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

K. Muto (TIT)

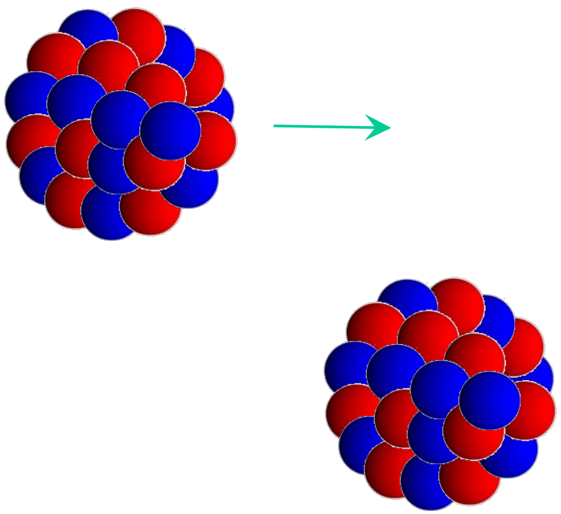
量子多体系のダイナミックス（原子核反応）

弾性散乱

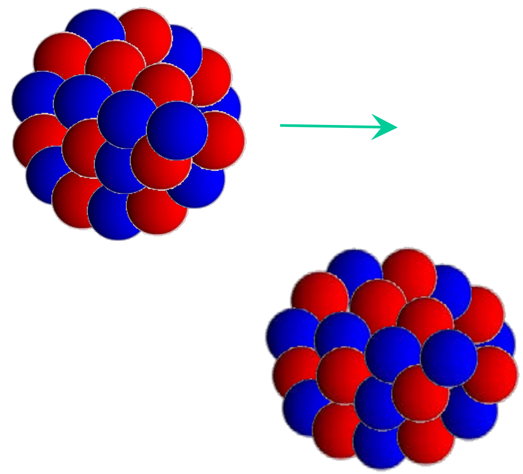


量子多体系のダイナミックス（原子核反応）

弾性散乱

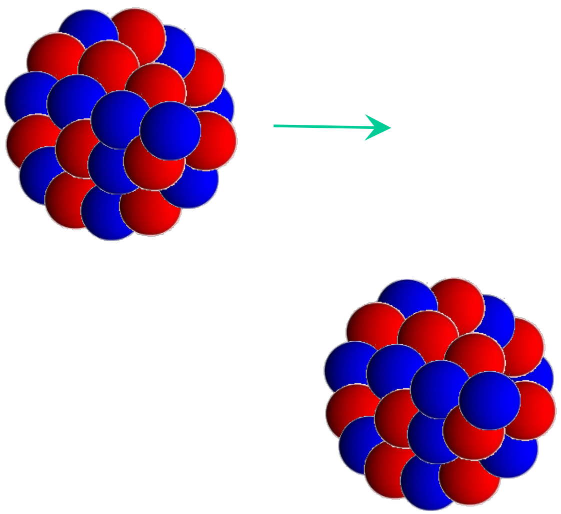


非弾性散乱

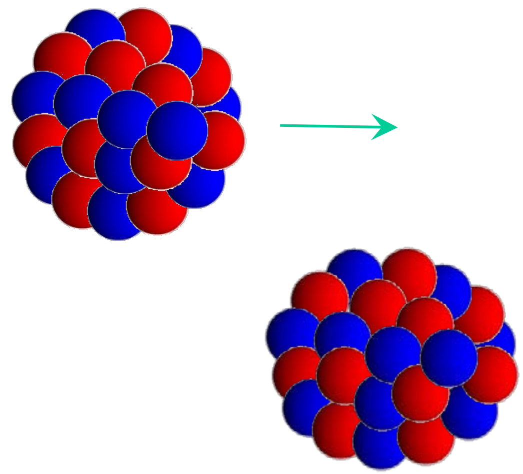


量子多体系のダイナミックス(原子核反応)

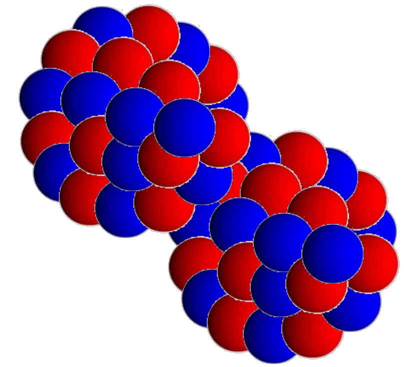
弾性散乱



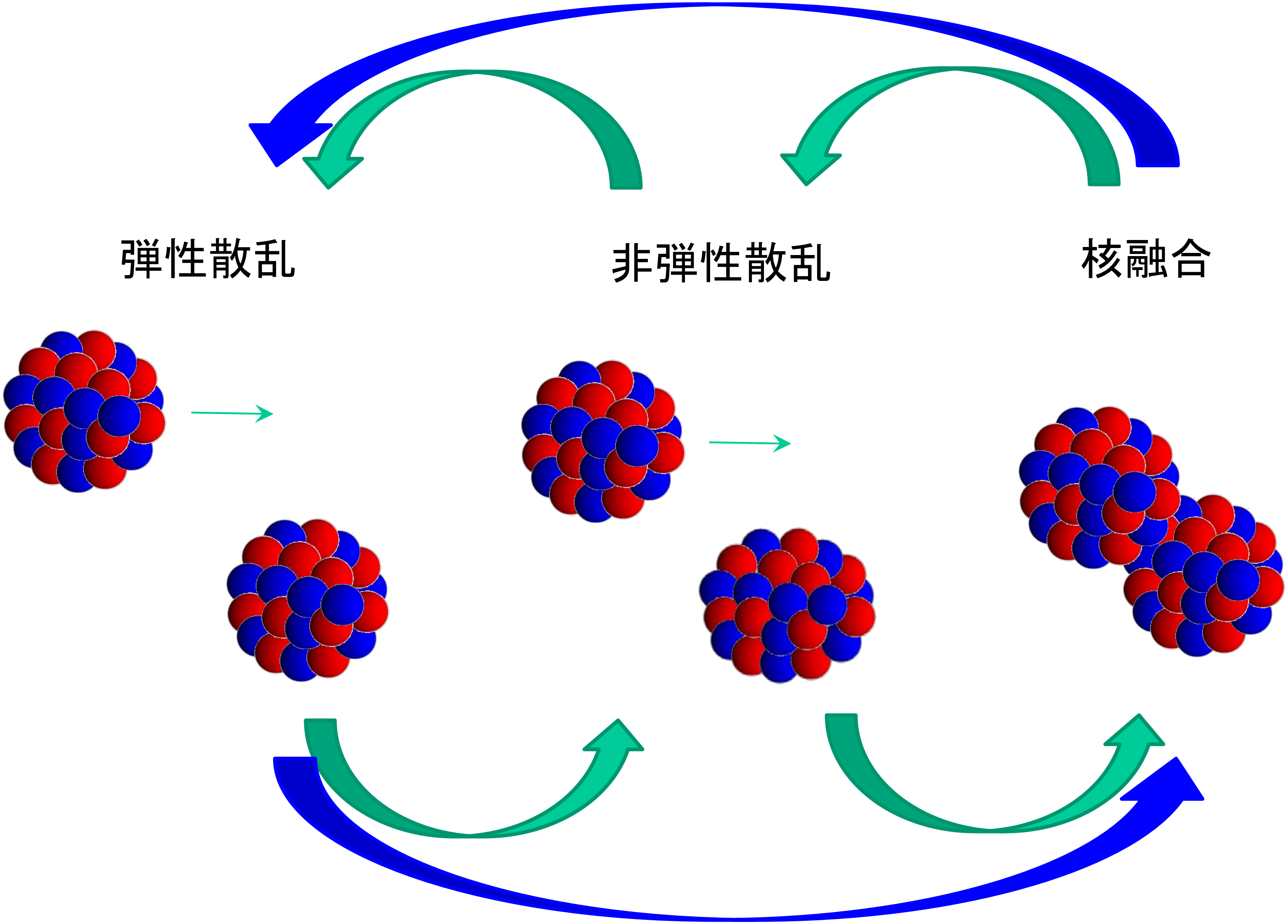
非弾性散乱



核融合

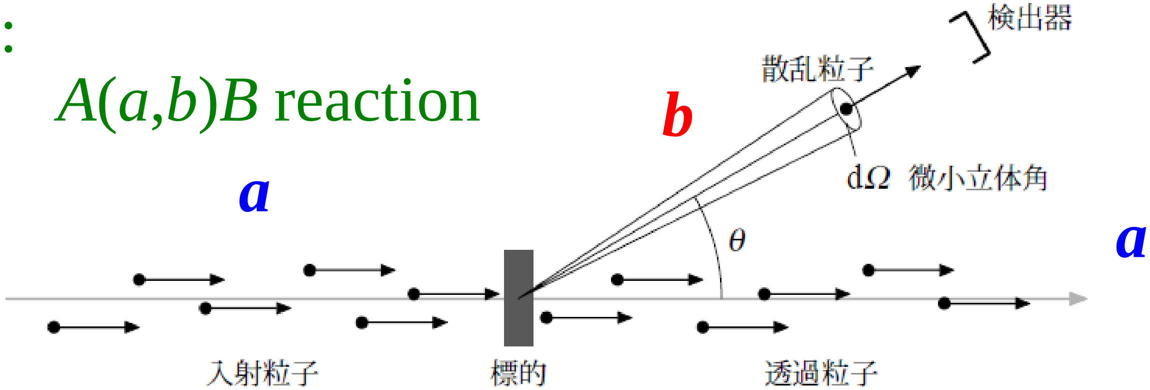


量子多体系のダイナミックス（原子核反応）



notation:

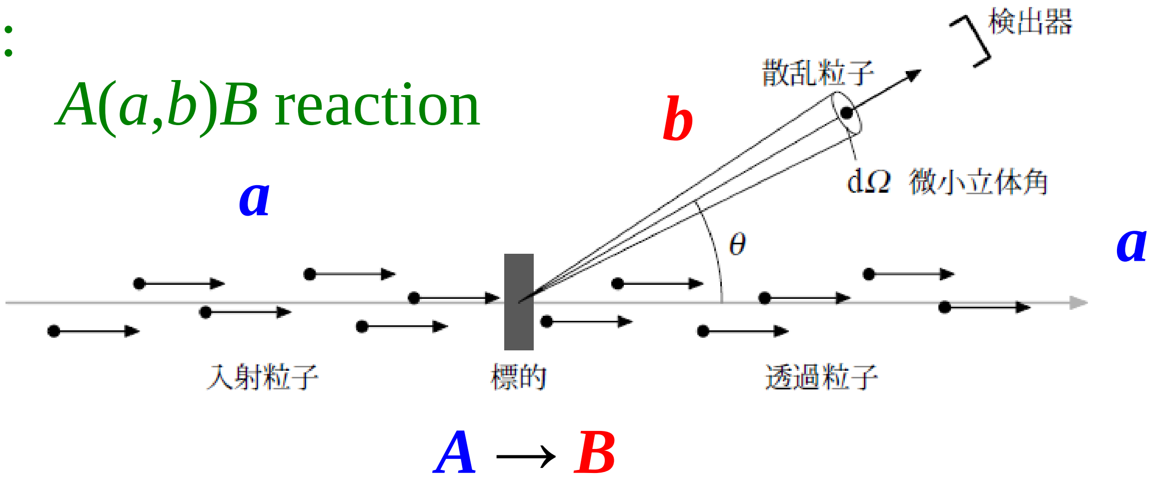
$A(a,b)B$ reaction



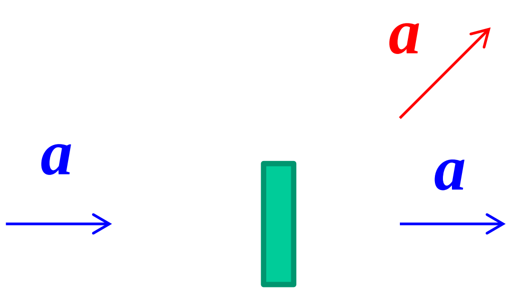
$A \rightarrow B$

notation:

$A(a,b)B$ reaction



✓elastic scattering

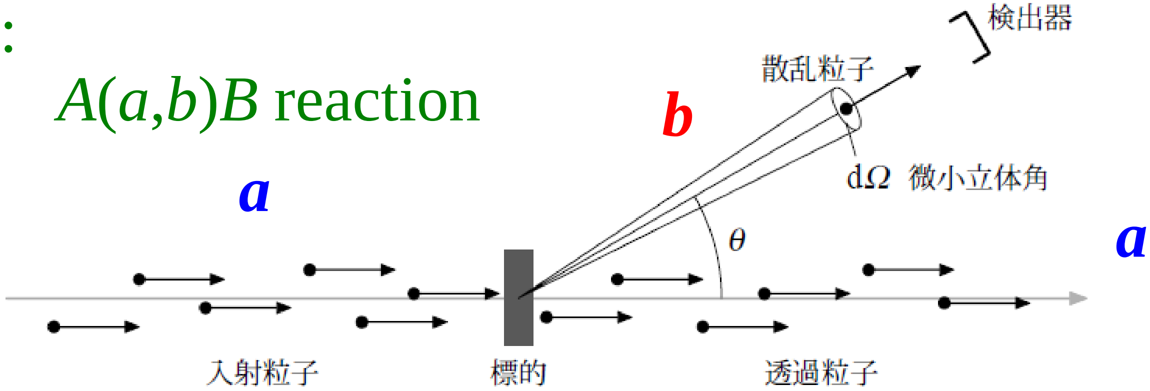


$A_{gs} \to A_{gs}$

fundamental interaction between a and A

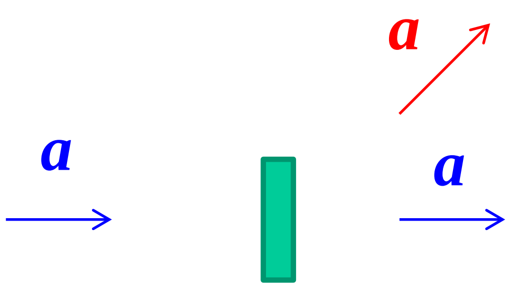
notation:

A(*a*,*b*)*B* reaction



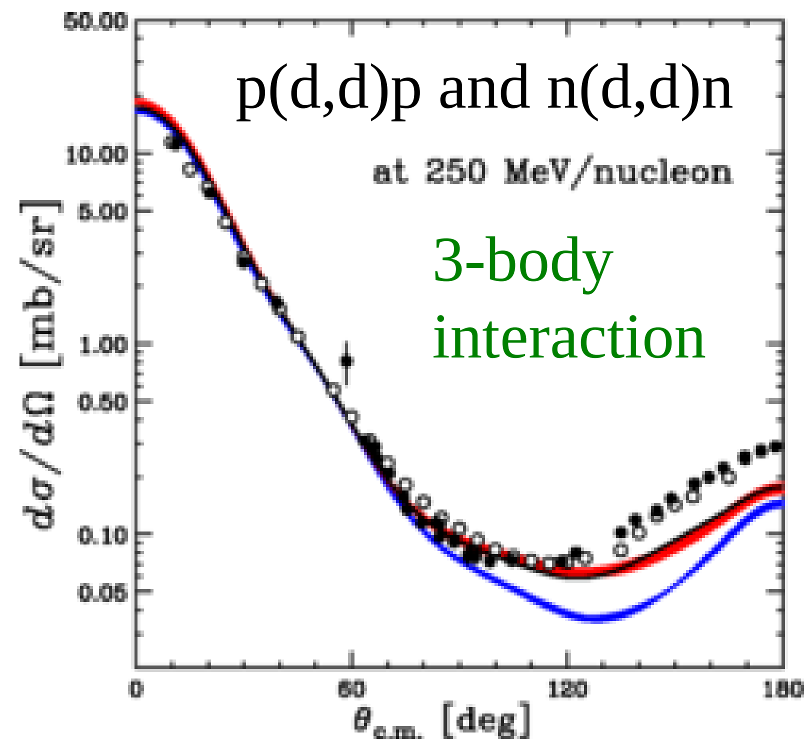
$A \rightarrow B$

✓elastic scattering



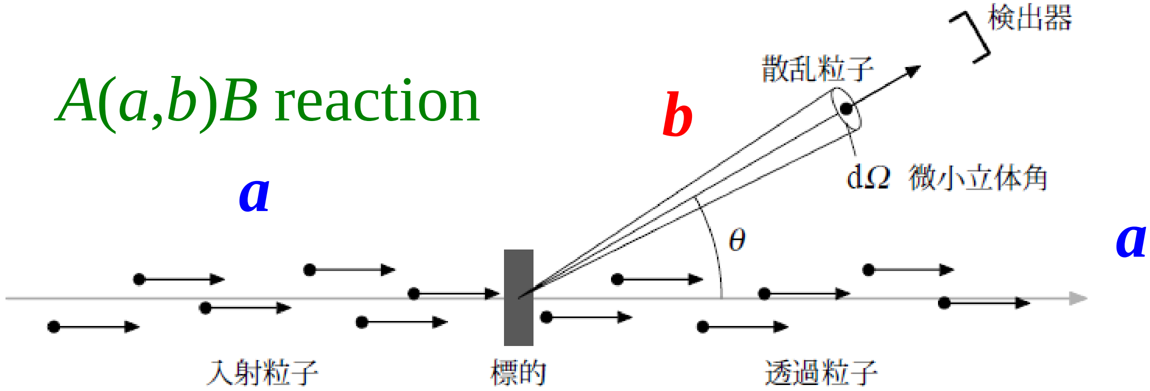
$A_{gs} \rightarrow A_{gs}$

fundamental interaction between *a* and *A*



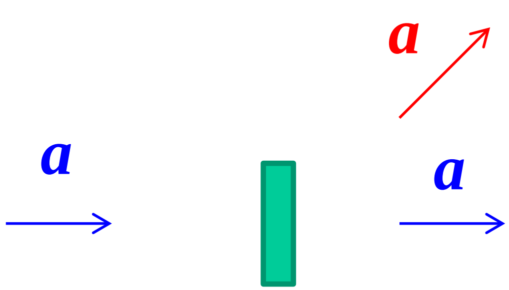
K. Sekiguchi et al., PRC89('14)064007

$A(a,b)B$ reaction



$A \rightarrow B$

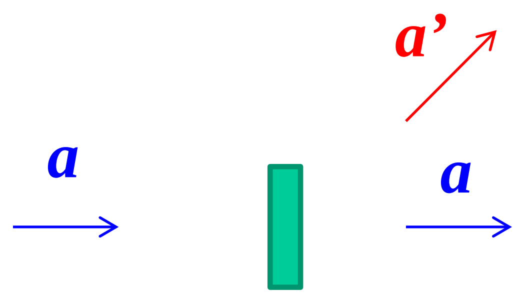
✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

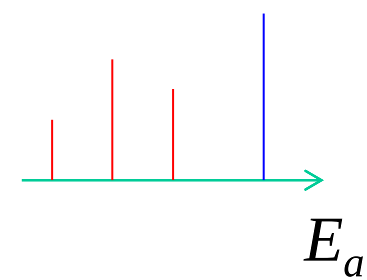
fundamental interaction between a and A

✓inelastic scattering

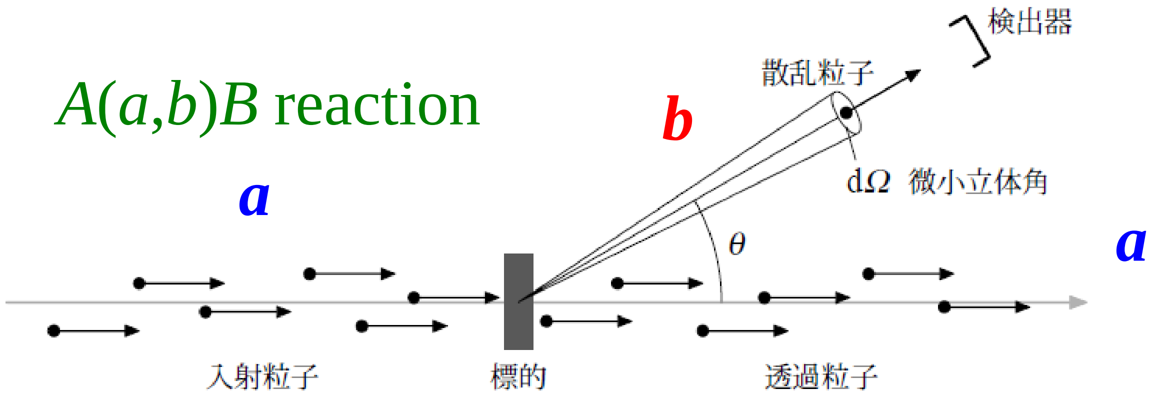


$A_{gs} \rightarrow A^*$

excitation spectrum of a nucleus A



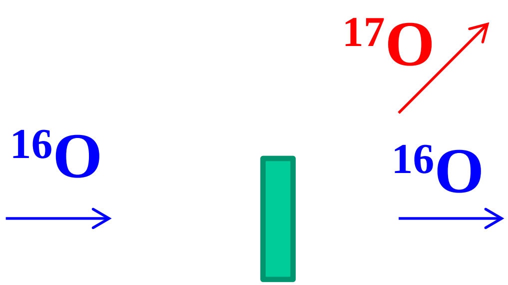
$A(a,b)B$ reaction



$A \to B$

transfer reactions

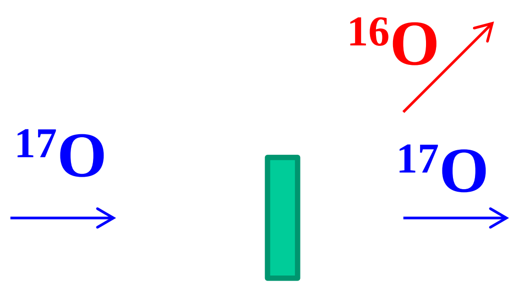
✓transfer reaction (pick-up reaction)



$^{208}Pb_{gs} \to {}^{207}Pb$

level schem of ^{207}Pb

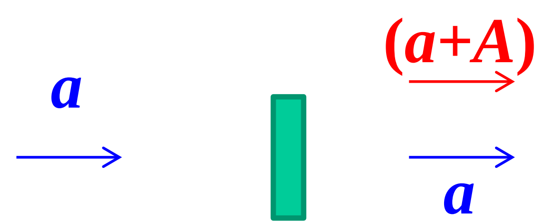
✓transfer reaction (stripping reaction)



$^{208}Pb_{gs} \to {}^{209}Pb$

level schem of ^{209}Pb

✓fusion reaction

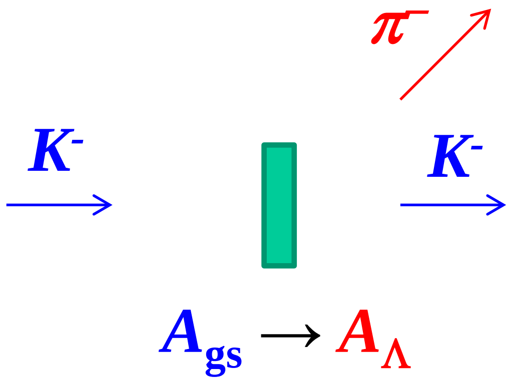


$A_{gs} \to X$

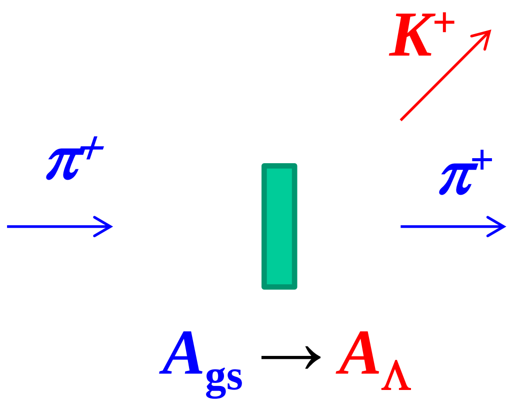
- interaction between *a* and *A*
- structure of *a* and *A*

hypernucleus production reactions

✓(K^-,π^-) reaction

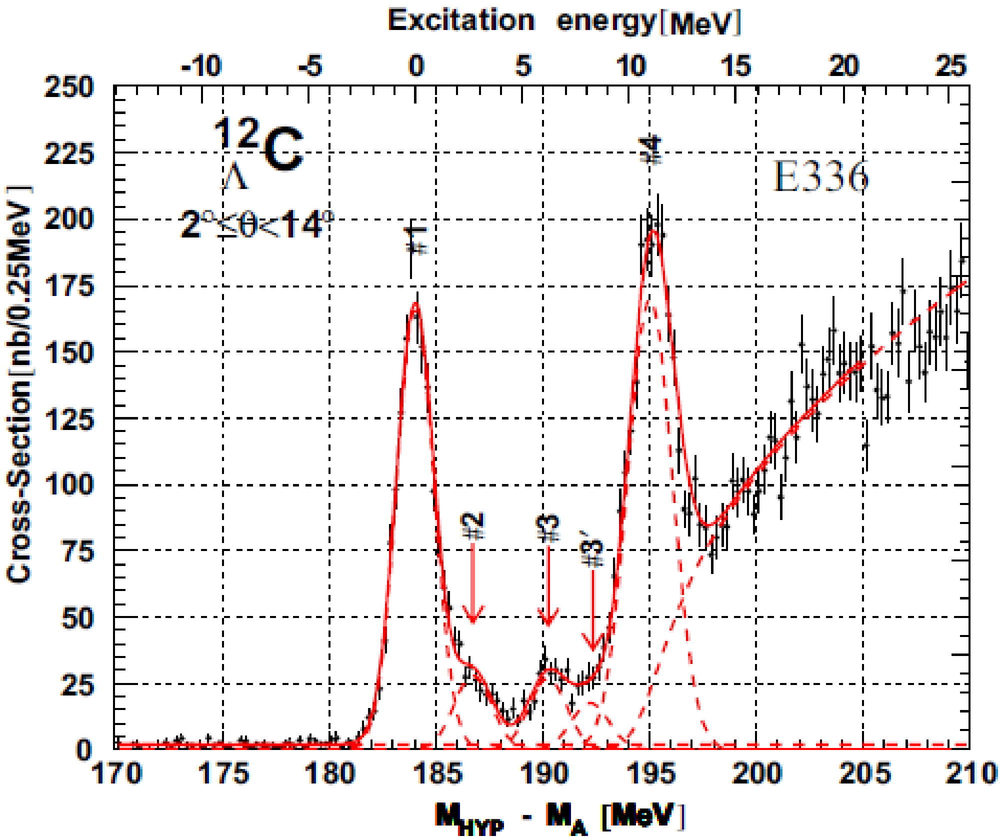


✓(π^+,K^+) reaction



excitation spectrum
of a hypernucleus A_Λ

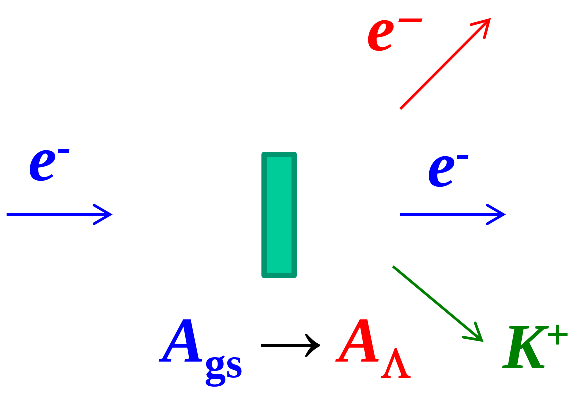
$^{12}C\ (\pi^+,K^+)\ ^{12}{}_{\Lambda}C$ reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

“reaction spectroscopy”

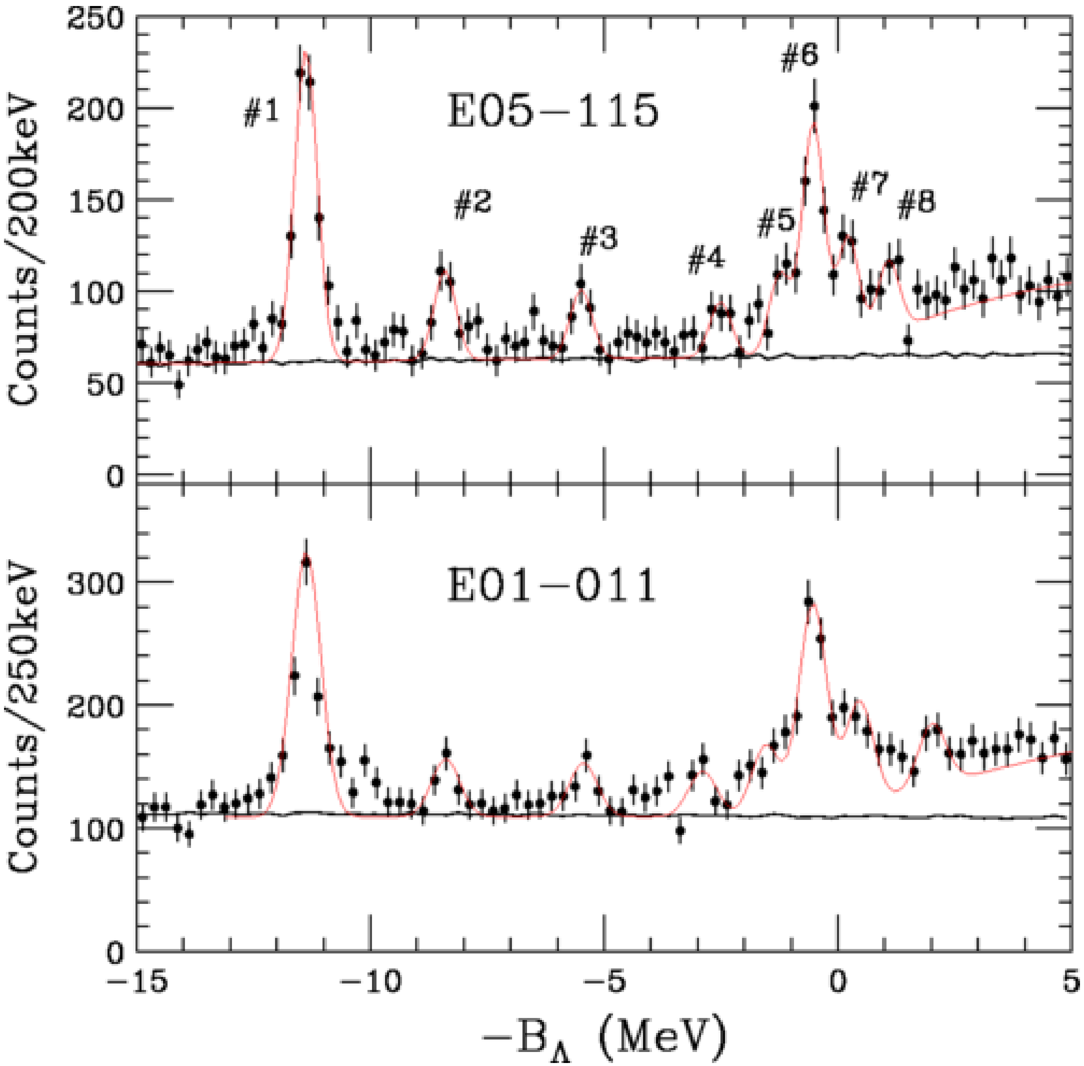
✓(e,e'K+) reaction



S.N. Nakamura et al., PRL110('13)012502

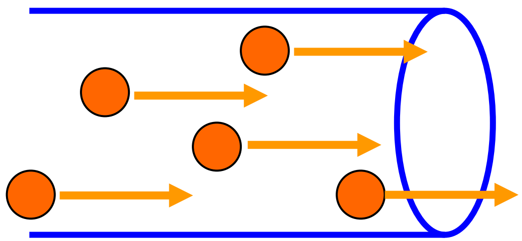
T. Gogami, Ph.D. Thesis (Tohoku U.) 2014

$^{12}C(e,e'K^+)\ ^{12}_{\Lambda}B$



L. Tang et al., PRC90('14)034320

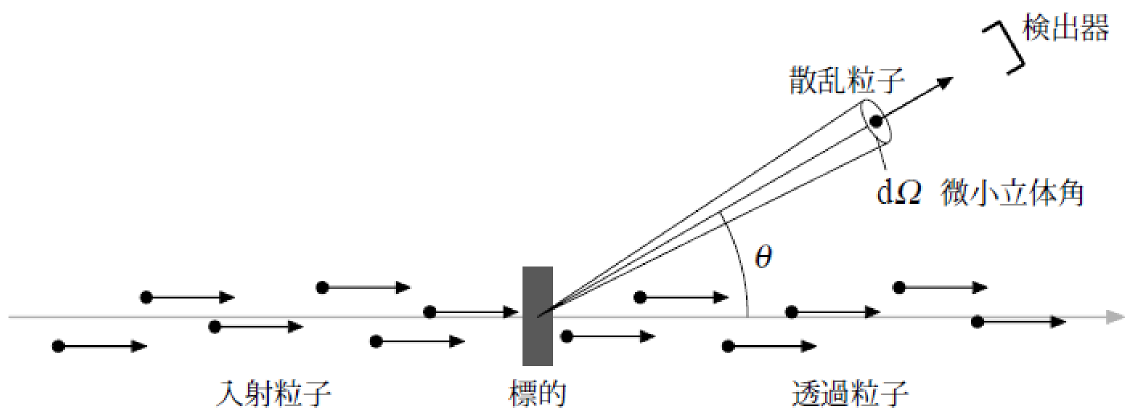
Cross sections



incident beam

flux = the number of particles crossing unit area per unit time

$$j = \rho_{\mathrm{P}} \cdot v$$

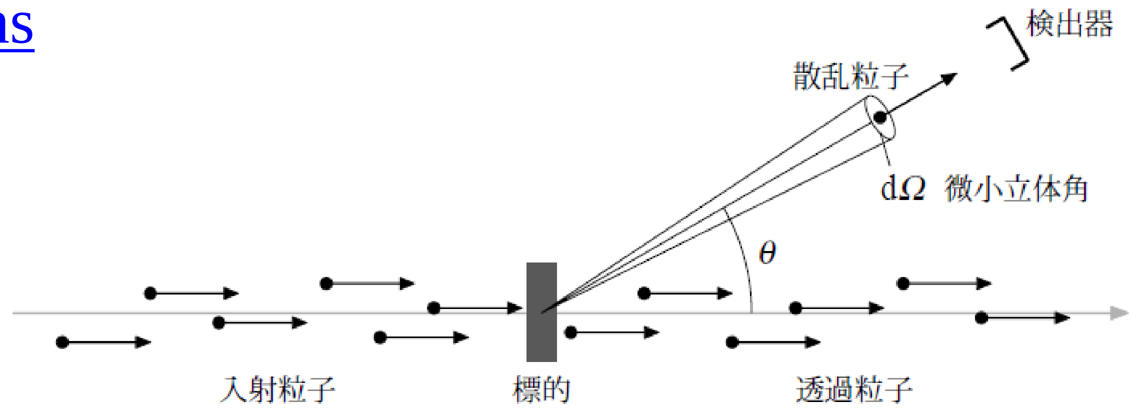


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow \quad R = N_{\mathrm{T}} \cdot \sigma \cdot j$$

cross section (σ)

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

→ $$R = N_T \cdot \sigma \cdot j$$

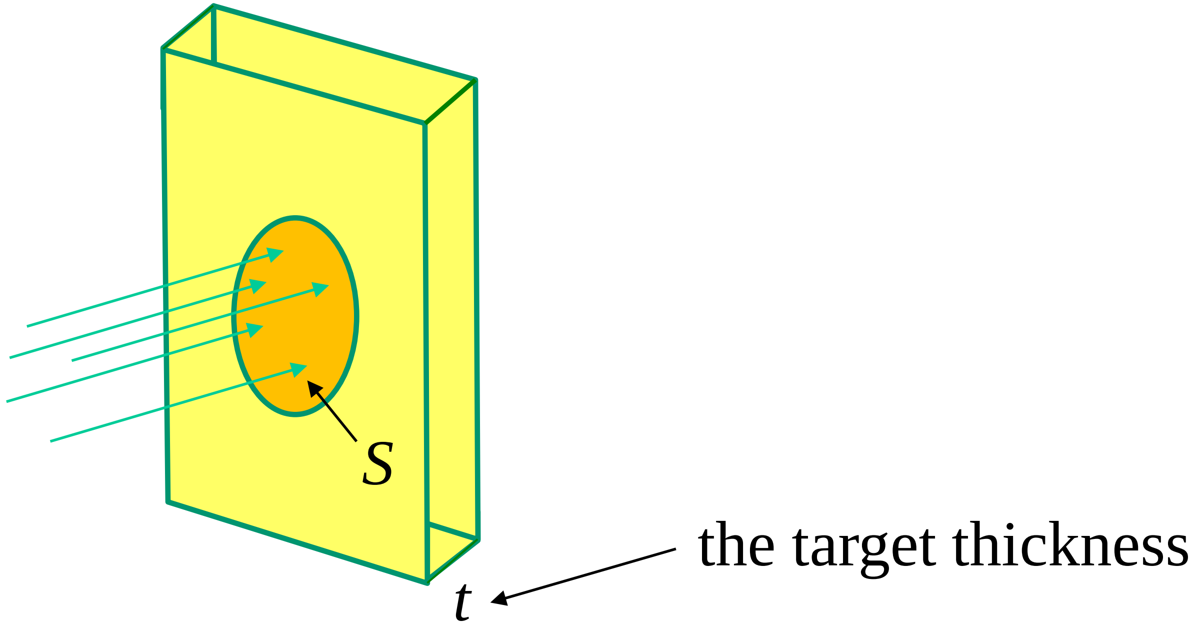
cross section

differential cross sections (angular distribution)

$$dR(\theta,\phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega, \qquad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

Cross sections (experiments)



$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

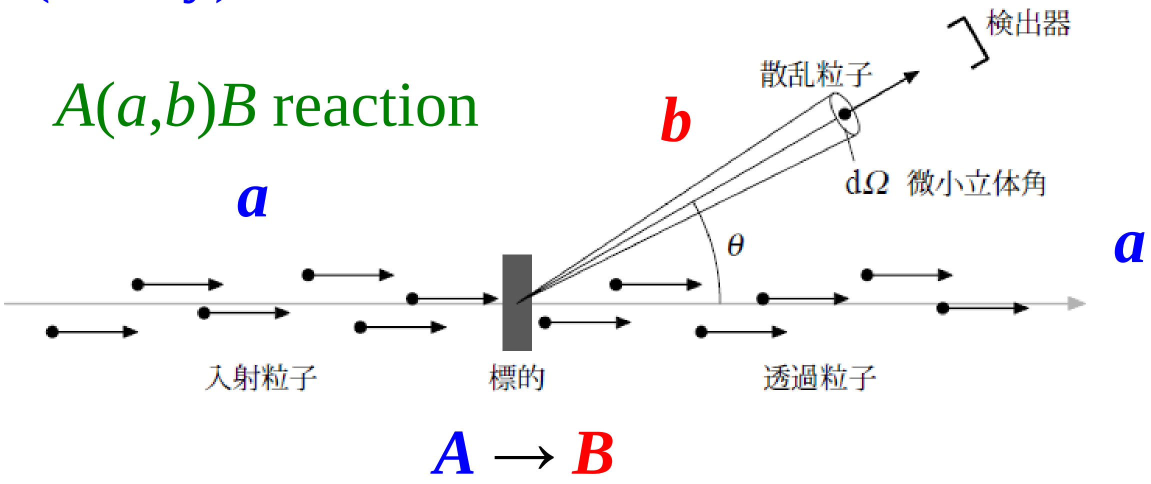
the number of target nucleus: $N_T = S \cdot t \cdot \rho_T$

beam intensity: $I = j \cdot S$

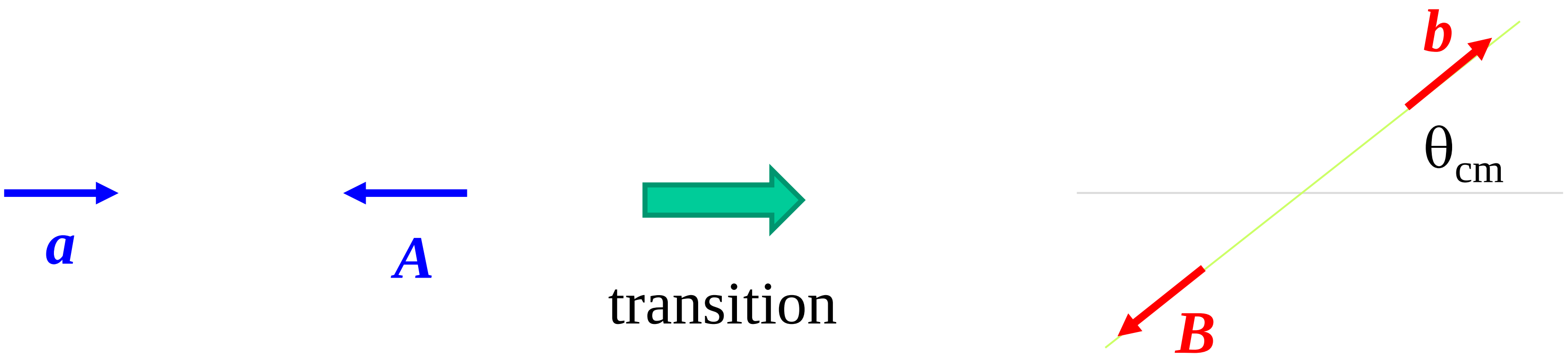
$$dR(\theta, \phi) = I \cdot \frac{d\sigma}{d\Omega} \cdot t\rho_T \cdot d\Omega \cdot \epsilon$$

detection efficiency

Cross sections (theory)



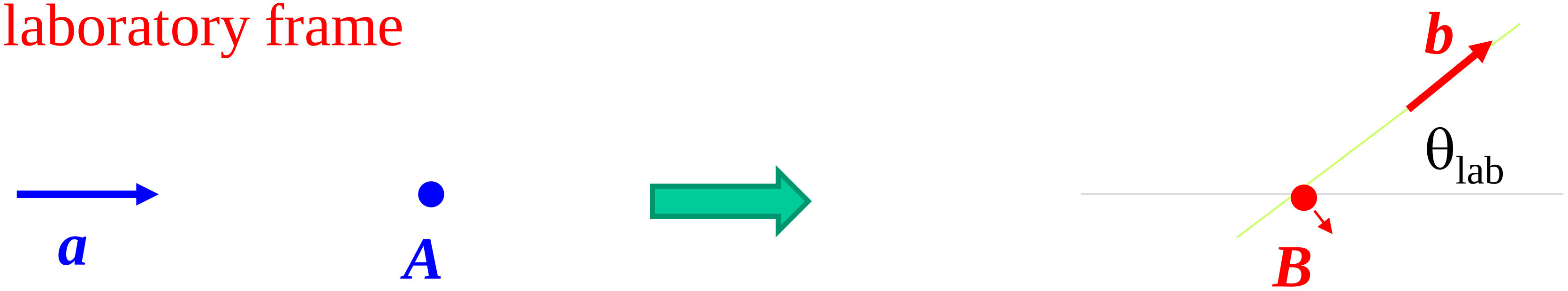
center of mass frame



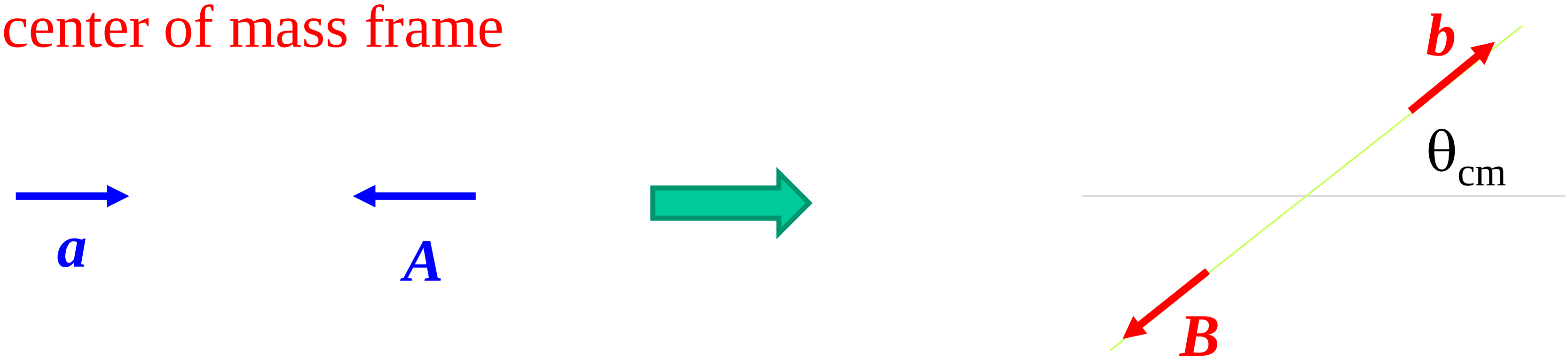
$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\rm in}}$$

Cross sections

✓ laboratory frame

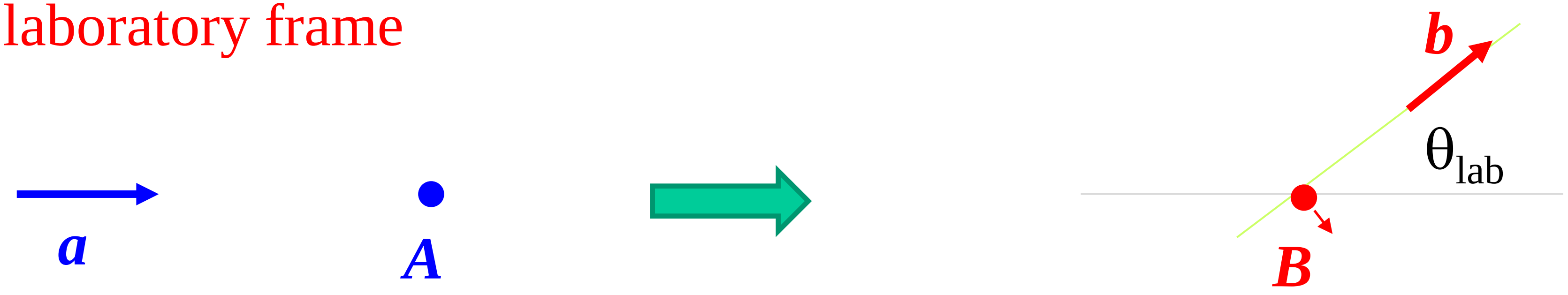


✓ center of mass frame

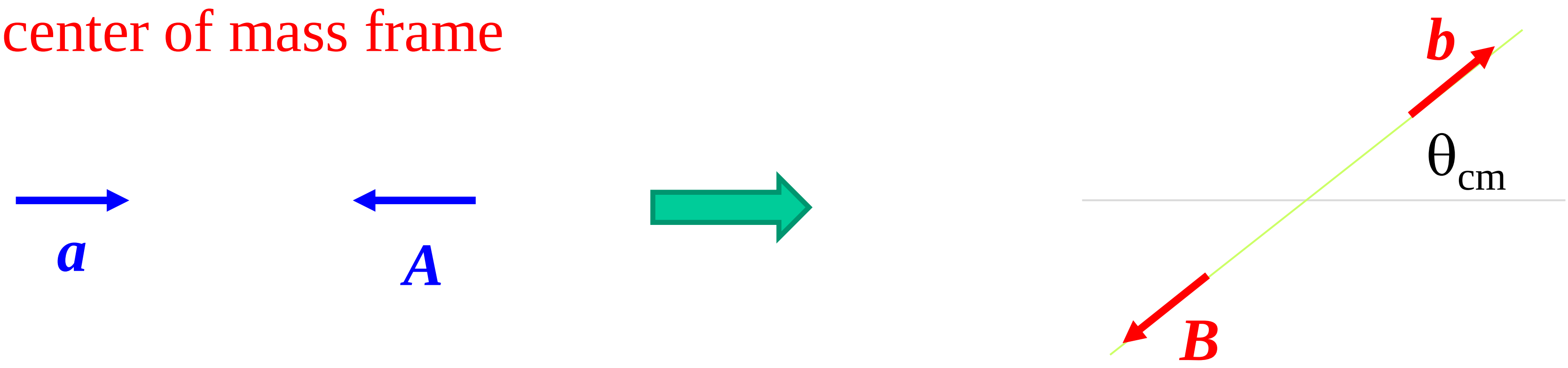


Cross sections

✓ laboratory frame



✓ center of mass frame



□ transformation ⟵ energy and momentum conservations

$$\tan\theta_{\rm lab} = \frac{\sin\theta_{\rm cm}}{\gamma+\cos\theta_{\rm cm}}, \qquad d\Omega_{\rm lab} = \frac{|1+\gamma\cos\theta_{\rm cm}|}{(1+\gamma^2+2\gamma\cos\theta_{\rm cm})^{3/2}}d\Omega_{\rm cm}$$

$$E_{\rm cm} = \frac{M_A}{M_a+M_A}E_{\rm lab}, \qquad \gamma = \sqrt{\frac{M_aM_b}{M_AM_B}\frac{E_{\rm cm}}{E_{\rm cm}+Q}}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

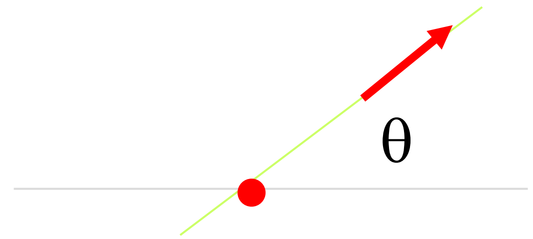
$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V(r) - E}\right)\psi(\boldsymbol{r}) = 0$$

perturbation

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$



$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

V(r)

$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

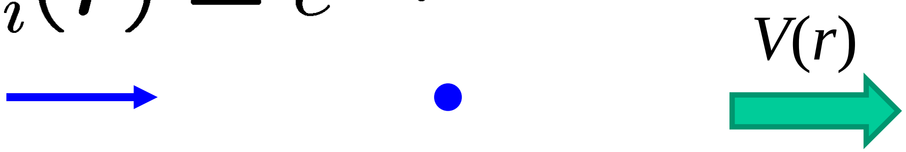
$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar}\int \frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3} \left|\langle\psi_f|V|\psi_i\rangle\right|^2 \delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2 \end{aligned}$$

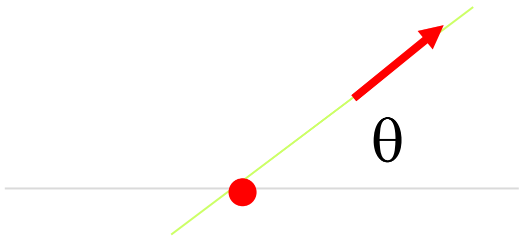
$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i - \boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

Born approximation

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$





$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

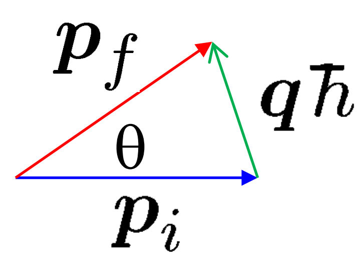
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\text{ch}}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

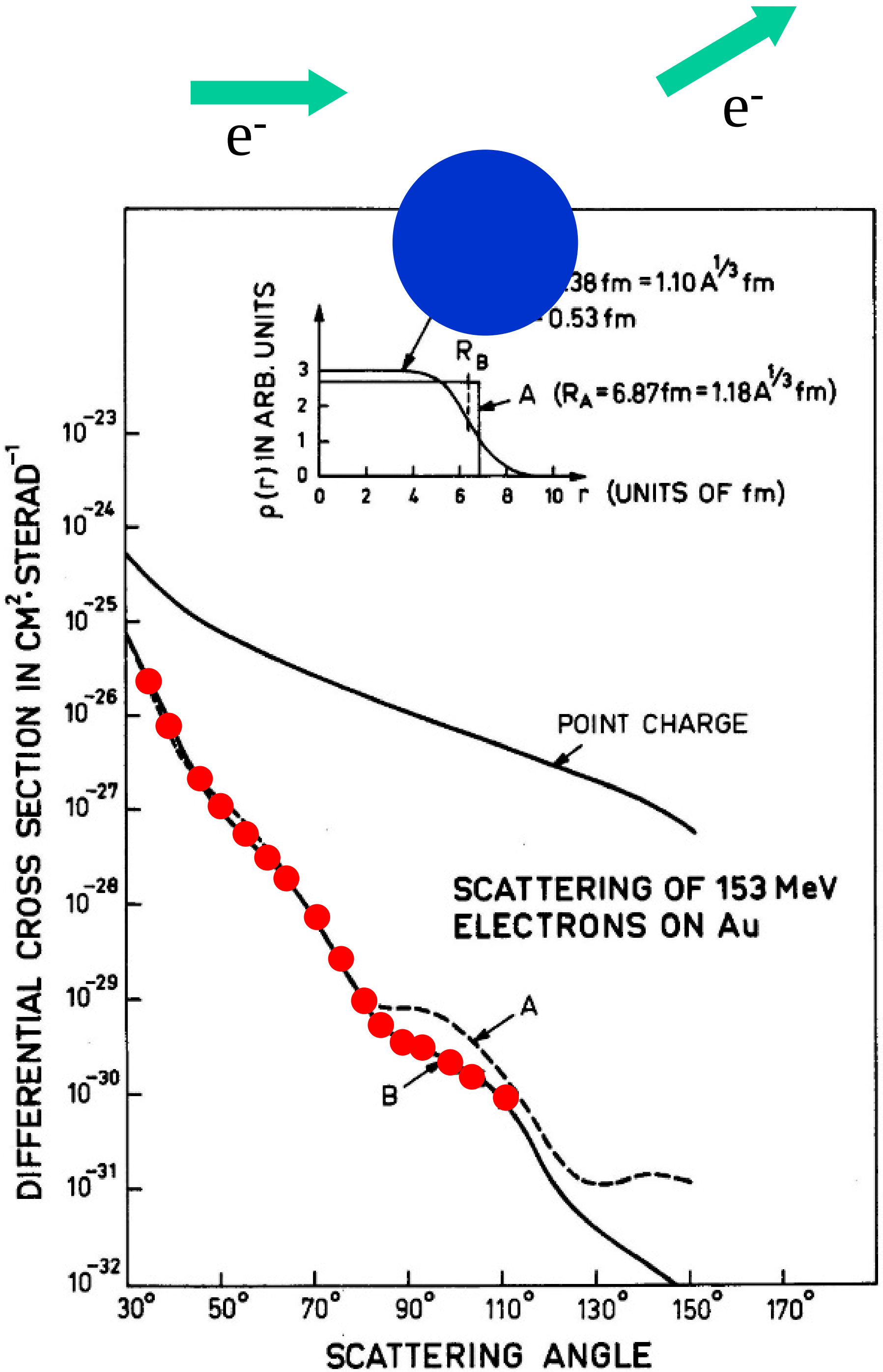
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2 = \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right)|F(\boldsymbol{q})|^2$$

Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho_{\text{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$

* relativistic correction:

$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \to \frac{d\sigma_{\text{Mott}}}{d\Omega} = \frac{d\sigma_{\text{Ruth}}}{d\Omega}\cdot\left(1-\frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right) \sim \frac{d\sigma_{\text{Ruth}}}{d\Omega}\cdot\cos^2\frac{\theta}{2} \qquad (v\to c)$$



レポート問題5（〆切：12月5日（土））

電子と原子核の相互作用が

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\mathrm{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

で与えられているとする。ここで、ρ_{ch} は原子核の電荷密度で

$$\int d\boldsymbol{r}\, \rho_{\mathrm{ch}}(\boldsymbol{r}) = Z$$

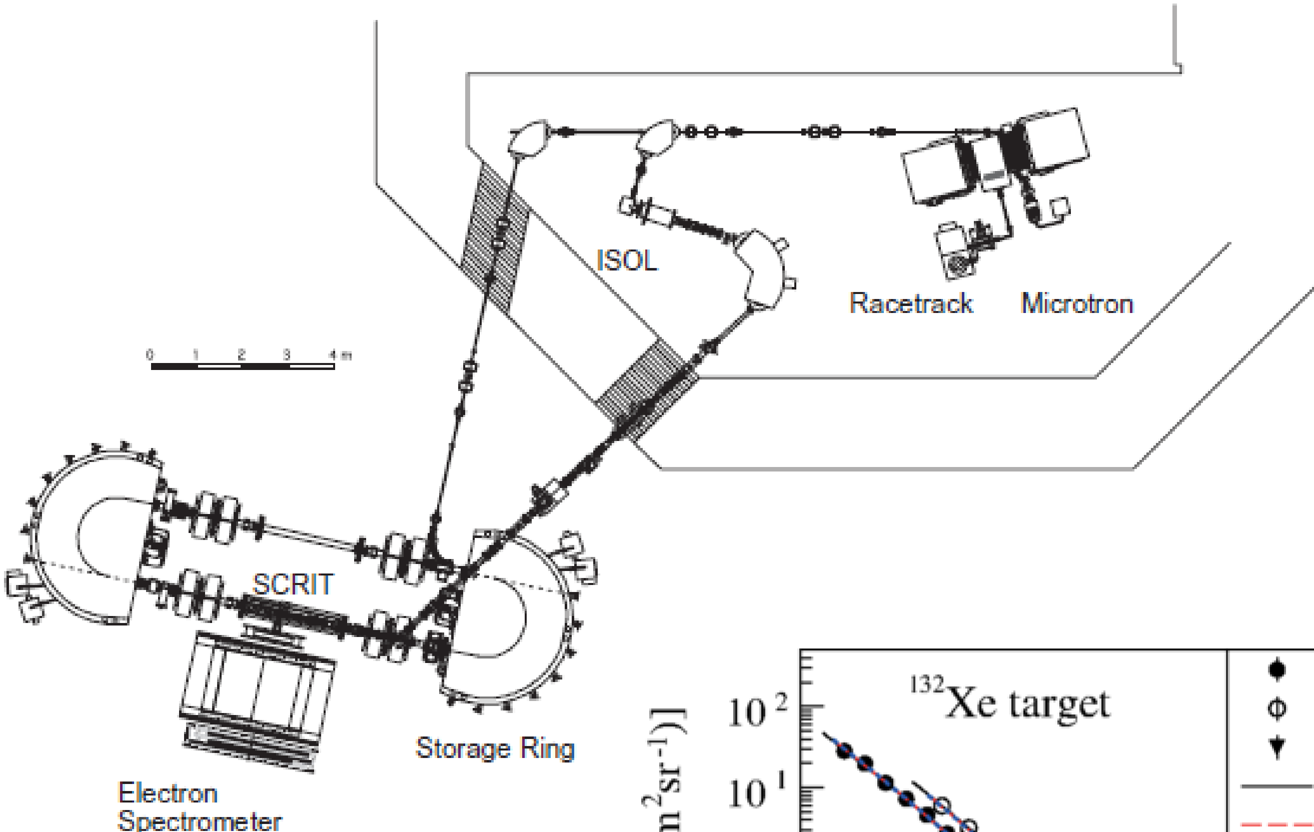
と規格化されているとする。ボルン近似を用いて弾性散乱の断面積を求め、

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E\sin^2\theta/2}\right)^2 |F(\boldsymbol{q})|^2$$

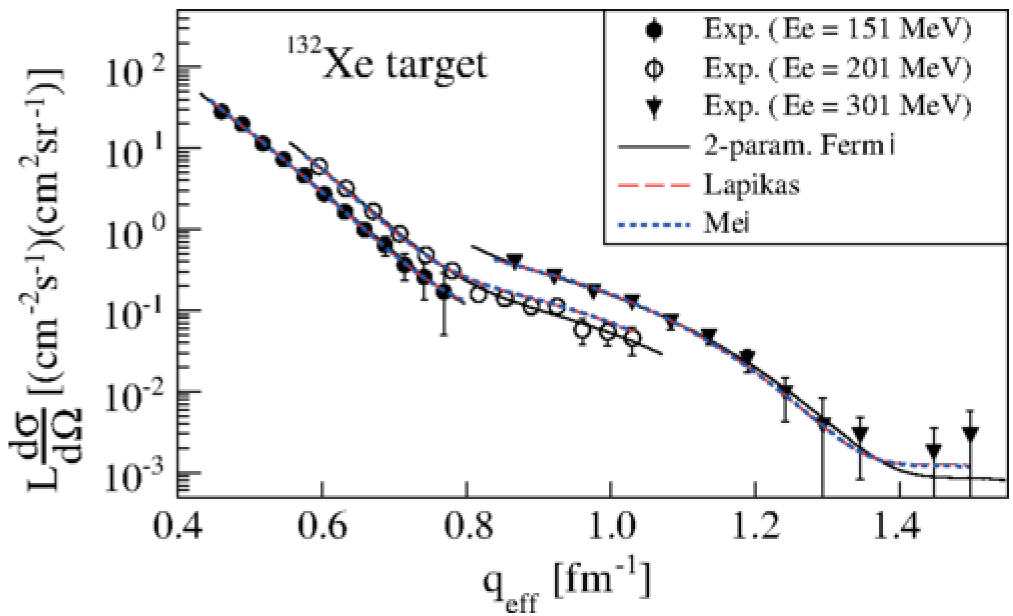
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \rho_{\mathrm{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$

となることを示せ。

cf. electron scattering off unstable nuclei (SCRIT)



K. Tsukada et al.,
PRL118, 262501 (2017)

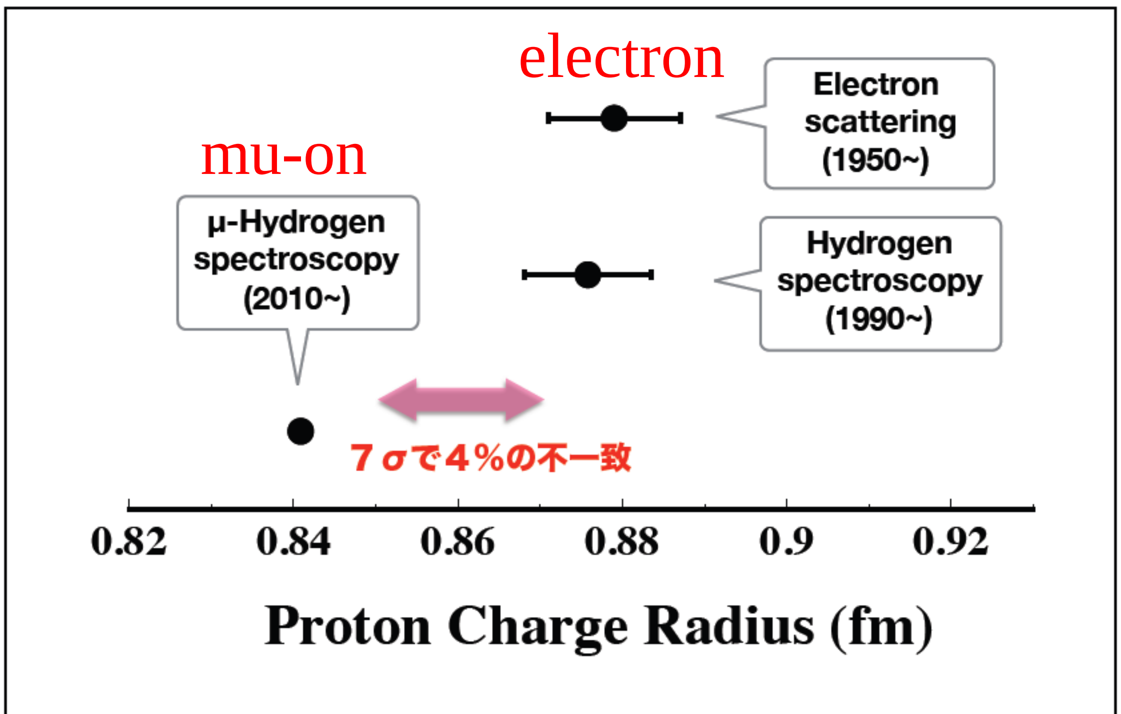


proton radius puzzle

$$
\begin{aligned}
F(\boldsymbol{q}) &= \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho_{\mathsf{ch}}(\boldsymbol{r})\,d\boldsymbol{r} \\
&\sim \int \left(1 - i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{(qr)^2}{2}\cos^2\theta + \cdots\right)\rho_{\mathsf{ch}}(\boldsymbol{r})\,d\boldsymbol{r} \\
&\sim Z\left(1 - \frac{q^2}{6}\langle r^2\rangle + \cdots\right)
\end{aligned}
$$

proton radius puzzle

$$
\begin{aligned}
F(\boldsymbol{q}) &= \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho_{\mathrm{ch}}(\boldsymbol{r})\,d\boldsymbol{r} \\
&\sim \int \left(1 - i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{(qr)^2}{2}\cos^2\theta + \cdots\right)\rho_{\mathrm{ch}}(\boldsymbol{r})\,d\boldsymbol{r} \\
&\sim Z\left(1 - \frac{q^2}{6}\langle r^2\rangle + \cdots\right)
\end{aligned}
$$



Distorted Wave Born approximation (DWBA)

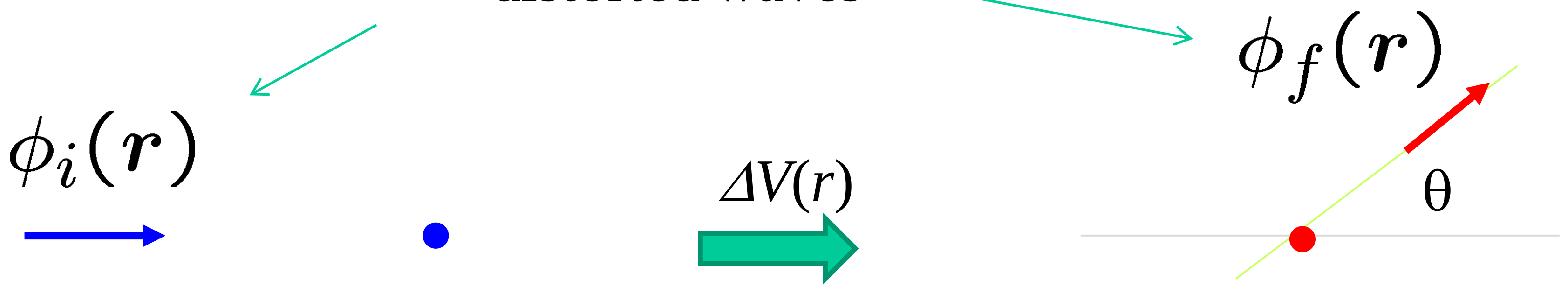
$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

$$\Longrightarrow\quad \left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V_0(r)} + \underline{V(r) - V_0(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

"distorted waves"

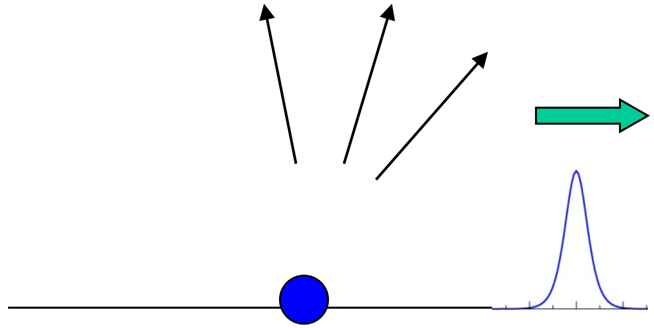


✓inelastic scattering
✓transfer reactions

How to choose $V_0(r)$? : Optical model

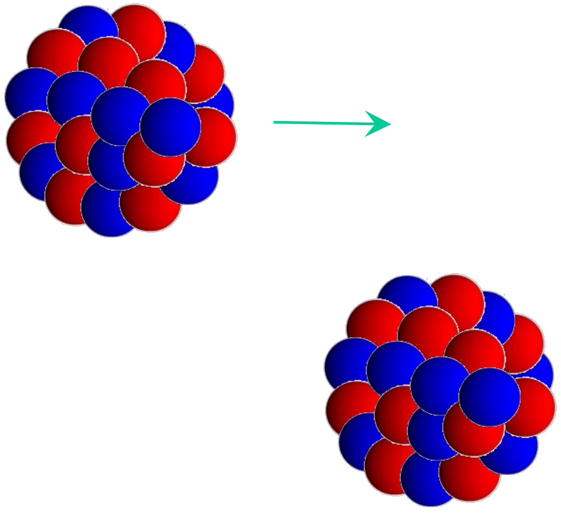
Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)

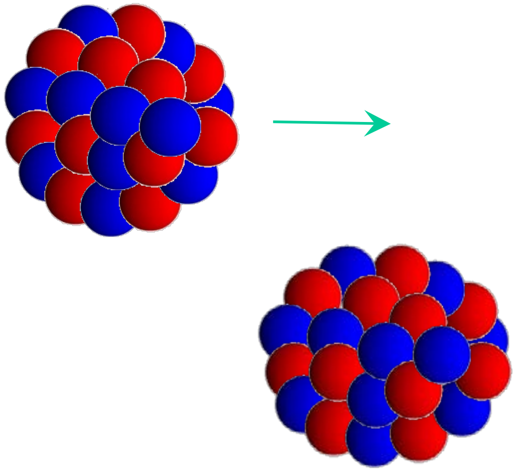


Loss of incident flux (absorption)

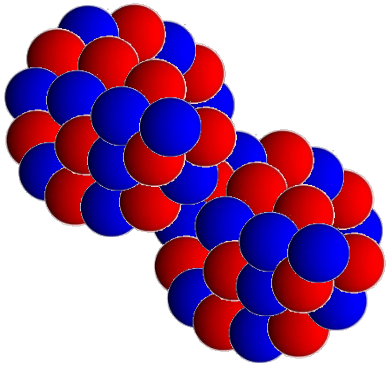
弾性散乱



非弾性散乱



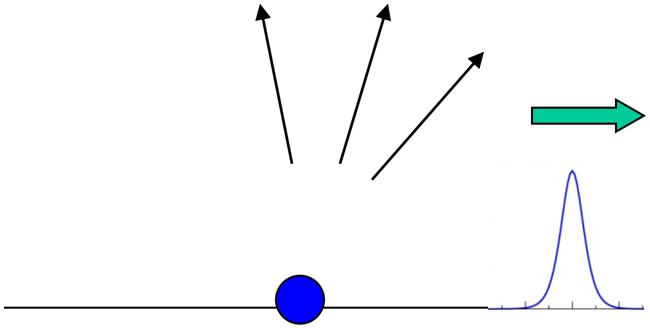
核融合



How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

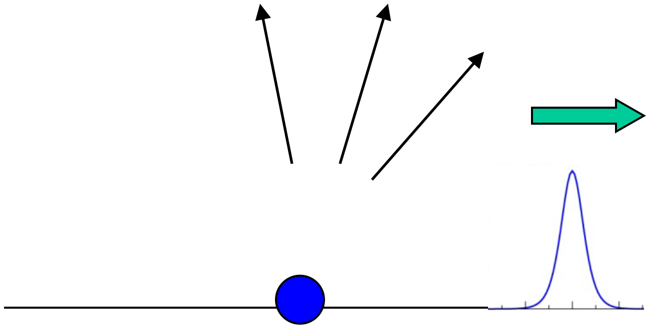
Optical potential

$$V_{\rm opt}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

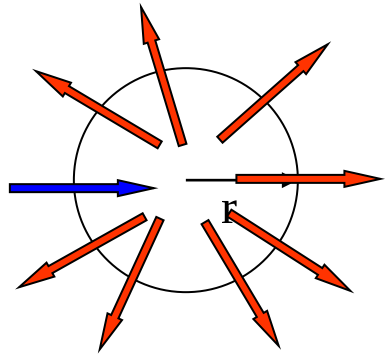
Optical potential

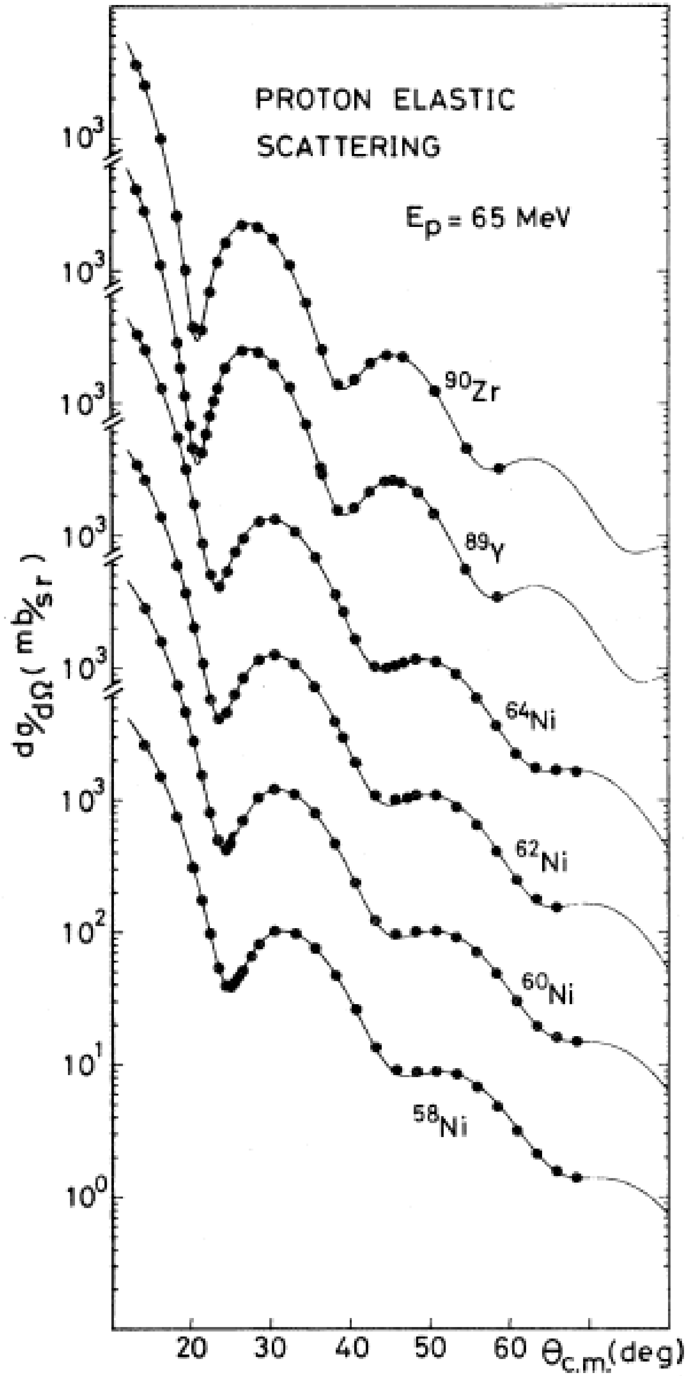
$$V_{\rm opt}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

$$\Rightarrow \quad \nabla \cdot \boldsymbol{j} = \cdots = -\frac{2}{\hbar} W |\psi|^2$$

(note) Gauss's theorem

$$\int_S \boldsymbol{j} \cdot \boldsymbol{n} \, dS = \int_V \nabla \cdot \boldsymbol{j} \, dV$$





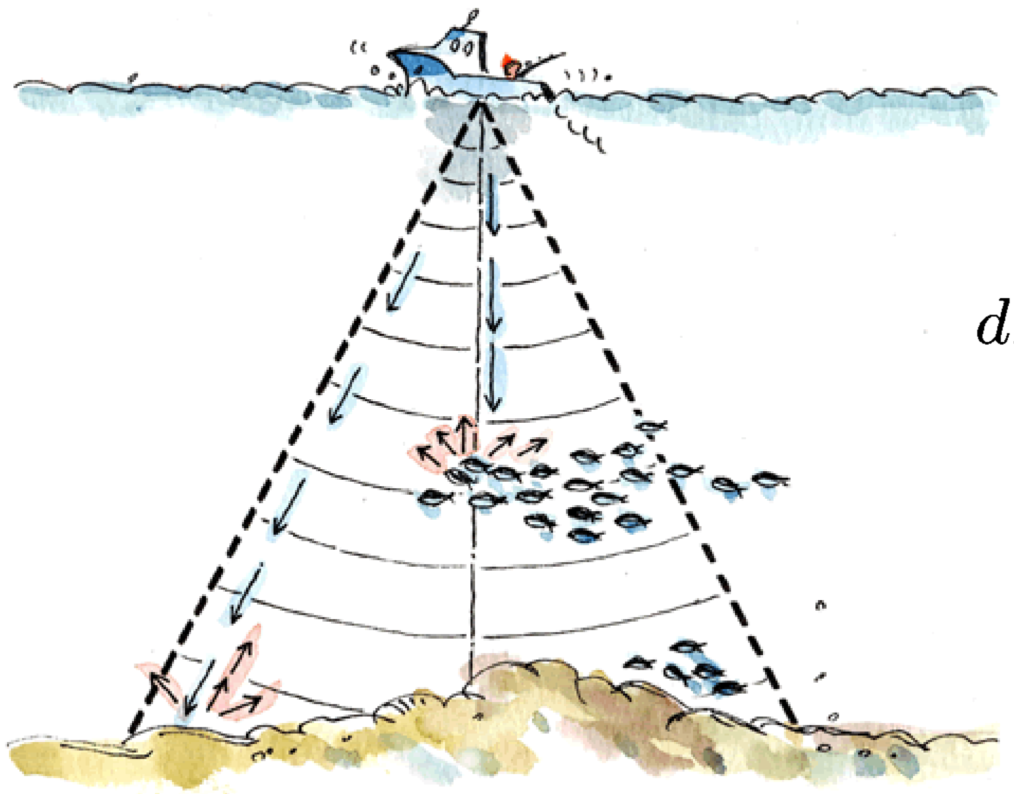
$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\rm opt}(r) - E\right)\psi(\boldsymbol{r}) = 0$$

Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al., PRC26 (1982) 944

Appendix: DWBA in ocean acoustics

Fishfinder　魚群探知機



https://www.furuno.co.jp/technology/about/fishfinder1.html

(backward) scattering of (ultra-)sonic waves due to fish etc.

$$dR(\theta,\phi) = N_{\mathsf{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

$$\downarrow$$

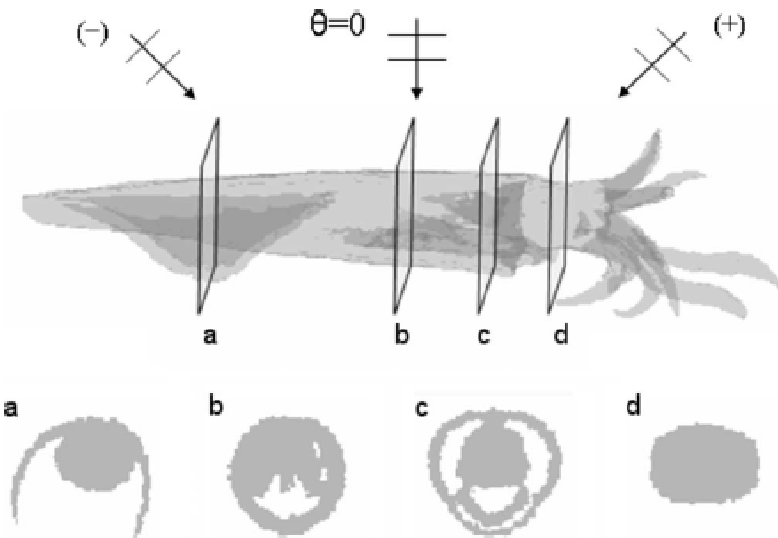
$$N_{\mathsf{T}} = \frac{\frac{dR}{d\Omega}}{j \cdot \frac{d\sigma}{d\Omega}}$$

one can know the number of fish N_{T} if one knows the differential cross sections

Use of the distorted wave Born approximation to predict scattering by inhomogeneous objects: Application to squid

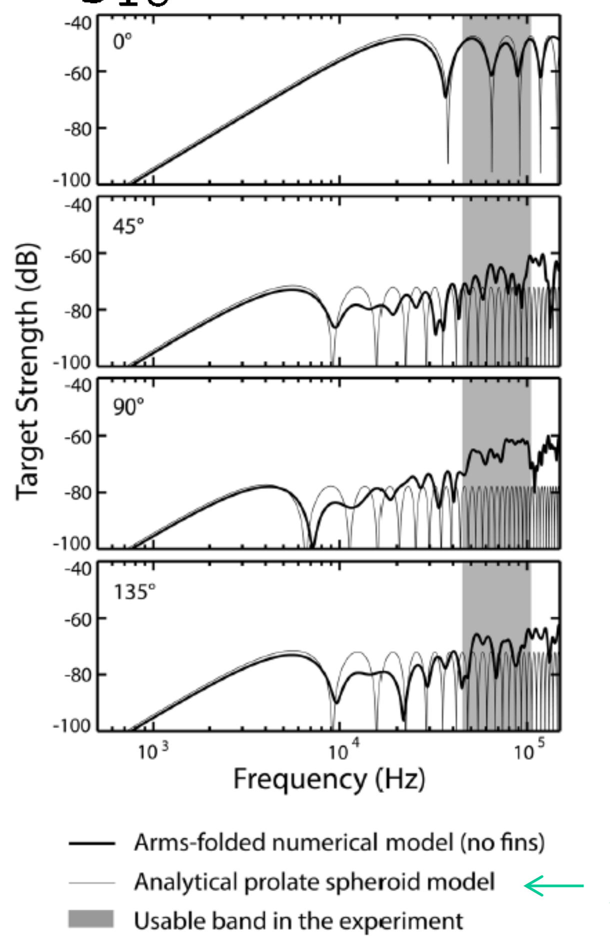
Benjamin A. Jones,[a] Andone C. Lavery, and Timothy K. Stanton
Department of Applied Ocean Physics and Engineering, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543-1053

J. Accoust. Soc. Am. 125 (‘09) 73



Modeling of squid

$10 \log_{10} \sigma$



DWBA: local wave number
inside a squid