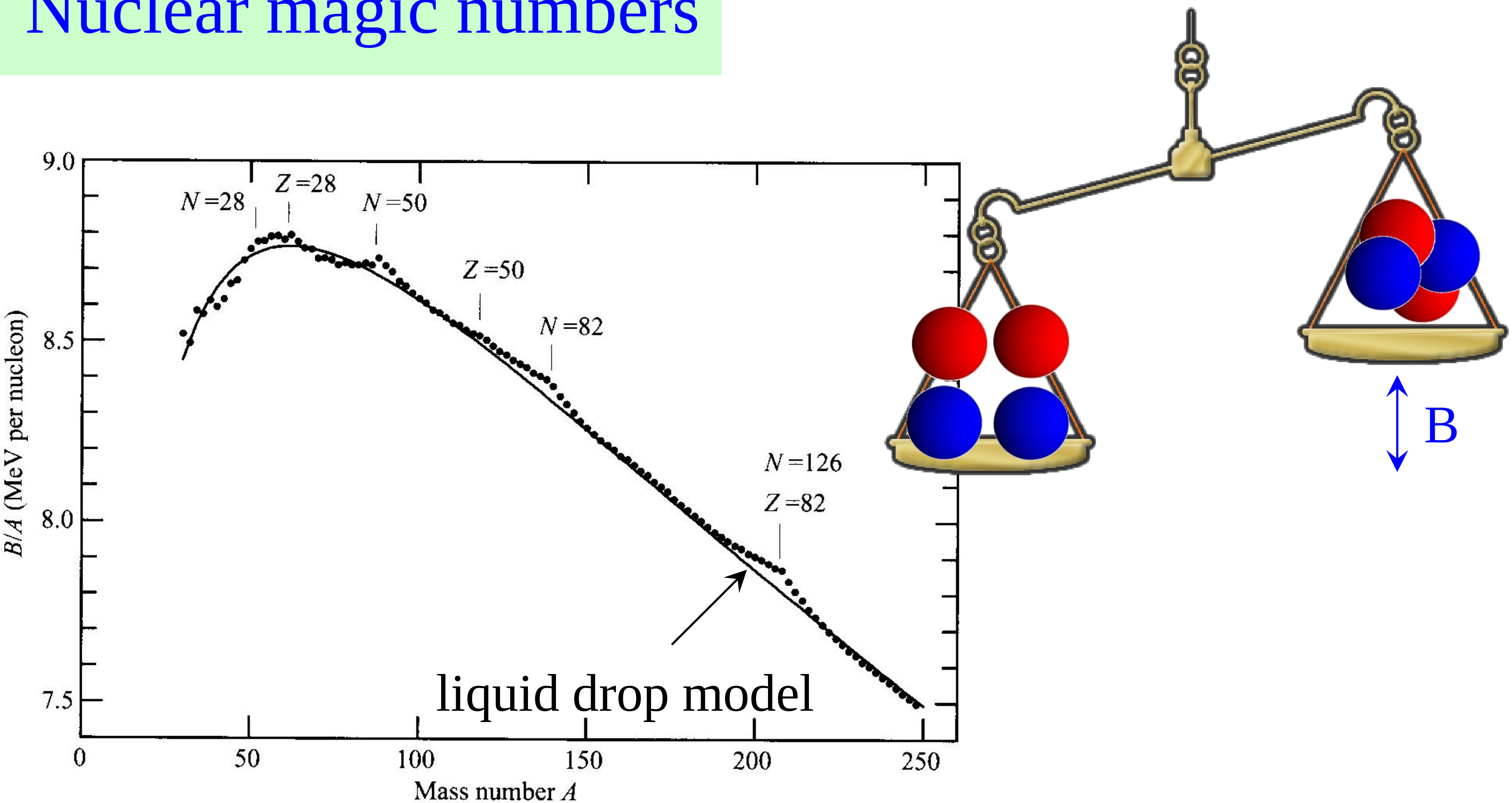
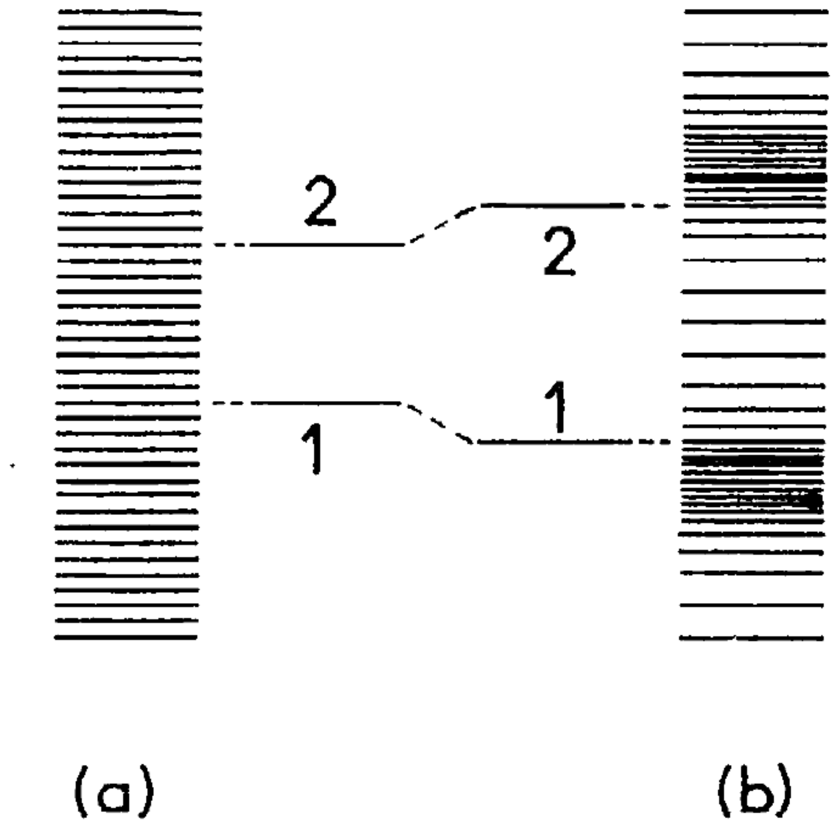
Nuclear magic numbers



Extra binding for *N* or *Z* = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

何故、閉殻の原子核は安定になるのか?

準位密度

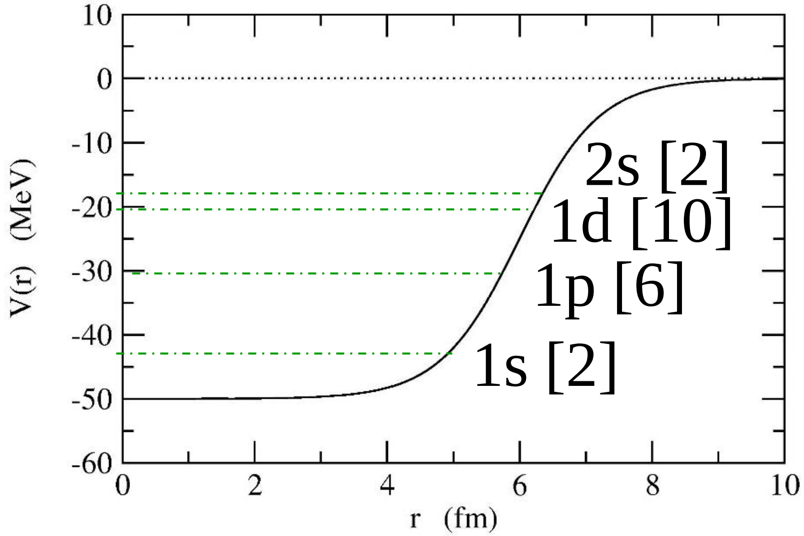


均一の場合　　濃淡がある場合

準位密度に濃淡があれば、下から数えて濃淡の終わりまで準位がつまると(図の1の場合)、均一の場合に比べてエネルギーが小さい

Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

An interpretation: independent particle motion in a potential well



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

degeneracy: $2*(2l+1)$

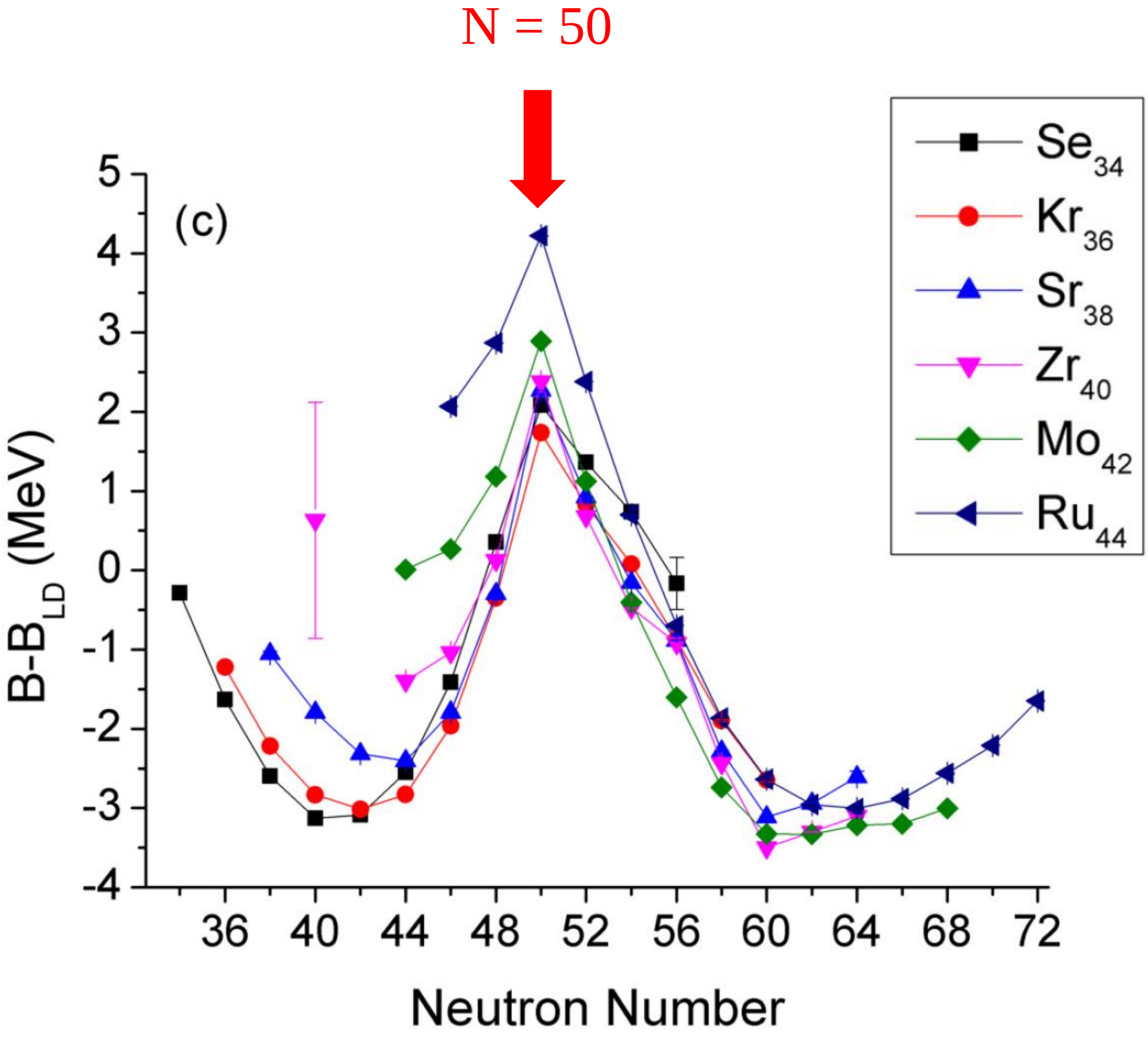
spin-orbit interaction

f[14]	34
s[2],d[10]	20
p[6]	8
s[2]	2

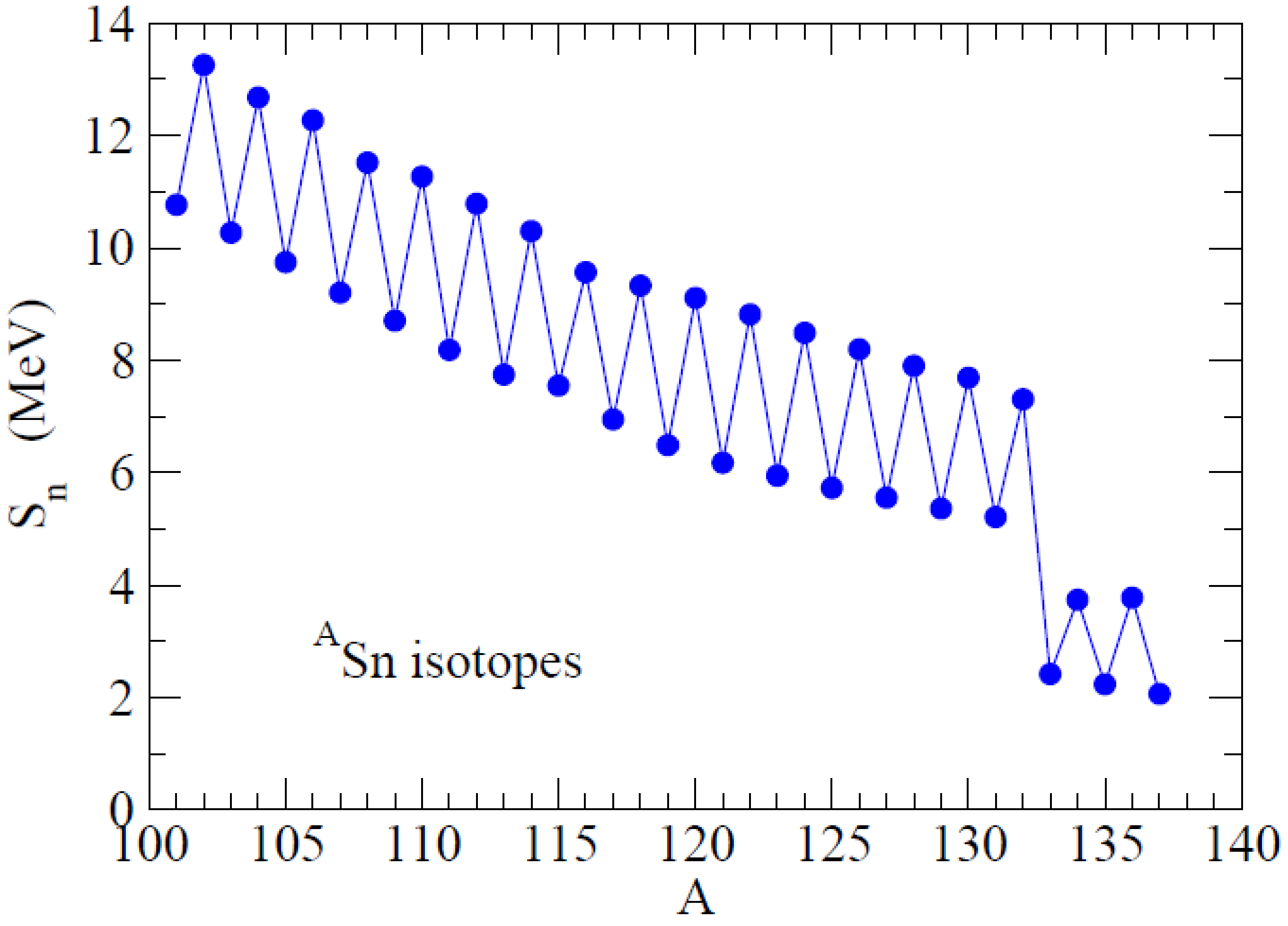
f[14] → $f_{5/2}$[6]

f[14] → $f_{7/2}$[8]

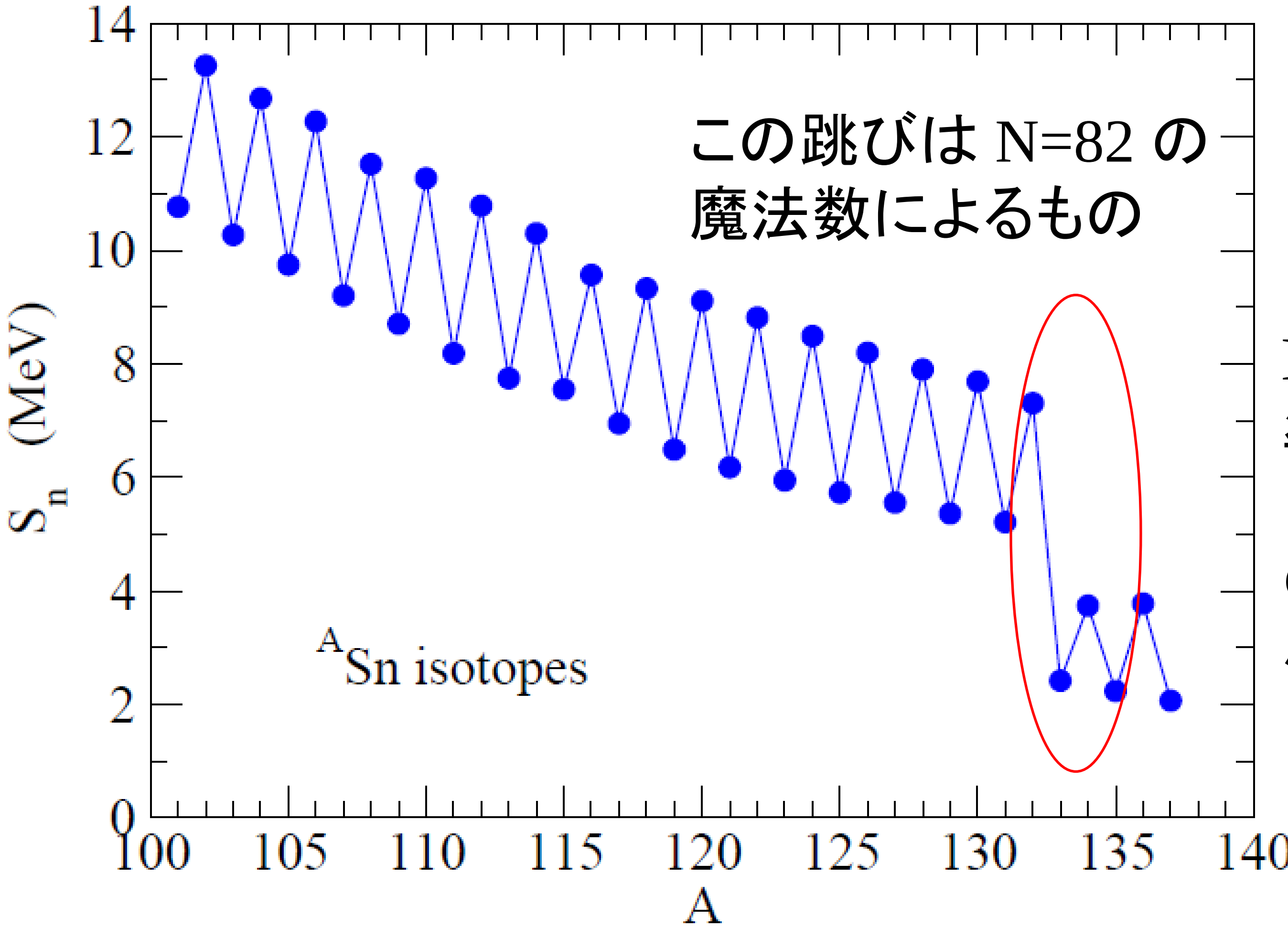
20+8 = 28



I. Bentley et al., PRC93 ('16) 044337



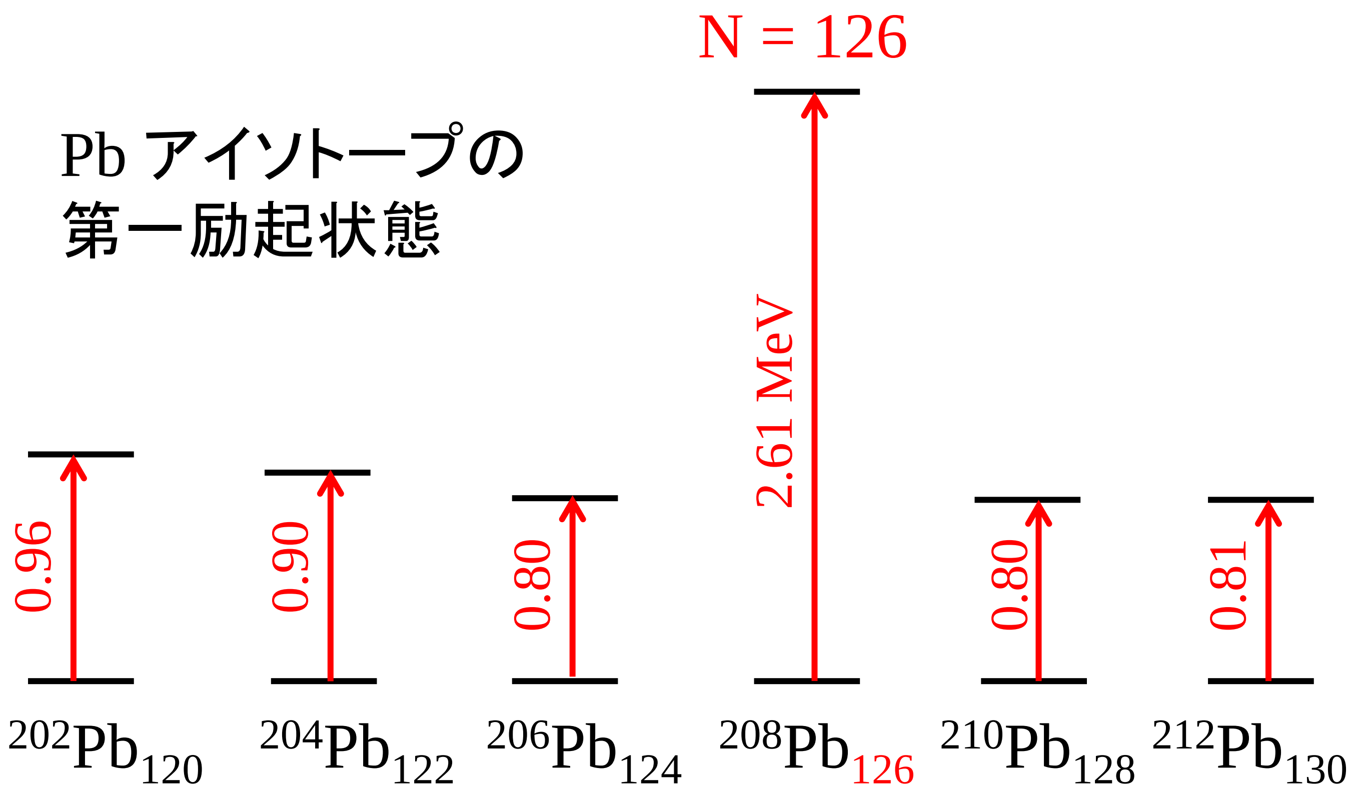
1n separation energy: S_n (A,Z) = B(A,Z) – B(A-1,Z)



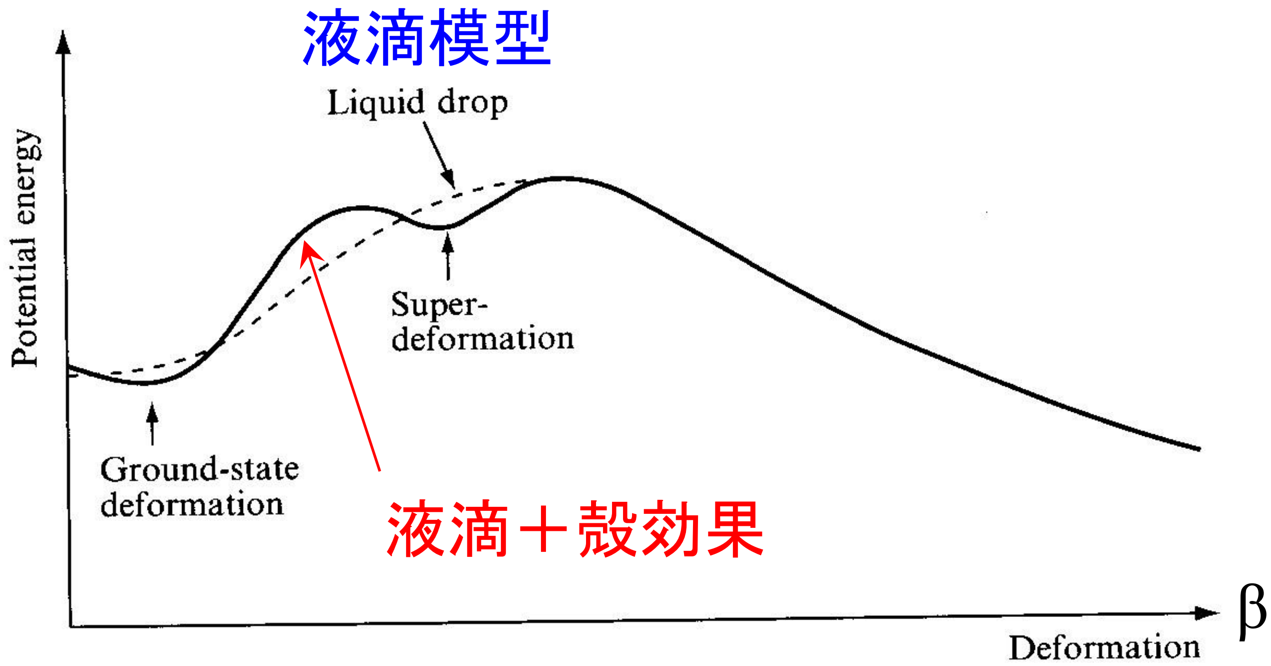
N=83から上の
準位がつまるため
中性子をとりのぞく
のにエネルギーが
小さくてすむ

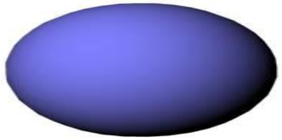
1n separation energy: $S_n (A,Z) = B(A,Z) - B(A-1,Z)$

他の証拠：第一励起状態の励起エネルギー



殻構造の帰結：原子核の変形



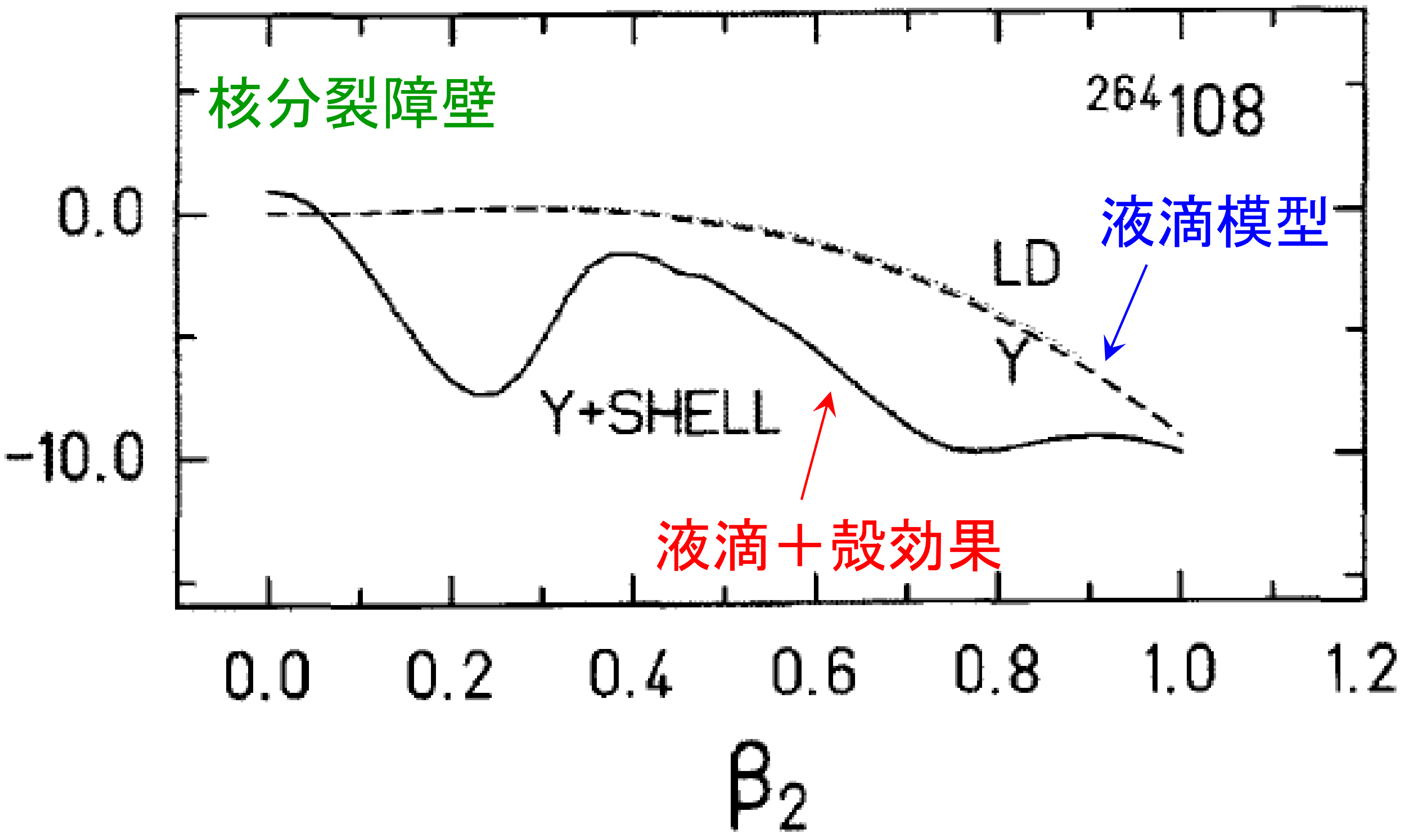


液滴模型 → 必ず球形

殻効果 → 変形状態が基底状態になる場合あり

＊後でもう少し詳しく解説します。

殻構造の帰結：超重核の安定化

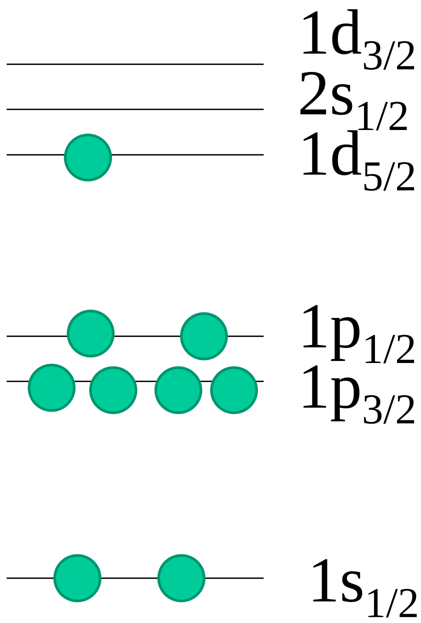


殻効果により核分裂障壁が高くなり原子核が安定化する

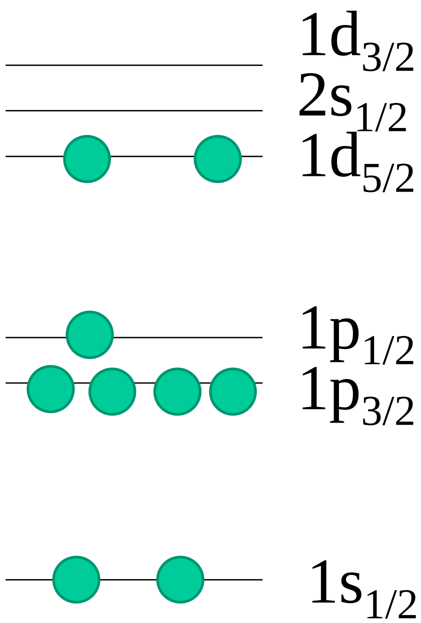
Z. Patyk et al., NPA491(‘89) 267

single-j model

shell model



configuration 1



configuration 2

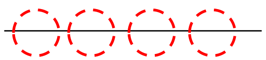
…… several others

angular momentum (spin) and parity for each configuration?

→ let us first investigate a single-j case single-j level: one level with an angular momentum j

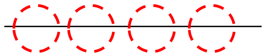
j

example: $j = p_{3/2}$



$p_{3/2}$

can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)

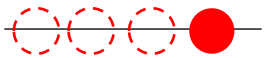


$p_{3/2}$

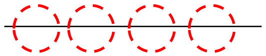
can accommodate 4 nucleons

(j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon



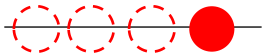
$p_{3/2}$



$p_{3/2}$ can accommodate 4 nucleons

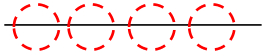
(j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon



$p_{3/2}$ ⇒ $I^\pi = 3/2^-$

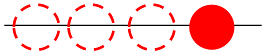
(there are 4 ways to occupy this level)



$p_{3/2}$ can accommodate 4 nucleons

(j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon

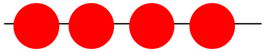


$p_{3/2}$

$I^{\pi} = 3/2^-$

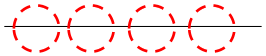
(there are 4 ways to occupy this level)

ii) 4 nucleons



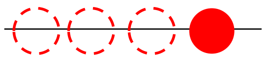
$p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

 $p_{3/2}$ can accommodate 4 nucleons

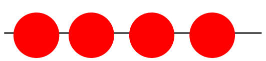
(j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon

 $p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

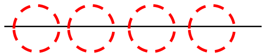
ii) 4 nucleons

 $p_{3/2}$ → $I^\pi = 0^+$

(there is only 1 way to occupy this level)

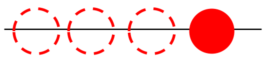
$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

parity: (-1) x (-1) x (-1) x (-1) = +1

 $p_{3/2}$

can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)

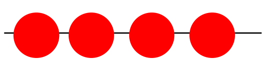
i) 1 nucleon

 $p_{3/2}$

$I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons

 $p_{3/2}$

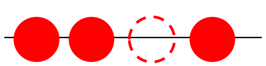
$I^\pi = 0^+$

(there is only 1 way to occupy this level)

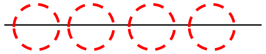
$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons

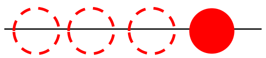
 $p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$



$p_{3/2}$ can accommodate 4 nucleons (j_z = +3/2, +1/2, -1/2, -3/2)

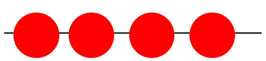
i) 1 nucleon



$p_{3/2}$ $\Longrightarrow$ $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

ii) 4 nucleons



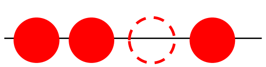
$p_{3/2}$ $\Longrightarrow$ $I^{\pi} = 0^{+}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons



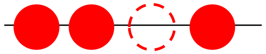
$p_{3/2}$ $\Longrightarrow$ $I^{\pi} = 3/2^{-}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iii) 3 nucleons



$p_{3/2}$

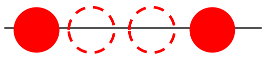
$I^{\pi} = 3/2^{-}$

(there are 4 ways to make a hole)

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

parity: (-1) x (-1) x (-1) = -1

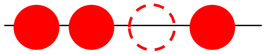
iv) 2 nucleons



$p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2$$

iii) 3 nucleons



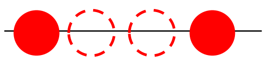
$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iv) 2 nucleons



$p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2$$

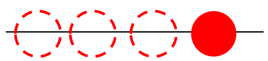
there are 4 x 3/2=6 ways to occupy this level with 2 nucleons.

→ $I^{\pi} = 0^{+}$ or 2^{+} (= 1+5)

3/2 + 3/2 → I = 0, ~~1~~, 2, ~~3~~

anti-symmetrization

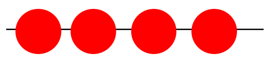
i) 1 nucleon



$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

ii) 4 nucleons

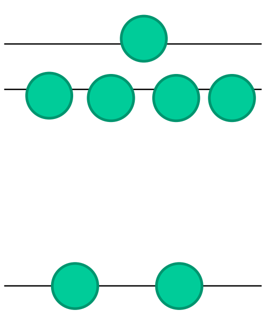


$p_{3/2}$ → $I^{\pi} = 0^{+}$

(there is only 1 way to occupy this level)

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

parity: (-1) x (-1) x (-1) x (-1) = +1



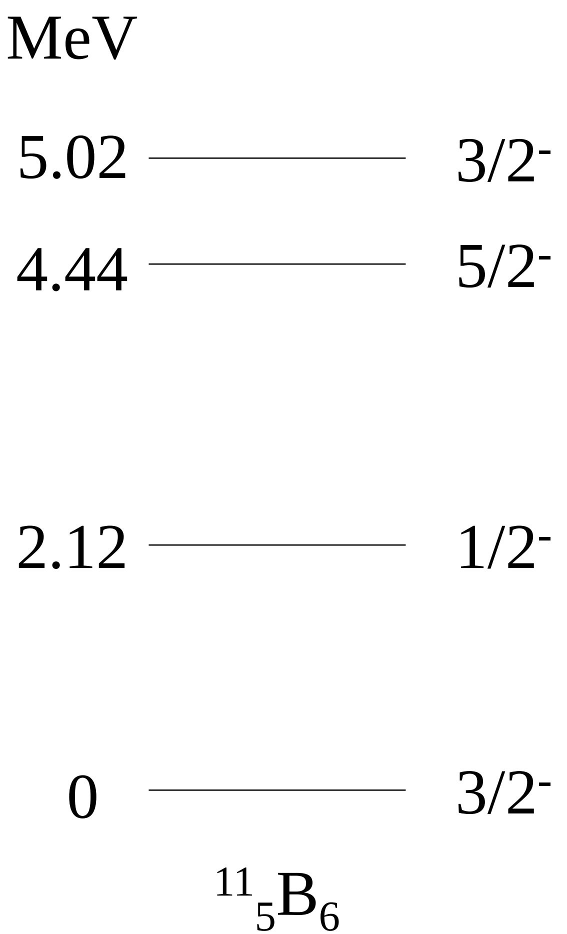
$1p_{1/2}$ → $I^{\pi} = 1/2^{-}$

$1p_{3/2}$ → $I^{\pi} = 0^{+}$

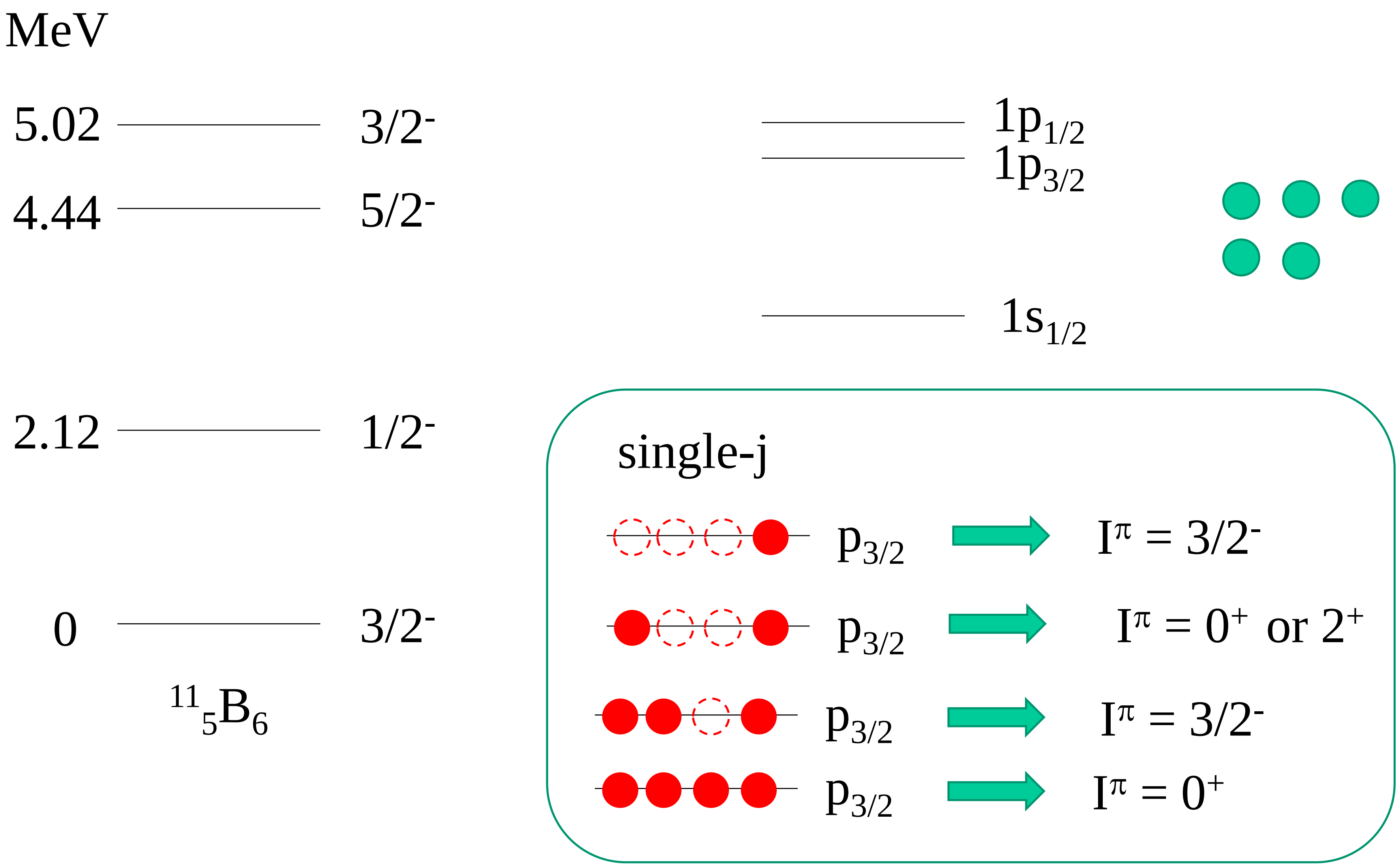
$1s_{1/2}$ → $I^{\pi} = 0^{+}$

in total,
$I^{\pi} = 1/2^{-}$

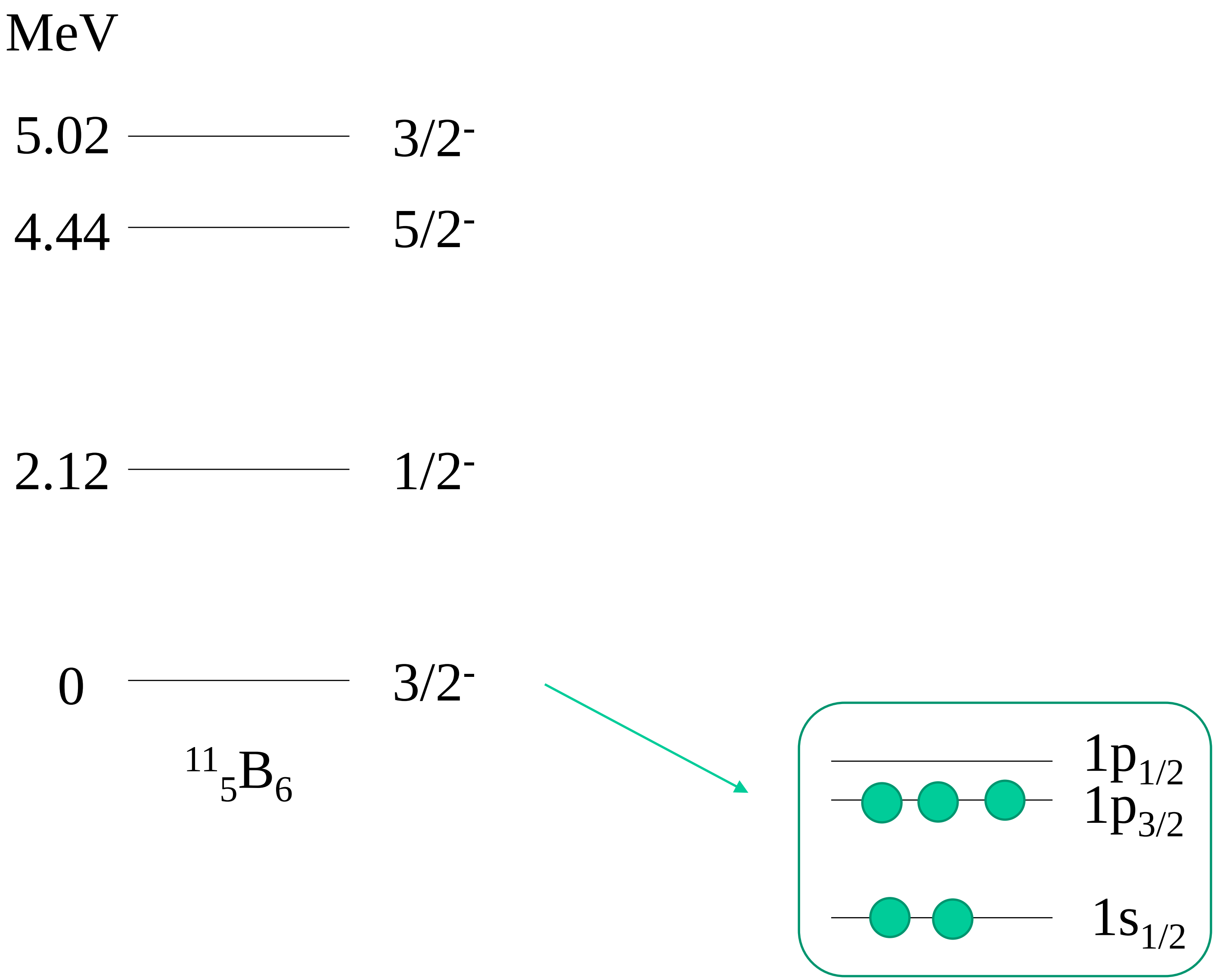
example: (main) shell model configurations for $^{11}_{5}B_6$



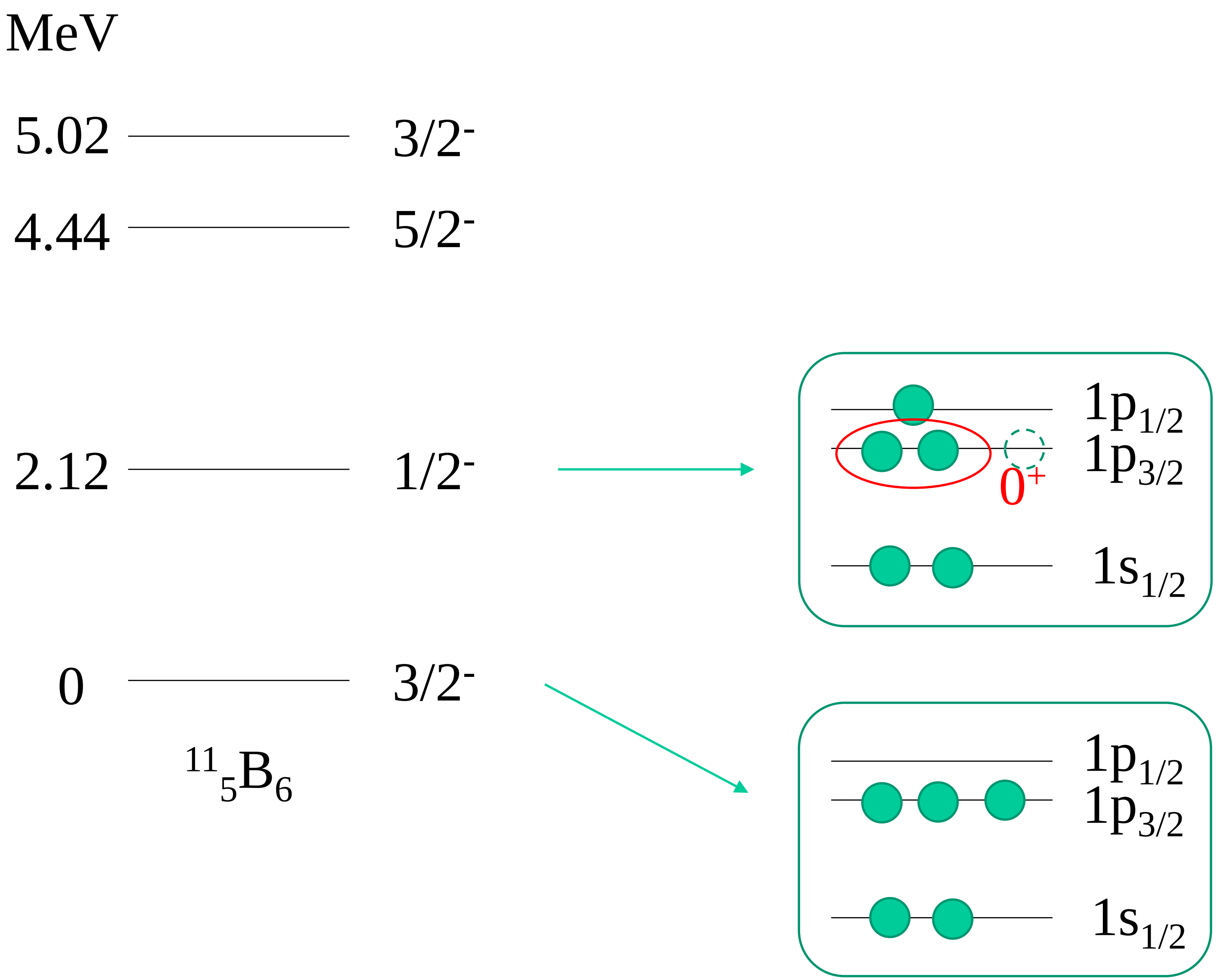
example: (main) shell model configurations for $^{11}_{5}B_6$



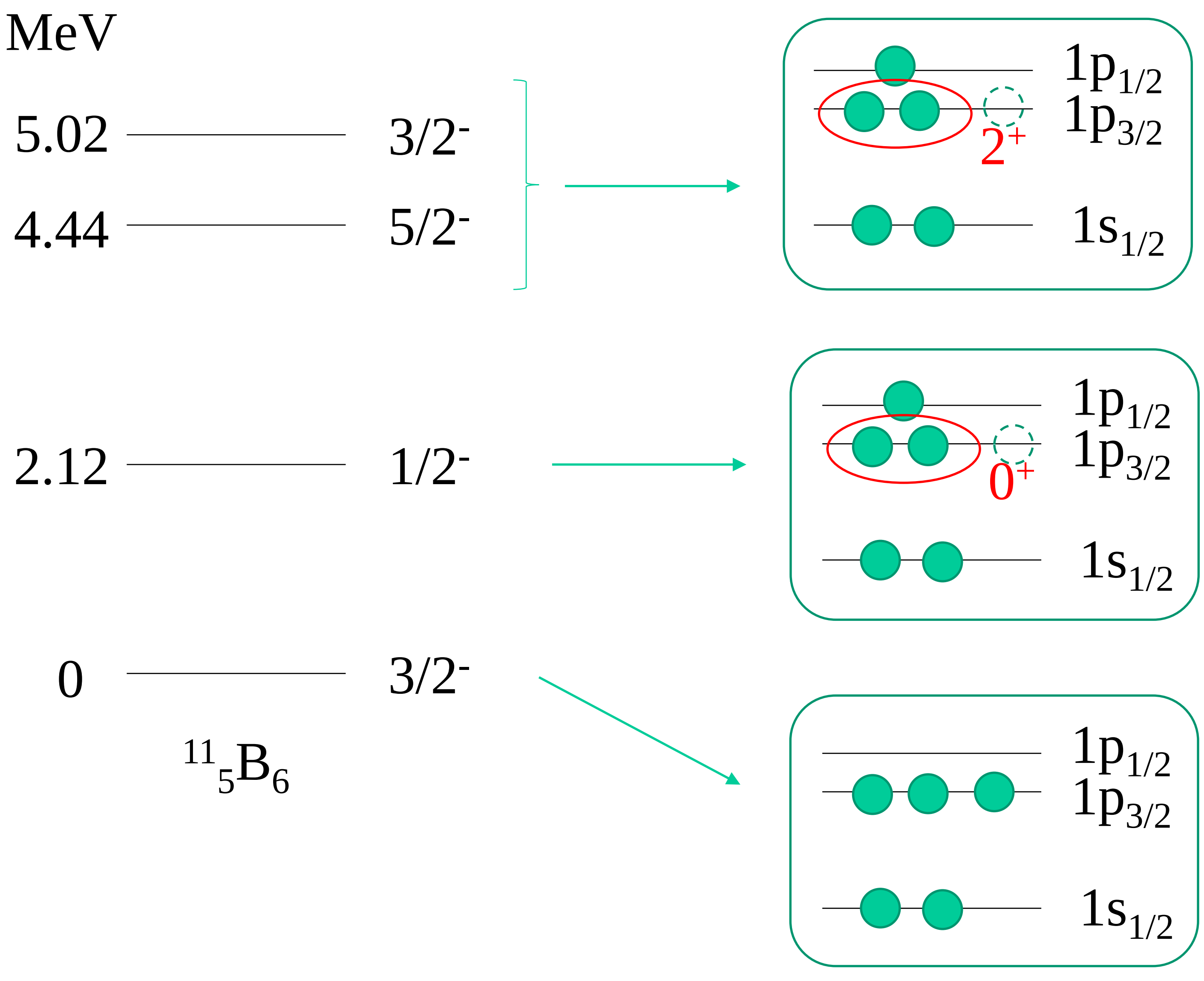
example: (main) shell model configurations for $^{11}_{5}B_{6}$



example: (main) shell model configurations for $^{11}_{5}B_{6}$

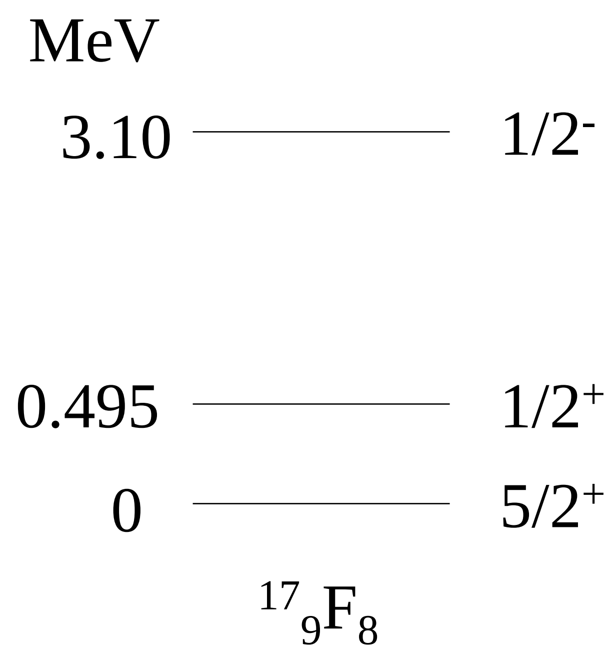


example: (main) shell model configurations for $^{11}_{5}B_{6}$



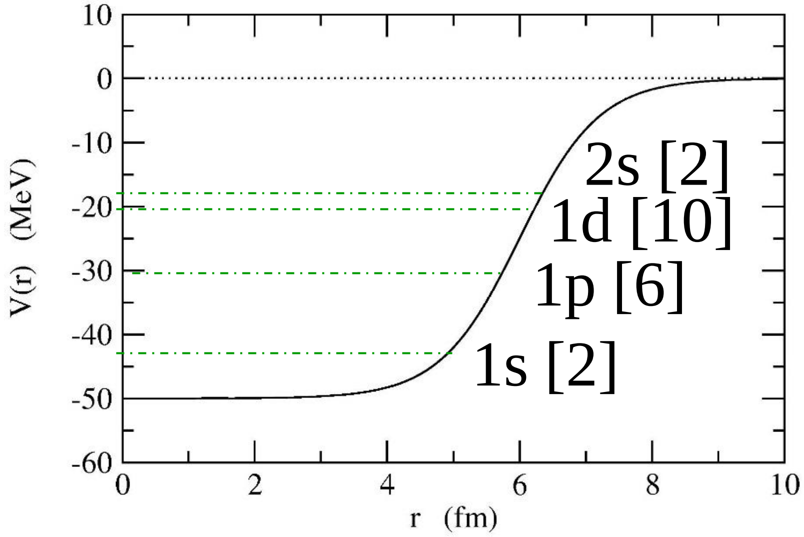
レポート問題１： 3次元調和振動子では、2s 軌道と 1d 軌道のエネルギーが縮退している。一方で、Woods-Saxon ポテンシャルや井戸型ポテンシャルでは 1d 軌道のエネルギーが下になる。定性的にそうなる理由を説明せよ。

レポート問題２： ^{17}Fの基底状態の陽子の配位を殻模型を使って説明せよ。第一励起状態、第二励起状態はどうなるか？



Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

An interpretation: independent particle motion in a potential well



+ spin-orbit interaction

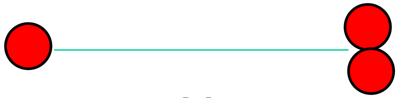
how to construct the potential well?

Mean-field (Hartree-Fock) Theory

nucleon-nucleon interaction

$\boldsymbol{r}$

$$v(\boldsymbol{r})$$



$\boldsymbol{r}$

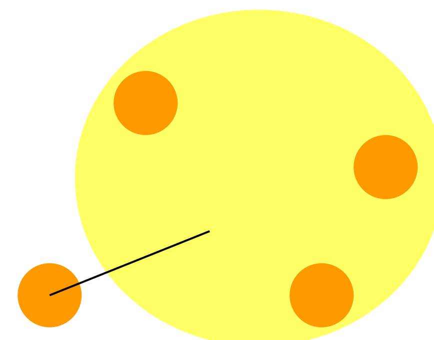
$$2v(\boldsymbol{r})$$

Mean-field (Hartree-Fock) Theory

nucleon-nucleon interaction

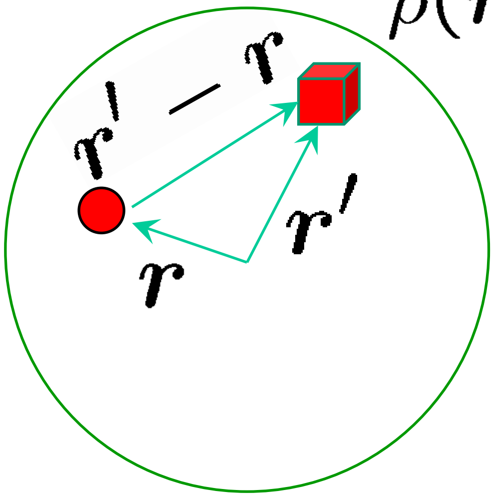
$\Rightarrow$ $v(\boldsymbol{r})$

平均場



interaction for a nucleon inside a nucleus:

$\rho(\boldsymbol{r}')d\boldsymbol{r}'$



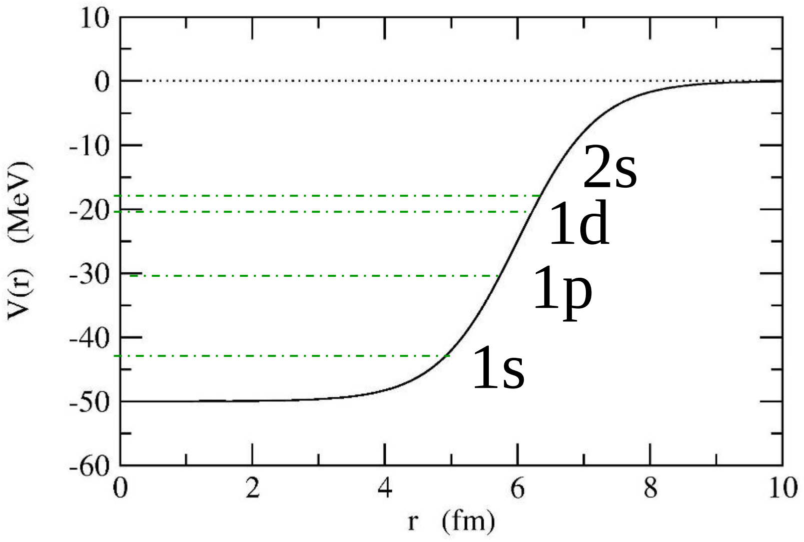
$$\Rightarrow \quad v(\boldsymbol{r}'-\boldsymbol{r})\cdot\underline{\rho(\boldsymbol{r}')d\boldsymbol{r}'}$$

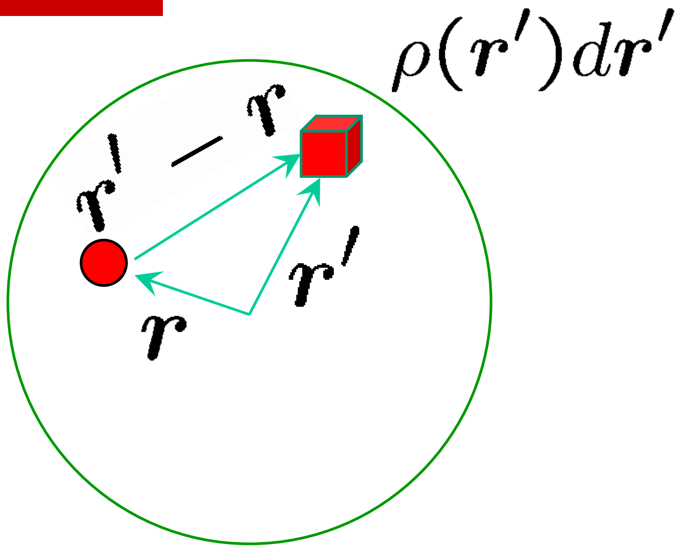
the number of nucleon at *r'*

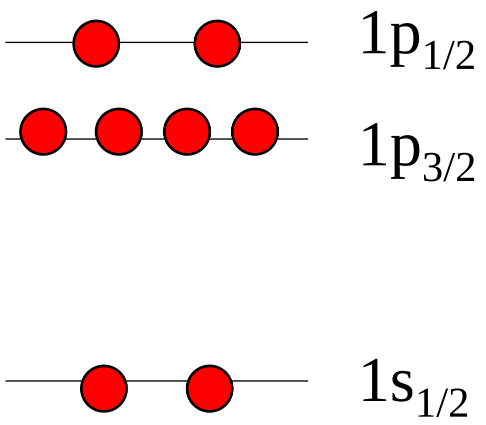
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

Mean-field (Hartree-Fock) Theory





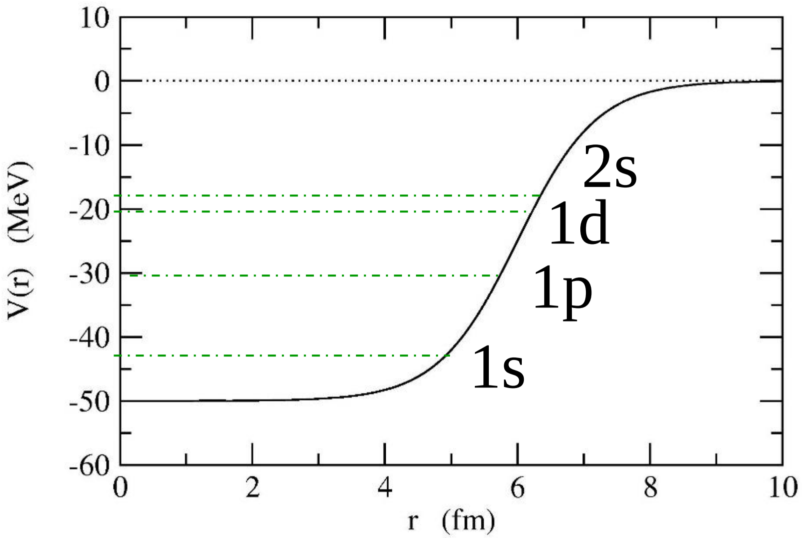


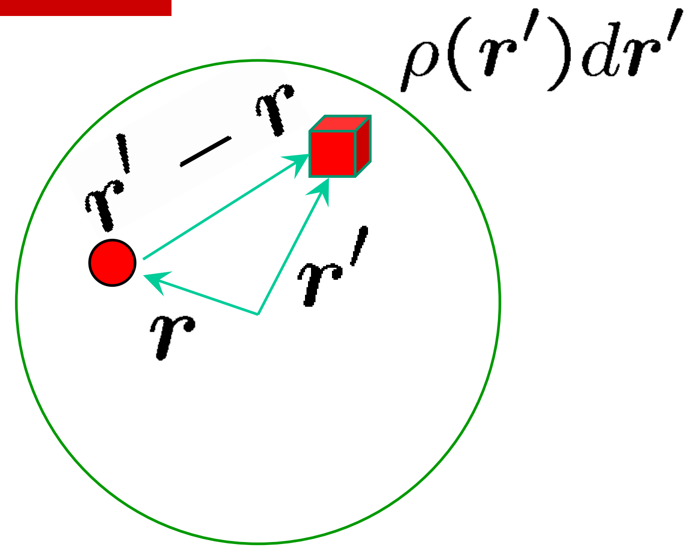
shell model

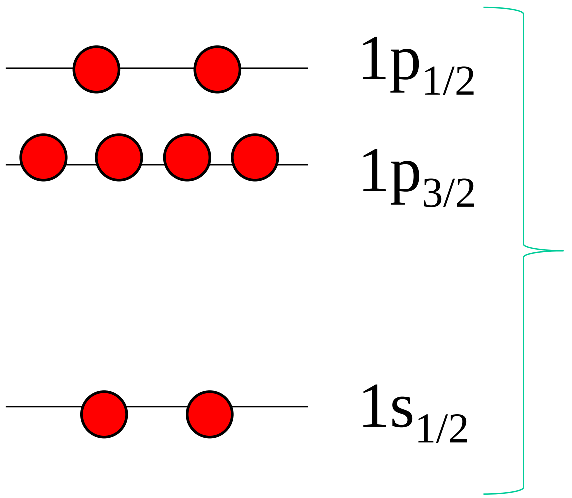
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r} - \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

Mean-field (Hartree-Fock) Theory







shell model

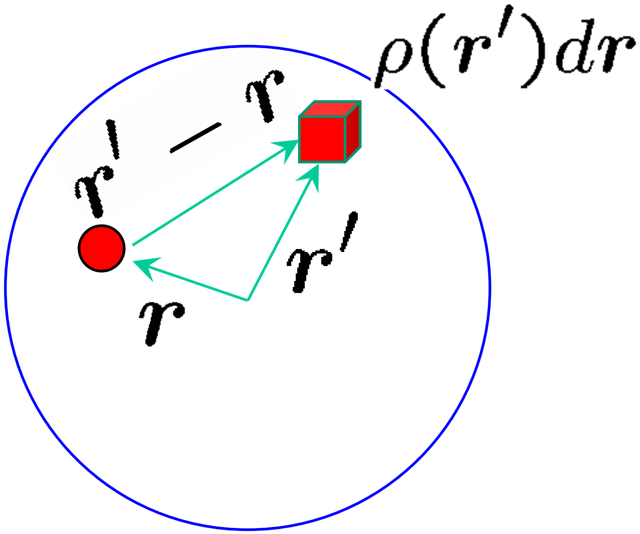
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

independent motion

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

Mean-field (Hartree-Fock) Theory



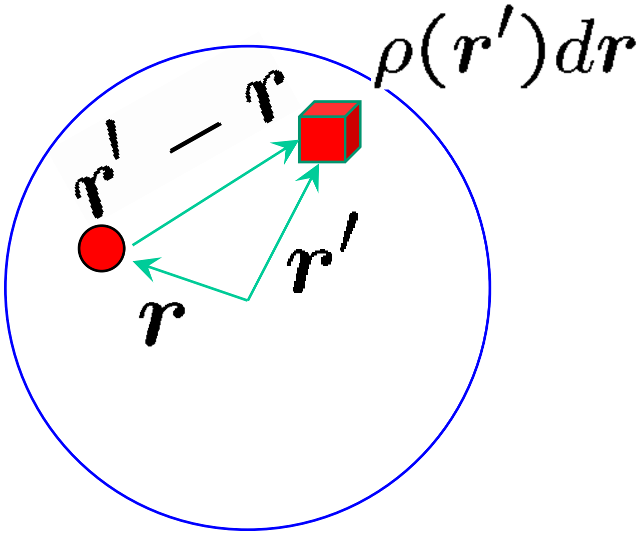
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r} - \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

$$0 = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

Mean-field (Hartree-Fock) Theory



naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

$$\begin{aligned} 0 &= \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\ &= \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right)d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \end{aligned}$$

the potential depends on the solutions

Mean-field (Hartree-Fock) Theory

$$0 = \left[-\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 + \int v(\boldsymbol{r} - \boldsymbol{r}') \left(\sum_j |\psi_j(\boldsymbol{r}')|^2 \right) d\boldsymbol{r}' - \epsilon_i \right] \psi_i(\boldsymbol{r})$$

the potential depends on the solutions

⟶ self-consistent solutions

Iteration: $\{\psi_i\} \to \rho \to V \to \{\psi_i\} \to \cdots$