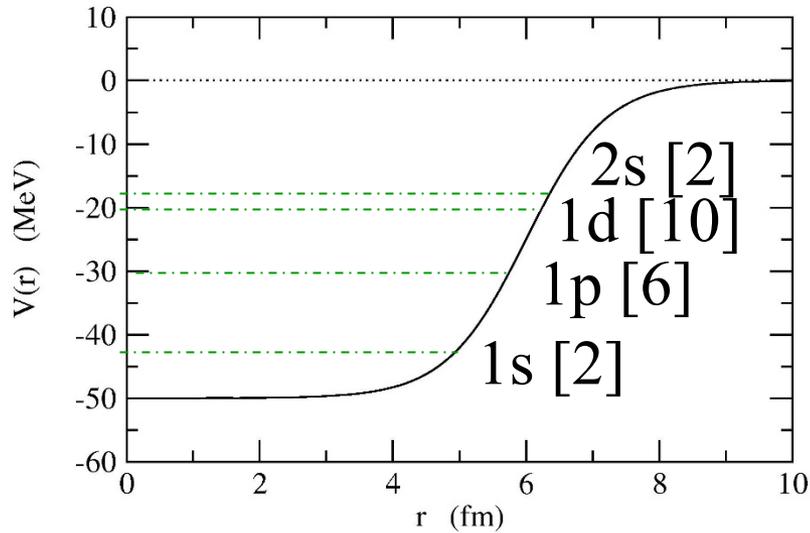


$N$  or  $Z = 2, 8, 20, 28, 50, 82, 126$  (魔法数)の時に特別に安定

解釈: ポテンシャル中の独立粒子運動



+ スピン軌道力

核内(媒質内)での核子間相互作用

Hartree-Fock 方程式

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + \int v(\mathbf{r} - \mathbf{r}') \left( \sum_j |\psi_j(\mathbf{r}')|^2 \right) d\mathbf{r}' - \epsilon_i \right] \psi_i(\mathbf{r}) - \int v(\mathbf{r} - \mathbf{r}') \left( \sum_j \psi_j^*(\mathbf{r}') \psi_i(\mathbf{r}') \right) d\mathbf{r}' \psi_j(\mathbf{r}) = 0$$

$$\begin{aligned}
v(\mathbf{r}, \mathbf{r}') &= t_0(1 + x_0\hat{P}_\sigma)\delta(\mathbf{r} - \mathbf{r}') \\
&+ \frac{1}{2}t_1(1 + x_1\hat{P}_\sigma)(\mathbf{k}^2\delta(\mathbf{r} - \mathbf{r}') + \delta(\mathbf{r} - \mathbf{r}')\mathbf{k}^2) \\
&+ t_2(1 + x_2\hat{P}_\sigma)\mathbf{k}\delta(\mathbf{r} - \mathbf{r}')\mathbf{k} \\
&+ \frac{1}{6}t_3(1 + x_3\hat{P}_\sigma)\delta(\mathbf{r} - \mathbf{r}')\rho^\alpha((\mathbf{r} + \mathbf{r}')/2) \\
&+ iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{k} \times \delta(\mathbf{r} - \mathbf{r}')\mathbf{k}
\end{aligned}$$

if  $x_i=0, t_1=t_2=0$ :

$$\mathbf{k} = (\nabla_1 - \nabla_2)/2i$$

$$v(\mathbf{r}, \mathbf{r}') = \underbrace{t_0\delta(\mathbf{r} - \mathbf{r}')}_{\text{short-range attraction}} + \underbrace{\frac{1}{6}t_3\delta(\mathbf{r} - \mathbf{r}')\rho^\alpha(\mathbf{r})}_{\text{repulsion to avoid collapse}}$$

$$+ \underbrace{iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{k} \times \delta(\mathbf{r} - \mathbf{r}')\mathbf{k}}_{\text{spin-orbit interaction}}$$

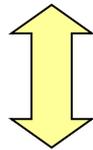
spin-orbit interaction

# Density Functional Theory

With Skyrme interaction:

$$\begin{aligned}\langle \Psi | H | \Psi \rangle &= E[\rho, \tau, J] \\ &= \int d\mathbf{r} \left( \frac{\hbar^2}{2m} \tau + \frac{1}{2} t_0 \left( 1 + \frac{1}{2} x_0 \right) \rho^2 \right. \\ &\quad \left. - \frac{1}{2} t_0 \left( x_0 + \frac{1}{2} \right) \sum_q \rho_q^2 \cdots \right)\end{aligned}$$

Energy functional in terms of local densities



Close analog to the Density Functional Theory (DFT)

密度汎関数法

## i) Hohenberg-Kohn Theorem

$$H = H_0 + V_{\text{ext}}$$

*Lemma* :  $\rho(\mathbf{r}) \rightarrow V_{\text{ext}}(\mathbf{r})$  (unique)



Density: the basic variable

(密度が分かれば原理的に全て分かる)

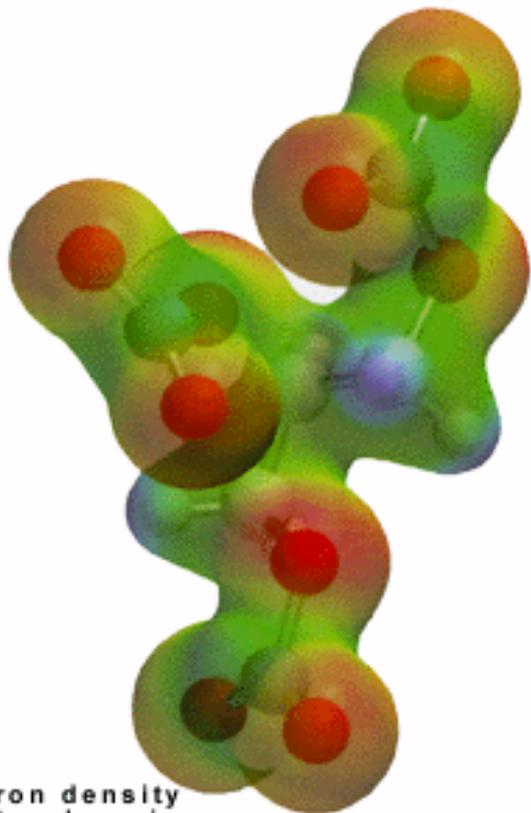
## ii) Hohenberg-Kohn variational principle

The existence of a functional  $E[\rho]$ , which gives the exact g.s. energy for a given g.s. density

$$\longrightarrow E[\rho] \geq E_{gs}$$

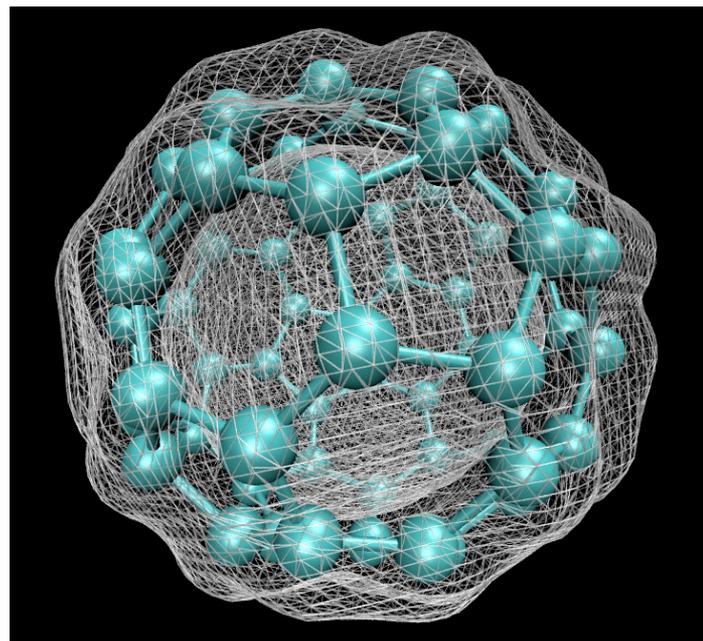
うまい方法で  $E[\rho]$  を作れば、それを使って多体計算が簡単に行える。

$$E[\rho] = E_{\text{HF}}[\rho] + E_{\text{corr}}[\rho]$$



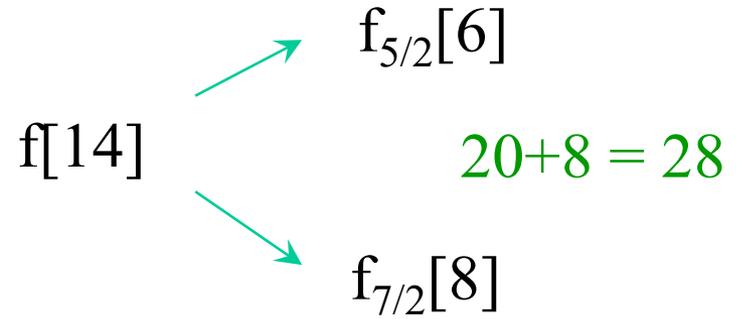
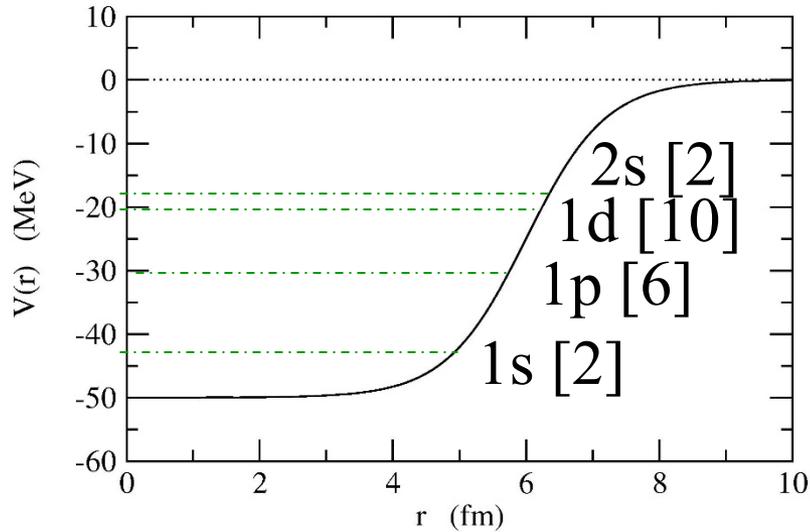
The electron density  
of nitroglycerine

ニトログリセリンの電子密度  
(Nobelprize.org より)

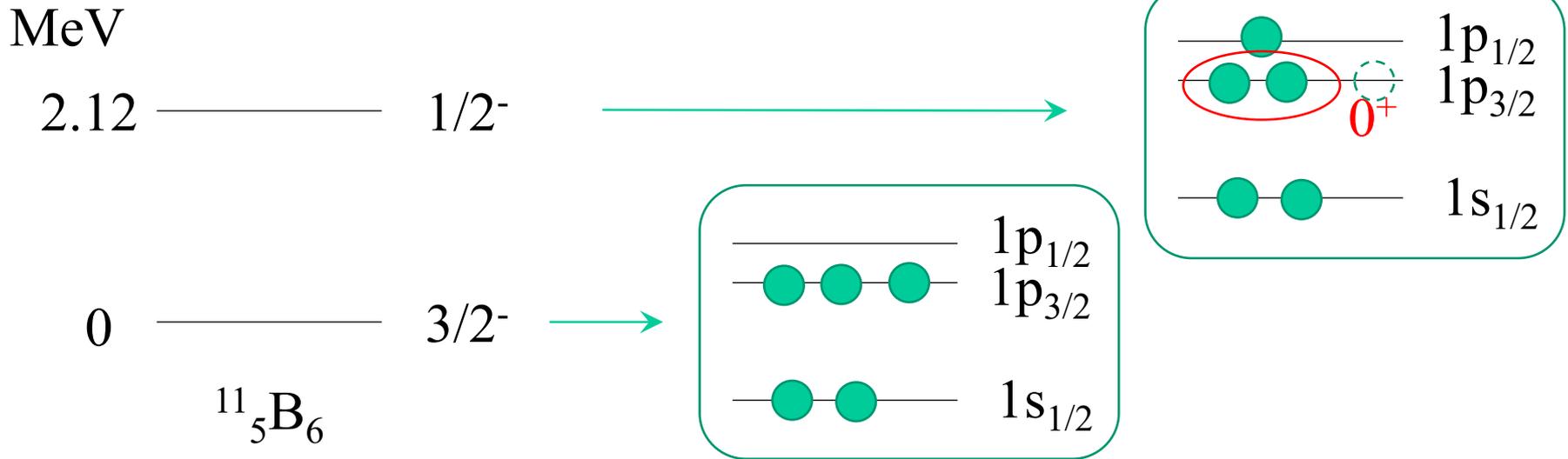


$\text{C}_{60}$  の電子密度  
(Wikipedia より)

# Mean-field approximation and deformation



example:

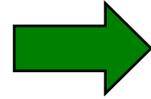


同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型(球形ポテンシャルの準位)で考えた場合:

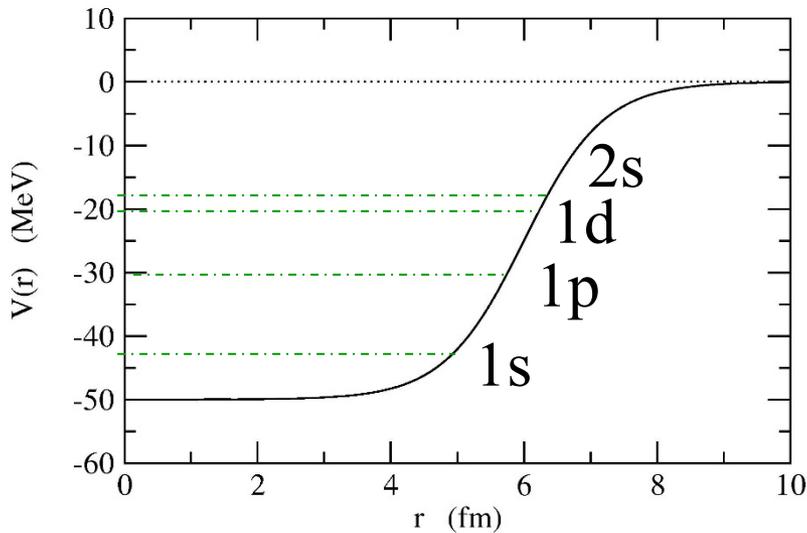
—●—  $1p_{1/2}$  [2]

●●●●  $1p_{3/2}$  [4]



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

—●—●—  $1s_{1/2}$  [2]



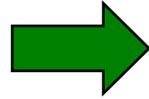
同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型(球形ポテンシャルの準位)で考えた場合:

—●—  $1p_{1/2}$  [2]

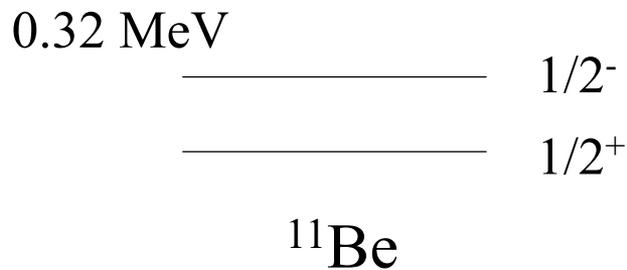
—●●●●—  $1p_{3/2}$  [4]

—●●—  $1s_{1/2}$  [2]



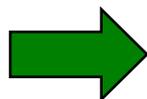
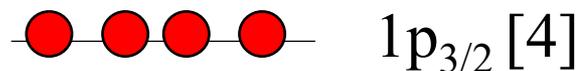
$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

実際の  $^{11}\text{Be}$  の準位を見てみると:



同じように殻模型で $^{11}_4\text{Be}_7$  のレベルを考えると。。。

殻模型 (球形ポテンシャルの準位) で考えた場合:



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

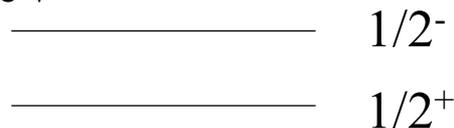


かなり無理

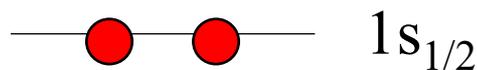
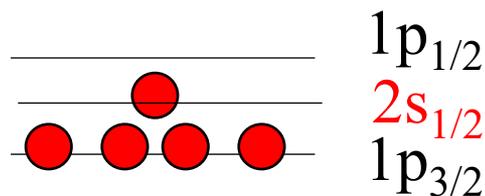
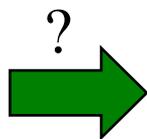


実際の  $^{11}\text{Be}$  の準位を見てみると:

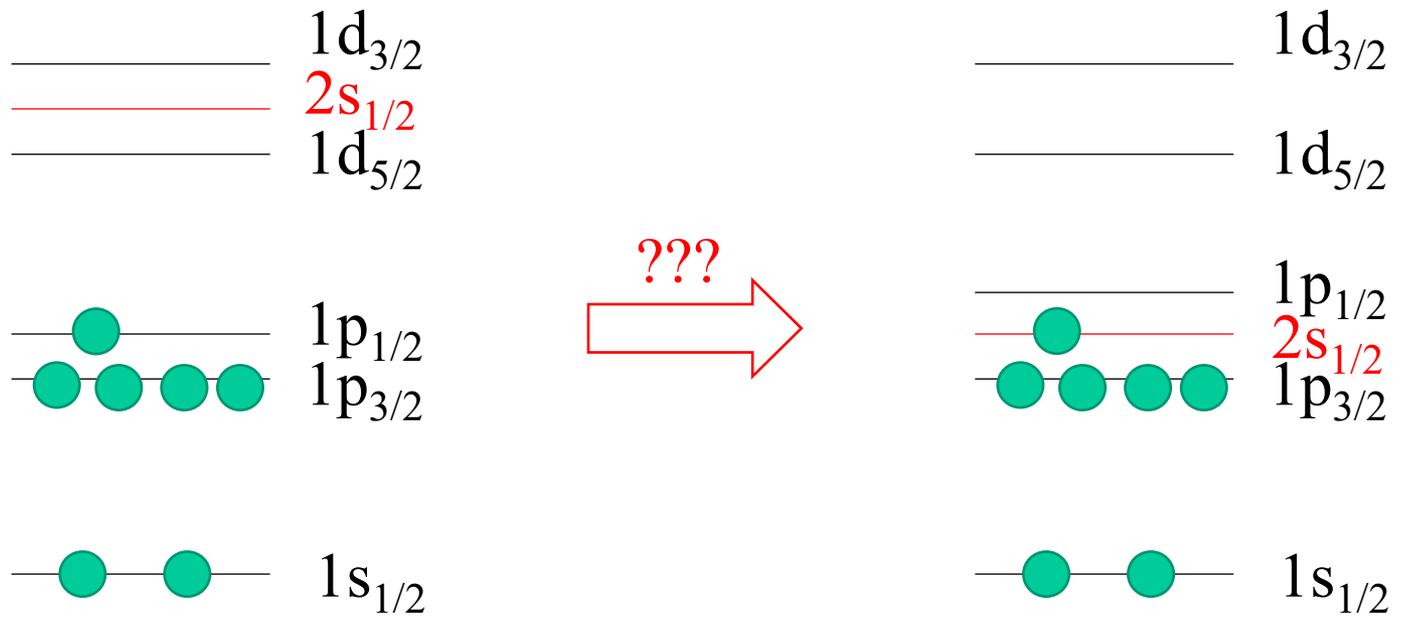
0.32 MeV



$^{11}\text{Be}$



“parity inversion”

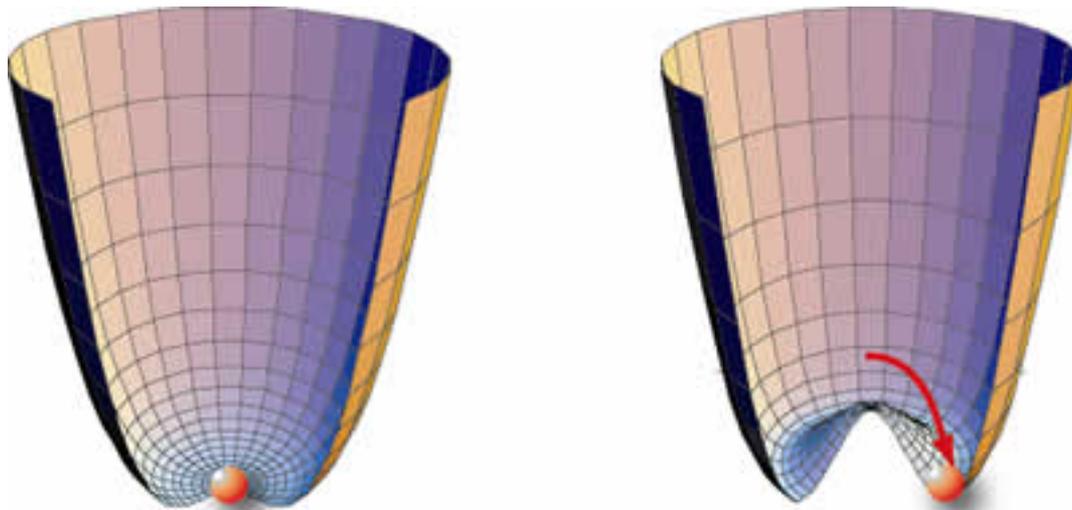


球形ポテンシャルに無理があるなら、変形させてみる?

# Mean-field approximation and deformation

$$H = \sum_{i=1}^A \left( -\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{MF}}(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i,j}^A v(\mathbf{r}_i, \mathbf{r}_j) - \sum_i V_{\text{MF}}(\mathbf{r}_i)$$

→  $\Psi_{\text{MF}}$  : ハミルトニアン  $H$  が持っている対称性を持たなくてもいい  
“対称性の自発的破れ”



$\Psi_{MF}$  : ハミルトニアン  $H$  が持っている対称性を持たなくてもいい

## 典型的な例

➤ 並進対称性: 原子核のDFTでは常に破れる

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} \sum_{i,j}^A v(\mathbf{r}_i - \mathbf{r}_j) \rightarrow \sum_{i=1}^A \left( -\frac{\hbar^2}{2m} \nabla_i^2 + \underline{V_{MF}(\mathbf{r}_i)} \right)$$

$\Psi_{MF}$  : ハミルトニアン  $H$  が持っている対称性を持たなくてもいい

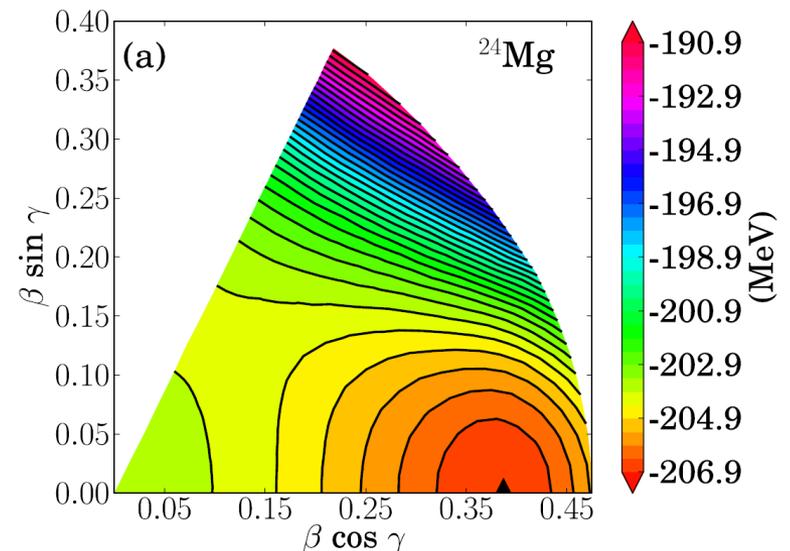
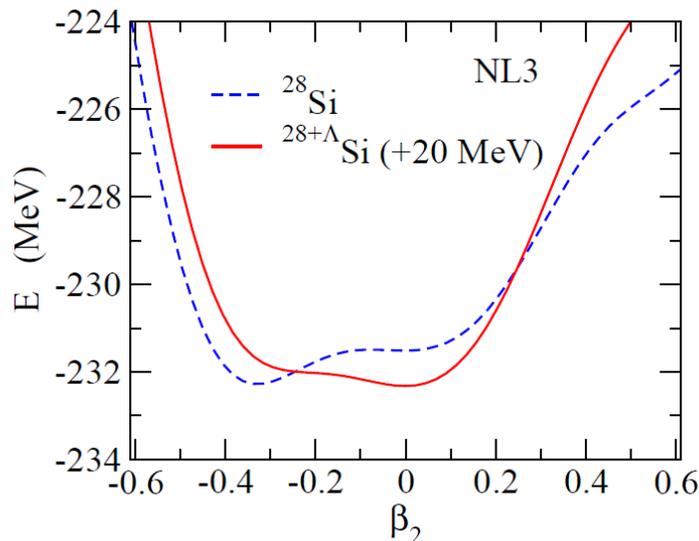
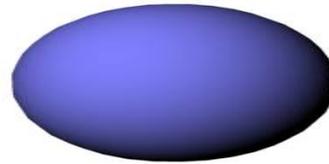
## 典型的な例

➤ 並進対称性: 原子核のDFTでは常に破れる

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} \sum_{i,j}^A v(r_i - r_j) \rightarrow \sum_{i=1}^A \left( -\frac{\hbar^2}{2m} \nabla_i^2 + \underline{V_{MF}(r_i)} \right)$$

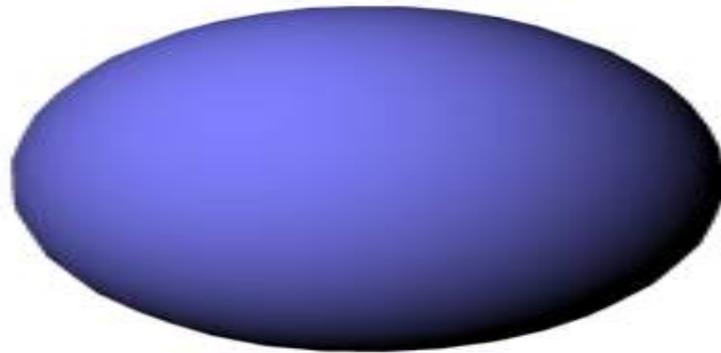
➤ 回転対称性

変形した基底状態



# Nuclear Deformation

実験的な証拠



# Nuclear Deformation

## Excitation spectra of $^{154}\text{Sm}$

(MeV)

0.903 —————  $8^+$

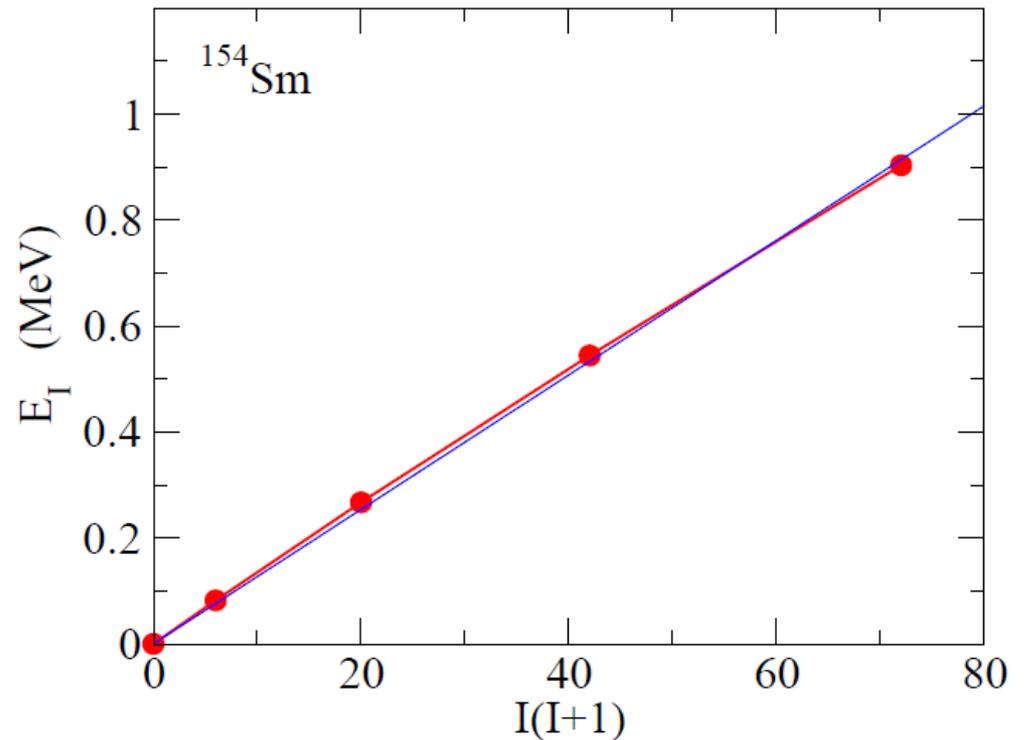
0.544 —————  $6^+$

0.267 —————  $4^+$

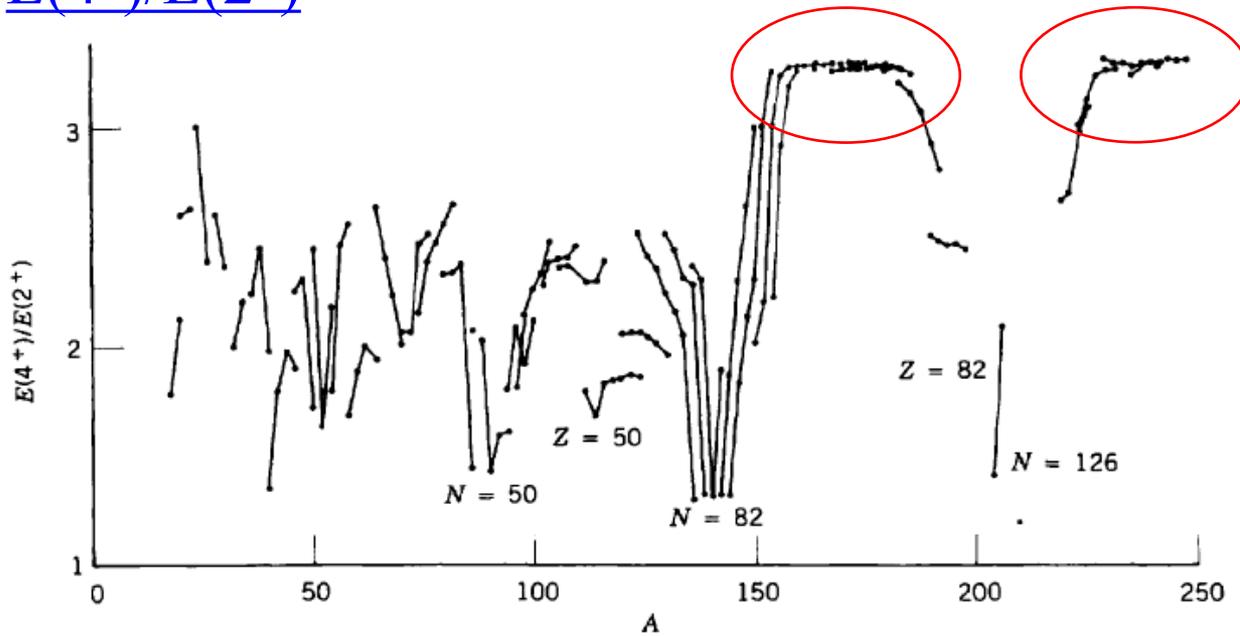
0.082 —————  $2^+$   
0 —————  $0^+$

$^{154}\text{Sm}$

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



# $E(4^+)/E(2^+)$

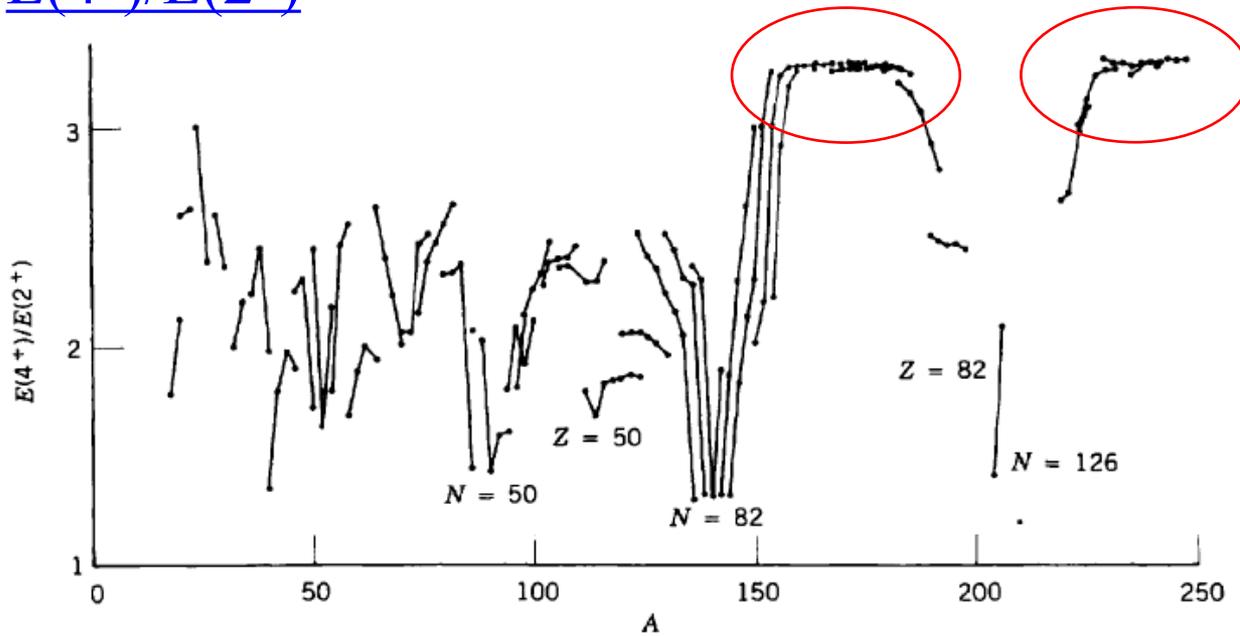


deformed nuclei:  
 $E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:  
 $E(4^+)/E(2^+) \sim 2$

K.S. Krane, "Introductory Nuclear Physics"

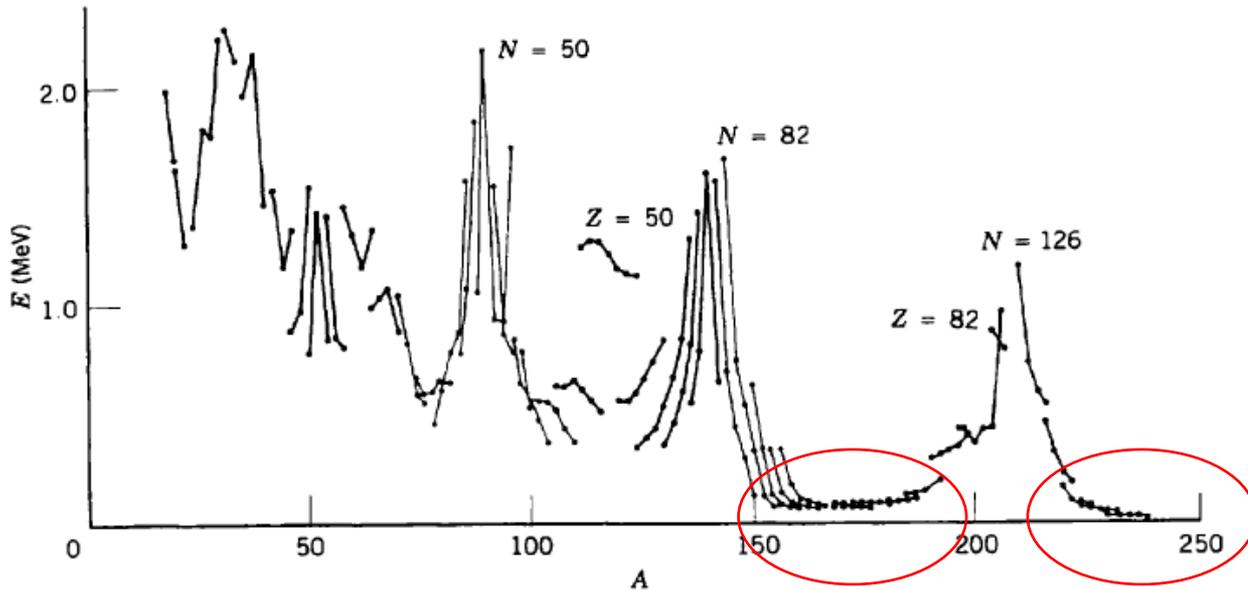
## $E(4^+)/E(2^+)$



deformed nuclei:  
 $E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:  
 $E(4^+)/E(2^+) \sim 2$

## $E(2^+)$



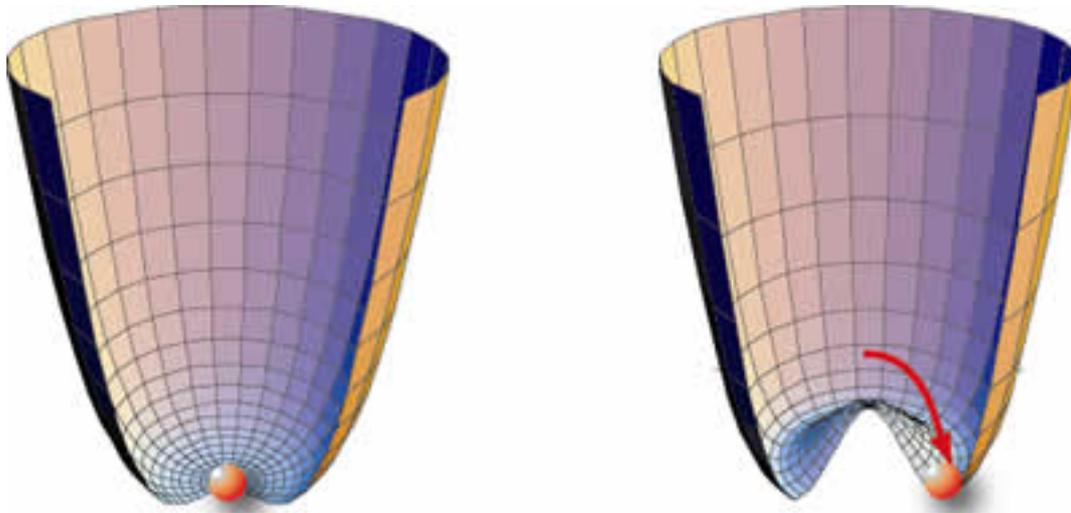
K.S. Krane. "Introductory Nuclear Physics"

a small energy  
→ spontaneously  
symm. breaking

deformed nuclei

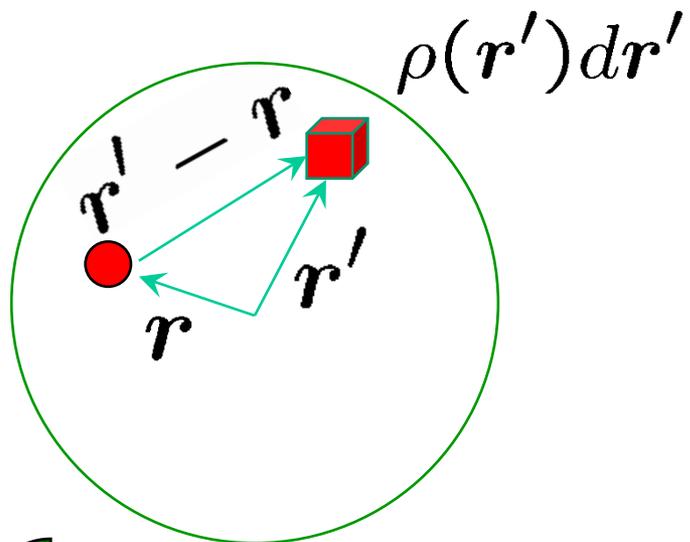
## Spontaneous symmetry breaking

The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.



Nambu-Goldstone mode (zero energy mode)  
to restore the symmetry

# One-particle motion in a deformed potential

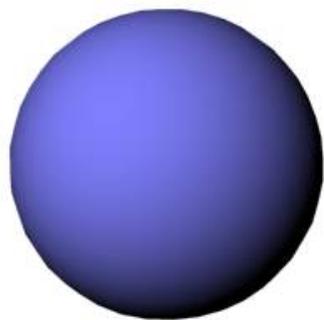


naively speaking,

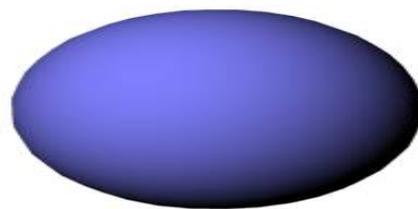
$$V(\mathbf{r}) \sim \int v(\mathbf{r} - \mathbf{r}')\rho(\mathbf{r}')d\mathbf{r}'$$



$$V(\mathbf{r}) \sim \int v(\mathbf{r}, \mathbf{r}')\rho(\mathbf{r}')d\mathbf{r}' \sim -g\rho(\mathbf{r}) \quad \text{if } v(\mathbf{r}, \mathbf{r}') = -g\delta(\mathbf{r} - \mathbf{r}')$$

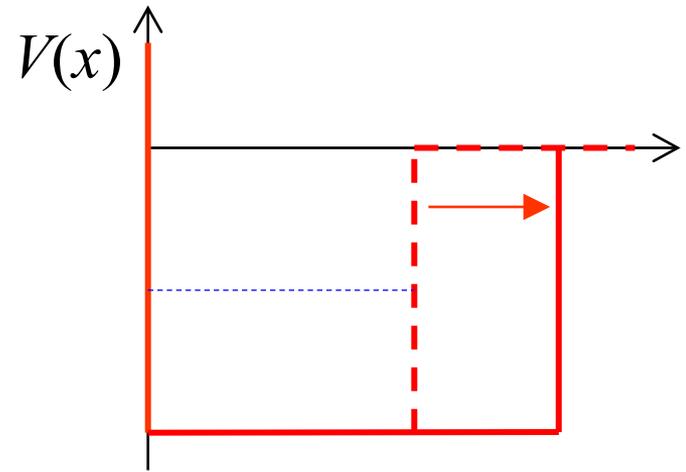
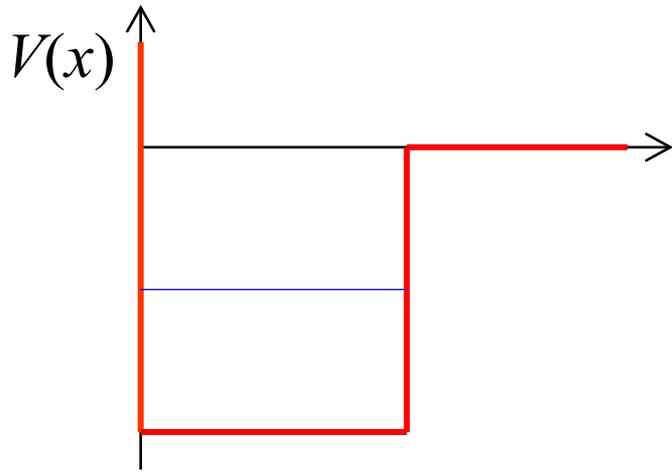


$\rightarrow V(r)$

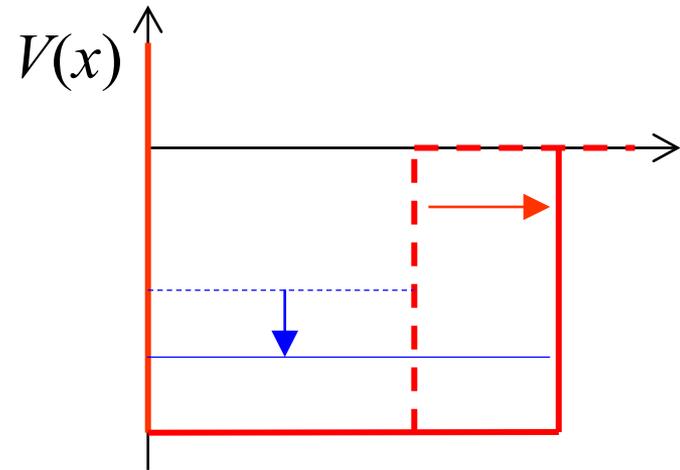
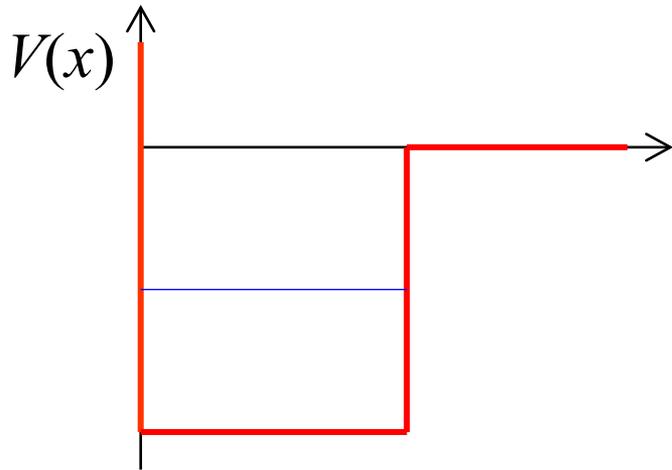


$\rightarrow V(r, \theta)$

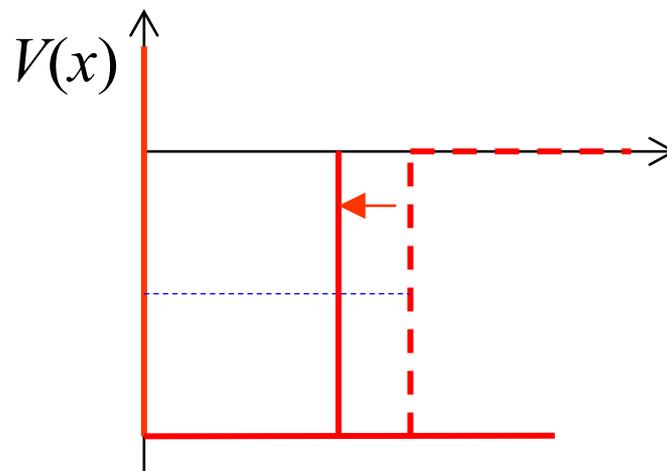
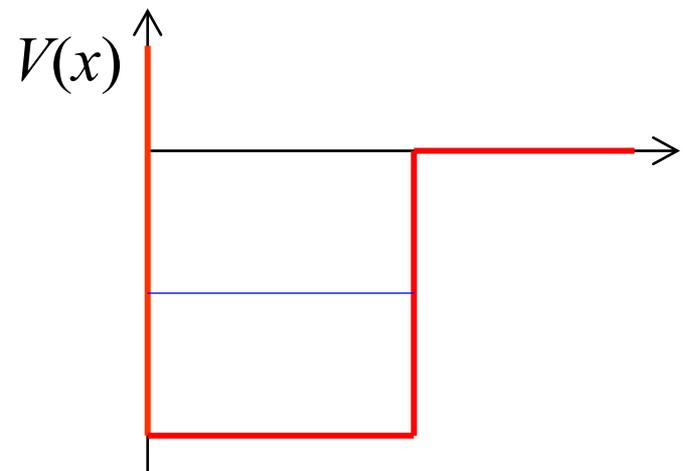
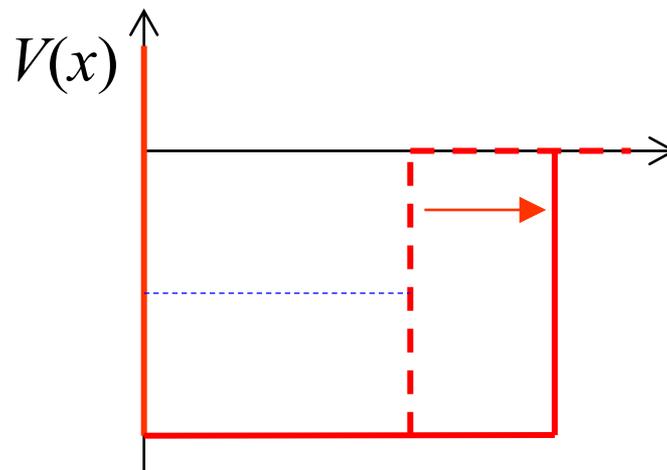
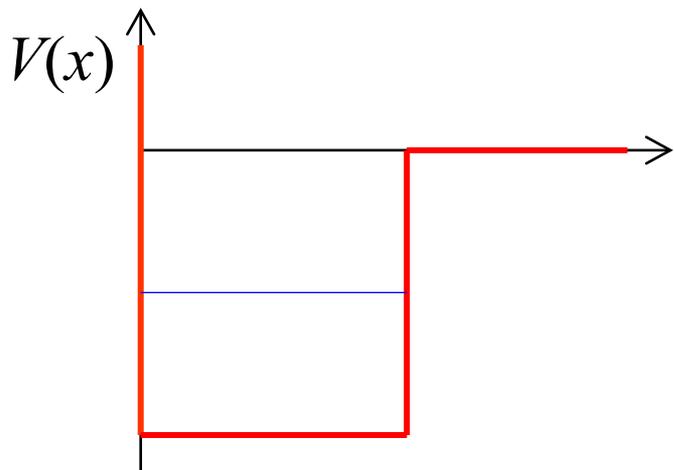
# (準備) 1次元井戸型ポテンシャル



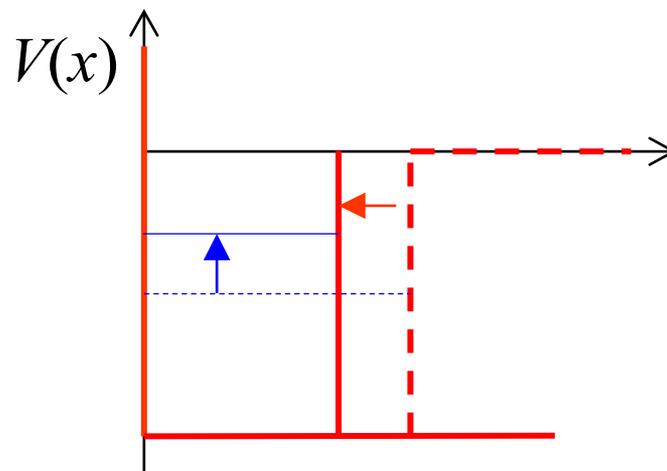
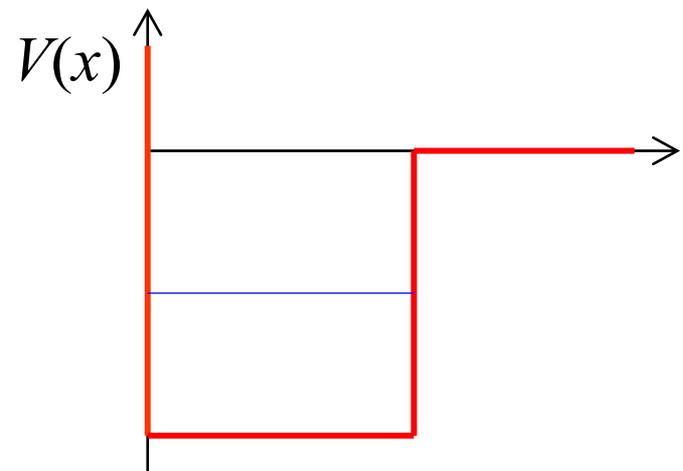
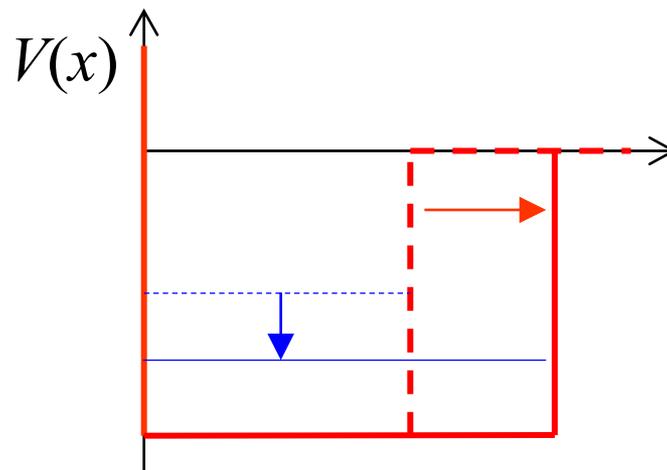
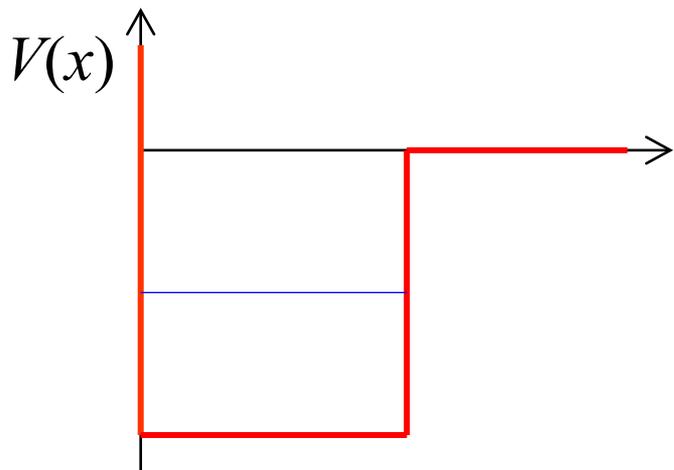
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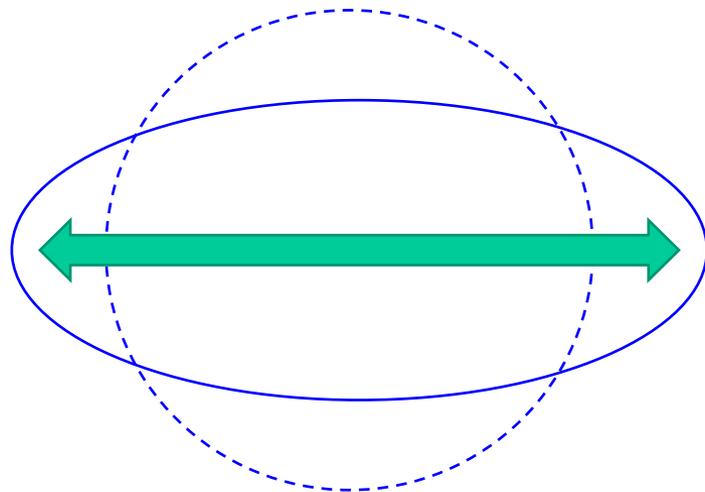


# (準備) 1次元井戸型ポテンシャル

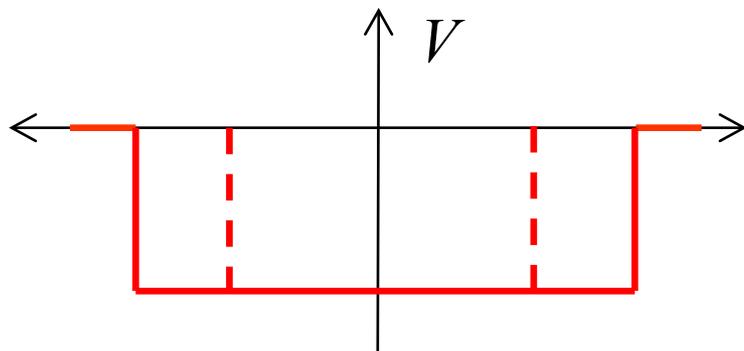
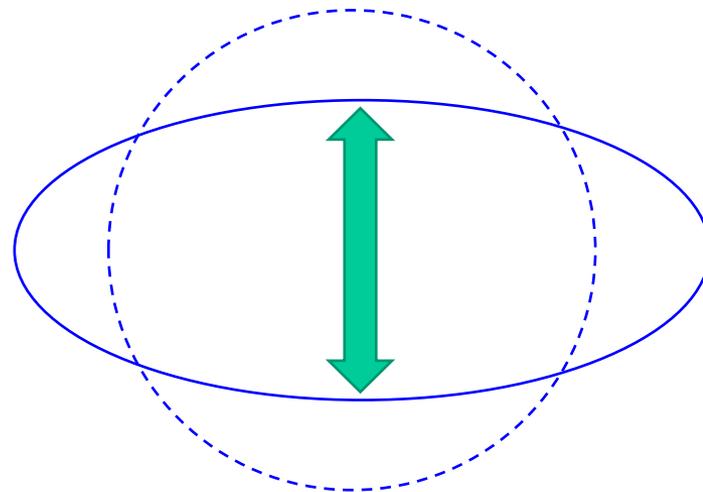


# One-particle motion in a deformed potential

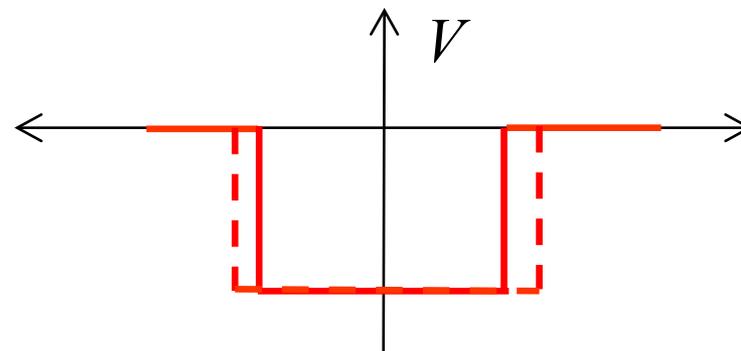
長軸に沿った運動



短軸に沿った運動



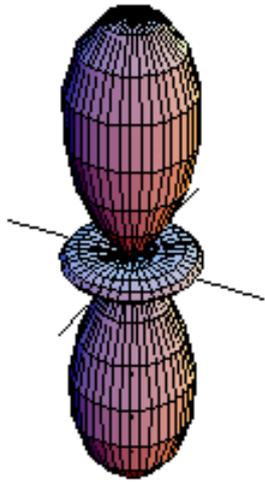
$E \rightarrow$  小



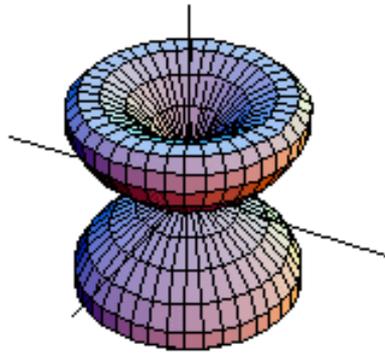
$E \rightarrow$  大

$l=2$

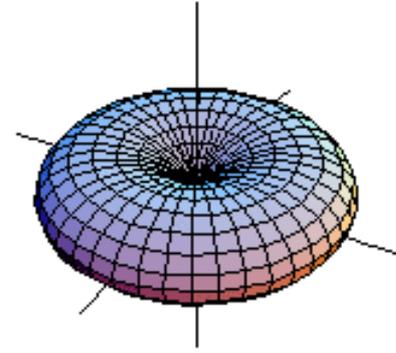
$z$



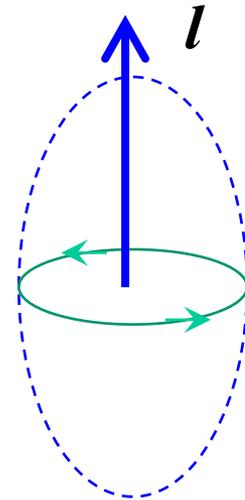
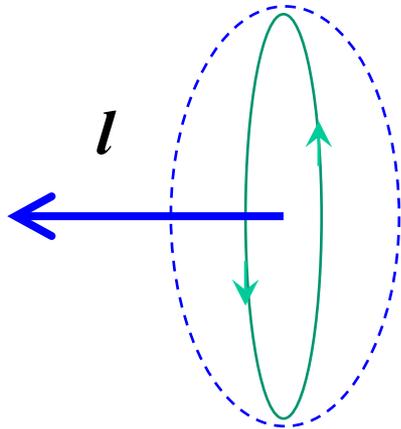
$r = Y_{20}$   
( $l_z = 0$ )



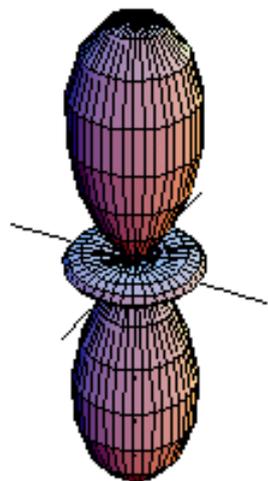
$r = Y_{21}$   
( $l_z = 1$ )



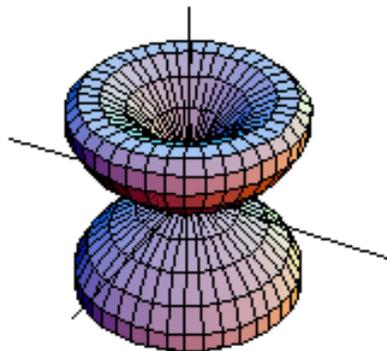
$r = Y_{22}$   
( $l_z = 2$ )



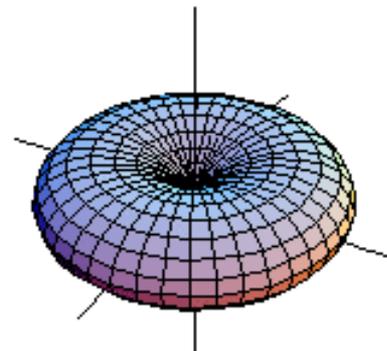
$l=2$



$$r = Y_{20}$$
$$(l_z = 0)$$



$$r = Y_{21}$$
$$(l_z = 1)$$



$$r = Y_{22}$$
$$(l_z = 2)$$



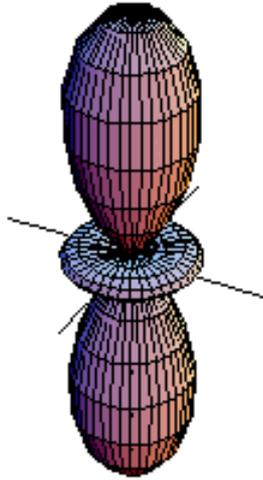
$E \rightarrow$  小

なら

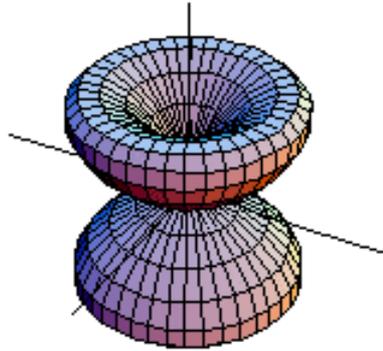
$E \rightarrow$  大

$l=2$

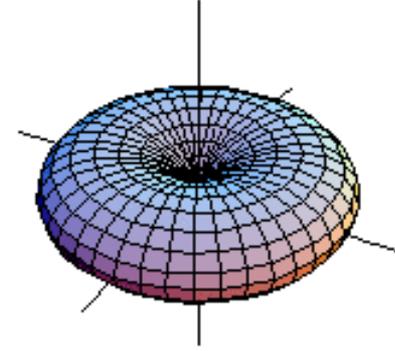
$z$



$$r = Y_{20}$$
$$(l_z = 0)$$

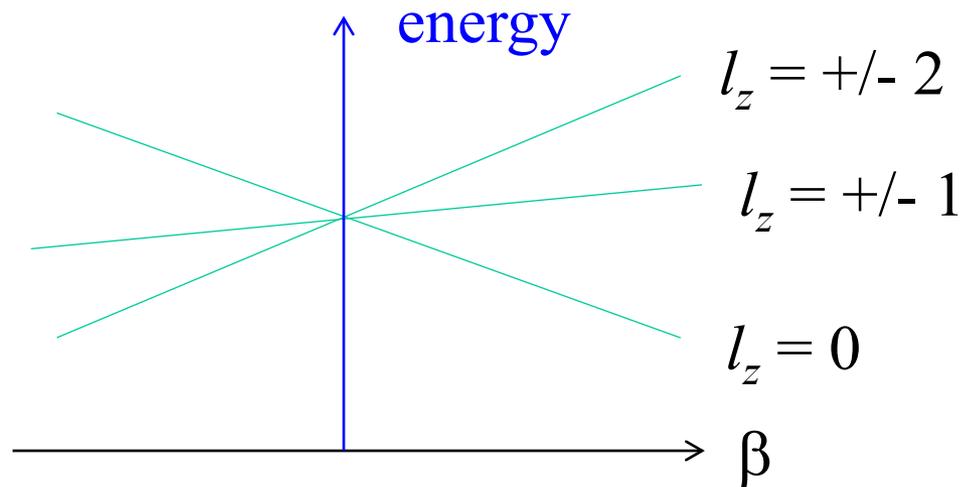


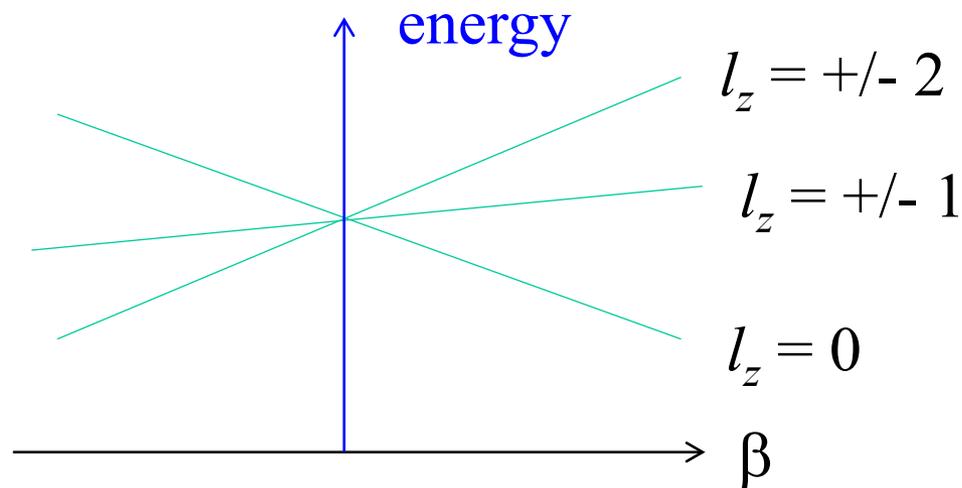
$$r = Y_{21}$$
$$(l_z = 1)$$



$$r = Y_{22}$$
$$(l_z = 2)$$

軌道が  
スプリット





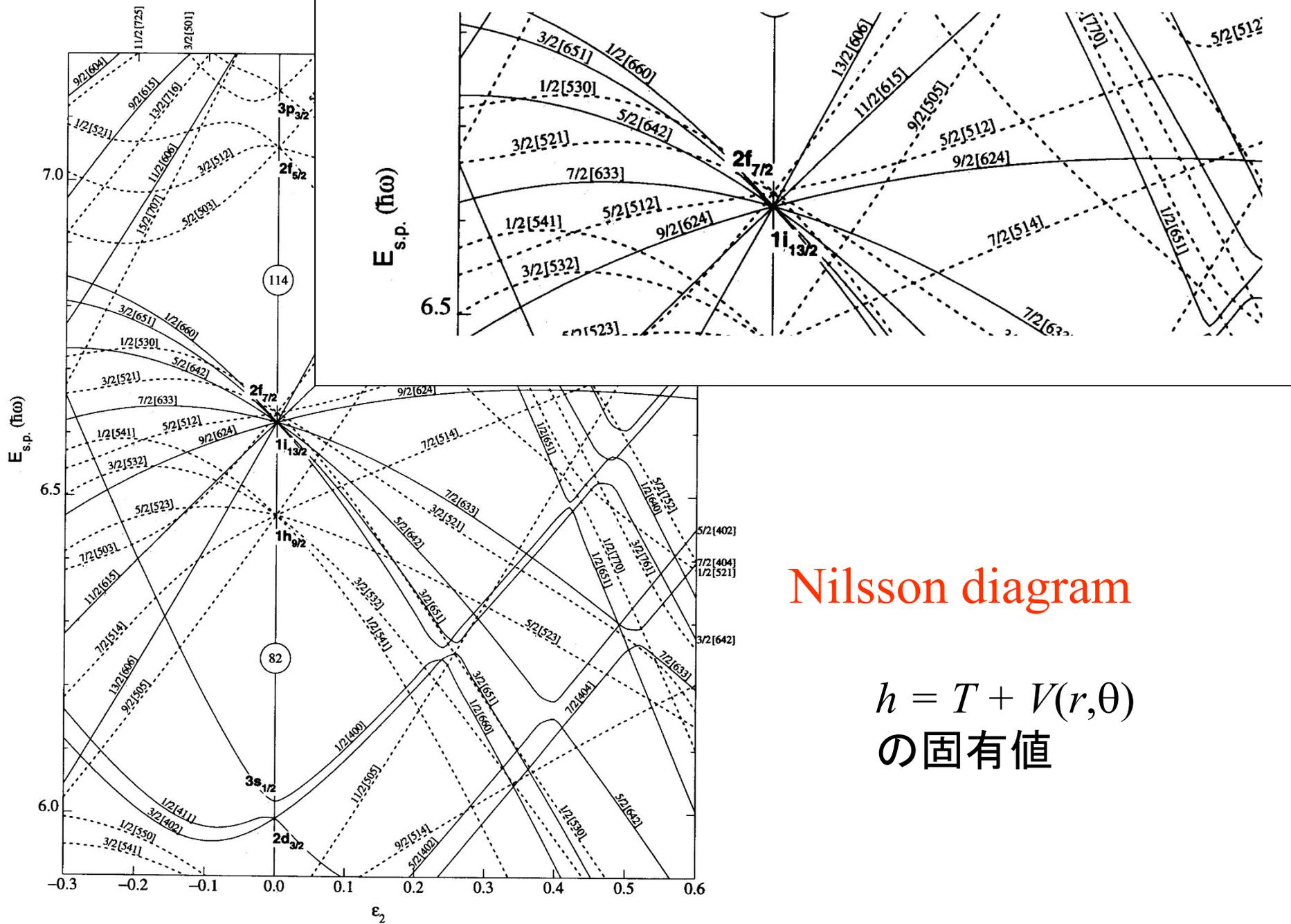
## 波動関数

- (note)  $V(r, \theta) \rightarrow$  回転対称性を持っていない  
 $\rightarrow$  角運動量がいい量子数ではない

$$\phi_{nll_z}(r, \theta, \phi) \rightarrow \phi_{nl_z}(r, \theta, \phi) = \sum_l \psi_{nl}(r) Y_{ll_z}(\theta, \phi)$$

いろいろな角運動量成分  
 が混じる

\* 軸対称変形であれば  $l_z$  は保存



# Nilsson diagram

$h = T + V(r, \theta)$   
の固有値

Figure 13. Nilsson diagram for protons,  $Z \geq 82$  ( $\epsilon_4 = \epsilon_2^2/6$ ).

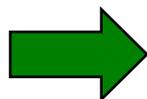
# 殻模型による $^{11}_4\text{Be}_7$ のレベル

殻模型 (球形ポテンシャルの準位) で考えた場合:

—●—  $1p_{1/2}$  [2]

●●●●  $1p_{3/2}$  [4]

—●—●—  $1s_{1/2}$  [2]



$^{11}\text{Be}$  の基底状態は  $I^\pi = 1/2^-$

かなり無理



実際の  $^{11}\text{Be}$  の準位を見てみると:

0.32 MeV

—  $1/2^-$   
—  $1/2^+$

$^{11}\text{Be}$



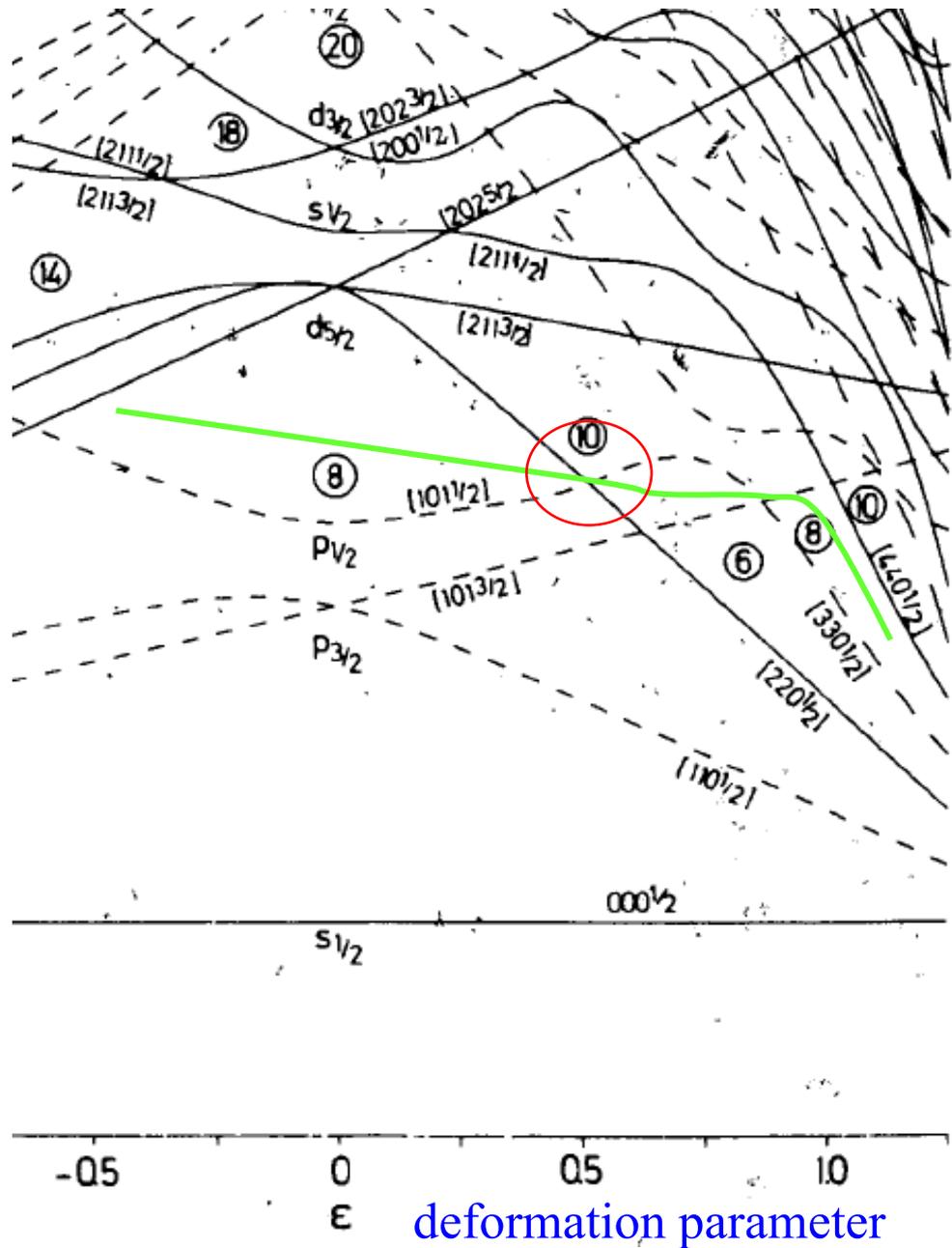
—●—  $1p_{1/2}$   
●●●●  $2s_{1/2}$   
—●—●—  $1p_{3/2}$

—●—●—  $1s_{1/2}$

“parity inversion”

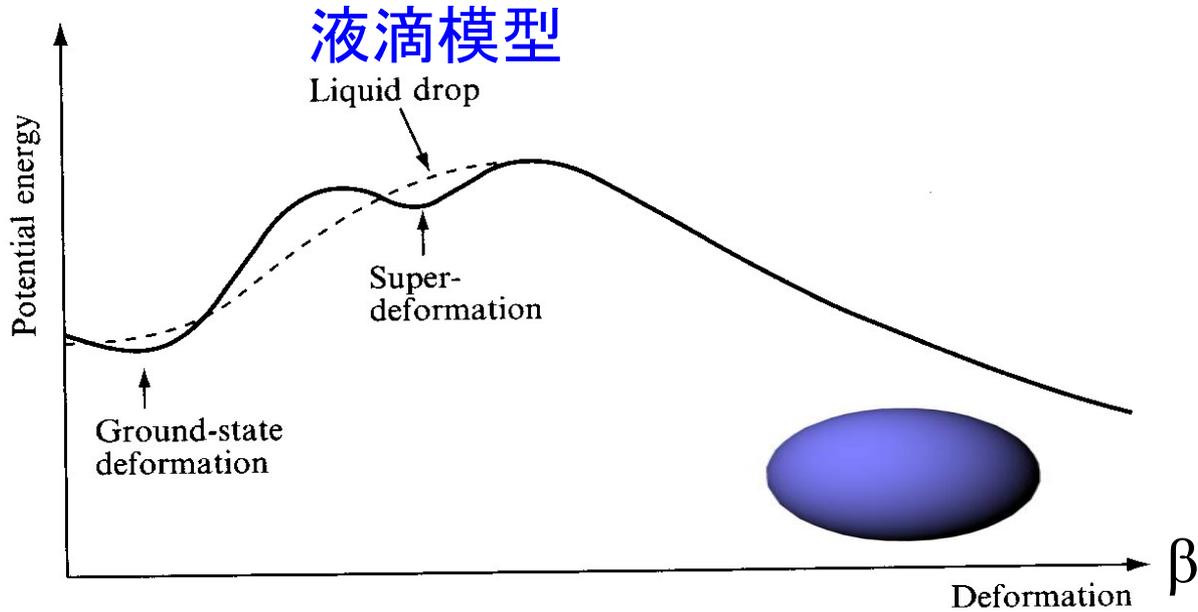


$^{11}_4\text{Be}_7$



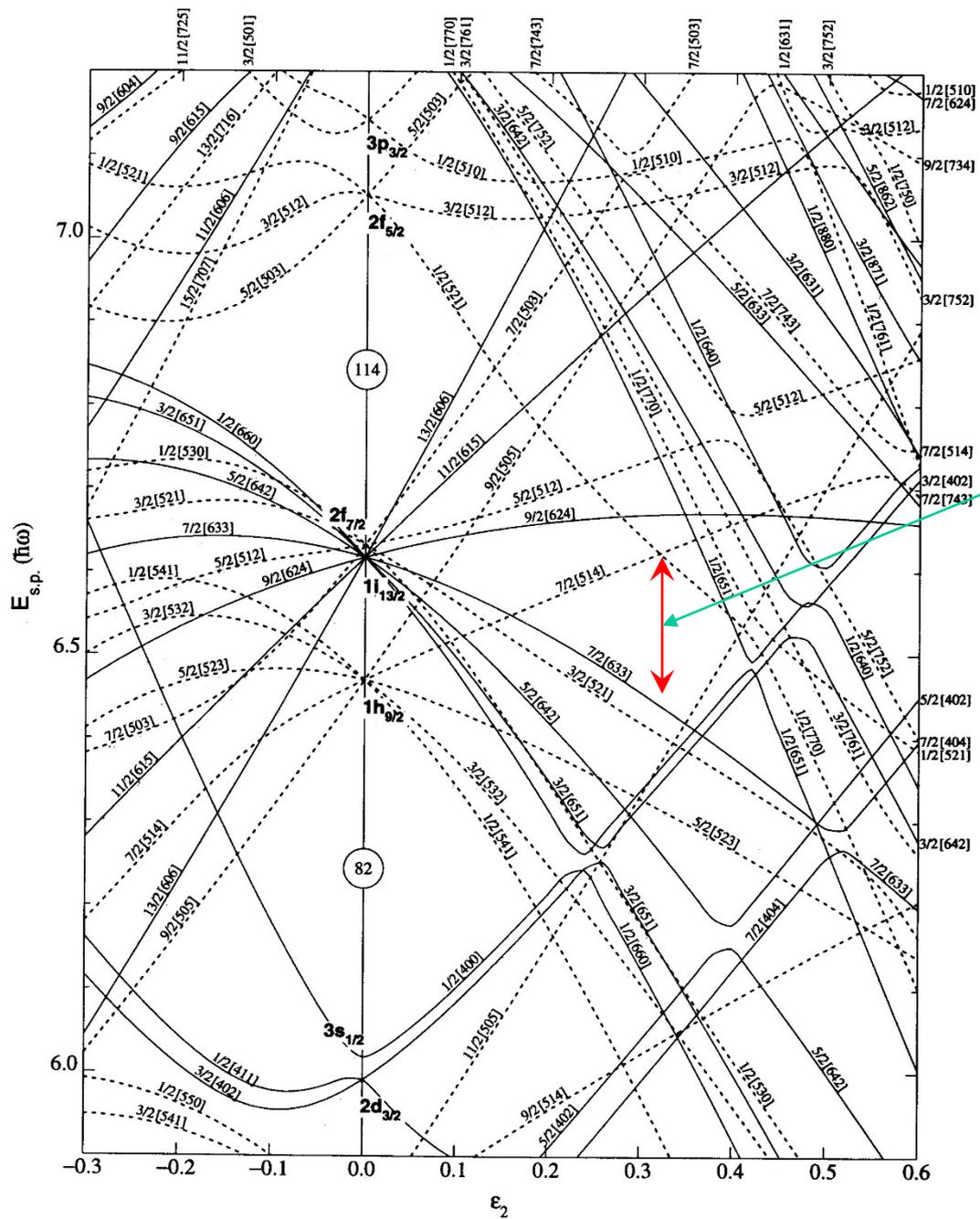


# 原子核の変形と殻効果



$$E(\beta) = E_{LDM}(\beta) + E_{shell}(\beta)$$

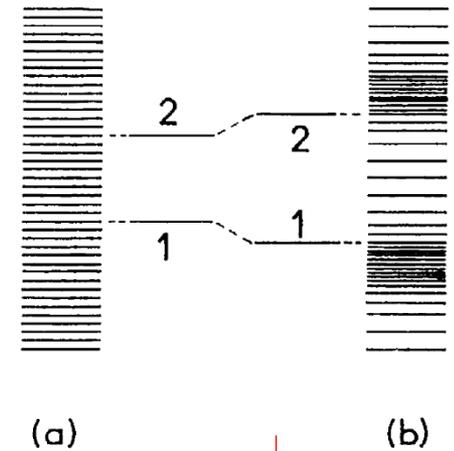
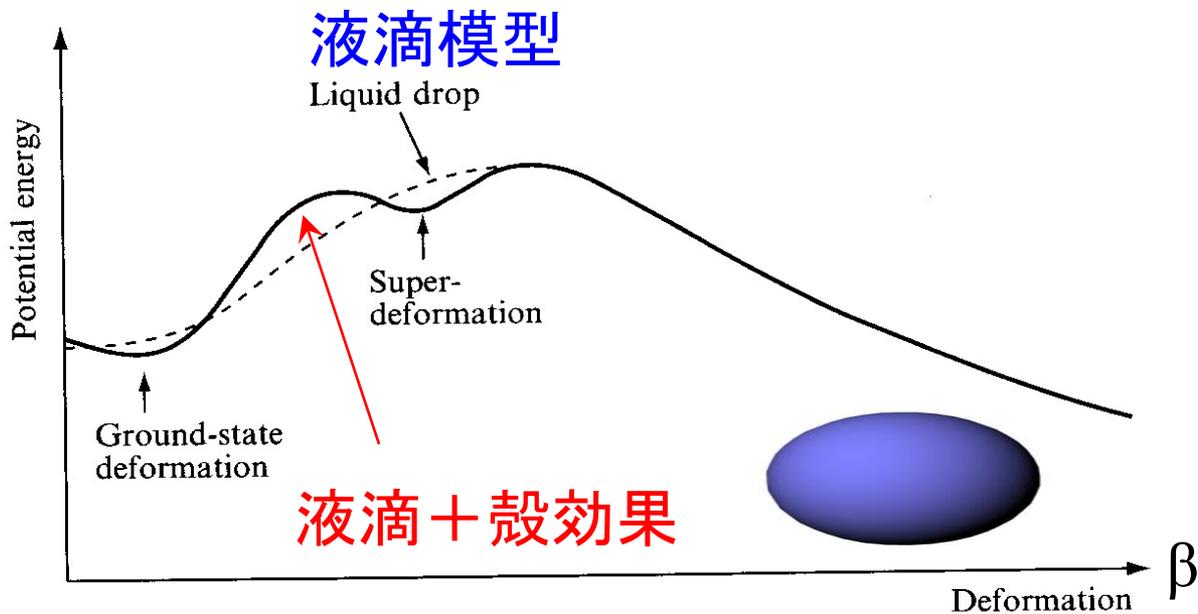
液滴模型→常に球形が基底状態



変形することにより  
ギャップが開く

Nilsson diagram

Figure 13. Nilsson diagram for protons,  $Z \geq 82$  ( $\epsilon_4 = \epsilon_2^2/6$ ).



準位にギャップ  
が開くと原子核が  
安定になる

$$E(\beta) = E_{LDM}(\beta) + E_{shell}(\beta)$$

原子核が変形

→ 核子が感じるポテンシャルも変形

→ 変形度によって異なる量子力学的補正(殻効果)

## レポート問題3

(i) 3次元非等方調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_z^2 z^2 + \frac{1}{2}m\omega_{\perp}^2 (x^2 + y^2)$$

$$\omega_{\perp} = \omega_0 \left(1 + \frac{\epsilon}{3}\right)$$

$$\omega_z = \omega_0 \left(1 - \frac{2}{3}\epsilon\right)$$

を考える。 $\epsilon$ を0から1まで変化させるとき、 $\epsilon = 0$ の時の基底状態、第一励起状態、第二励起状態のエネルギーはどのように変化するか図を使って議論せよ。

(ii) 同様に  $\epsilon$ を0から-1まで変化させるとどうなるか?

\* (i) と (ii) をまとめて答えてもOK

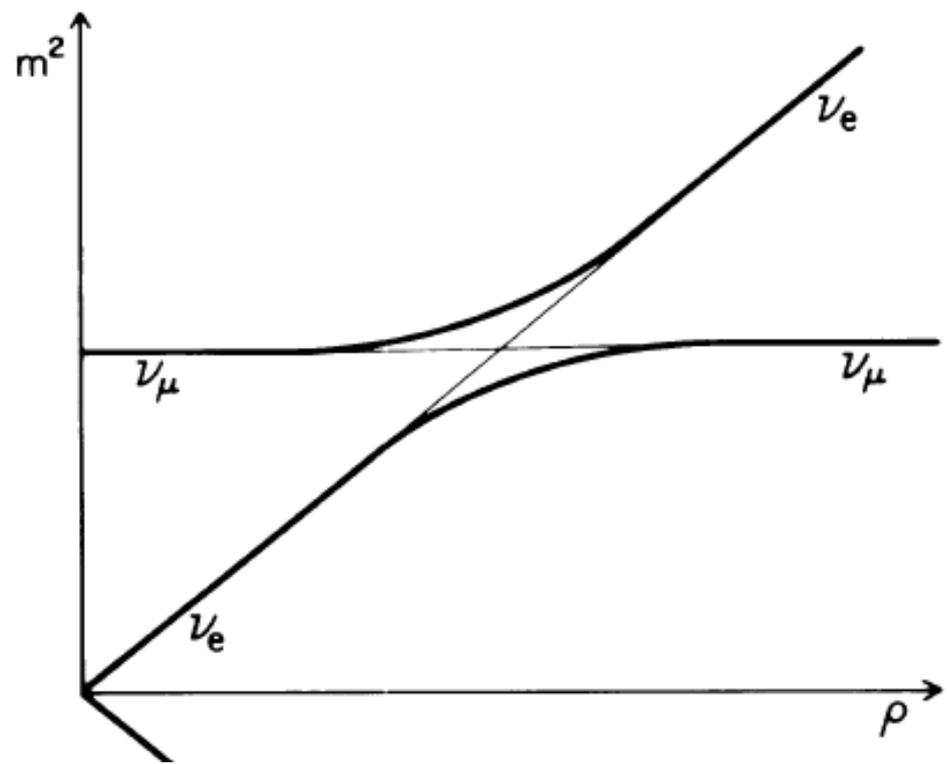


cf. ニュートリノ振動と準位交差問題

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \Psi_e \\ \Psi_\mu \end{pmatrix} = \left[ \begin{pmatrix} E + A(r) & 0 \\ 0 & E \end{pmatrix} + a \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \right] \begin{pmatrix} \Psi_e \\ \Psi_\mu \end{pmatrix}$$

$$E = \frac{1}{2}(m_1^2 + m_2^2), \quad a = \frac{1}{2}(m_2^2 - m_1^2)$$

電子ニュートリノと物質中の電子との相互作用



物質中で共鳴的にニュートリノ振動が起こる = MSW 効果

Ref.  
H.A. Bethe, PRL56('86)1305,  
W.C. Haxton, PRL57('86)1271