

原子核基礎論B

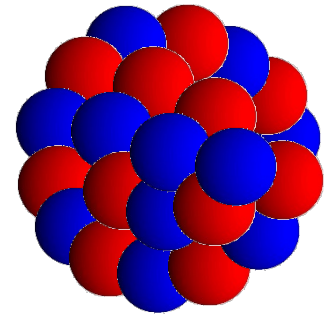
原子核理論研究室(物2) 萩野浩一

シラバス

1. 原子核の集団運動とその微視的理解(3コマ)
2. 原子核反応論基礎(1コマ)
3. 核融合反応(1コマ)
4. 核分裂:現象論と微視的理論(1コマ)
5. ニホニウムと超重元素の物理(1コマ)
6. 高温・高密度核物質外観(1コマ)
7. 高エネルギー重イオン衝突(2コマ)
8. 有限温度・密度における場の理論入門(2コマ)
9. QCD有効模型における相転移と相図(1コマ)
10. 有限温度・密度格子QCDと符号問題(1コマ)

萩野

大西



シラバス(基礎論A)

5. 殻模型と魔法数(1コマ)
6. 平均場理論と核変形(3コマ)
7. 中性子過剰核の物理(2コマ)
8. 非束縛核と共鳴散乱理論(1コマ)

主に基底状態

→ 基礎論Bで

シラバス(基礎論B)

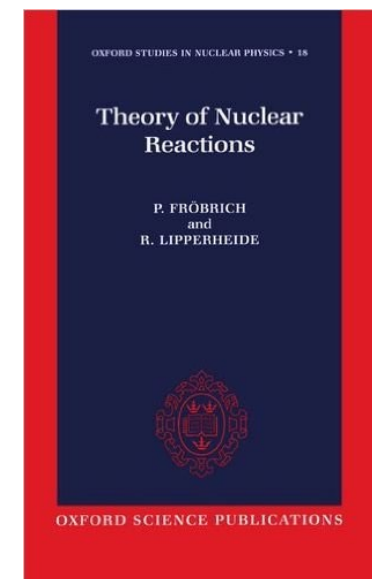
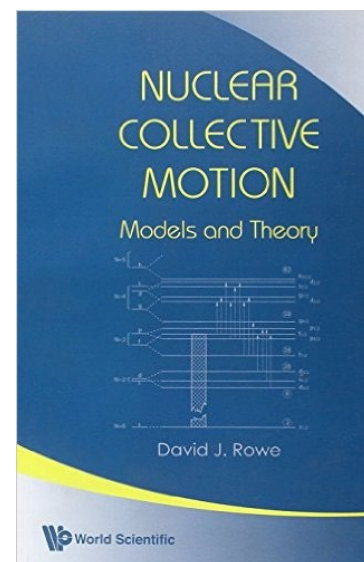
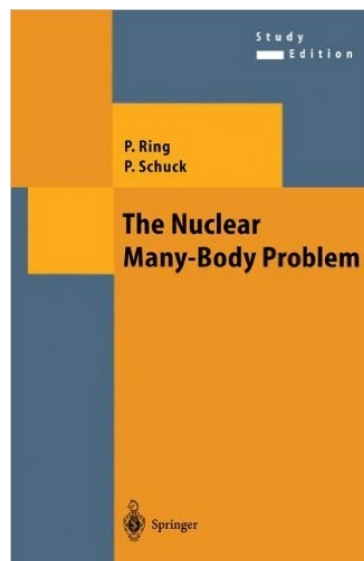
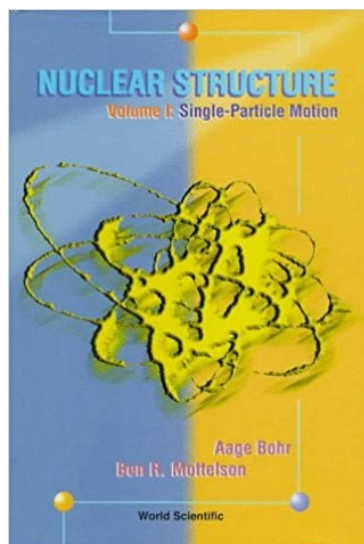
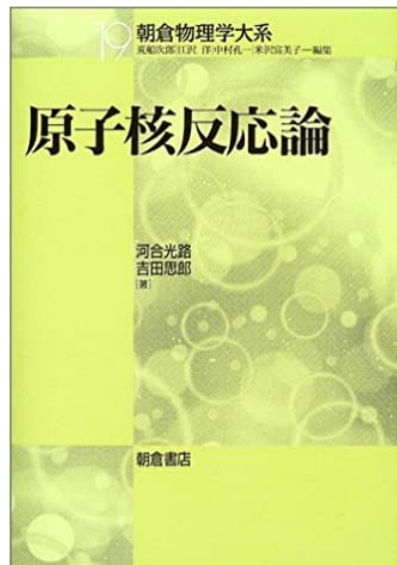
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主に励起状態
と核反応

成績： レポート80%、出席など20%

質問： 適宜聞いて下さい。
チャットに書き込んでもOK。

参考書



Bohr-Mottelson

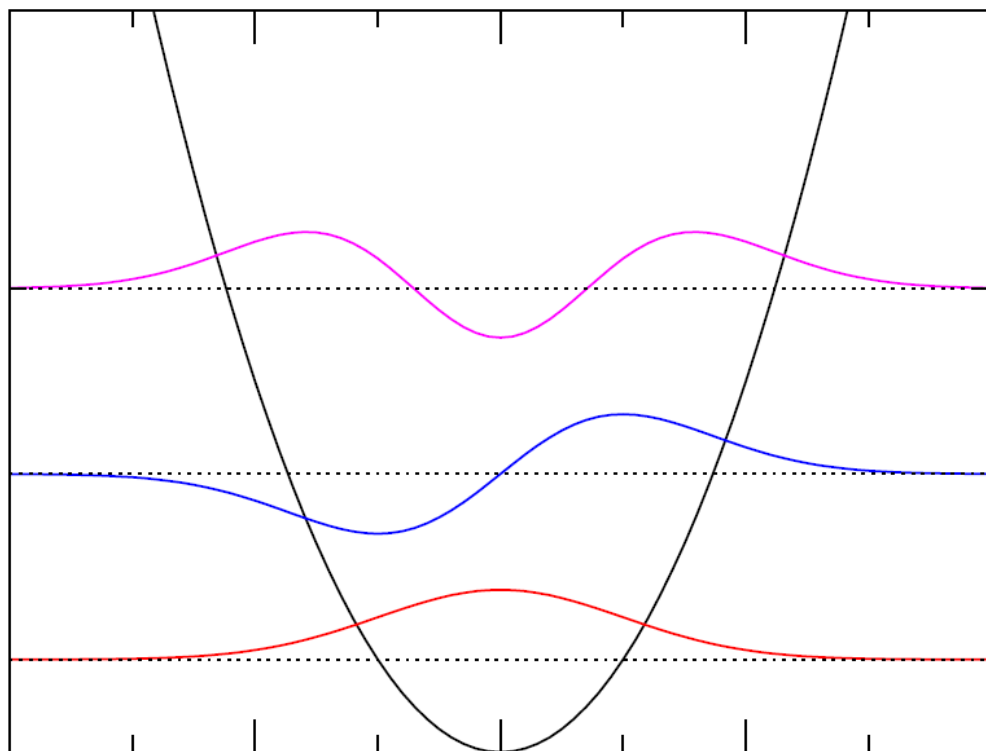
Ring-Schuck

Rowe

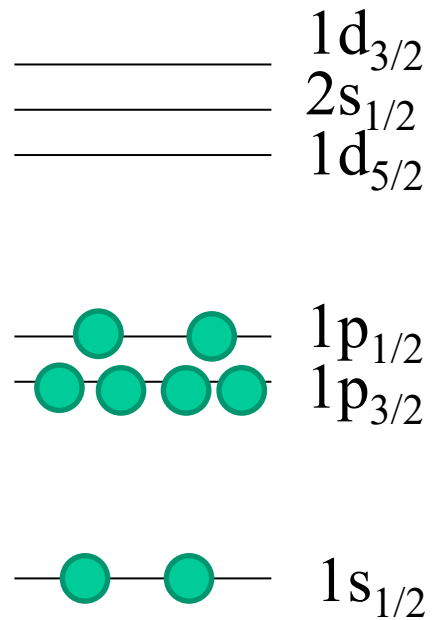
Frobrich
-Lipperheide

励起状態

ポテンシャル中の1粒子の場合

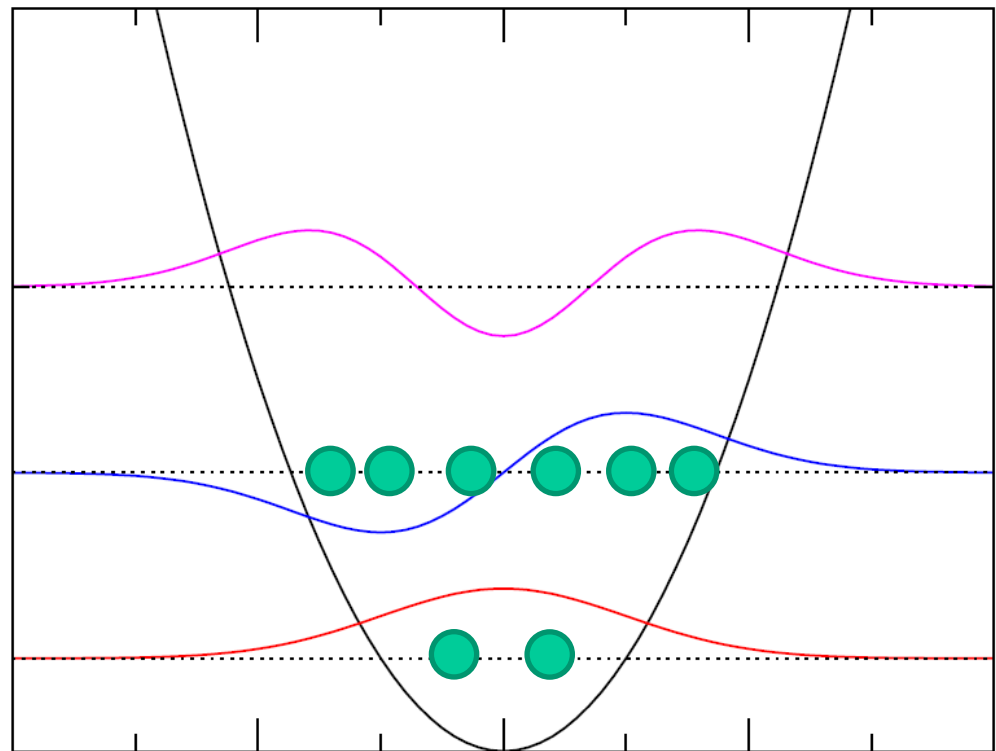


原子核の励起状態



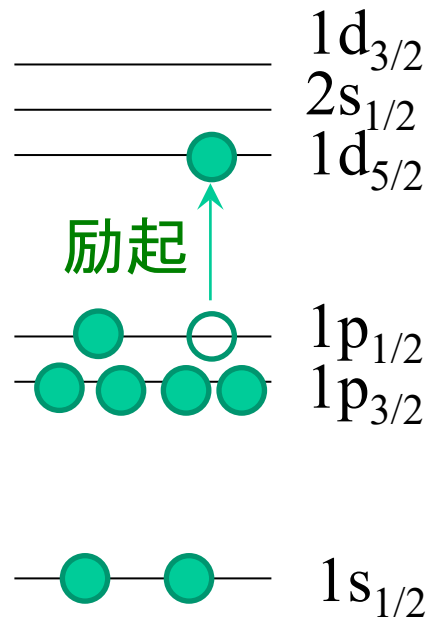
基底状態

多体系の場合

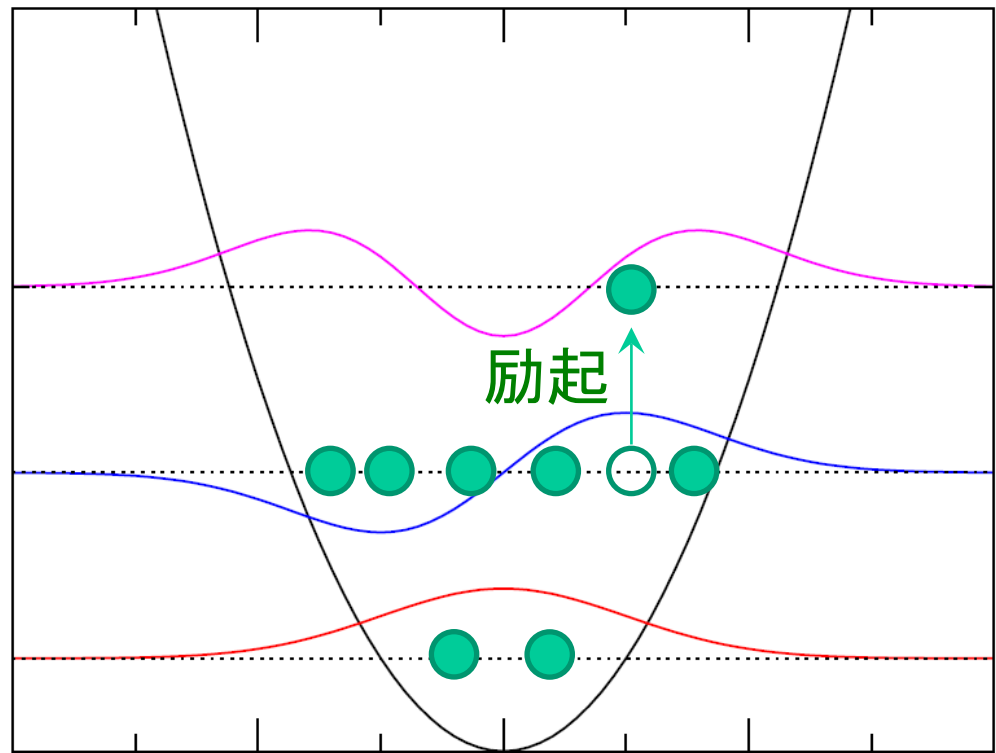


基底状態

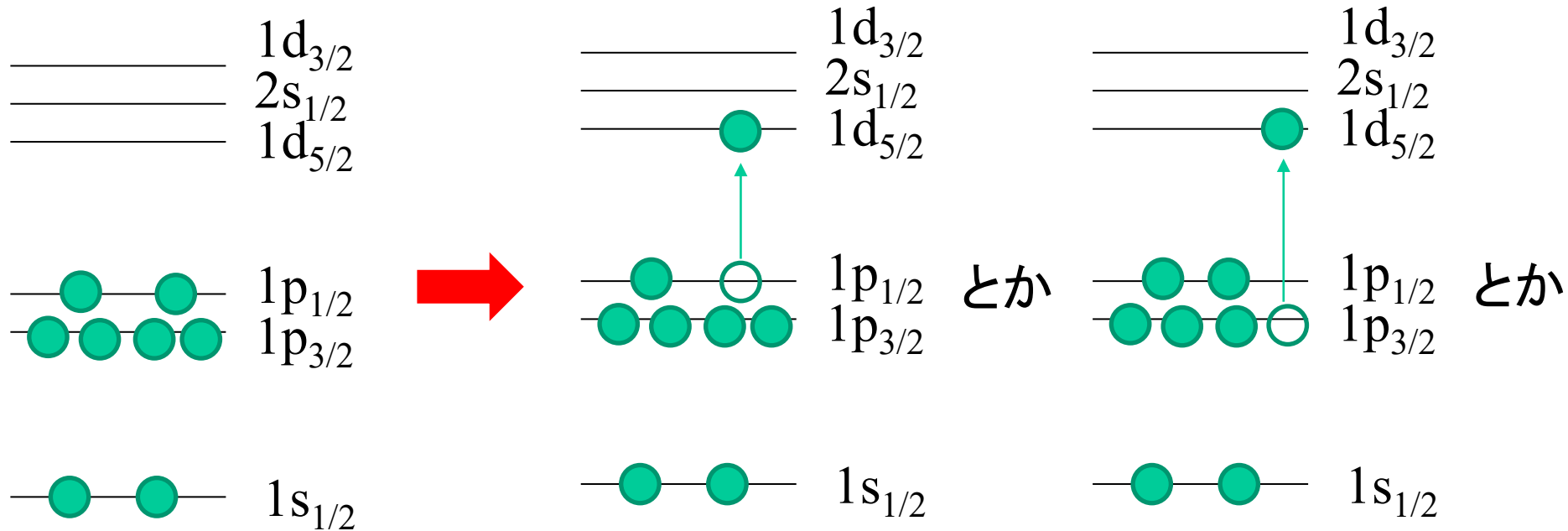
原子核の励起状態



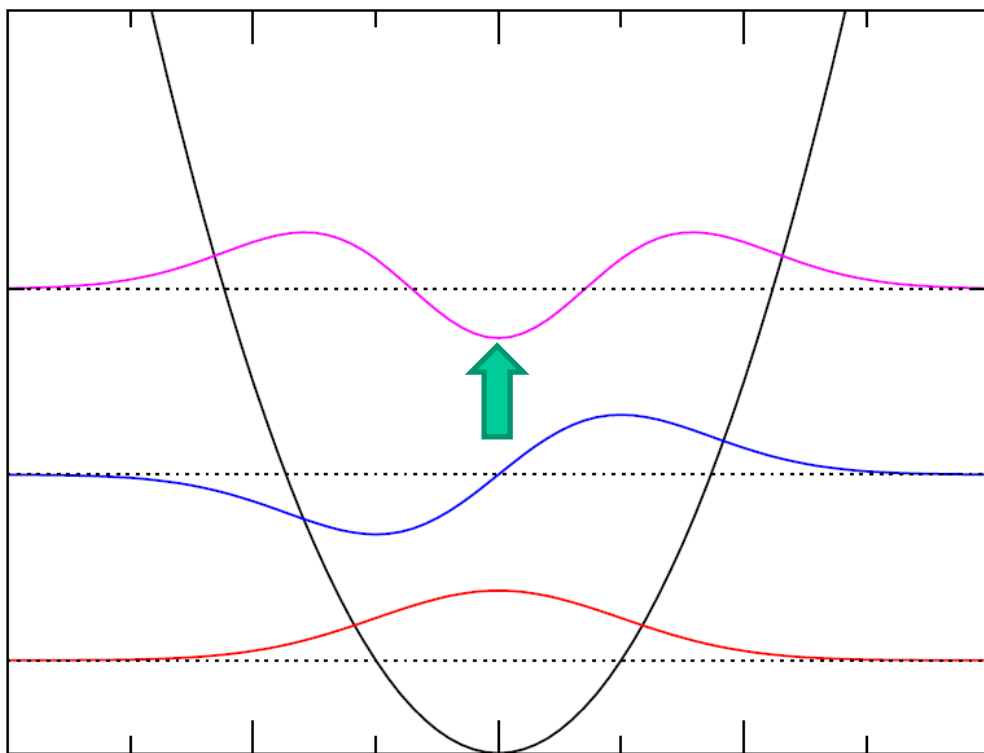
多体系の場合



原子核の励起状態



基底状態



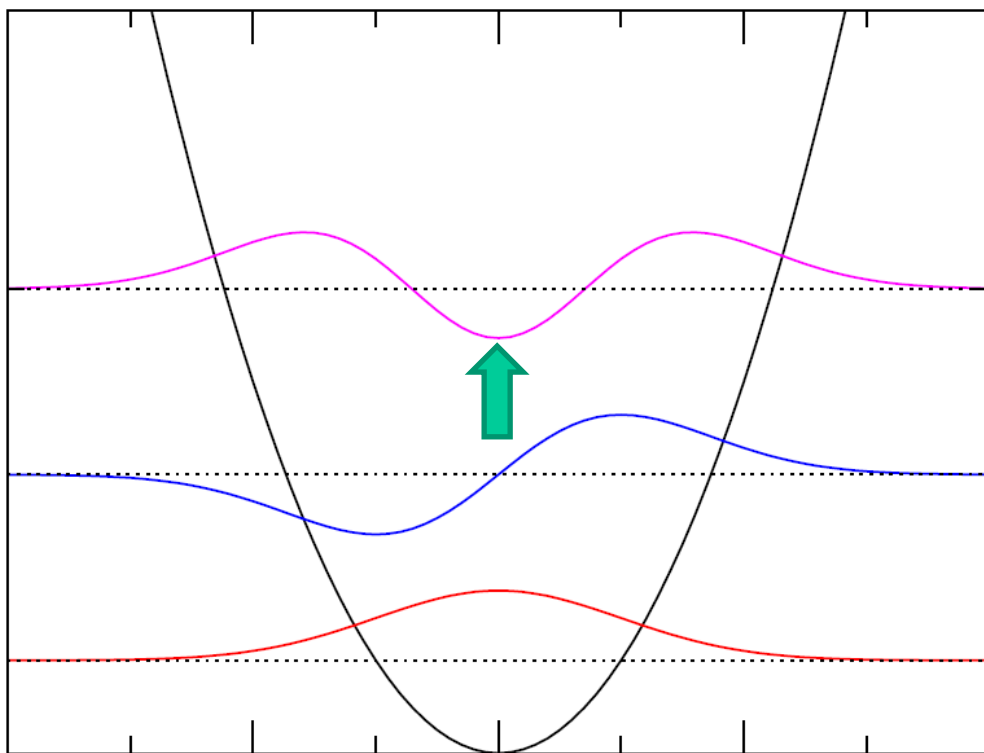
原子核では

$$\hbar\omega \sim 41 A^{-1/3} \quad (\text{MeV})$$

$$\leftarrow R \sim 1.2 A^{1/3} \quad (\text{fm})$$

$A = 16$ だと 16.27 MeV

cf. 実際に、 ^{16}O の 16.2 MeV
に 1^- 状態



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cf. 実際に、 ^{16}O の16.2 MeV
に 1- 状態

.....でも実際にはこのようには理解できない励起状態
も多数存在する(集団励起)

Giant Dipole Resonance (GDR) 巨大双極子共鳴

光吸収の
スペクトル

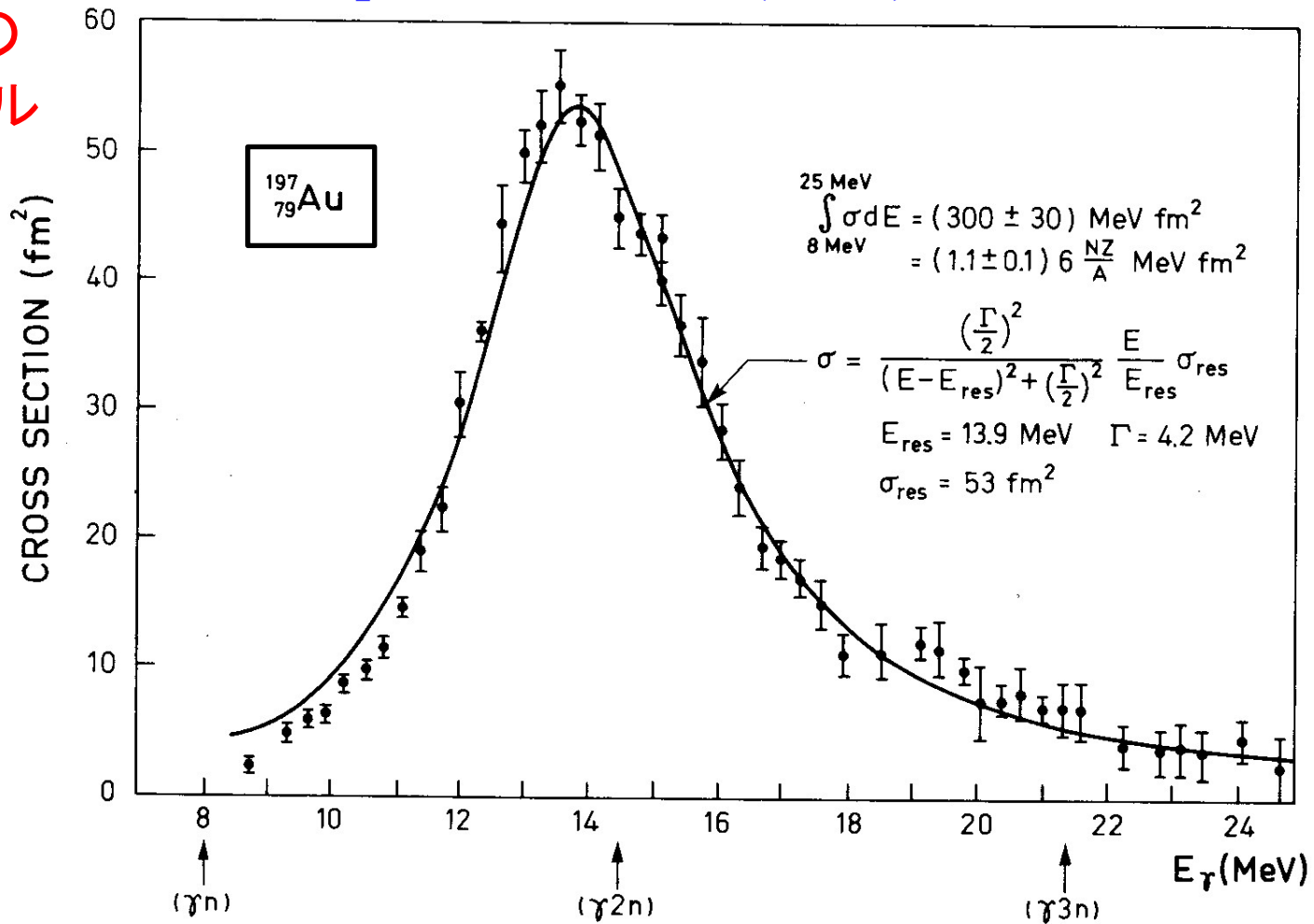


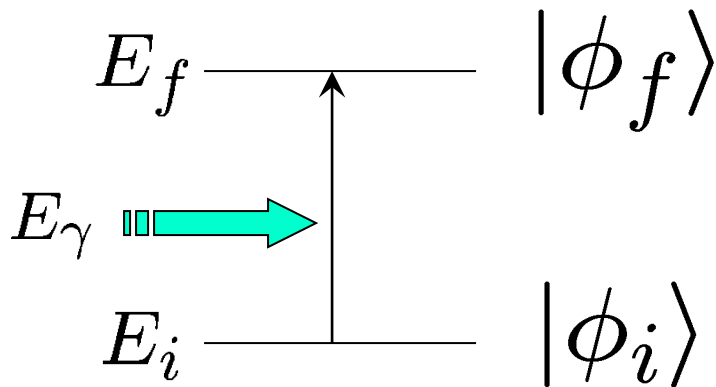
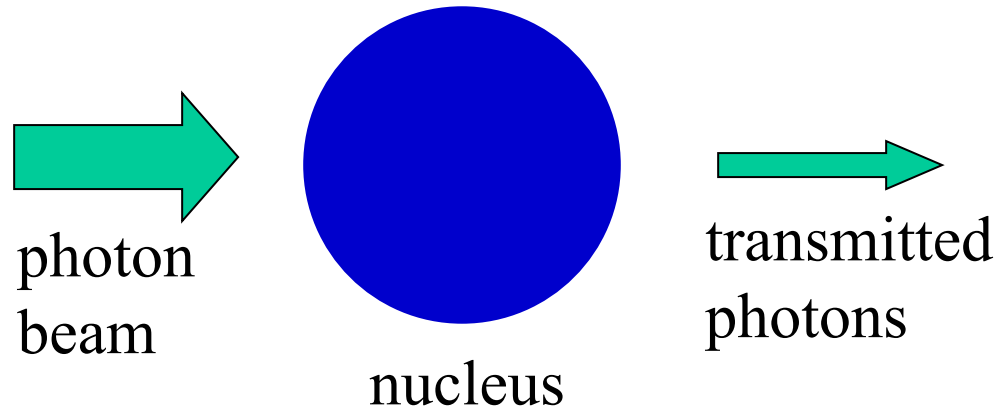
Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

$$\text{cf. } 41 \times 197^{-1/3} = 7.05 \text{ MeV}$$

Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section



The state is strongly excited when
 $E_f - E_i = E_\gamma$.

Giant Dipole Resonance (GDR) 巨大双極子共鳴

光吸収の
スペクトル

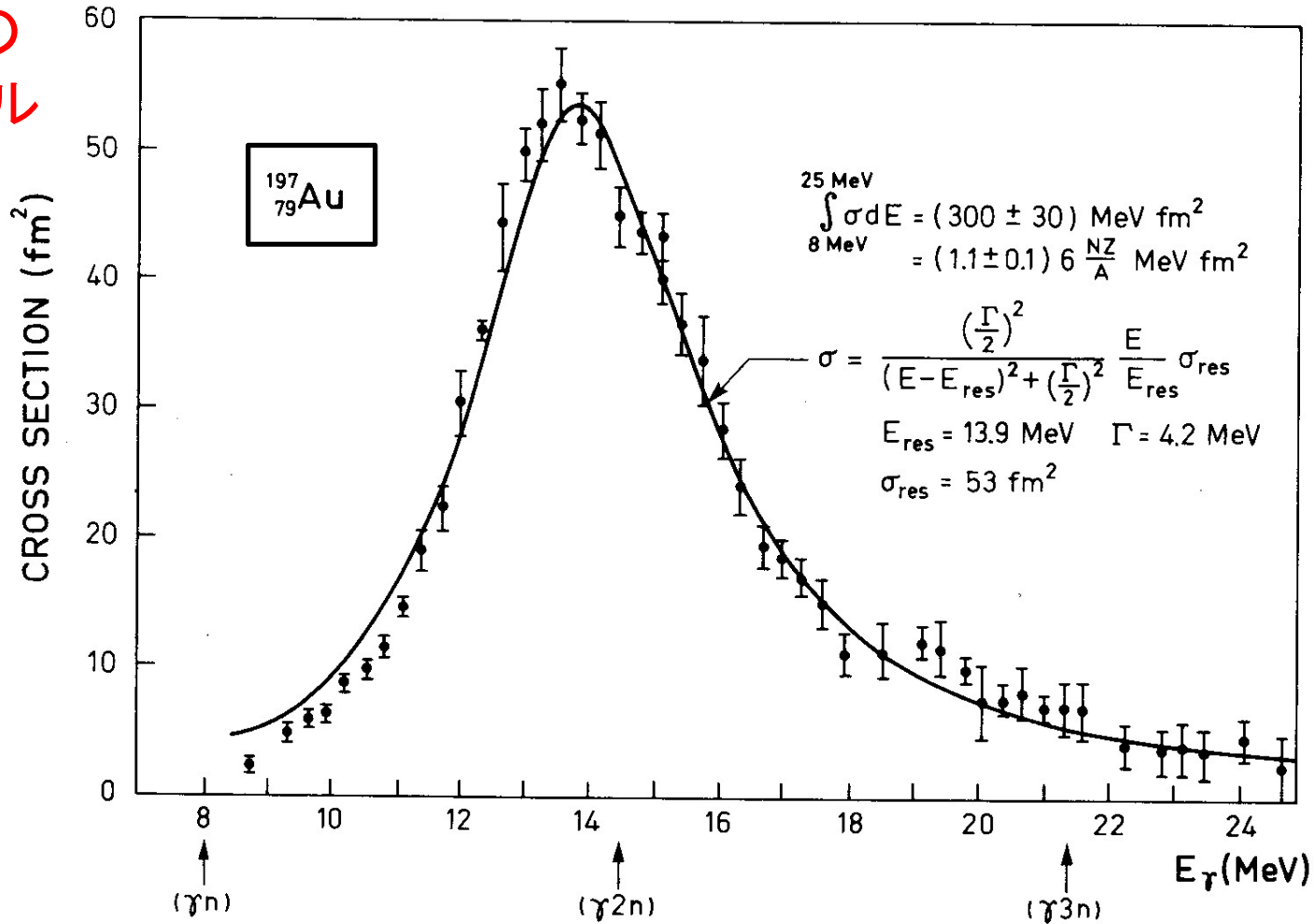


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Remarks

i) Photon interaction \longleftrightarrow dipole excitation

$$H_{\text{int}} = \frac{1}{2m} \frac{e}{c} (\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p})$$

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\alpha=1,2} \sqrt{\frac{2\pi c^2 \hbar}{\omega V}} (a_{\mathbf{k}\alpha} \boldsymbol{\epsilon}_{\alpha} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}} t} + h.c.)$$

$$e^{i\mathbf{k} \cdot \mathbf{r}} \sim 1 \quad (\text{dipole approximation})$$

Remarks

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$$E_\gamma = 10 \text{ MeV}, R = 5 \text{ fm}$$

だと $kR \sim 0.25$

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$$\sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_\gamma - E_f + E_i)$$

$$\tilde{z} = \sum_p (z_p - Z_{\text{cm}})$$

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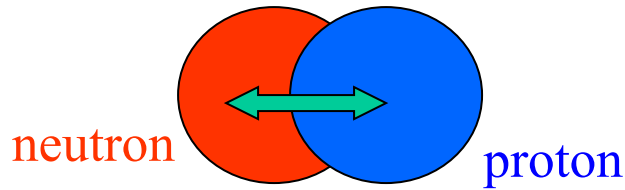
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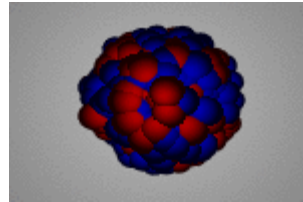
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ii) Isospin

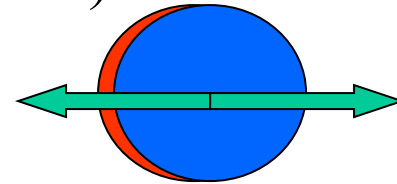


Isovector type



(note)

$$\tilde{z} = \sum_p (z_p - Z_{cm})$$



Isoscalar dipole motion

\longleftrightarrow c.m. motion (to the first order)

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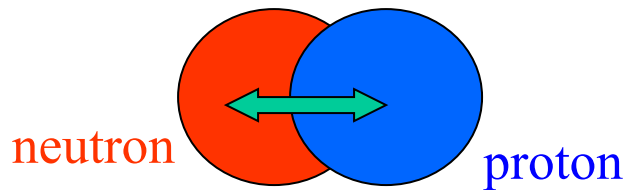
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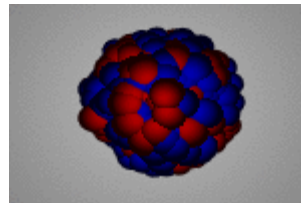
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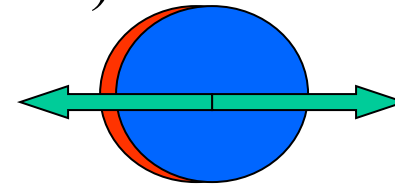


Isovector type



(note)

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Isoscalar dipole motion

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iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

Giant Dipole Resonance (GDR) 巨大双極子共鳴

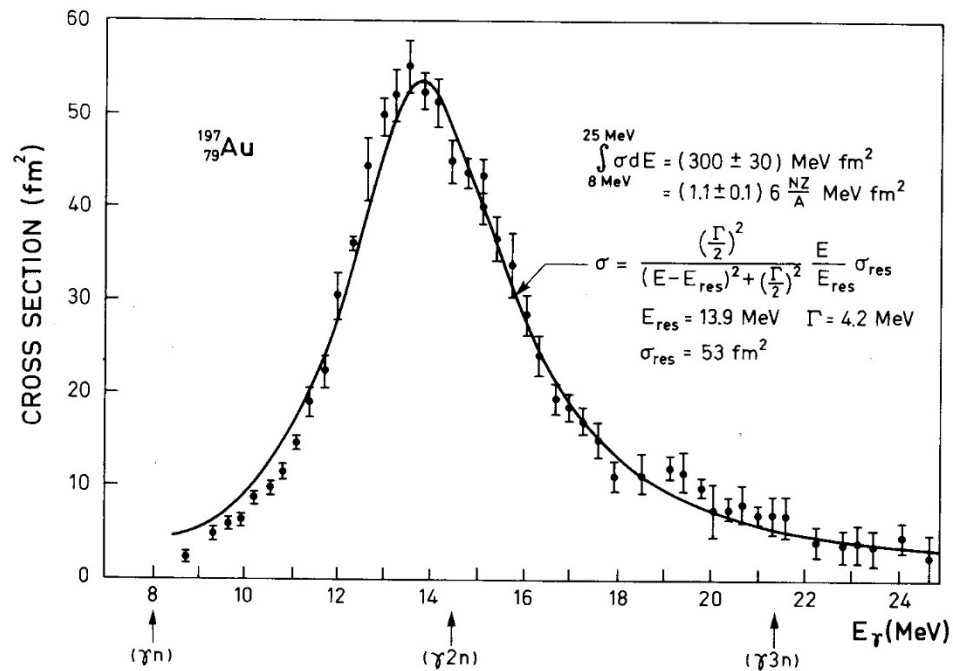
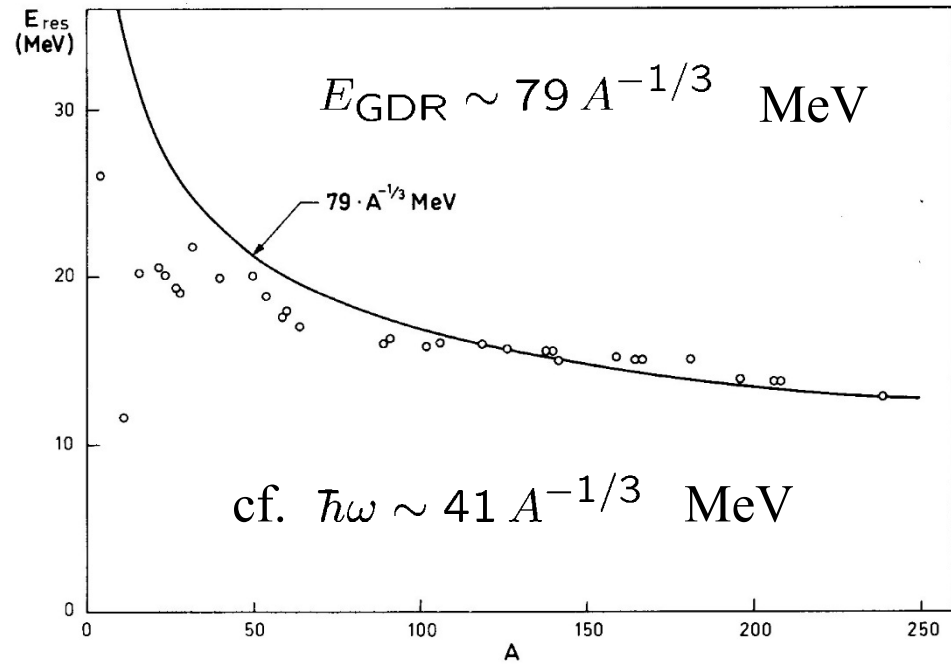
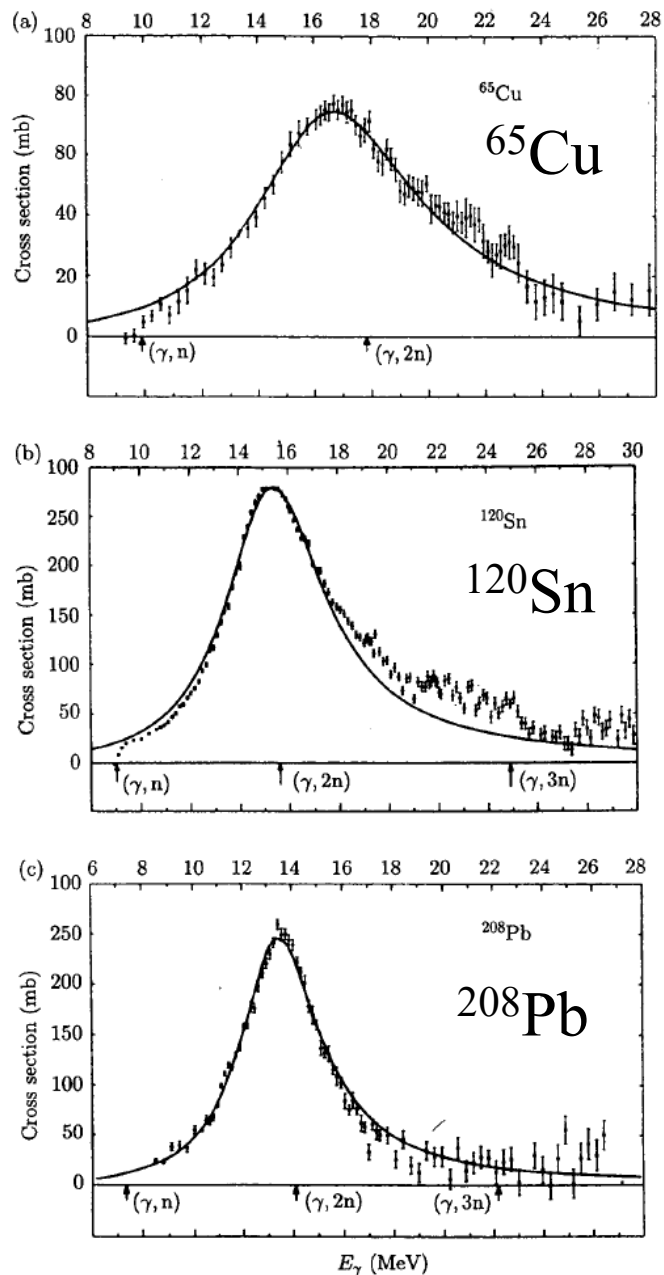


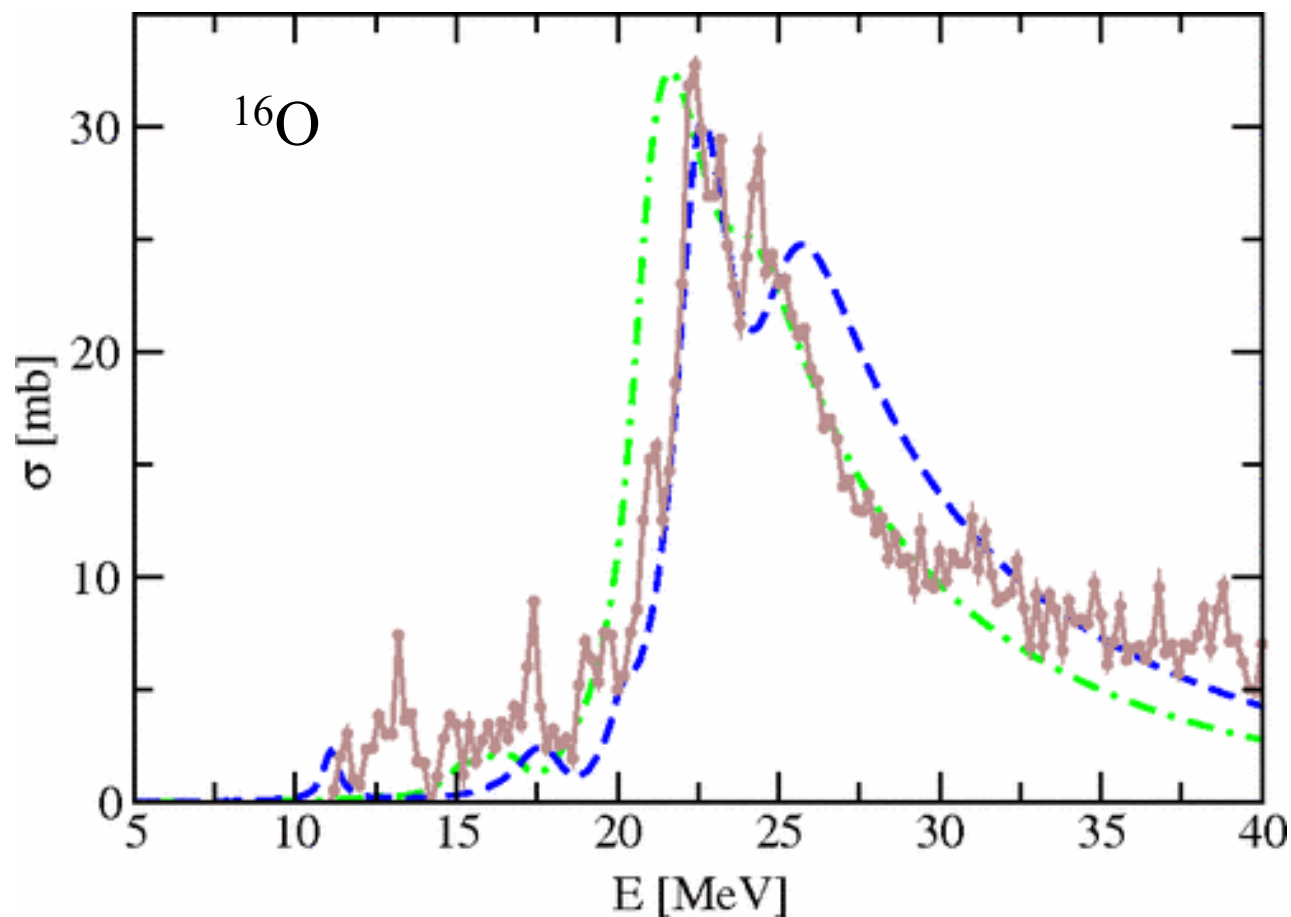
Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



Bohr-Mottelson
 “Nuclear Structure vol. II”

M.N. Harakeh and A. van der Woude,
 “Giant Resonances”

FIG. 1.2. The photo-neutron cross section $\sigma(\gamma, n)$ as a function of the photon energy for the three nuclei ^{208}Pb , ^{120}Sn and ^{65}Cu . Note that for these nuclei $\sigma(\gamma, n) \approx \sigma_{\text{abs}}(\gamma)$. From reference (BER75).

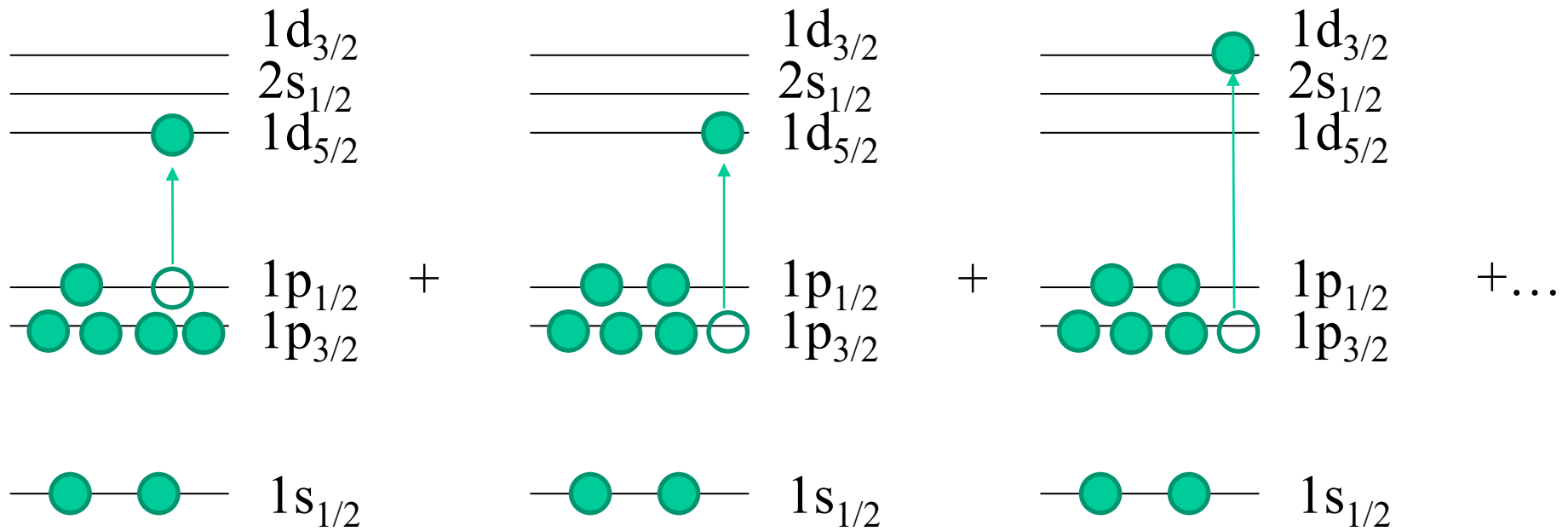


実験データ: 茶色

N. Lyutorovich et al., Phys. Rev. Lett. 109 (2012) 092502

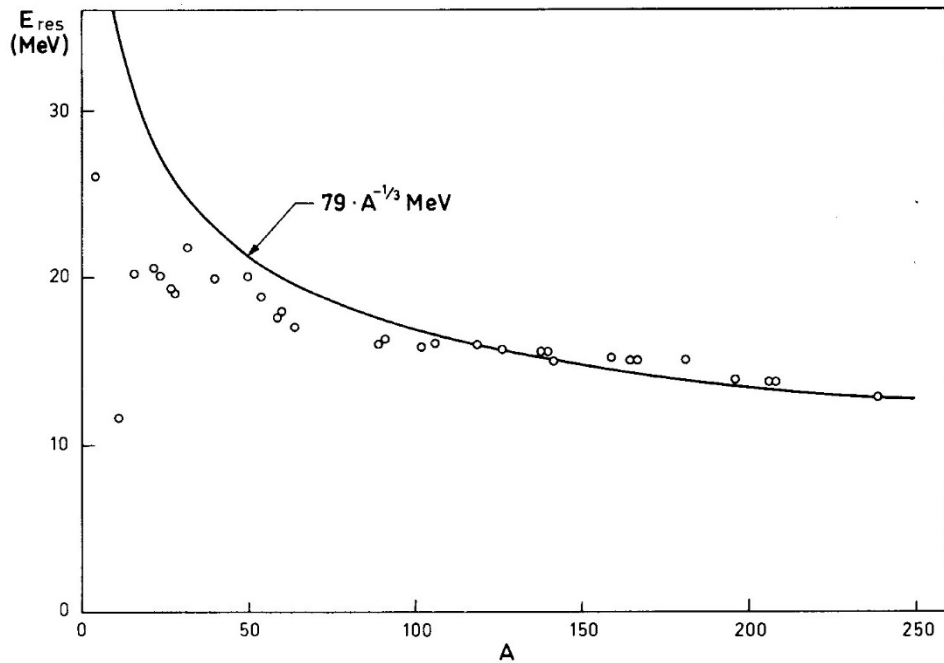
cf. $41 \times 16^{-1/3} = 16.27 \text{ MeV}$

何故励起エネルギーが大きくなるのか？

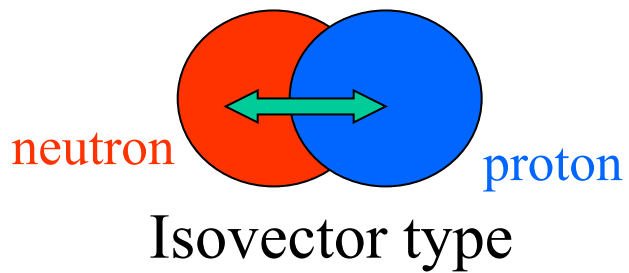


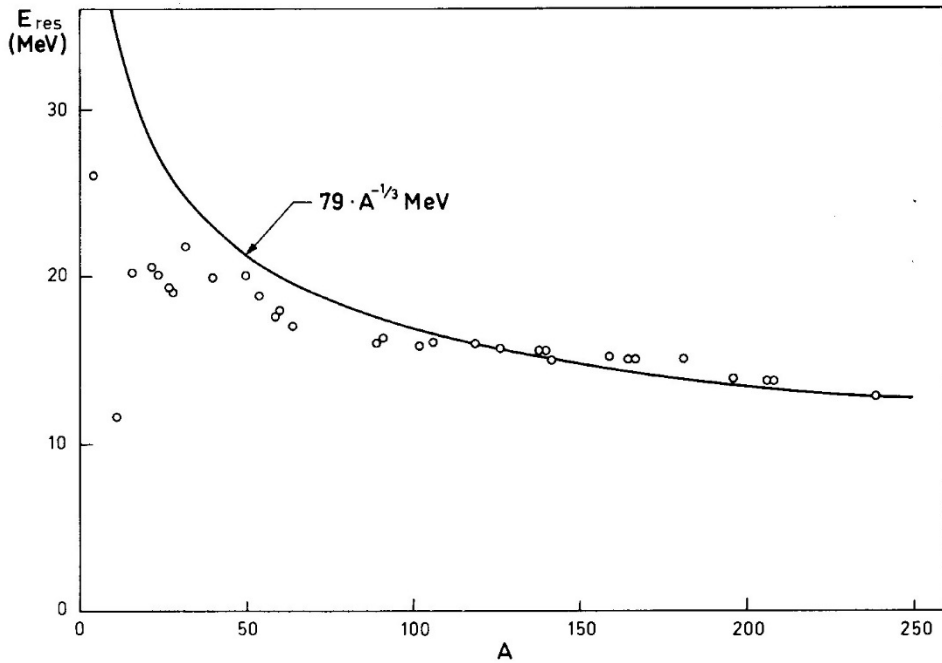
様々な励起状態がコヒーレントに重ね合わさることにより
「集団的」になる。→(次回もう少し詳しく)

残留相互作用が大きな役割



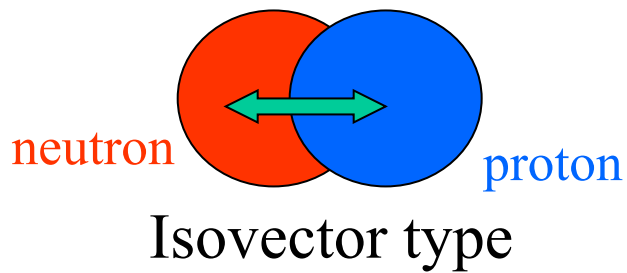
Bohr-Mottelson
“Nuclear Structure vol. II”



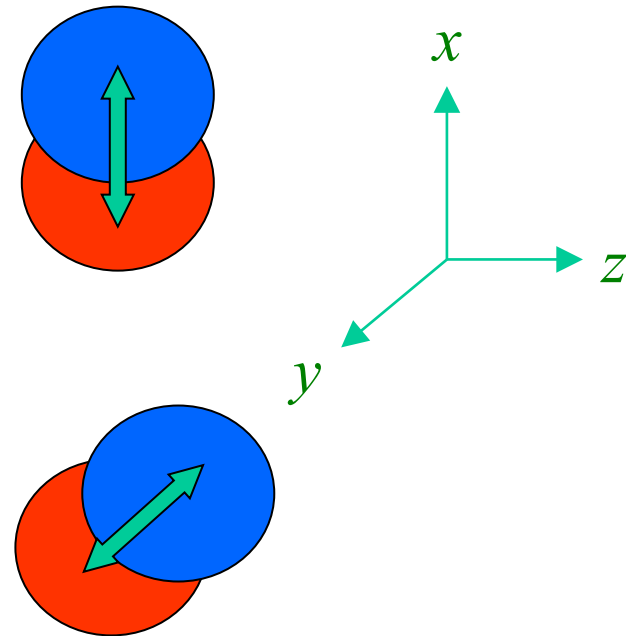


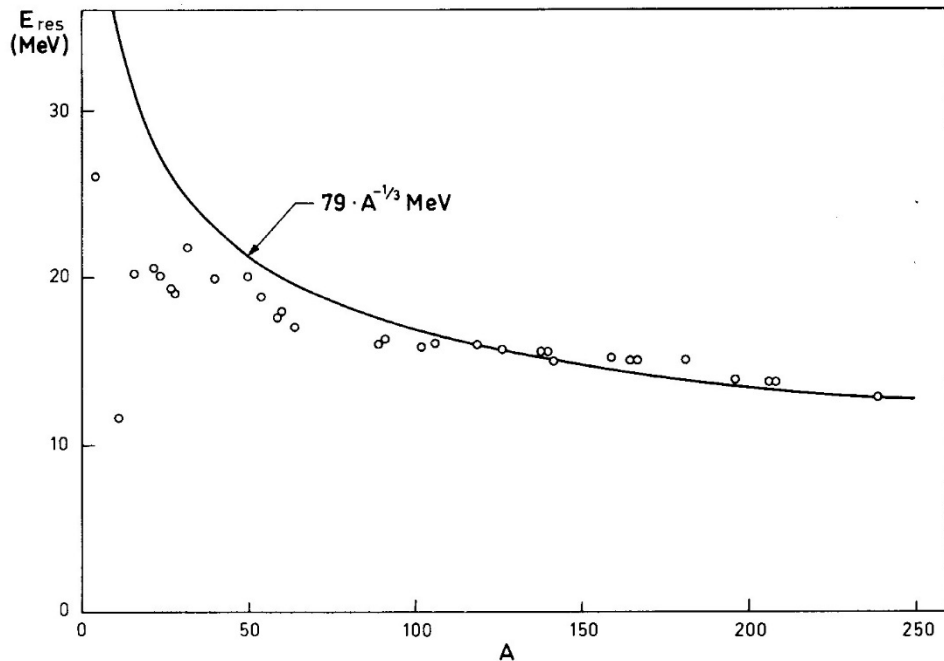
$$E_{GDR} \propto A^{-1/3}$$

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3つのモード

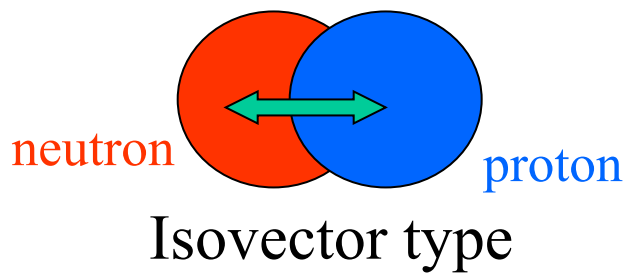




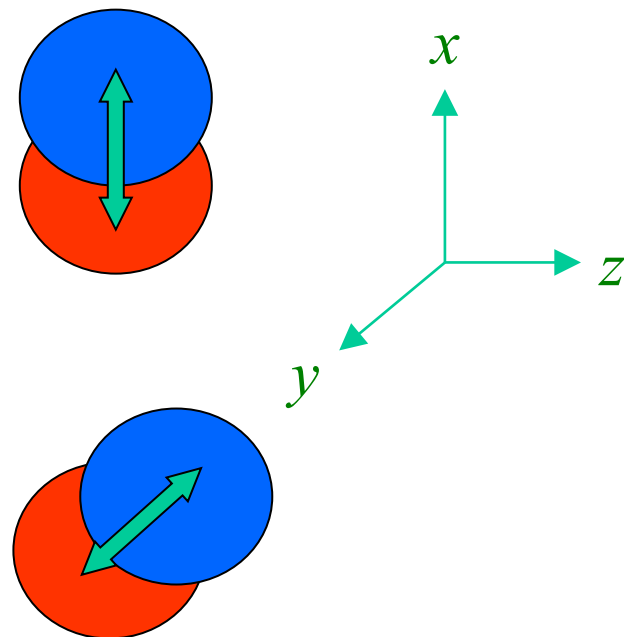
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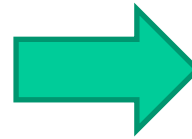
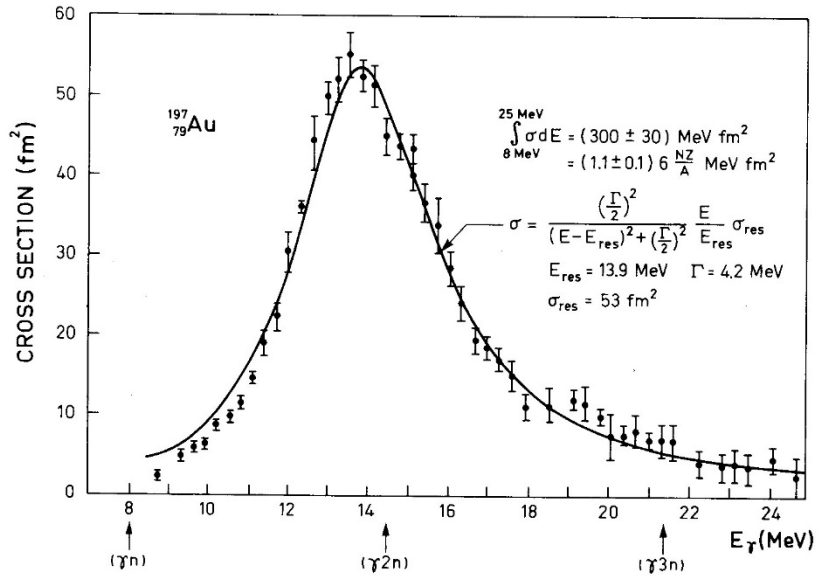
$$\propto 1/R$$

Bohr-Mottelson
 “Nuclear Structure vol. II”



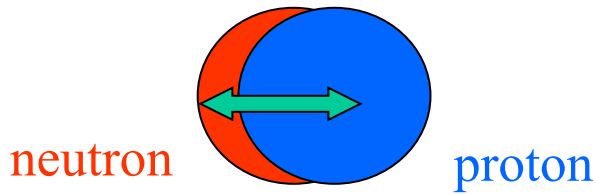
3つのモード





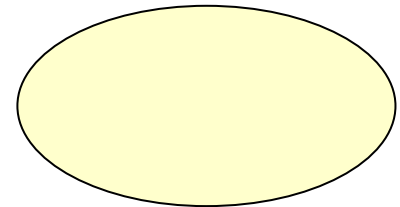
?

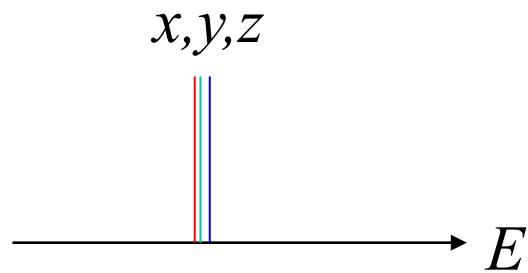
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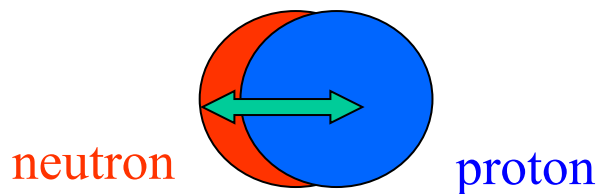
$$E_{\text{GDR}} \propto 1/R$$

deformed nucleus



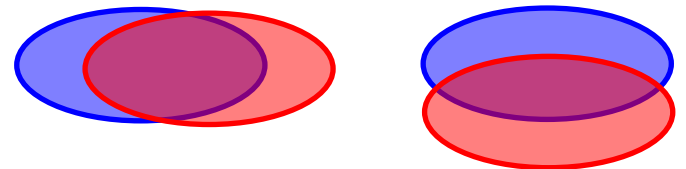
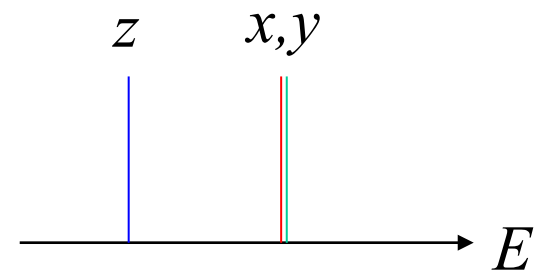
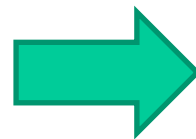


spherical nucleus

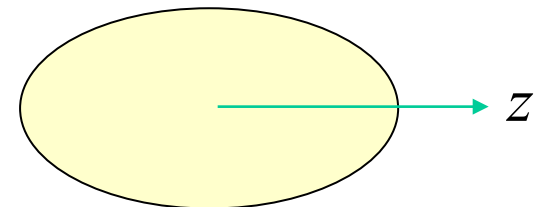


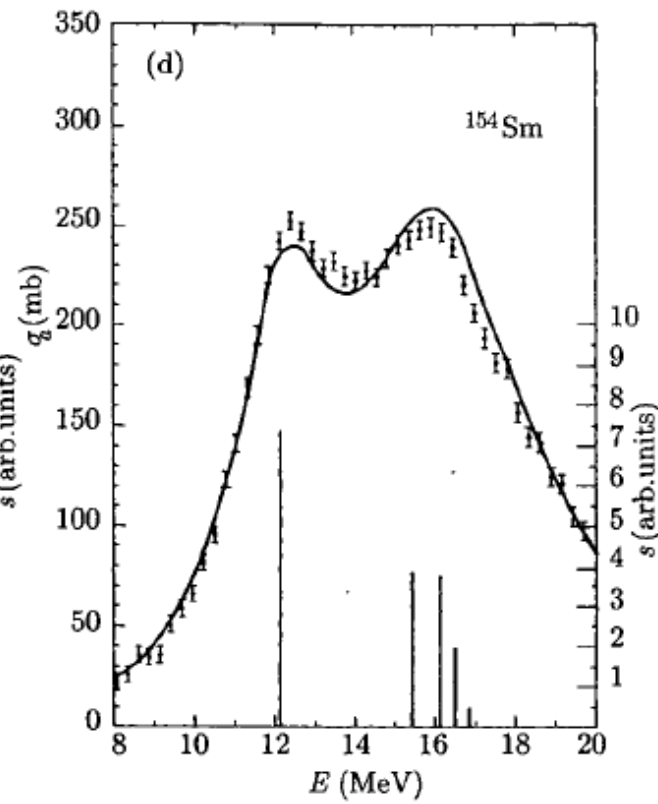
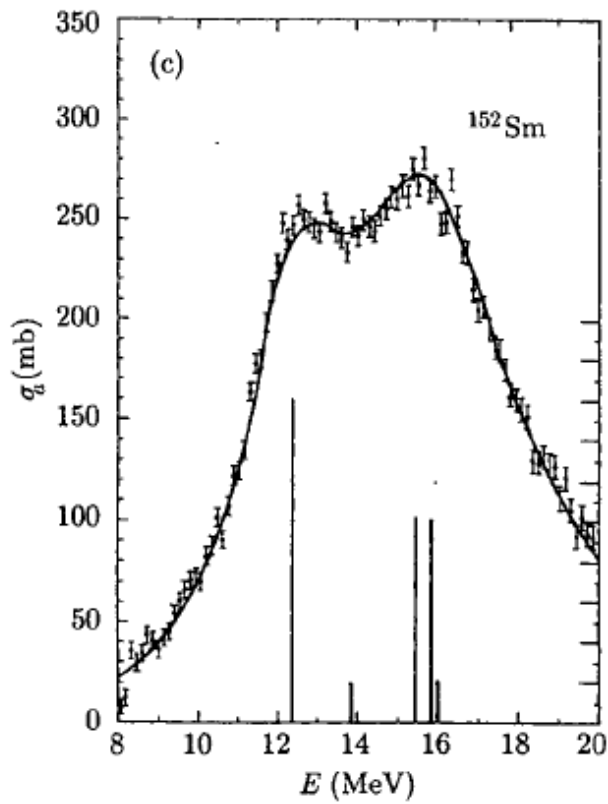
$$E_{\text{GDR}} \propto 1/R$$

(prolate deformation)



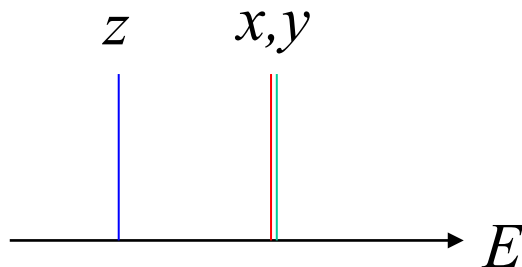
deformed nucleus



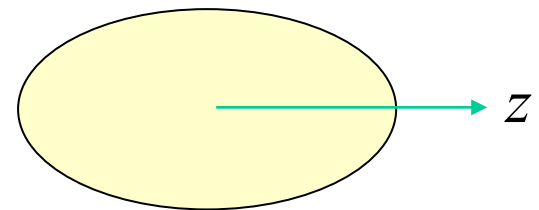


M.N. Harakeh and
A. van der Woude,
“Giant Resonances”

(prolate deformation)



deformed nucleus



Deformation effect

$$E_{\text{GDR}} \sim A^{-1/3} \sim 1/R$$

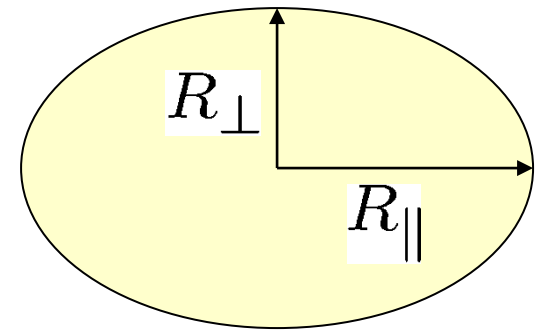
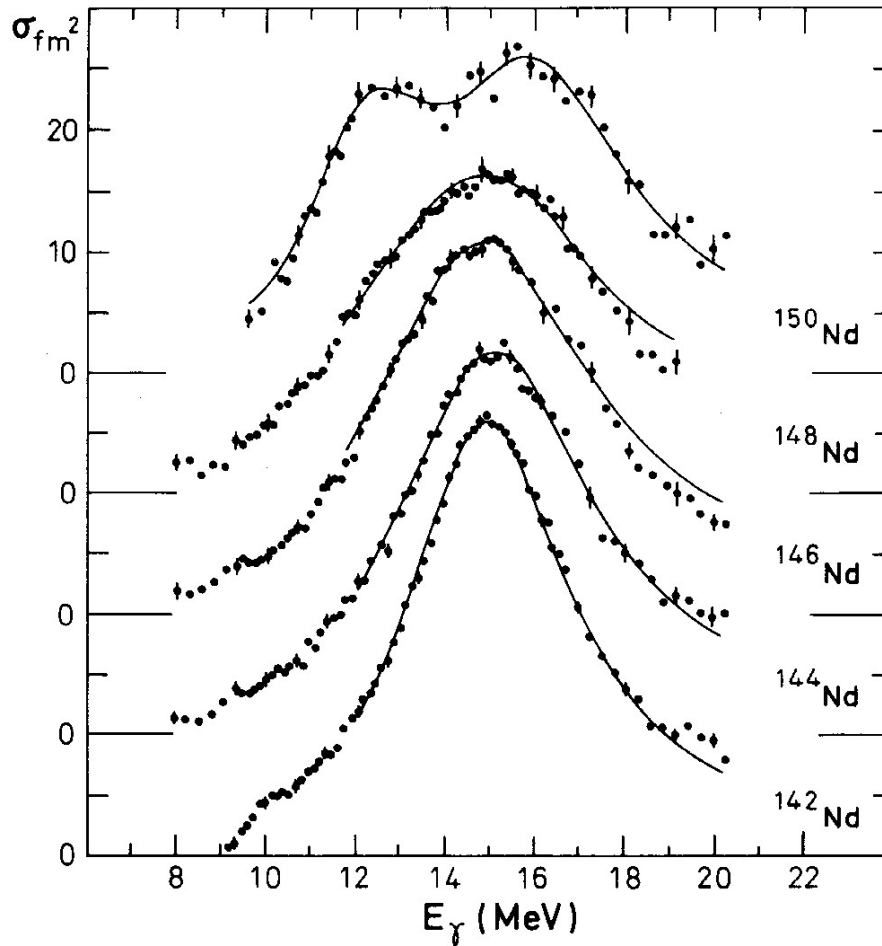
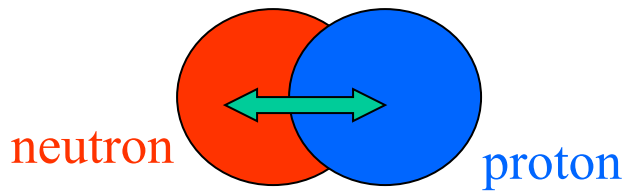


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys. A172*, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.

Giant Dipole Resonances

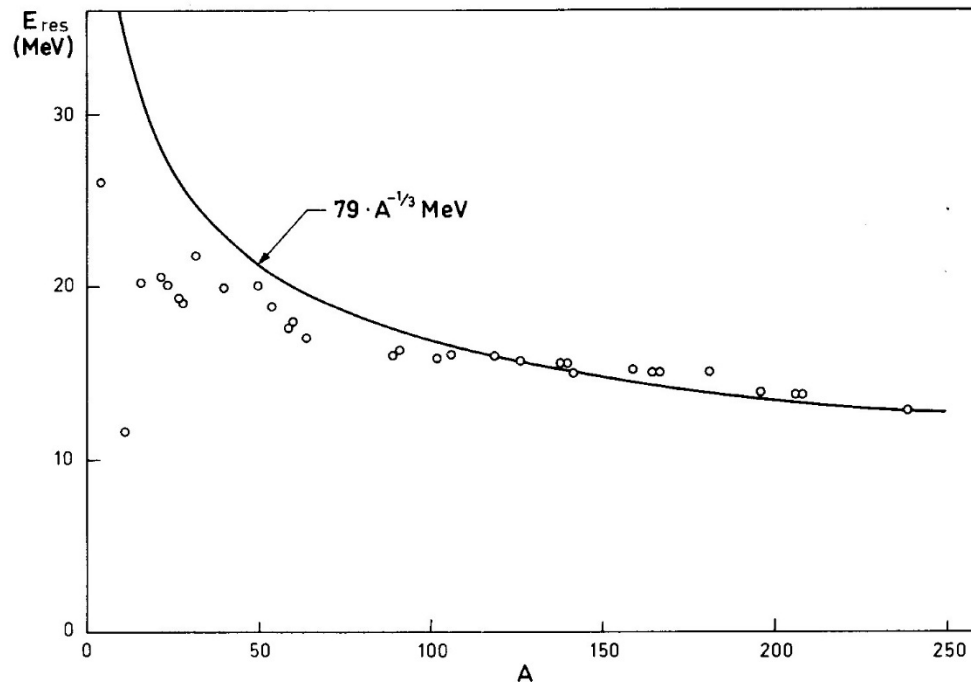
• Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

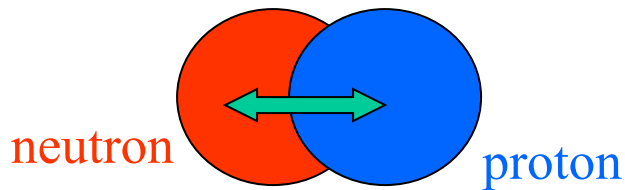
→ $\hbar\omega \sim A^{-1/6}$

→ Inconsistent with expt.
(except for light nuclei)



Giant Dipole Resonances

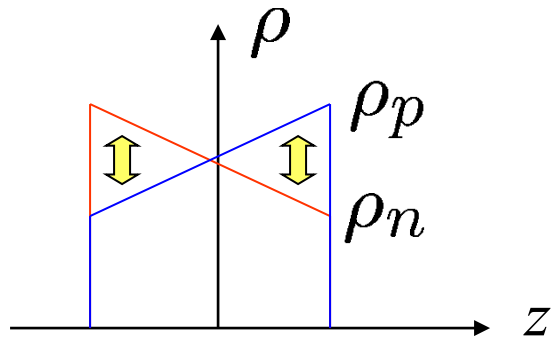
• Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/6}$$

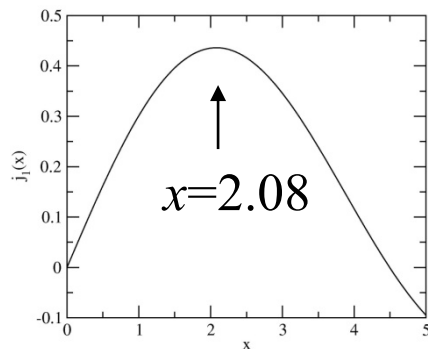
• Steinwedel-Jensen type



$$\hat{Q} = j_1(kr) Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/3}$$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x \cos x) / x^2$$

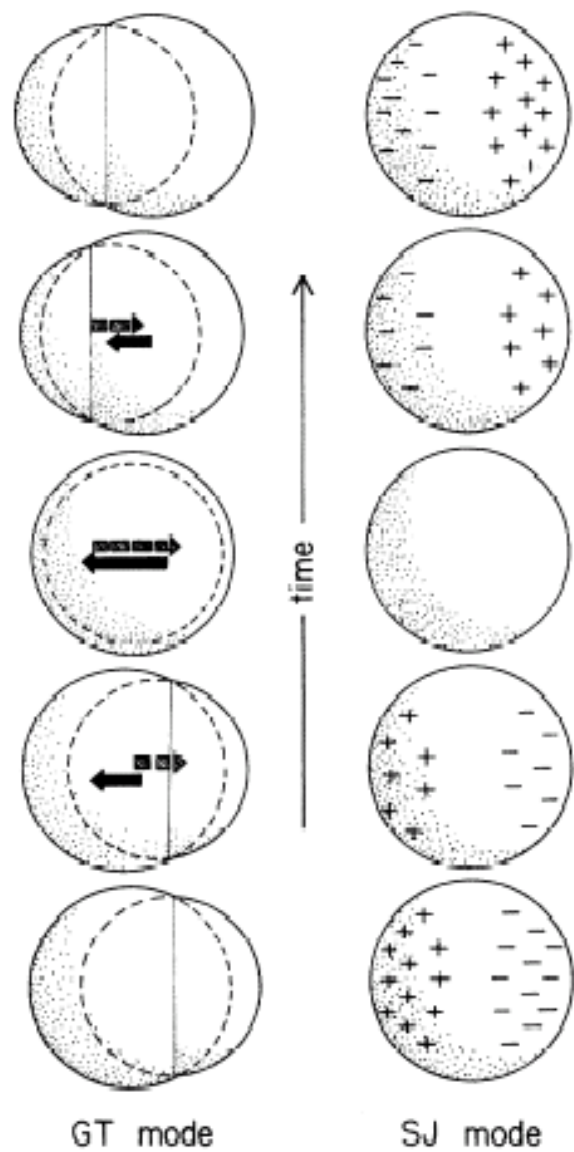


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.

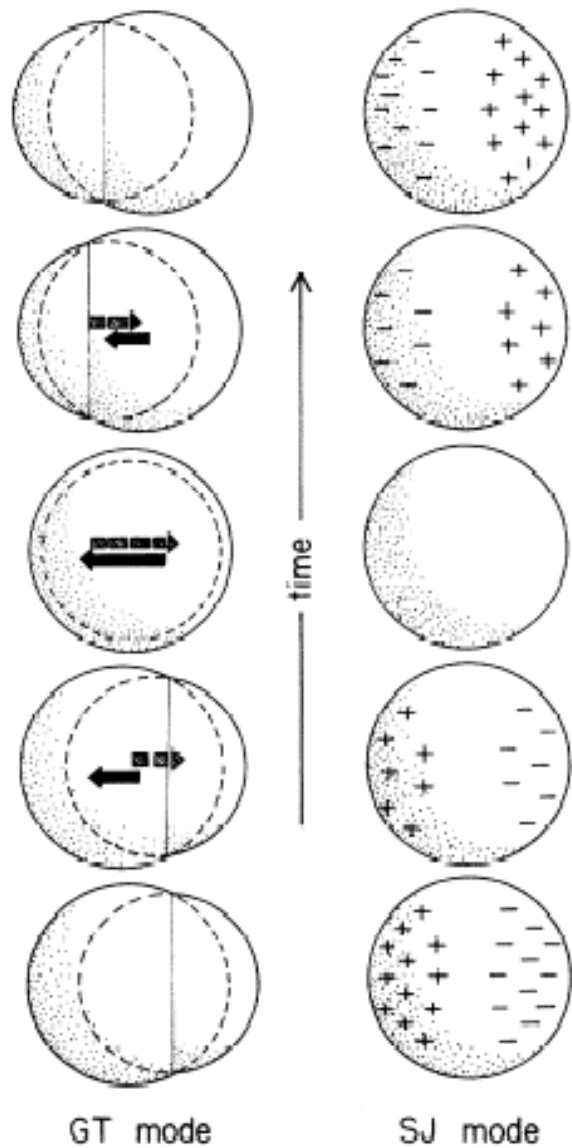
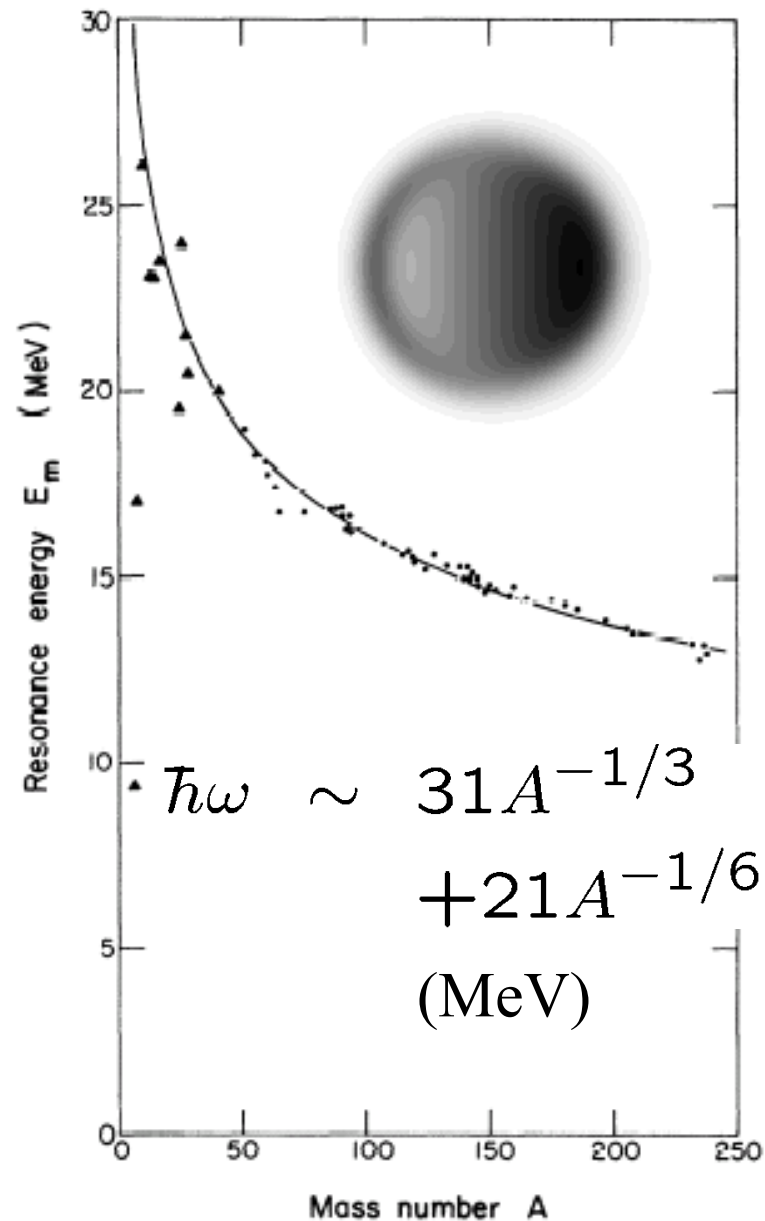
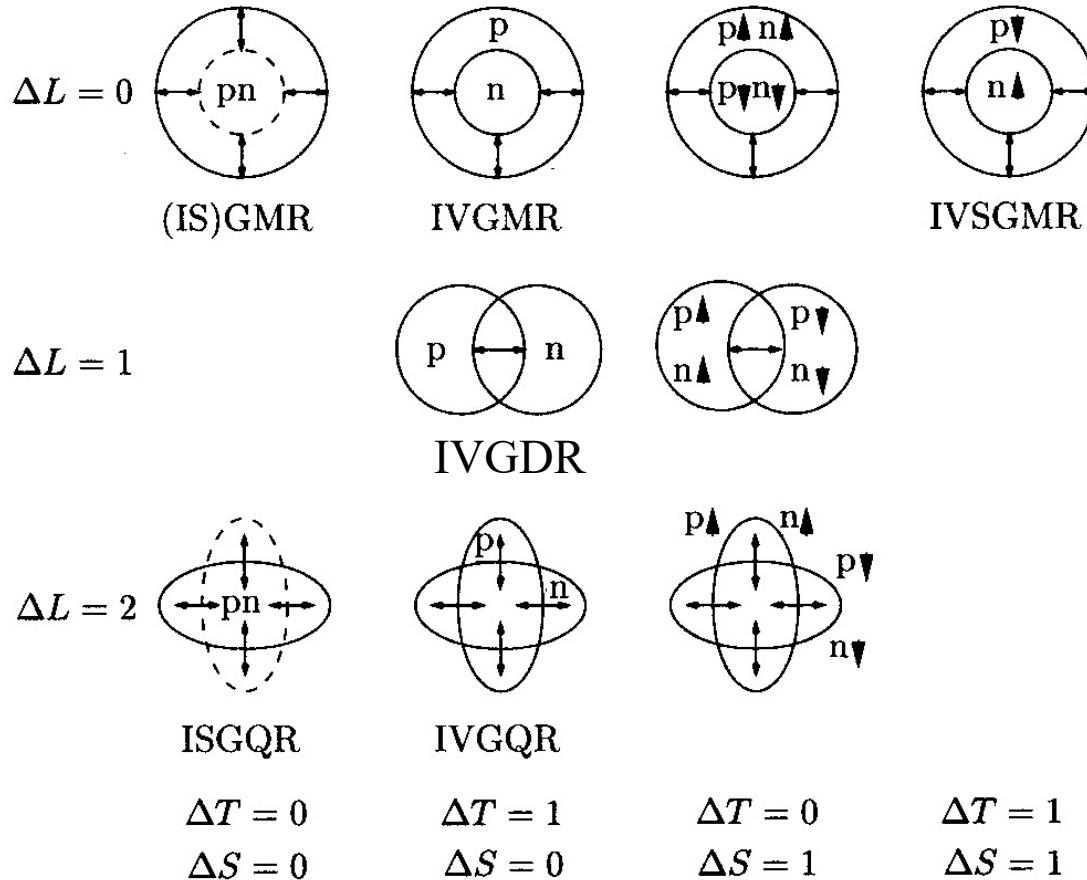


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.



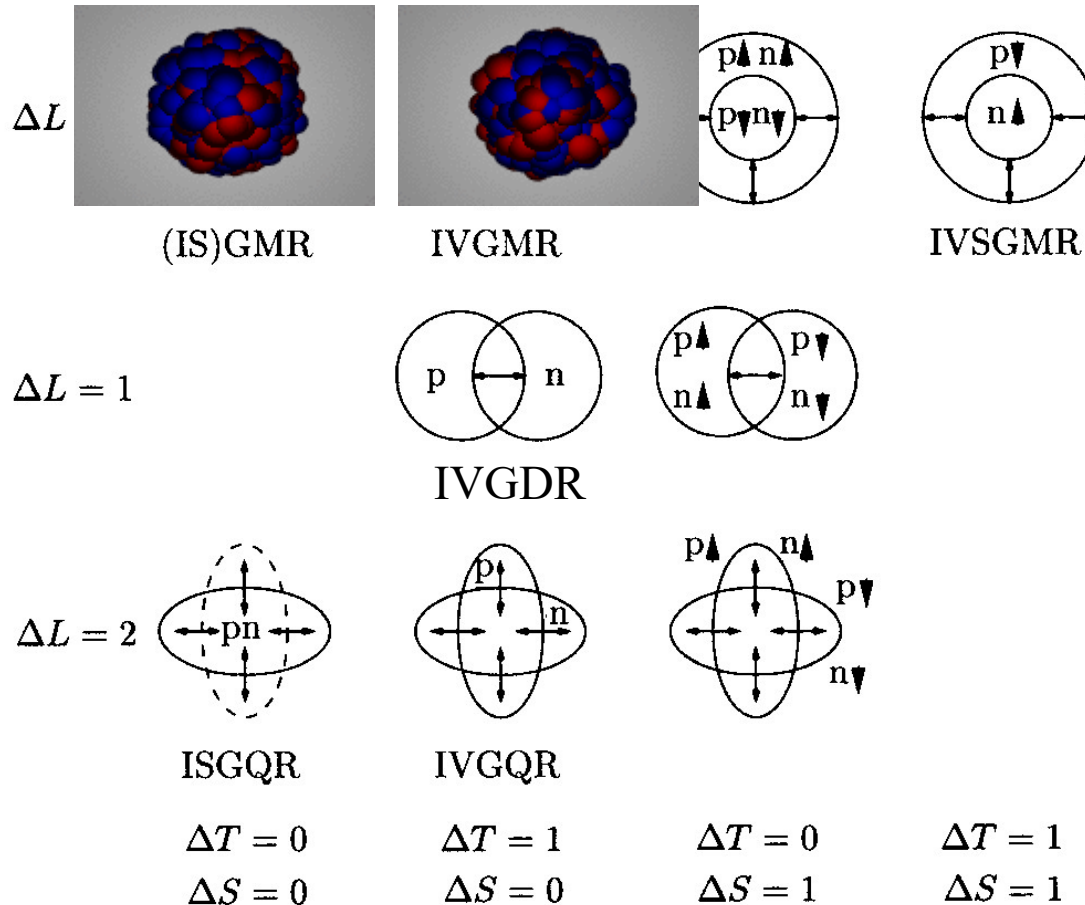
ii) Inelastic scattering

(e,e'), (p,p'), (α,α'), Heavy-ion \longrightarrow Higher multipolarities



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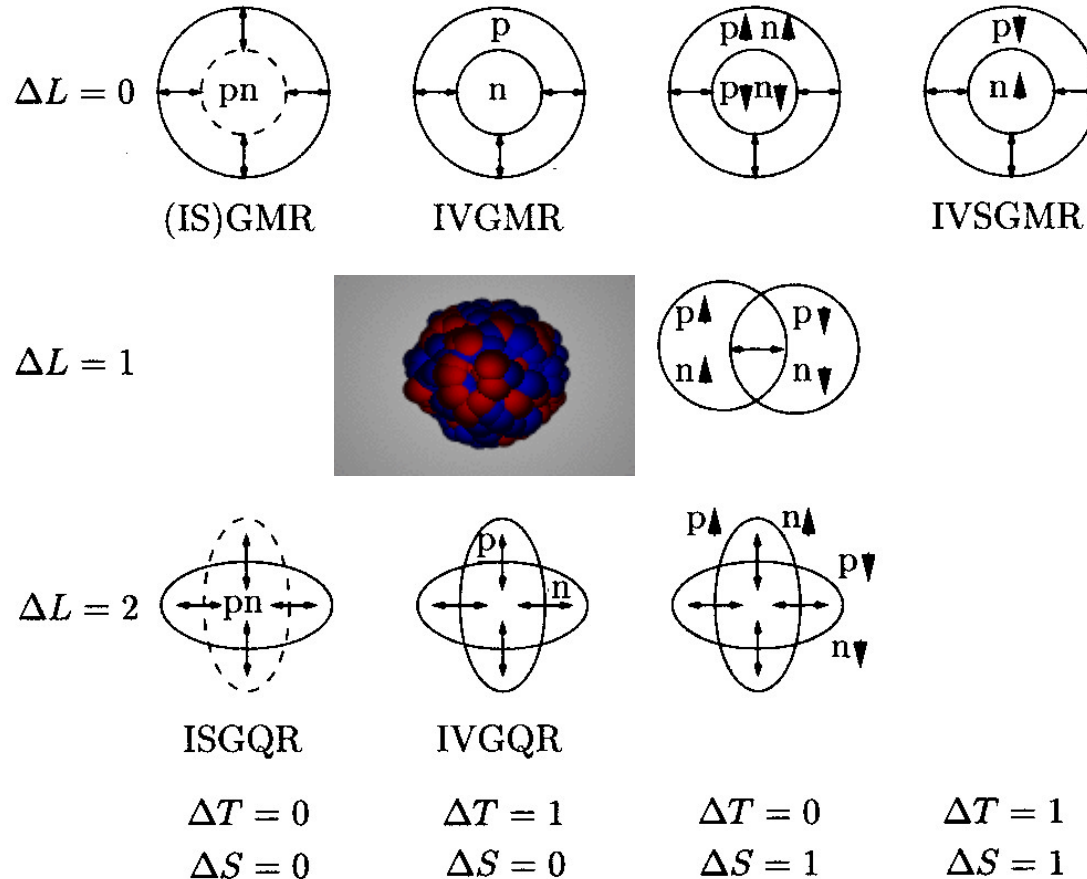


movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

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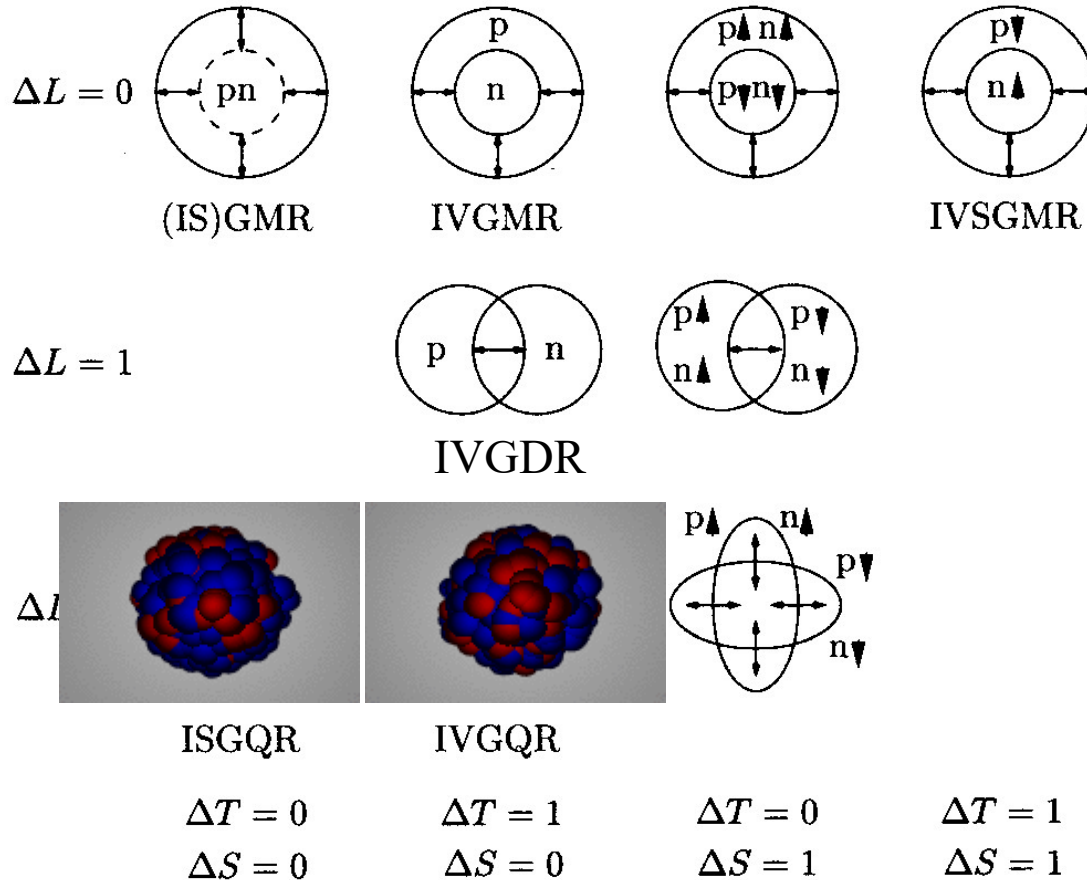


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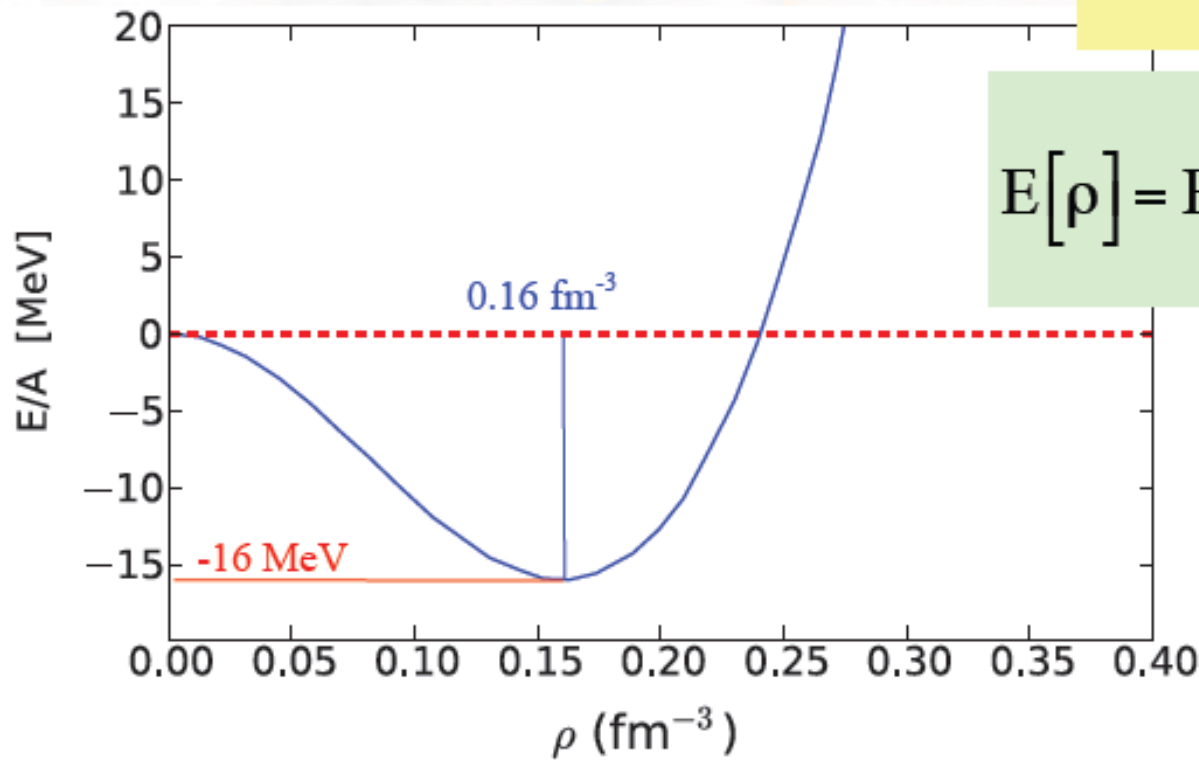
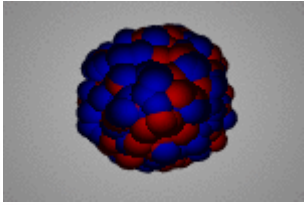
(e,e'), (p,p'), (α,α'), Heavy-ion \longrightarrow Higher multipolarities



(note) $\Delta L = 2 \longrightarrow \Delta N = 2$ Giant Resonance (GQR)

$\Delta N = 0$ Low-lying state

EOS of infinite nuclear matter

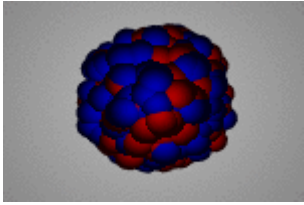


$$K_{\infty} = 9\rho^2 \left. \frac{d^2[E(\rho)/\rho]}{d\rho^2} \right|_{\rho_0}$$

$$E[\rho] = E[\rho_0] + \frac{1}{18} K_{\infty} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2$$

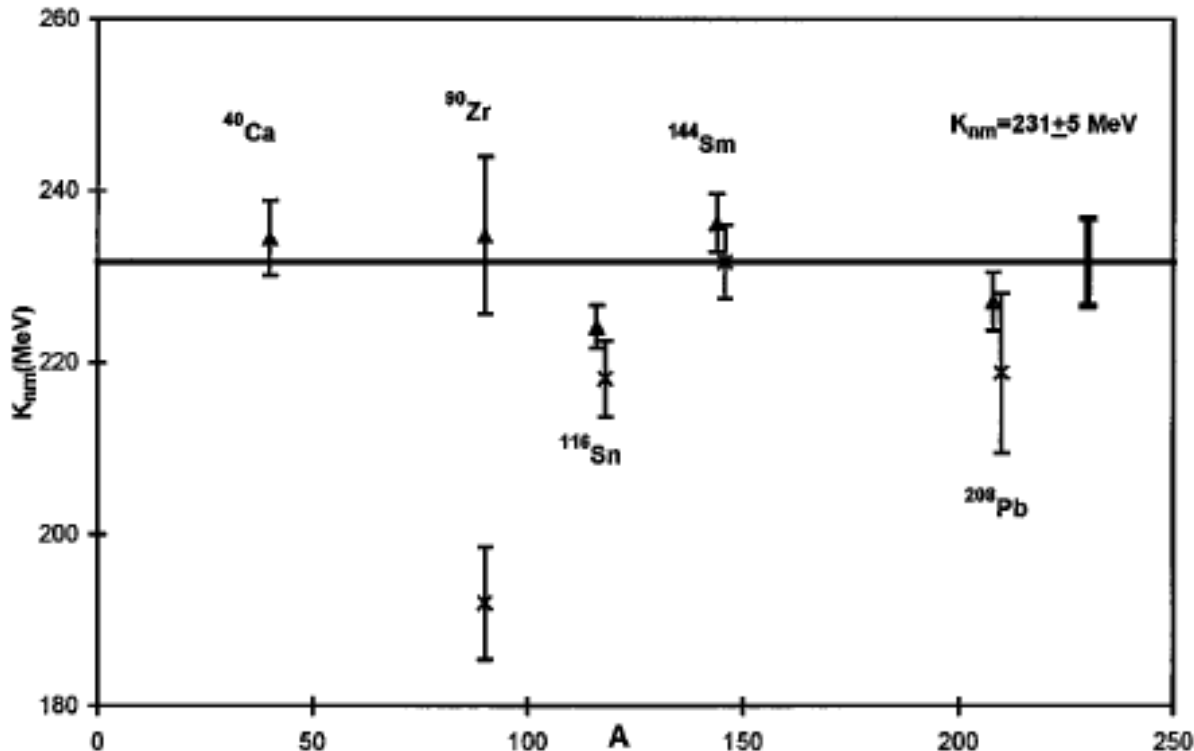
cf. 中性子星の大きさ
や重さ(MR曲線)

Isoscalar giant monopole resonances (breathing mode)



$$E_{\text{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m \langle r^2 \rangle}}$$

J.P. Blaizot,
Phys. Rep. 64 ('80) 171



$K \sim 231 \pm 5 \text{ MeV}$

レポート問題1 (×切: 12月4日(土))

「振動状態」の量子力学的意味を考えるために $H|\psi_n\rangle = E_n|\psi_n\rangle$ で記述される系を考えよう。

- 1) 時刻 $t=0$ で系が基底状態 ($n=0$) にいる場合、時刻 t において波動関数の2乗

$$\rho(x, t) = |\Psi(x, t)|^2$$

が t に依らないことを示せ。

- 2) 時刻 $t=0$ で系が $\Psi(x, t=0) = \psi_0(x) + \alpha\psi_1(x)$ という状態にいるとき、時刻 t における

$$\rho(x, t) = |\Psi(x, t)|^2$$

を求めよ。ただし、 $|\alpha|$ は1に比べて十分小さいとし、 $|\alpha|^2$ の項は無視してよい。

このとき、 $\delta\rho(x, t) \equiv |\Psi(x, t)|^2 - |\psi_0(x)|^2$

は t の関数として振動するが、その周期を求めよ。