

# 原子核基礎論B

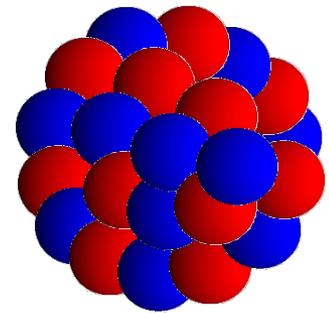
## 原子核理論研究室(物2) 萩野浩一

### シラバス

1. 原子核の集団運動とその微視的理解(3コマ)
2. 原子核反応論基礎(1コマ)
3. 核融合反応(1コマ)
4. 核分裂:現象論と微視的理論(1コマ)
5. ニホニウムと超重元素の物理(1コマ)
6. 高温・高密度核物質外観(1コマ)
7. 高エネルギー重イオン衝突(2コマ)
8. 有限温度・密度における場の理論入門(2コマ)
9. QCD有効模型における相転移と相図(1コマ)
10. 有限温度・密度格子QCDと符号問題(1コマ)

萩野

大西



## シラバス(基礎論A)

5. 殻模型と魔法数(1コマ)
6. 平均場理論と核変形(3コマ)
7. 中性子過剰核の物理(2コマ)
8. 非束縛核と共鳴散乱理論(1コマ)

主に基底状態

→ 基礎論Bで

## シラバス(基礎論B)

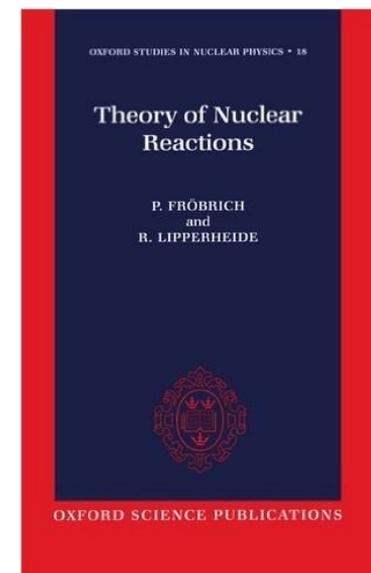
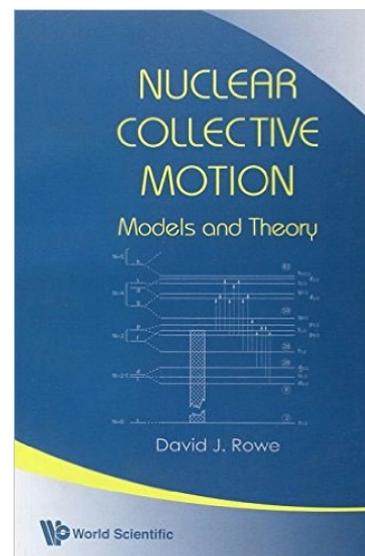
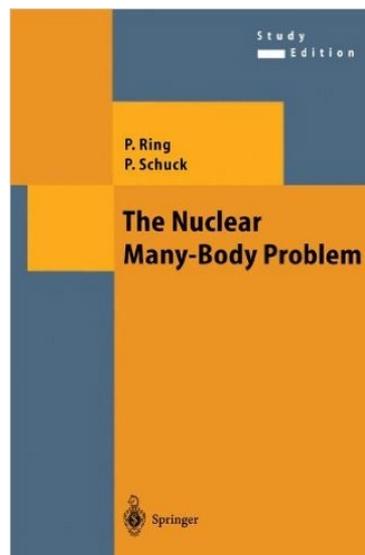
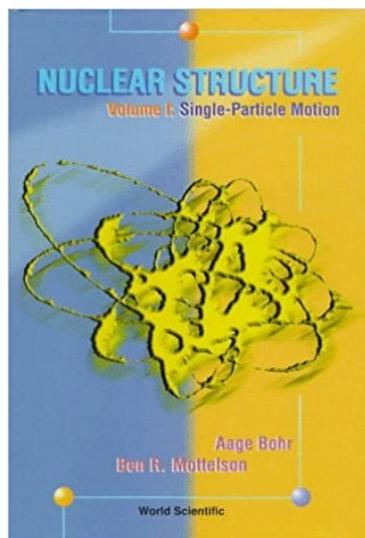
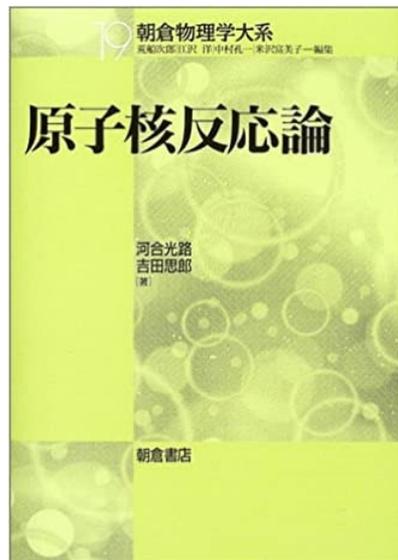
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主に励起状態  
と核反応

成績： レポート80%、出席など20%

質問： 適宜聞いて下さい。  
チャットに書き込んでもOK。

# 参考書



Bohr-Mottelson

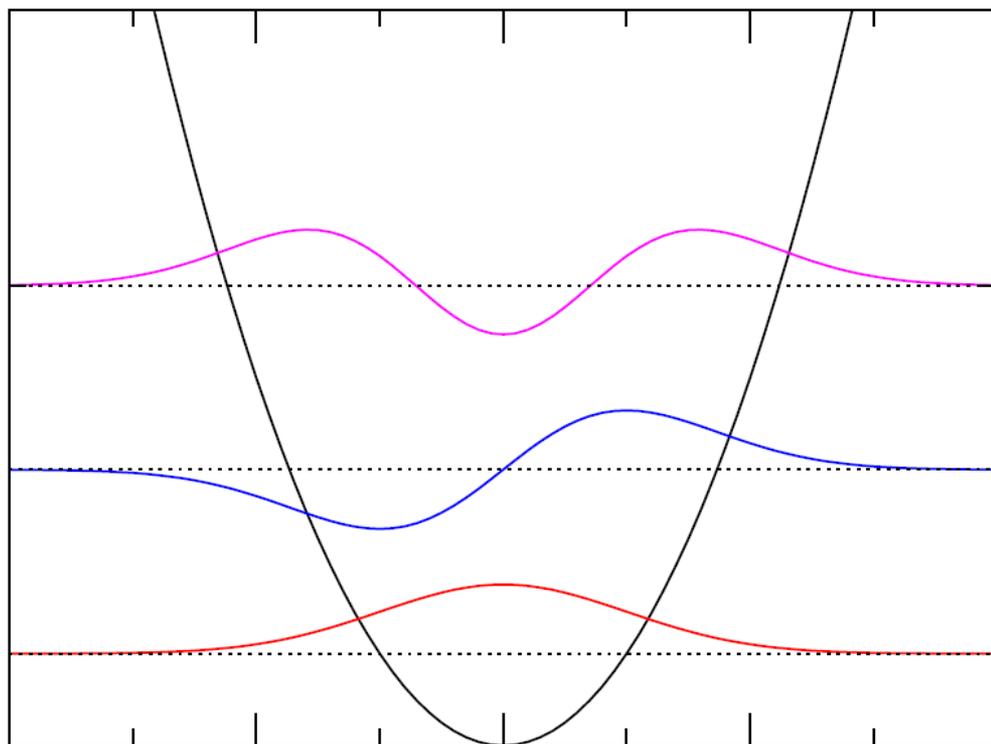
Ring-Schuck

Rowe

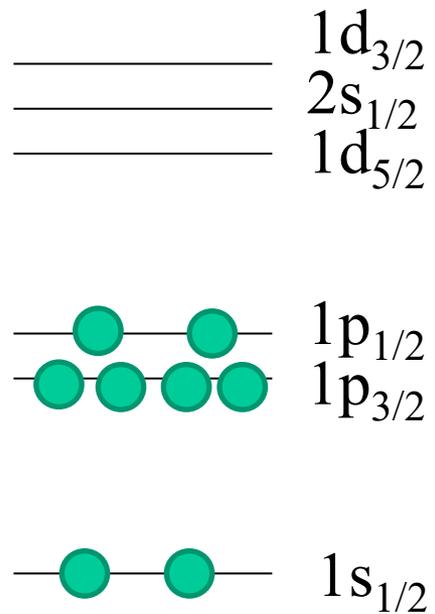
Frobrich  
-Lipperheide

# 励起状態

## ポテンシャル中の1粒子の場合

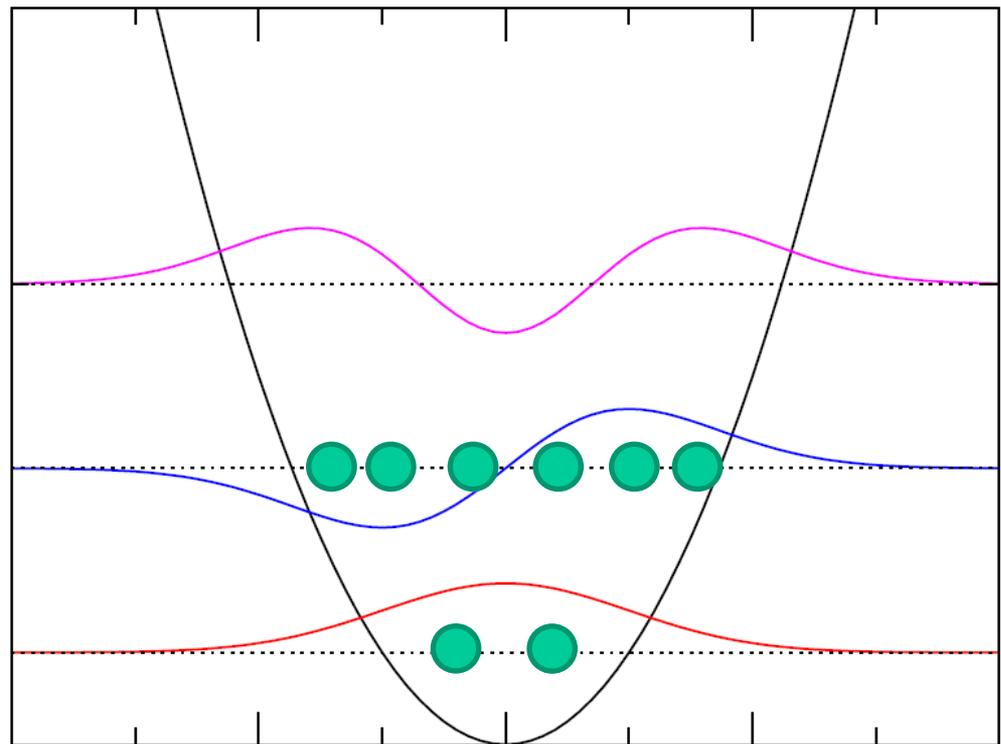


# 原子核の励起状態



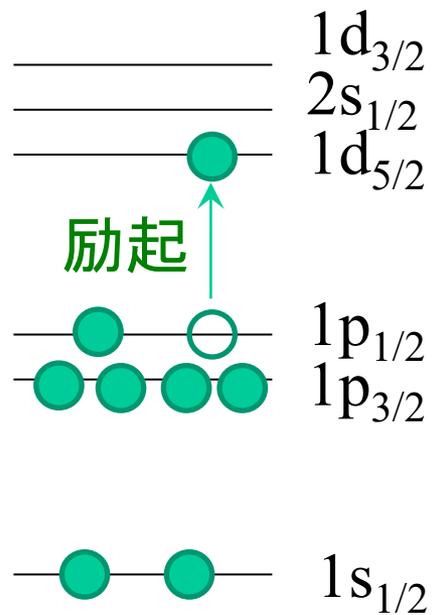
基底状態

# 多体系の場合

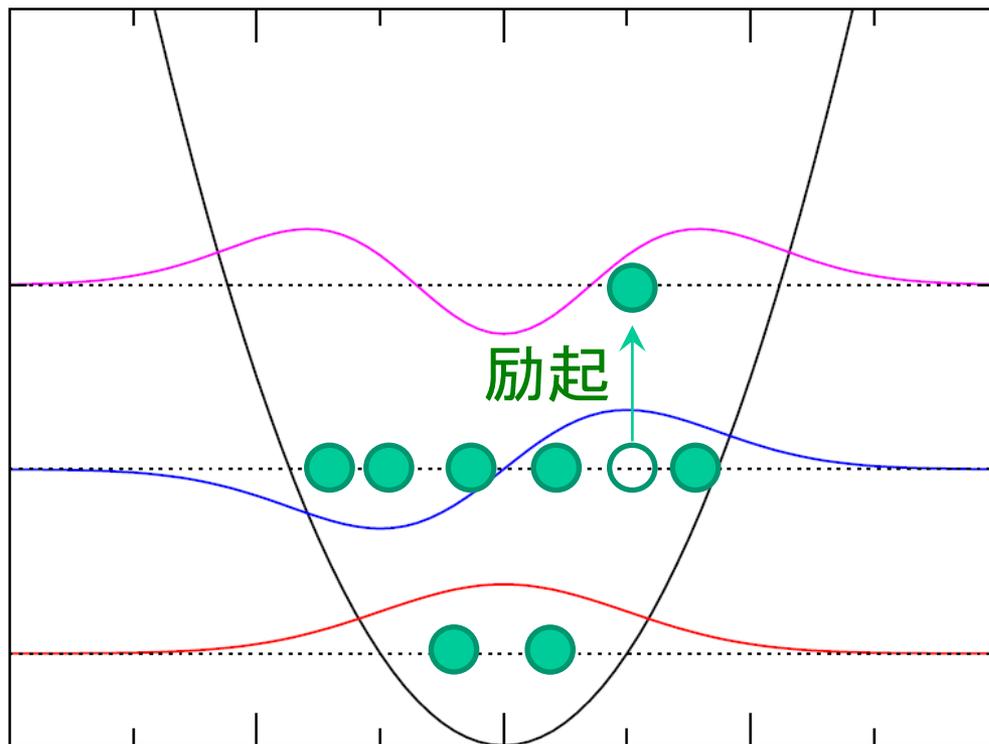


基底状態

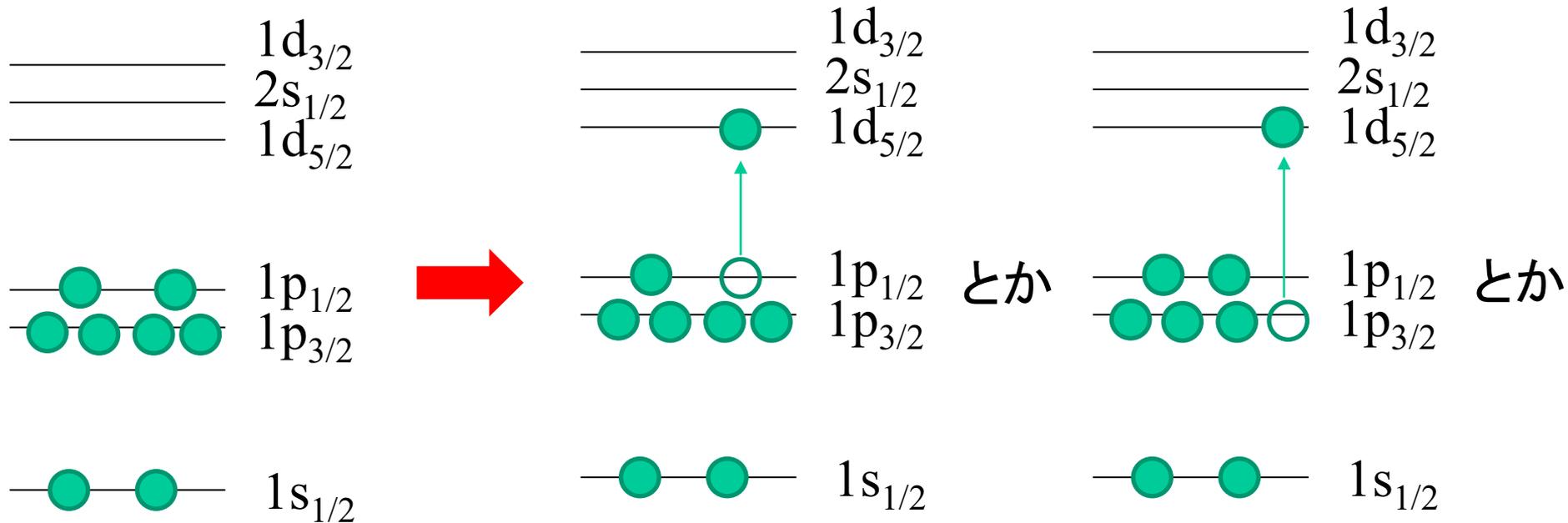
# 原子核の励起状態



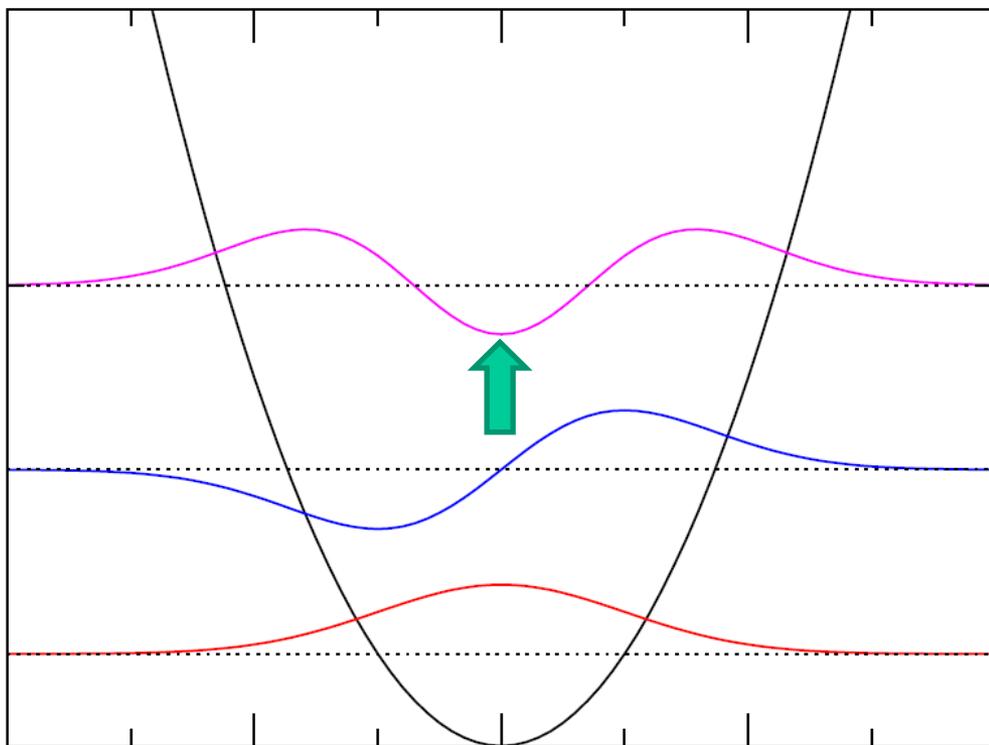
# 多体系の場合



# 原子核の励起状態



基底状態



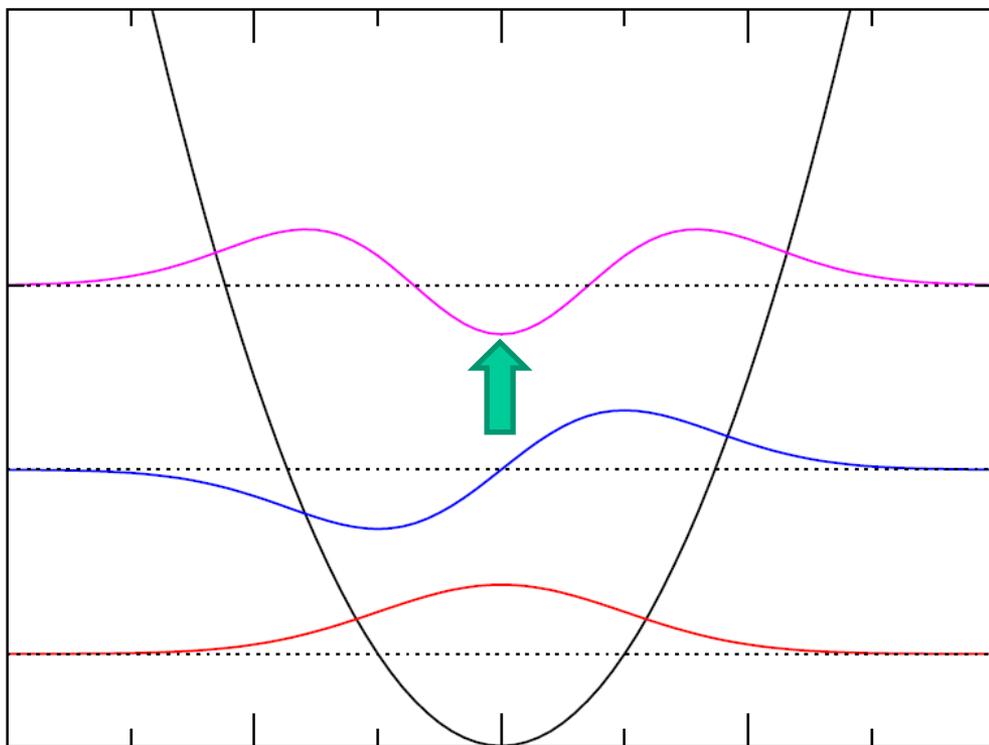
原子核では

$$\hbar\omega \sim 41 A^{-1/3} \quad (\text{MeV})$$

$$\leftarrow R \sim 1.2 A^{1/3} \quad (\text{fm})$$

$A = 16$  だと 16.27 MeV

cf. 実際に、 $^{16}\text{O}$  の 16.2 MeV  
に 1- 状態



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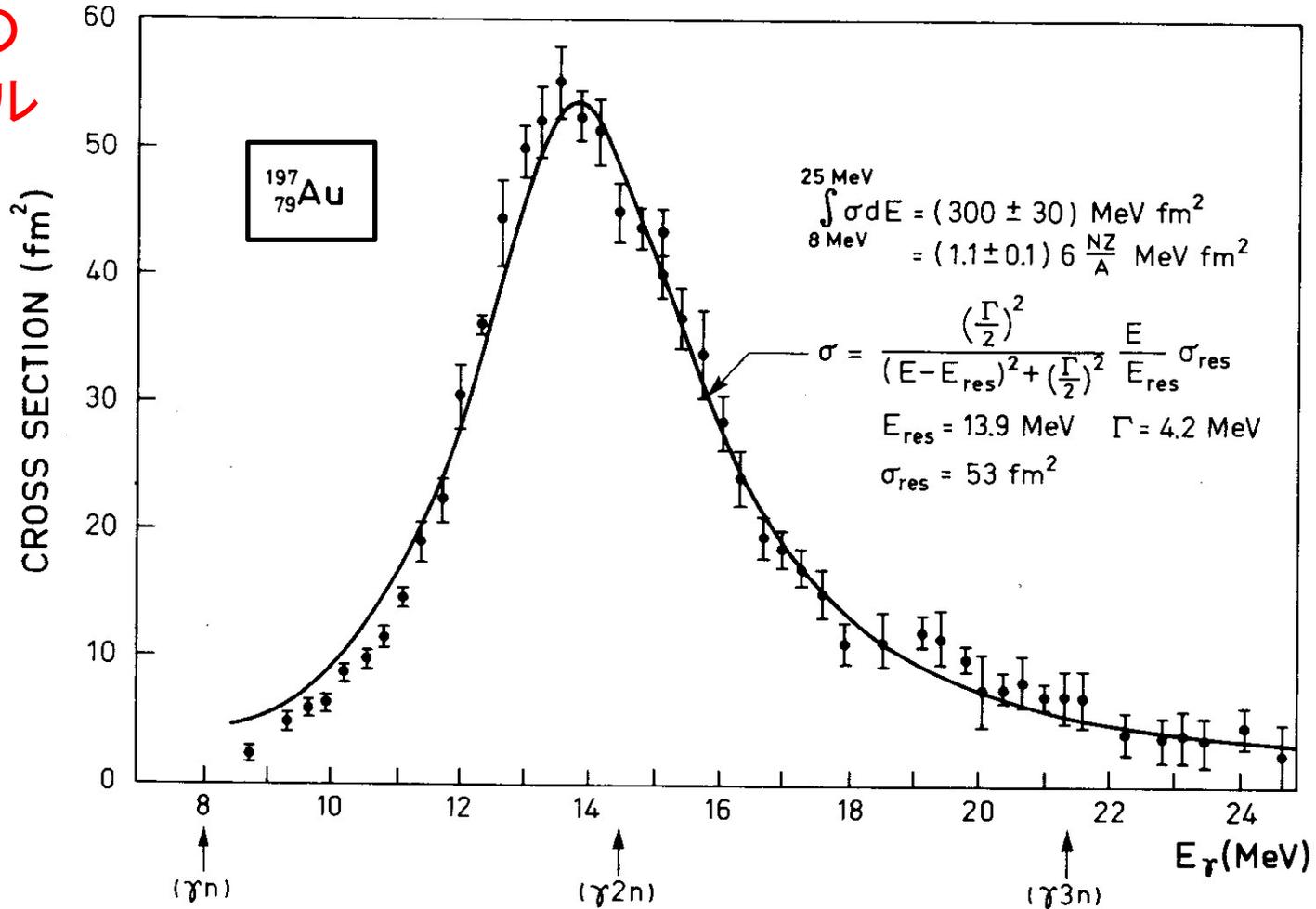
$$A = 16 \text{ だと } 16.27 \text{ MeV}$$

cf. 実際に、 $^{16}\text{O}$ の16.2 MeV  
に 1- 状態

.....でも実際にはこのようには理解できない励起状態  
も多数存在する(集団励起)

# Giant Dipole Resonance (GDR) 巨大双極子共鳴

光吸収の  
スペクトル



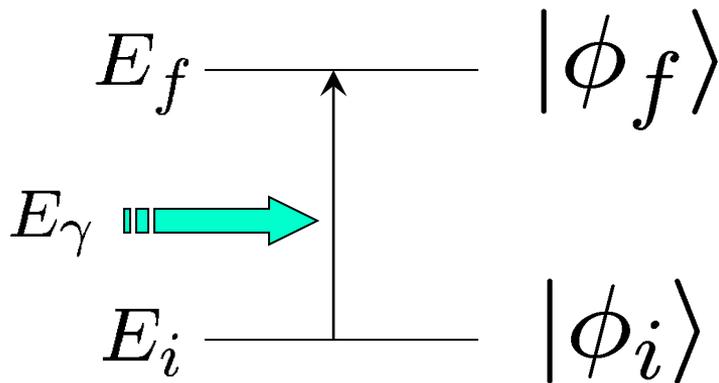
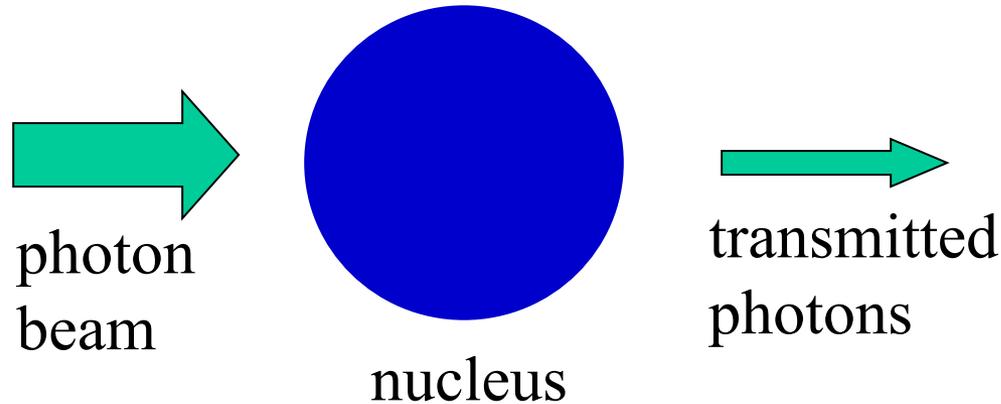
**Figure 6-18** Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

$$\text{cf. } 41 \times 197^{-1/3} = 7.05 \text{ MeV}$$

# Collective Vibrations

How does a nucleus respond to an external perturbation?

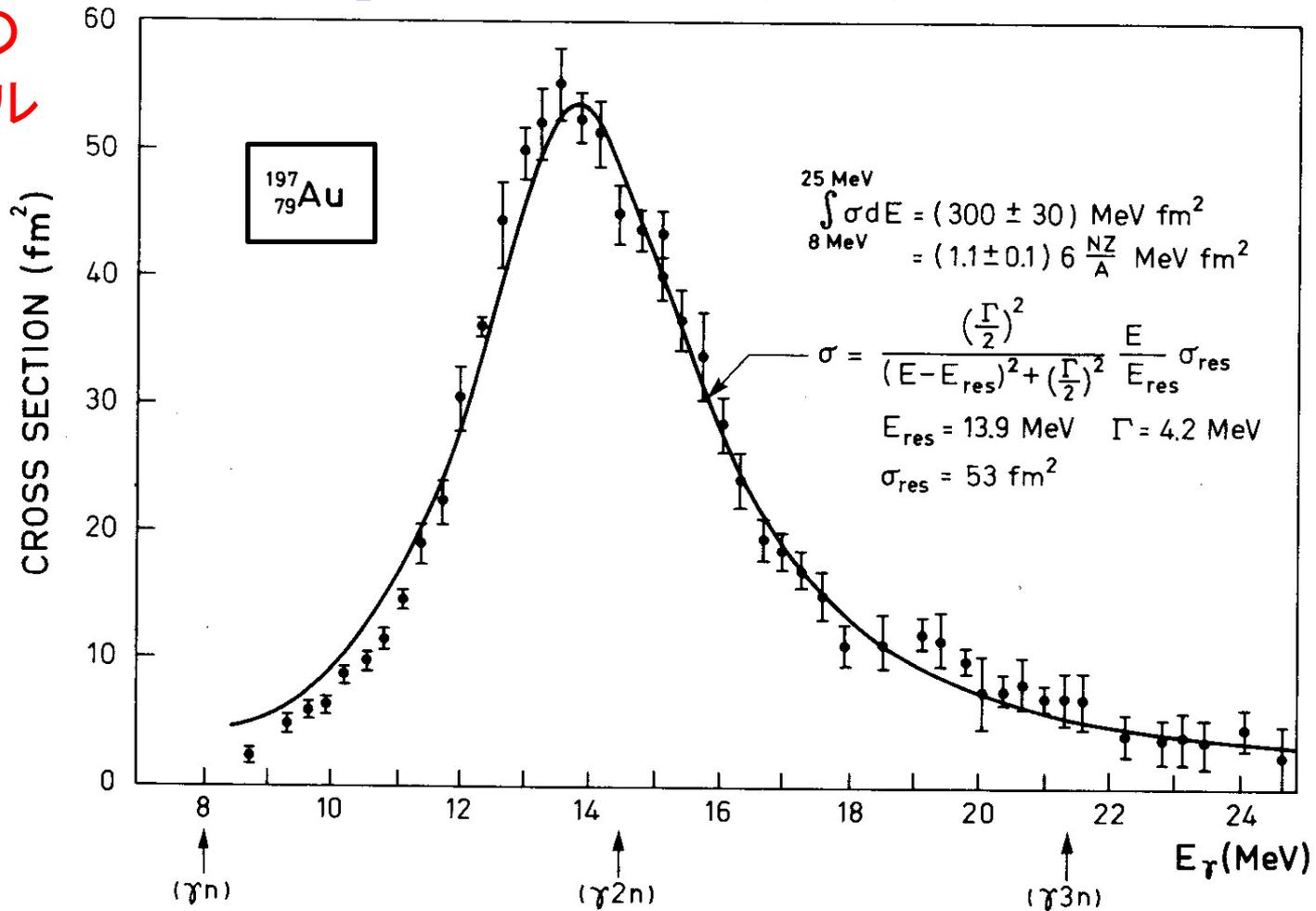
## i) Photo absorption cross section



The state is strongly excited when  
 $E_f - E_i = E_\gamma$ .

# Giant Dipole Resonance (GDR) 巨大双極子共鳴

光吸収の  
スペクトル



**Figure 6-18** Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

$$\text{cf. } 41 \times 197^{-1/3} = 7.05 \text{ MeV}$$

## Remarks

i) Photon interaction  $\longleftrightarrow$  dipole excitation

$$H_{\text{int}} = \frac{1}{2m} \frac{e}{c} (\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p})$$

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\alpha=1,2} \sqrt{\frac{2\pi c^2 \hbar}{\omega V}} (a_{\mathbf{k}\alpha} \boldsymbol{\epsilon}_{\alpha} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}} t} + h.c.)$$

$$e^{i\mathbf{k} \cdot \mathbf{r}} \sim 1 \quad (\text{dipole approximation})$$

## Remarks

i) Photon interaction  $\longleftrightarrow$  dipole excitation

$$E_\gamma = 10 \text{ MeV}, R = 5 \text{ fm}$$

だと  $kR \sim 0.25$

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$$\sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_\gamma - E_f + E_i)$$

$$\tilde{z} = \sum_p (z_p - Z_{\text{cm}})$$

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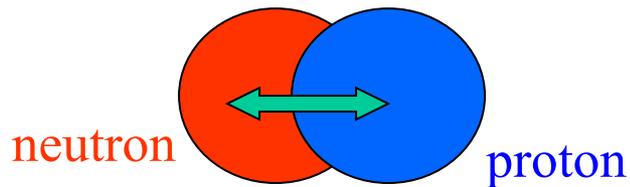
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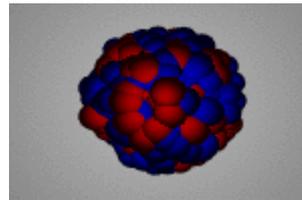
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### ii) Isospin

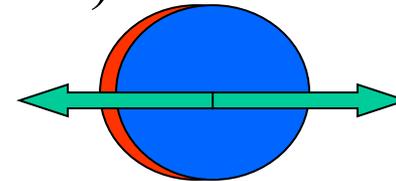


Isovector type



(note)

$$\tilde{z} = \sum_p (z_p - Z_{cm})$$



Isoscalar dipole motion

$\longleftrightarrow$  c.m. motion (to the first order)

## Remarks

### i) Photon interaction $\longleftrightarrow$ dipole excitation

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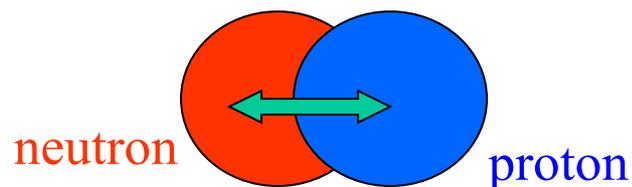
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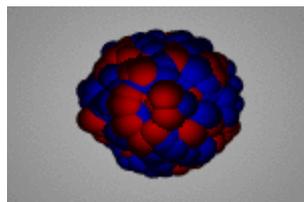
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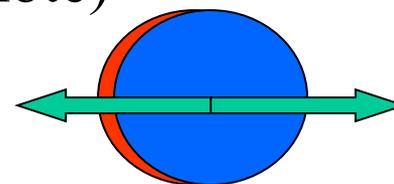


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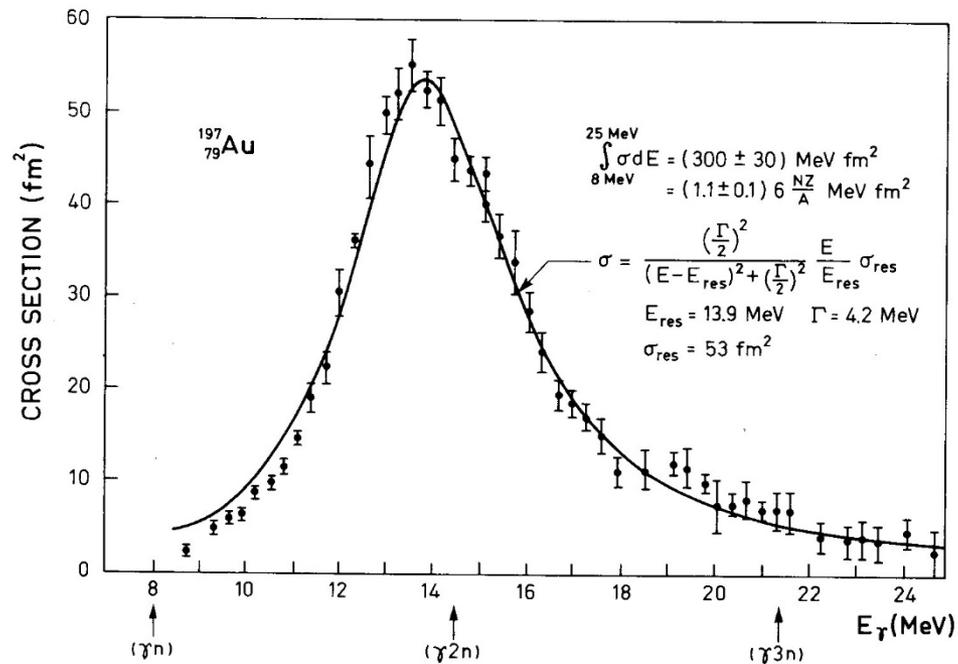
Isoscalar dipole motion

$\longleftrightarrow$  c.m. motion (to the first order)

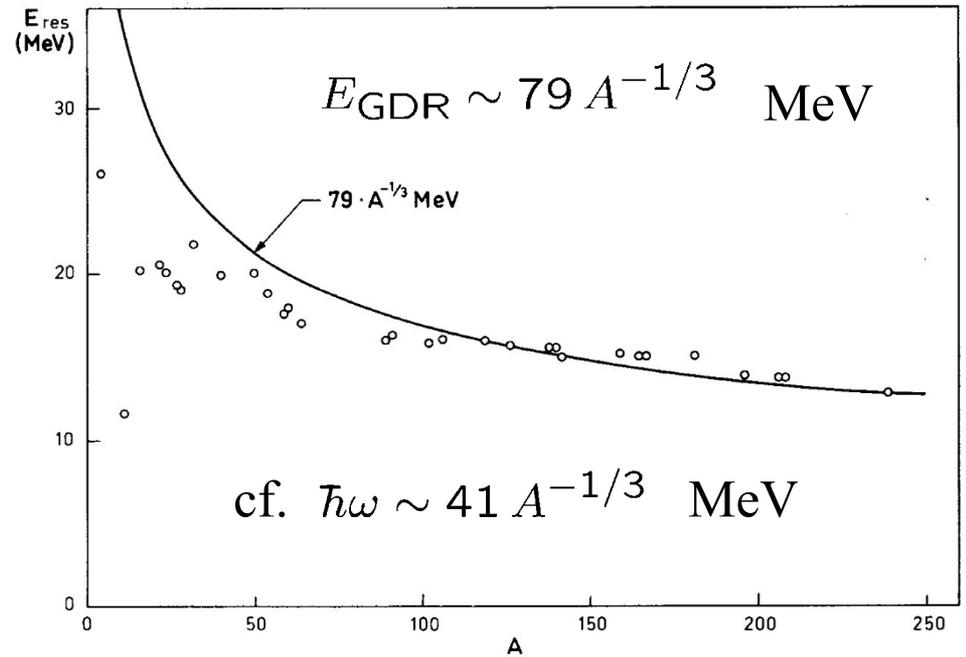
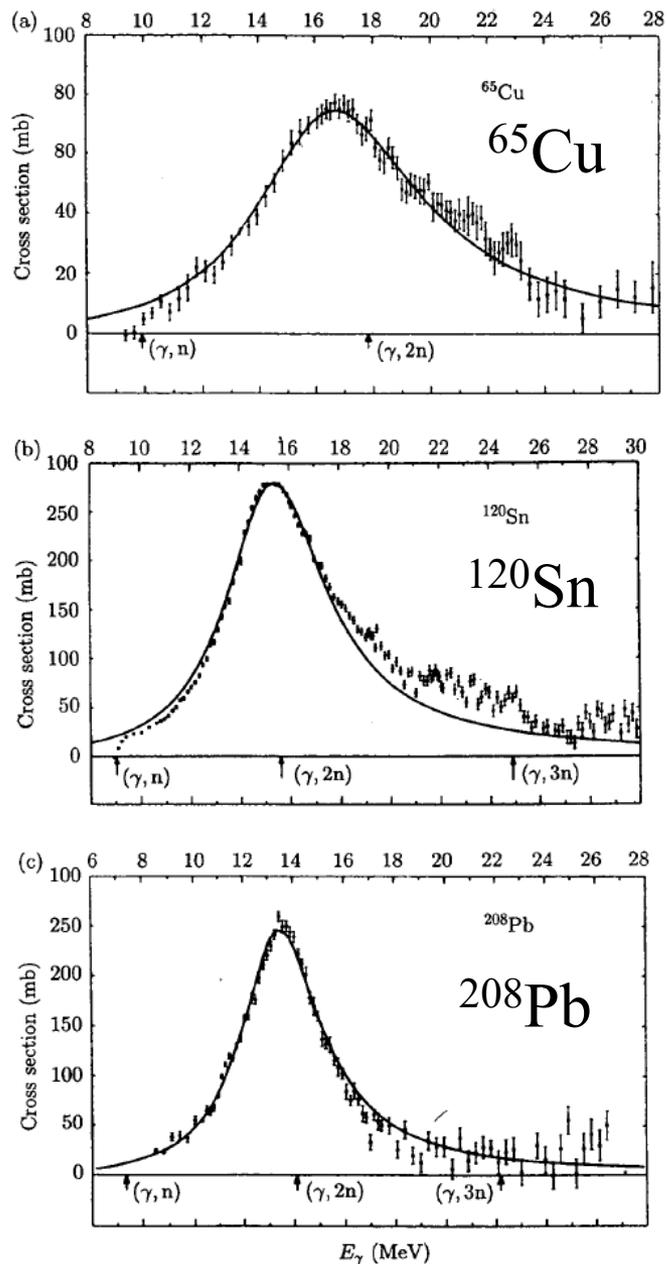
### iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

# Giant Dipole Resonance (GDR) 巨大双極子共鳴



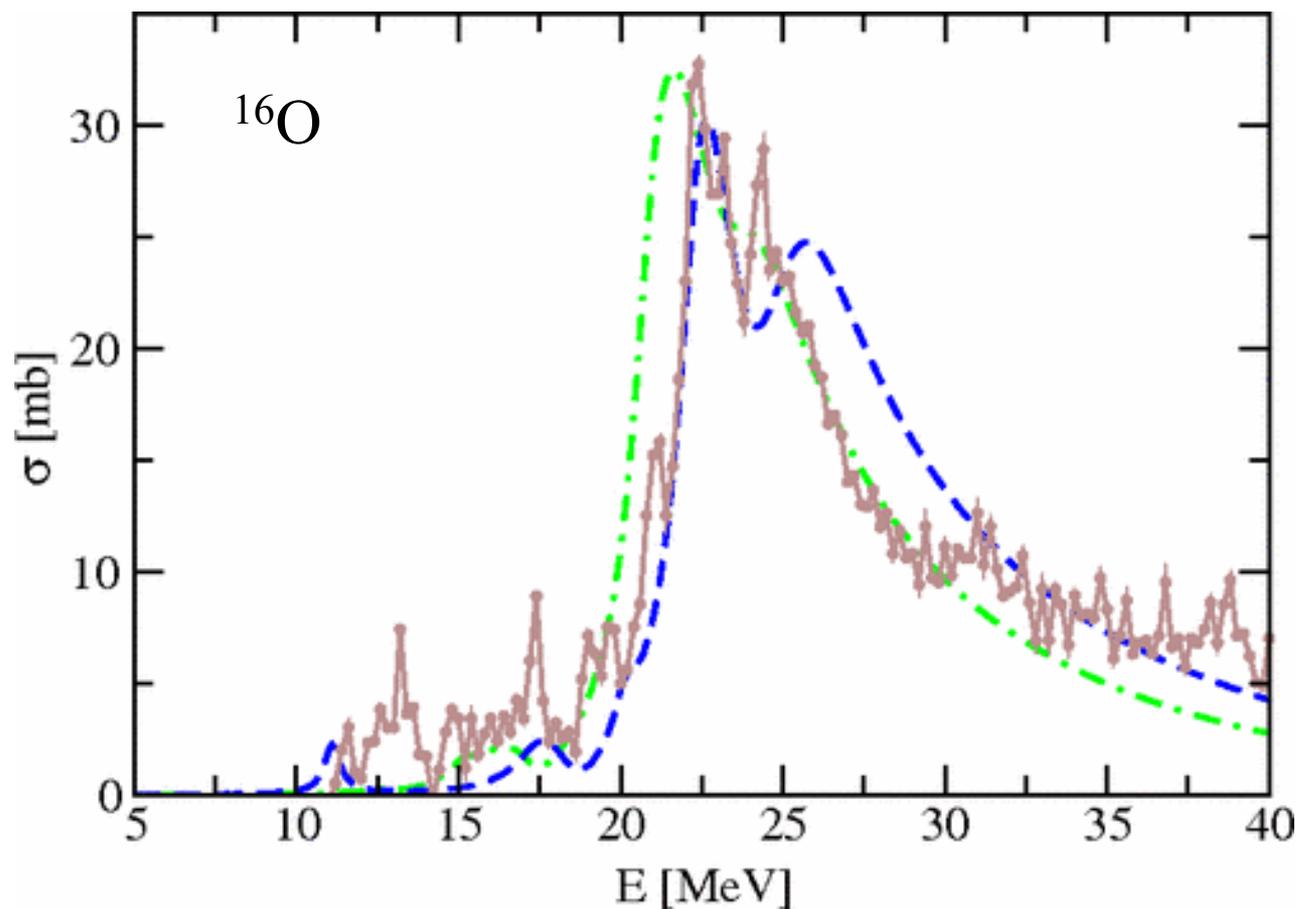
**Figure 6-18** Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



Bohr-Mottelson  
 “Nuclear Structure vol. II”

M.N. Harakeh and A. van der Woude,  
 “Giant Resonances”

FIG. 1.2. The photo-neutron cross section  $\sigma(\gamma, n)$  as a function of the photon energy for the three nuclei  $^{208}\text{Pb}$ ,  $^{120}\text{Sn}$  and  $^{65}\text{Cu}$ . Note that for these nuclei  $\sigma(\gamma, n) \approx \sigma_{\text{abs}}(\gamma)$ . From reference (BER75).

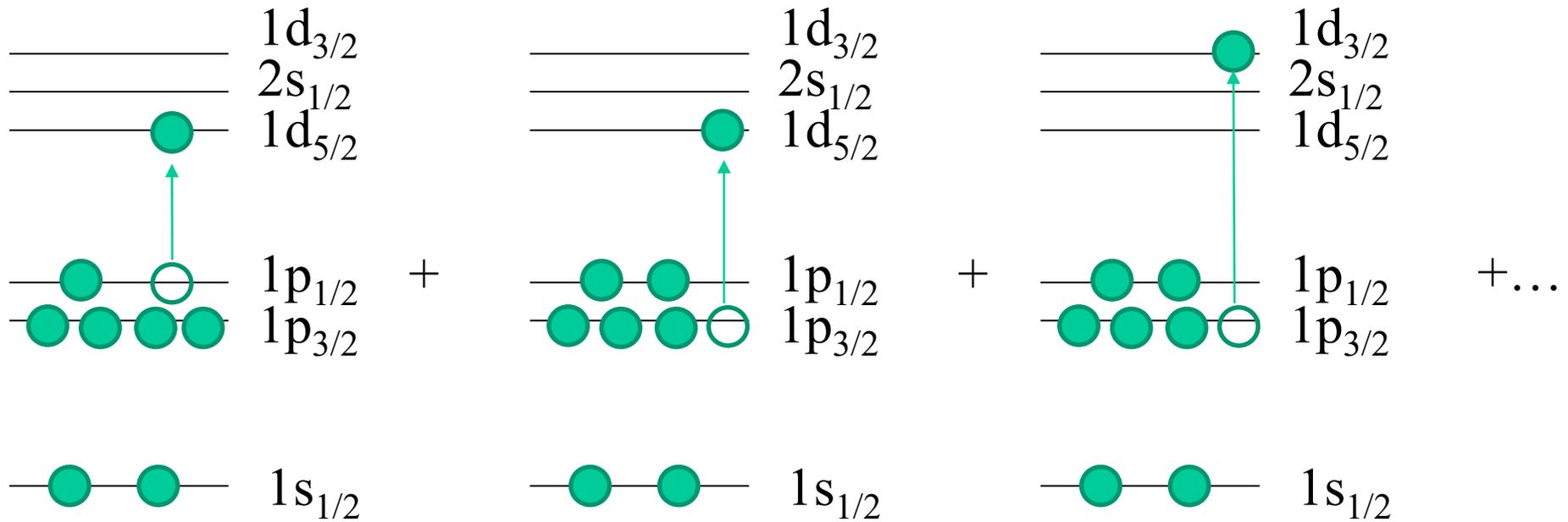


実験データ: 茶色

N. Lyutorovich et al., Phys. Rev. Lett. 109 (2012) 092502

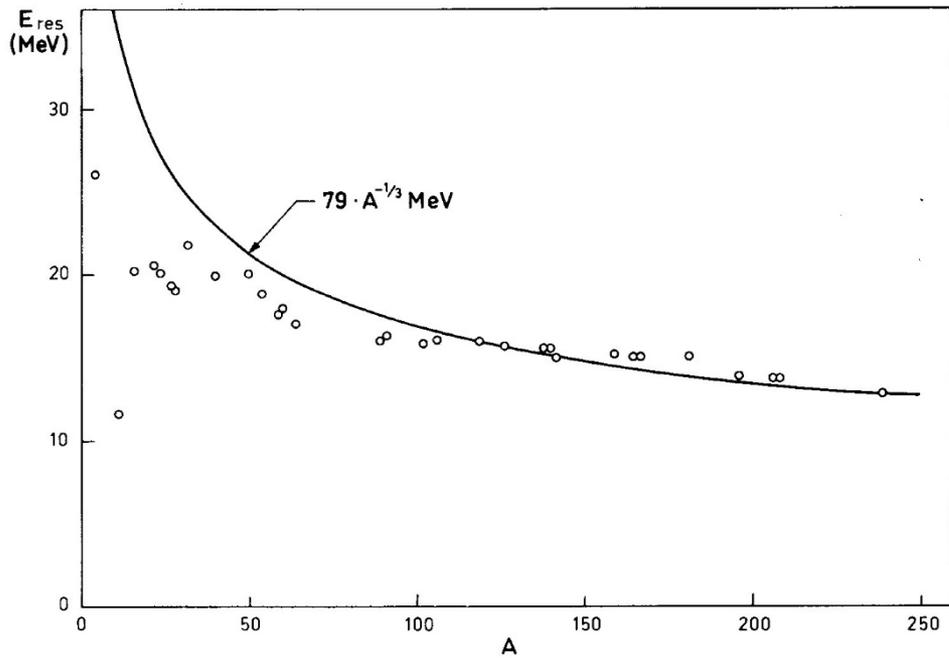
cf.  $41 \times 16^{-1/3} = 16.27 \text{ MeV}$

# 何故励起エネルギーが大きくなるのか？

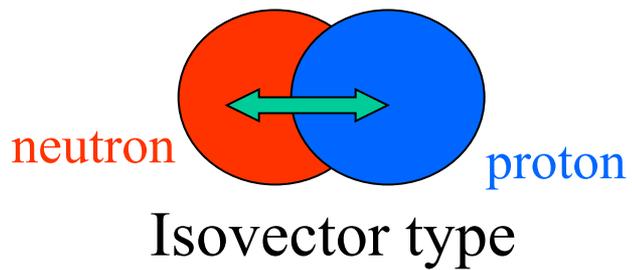


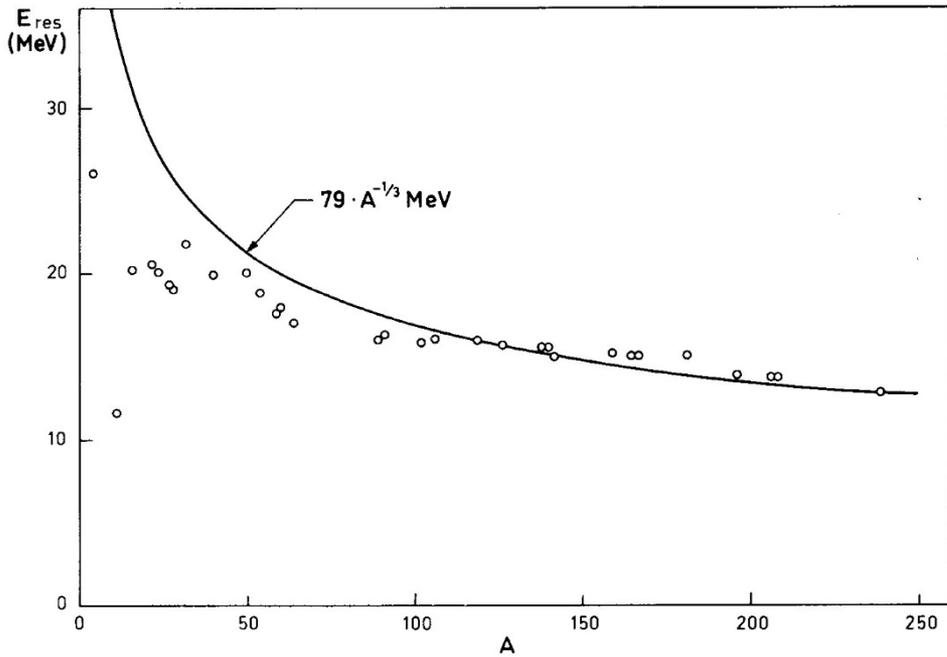
様々な励起状態がコヒーレントに重ね合わさることにより  
「集団的」になる。→(次回もう少し詳しく)

残留相互作用が大きな役割



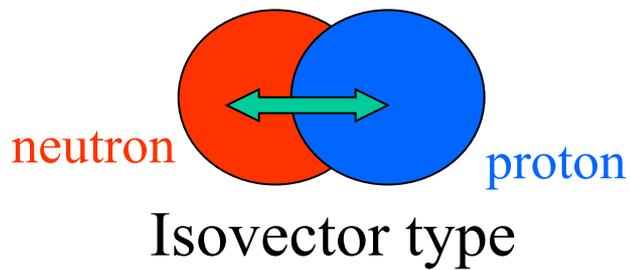
Bohr-Mottelson  
“Nuclear Structure vol. II”



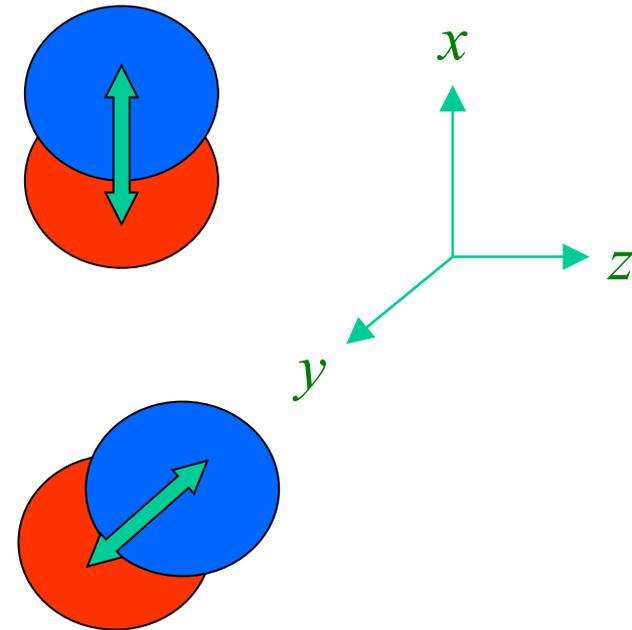


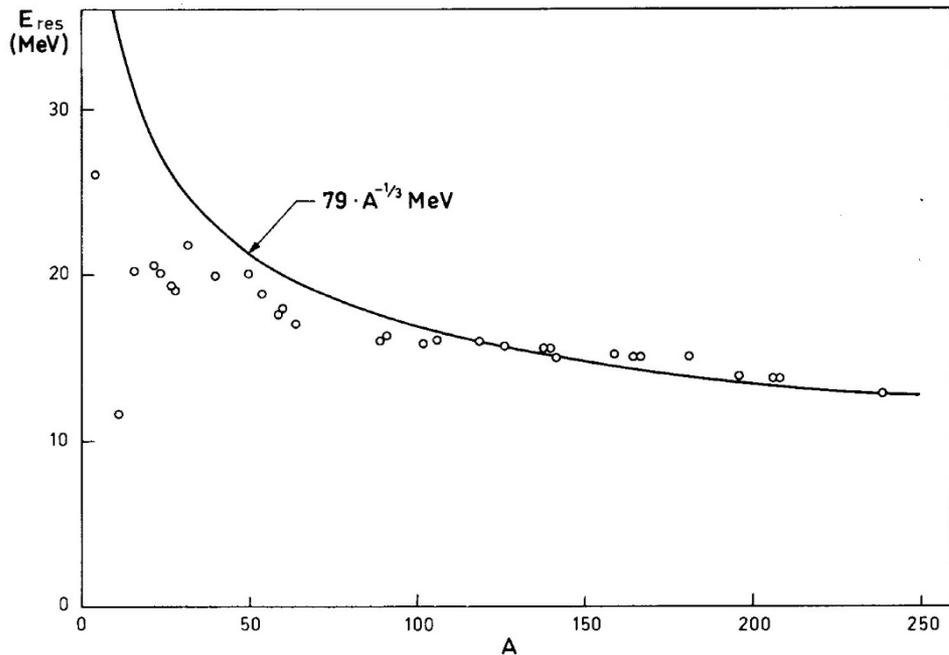
$$E_{GDR} \propto A^{-1/3}$$

Bohr-Mottelson  
 “Nuclear Structure vol. II”



3つのモード

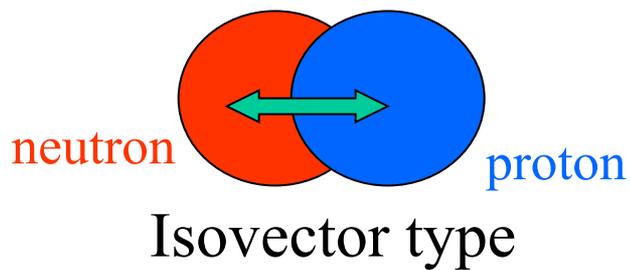




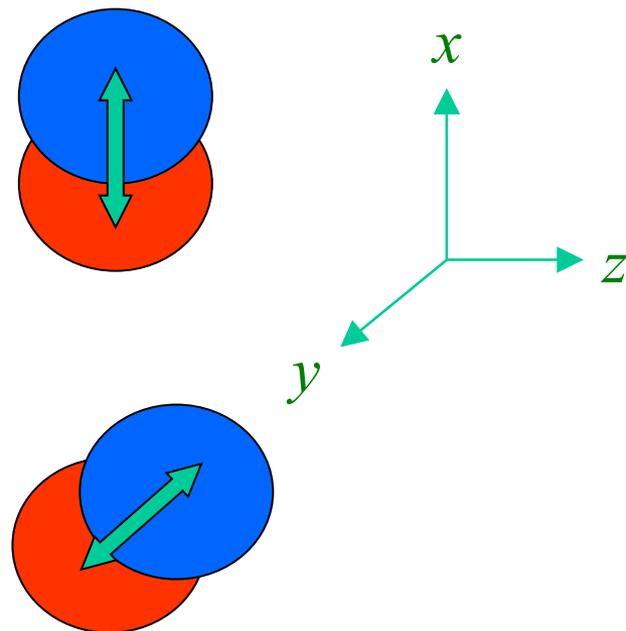
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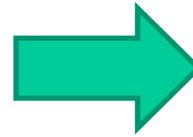
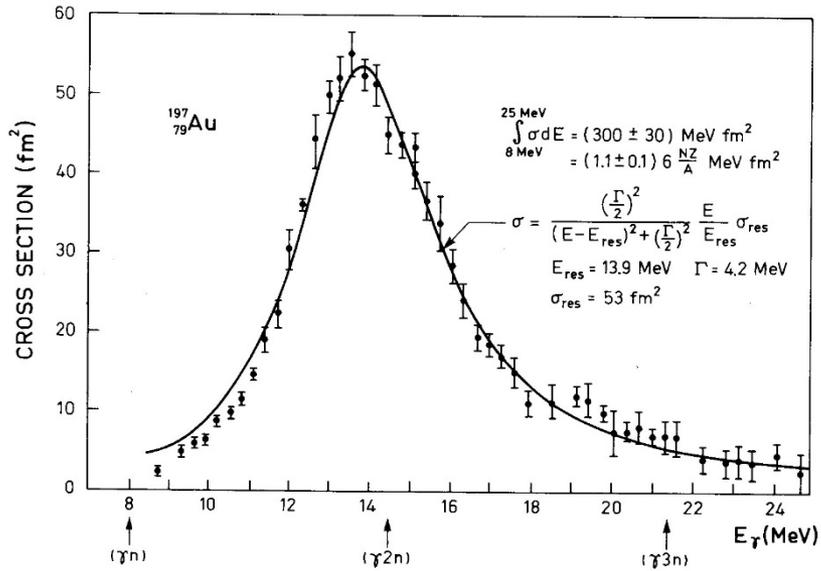
$$\propto 1/R$$

Bohr-Mottelson  
 “Nuclear Structure vol. II”



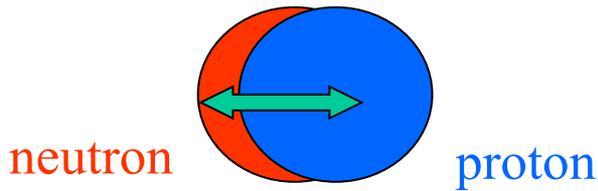
3つのモード





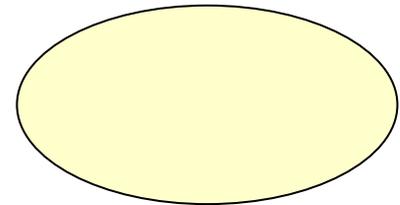
?

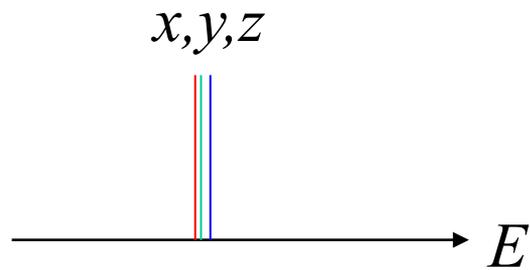
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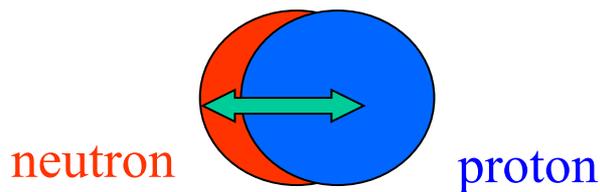
$$E_{\text{GDR}} \propto 1/R$$

deformed nucleus



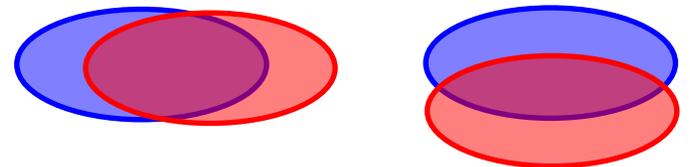
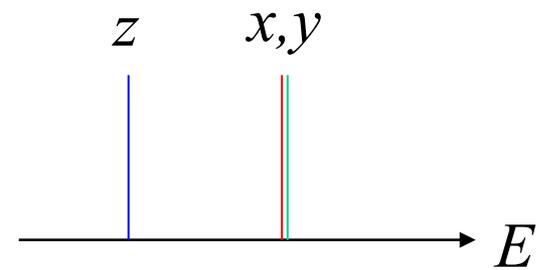
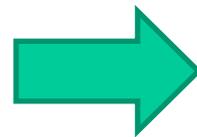


spherical nucleus

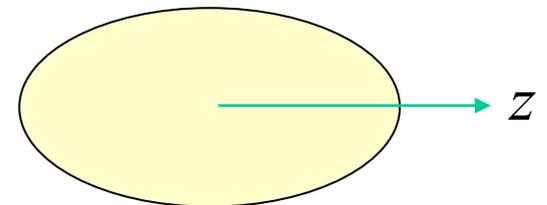


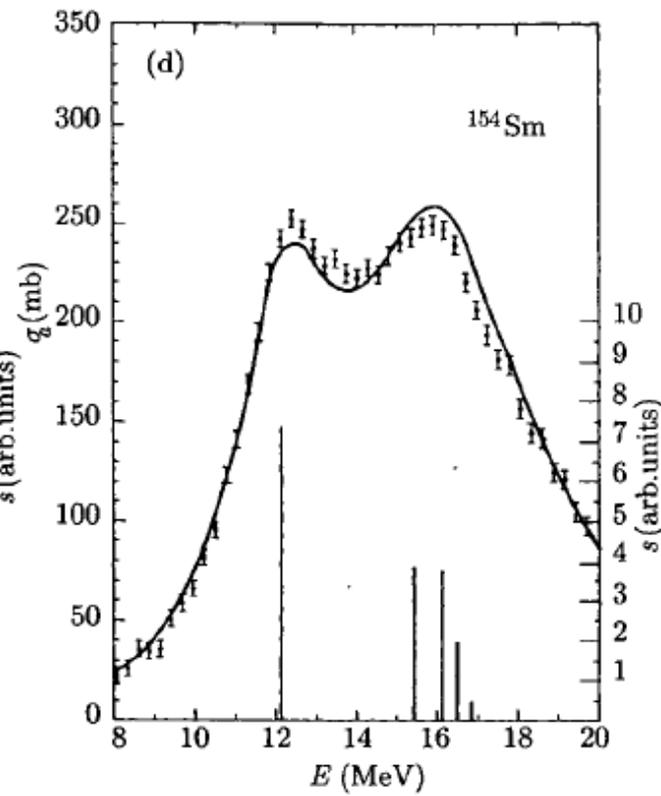
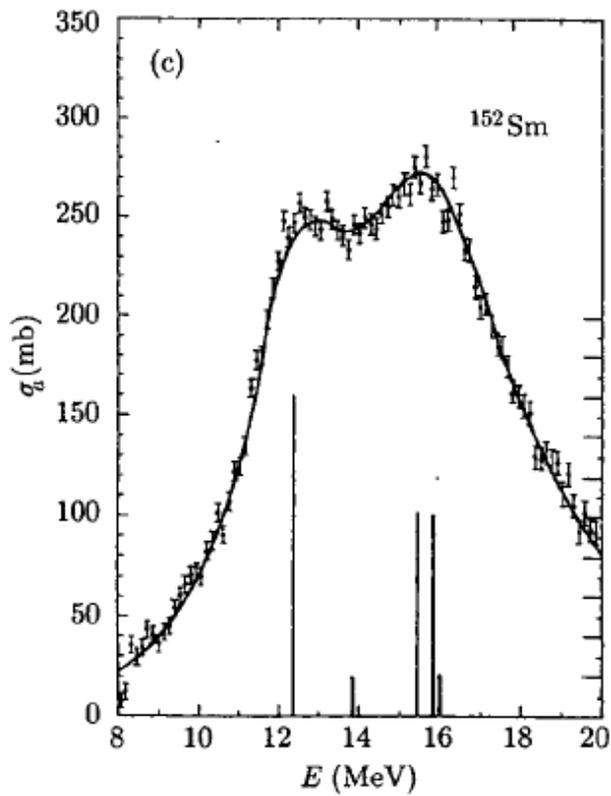
$$E_{\text{GDR}} \propto 1/R$$

(prolate deformation)



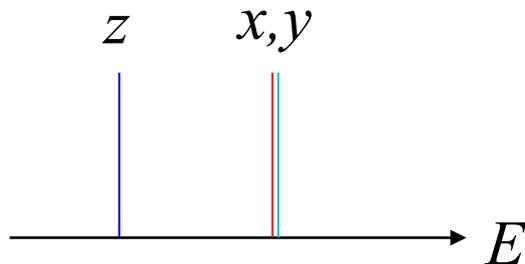
deformed nucleus



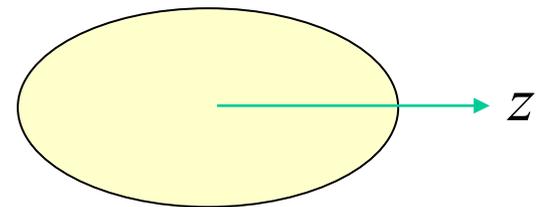


M.N. Harakeh and  
A. van der Woude,  
“Giant Resonances”

(prolate deformation)

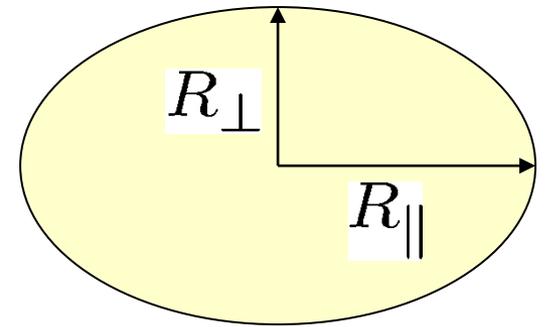
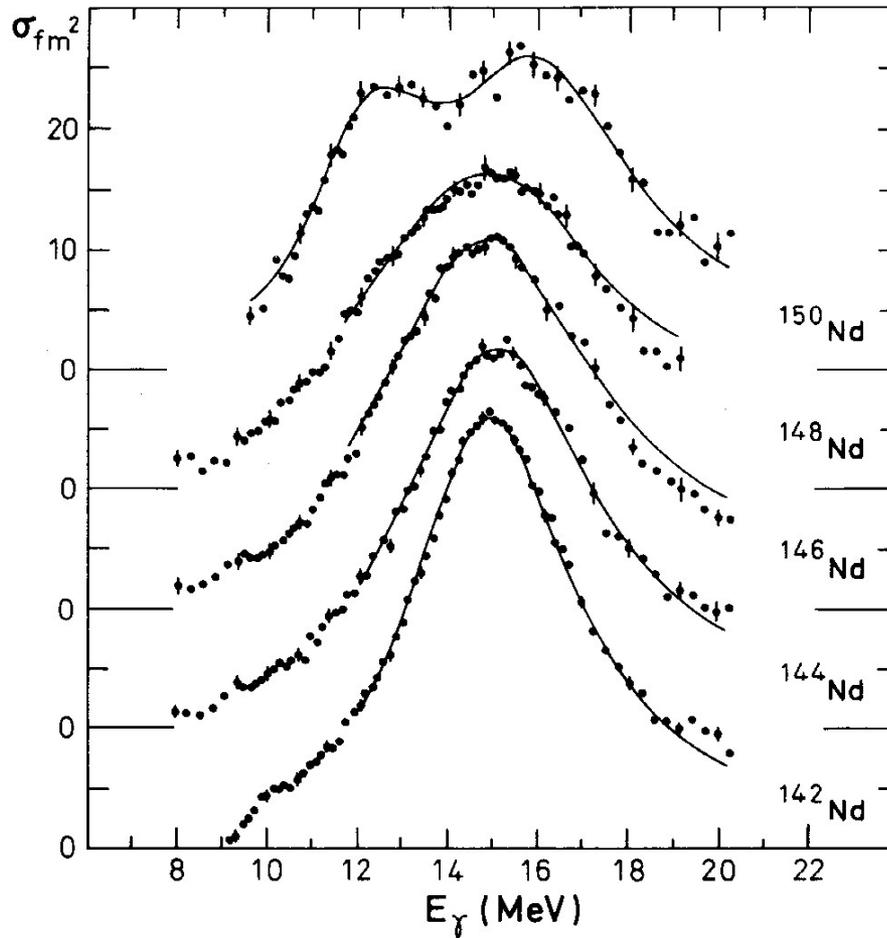


deformed nucleus



## Deformation effect

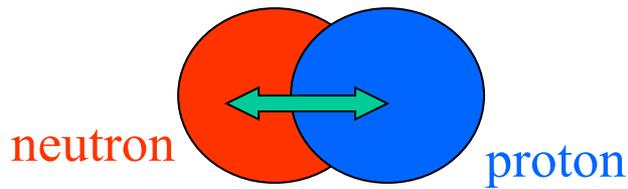
$$E_{\text{GDR}} \sim A^{-1/3} \sim 1/R$$



**Figure 6-21** Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys. A172*, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.

# Giant Dipole Resonances

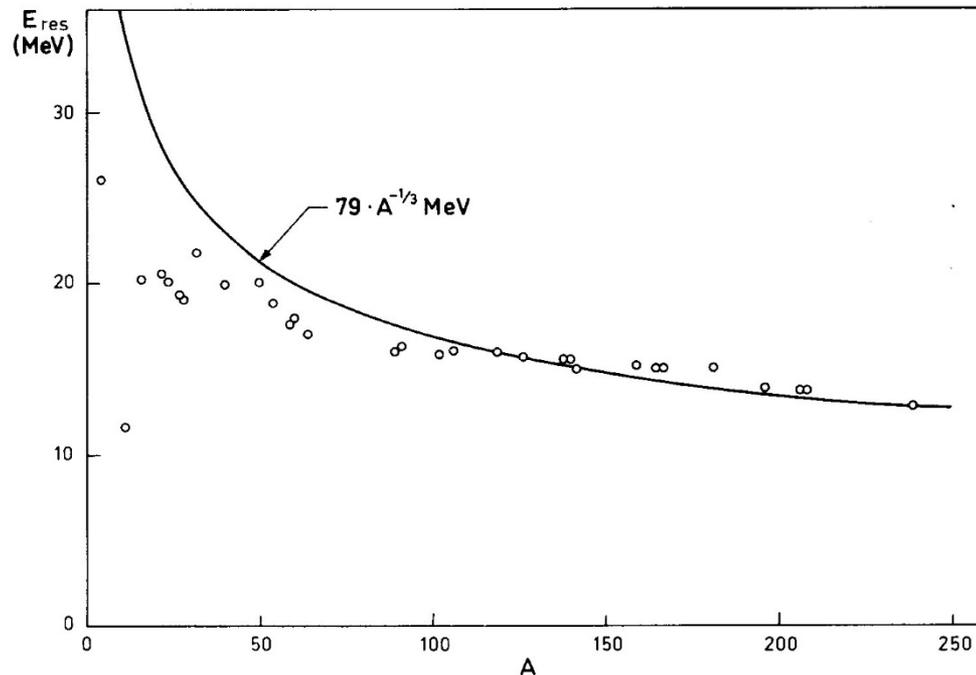
## • Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

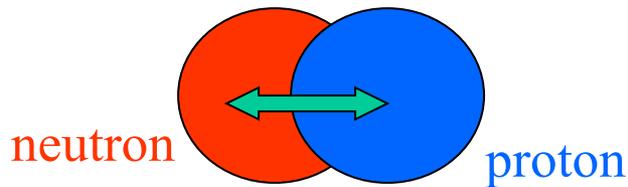
→  $\hbar\omega \sim A^{-1/6}$

→ Inconsistent with expt.  
(except for light nuclei)



# Giant Dipole Resonances

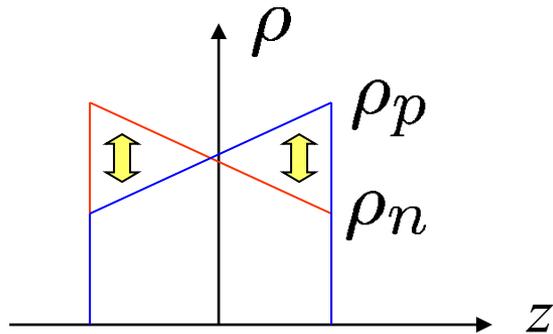
## • Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/6}$$

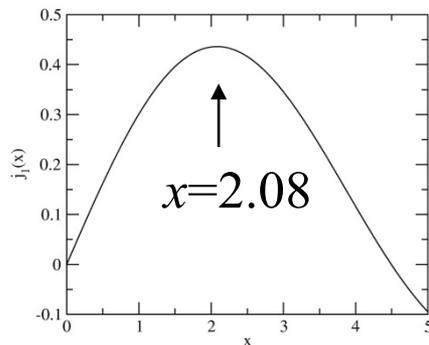
## • Steinwedel-Jensen type



$$\hat{Q} = j_1(kr) Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/3}$$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x \cos x) / x^2$$

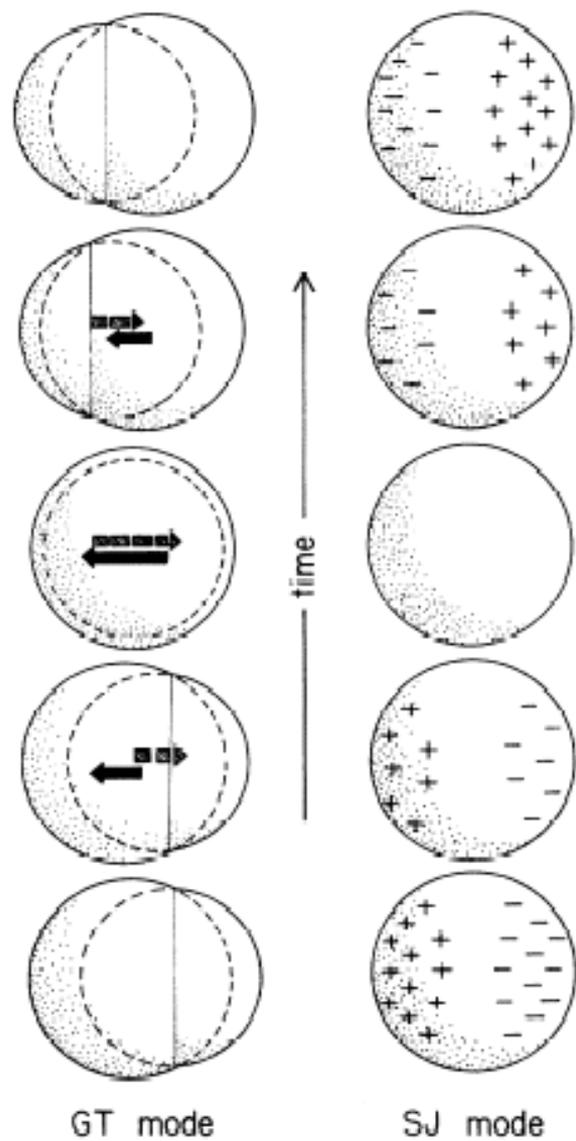


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.

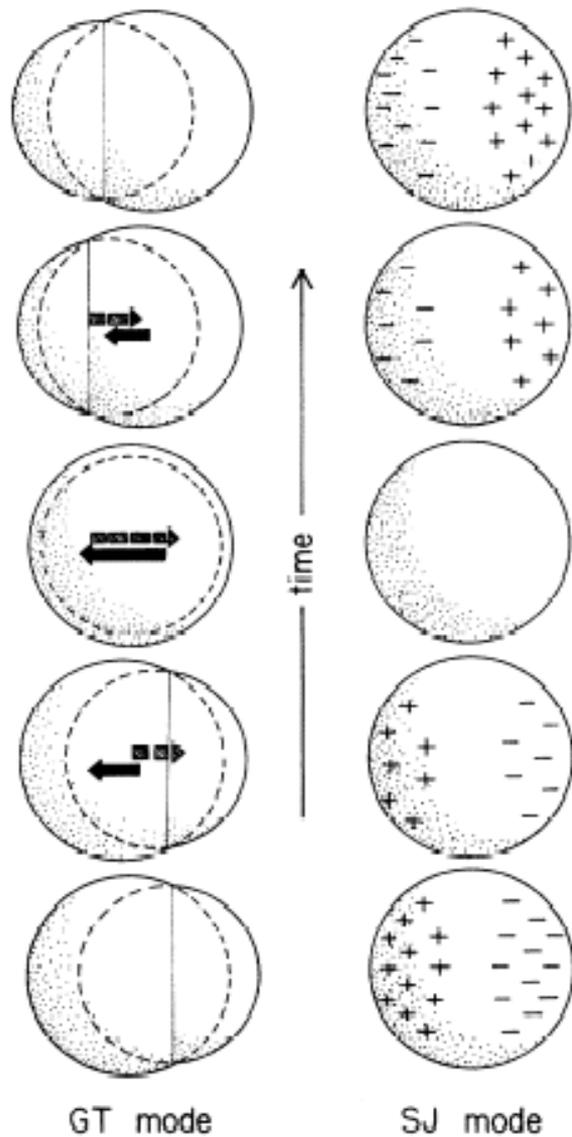
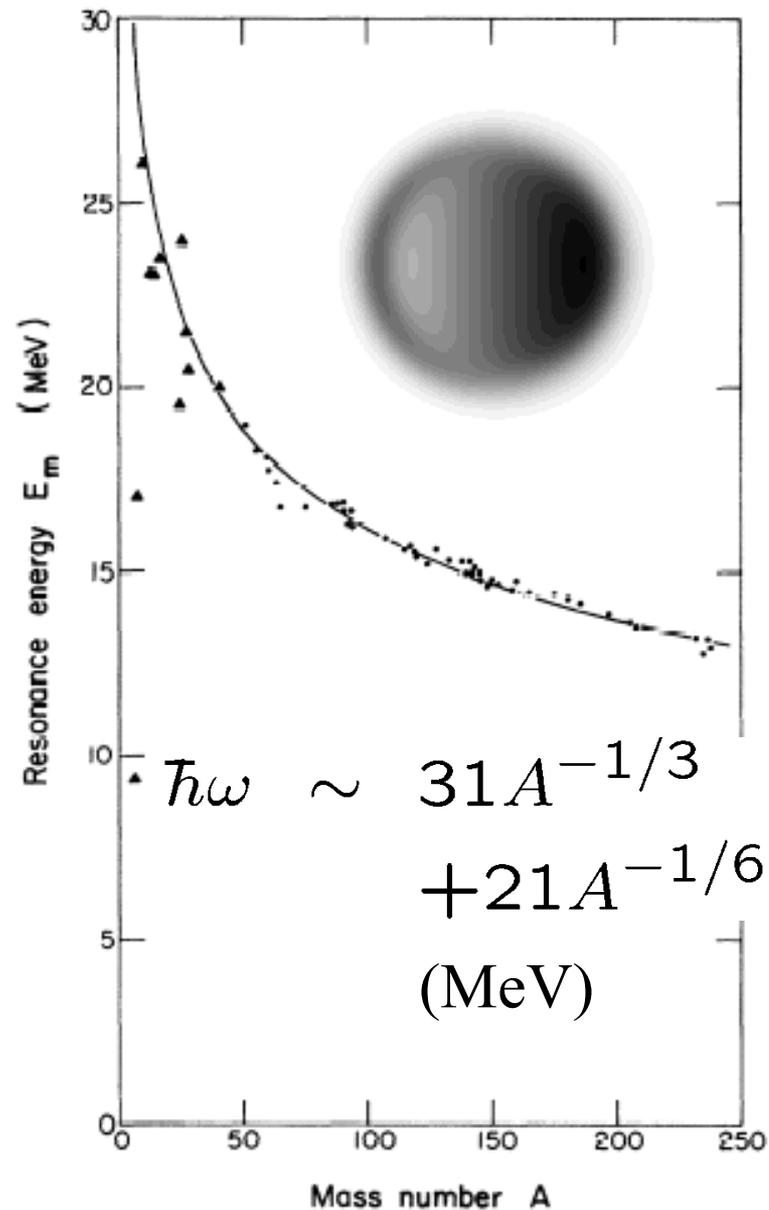
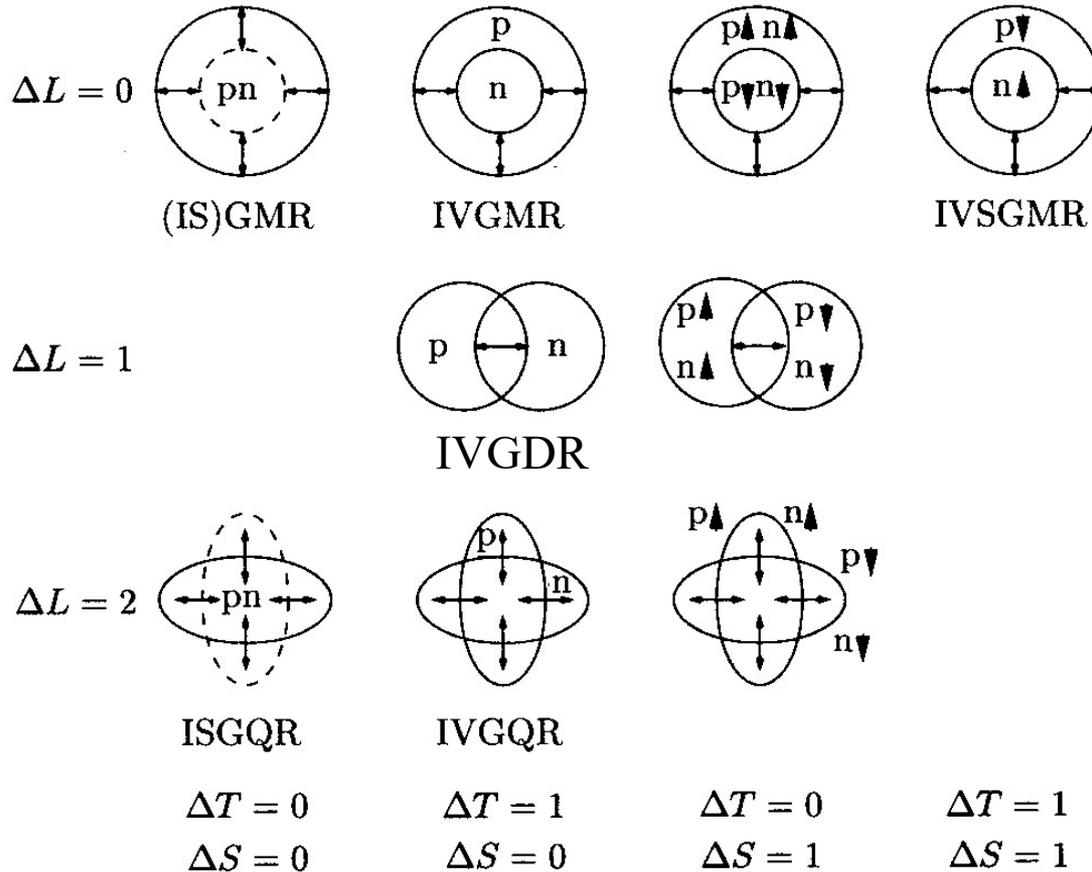


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.



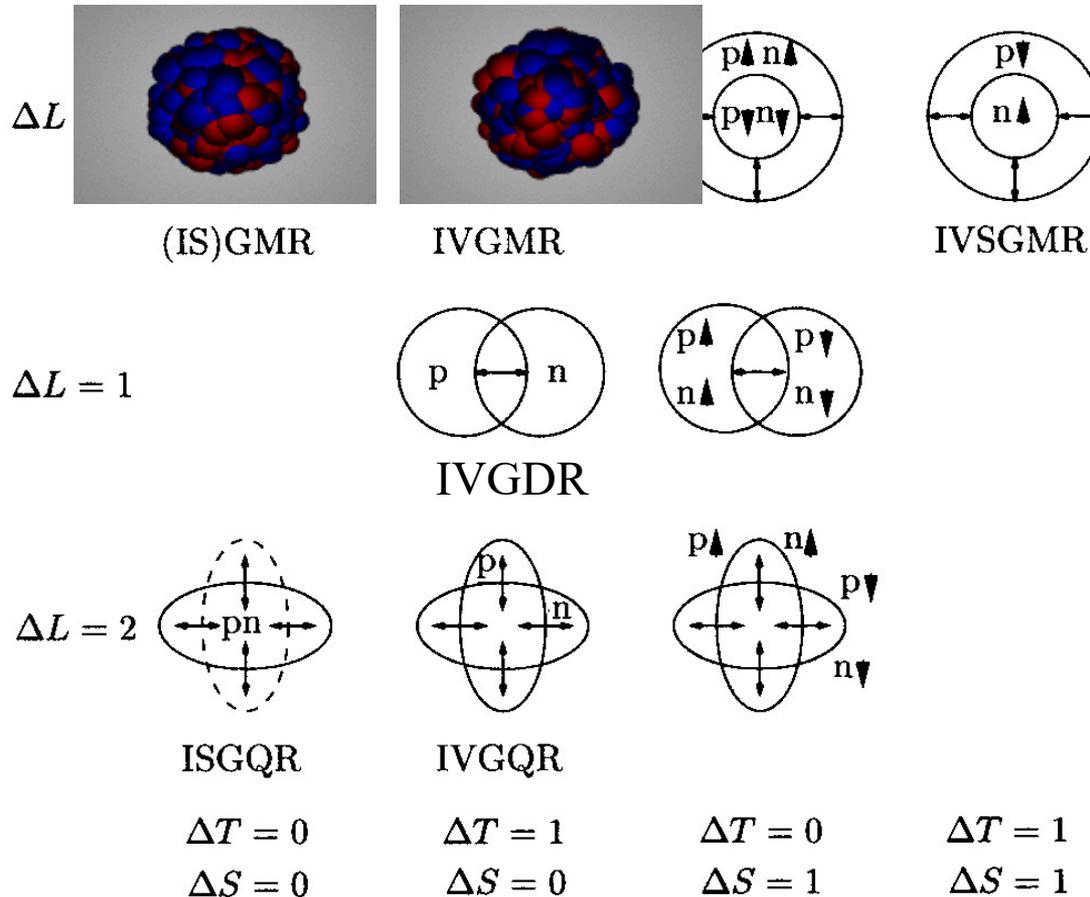
## ii) Inelastic scattering

(e,e'), (p,p'), ( $\alpha,\alpha'$ ), Heavy-ion  $\longrightarrow$  Higher multipolarities



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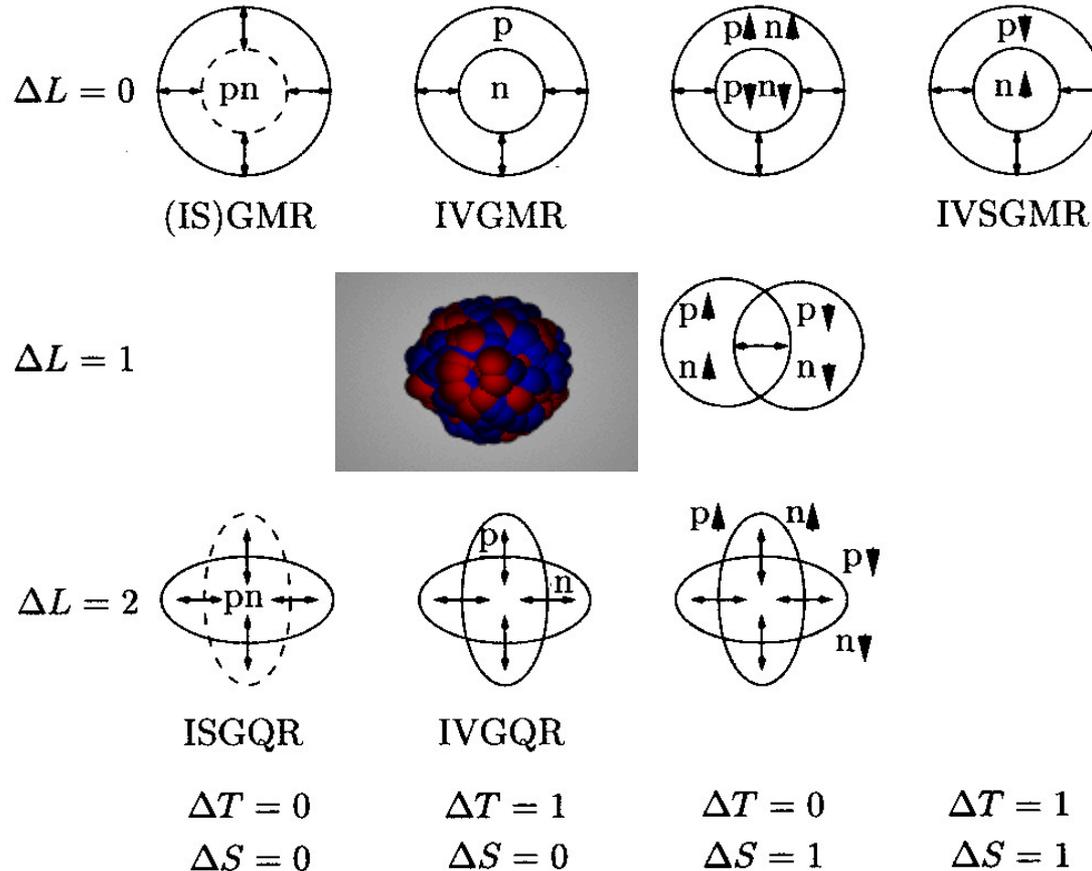


movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

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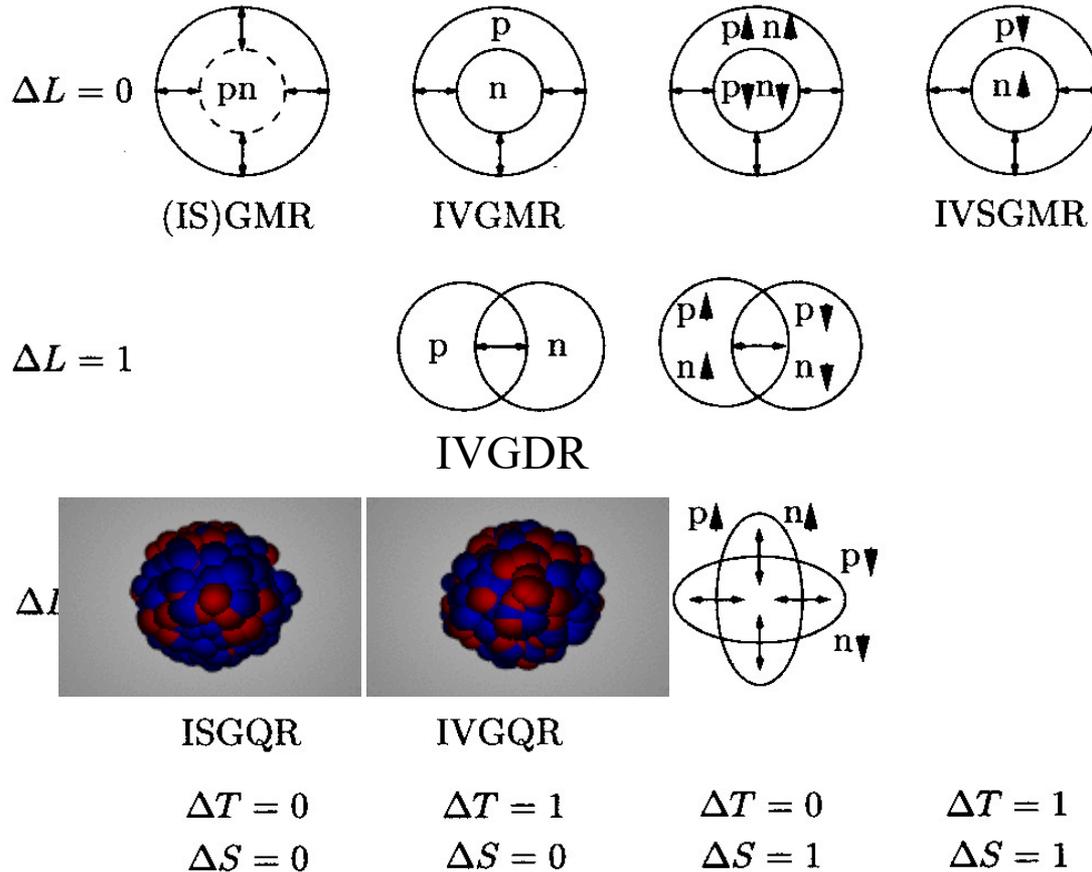


movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

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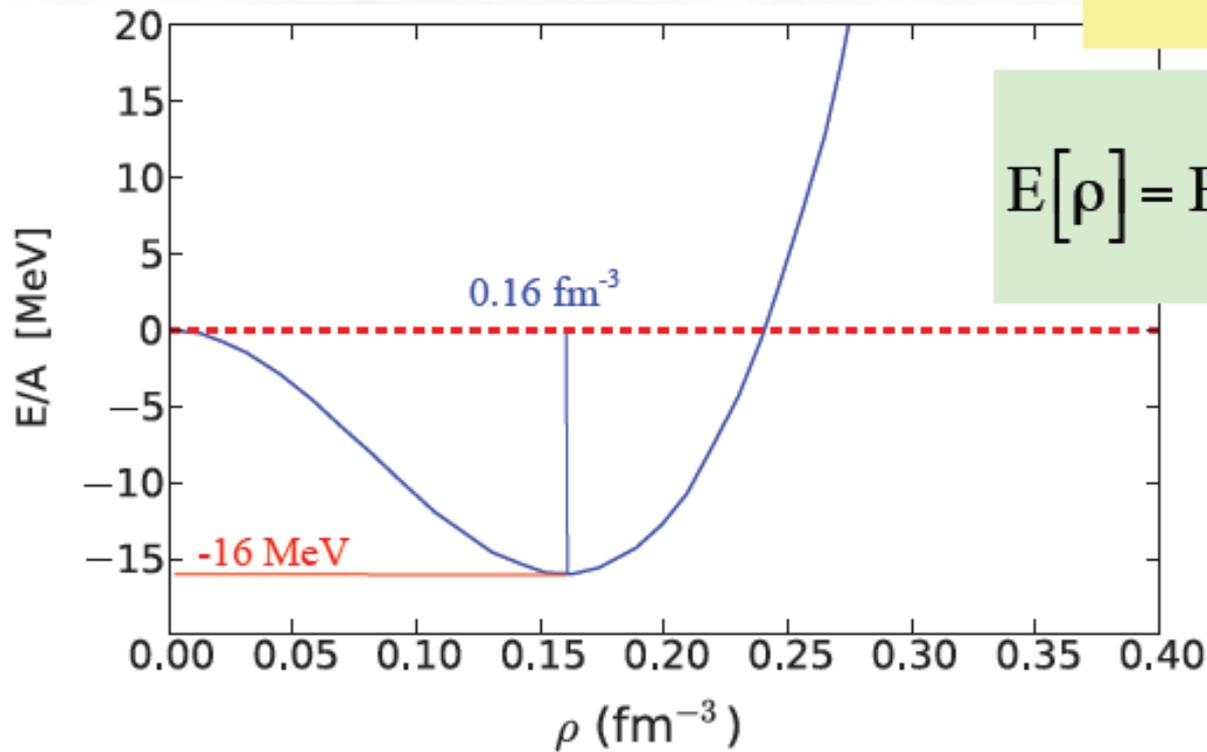
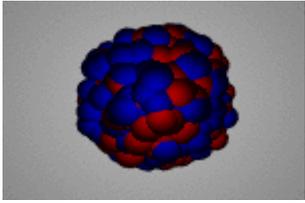
(e,e'), (p,p'), ( $\alpha,\alpha'$ ), Heavy-ion  $\longrightarrow$  Higher multipolarities



(note)  $\Delta L = 2 \longrightarrow \Delta N = 2$  Giant Resonance (GQR)

$\Delta N = 0$  Low-lying state

# EOS of infinite nuclear matter

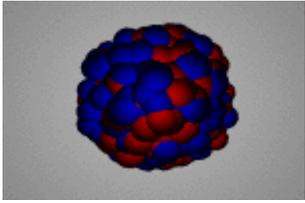


$$K_{\infty} = 9\rho^2 \left. \frac{d^2[E(\rho)/\rho]}{d\rho^2} \right|_{\rho_0}$$

$$E[\rho] = E[\rho_0] + \frac{1}{18} K_{\infty} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2$$

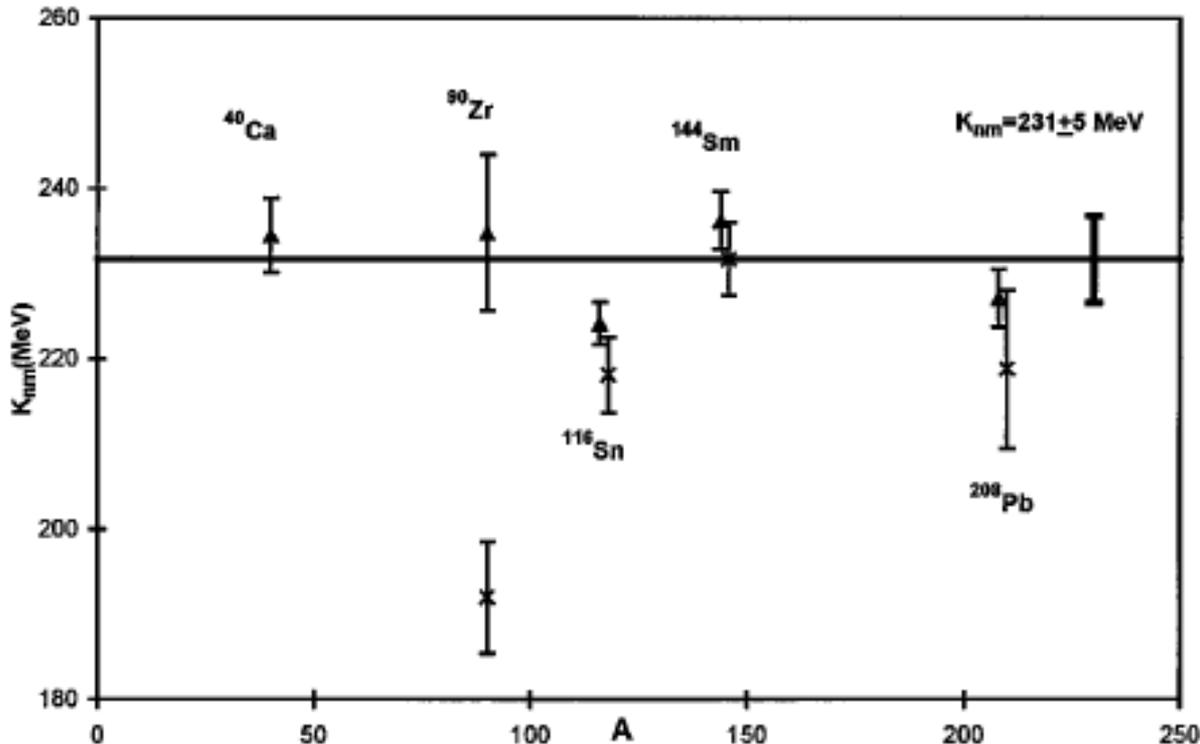
cf. 中性子星の大きさ  
や重さ(MR曲線)

# Isoscalar giant monopole resonances (breathing mode)



$$E_{\text{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m \langle r^2 \rangle}}$$

J.P. Blaizot,  
Phys. Rep. 64 ('80) 171



$K \sim 231 \pm 5 \text{ MeV}$

## レポート問題1 (×切: 12月4日(土))

「振動状態」の量子力学的意味を考えるために  $H|\psi_n\rangle = E_n|\psi_n\rangle$  で記述される系を考えよう。

- 1) 時刻  $t=0$  で系が基底状態 ( $n=0$ ) にいる場合、時刻  $t$  において波動関数の2乗

$$\rho(x, t) = |\Psi(x, t)|^2$$

が  $t$  に依らないことを示せ。

- 2) 時刻  $t=0$  で系が  $\Psi(x, t=0) = \psi_0(x) + \alpha\psi_1(x)$  という状態にいるとき、時刻  $t$  における

$$\rho(x, t) = |\Psi(x, t)|^2$$

を求めよ。ただし、 $|\alpha|$  は1に比べて十分小さいとし、 $|\alpha|^2$  の項は無視してよい。

このとき、 $\delta\rho(x, t) \equiv |\Psi(x, t)|^2 - |\psi_0(x)|^2$

は  $t$  の関数として振動するが、その周期を求めよ。