

集団励起の微視的理論

原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に参与)
- ✓ 集団励起(多くの核子が集団として励起に参与)

集団励起を微視的に理解
してみる
(集団励起をミクロに見て
みるとどうなっているのか?)

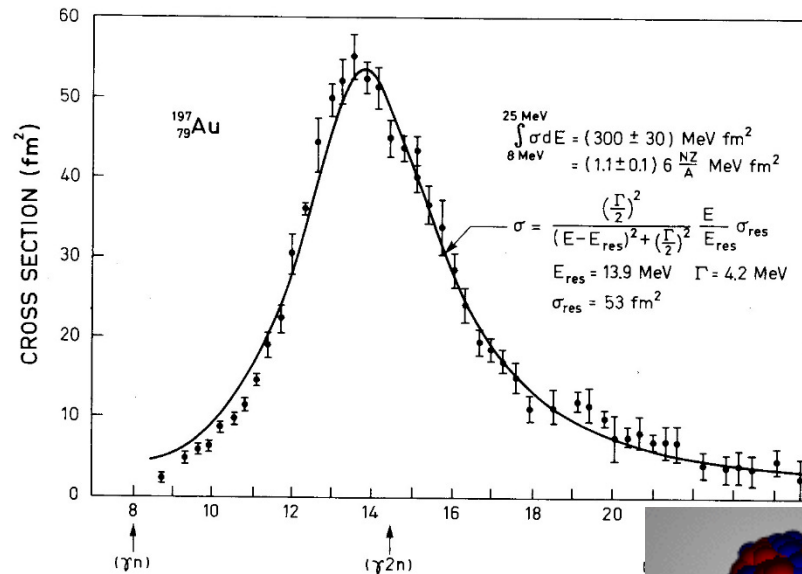
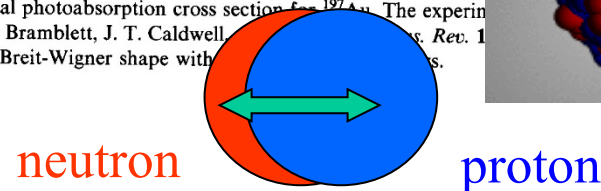
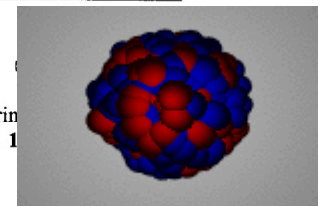


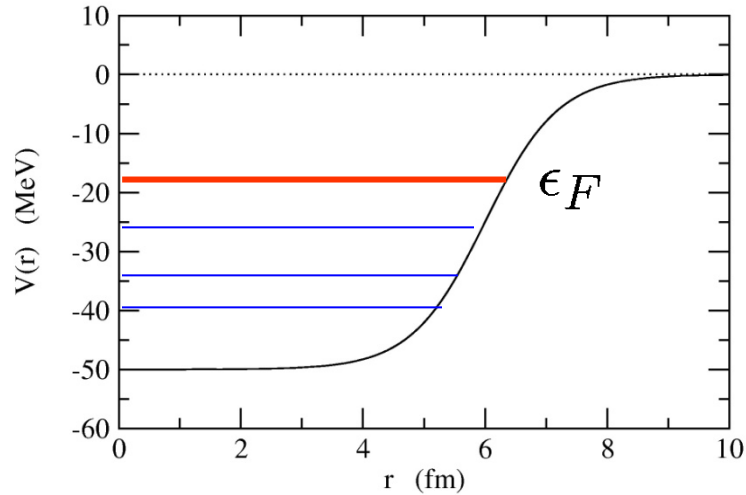
Figure 6-18 Total photoabsorption cross section for ¹⁹⁷Au. The experim. data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, Phys. Rev. 171, 1055 (1968). The solid curve is of Breit-Wigner shape with $E_{res} = 13.9$ MeV, $\Gamma = 4.2$ MeV, and $\sigma_{res} = 53$ fm².



集団励起の例: 巨大双極子共鳴

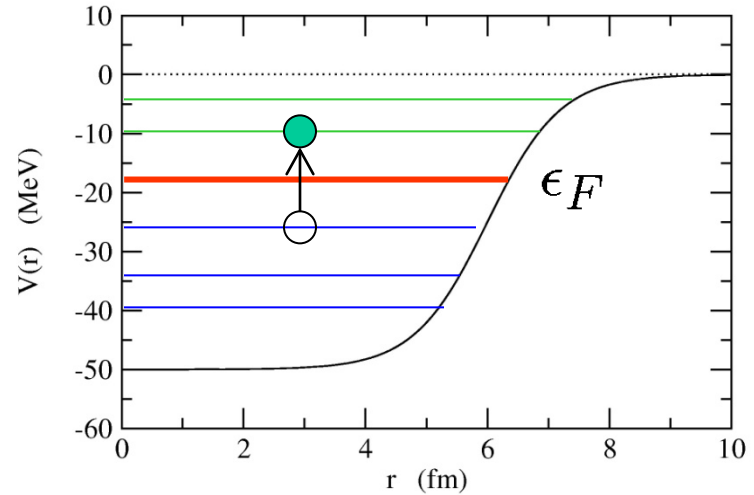
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle = \prod_h a_h^\dagger |0\rangle$$

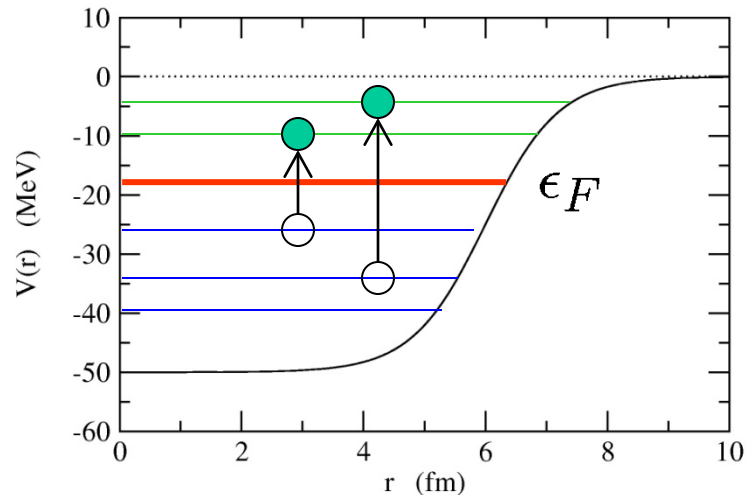
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$

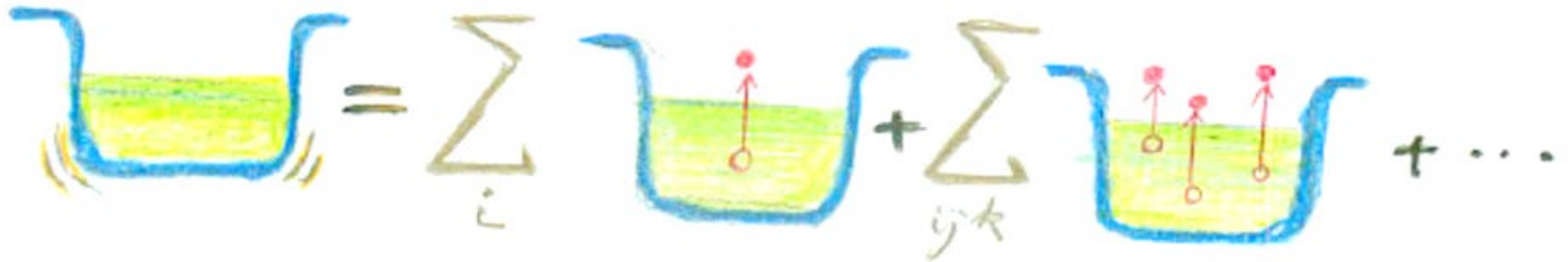
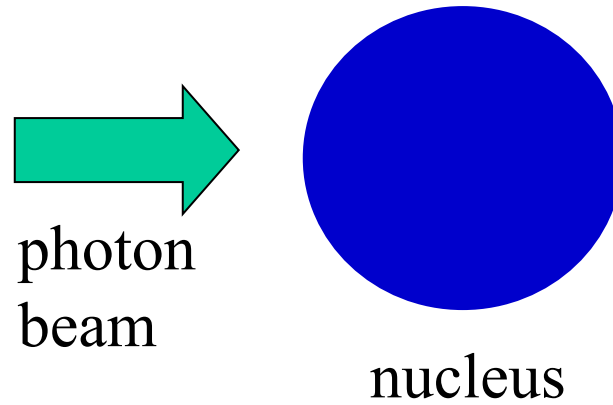


Tamm-Dancoff Approximation

$$\begin{aligned}\text{Assume: } |\nu\rangle = Q_\nu^\dagger |HF\rangle &= \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \\ &\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle\end{aligned}$$

(superposition of 1p1h states)

原子核を外場により揺らしてみると何が起こるのか？



スライド：松柳研一氏

Tamm-Dancoff Approximation

$$\begin{aligned}\text{Assume: } |\nu\rangle = Q_\nu^\dagger |HF\rangle &= \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \\ &\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle\end{aligned}$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$



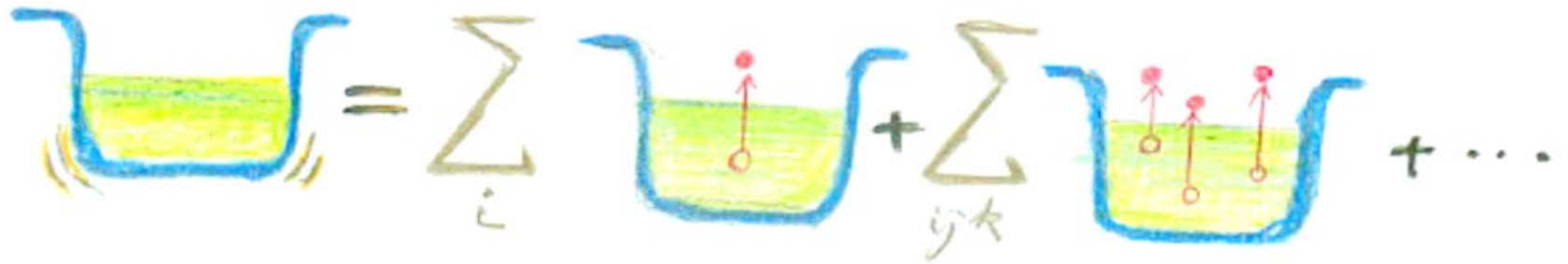
$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

residual
interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

Tamm-Dancoff equation; 1p1h の空間でハミルトニアンを対角化

残留相互作用の意味



スライド: 松柳研一氏

$$V(\mathbf{r}) \sim \int d\mathbf{r}' v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}')$$

vibration: $\rho = \rho_0(\mathbf{r}) \rightarrow \rho_0(\mathbf{r}) + \delta\rho(\mathbf{r}, t)$

residual
interaction

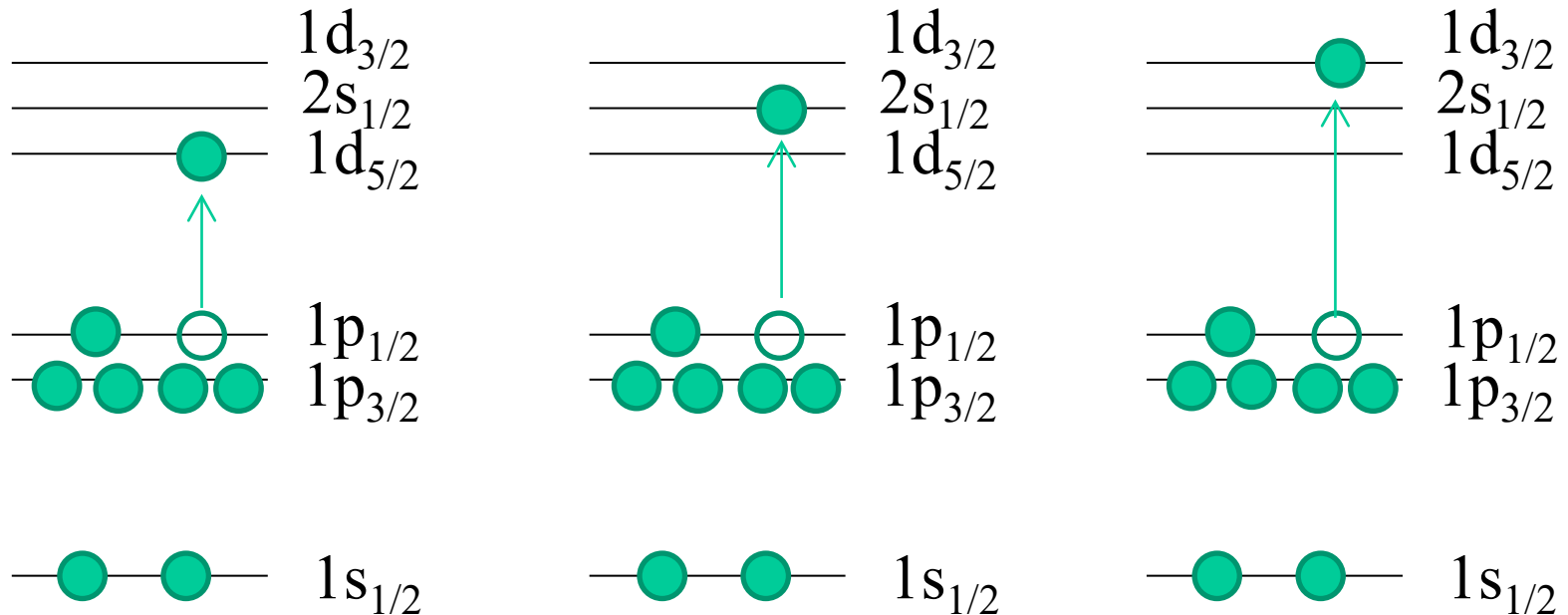
TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph' | \bar{v} | hp' \rangle = g$$

for 3 ph configurations:

(例えば)



TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph' | \bar{v} | hp' \rangle = g$$

for 3 ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization:

$$\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$$

TDA on a schematic model

$$\begin{pmatrix} \epsilon + g & g & g \\ g & \epsilon + g & g \\ g & g & \epsilon + g \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\epsilon + 3g) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

全ての状態が同位相で寄与
=コヒーレントな重ね合わせ

他の固有状態:

$$\begin{pmatrix} \epsilon + g & g & g \\ g & \epsilon + g & g \\ g & g & \epsilon + g \end{pmatrix} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

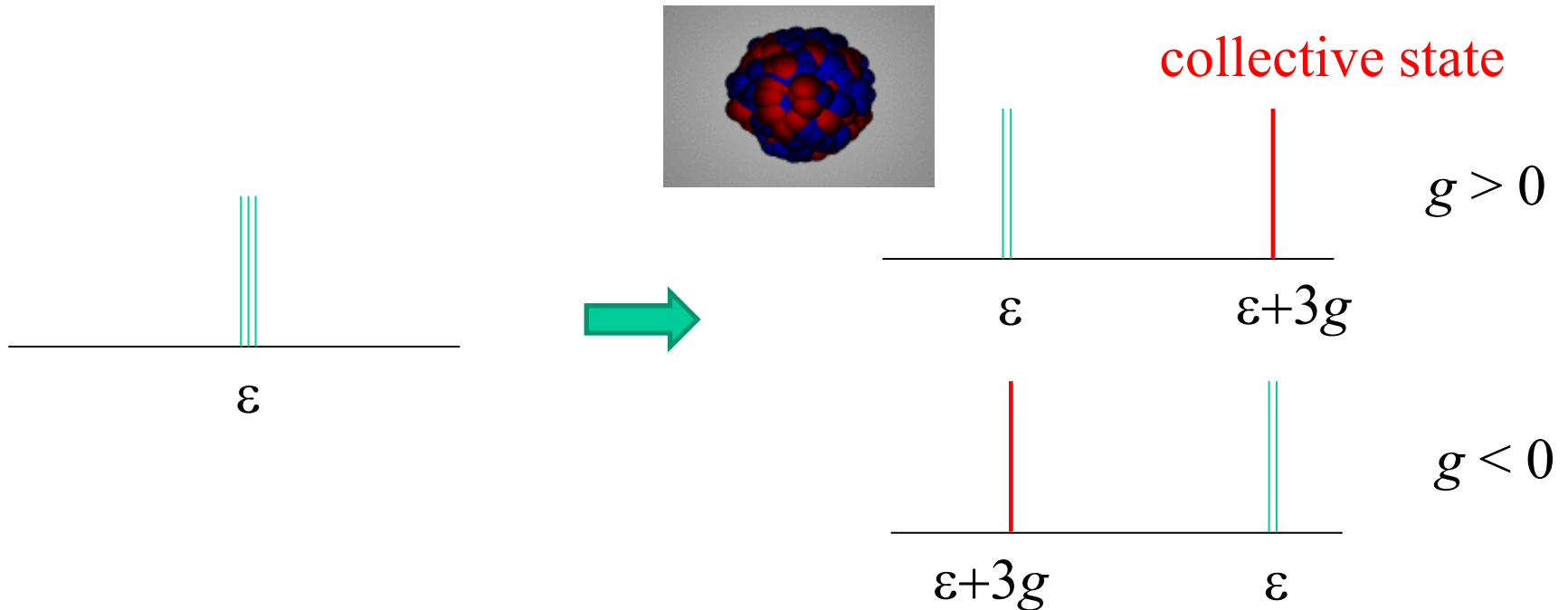
$$\begin{pmatrix} \epsilon + g & g & g \\ g & \epsilon + g & g \\ g & g & \epsilon + g \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

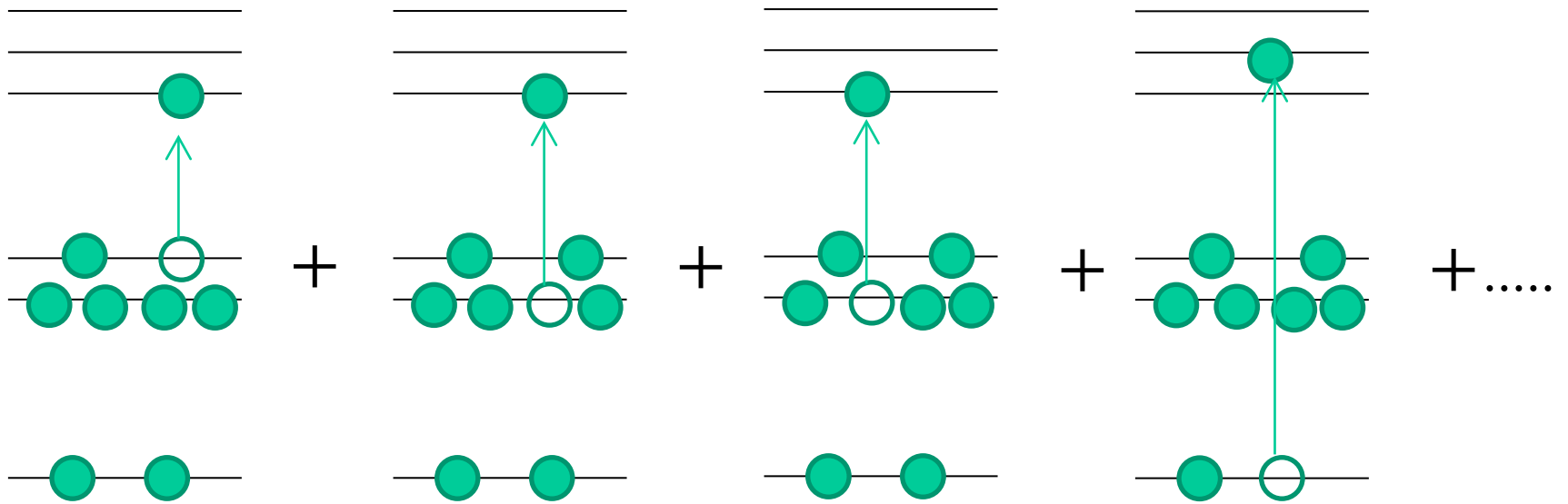
位相がそろっていない

TDA on a schematic model

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization: $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$





複数の粒子・空孔状態を**コヒーレント**に重ね合わせることによって
多数の核子が励起に関与していることを表現する

$$|\nu\rangle = \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

➡ $\left| \left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle \right|^2 = \left(\sum_{ph} f_{ph} X_{ph} \right)^2$ 干渉項がすべて同符号で寄与

TDA on a separable interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$


Separable interaction: $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$


(separable interaction)


$$H\psi = E\psi; \quad \psi = \sum_i C_i \phi_i$$


$$\longrightarrow \sum_j H_{ij} C_j = E C_i; \quad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j$ (separable form)


$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$


$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$


$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T$$


$$\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}$$

(separable interaction)

$$H\psi = E\psi; \quad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \sum_j H_{ij} C_j = E C_i; \quad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j \longrightarrow$

$$\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}$$

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \lambda D_{ph} D_{p'h'}^*$$

$$\longrightarrow \frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$


(TDA dispersion relation)


TDA on a schematic model


Separable interaction: $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$

Tamm-Dancoff equation: $\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$


$$(\epsilon_{ph} - E) X_{ph} + \lambda D_{ph} \cdot T = 0 \quad T \equiv \sum_{ph} D_{ph}^* X_{ph}$$


$$X_{ph} = -\lambda \frac{D_{ph} T}{\epsilon_{ph} - E}$$


$$T = \sum_{ph} D_{ph}^* X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

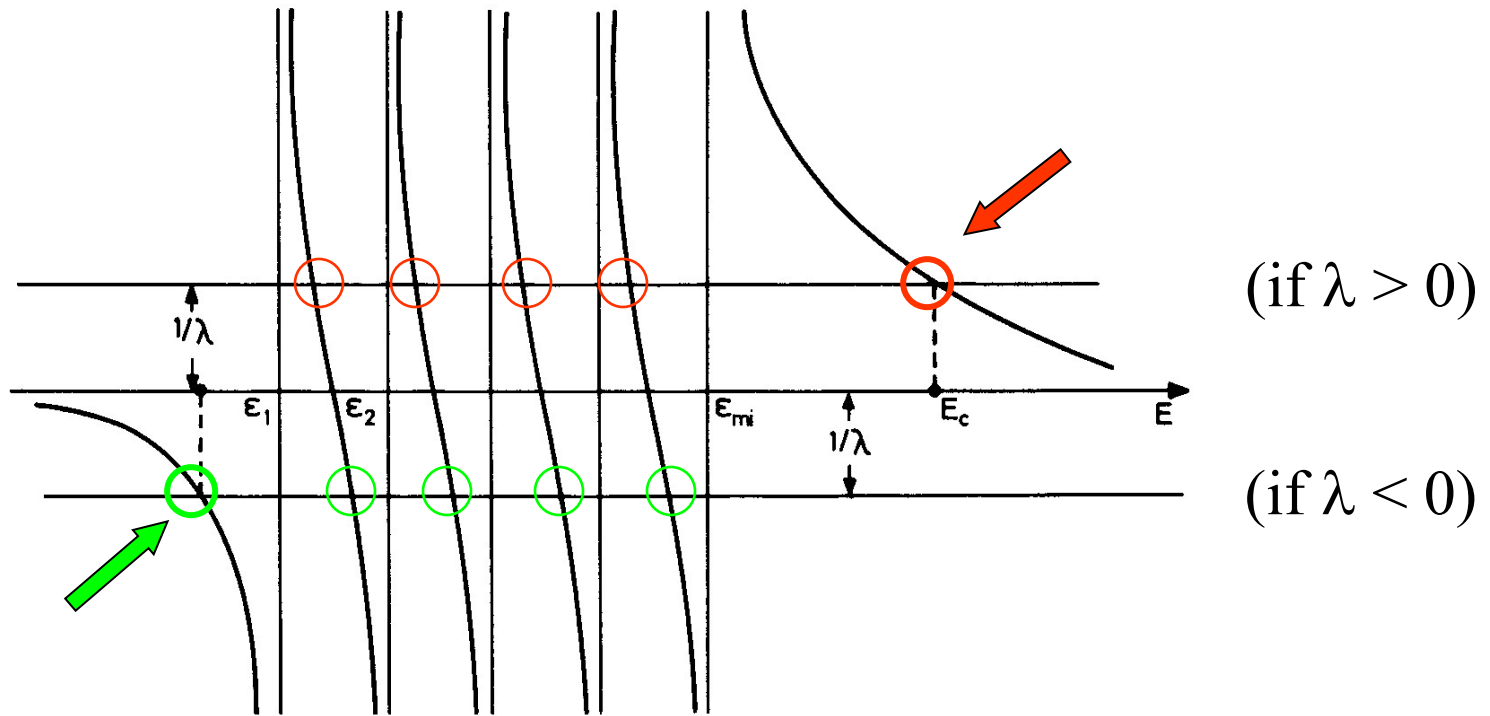


Figure 8.4. Graphical solution of Eq. (8.18).

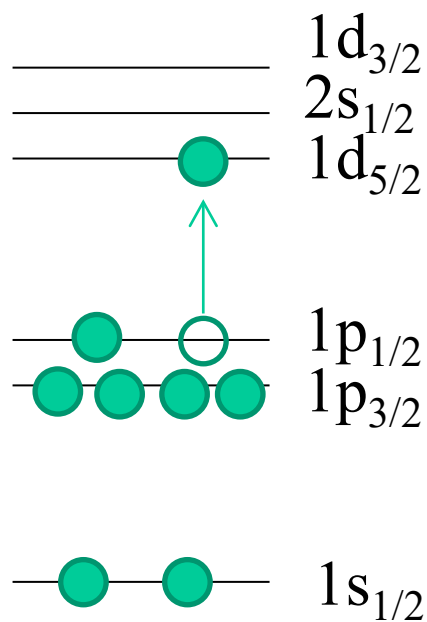
(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon + \lambda \sum_{ph} |D_{ph}|^2, \quad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superposition of 1p1h states

原子核の励起状態

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一粒子励起の例

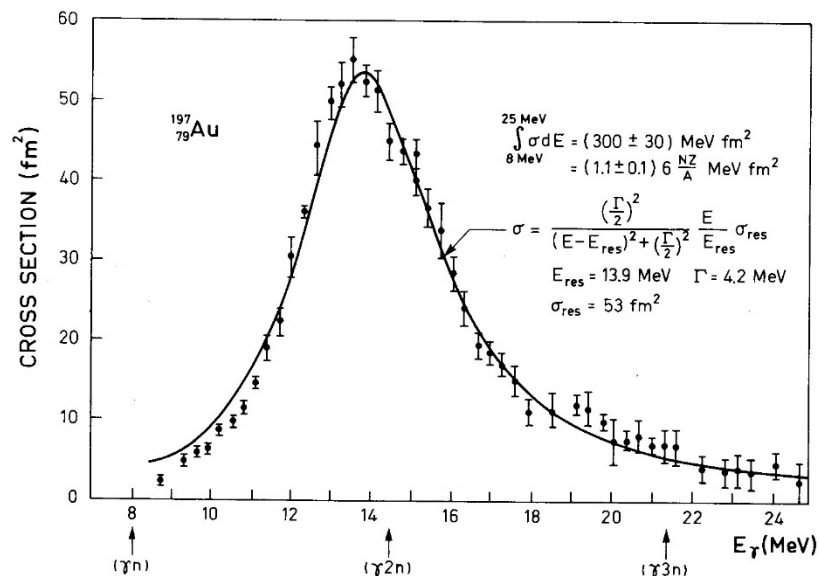
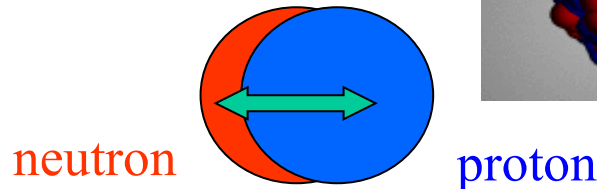


Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



集団励起の例: 巨大双極子共鳴

Giant Dipole Resonance (GDR) 巨大双極子共鳴

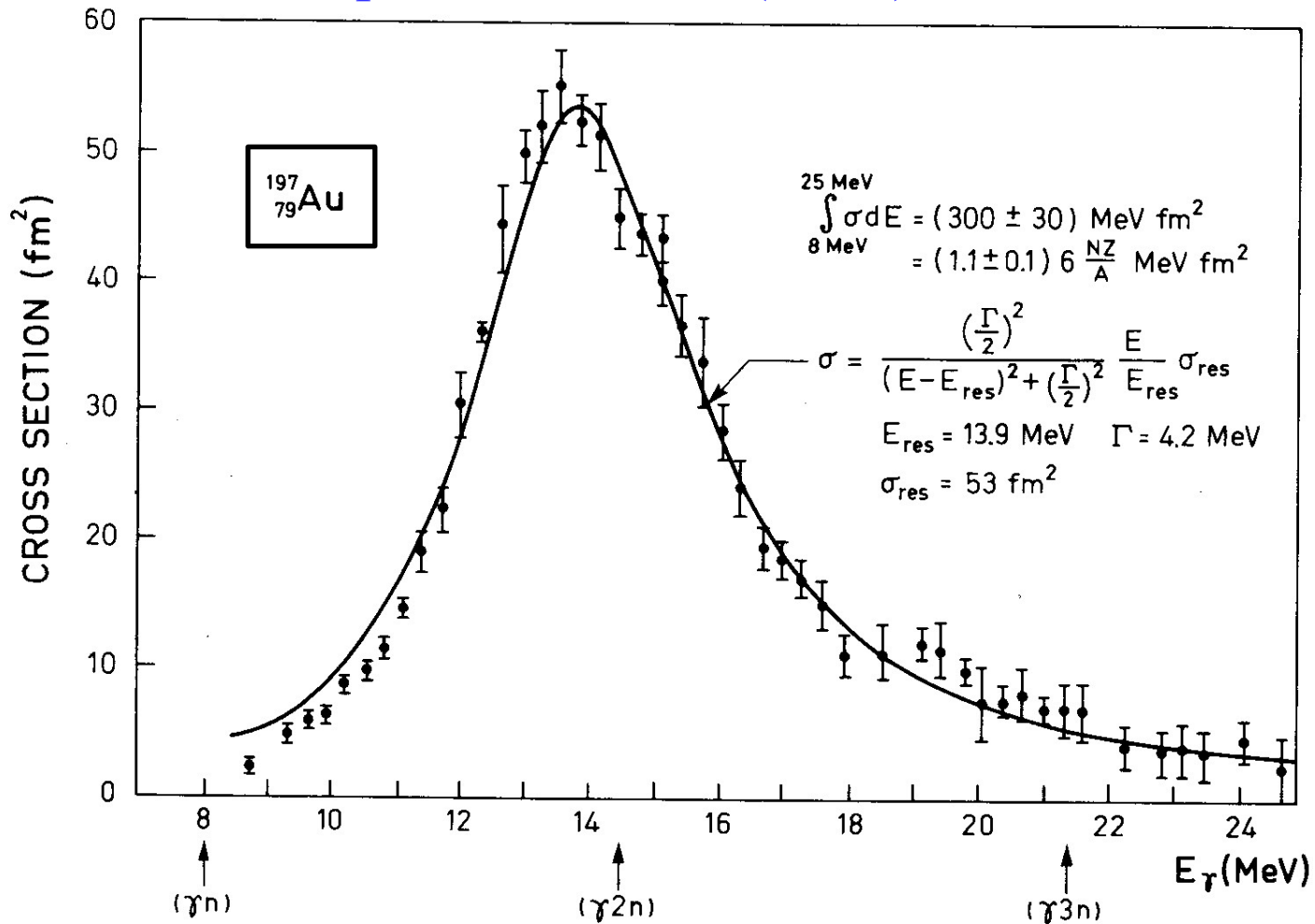


Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

$$\text{cf. } 41 \times 197^{-1/3} = 7.05 \text{ MeV} \rightarrow 14 \text{ MeV}$$

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

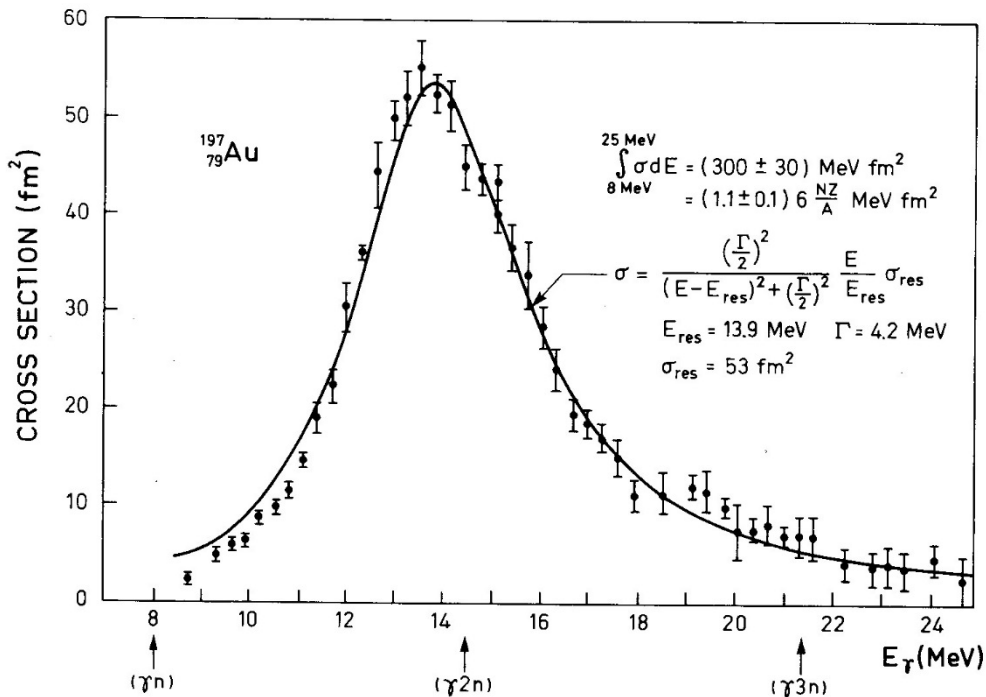
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 41 A^{-1/3}$

IS GQR: $E \sim 65 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41 A^{-1/3}$ (MeV)



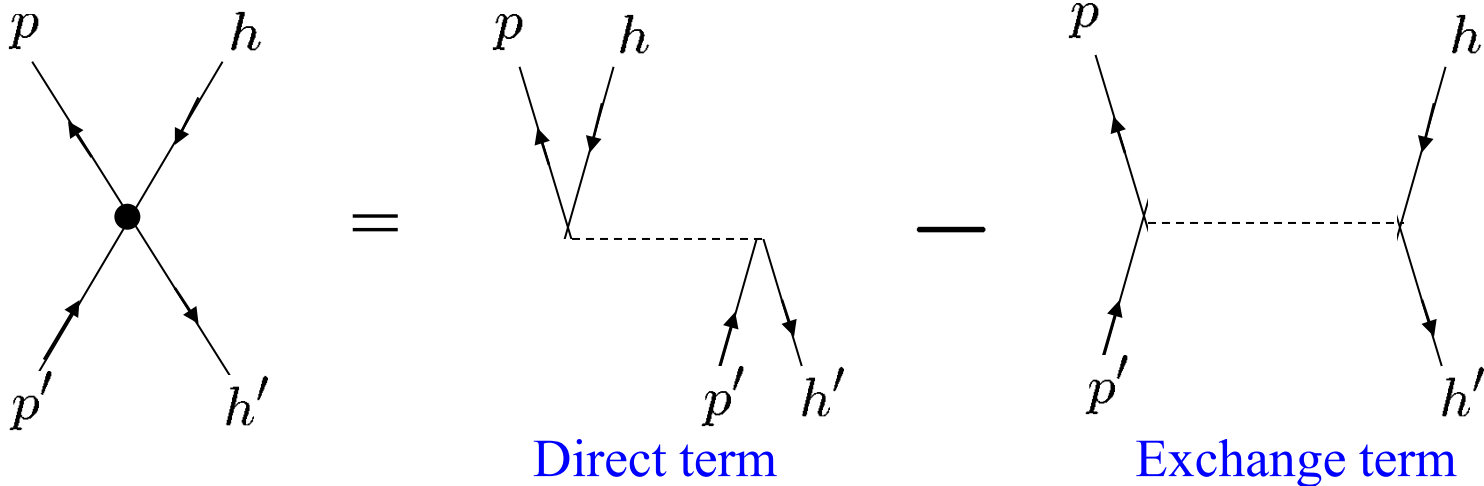
^{197}Au

$E_{\text{GDR}} = 14$ (MeV)

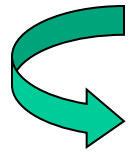
$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$

~ 7 (MeV)

$$\langle ph^{-1} | \bar{v} | p'h'^{-1} \rangle = \langle ph' | \bar{v} | hp' \rangle = \langle ph' | v | hp' \rangle - \langle ph' | v | p'h \rangle$$



$$\left\{ \begin{array}{l} \langle PP^{-1} | \bar{v} | PP^{-1} \rangle \sim \langle NN^{-1} | \bar{v} | NN^{-1} \rangle = D - E \\ \langle PP^{-1} | \bar{v} | NN^{-1} \rangle = D \quad (\text{no charge exchange}) \end{array} \right.$$



$$\langle IS | \bar{v} | IS \rangle = 2D - E \sim D$$

$$\langle IV | \bar{v} | IV \rangle = -E \sim -D$$

$$|IS\rangle \propto |NN^{-1}\rangle + |PP^{-1}\rangle$$

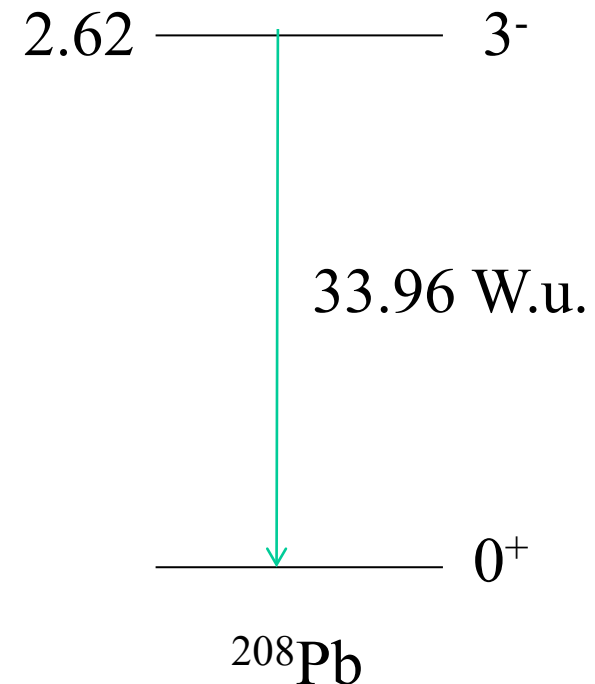
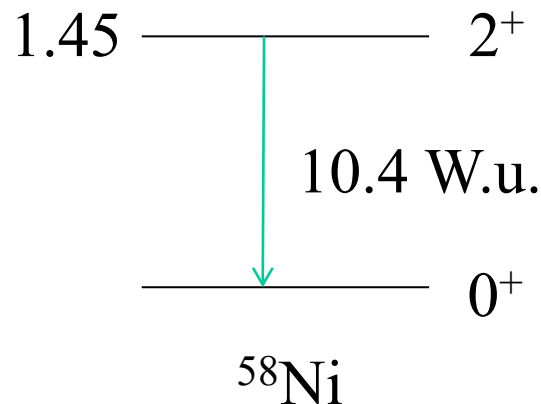
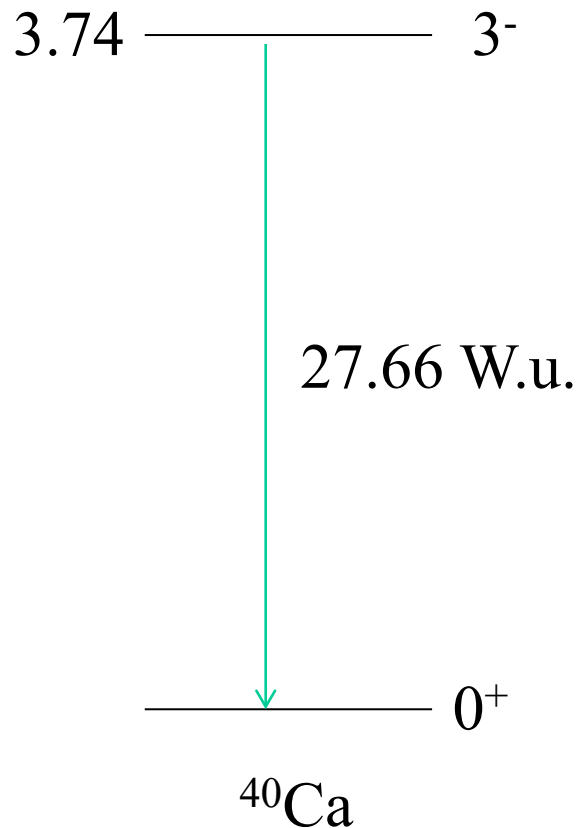
$$|IV\rangle \propto |NN^{-1}\rangle - |PP^{-1}\rangle$$

どれだけの核子が励起に関与しているのか?

Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \rightarrow I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi} \left(\frac{3}{\lambda + 3} \right)^2 \quad (e^2\text{fm}^{2\lambda})$$

exp data:



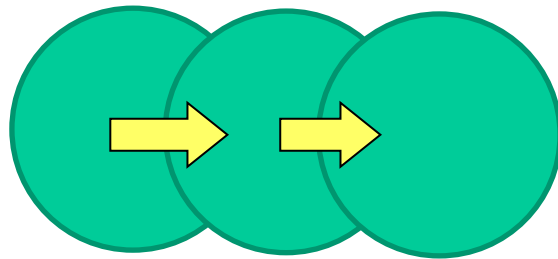
Spurious motion and RPA

Mean-Field Approximation \longleftrightarrow Broken symmetries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

\longrightarrow Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy \rightarrow zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} X_{ph} a_p^\dagger a_h |0\rangle \quad (\text{TDA})$$

 A better approximation:

the random phase approximation (RPA)

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left(X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

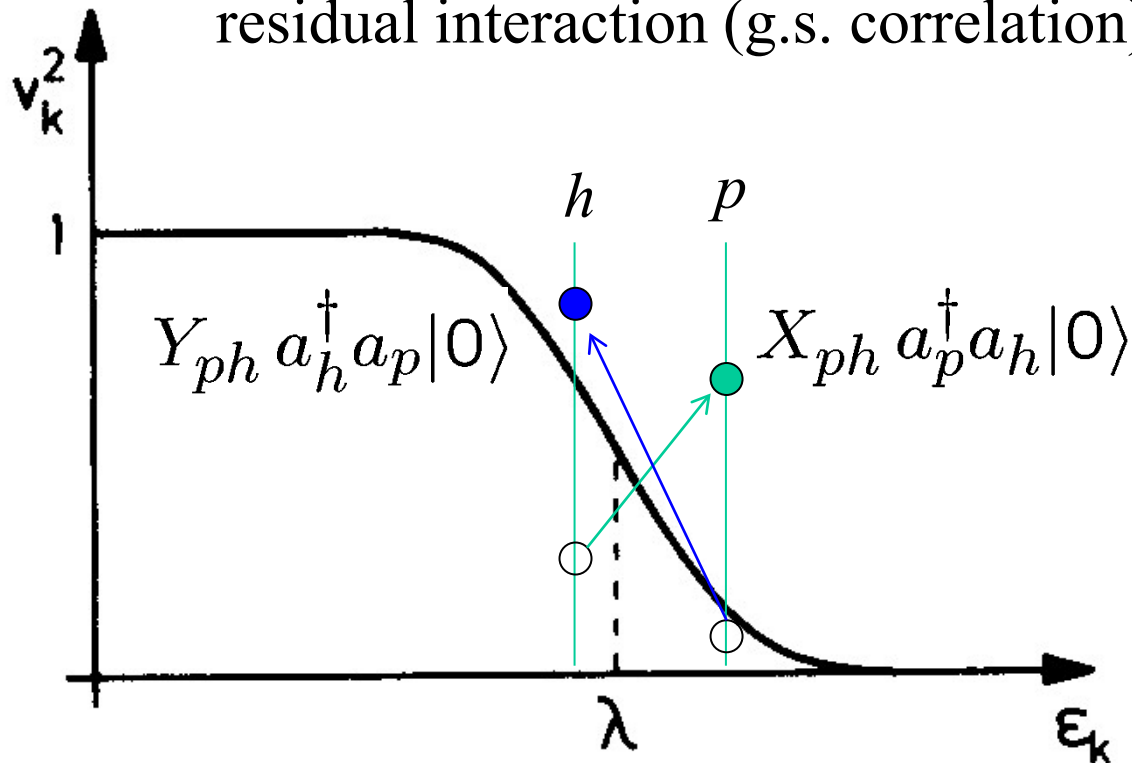
(superposition of 1p1h states)

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left(X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



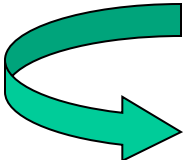
Random Phase Approximation: Historical note

D. Bohm and D. Pines, Phys. Rev. 92('53)609

The plasma oscillation in an infinite electron gas

$$H = \frac{1}{2m} \sum_i \left(\mathbf{p}_i + \frac{e}{c} \mathbf{A}(\mathbf{x}_i) \right)^2 + \frac{1}{8\pi} \int \mathbf{E}(\mathbf{x})^2 d\mathbf{x} - 2\pi n e^2 \sum_k \frac{1}{k^2}$$

$$\mathbf{A}(\mathbf{x}) = \sqrt{4\pi c^2} \sum_k \mathbf{q}_k \boldsymbol{\epsilon}_k e^{i\mathbf{k} \cdot \mathbf{x}}$$


$$\begin{aligned} \sum_i A(\mathbf{x}_i)^2 &= \sum_{ikl} \boldsymbol{\epsilon}_k \cdot \boldsymbol{\epsilon}_l q_k q_l e^{i(\mathbf{k} + \mathbf{l}) \cdot \mathbf{x}_i} \\ &\rightarrow \sum_{ik} q_k q_{-k} \quad (\mathbf{k} + \mathbf{l} = 0 \text{ only}) \end{aligned}$$