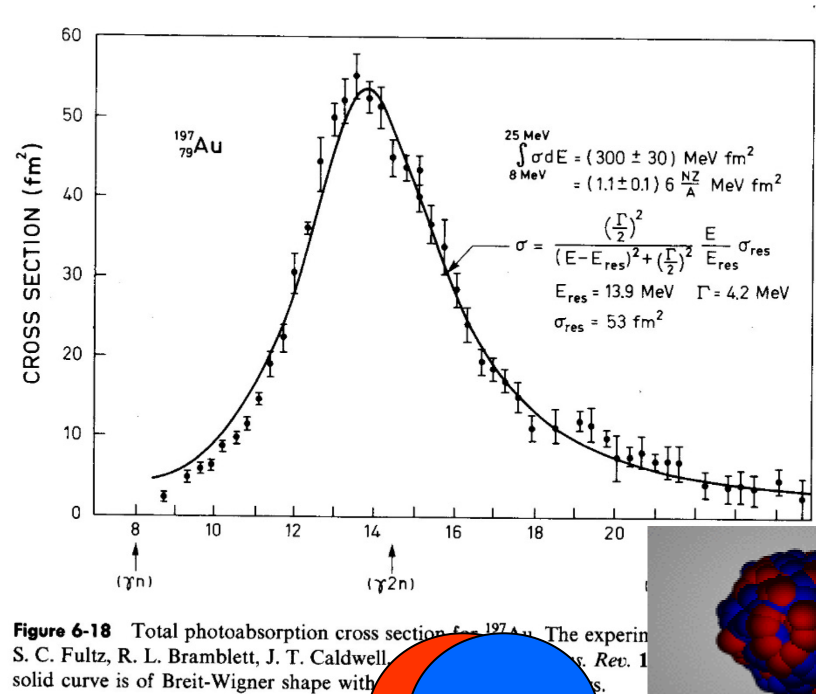
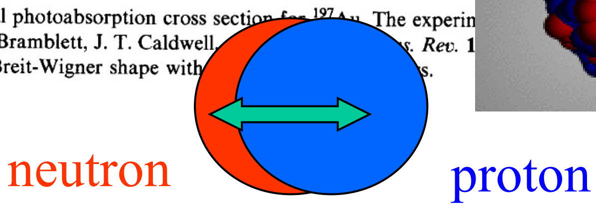
集団励起の微視的理論

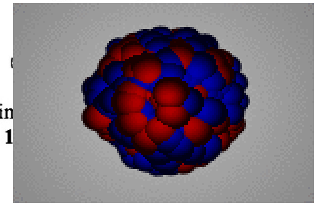
原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に関与)
- ✓ 集団励起(多くの核子が集団として励起に関与)

集団励起を微視的に理解してみる
(集団励起をミクロに見てみるとどうなっているのか?)

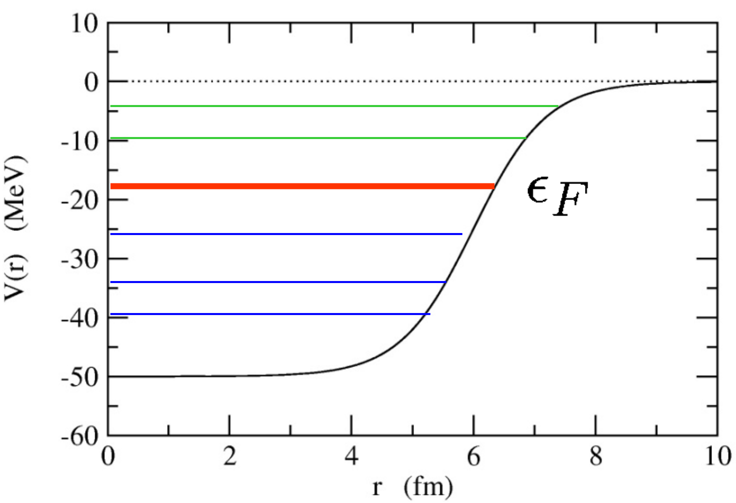






集団励起の例:巨大双極子共鳴

一般に:　$$|\Psi_k\rangle = \sum_i C_i |\Phi(SD)_i\rangle$$



一般に：$$|\Psi_k\rangle = \sum_i C_i |\Phi(SD)_i\rangle$$

タム・ダンコフ近似

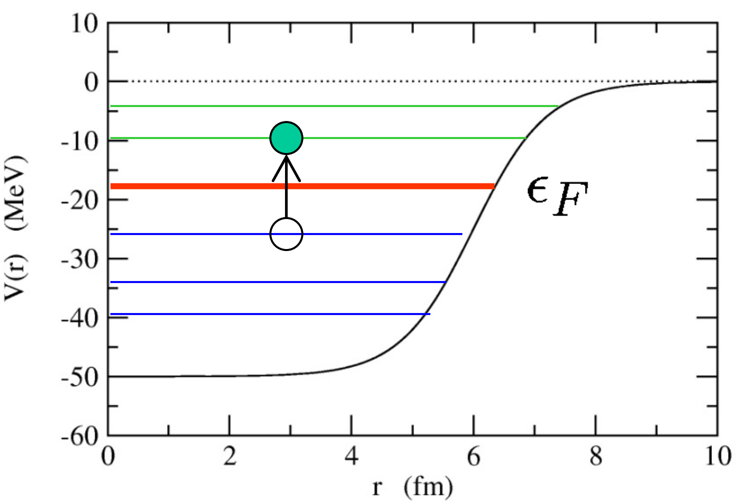
基底状態：$|HF\rangle$

励起状態：$$|\nu\rangle = Q^\dagger_\nu |HF\rangle = \sum_{ph} X_{ph}\, a^\dagger_p a_h |HF\rangle$$

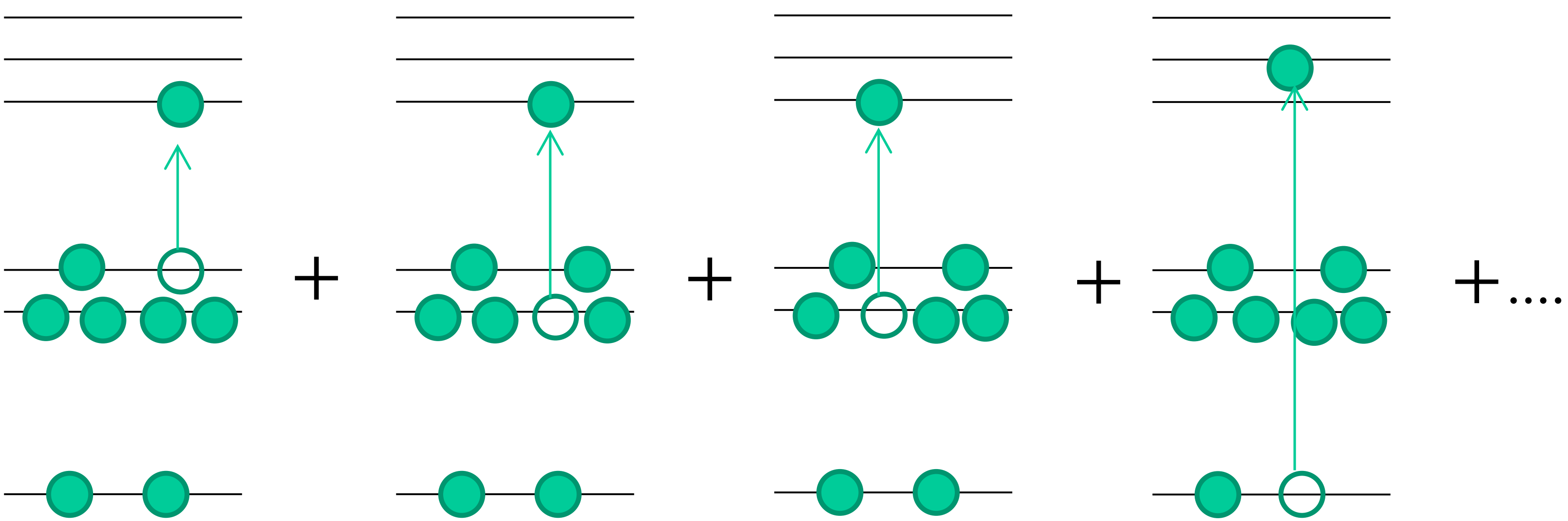
1 particle-1 hole (1p1h) state

$$\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

（1p1h 状態の重ね合わせ）



$$a^\dagger_p a_h |HF\rangle$$



複数の粒子・空孔状態をコヒーレントに重ね合わせることによって
多数の核子が励起に関与していることを表現する

$$\left|\left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle\right|^2 = \left(\sum_{ph} f_{ph} X_{ph} \right)^2$$

干渉項がすべて
同符号で寄与

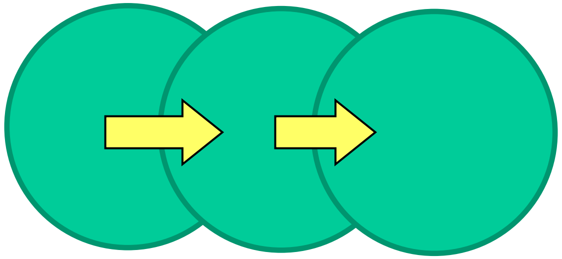
Spurious motion and RPA

Mean-Field Approximation Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy → zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|0\rangle \qquad \text{(TDA)}$$

A better approximation:

the random phase approximation (RPA)

$$\boxed{|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle}$$

(superposition of 1p1h states)

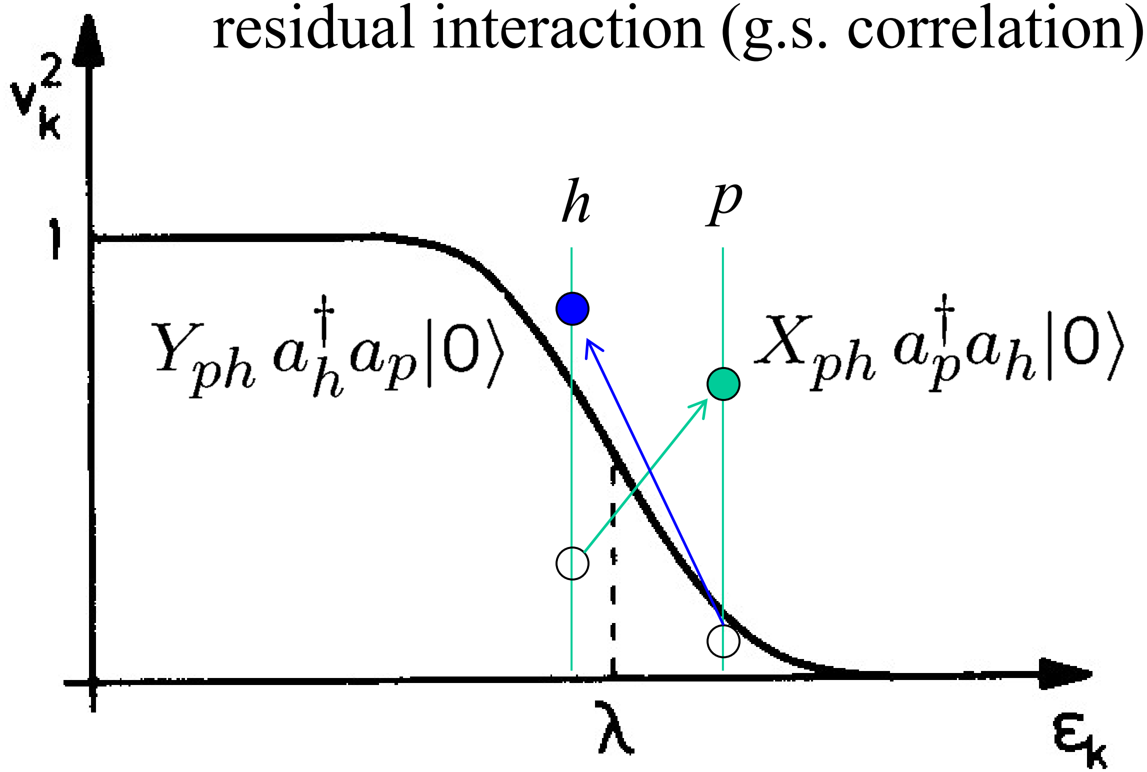
基底状態:　$Q_\nu|0\rangle = 0$　で定義。

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q^\dagger_\nu|0\rangle = \sum_{ph}\left(X_{ph}\,a^\dagger_p a_h - Y_{ph}\,a^\dagger_h a_p\right)|0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



A better approximation: **the random phase approximation (RPA)**

$$\boxed{|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle}$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

$$\Downarrow$$

$$\langle HF|[\delta Q, [H, Q_\nu^\dagger]]|HF\rangle = E_\nu\langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

$\rightarrow$ coupled equations for X and Y

$$\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu\langle HF|[\delta Q,Q_\nu^\dagger]|HF\rangle$$

$$Q_\nu^\dagger = \sum_{ph} X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p \qquad \delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

RPA equation:

$$\boxed{\begin{aligned} \sum_{p'h'} A_{ph,p'h'}X_{p'h'} + B_{ph,p'h'}Y_{p'h'} &= E_\nu X_{ph} \\ \sum_{p'h'} B^*_{ph,p'h'}X_{p'h'} + A^*_{ph,p'h'}Y_{p'h'} &= -E_\nu Y_{ph} \end{aligned}}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

or

$$\boxed{\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}}$$

Spurious motion in RPA

Mean-Field Approximation $\longleftrightarrow$ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

$\Rightarrow$ Zero mode (Nambu-Goldstone mode)

$$[H, \hat{O}] = 0$$

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

$\hat{O}$ is a solution of RPA with E=0

$$Q^\dagger = \hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

(note) $Q^\dagger_{\mathrm{TDA}} = \sum_{ph} O_{ph} a_p^\dagger a_h \Longrightarrow [H, Q^\dagger_{\mathrm{TDA}}] \neq 0$

Spurious motion in RPA

Mean-Field Approximation $\Longleftrightarrow$ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

$\Rightarrow$ Zero mode (Nambu-Goldstone mode)

RPA $$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

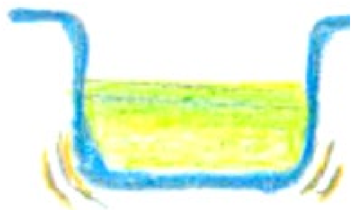
if $[H, \hat{O}] = 0$

Then $\hat{O}$ is a solution of RPA with E=0

The physical solutions are completely separated out from the spurious modes.

他のRPAの定式化

- 線形応答理論

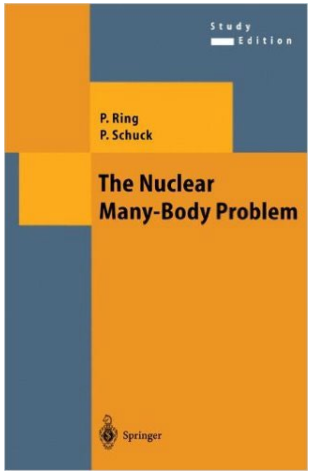


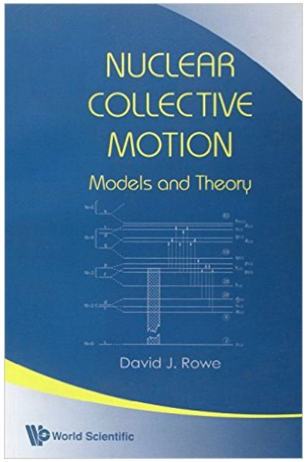
外場で原子核を揺すった時に、
原子核がどのように応答するか摂動論
を使って議論する
→固有モードを見つける

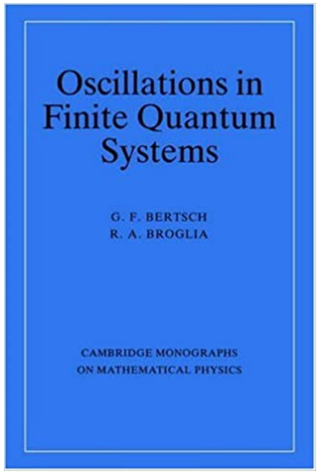
- 時間に依存するハートリー・フォック(TDHF)方程式を線形化

$$i\hbar\dot{\rho}(t) = [h[\rho], \rho] \quad \longleftarrow \quad \rho(t) = \rho_0 + \delta\rho(t)$$

詳しくは：







Comparison between Skyrme-(Q)RPA calculation and exp. data

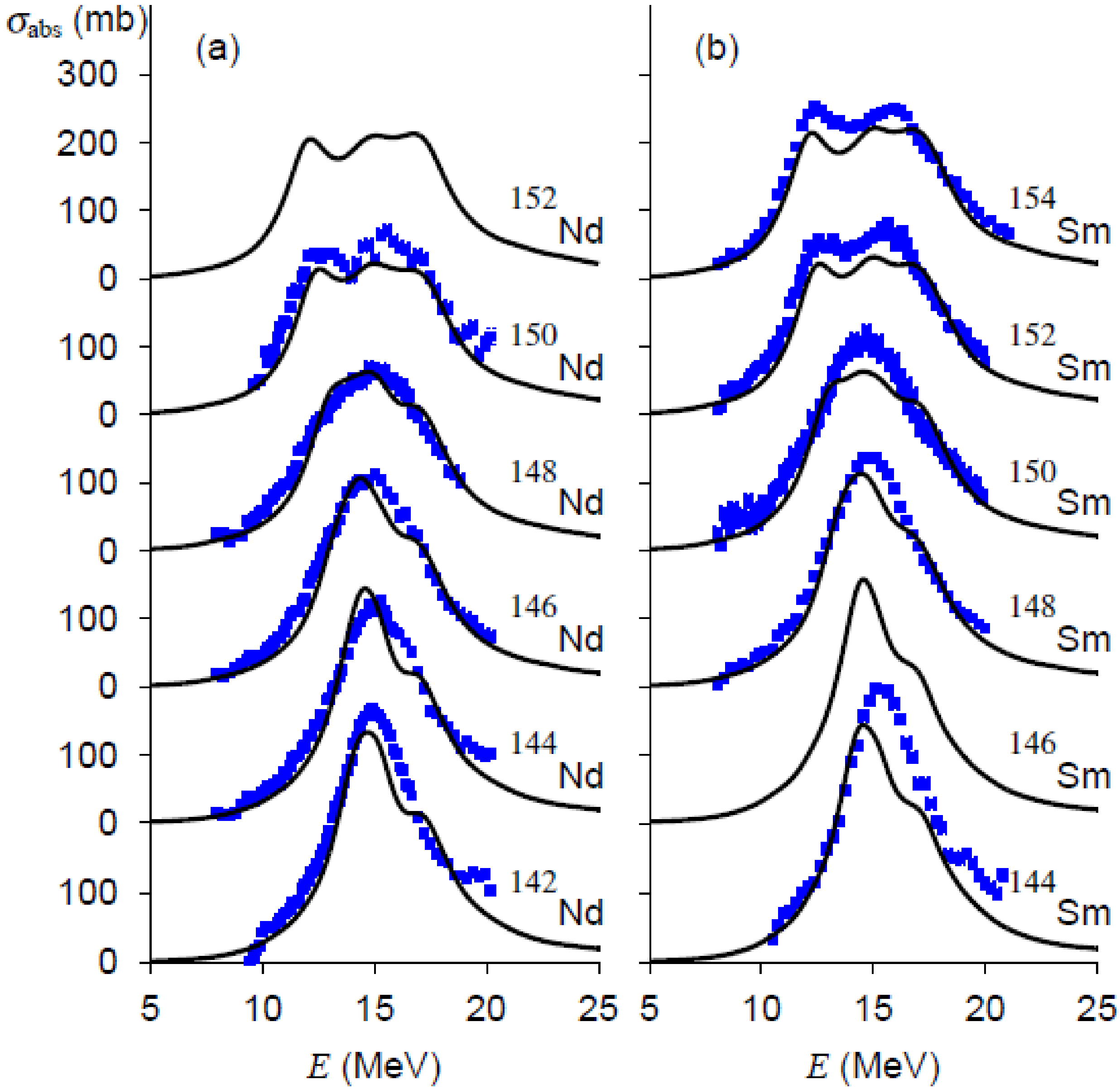
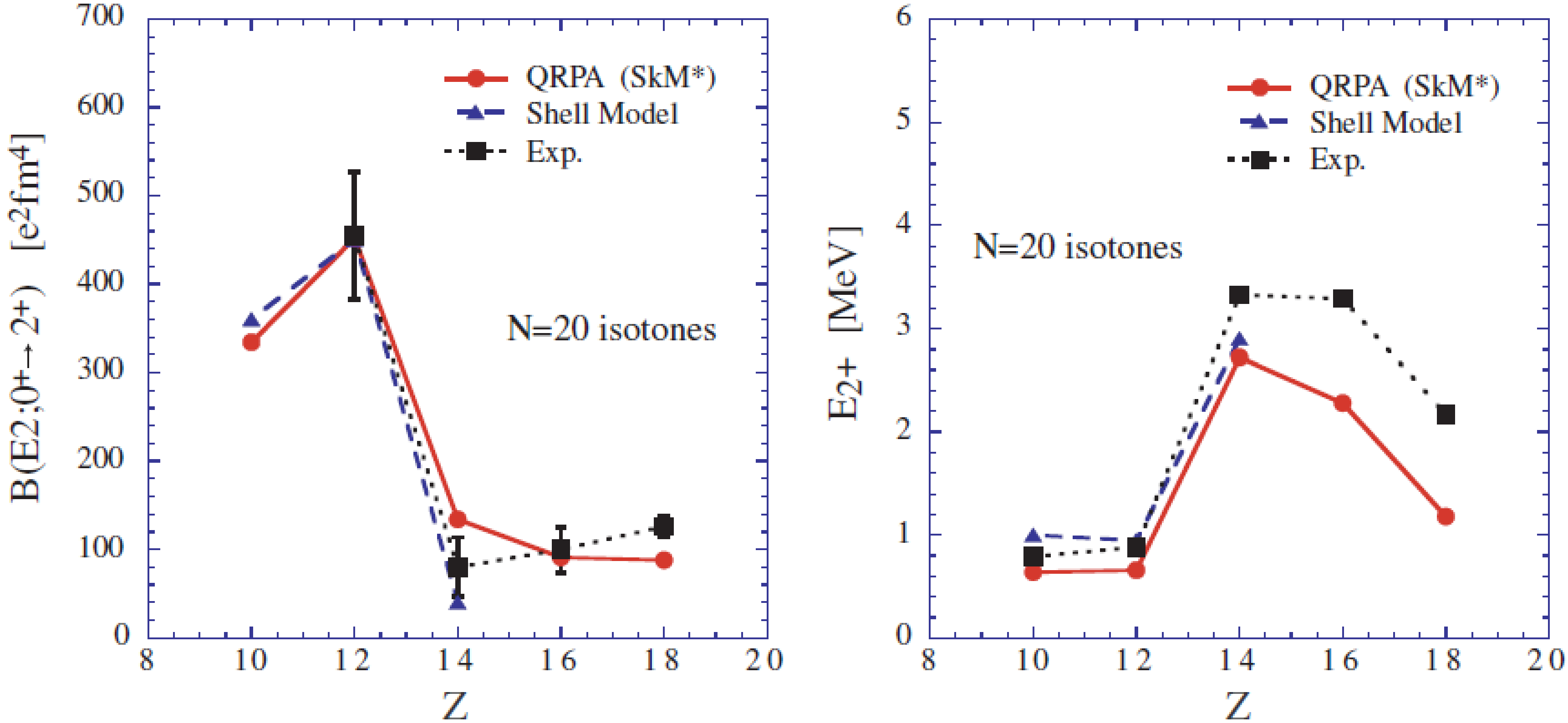


photo-absorption cross section (GDR)

K. Yoshida and T. Nakatsukasa, PRC83('11)021304



M. Yamagami and Nguyen Van Giai, PRC69 ('04) 034301

RPA on a schematic model

$$\boxed{\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E\begin{pmatrix} X \\ Y \end{pmatrix}}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

Separable interaction:

$$\begin{aligned} \langle ph'|\bar{v}|hp'\rangle &= \lambda D_{ph}D^*_{p'h'} \\ \langle pp'|\bar{v}|hh'\rangle &= \lambda D_{ph}D_{p'h'} \end{aligned}$$

$$\boxed{\frac{1}{\lambda} = \sum_{ph}\frac{|D_{ph}|^2}{E-\epsilon_{ph}} - \frac{|D_{ph}|^2}{E+\epsilon_{ph}}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation: $$\frac{1}{\lambda} = \sum_{ph}\frac{|D_{ph}|^2}{E-\epsilon_{ph}}$$

cf. タム・ダンコフ近似の場合：

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

Tamm-Dancoff equation: $\displaystyle\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$

$$(\epsilon_{ph} - E) X_{ph} + \lambda D_{ph} \cdot T = 0 \qquad T \equiv \sum_{ph} D^*_{ph} X_{ph}$$

$$X_{ph} = -\lambda \frac{D_{ph} T}{\epsilon_{ph} - E}$$

$$T = \sum_{ph} D^*_{ph} X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\boxed{\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}}$$

(TDA dispersion relation)

RPA on a schematic model

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$$
$$\langle pp'|\bar{v}|hh'\rangle = \lambda D_{ph} D_{p'h'}$$

Separable interaction:

$$\boxed{\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}}$$

(RPA dispersion relation)

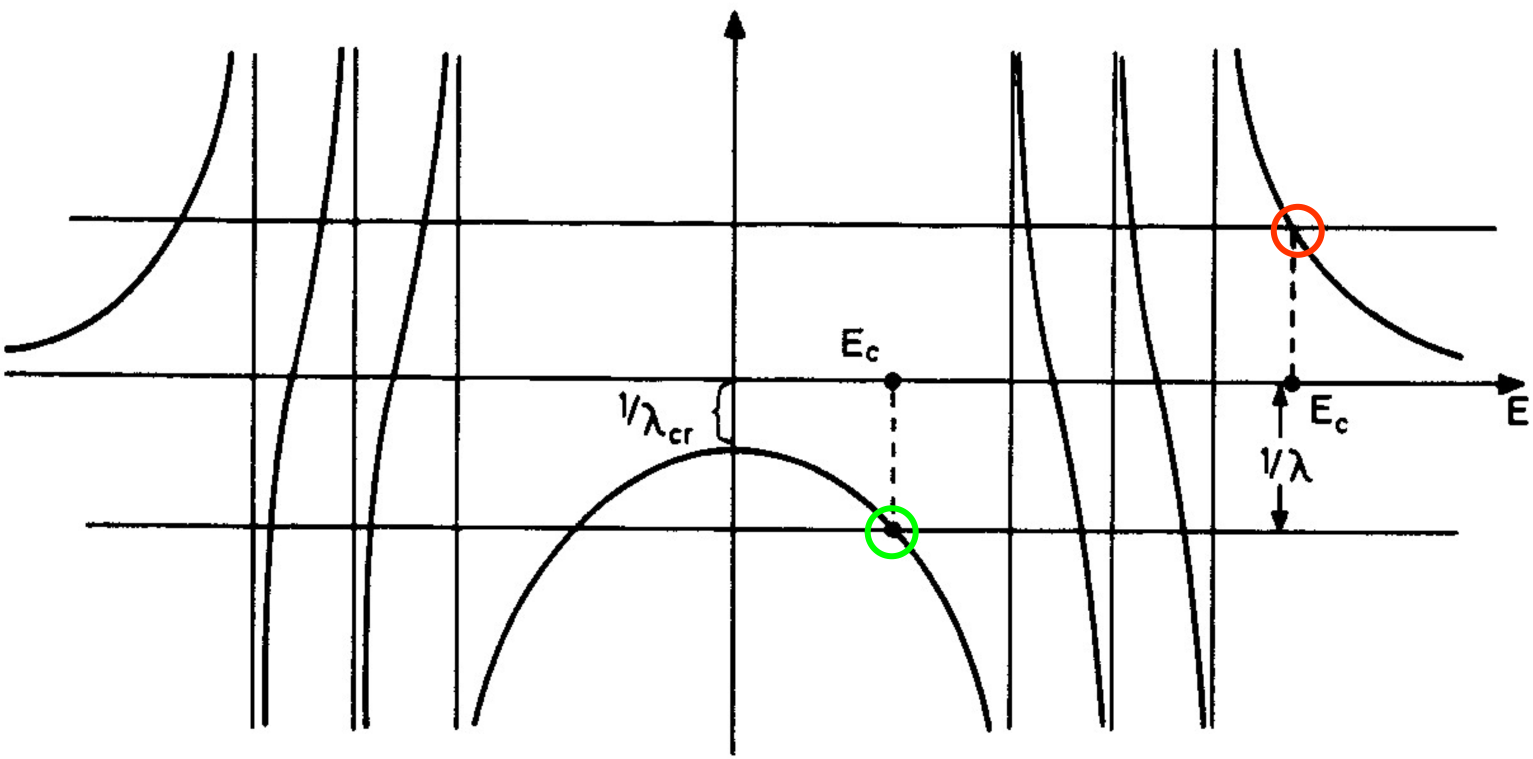


Figure 8.11. Graphical solution of the dispersion relation (8.135).

RPA on a schematic model

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph}D^*_{p'h'}$$
$$\langle pp'|\bar{v}|hh'\rangle = \lambda D_{ph}D_{p'h'}$$

Separable interaction:

$$\boxed{\frac{1}{\lambda} = \sum_{ph}\frac{|D_{ph}|^2}{E-\epsilon_{ph}} - \frac{|D_{ph}|^2}{E+\epsilon_{ph}}}$$

(RPA dispersion relation)

$\epsilon_{ph} = \epsilon$ のとき、

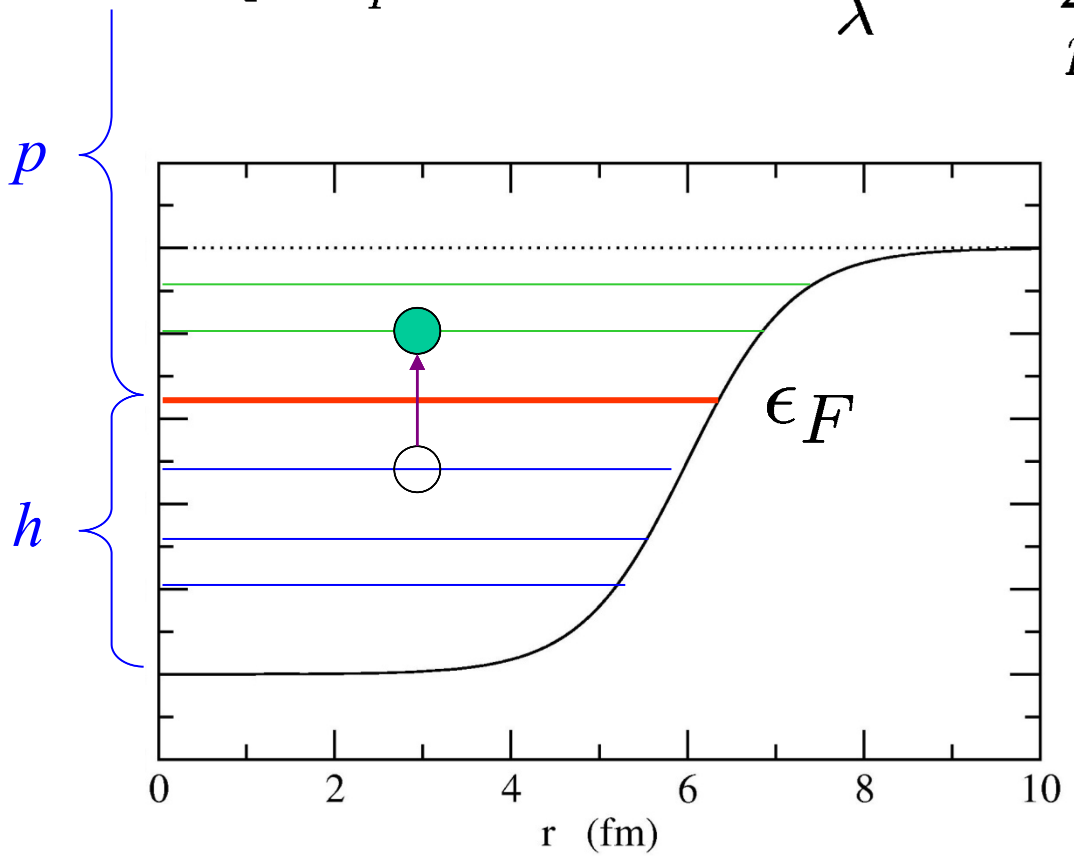
$$\frac{1}{\lambda} = \sum_{ph}|D_{ph}|^2\left(\frac{1}{E-\epsilon} - \frac{1}{E+\epsilon}\right) = \sum_{ph}|D_{ph}|^2\frac{2\epsilon}{E^2-\epsilon^2}$$

$$\longrightarrow \quad E^2 = \epsilon^2 + 2\epsilon\lambda\sum_{ph}|D_{ph}|^2$$

λ が負（引力）だと、どこかで $E^2 < 0$ となる

Continuum Excitations

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'} \quad \Longrightarrow \quad \frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$



h: all the occupied (bound) states

p: the bound excited states + continuum states

$$\frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E} = -\sum_{ph} \langle \phi_h | D^\dagger | \phi_p \rangle \frac{1}{\epsilon_p - \epsilon_h - E} \langle \phi_p | D | \phi_h \rangle$$

(note) $\hat{h}\phi_p = \epsilon_p \phi_p$

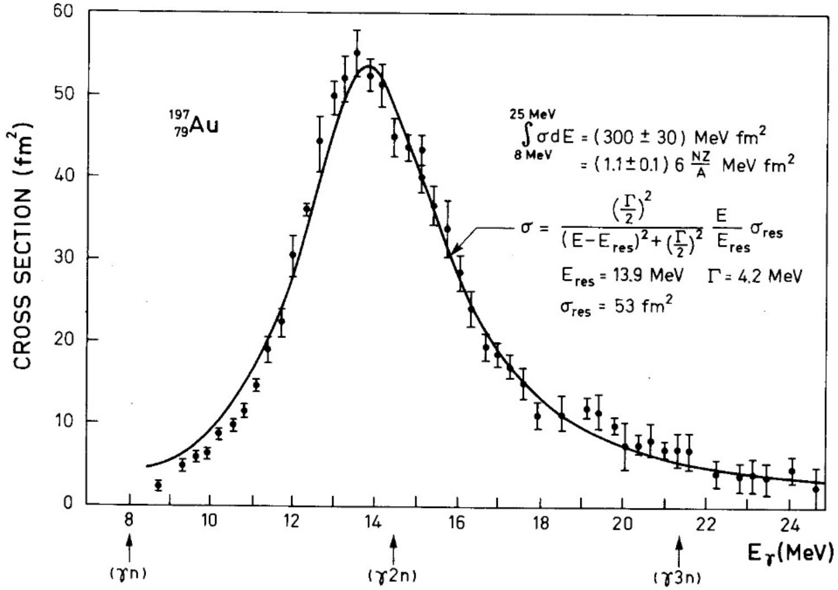
$$\frac{1}{\lambda} = -\sum_{ph} \langle \phi_h | D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} | \phi_p \rangle \langle \phi_p | D | \phi_h \rangle$$

$$1 = \sum_i |\phi_i\rangle\langle\phi_i| = \sum_p |\phi_p\rangle\langle\phi_p| + \sum_h |\phi_h\rangle\langle\phi_h|$$

$$\frac{1}{\lambda} = -\sum_h \langle \phi_h | D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} \left[1 - \sum_{h'} |\phi_{h'}\rangle\langle\phi_{h'}| \right] D | \phi_h \rangle$$

particle 状態の和がなくなった→連続状態もすべて自動的に入る

巨大共鳴の幅



i) 連続状態との結合（粒子放出）

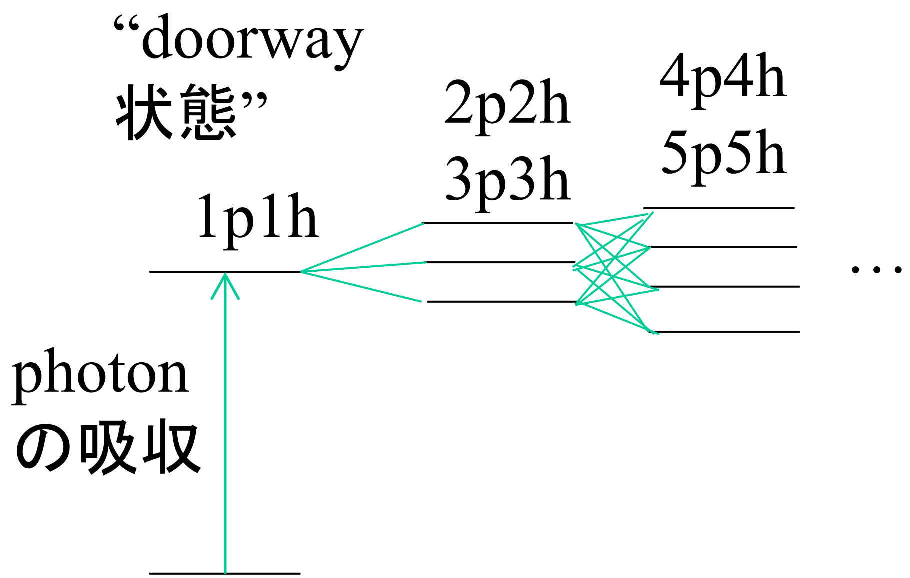
escape width $\Gamma^{\uparrow}$

continuum RPA

ii) より複雑な状態との結合

spreading width $\Gamma^{\downarrow}$

軽い核を除き
幅の主成分



1体演算子

レポート問題4（〆切：12月3日（土））

１．分離型相互作用

$$\begin{aligned}\langle ph'|\bar{v}|hp'\rangle &= \lambda D_{ph} D^*_{p'h'} \\ \langle pp'|\bar{v}|hh'\rangle &= \lambda D_{ph} D_{p'h'}\end{aligned}$$

の場合にRPA方程式を解き、RPA dispersion relation を導け。

２．RPA のA 行列、B行列が

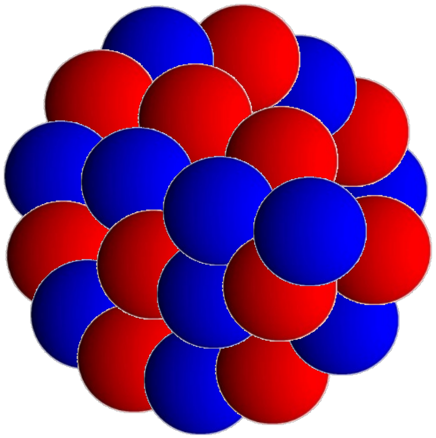
$$A = \begin{pmatrix} \epsilon & g \\ g & \epsilon \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & g \\ g & 0 \end{pmatrix}$$

で与えられているときに RPA方程式を解き、（正の）固有値を求めよ。

原子核反応論

- 原子核物理：核子多体系としての原子核の振る舞い

  ⟵ 核子間相互作用から理解する

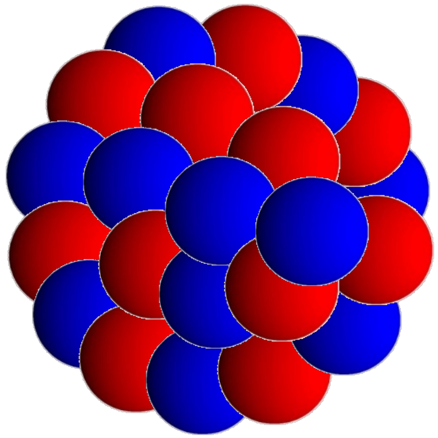


- 静的な振る舞い：原子核構造論
  - ✓ 基底状態の性質
    （質量、大きさ、形など）
  - ✓ 励起状態の性質
- ダイナミックス：原子核反応論

原子核反応論

- 原子核物理:核子多体系としての原子核の振る舞い

　　← 核子間相互作用から理解する



- 静的な振る舞い:原子核構造論
  - ✓ 基底状態の性質
    （質量、大きさ、形など）
  - ✓ 励起状態の性質

- ダイナミックス:原子核反応論

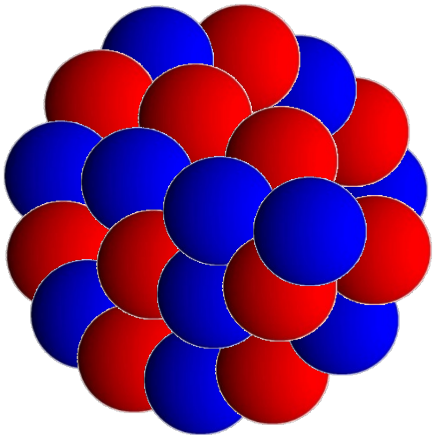
原子核は複合粒子
✓ 豊富な反応様式

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応

原子核反応論

- 原子核物理：核子多体系としての原子核の振る舞い

  ⟵ 核子間相互作用から理解する



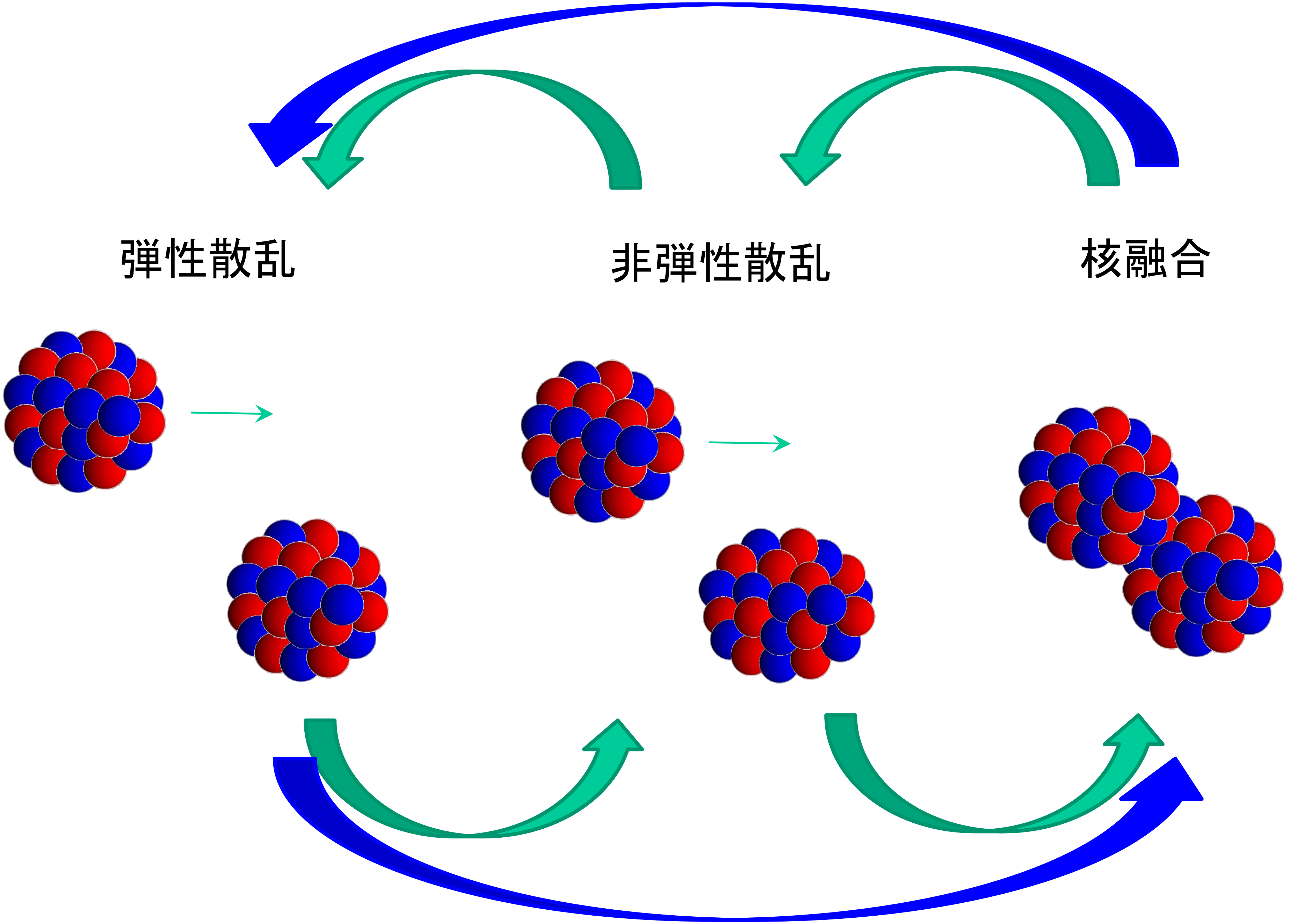
- 静的な振る舞い：原子核構造論
  - ✓ 基底状態の性質
    （質量、大きさ、形など）
  - ✓ 励起状態の性質

- ダイナミックス：原子核反応論

原子核は複合粒子
✓ 豊富な反応様式
✓ 核構造と核反応の織り成す様々なインタープレイ

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応

量子多体系のダイナミックス（原子核反応）



Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

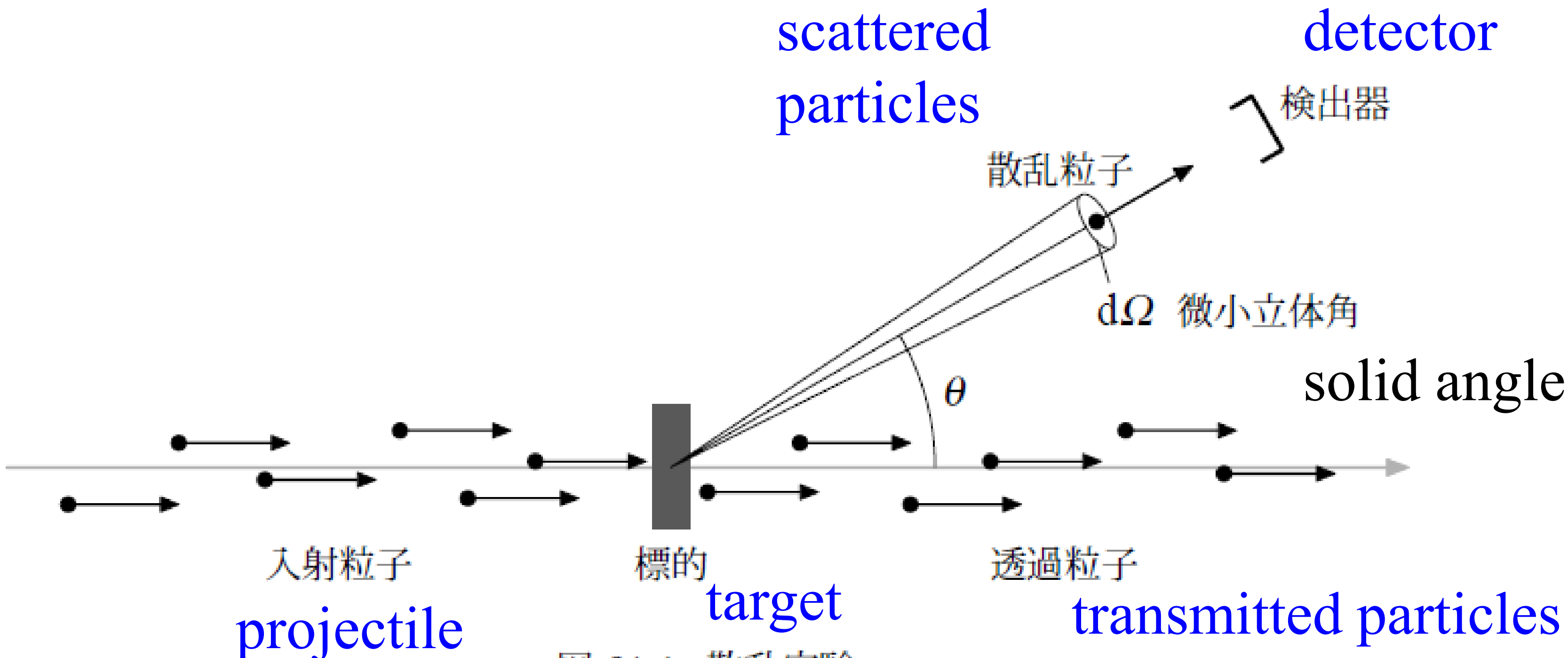


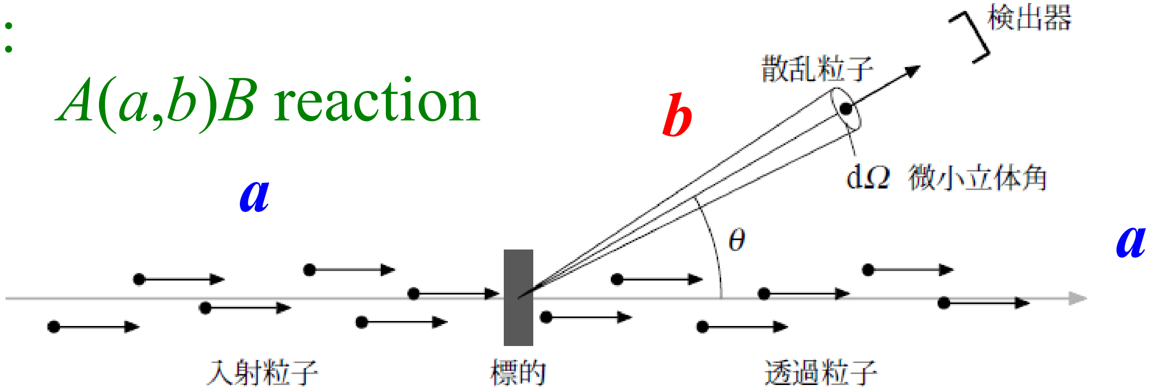
図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

K. Muto (TIT)

notation:

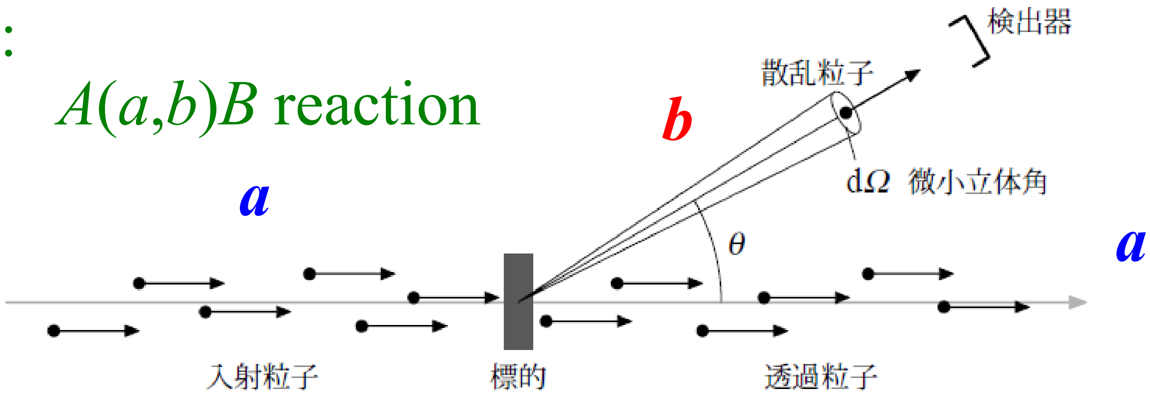
$A(a,b)B$ reaction



$A \rightarrow B$

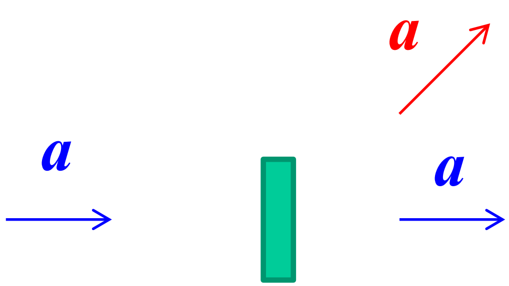
notation:

$A(a,b)B$ reaction



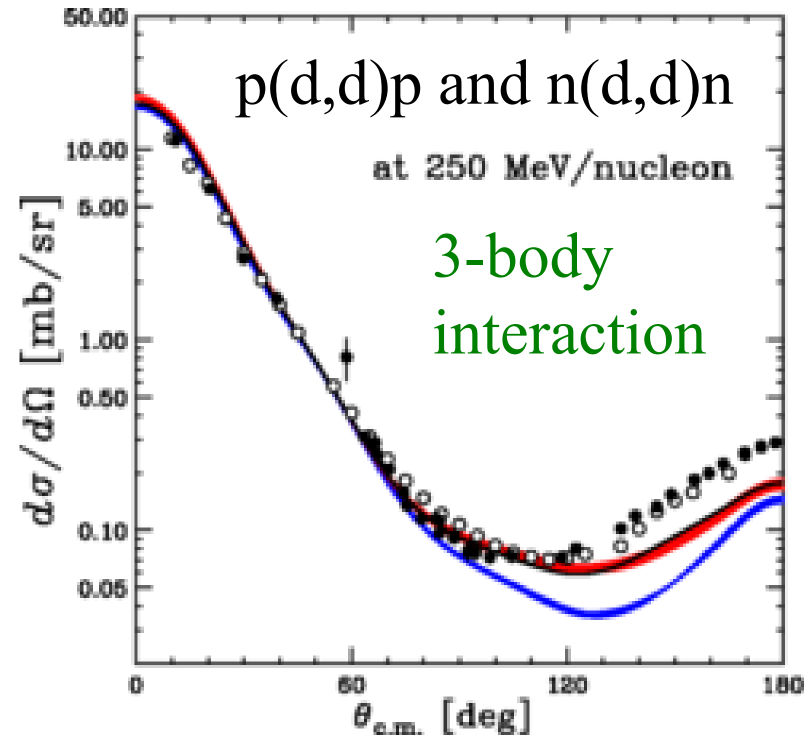
$A \rightarrow B$

✓elastic scattering



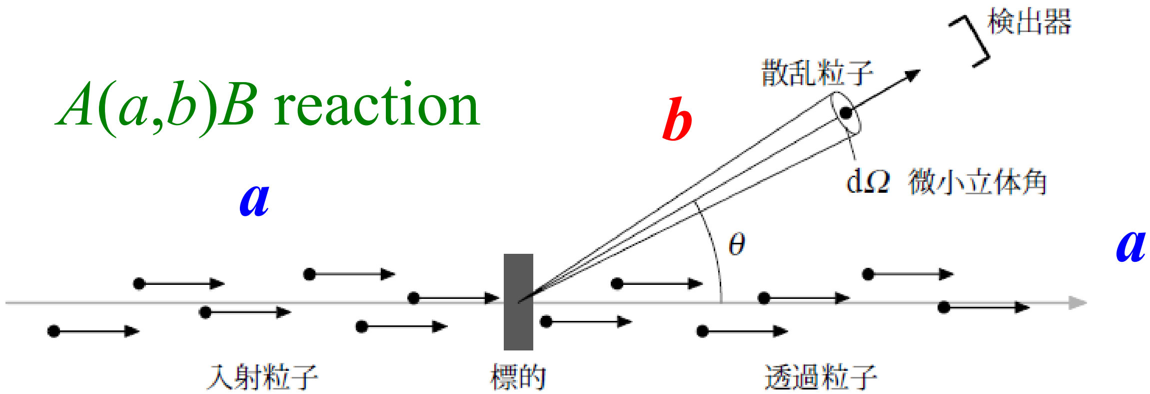
$A_{gs} \rightarrow A_{gs}$

fundamental interaction between a and A



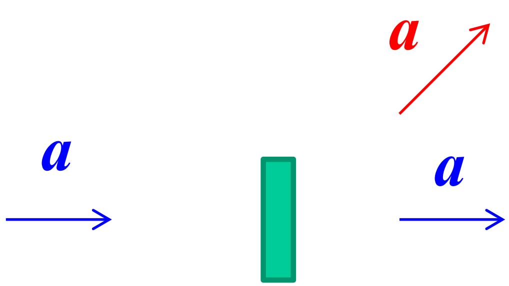
K. Sekiguchi et al., PRC89(‘14)064007

$A(a,b)B$ reaction



$A \rightarrow B$

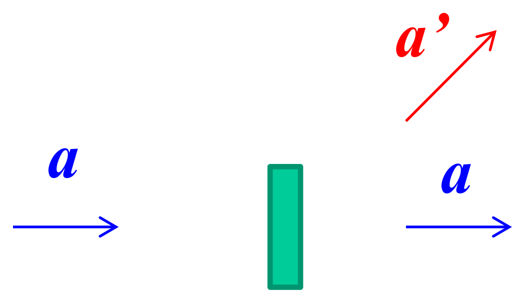
✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

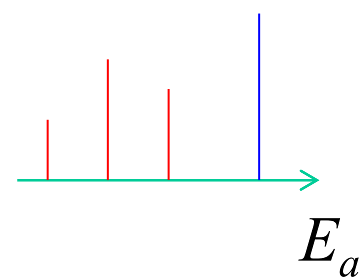
fundamental interaction between a and A

✓inelastic scattering

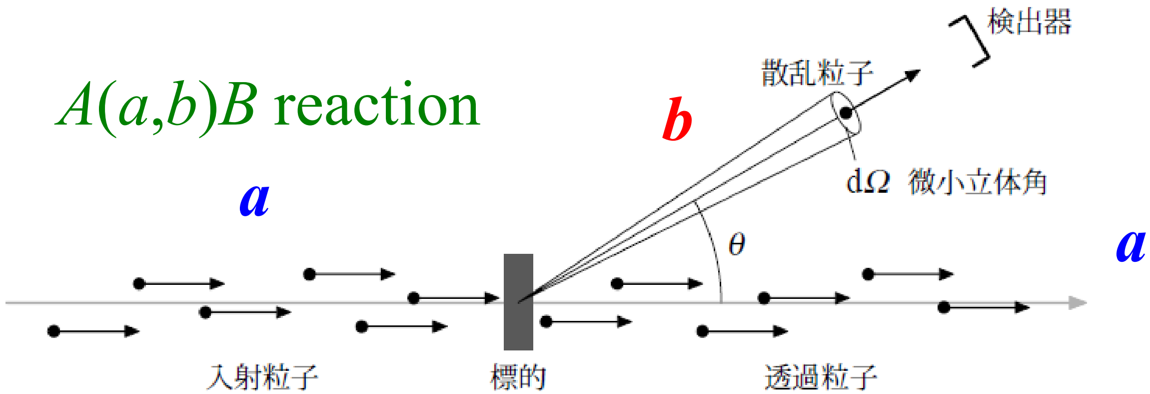


$A_{gs} \rightarrow A^*$

excitation spectrum of a nucleus A



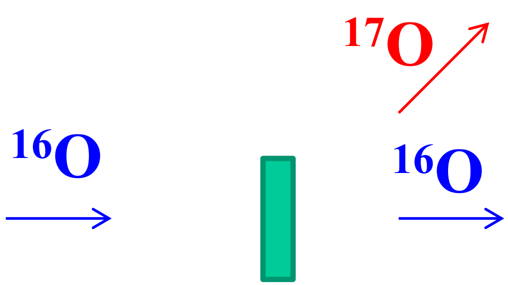
$A(a,b)B$ reaction



$A \rightarrow B$

transfer reactions

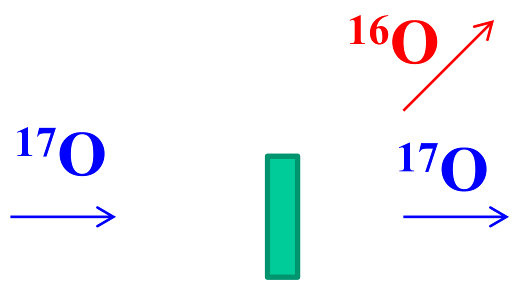
✓transfer reaction (pick-up reaction)



$^{208}\mathrm{Pb}_{gs} \rightarrow {}^{207}\mathrm{Pb}$

level schem of ^{207}Pb

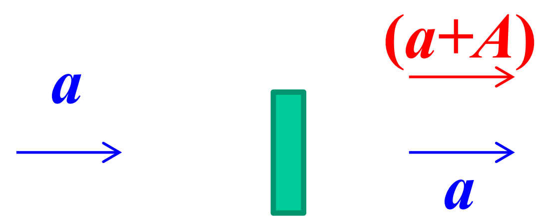
✓transfer reaction (stripping reaction)



$^{208}\mathrm{Pb}_{gs} \rightarrow {}^{209}\mathrm{Pb}$

level schem of ^{209}Pb

✓fusion reaction

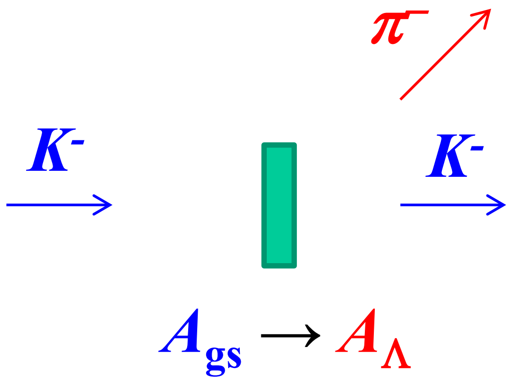


$A_{gs} \rightarrow X$

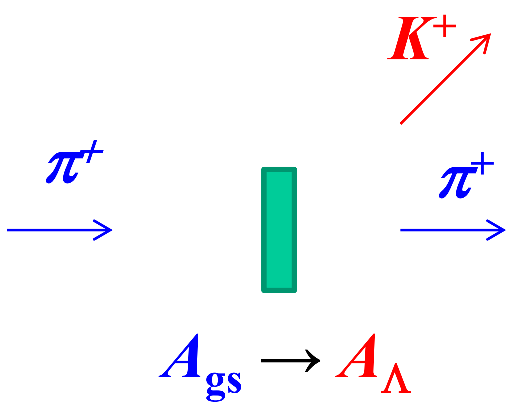
- interaction between a and A
- structure of a and A

hypernucleus production reactions

✓(K^-,π^-) reaction

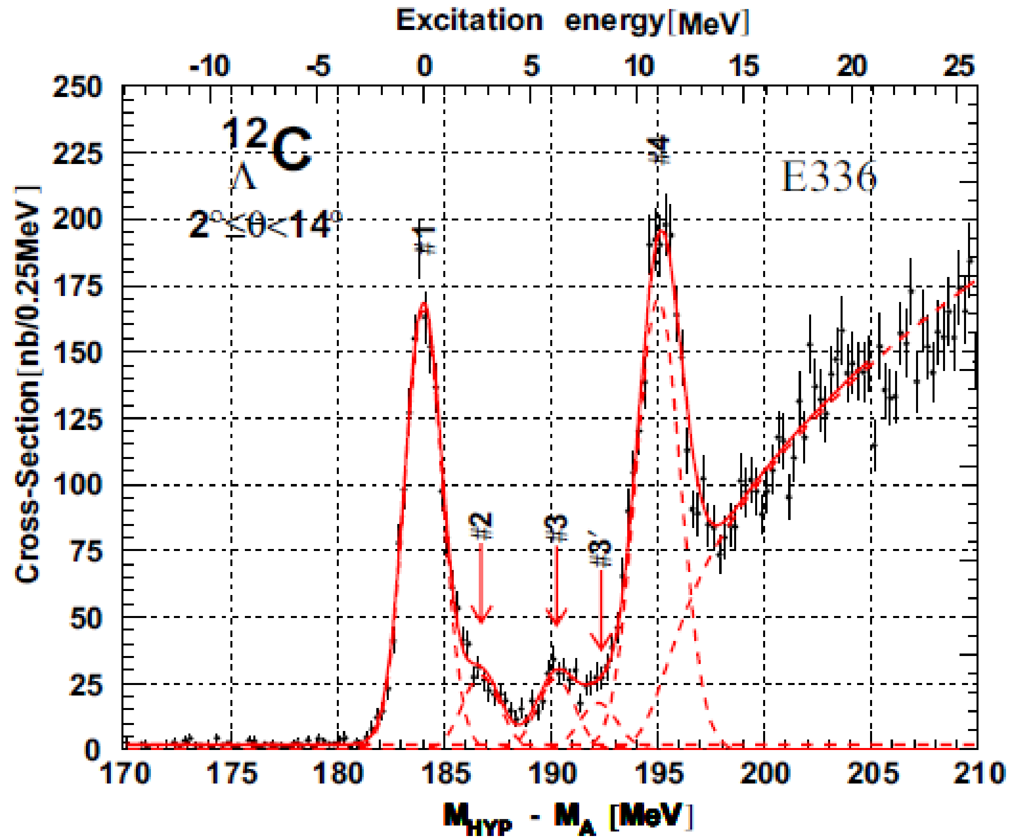


✓(π^+,K^+) reaction



excitation spectrum
of a hypernucleus A_Λ

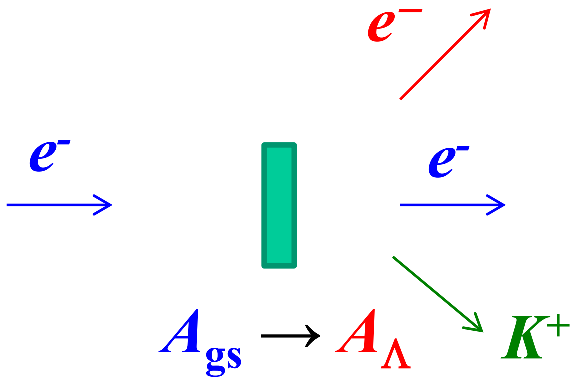
$^{12}C\ (\pi^+,K^+)\ ^{12}{}_{\Lambda}C$ reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

“reaction spectroscopy”

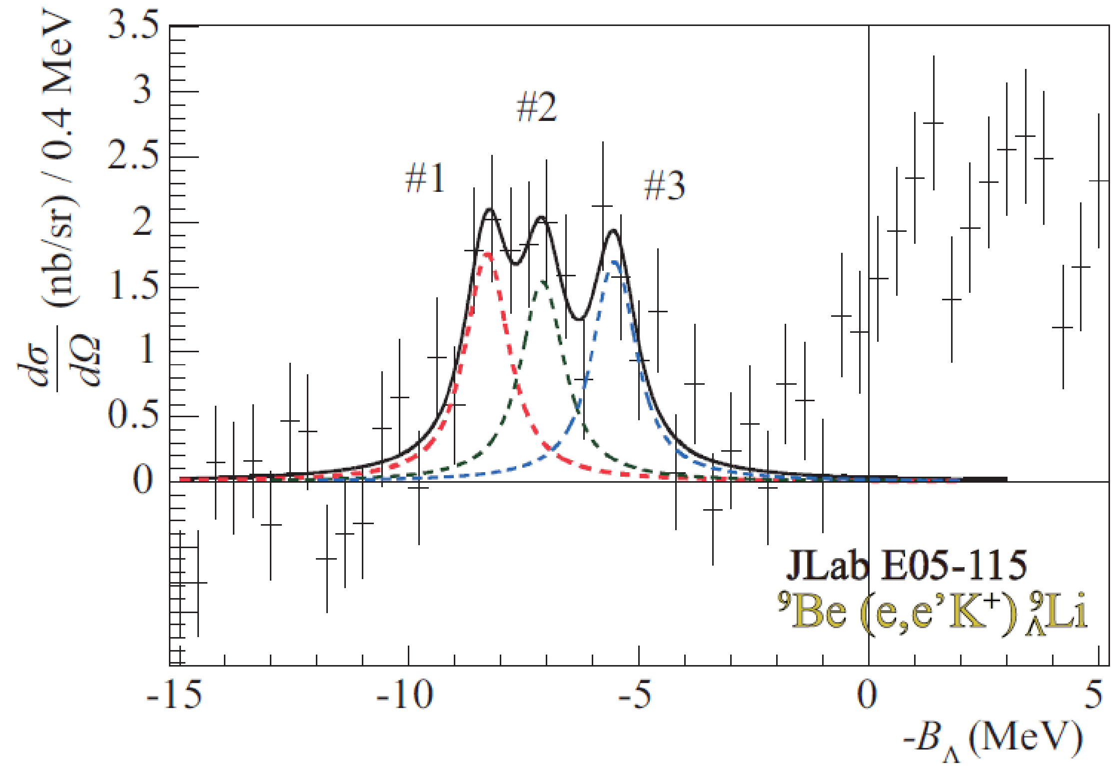
✓(e,e'K$^+$) reaction



S.N. Nakamura et al.,
PRL110('13)012502

T. Gogami,
Ph.D. Thesis (Tohoku U.)
2014

^{9}Be(e,e'K$^+$) $^{9}{}_{\Lambda}$Li



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PTEP2022, 013D01 (2021). ^{3}H(e,e'K$^+$)nnΛ