

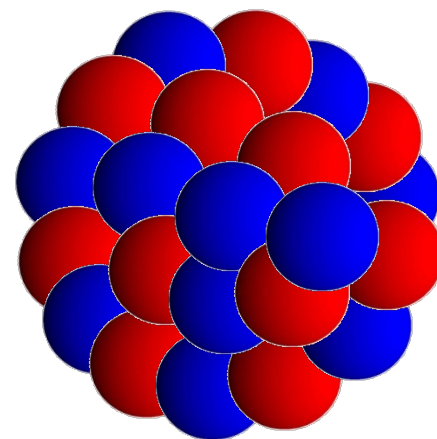
原子核反応論

□ 原子核物理: 核子多体系としての原子核の振る舞い

← 核子間相互作用から理解する

➤ 静的な振る舞い: 原子核構造論

- ✓ 基底状態の性質
(質量、大きさ、形など)
- ✓ 励起状態の性質



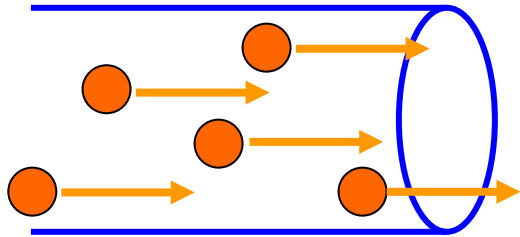
➤ ダイナミクス: 原子核反応論

原子核は複合粒子

- ✓ 豊富な反応様式
- ✓ 核構造と核反応
の織り成す様々な
インタープレイ

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応

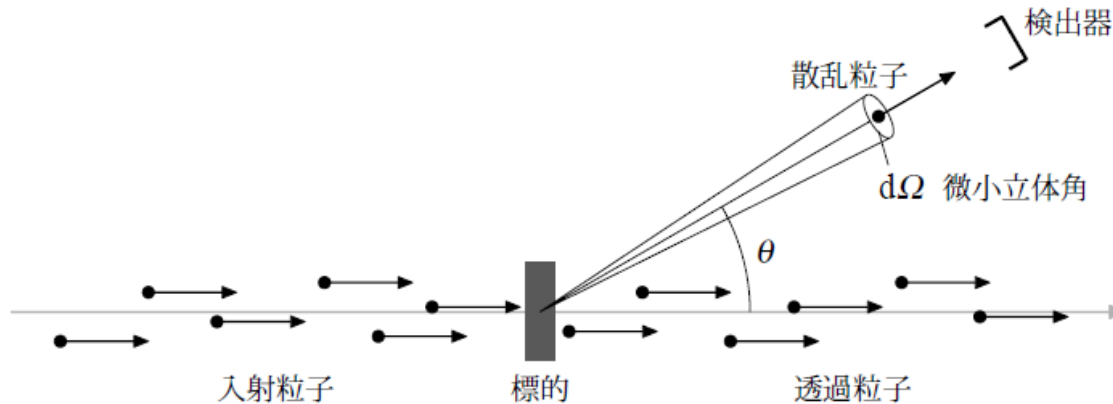
Cross sections



incident beam

flux = the number of particles
crossing unit area
per unit time

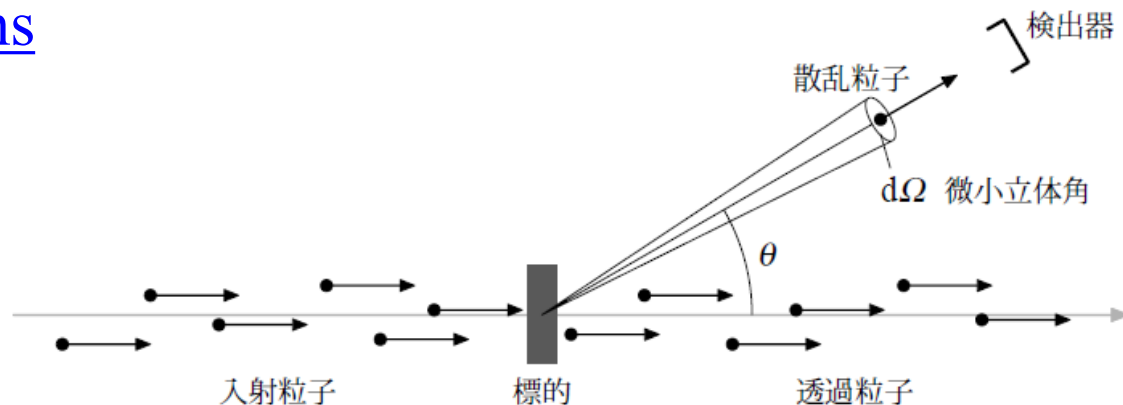
$$j = \rho_P \cdot v$$



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

→ $R = N_T \cdot \sigma \cdot j$ ← cross section

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

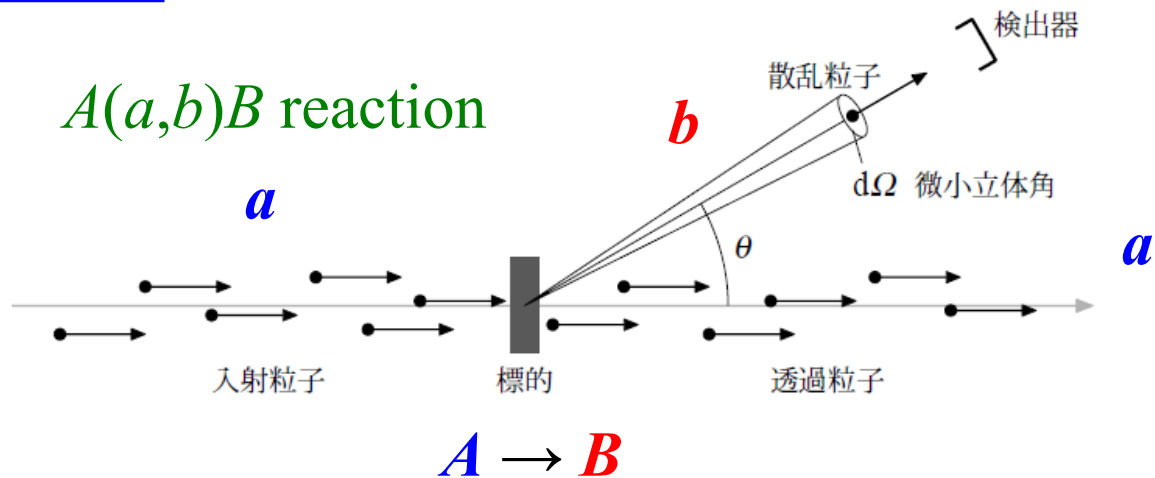
$$\longrightarrow R = N_T \cdot \sigma \cdot j \quad \text{cross section}$$

differential cross sections (angular distribution)

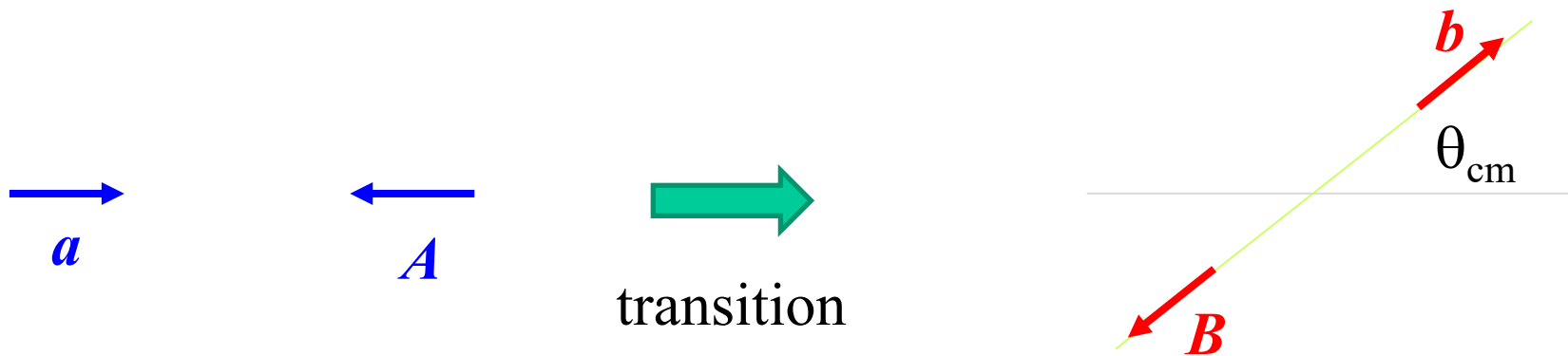
$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega \quad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm² = 100 fm² (1 mb = 10^{-3} b = 0.1 fm²)

断面積 (量子力学)



center of mass frame



$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\text{in}}}$$

Cross sections

✓ laboratory frame



✓ center of mass frame



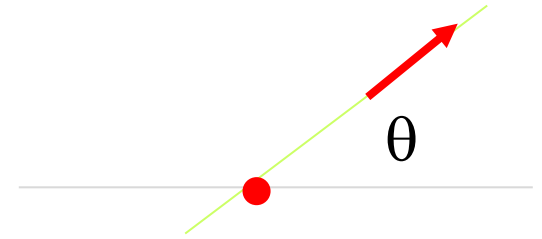
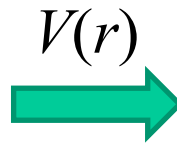
□ transformation ← energy and momentum conservations

$$\tan \theta_{\text{lab}} = \frac{\sin \theta_{\text{cm}}}{\gamma + \cos \theta_{\text{cm}}}, \quad d\Omega_{\text{lab}} = \frac{|1 + \gamma \cos \theta_{\text{cm}}|}{(1 + \gamma^2 + 2\gamma \cos \theta_{\text{cm}})^{3/2}} d\Omega_{\text{cm}}$$
$$E_{\text{cm}} = \frac{M_A}{M_a + M_A} E_{\text{lab}}, \quad \gamma = \sqrt{\frac{M_a M_b}{M_A M_B} \frac{E_{\text{cm}}}{E_{\text{cm}} + Q}}$$

Born approximation

$$\psi_f(\mathbf{r}) = e^{i\mathbf{p}_f \cdot \mathbf{r} / \hbar}$$

$$\psi_i(\mathbf{r}) = e^{i\mathbf{p}_i \cdot \mathbf{r} / \hbar}$$



$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \underline{V(r)} - E \right) \psi(\mathbf{r}) = 0$$

perturbation

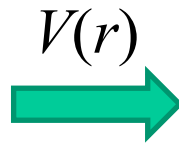
transition rate for elastic scattering:

$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}_f}{(2\pi\hbar)^3} |\langle \psi_f | V | \psi_i \rangle|^2 \delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2 \hbar^4} \int d\Omega |\tilde{V}(\mathbf{q})|^2 \end{aligned}$$

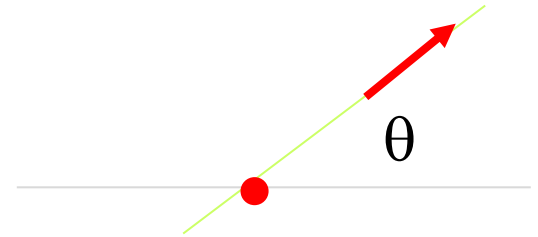
$$\tilde{V}(\mathbf{q}) = \int d\mathbf{r} e^{i(\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{r} / \hbar} V(r) \equiv \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} V(r)$$

Born approximation

$$\psi_i(\mathbf{r}) = e^{i\mathbf{p}_i \cdot \mathbf{r} / \hbar}$$



$$\psi_f(\mathbf{r}) = e^{i\mathbf{p}_f \cdot \mathbf{r} / \hbar}$$



$$W_{fi} = \frac{\mu p_i}{4\pi^2 \hbar^4} \int d\Omega |\tilde{V}(\mathbf{q})|^2$$

$$\tilde{V}(\mathbf{q}) = \int d\mathbf{r} e^{i(\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{r} / \hbar} V(r) \equiv \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} V(r)$$

momentum transfer

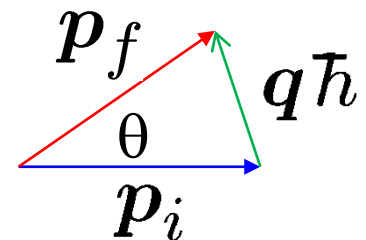


incident flux: $j_{\text{inc}} = \rho_i v = p_i / \mu$



$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \frac{\mu^2}{4\pi^2 \hbar^4} |\tilde{V}(\mathbf{q})|^2$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin \frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\mathbf{r}' \frac{\rho_{\text{ch}}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

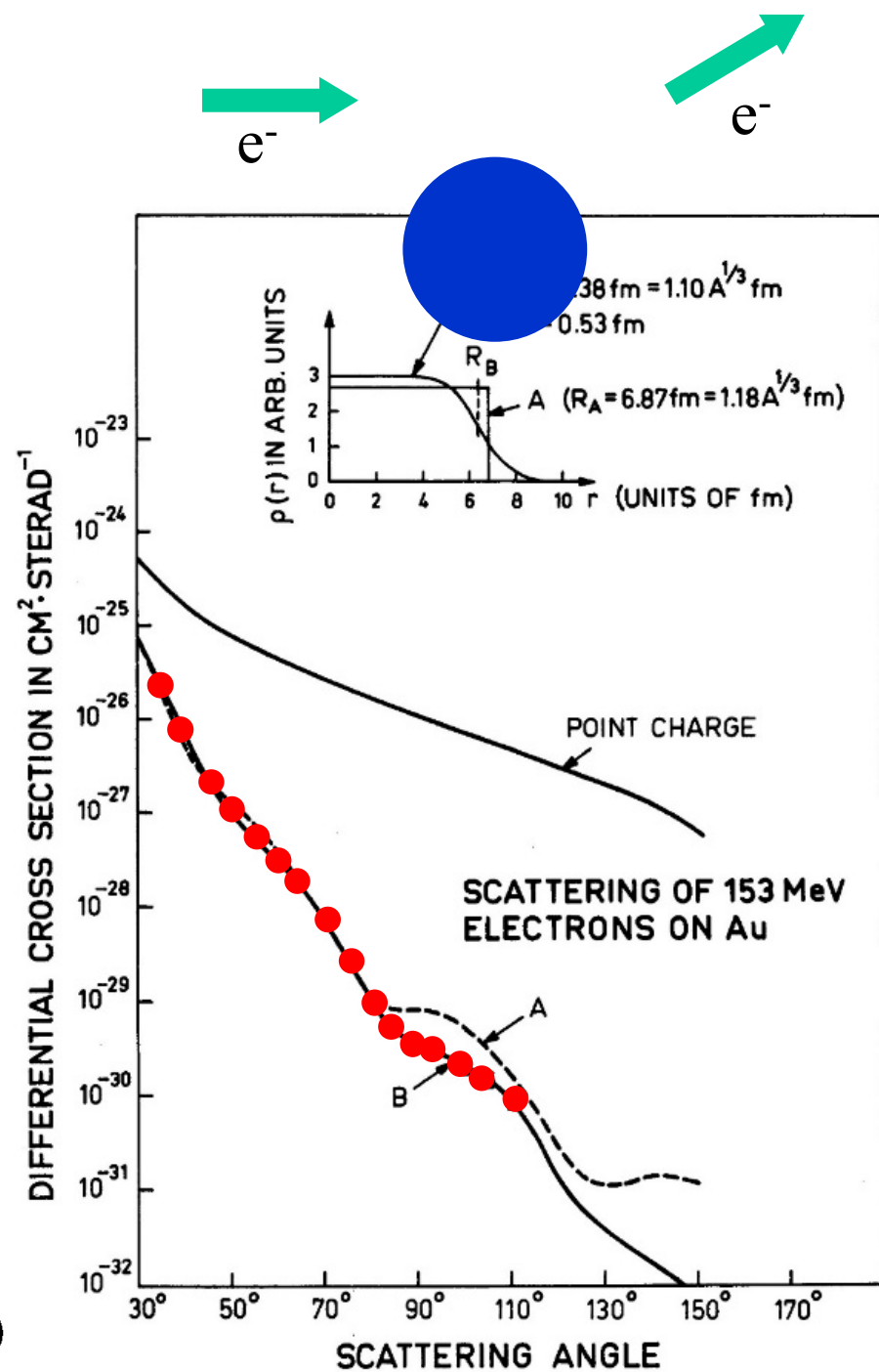
$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{e^4}{(4E \sin^2 \theta/2)^2} |F(\mathbf{q})|^2 \\ &= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega} \right) |F(\mathbf{q})|^2 \end{aligned}$$

Form factor

$$F(\mathbf{q}) = \int e^{-i\mathbf{q}\cdot\mathbf{r}} \rho_{\text{ch}}(\mathbf{r}) d\mathbf{r}$$

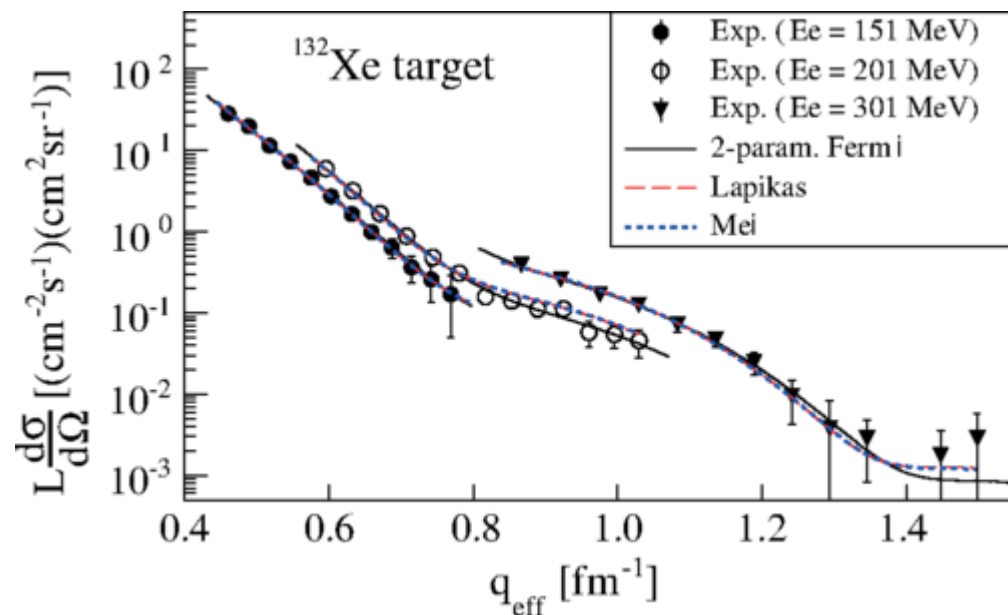
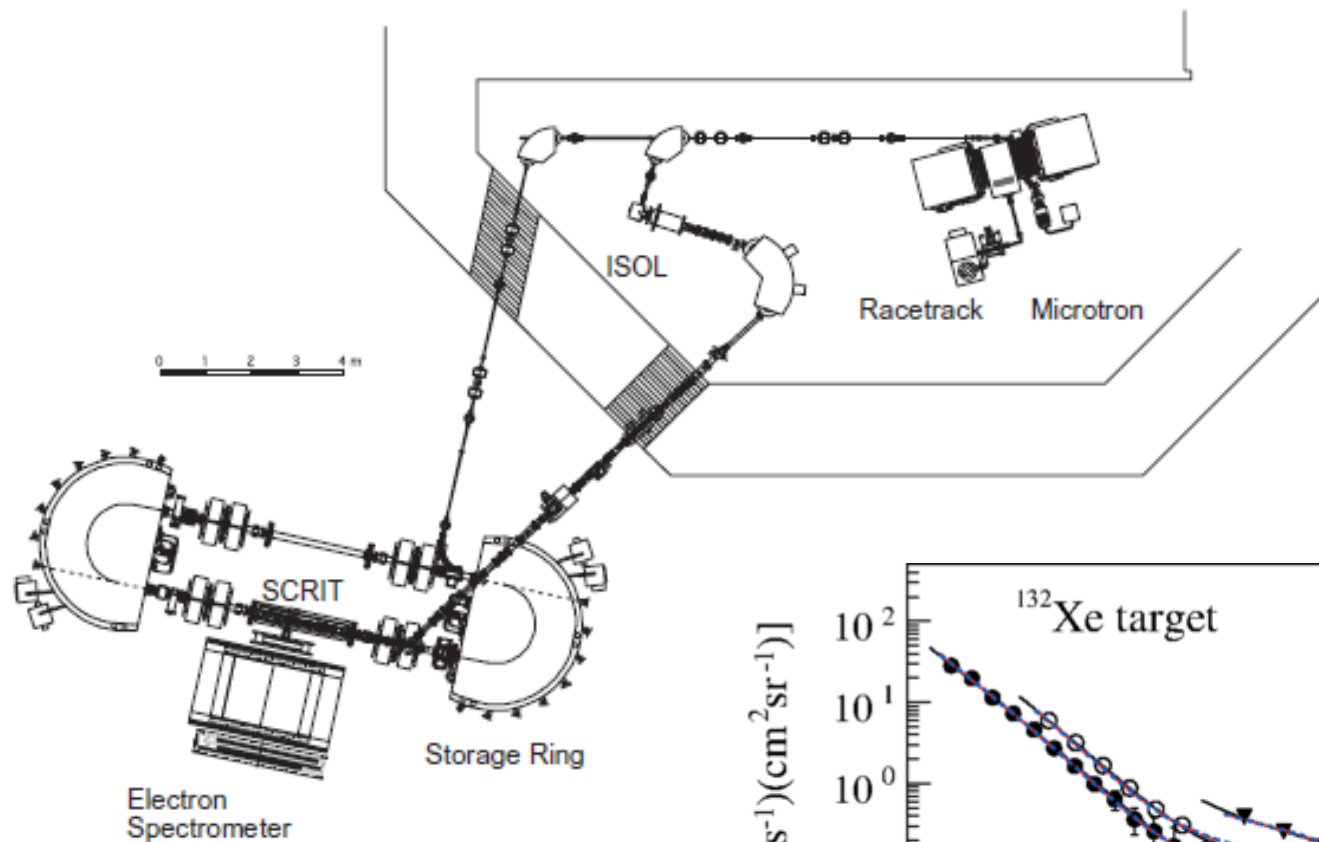
* relativistic correction:

$$\begin{aligned} \frac{d\sigma_{\text{Ruth}}}{d\Omega} &\rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega} \\ &= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2 \frac{\theta}{2} \right) \\ &\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2 \frac{\theta}{2} \quad (v \rightarrow c) \end{aligned}$$



cf. electron scattering off unstable nuclei (SCRIT)

若杉さん、塚田さん(化研)、須田さん(東北大)など



K. Tsukada et al.,
PRL118, 262501 (2017)

レポート問題5(⌘切:12月3日(土))

電子と原子核の相互作用が

$$V(\boldsymbol{r}) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\text{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

で与えられているとする。ここで、 ρ_{ch} は原子核の電荷密度で

$$\int d\boldsymbol{r} \rho_{\text{ch}}(\boldsymbol{r}) = Z$$

と規格化されているとする。ボルン近似を用いて弾性散乱の断面積を求め、

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E \sin^2 \theta/2} \right)^2 |F(\boldsymbol{q})|^2$$

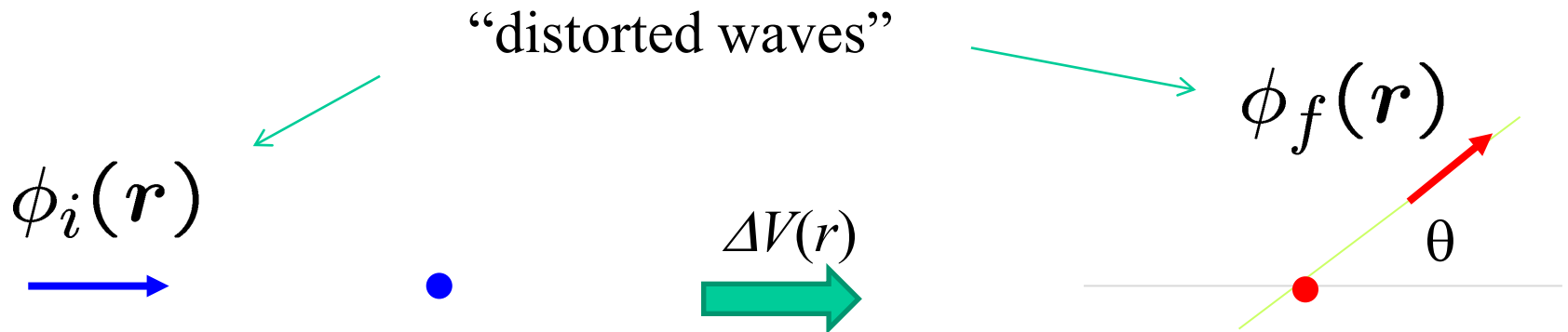
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho_{\text{ch}}(\boldsymbol{r}) d\boldsymbol{r}$$

となることを示せ。

Distorted Wave Born approximation (DWBA)

$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \underbrace{V(r)}_{\text{perturbation}} - E \right) \psi(\mathbf{r}) = 0$$

→
$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \underbrace{V_0(r)}_{\text{unperturbed}} + \underbrace{V(r) - V_0(r)}_{\text{perturbation}} - E \right) \psi(\mathbf{r}) = 0$$

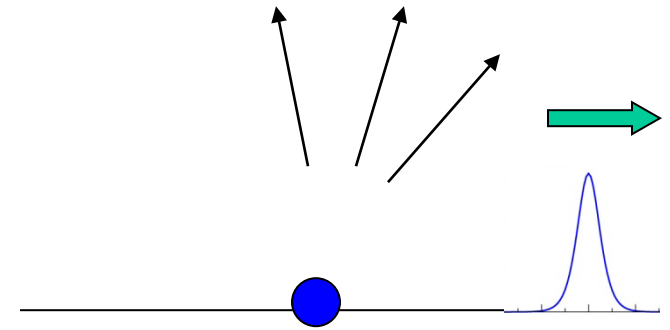
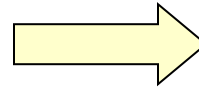


- ✓ inelastic scattering
- ✓ transfer reactions

How to choose $V_0(r)$? : Optical model

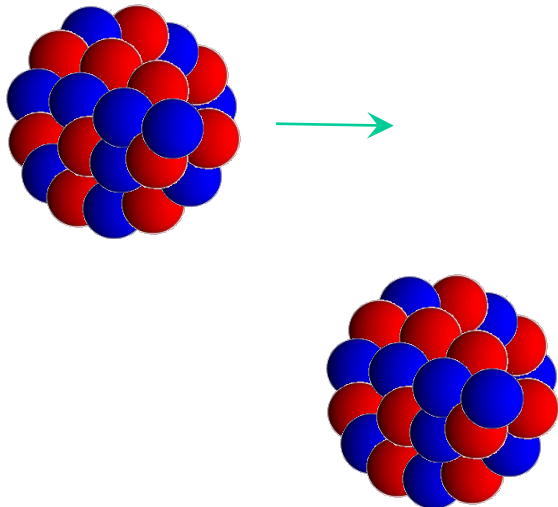
Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)

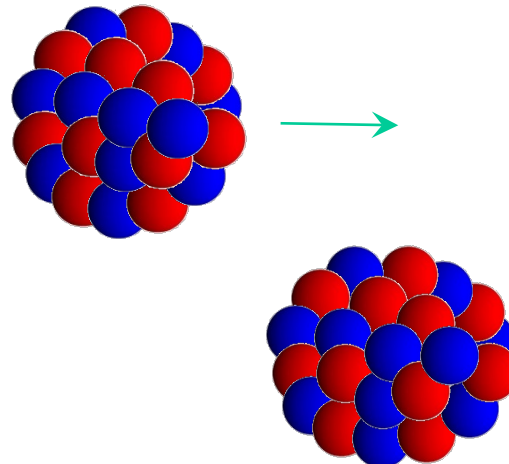


Loss of incident flux
(absorption)

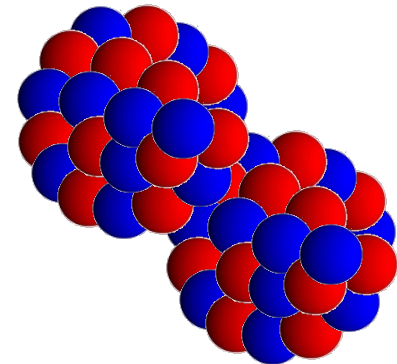
弹性散乱



非弹性散乱



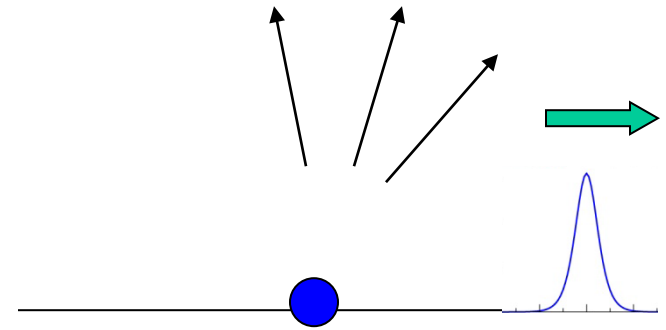
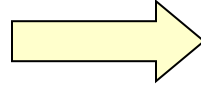
核融合



How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux
(absorption)

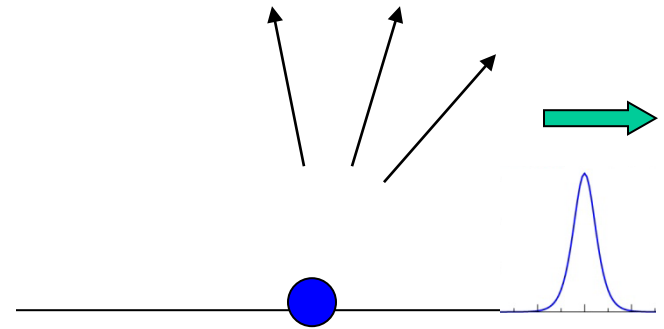
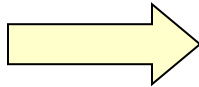
Optical potential

$$V_{\text{opt}}(\mathbf{r}) = V(\mathbf{r}) - iW(\mathbf{r}) \quad (W > 0)$$

How to choose $V_0(r)$? : Optical model

Reaction processes

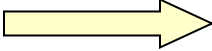
- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux
(absorption)

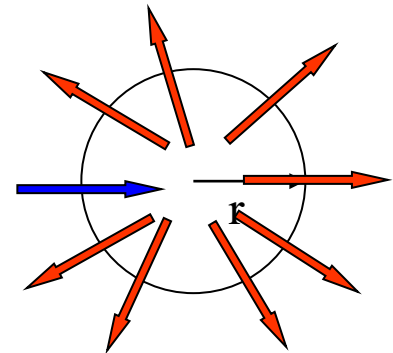
Optical potential

$$V_{\text{opt}}(\mathbf{r}) = V(\mathbf{r}) - iW(\mathbf{r}) \quad (W > 0)$$


$$\nabla \cdot \mathbf{j} = \dots = -\frac{2}{\hbar}W|\psi|^2$$

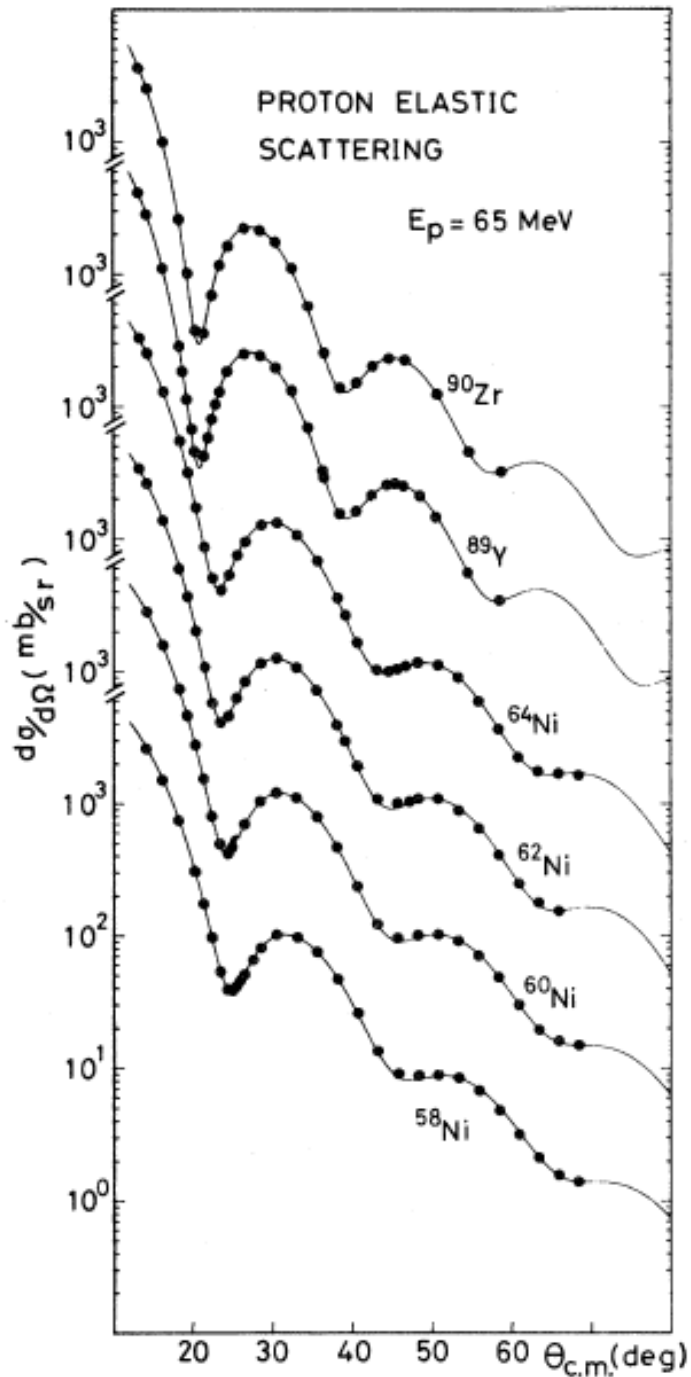
(note) Gauss's theorem

$$\int_S \mathbf{j} \cdot \mathbf{n} dS = \int_V \nabla \cdot \mathbf{j} dV$$



$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\text{opt}}(r) - E \right) \psi(\mathbf{r}) = 0$$

Woods-Saxon + volume & surface
imaginary parts

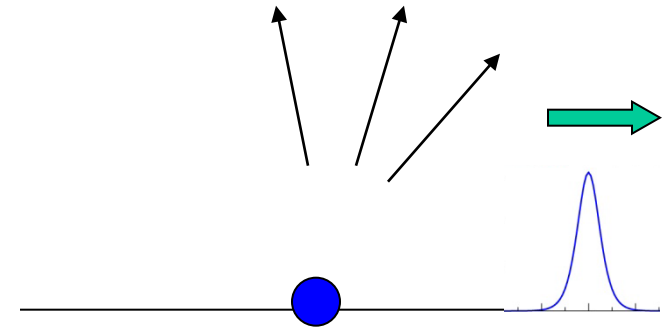
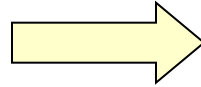


H. Sakaguchi et al.,
PRC26 (1982) 944

Absorption cross sections

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux
(absorption)

reaction cross sections

total scattering cross section minus elastic cross section

$$\sigma_R = \sigma_{\text{tot}} - \sigma_{\text{el}}$$

- fusion
- inelastic
- transfer



超重元素と核融合反応

10 11 12 13 14 15 16 17 18

元素の周期表

1	1 H																2 He	
2	3 Li	4 Be									5 B	6 C	7 N	8 O	9 F	10 Ne		
3	11 Na	12 Mg									13 Al	14 Si	15 P	16 S	17 Cl	18 Ar		
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	* 72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
			* 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
			* 90 Th	91 Pa	92 U	93 Np	94 Pu	* 95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

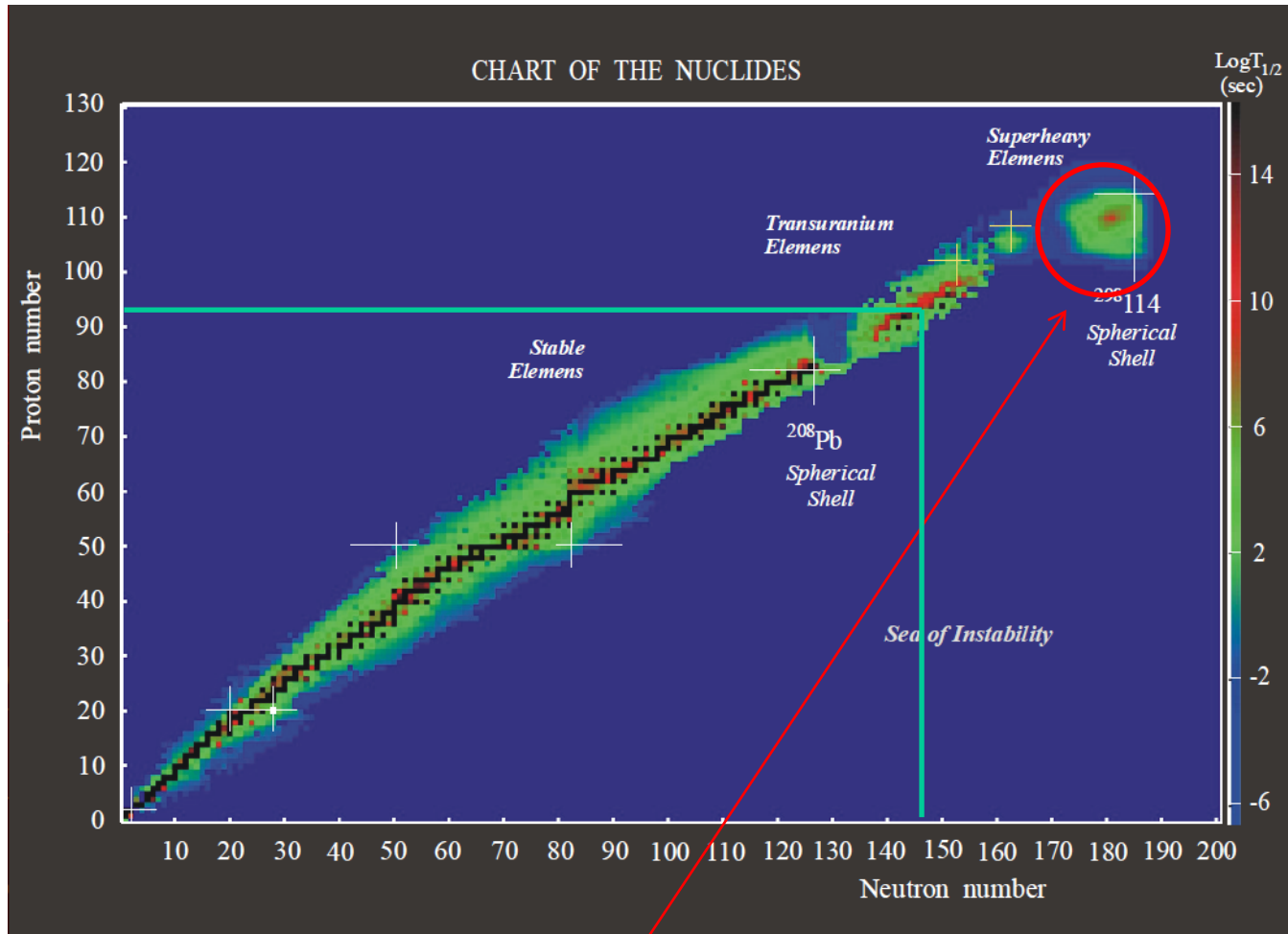
人工元素

← 原子核反応

超重元素

(アクチノイド以降の元素)

安定の島の予言: 超重元素探索の重要な動機

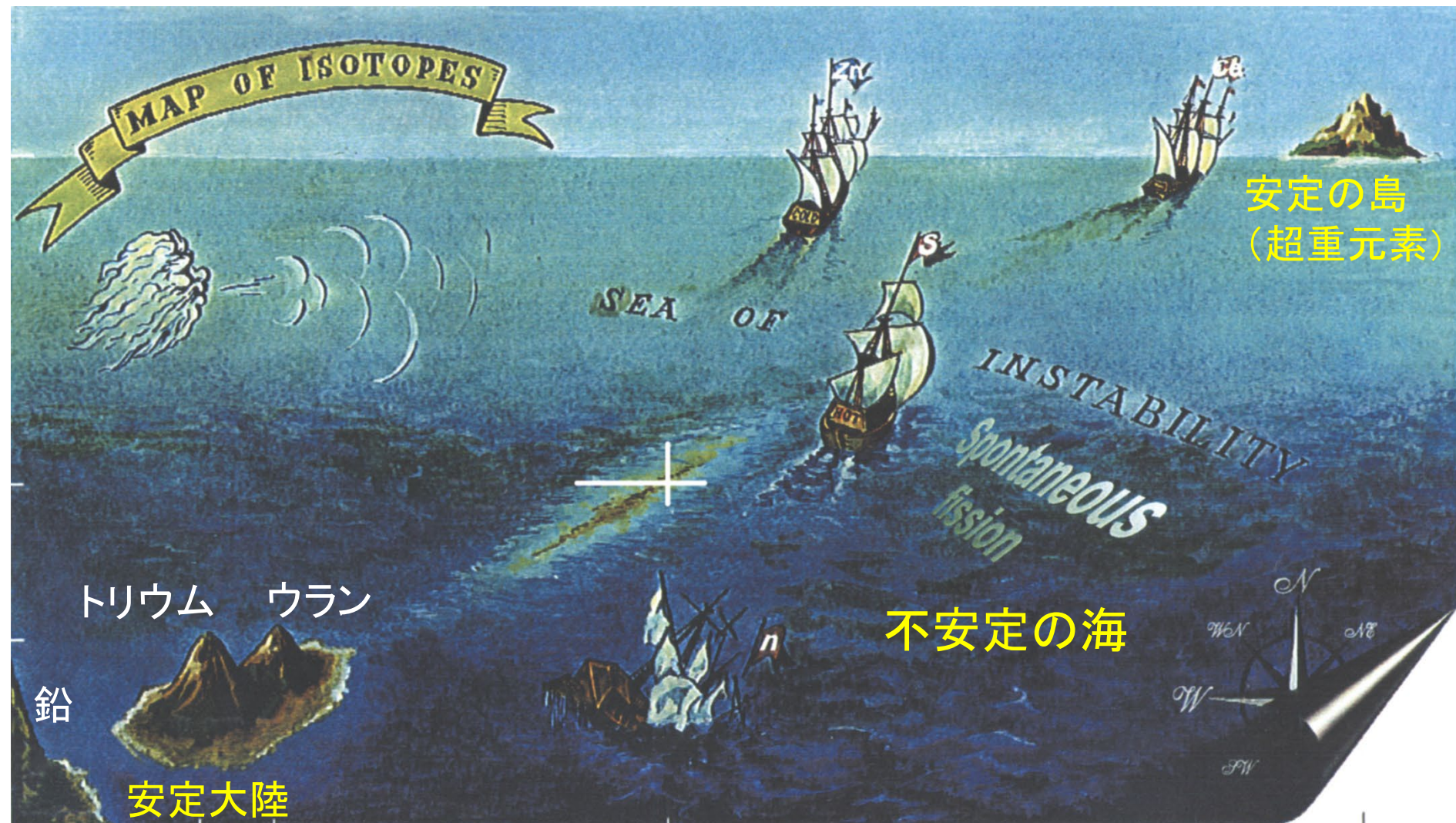


安定の島: $Z=114$, $N=184$ の周辺

Yuri Oganessian

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

安定の島(超重元素)を目指して



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Og
oganeson

Yuri Oganessian

超重元素の核融合反応

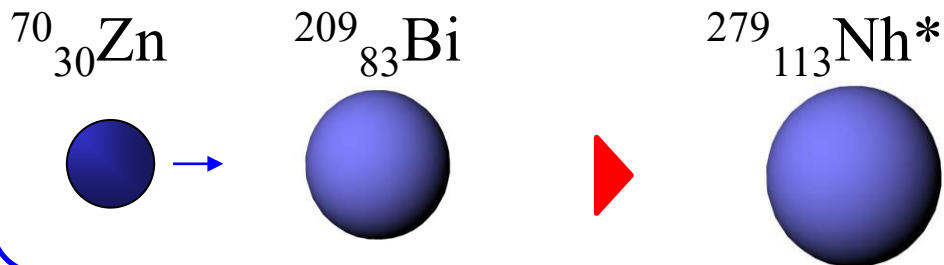
113番元素: ニホニウムNh

113 Nh nihonium	115 Mc moscovium
117 Ts tennessine	118 Og oganesson

2016年11月



Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be										5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg										13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac*	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu				
	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr				



重イオン核融合反応

who is she?

7

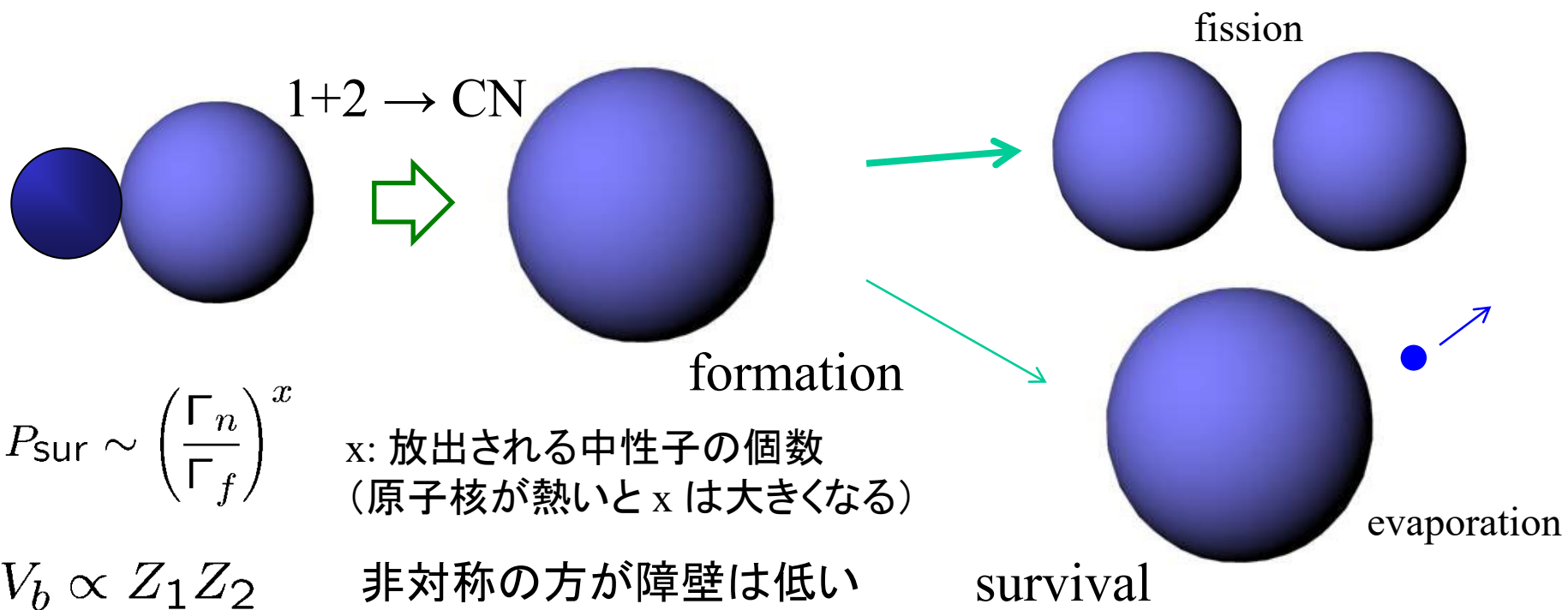
87	88	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
Fr	Ra	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo

Z=110	Darmstadtium (Ds)	1994	Germany
Z=111	Roentgenium (Rg)	1994	Germany
Z=112	Copernicium (Cn)	1996	Germany
Z=113	Nihonium (Nh)	2003	Russia / 2004 Japan
Z=114	Flerovium (Fl)	1999	Russia
Z=115	Moscovium (Mc)	2003	Russia
Z=116	Livermorium (Lv)	2000	Russia
Z=117	Tennessine (Ts)	2010	Russia
Z=118	Oganesson (Og)	2002	Russia

113 Nh nihonium	115 Mc moscovium
117 Ts tennessine	118 Og oganeson

ドイツ、日本: 冷たい融合反応 (cold fusion)

ロシア: 熱い融合反応 (hot fusion)

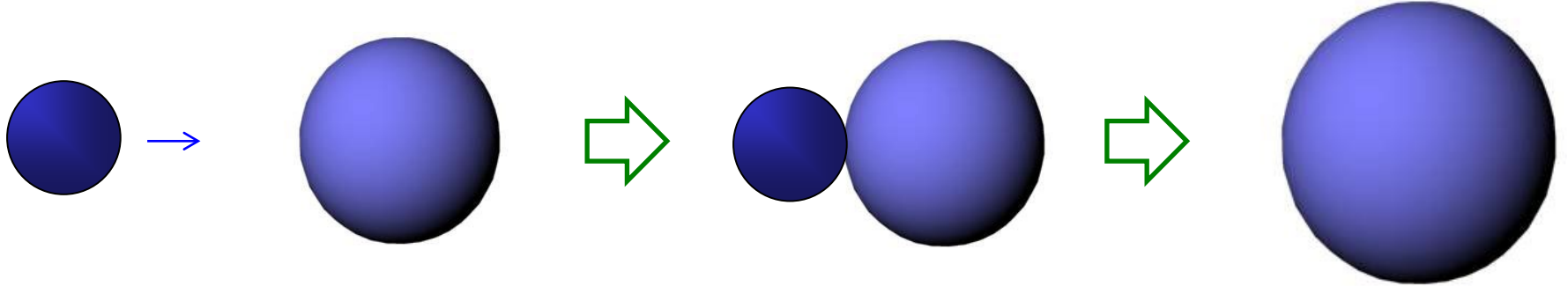


	Hot Fusion	Cold Fusion
Example	$^{48}\text{Ca} + ^{243}\text{Am} \rightarrow ^{287}\text{Nh} + 4n$	$^{70}\text{Zn} + ^{209}\text{Bi} \rightarrow ^{278}\text{Nh} + 1n$
asymmetry	large	small
Capture	large	small
Survival	small	large

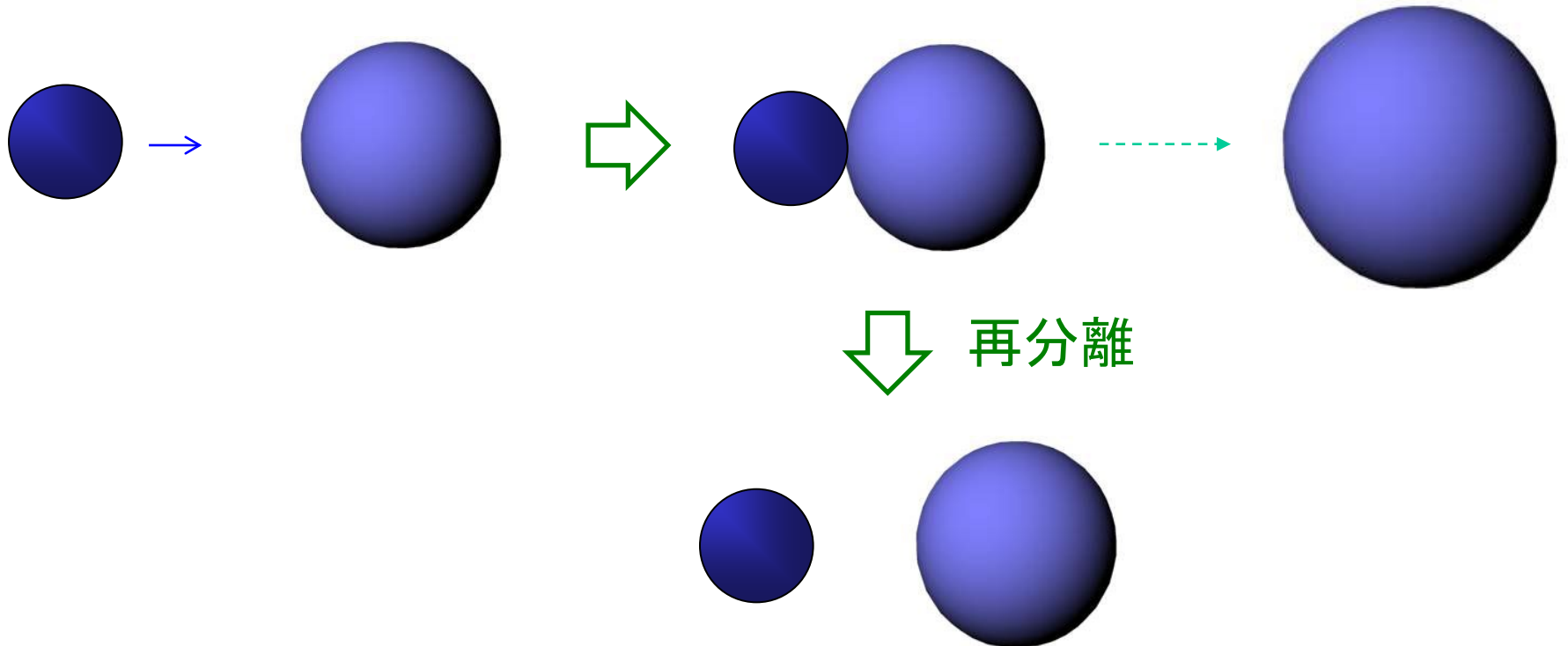
どうやって超重核を作る?

核融合反応

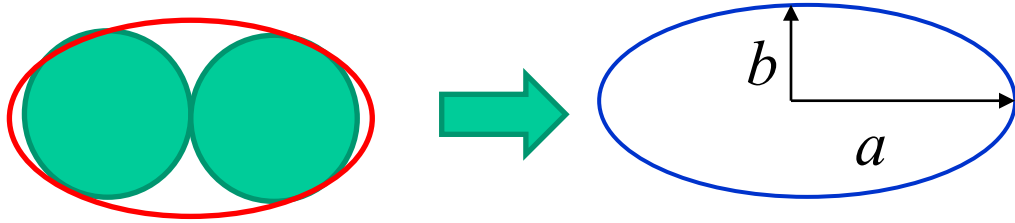
➤ 中重核の核融合反応:



➤ 重核や超重核の核融合反応:



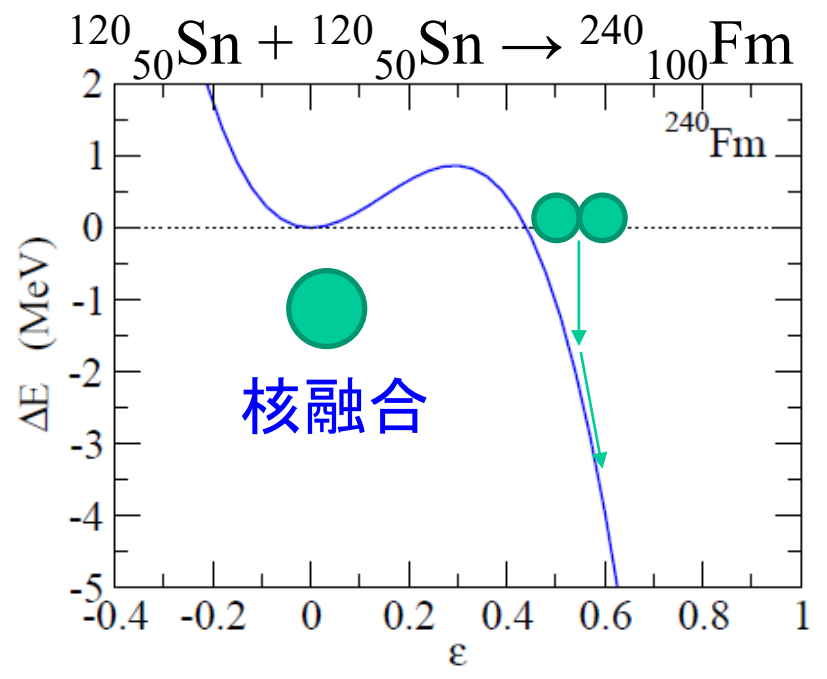
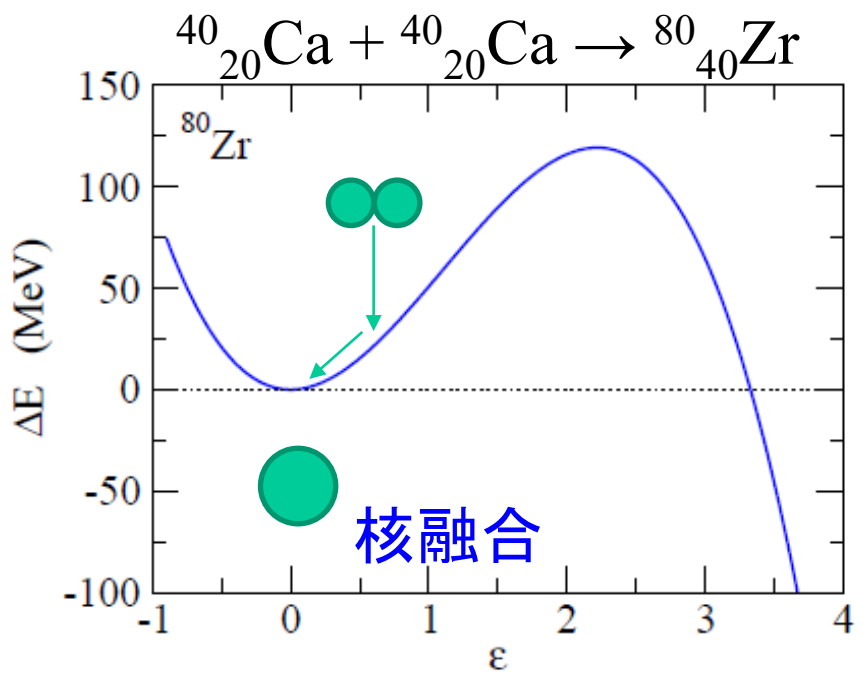
同じ原子核が接触すると:

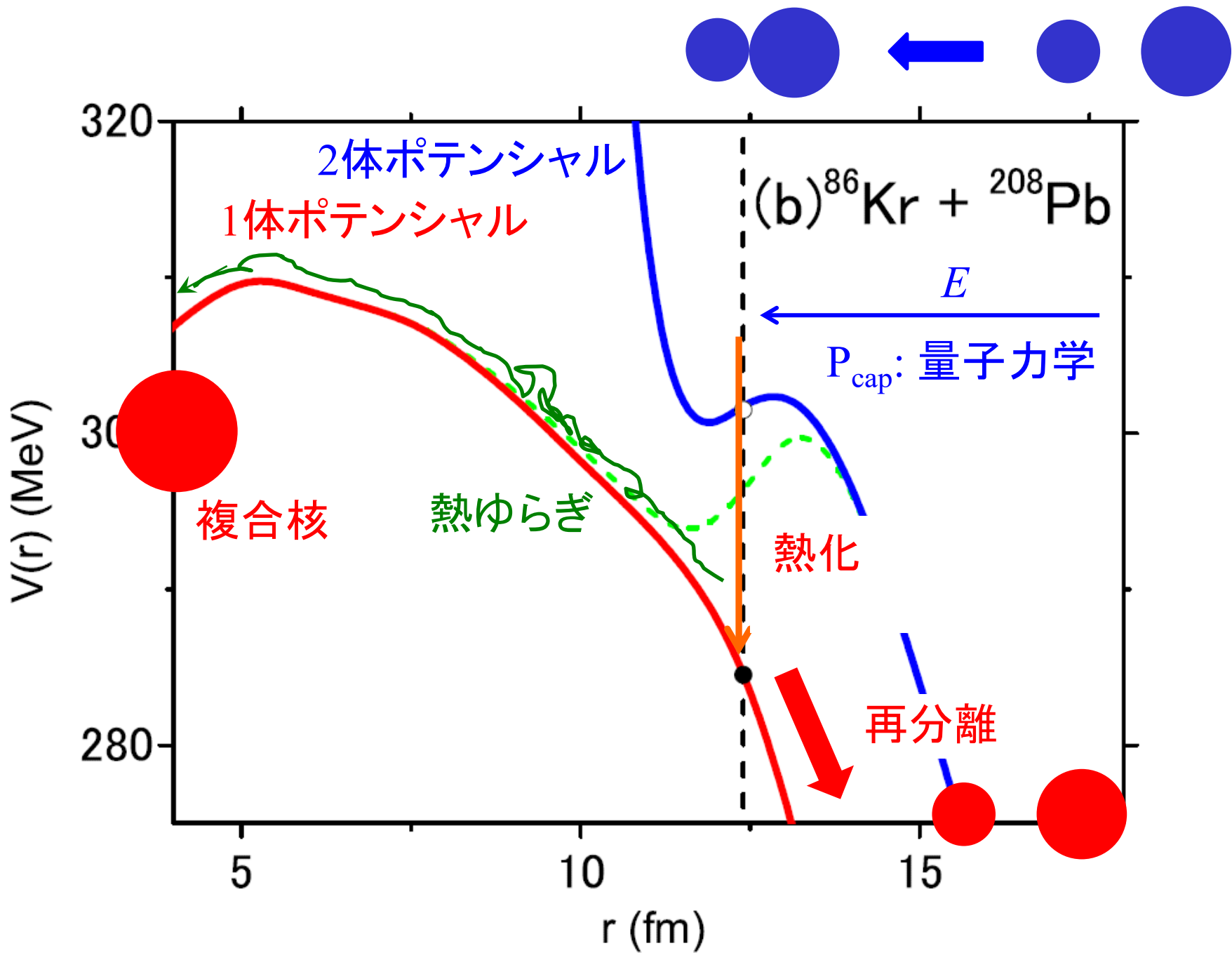


$$a = R_0 \cdot (1 + \epsilon)$$

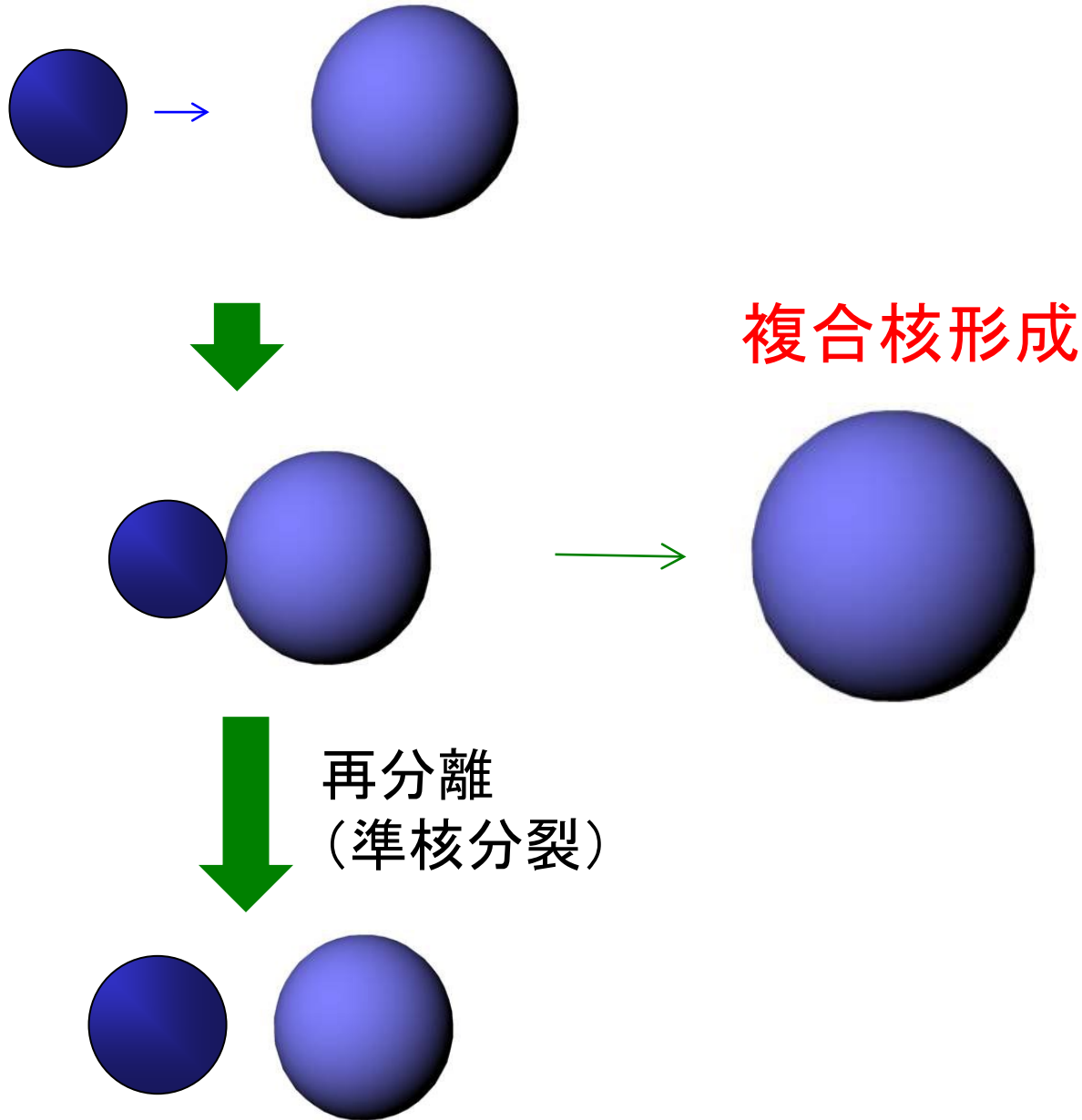
$$b = R_0 \cdot (1 + \epsilon)^{-1/2}$$

$$\frac{a}{b} \sim \frac{2R}{R} = 2 \rightarrow \epsilon \sim 0.587$$

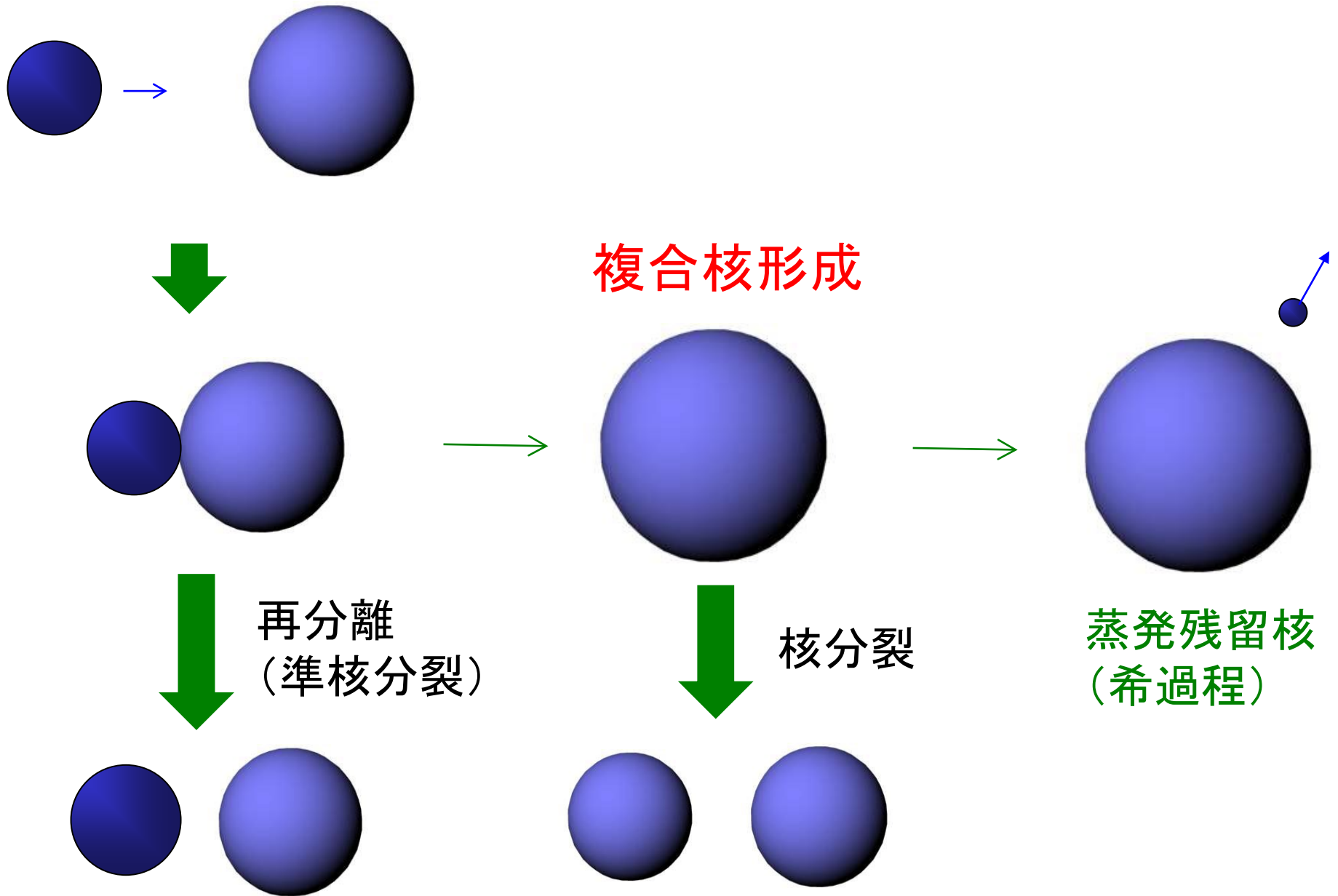




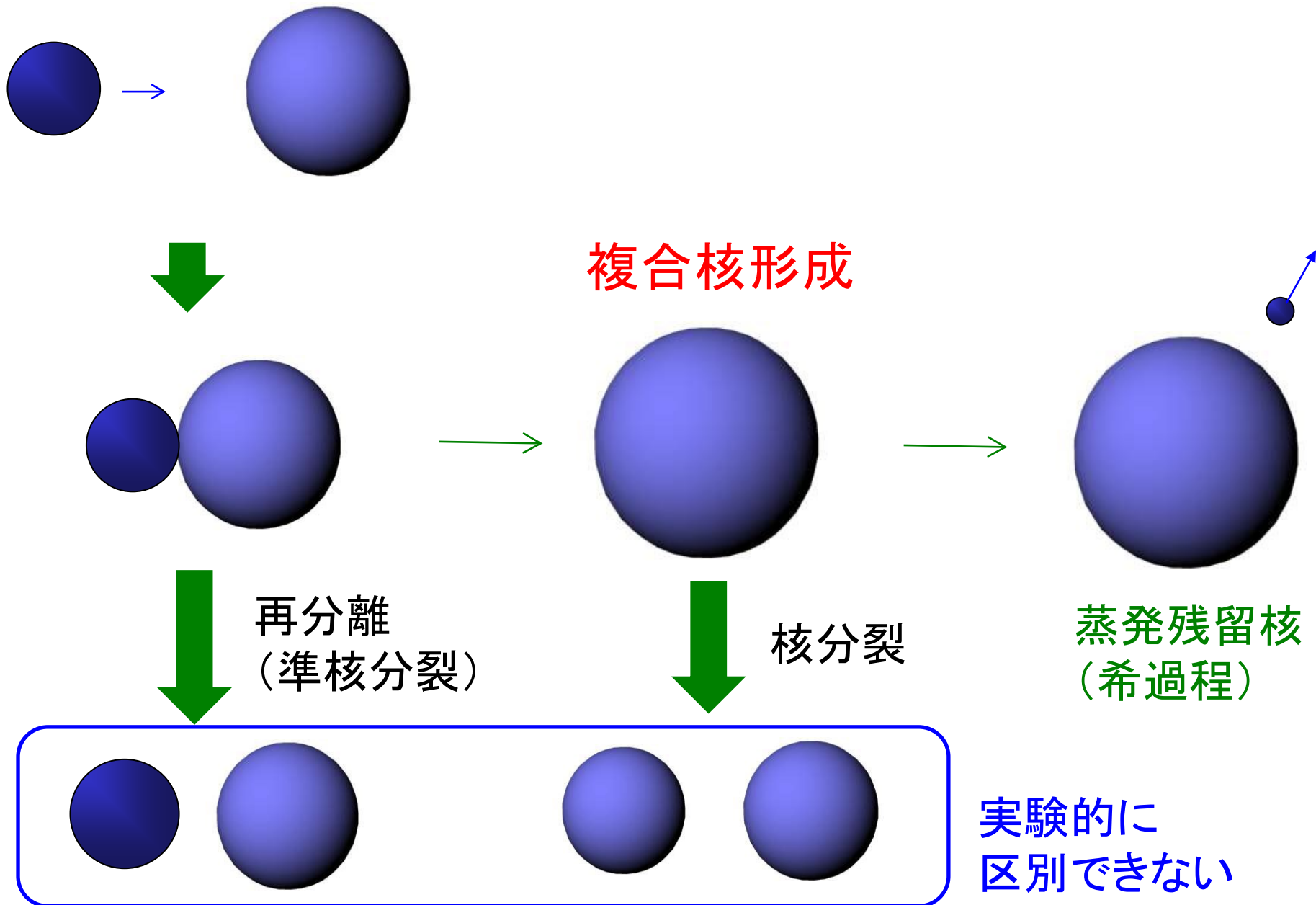
超重元素領域における重イオン核融合反応



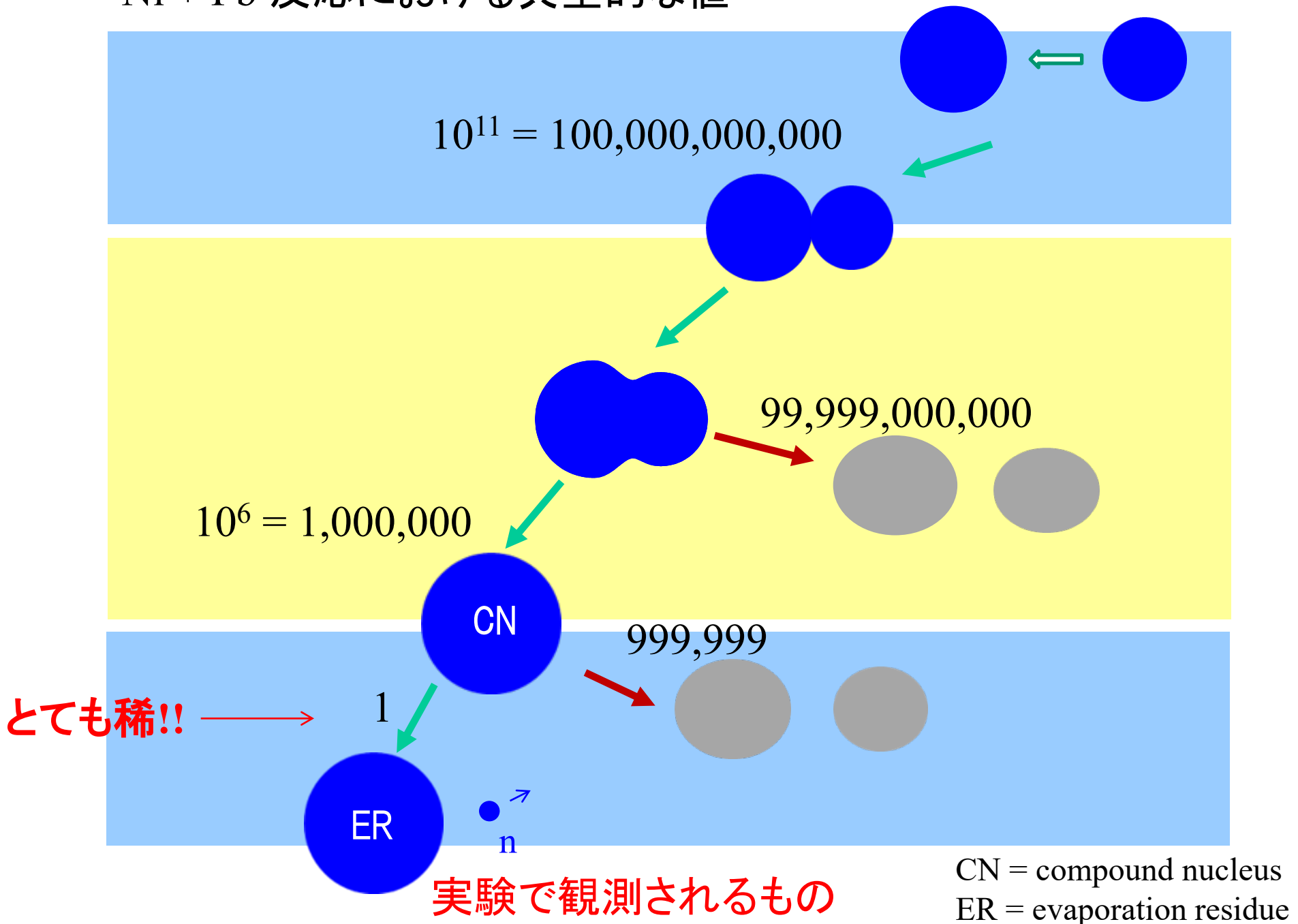
超重元素領域における重イオン核融合反応



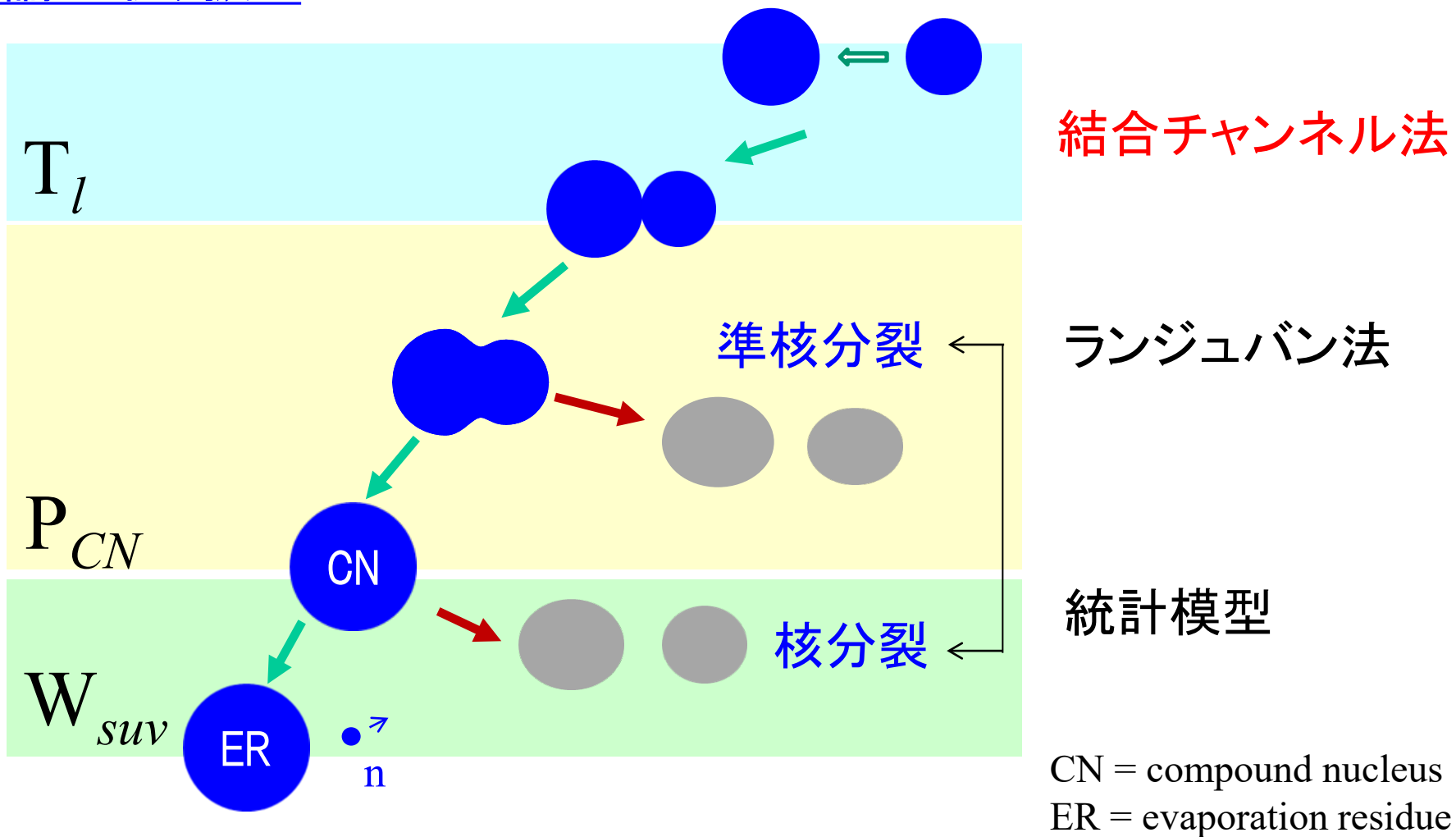
超重元素領域における重イオン核融合反応



Ni + Pb 反応における典型的な値

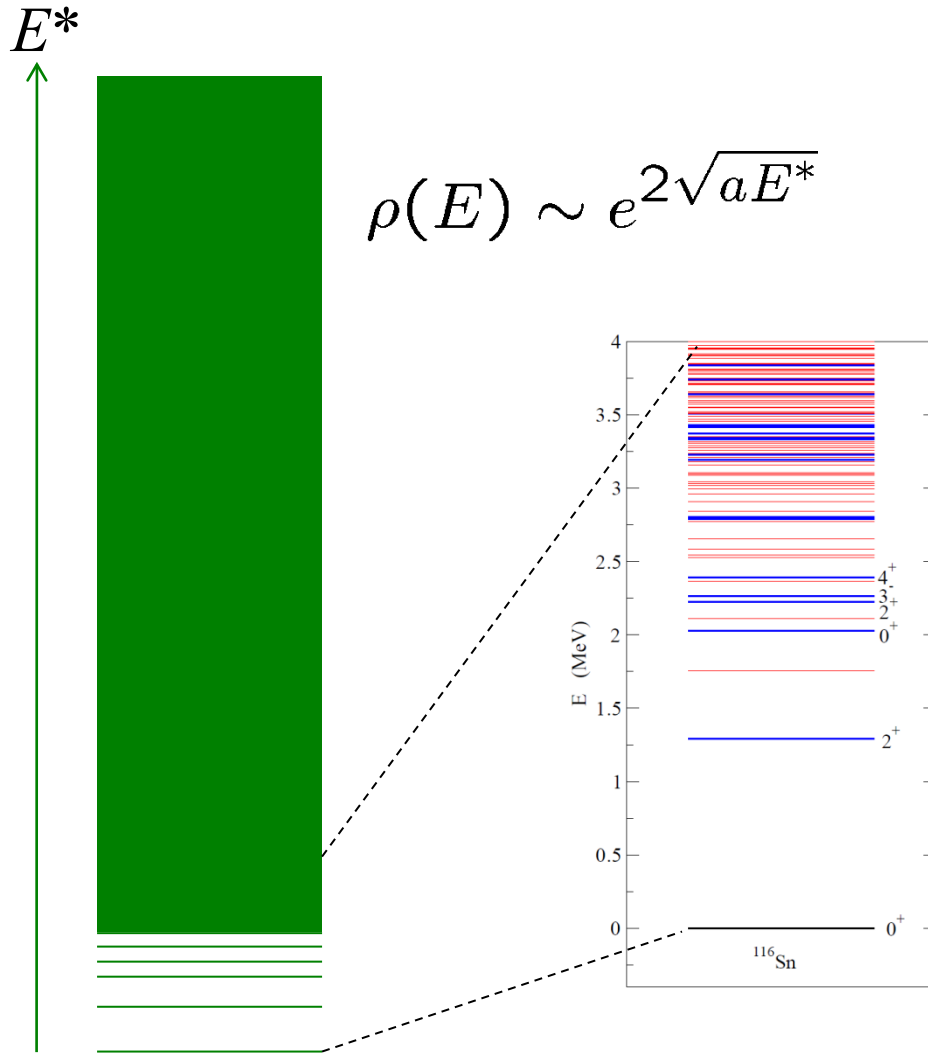


理論的取り扱い

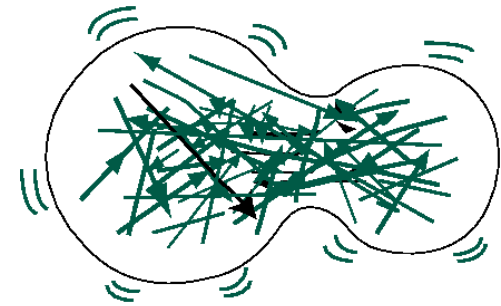


$$\sigma_{ER}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E) P_{CN}(E, l) W_{suv}(E^*, l)$$

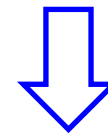
重イオン核融合反応と量子開放系



原子核のスペクトル



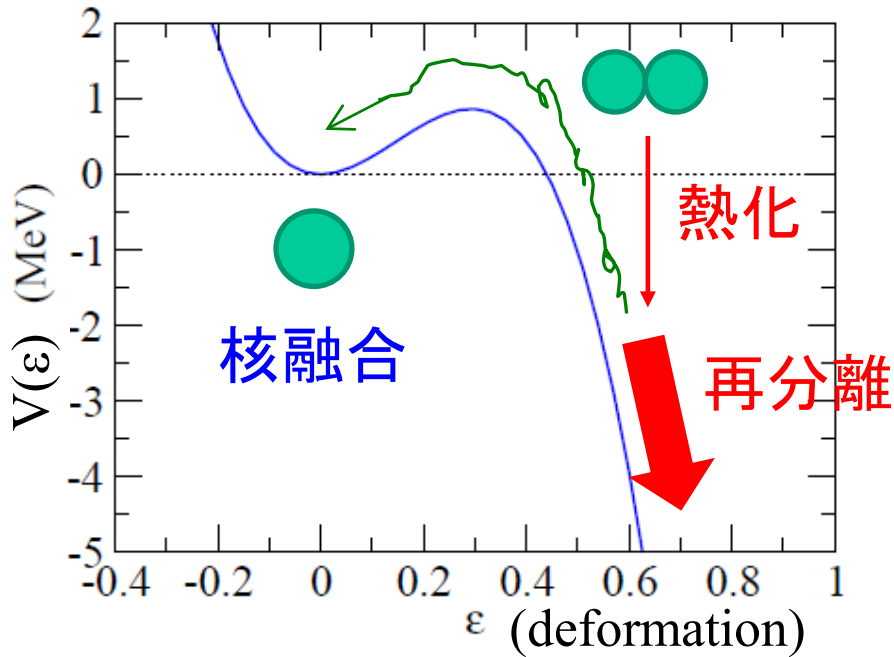
核反応の途中で複雑に励起



原子核の内部自由度:「環境」
「内的環境自由度」

→量子開放系の物理

ランジュバン法



熱的なゆらぎ

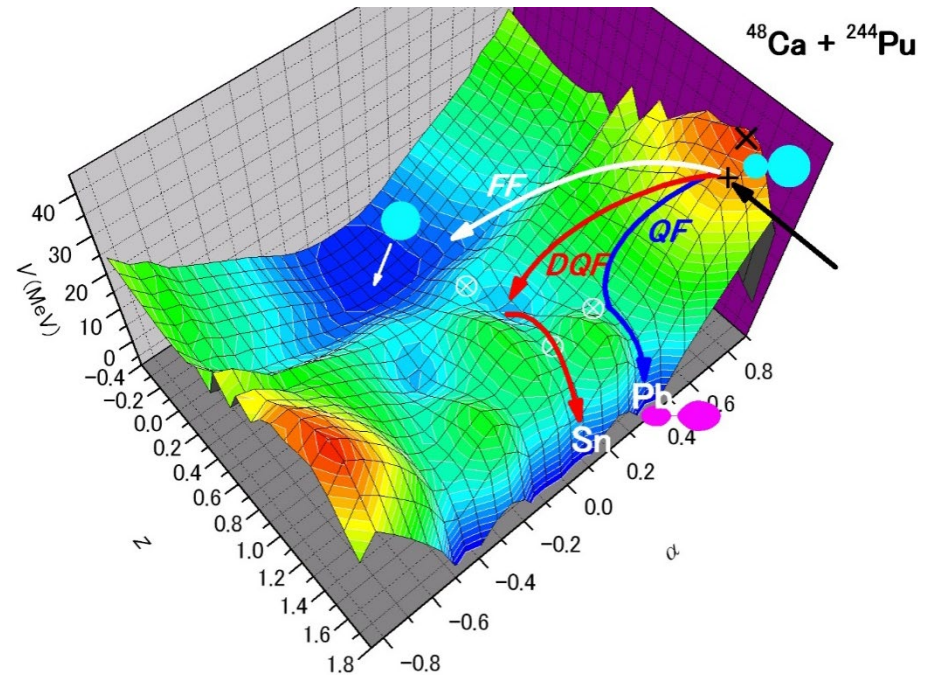
→ ランジュバン法
(ブラウン運動)

$$m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

γ : friction coefficient
 $R(t)$: random force

多次元化

- q : 核間距離
- 変形
- フラグメントの質量



理論: ランジュバン法

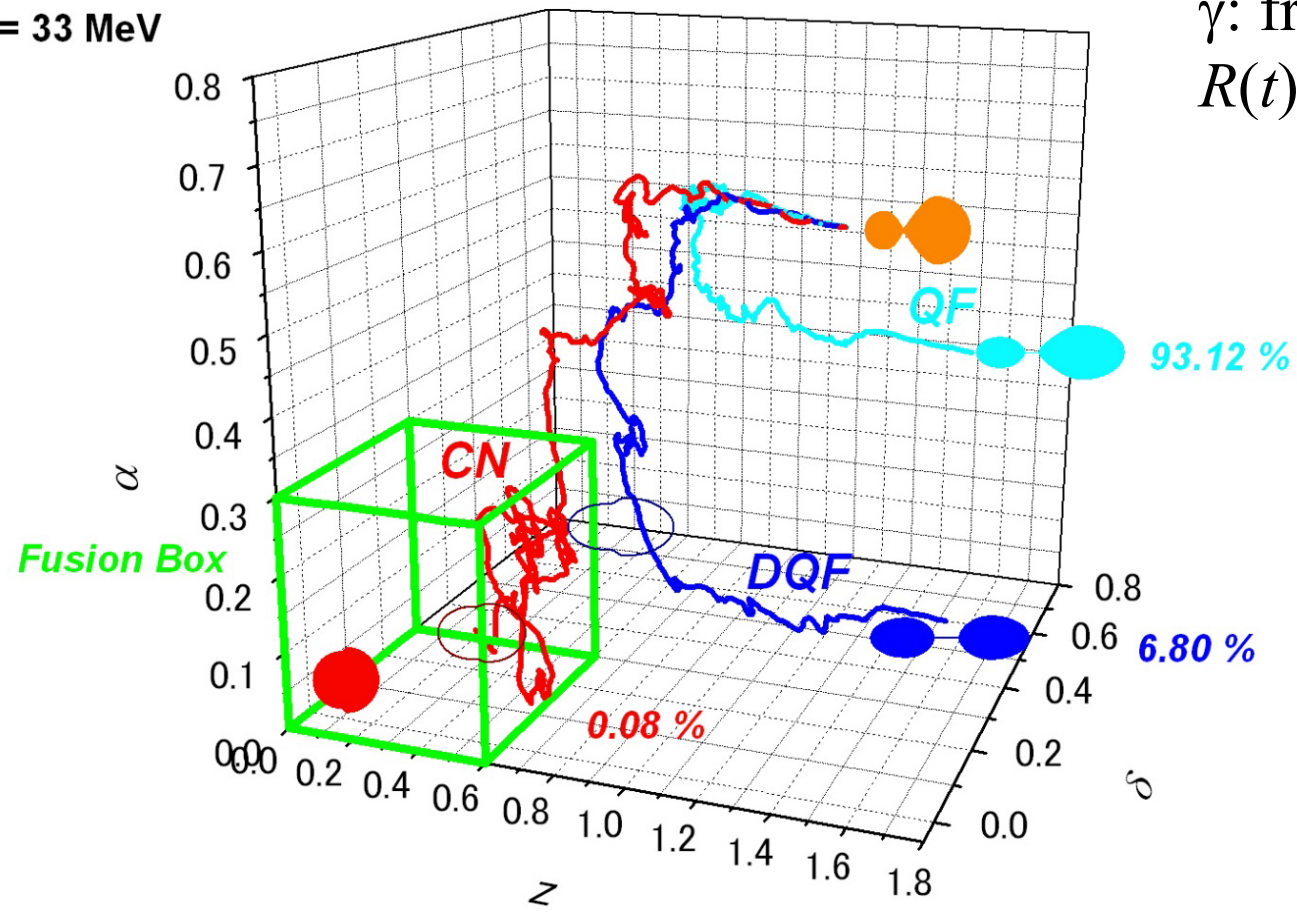
multi-dimensional extension of:

$$m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

γ : friction coefficient
 $R(t)$: random force



$E^* = 33 \text{ MeV}$



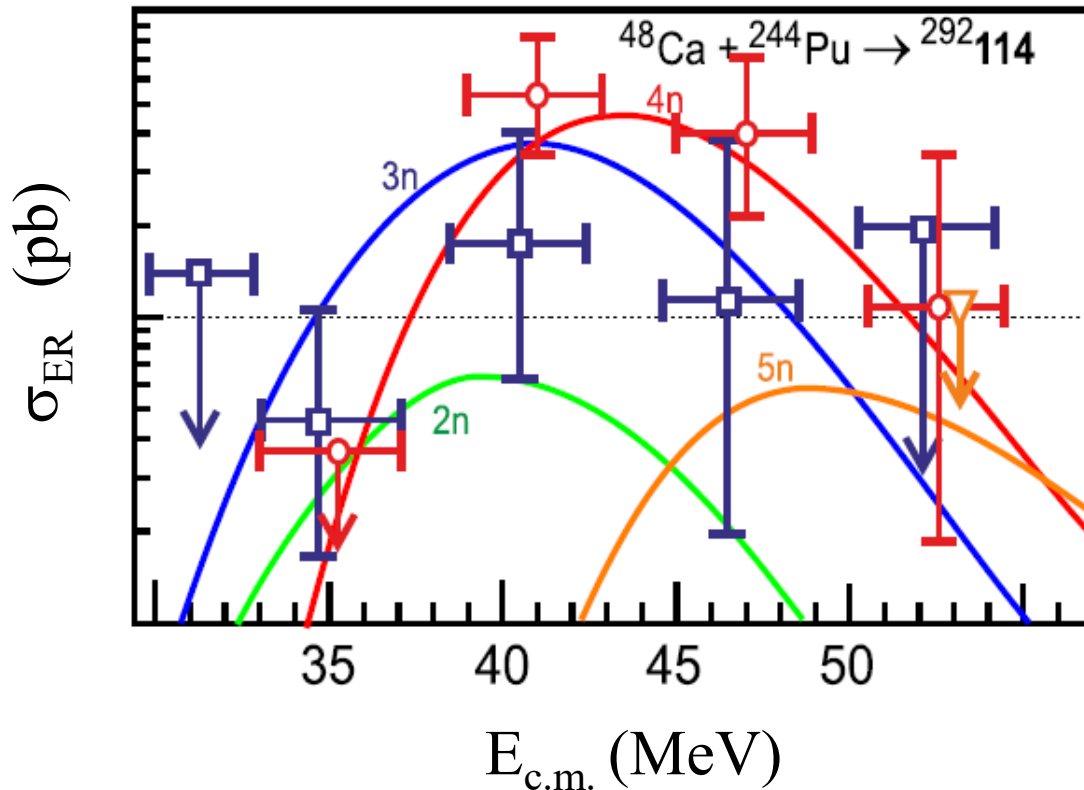
理論: ランジュバン法

multi-dimensional extension of:

$$m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

現象論的には成功

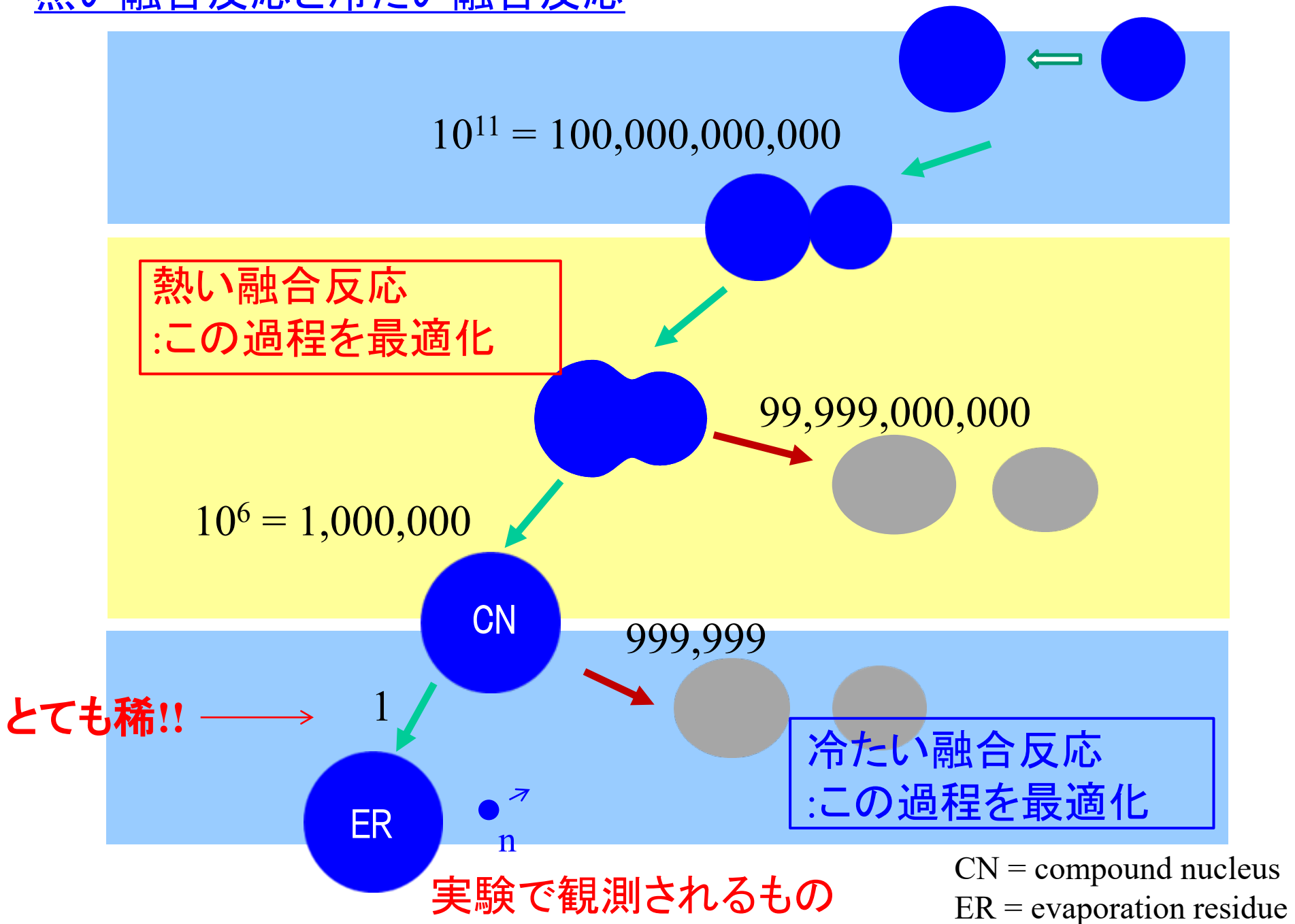
γ : friction coefficient
 $R(t)$: random force



量子効果？

V.I. Zagrebaev and W. Greiner (2015)

熱い融合反応と冷たい融合反応



レポート問題6(×切:12月3日(土))

113番元素 Nh を



で作る場合で、できた原子核の励起エネルギーを比較せよ。ただし、反応の入射エネルギーは重心系でクーロン障壁の高さと同じとせよ。また、それぞれの原子核が平均で何個の中性を放出するか見積もれ(中性子の平均的な分離エネルギーを 8 MeV とする)。

必要な情報:

✓ 束縛エネルギー (MeV)	${}^{48}\text{Ca}$: 416.0	${}^{70}\text{Zn}$: 611.1
	${}^{243}\text{Am}$: 1829.84	${}^{209}\text{Bi}$: 1640.23
	${}^{291}\text{Nh}$: 2086.85	${}^{279}\text{Nh}$: 2004.25

* ${}^{291,279}\text{Nh}$ の束縛エネルギーは理論値

✓ クーロン障壁の高さ (MeV)

$${}^{48}\text{Ca}+{}^{243}\text{Am}: 192.7, \quad {}^{70}\text{Zn}+{}^{209}\text{Bi}: 256.1$$

* グローバルポテンシャルによる理論値

$$\sigma \sim \text{fb} = 10^{-39} \text{ cm}^2$$

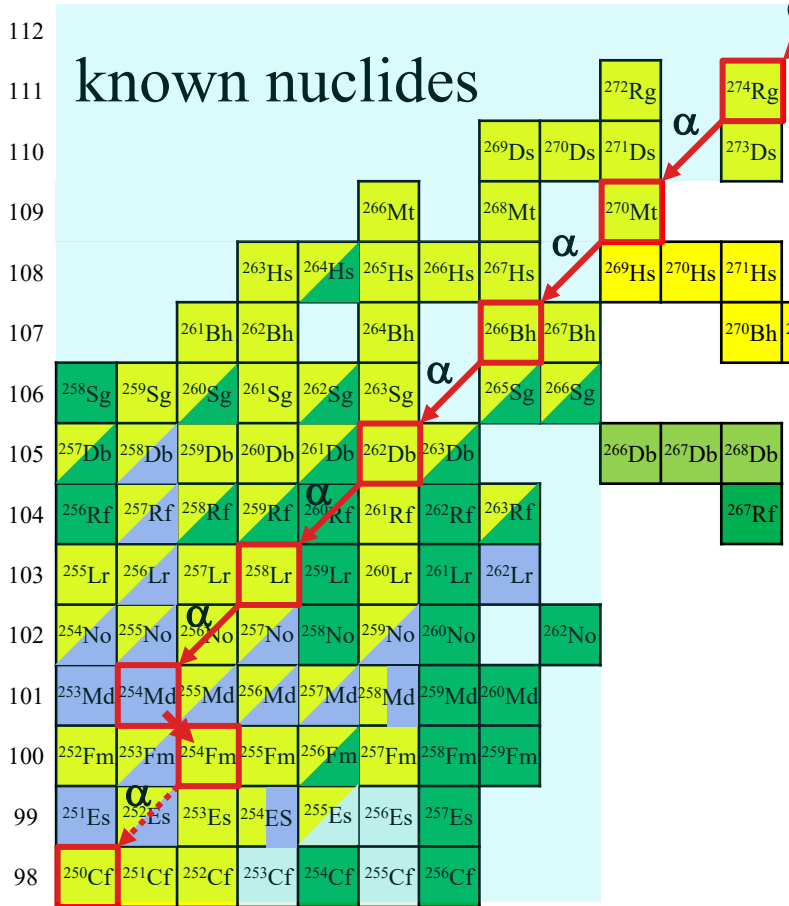
RIKEN

(Cold fusion)



3rd event Aug. 12 2012

113

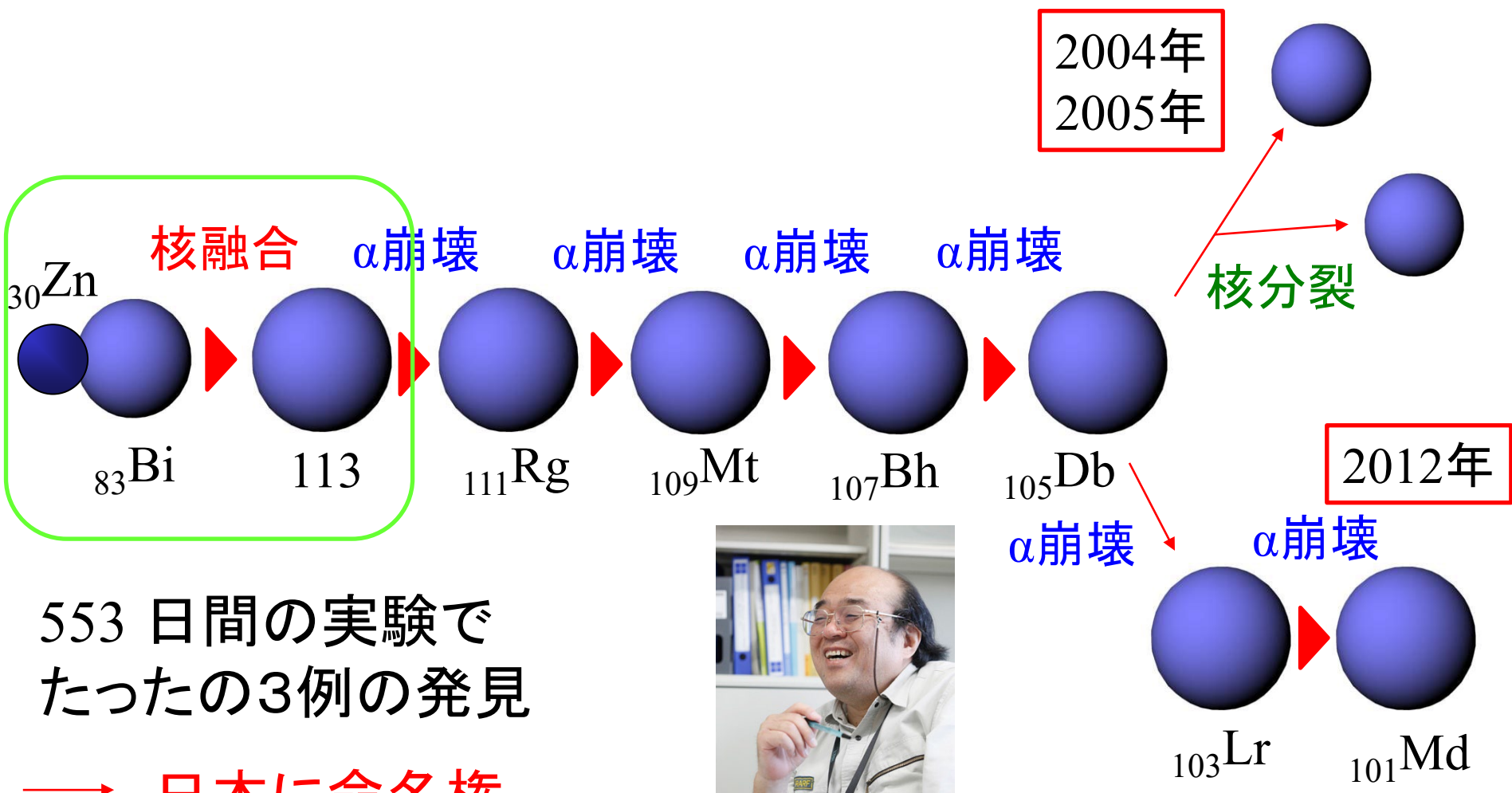
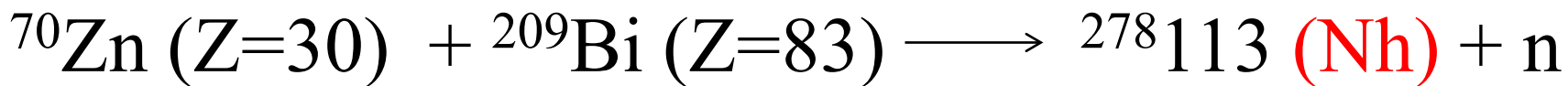


Dubna
(Hot fusion)
 $\sigma \sim \text{pb} = 10^{-36} \text{ cm}^2$

Cold Fusion:
既知核とつながる
(不定性がより少ない)

Hot Fusion:
より中性子過剰な複合核
が作れる。

新元素113番 ニホニウム (Nh)

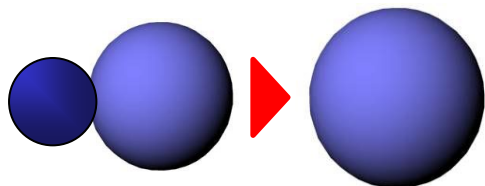
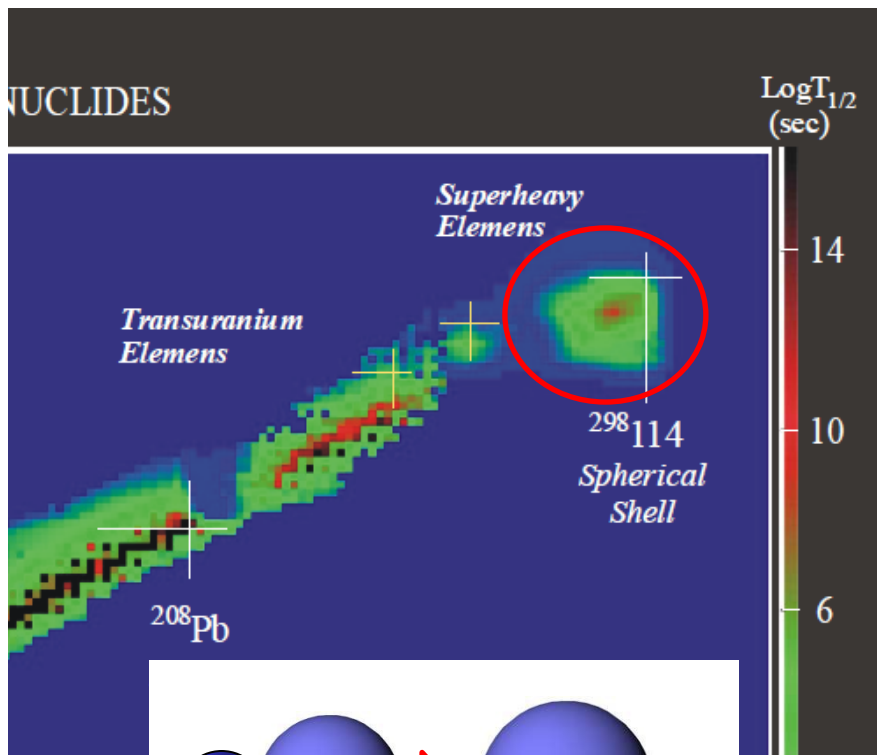


553 日間の実験で
たったの3例の発見

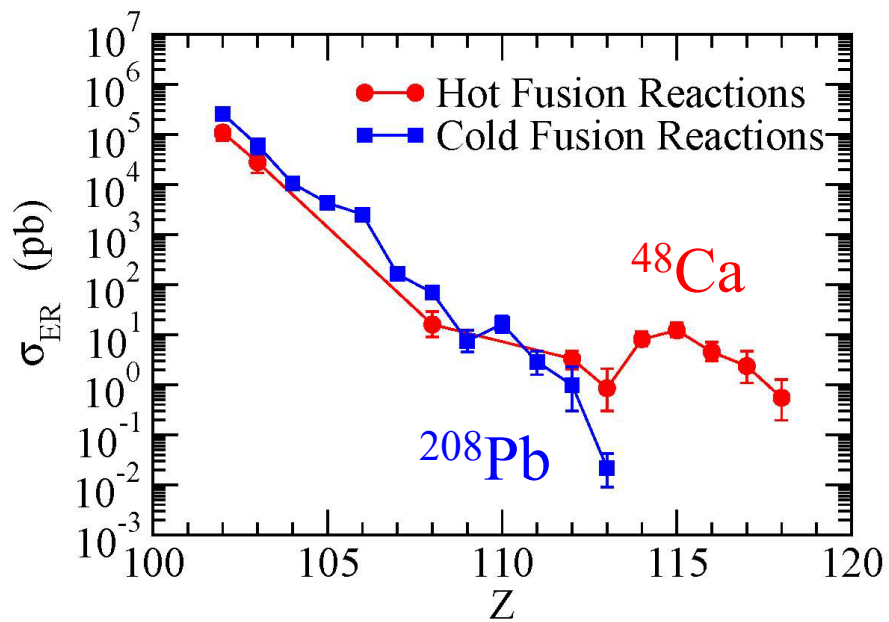
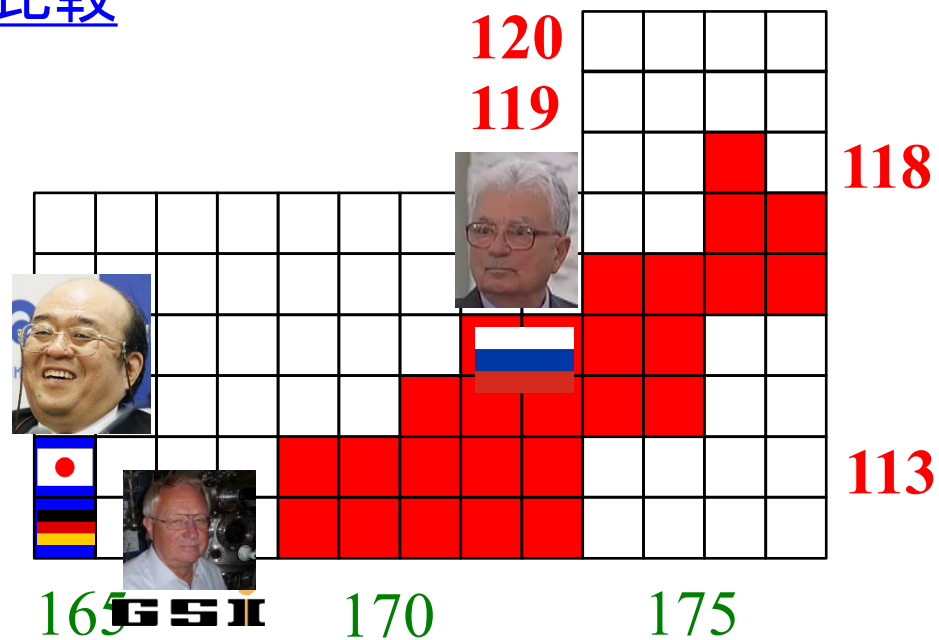
→ 日本に命名権
ニホニウム Nh



熱い融合と冷たい融合：断面積の比較

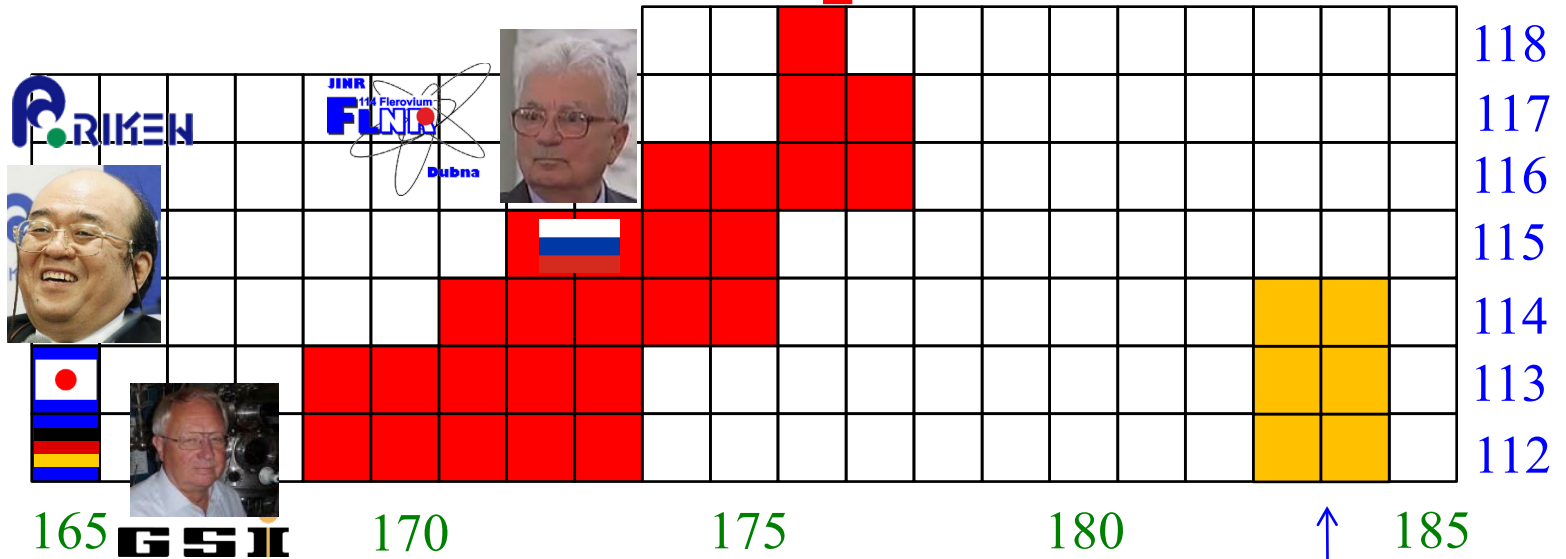


核融合反応

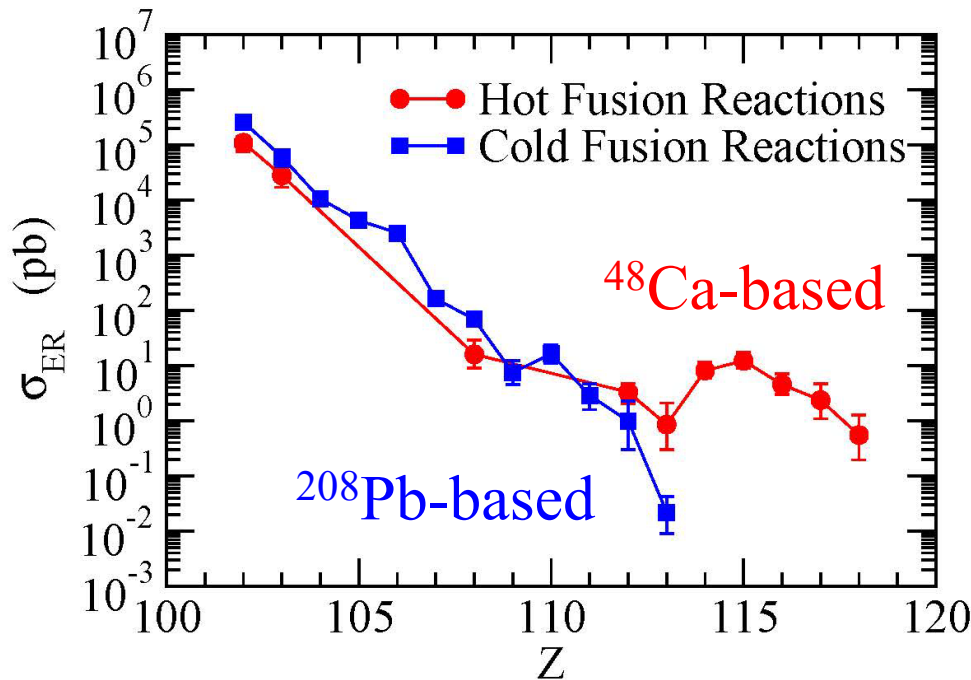


熱い融合反応: ^{48}Ca +actinides

Z = 119 and 120



安定の島?



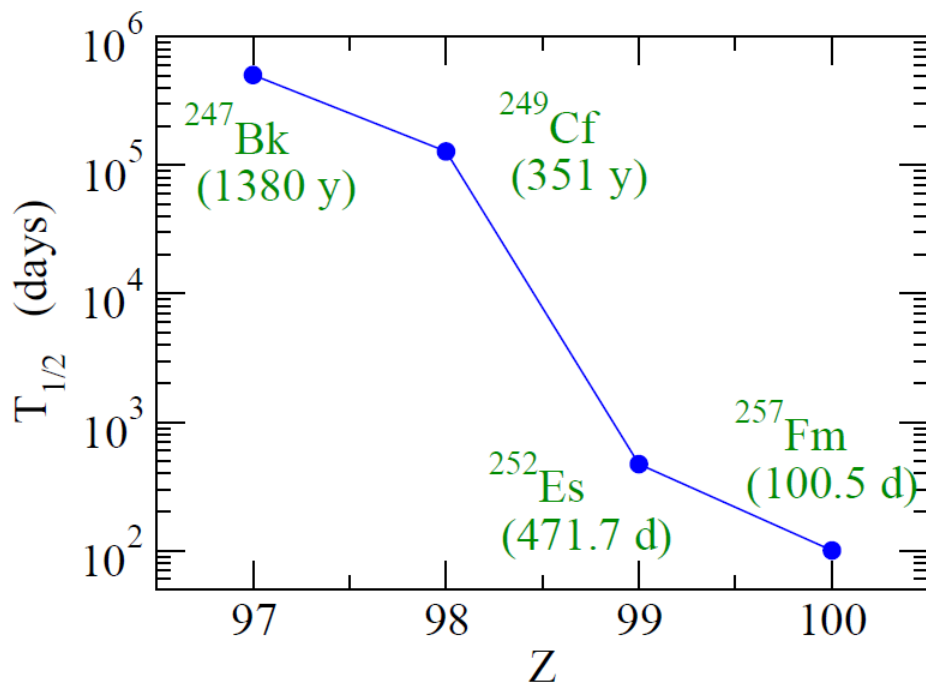
119番、120番元素
を熱い融合反応で作る

119番及び120番元素に向けて

^{48}Ca を使った熱い融合反応:



短寿命→標的にできるほど大量に
ない



$^{48}\text{Ca} \rightarrow {}^{50}_{22}\text{Ti}, {}^{51}_{23}\text{V}, {}^{54}_{24}\text{Cr}$ 核

二重閉殻 → 開殻

どのくらい断面積は
影響を受けるのか?

超重元素反応の新モデル: TDHF + Langevin アプローチ

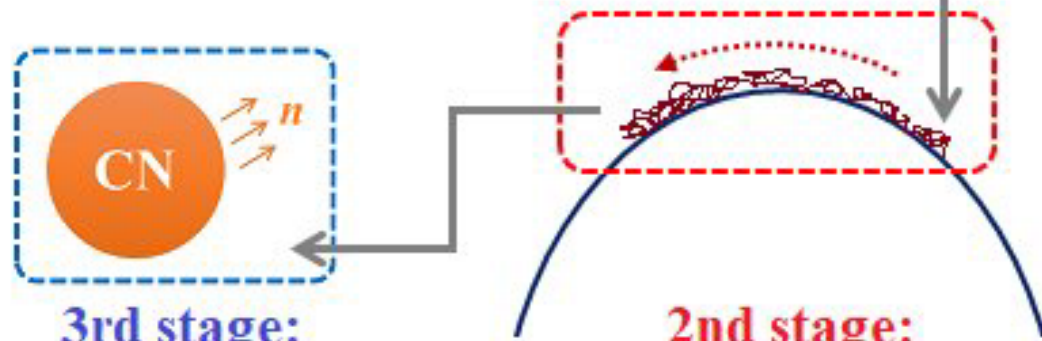
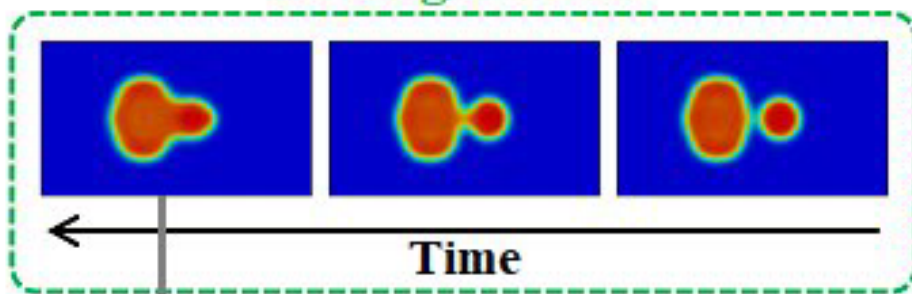
K. Sekizawa and K.H., PRC99 (2019) 051602(R)



新しい核反応モデルの開発

時間に依存する
密度汎関数理論

1st stage: TDHF



3rd stage:
statistical model

統計力学的手法

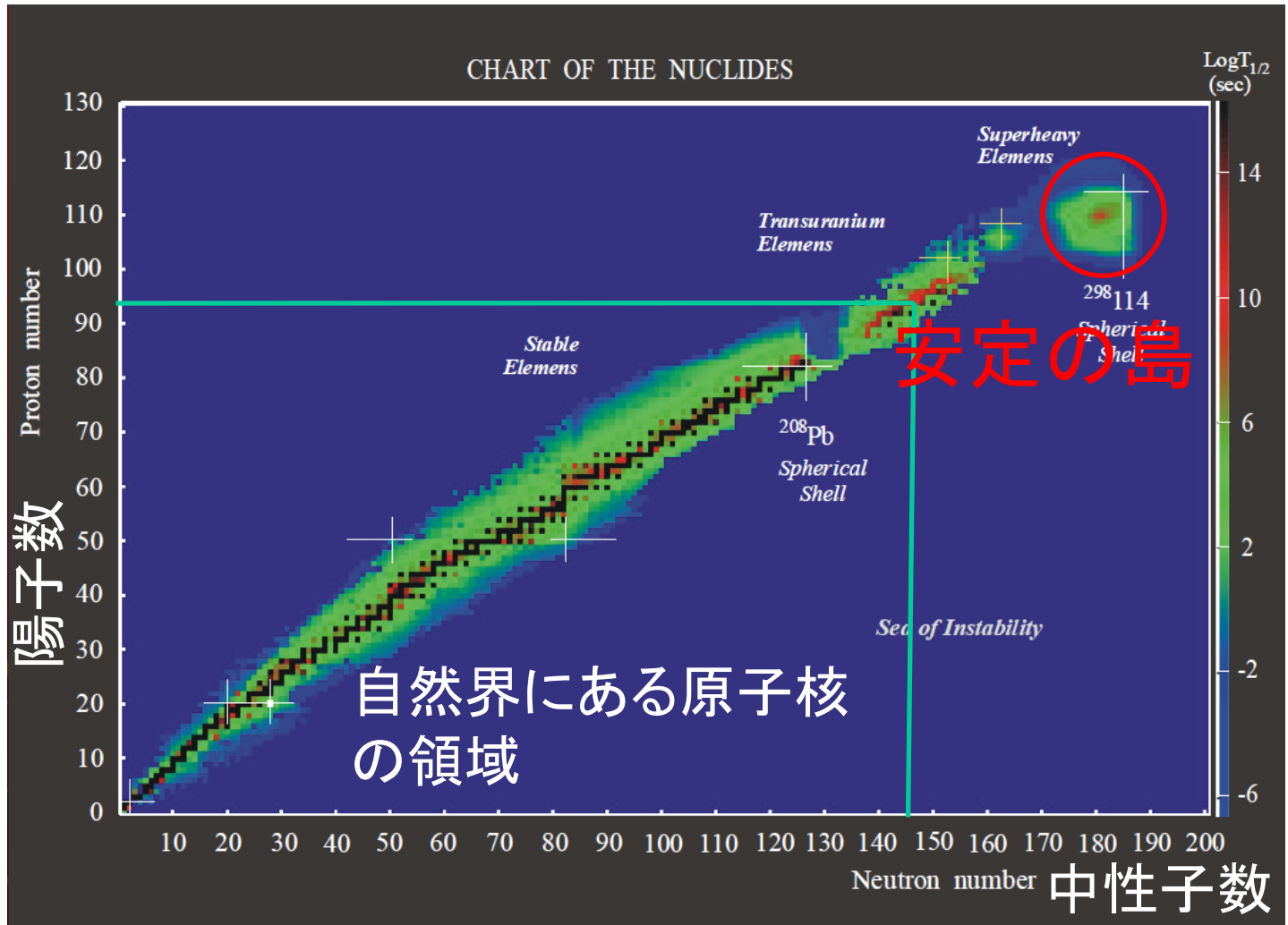
2nd stage:
Langevin model

ランジュバン方程式



^{48}Ca から ^{54}Cr に変えると2桁落ちくらい(120番元素生成)

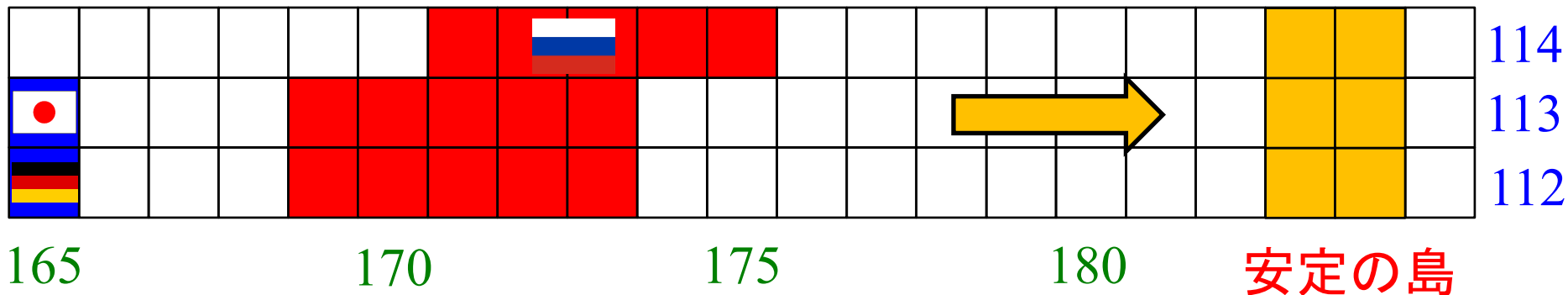
もう一つの重要な課題



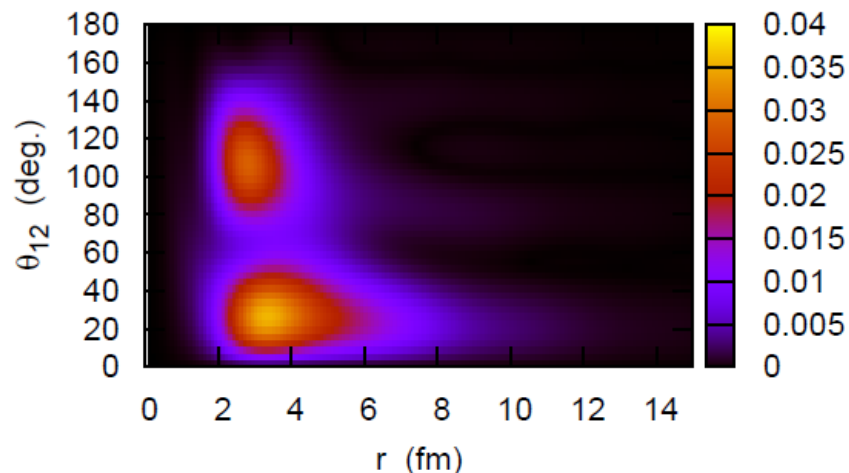
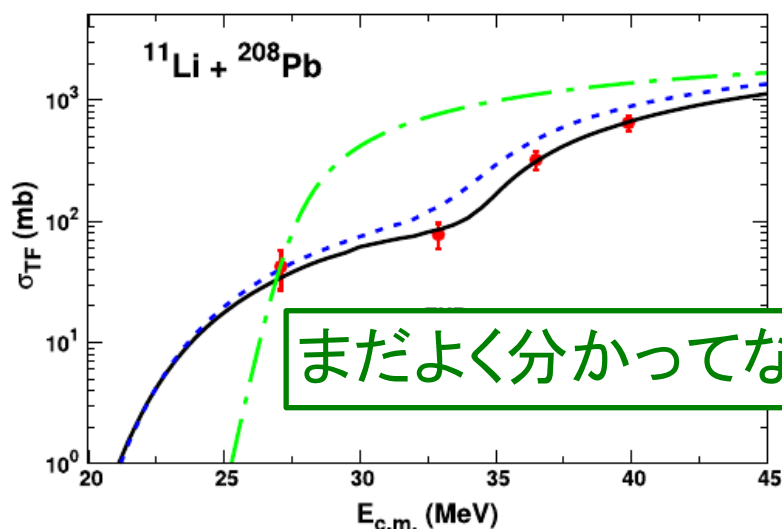
Yuri Oganessian

安定の島にどのように到達するか?

中性子過剰核の核融合反応



中性子過剰核ビームが必須→反応ダイナミクス?

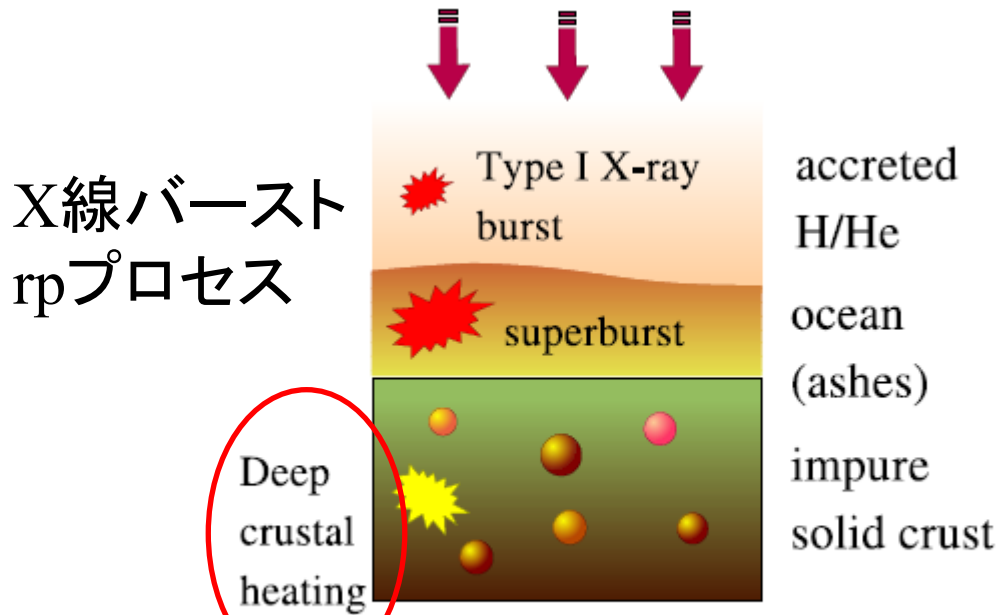


K.H. and H. Sagawa, PRC72('05)044321

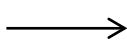
K.-S. Choi, K. Hagino et al.,
Phys. Lett. B780 ('18) 455

中性子過剰核の構造の理解も必要

質量降着を伴う中性子星で起こる 中性子過剰核の核融合反応

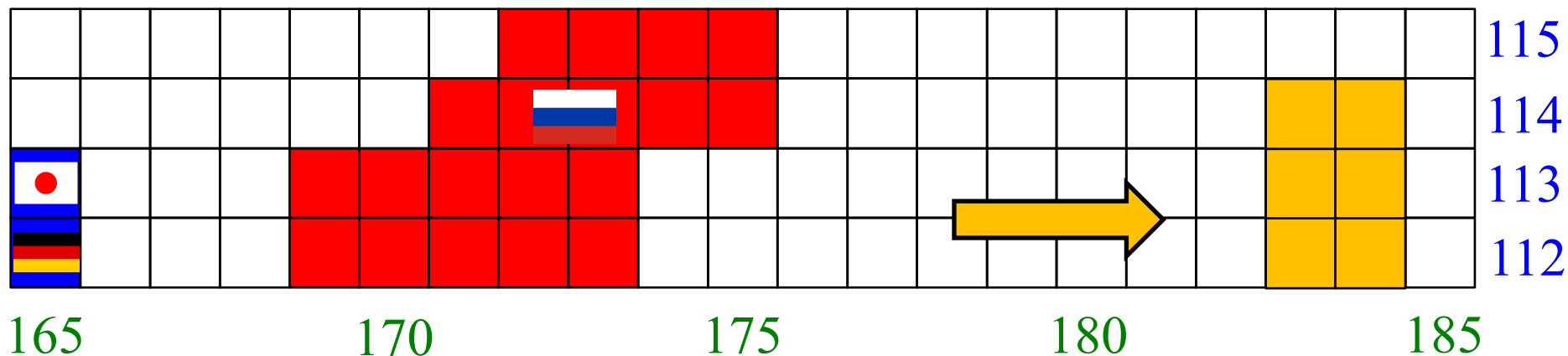


中性子過剰核の核融合反応



X線連星の静穏期
におけるX線の起源

安定の島に向けて



中性子過剰核ビームが必須

- 低いビーム強度をどうするか?
- 中性子過剰核を用いた核反応のダイナミクス?
 - ✓ 分解や多核子移行反応
 - ✓ 原子核の形状の時間発展中の中性子放出
 - ✓ 統計モデルはどれだけいいのか?

中性子過剰核の構造

課題はいっぱい