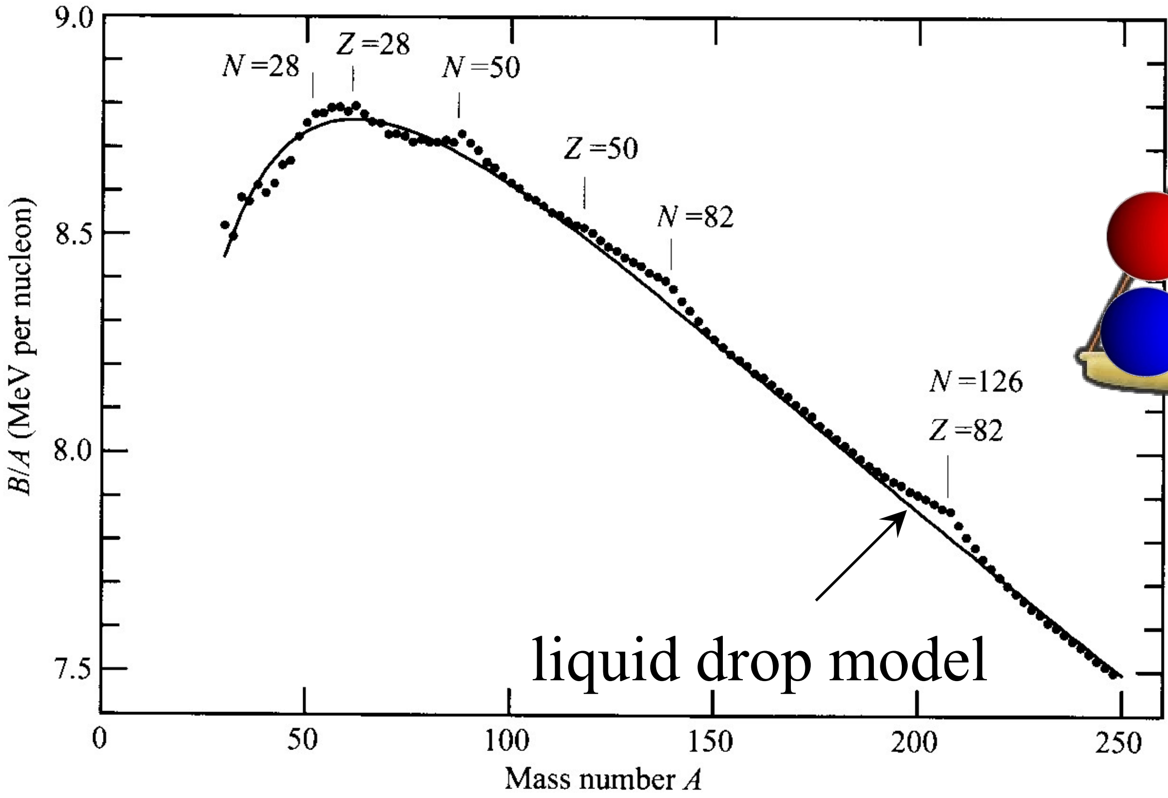
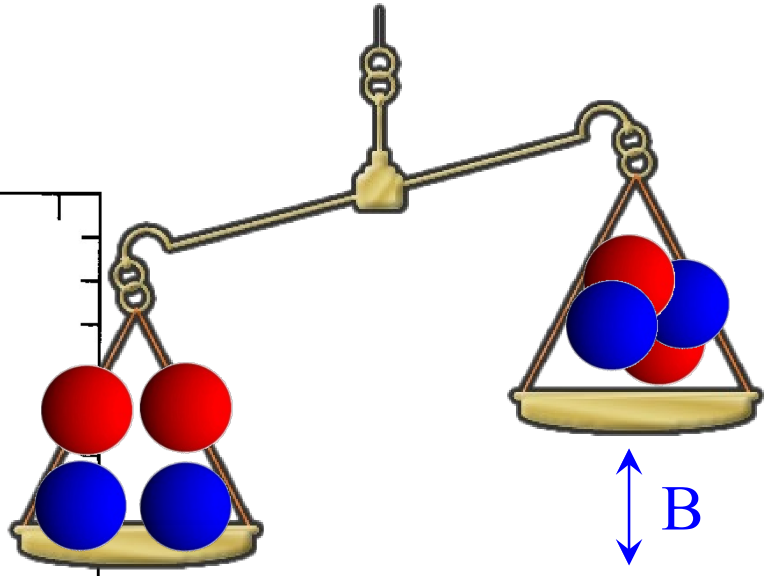
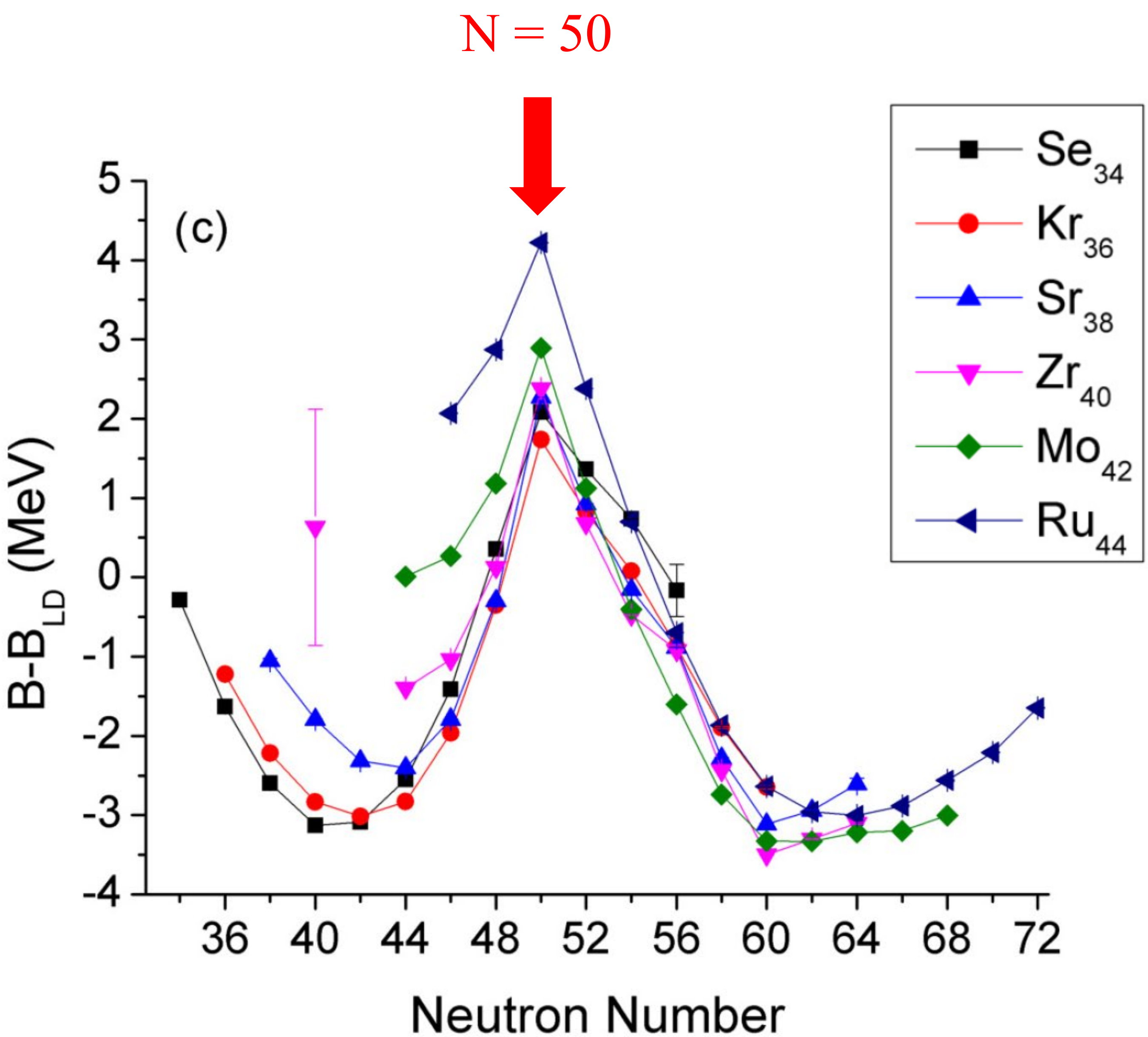
Nuclear magic numbers





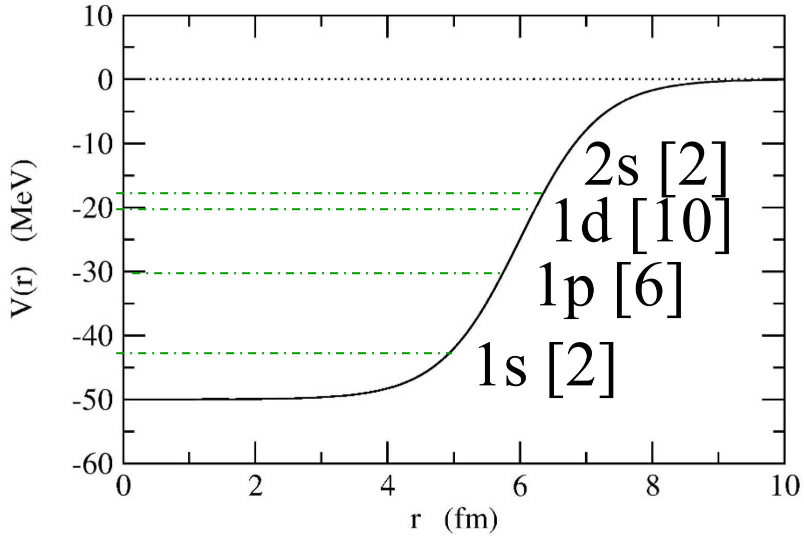
Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)



I. Bentley et al., PRC93 ('16) 044337

Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

An interpretation: independent particle motion in a potential well



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

degeneracy: 2*(2l+1)

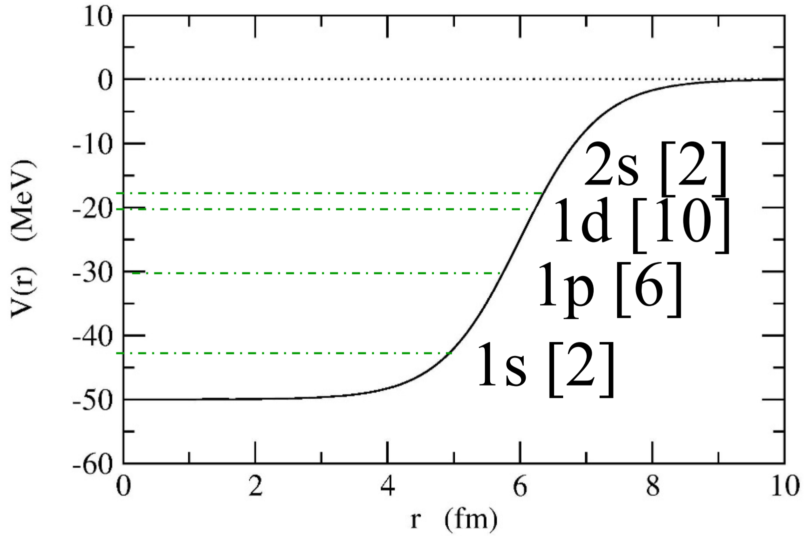
spin-orbit interaction

f[14]	34
s[2],d[10]	20
p[6]	8
s[2]	2

f[14] → $f_{5/2}$[6], $f_{7/2}$[8]

20+8 = 28

Shell Model: independent particle motion in a potential well



+ spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

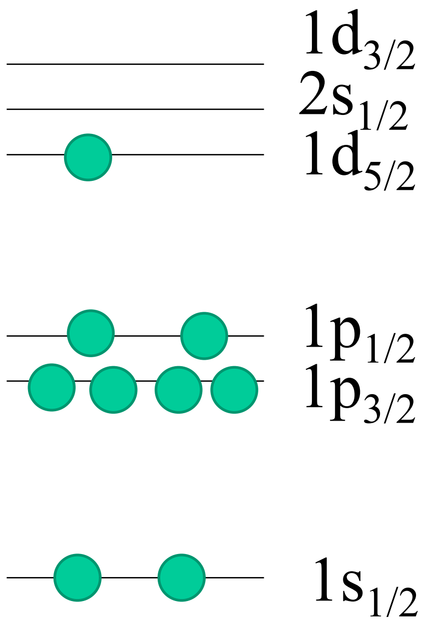
$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) = \sum_{m_l, m_s} \langle l\ m_l\ 1/2\ m_s | j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k$$

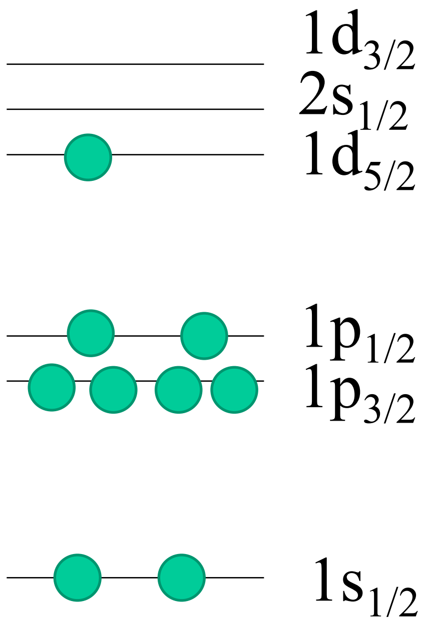
shell model

$$H = \sum_k \epsilon_k a_k^\dagger a_k$$

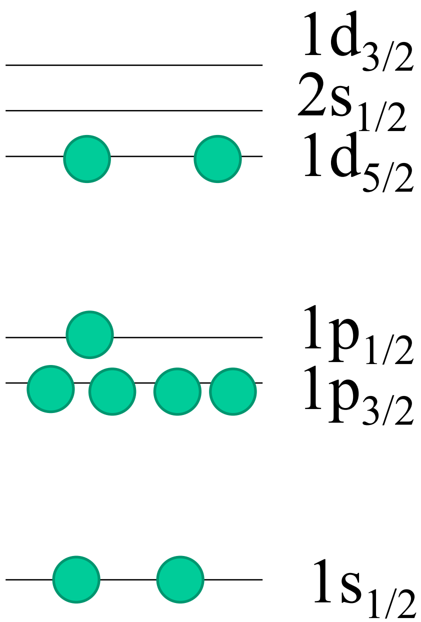


shell model

$$H = \sum_k \epsilon_k a_k^\dagger a_k$$



configuration 1



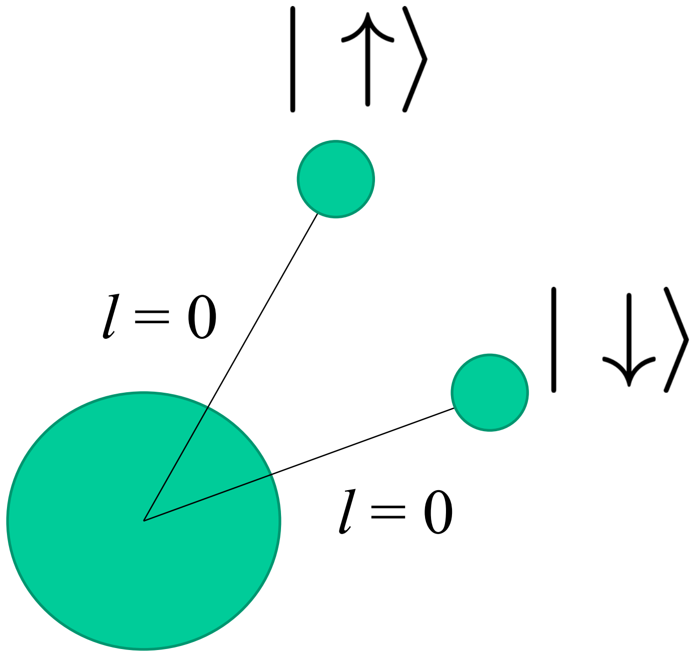
configuration 2

…… several others

angular momentum (spin) and parity for each configuration?

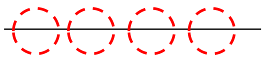
→ let us first investigate a single-j case

the first example: j = $s_{1/2}$



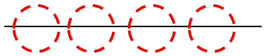
この系の全スピンは何か？

the next example: $j = p_{3/2}$



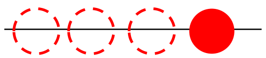
$p_{3/2}$

can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)

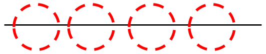


$p_{3/2}$ can accommodate 4 nucleons (j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon

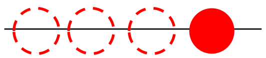


$p_{3/2}$



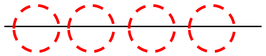
$p_{3/2}$ can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon



$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

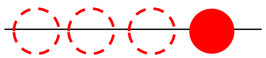
(there are 4 ways to occupy this level)



$p_{3/2}$ can accommodate 4 nucleons

(j_z= +3/2, +1/2, -1/2, -3/2)

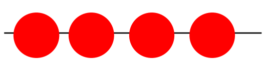
i) 1 nucleon



$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

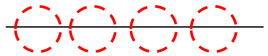
(there are 4 ways to occupy this level)

ii) 4 nucleons



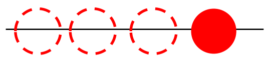
$p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

 $p_{3/2}$ can accommodate 4 nucleons

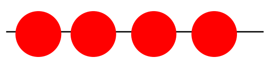
(j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon

 $p_{3/2}$ → $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

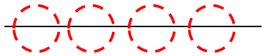
ii) 4 nucleons

 $p_{3/2}$ → $I^{\pi} = 0^{+}$

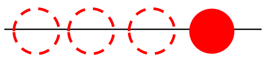
$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1

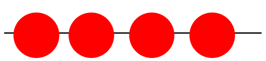
 $p_{3/2}$ can accommodate 4 nucleons (j_z= +3/2, +1/2, -1/2, -3/2)

i) 1 nucleon

 $p_{3/2}$ $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons

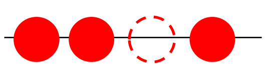
 $p_{3/2}$ $I^\pi = 0^+$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

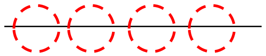
(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons

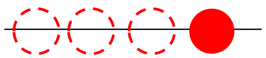
 $p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$



$p_{3/2}$ can accommodate 4 nucleons (j_z = +3/2, +1/2, -1/2, -3/2)

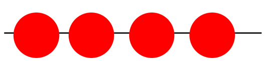
i) 1 nucleon



$p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons



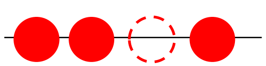
$p_{3/2}$ → $I^\pi = 0^+$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons



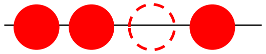
$p_{3/2}$ → $I^\pi = 3/2^-$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iii) 3 nucleons



$p_{3/2}$

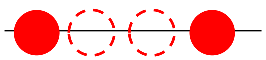
$I^\pi = 3/2^-$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

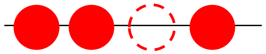
iv) 2 nucleons



$p_{3/2}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2$$

iii) 3 nucleons


$p_{3/2}$

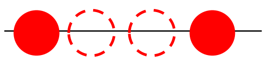
$$I = j_1 + j_2 + j_3$$

$\Rightarrow$ $I^\pi = 3/2^-$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iv) 2 nucleons


$p_{3/2}$

$$I = j_1 + j_2$$

there are 4 x 3/2=6 ways to occupy this level with 2 nucleons.

$\Rightarrow$ $I^\pi = 0^+$ or 2^+ (= 1+5)

3/2 + 3/2 $\rightarrow$ I = 0, ~~1~~, 2, ~~3~~

anti-symmetrization

レポート問題1：

角運動量 j を持つ軌道(j は半整数)にフェルミオン2つを生成する以下の演算子を考える。

$$[a_j^\dagger a_j^\dagger]^{(JM)} = \sum_{m,m'} \langle jmjm'|JM\rangle\, a_{jm}^\dagger a_{jm'}^\dagger$$

ここで、J は2粒子系の全角運動量、M はその z 成分である。

フェルミオン演算子の反交換関係

$$\{a_{jm}^\dagger, a_{jm'}^\dagger\} = 0$$

及び Clebsch-Gordan 係数の性質

$$\langle jmjm'|JM\rangle = (-1)^{j+j-J}\langle jm'jm|JM\rangle$$

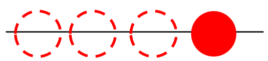
を用いて、角運動量 J は偶数の値しかとらないことを示せ。

レポート問題２：

前問で、角運動量 j が整数のボゾンの場合、全角運動量 J が
どのような値を取るか議論せよ（偶数か、奇数か、全て可か。
あるいは、他に何らかの制限がつくのか、など）。

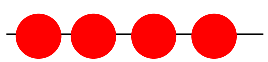
$$[a_j^\dagger a_j^\dagger]^{(JM)} = \sum_{m,m'} \langle jmjm'|JM\rangle \, a_{jm}^\dagger a_{jm'}^\dagger$$

i) 1 nucleon

 $p_{3/2}$ → $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

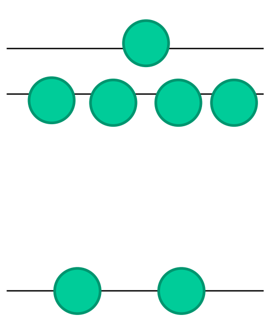
ii) 4 nucleons

 $p_{3/2}$ → $I^{\pi} = 0^{+}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1



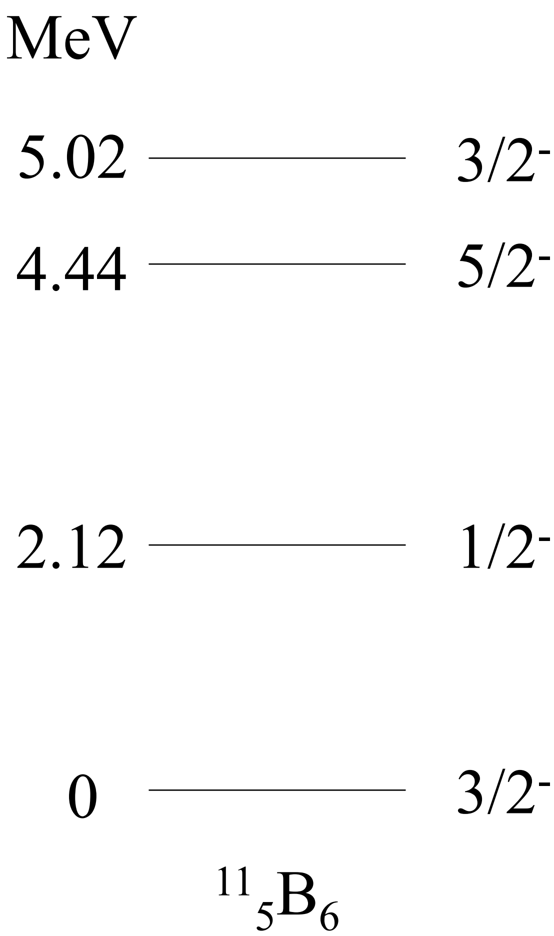
$1p_{1/2}$ → $I^{\pi} = 1/2^{-}$

$1p_{3/2}$ → $I^{\pi} = 0^{+}$

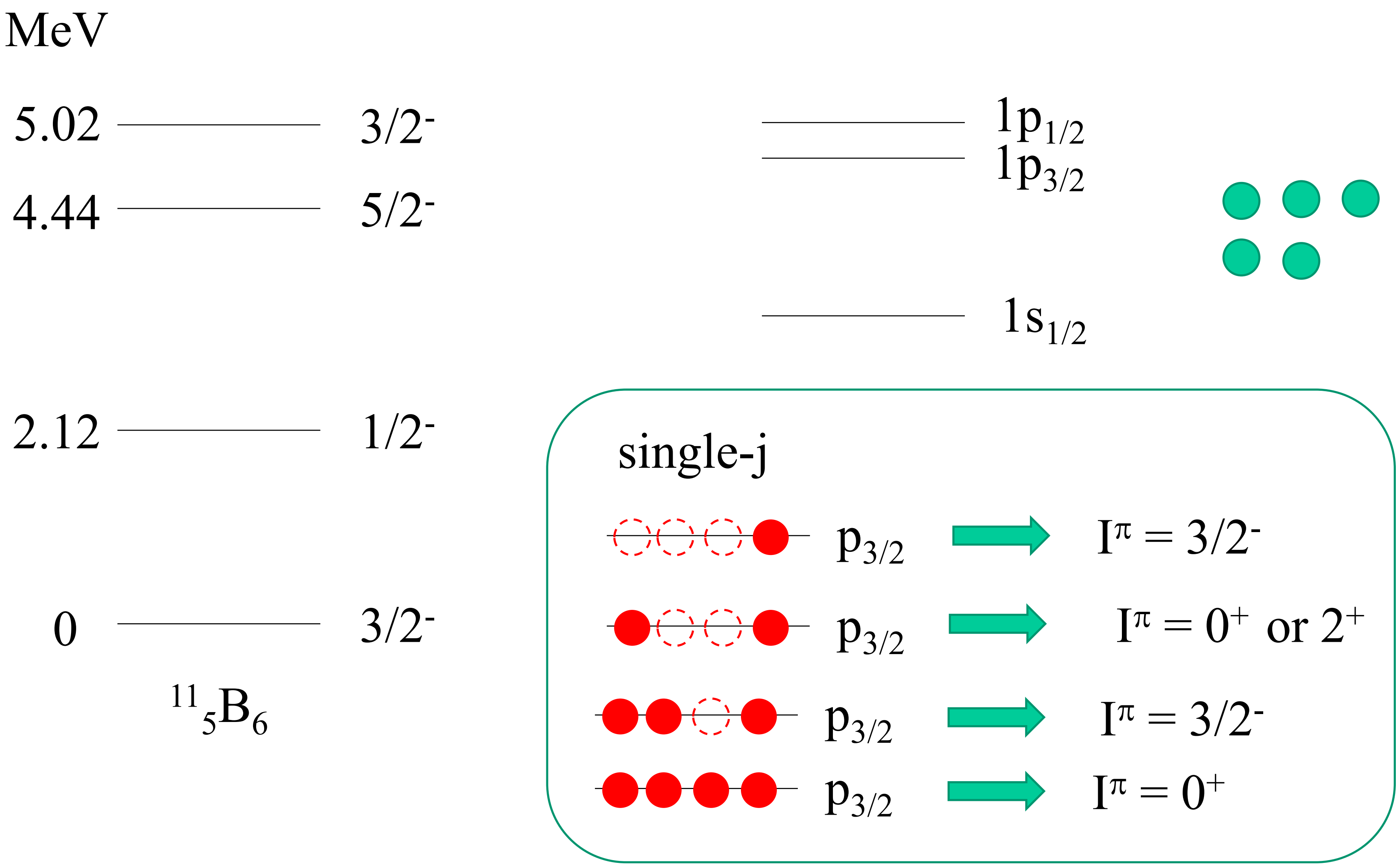
$1s_{1/2}$ → $I^{\pi} = 0^{+}$

in total,
$I^{\pi} = 1/2^{-}$

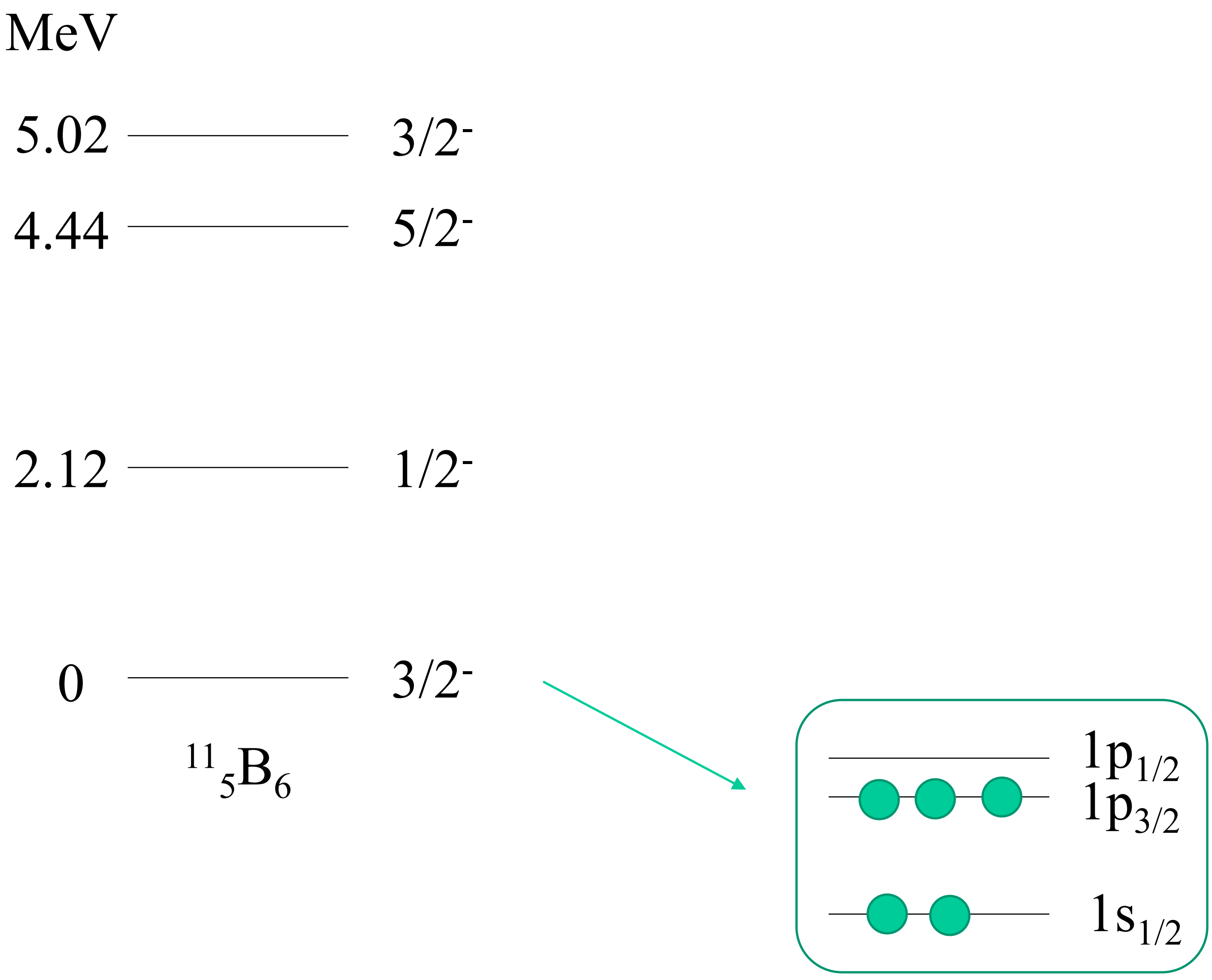
example: (main) shell model configurations for $^{11}_{5}B_6$



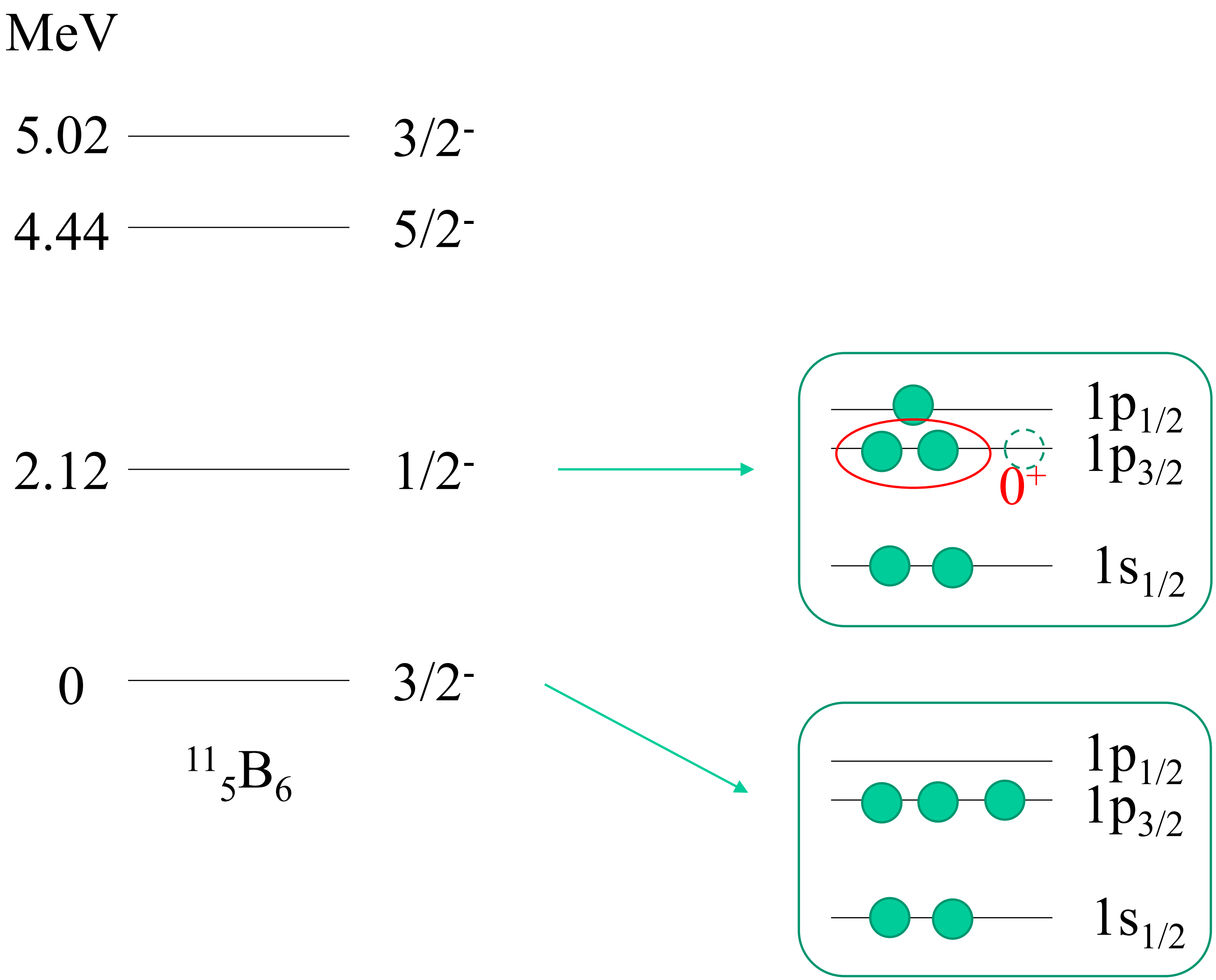
example: (main) shell model configurations for $^{11}{}_5B_6$



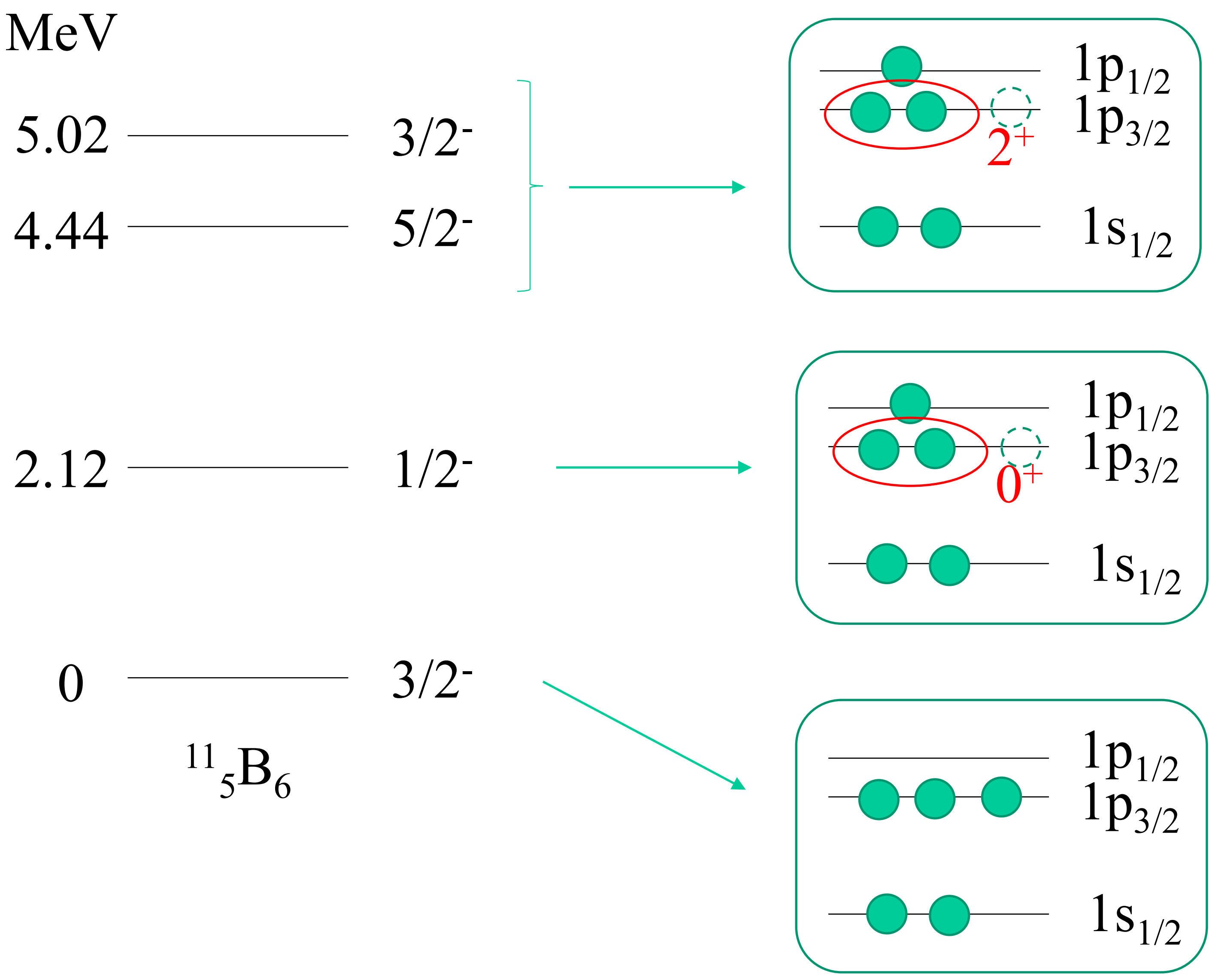
example: (main) shell model configurations for $^{11}{}_5B_6$



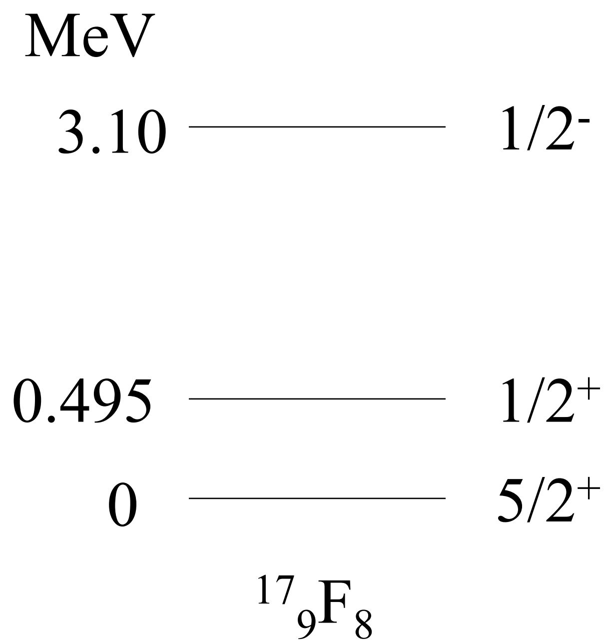
example: (main) shell model configurations for $^{11}{}_{5}B_{6}$



example: (main) shell model configurations for $^{11}_{5}B_{6}$

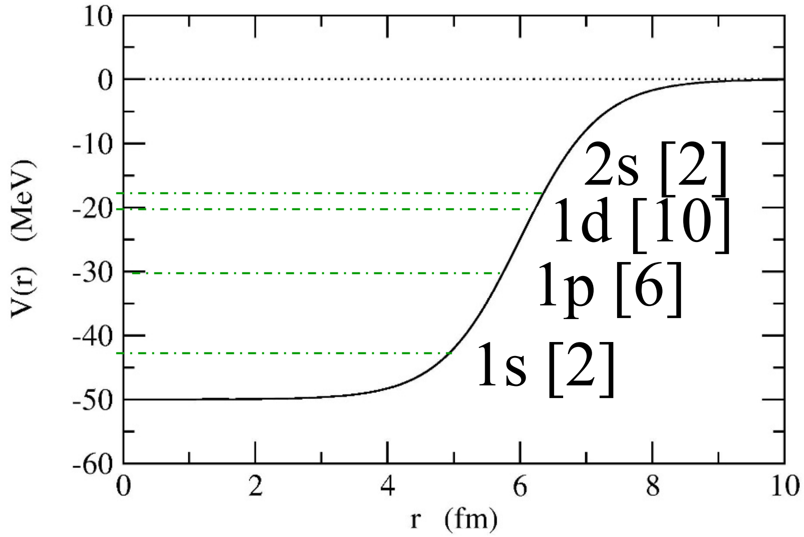


レポート問題3：${}^{17}_{9}F_8$の基底状態の陽子の配位を殻模型を使って説明せよ。第一励起状態、第二励起状態はどうなるか？



Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

An interpretation: independent particle motion in a potential well

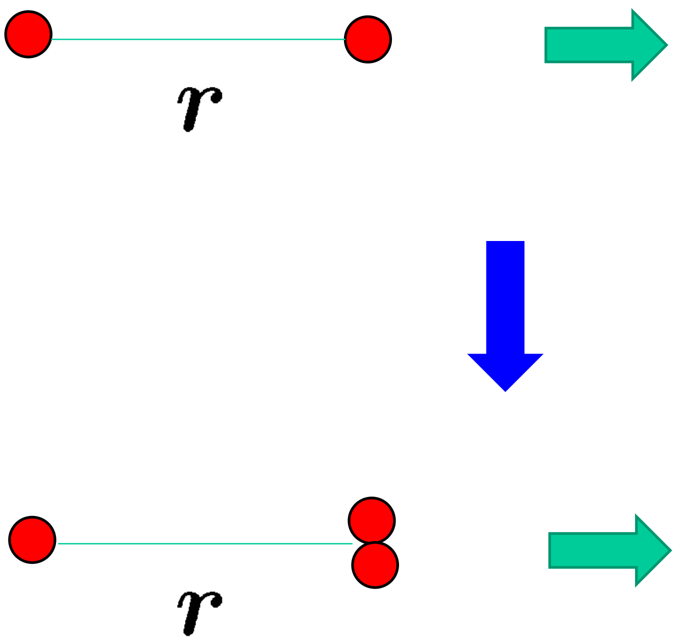


+ spin-orbit interaction

how to construct the potential well?

Mean-field (Hartree-Fock) Theory

nucleon-nucleon interaction

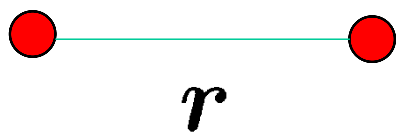


$$v(\boldsymbol{r})$$

$$2v(\boldsymbol{r})$$

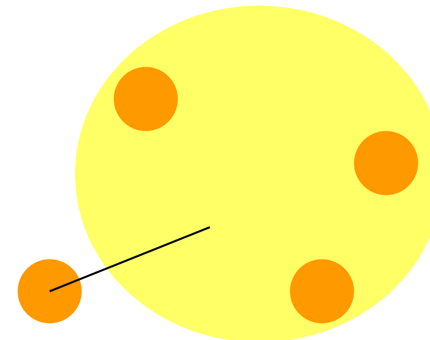
Mean-field (Hartree-Fock) Theory

nucleon-nucleon interaction



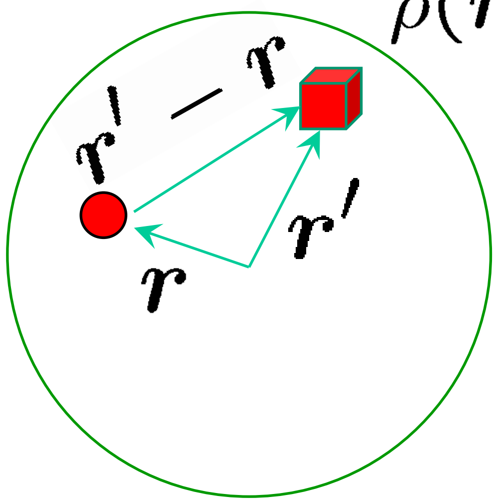
$$v(\boldsymbol{r})$$

平均場



interaction for a nucleon inside a nucleus:

$$\rho(\boldsymbol{r}')d\boldsymbol{r}'$$



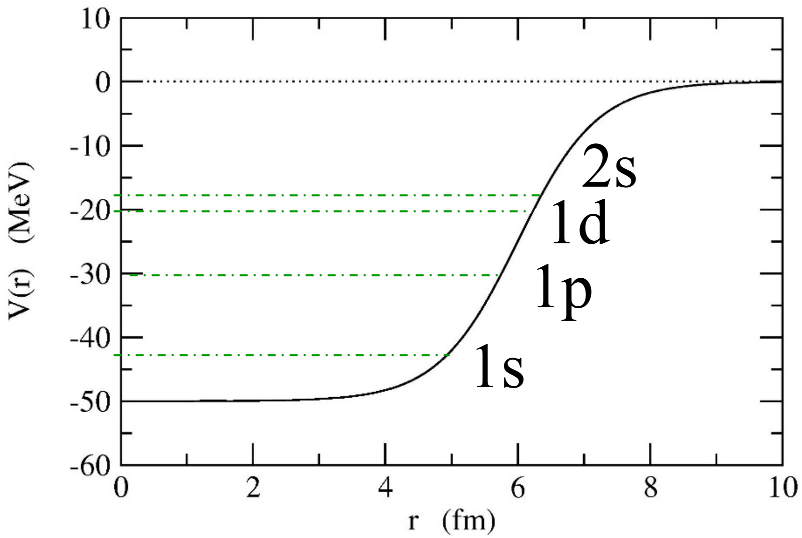
$$v(\boldsymbol{r}'-\boldsymbol{r})\cdot\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

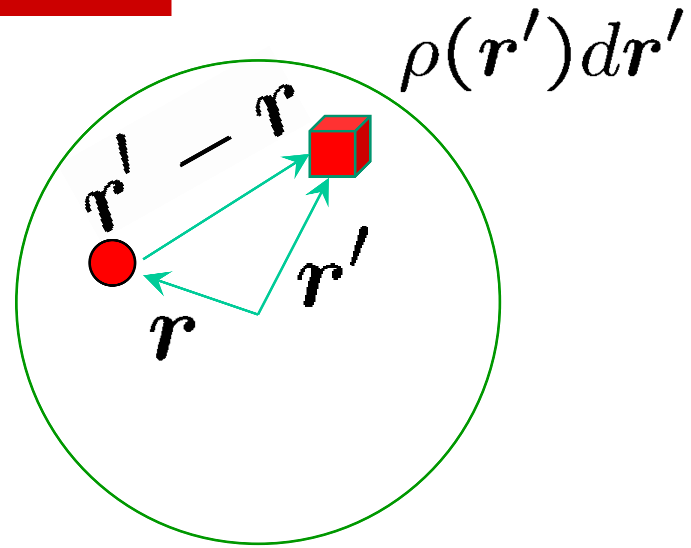
the number of nucleon at *r*'

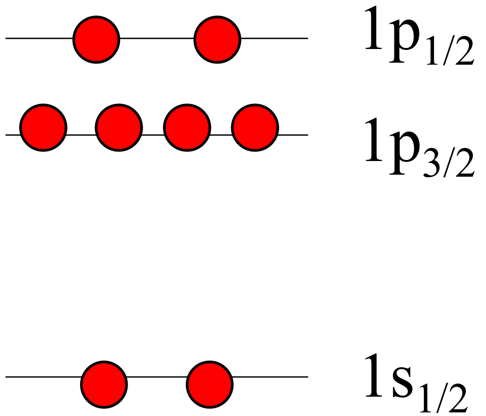
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

Mean-field (Hartree-Fock) Theory





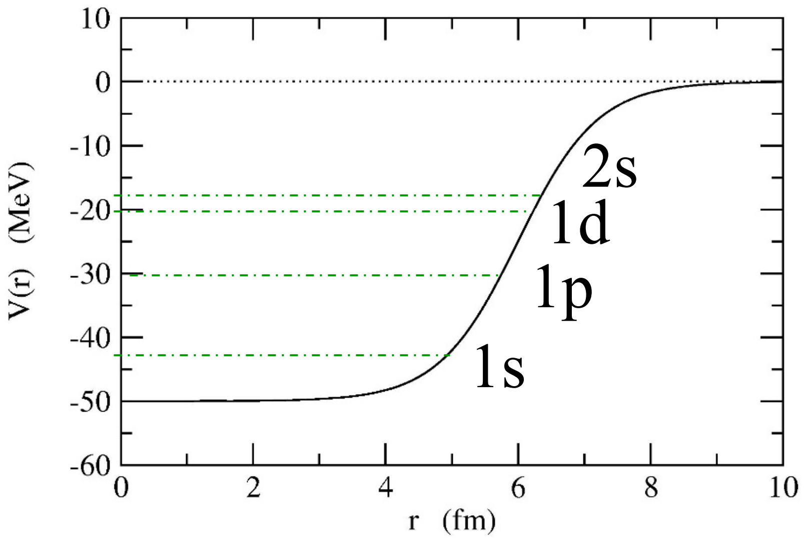


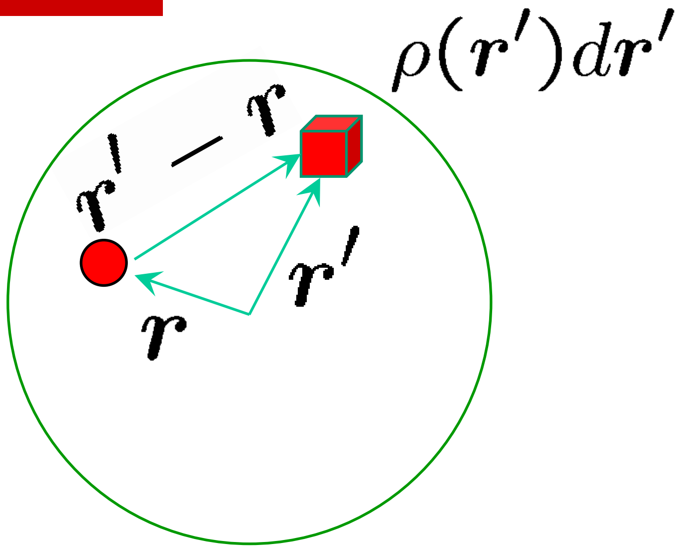
naively speaking,

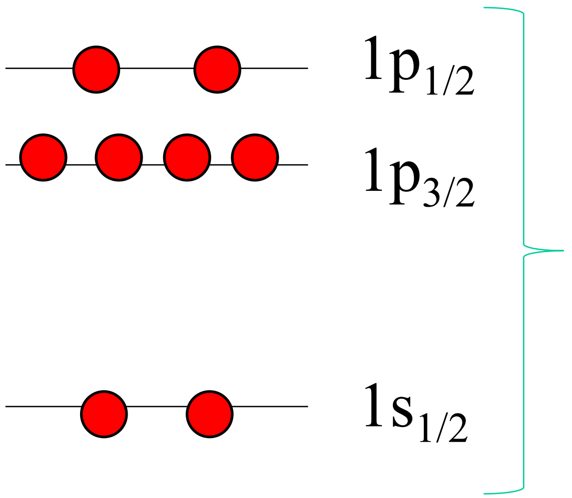
$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r} - \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

shell model

Mean-field (Hartree-Fock) Theory







shell model

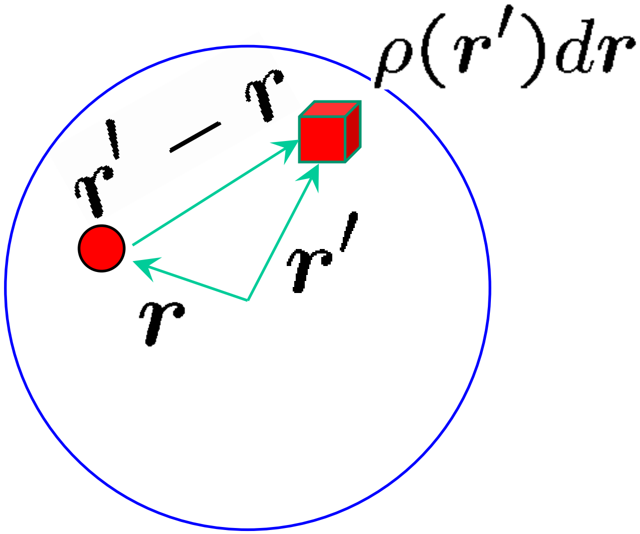
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

independent motion

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

Mean-field (Hartree-Fock) Theory



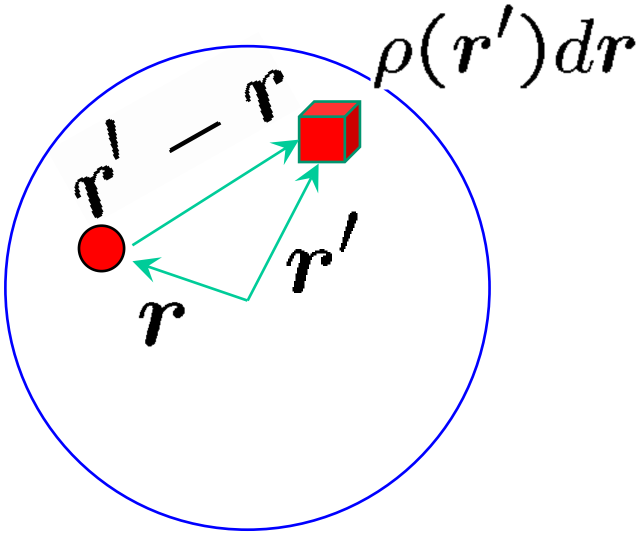
naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

$$0 = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

Mean-field (Hartree-Fock) Theory



naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

$$\begin{aligned} 0 &= \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\ &= \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \end{aligned}$$

the potential depends on the solutions

Mean-field (Hartree-Fock) Theory

$$0 = \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

the potential depends on the solutions

$\longrightarrow$ self-consistent solutions

Iteration: $\{\psi_i\} \rightarrow \rho \rightarrow V \rightarrow \{\psi_i\} \rightarrow \cdots$

Mean-field (Hartree-Fock) Theory

$$
\begin{aligned}
0 &= \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\
&= \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})
\end{aligned}
$$

the potential depends on the solutions

⟶ self-consistent solutions

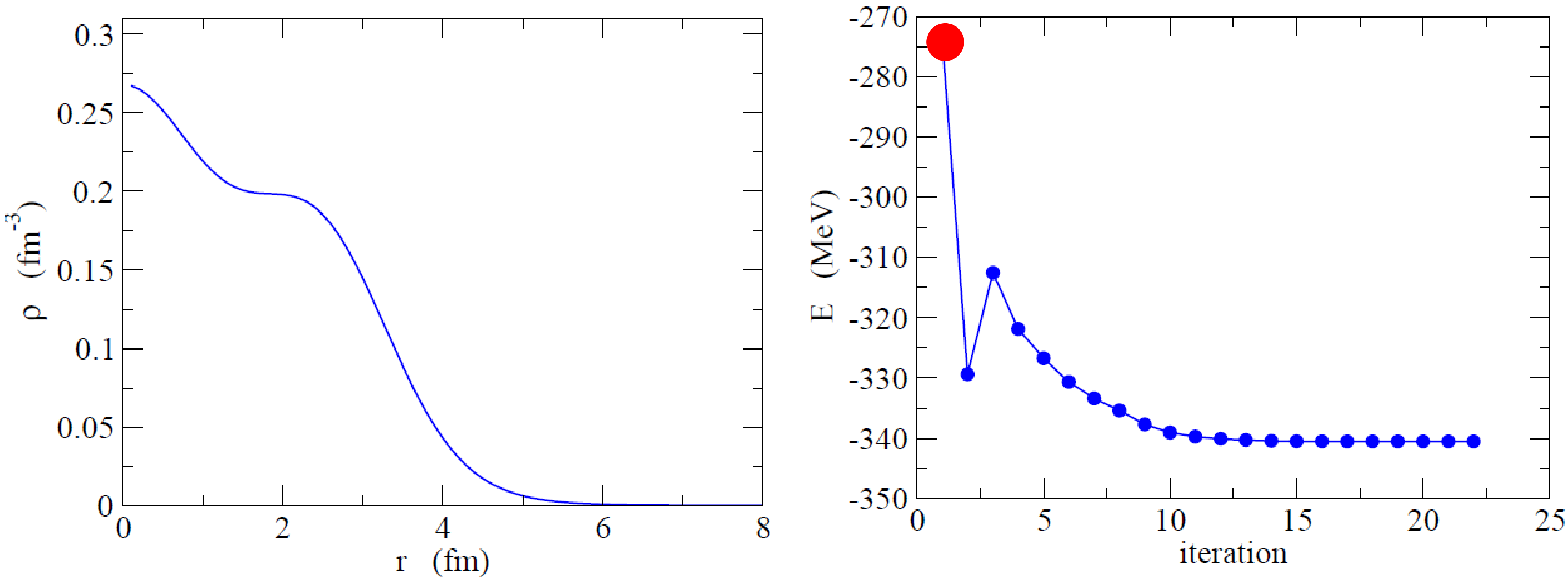
Iteration: $\{\psi_i\} \to \rho \to V \to \{\psi_i\} \to \cdots$

$$
\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2, \quad V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'
$$

repeat until the first and the last wave functions are the same.

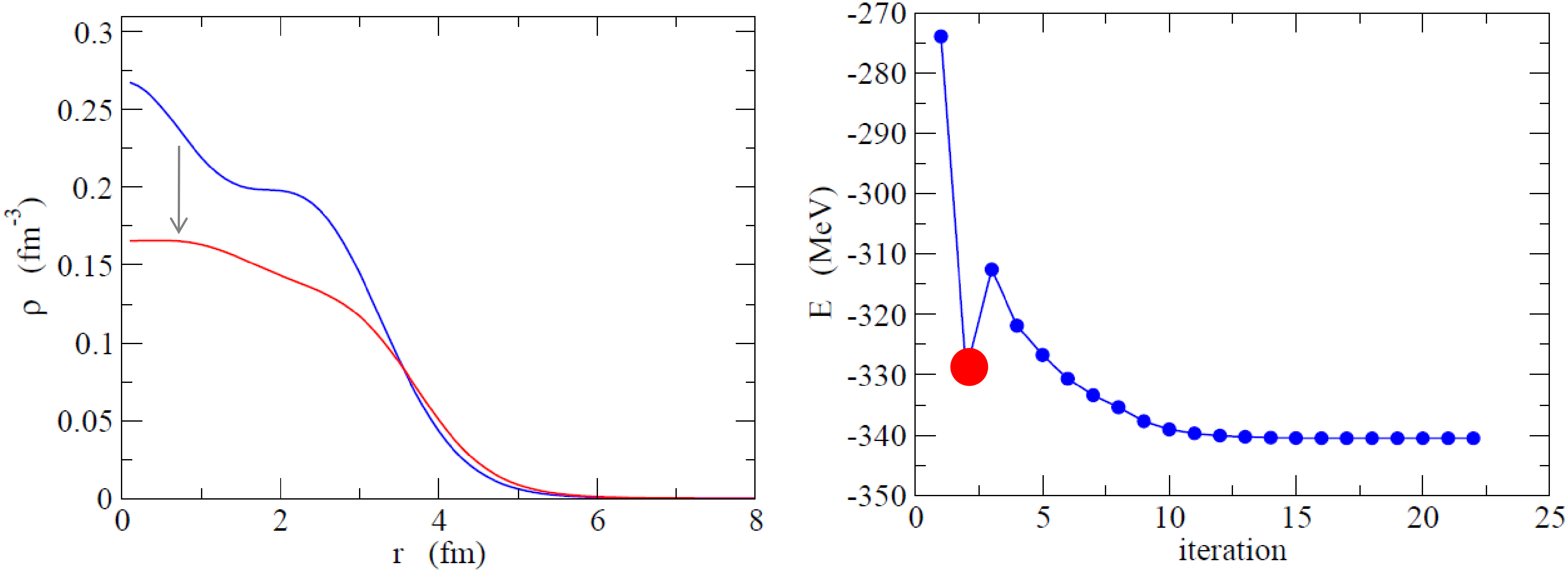
"self-consistent solutions"

Skyrme-Hartree-Fock calculations for ^{40}Ca



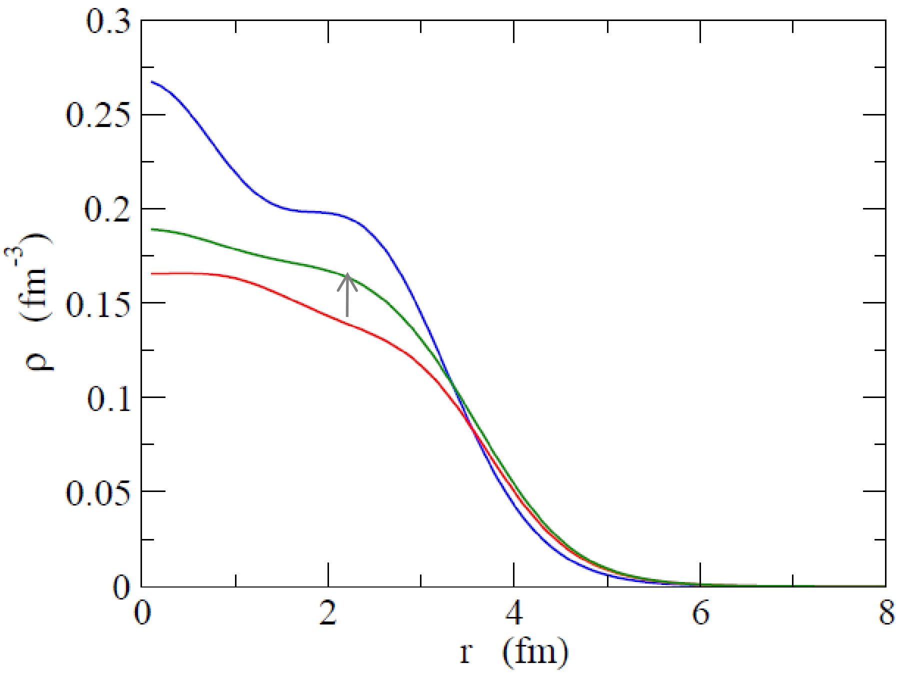
optimize the density by taking into account the nucleon-nucleon interaction

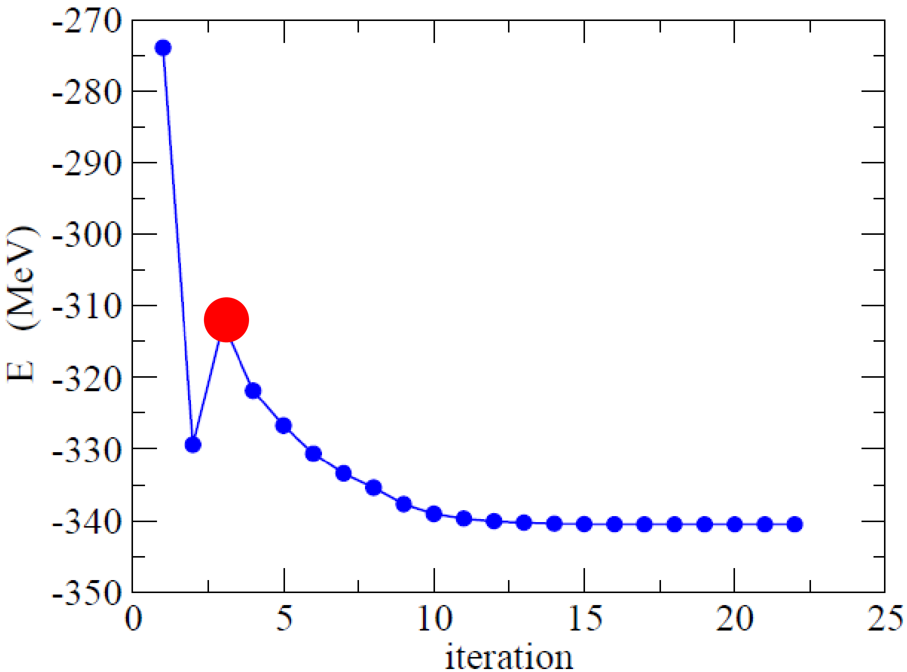
Skyrme-Hartree-Fock calculations for ^{40}Ca



optimize the density by taking into account the nucleon-nucleon interaction

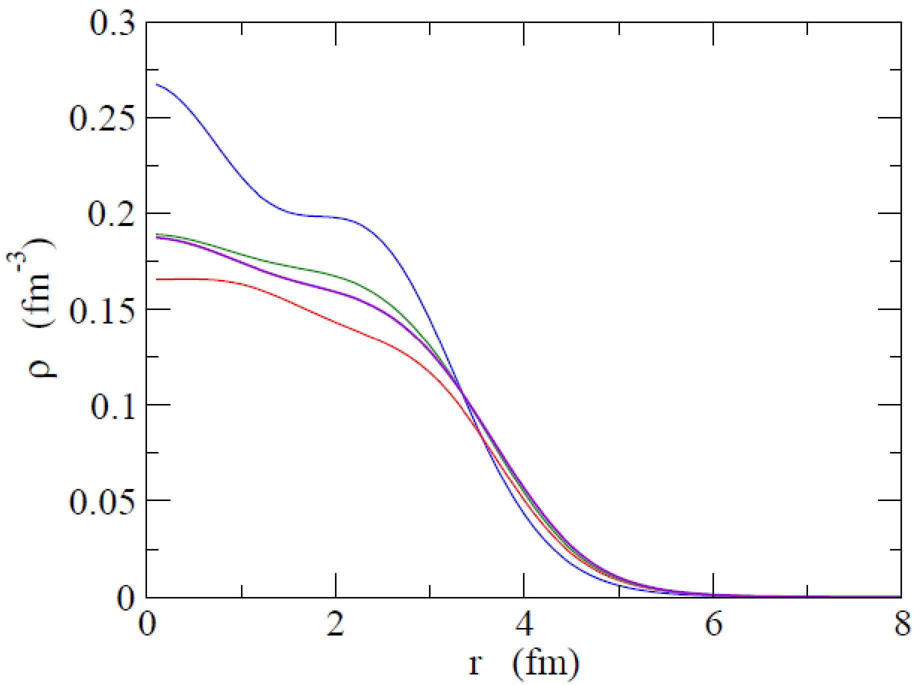
Skyrme-Hartree-Fock calculations for ^{40}Ca

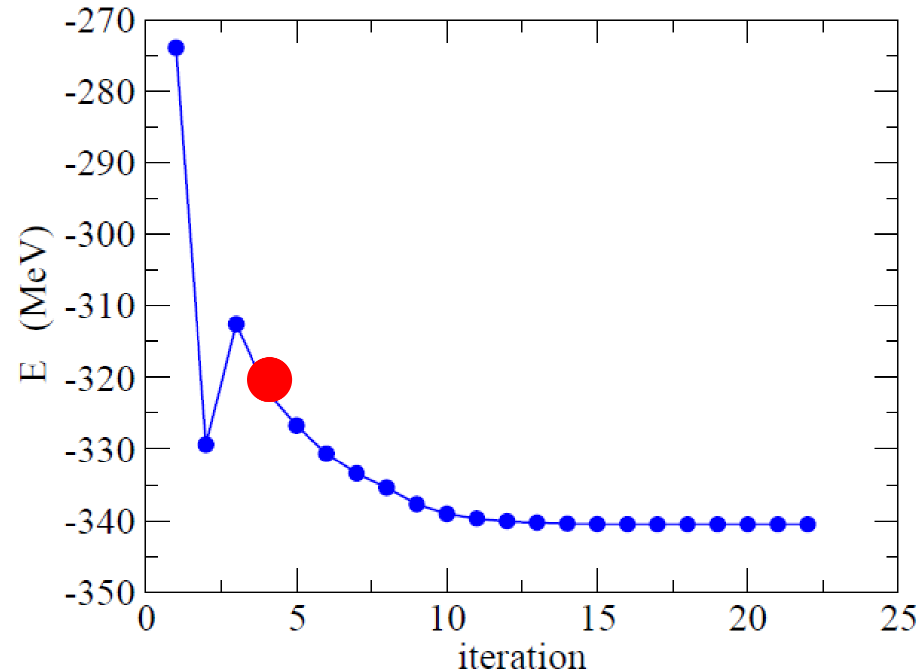




optimize the density by taking into account the nucleon-nucleon interaction

Skyrme-Hartree-Fock calculations for ^{40}Ca



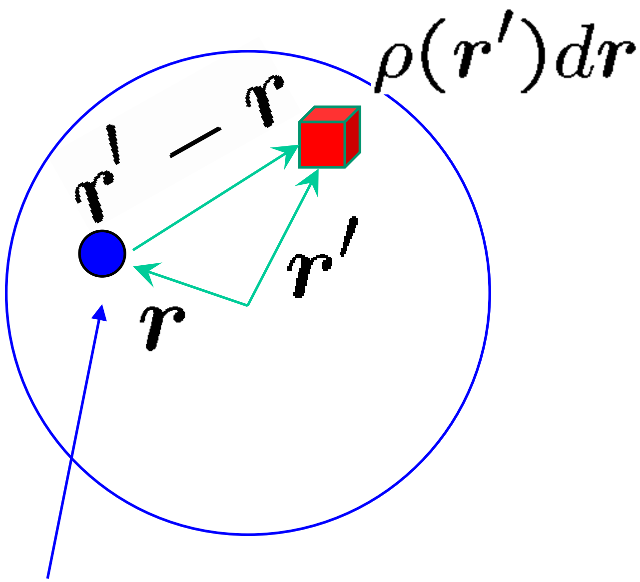


optimize the density by taking into account the nucleon-nucleon interaction

➡ 密度を少しずつ変えながらエネルギーを最適化している

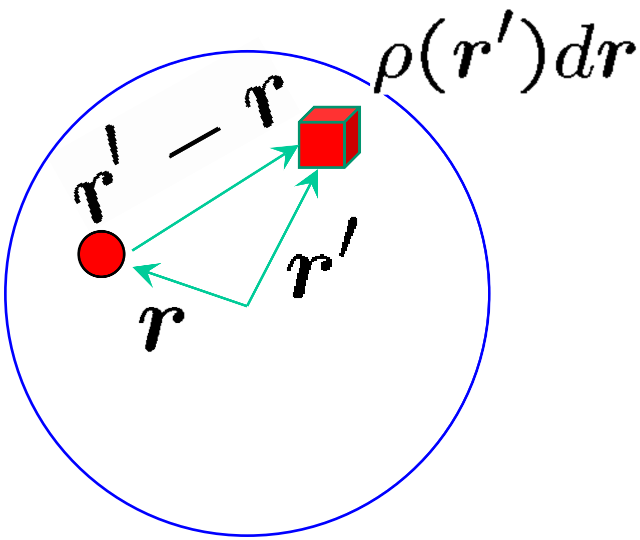
Mean-field (Hartree-Fock) Theory

<u>electro-static potential</u>



test charge

<u>nucleus</u>

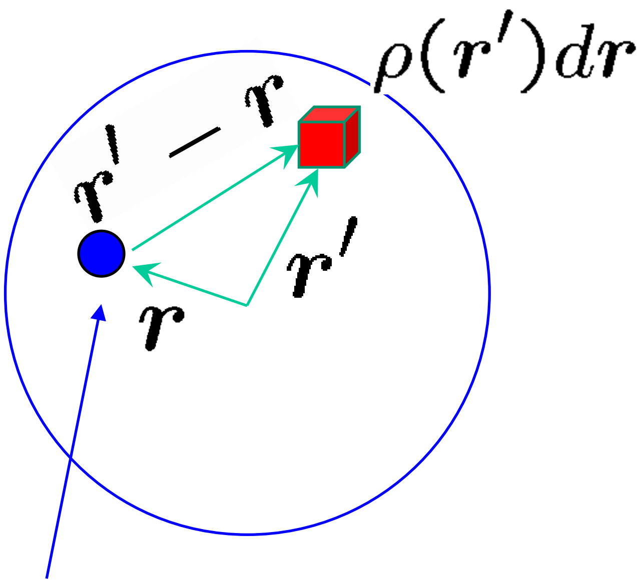


interaction between identical particles

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r} - \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

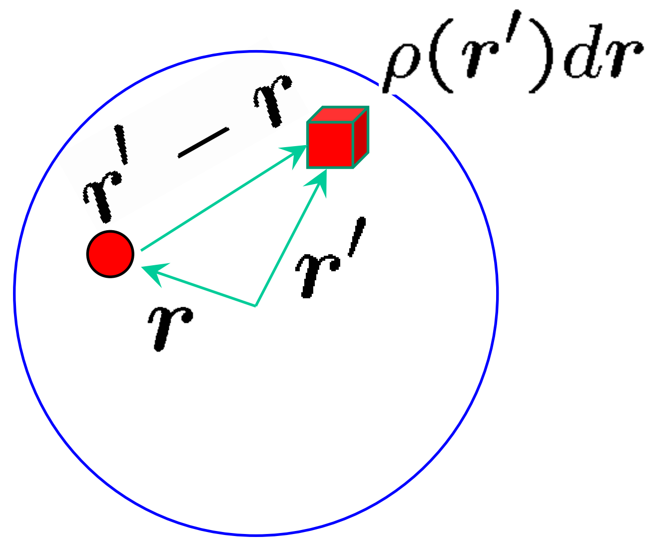
Mean-field (Hartree-Fock) Theory

electro-static potential



test charge

nucleus



interaction between identical particles
$\rightarrow$ needs anti-symmetrization

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r} - \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

anti-symmetrization

nucleon: fermion

$$\Psi(x_1, x_2, x_3 \cdots) = -\Psi(x_2, x_1, x_3 \cdots)$$

$$\psi_1(x_1)\psi_2(x_2) \to \frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)]$$

Slater determinat

$$0 = \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

$$\psi_j^*(\boldsymbol{r}')\psi_j(\boldsymbol{r}')\psi_i(\boldsymbol{r}) \to \psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\psi_j(\boldsymbol{r})$$

anti-symmetrization

nucleon: fermion

$$\Psi(x_1, x_2, x_3 \cdots) = -\Psi(x_2, x_1, x_3 \cdots)$$

$$\psi_1(x_1)\psi_2(x_2) \to \frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)]$$

Slater determinat

$$0 = \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

$$\psi_j^*(\boldsymbol{r}')\psi_j(\boldsymbol{r}')\psi_i(\boldsymbol{r}) \to \psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\psi_j(\boldsymbol{r})$$

$$\to \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

$$-\int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j \psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\right) d\boldsymbol{r}'\psi_j(\boldsymbol{r})$$

exchange term

Hartree-Fock theory

anti-symmetrization

$$
\begin{aligned}
0 &= \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\
&\to \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\
&\quad - \int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j \psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\right) d\boldsymbol{r}'\psi_j(\boldsymbol{r}) \\
&= \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r}) + \int d\boldsymbol{r}' V_{\mathrm{NL}}(\boldsymbol{r},\boldsymbol{r}')\psi_i(\boldsymbol{r}')
\end{aligned}
$$

non-local potential

Non-local potentials

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - E\right]\psi(\boldsymbol{r}) + \int d\boldsymbol{r}' V_{\mathrm{NL}}(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}') = 0$$

- Local equivalent potential

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - E\right]\psi(\boldsymbol{r}) + \left[\frac{1}{\psi(\boldsymbol{r})}\int d\boldsymbol{r}' V_{\mathrm{NL}}(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}')\right]\psi(\boldsymbol{r}) = 0$$

E-dep. potential

- Wigner 変換

$$V_W(\boldsymbol{r},\boldsymbol{p}) = \int V_{\mathrm{NL}}(\boldsymbol{r}-\boldsymbol{s}/2, \boldsymbol{r}+\boldsymbol{s}/2)\, e^{i\boldsymbol{p}\cdot\boldsymbol{s}/\hbar} d\boldsymbol{s}$$

✓ momentum expansion
✓ effective mass approximation

cf. Perrey-Buck 型 $$V_{\mathrm{NL}}(\boldsymbol{r},\boldsymbol{r}') = U\left(\frac{1}{2}|\boldsymbol{r}+\boldsymbol{r}'|\right)\exp\left[-\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{\beta}\right)^2\right]$$

Variational Principle (Rayleigh-Ritz method)

optimization $\longleftrightarrow$ variational principle

$$\frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq E_{\mathrm{g.s.}}$$

Variational Principle (Rayleigh-Ritz method)

optimization $\longleftrightarrow$ variational principle

$$\frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq E_{\text{g.s.}}$$

$$|\Psi\rangle = \sum_n C_n |\phi_n\rangle$$

$$\longrightarrow \quad \text{lhs} = \frac{\sum_n C_n^2 E_n}{\sum_n C_n^2} \geq E_0$$

H : many-body Hamiltonian

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots) = \psi_1(\boldsymbol{r}_1) \cdot \psi_2(\boldsymbol{r}_2) \cdot \psi_3(\boldsymbol{r}_3) \cdots$$

$\longleftarrow$ many-body wave function for independent particles

Variational Principle (Rayleigh-Ritz method)

optimization $\longleftrightarrow$ variational principle

$$\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \geq E_{\mathrm{g.s.}}$$

$$|\Psi\rangle = \sum_n C_n |\phi_n\rangle$$

$$\longrightarrow \quad \mathrm{lhs} = \frac{\sum_n C_n^2 E_n}{\sum_n C_n^2} \geq E_0$$

H : many-body Hamiltonian

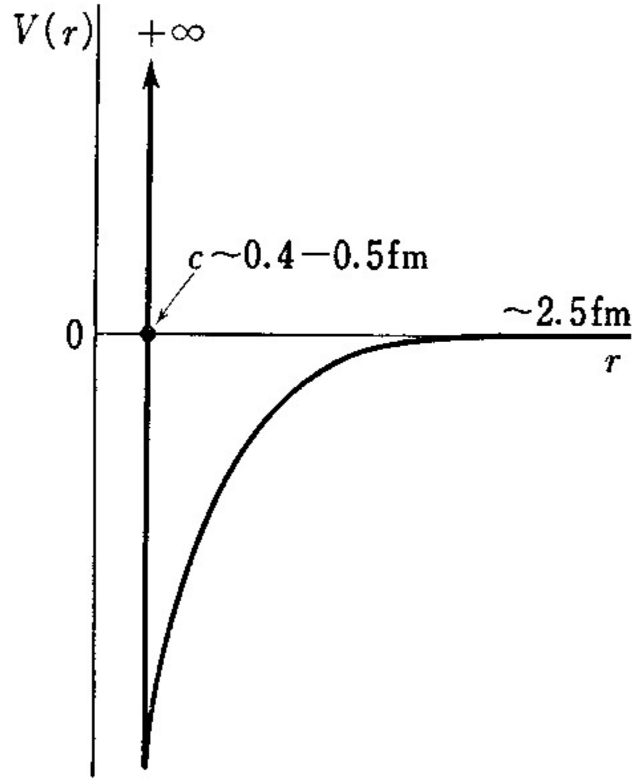
$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots) = \psi_1(\boldsymbol{r}_1)\cdot\psi_2(\boldsymbol{r}_2)\cdot\psi_3(\boldsymbol{r}_3)\cdots$$

$\longleftarrow$ many-body wave function for independent particles

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) = 0$$

change gradually the single-particle potential so that the total energy becomes minimum

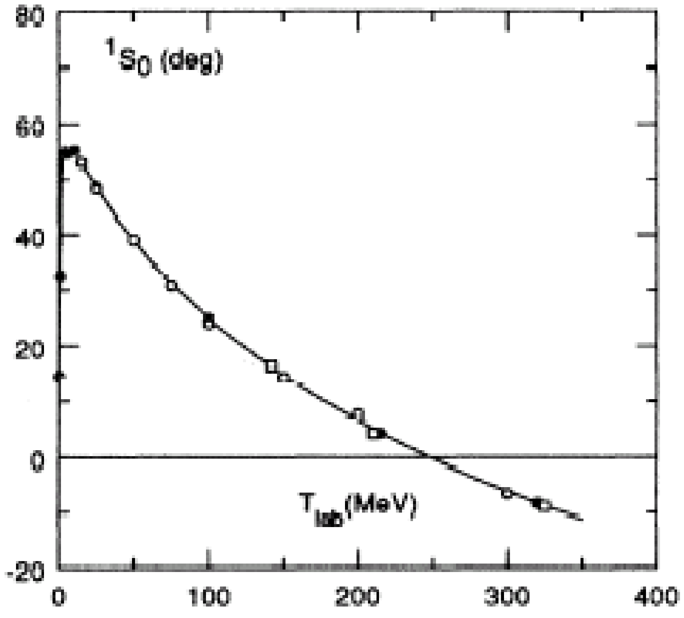
Bare nucleon-nucleon interaction

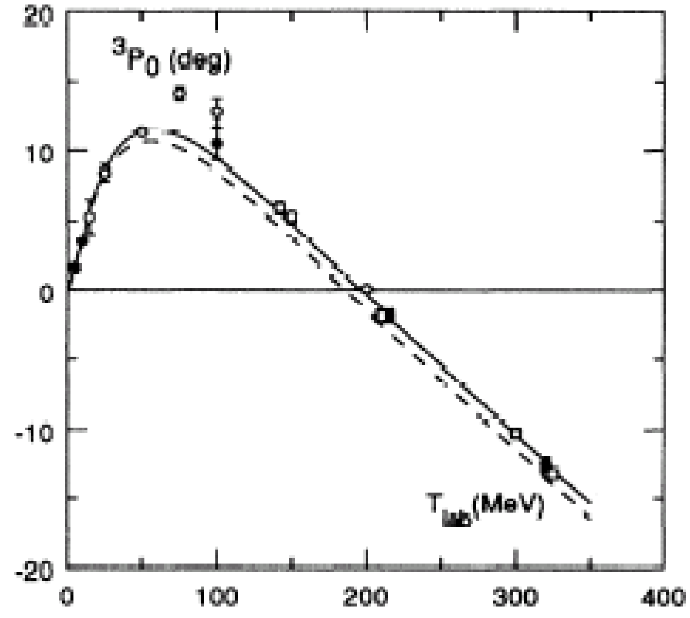


Existence of short range repulsive core

Bare nucleon-nucleon interaction

Phase shift for p-p scattering





(V.G.J. Stoks et al., PRC48('93)792)

Phase shift: +ve ⟶ -ve

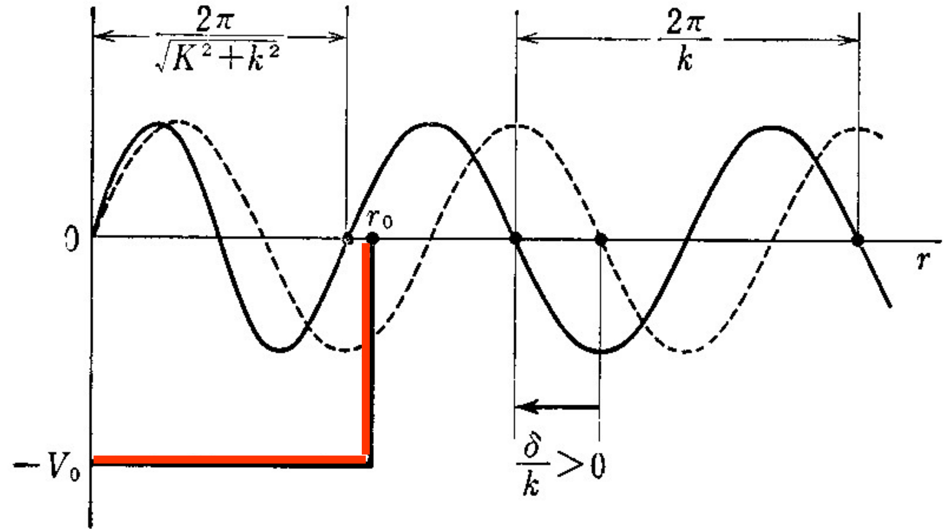
at high energies

Phase shift:

Radial wave function

$$\Psi_l(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}})$$

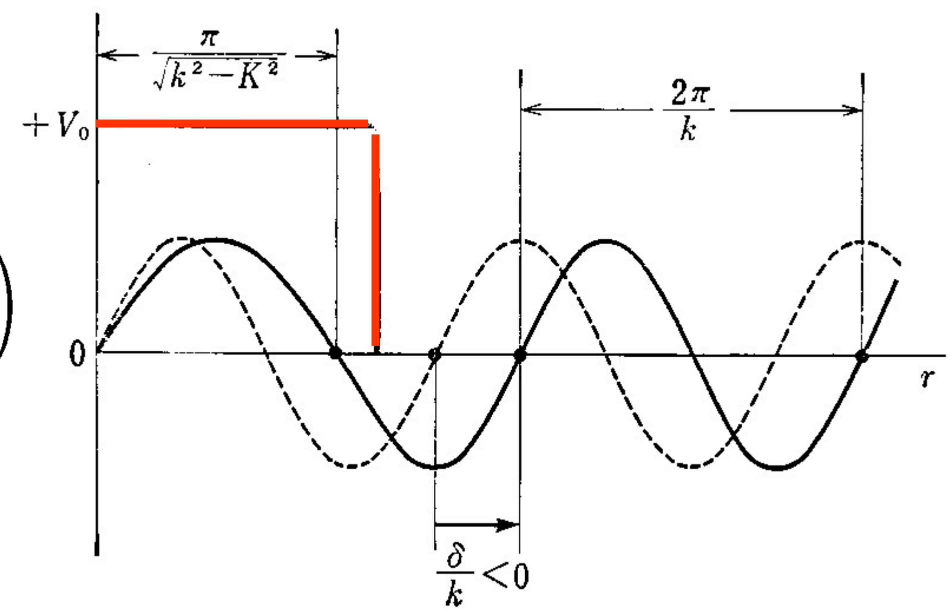
$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V(r) + \frac{l(l+1)\hbar^2}{2mr^2} - E\right] u_l(r) = 0$$



（a） 引　力　attraction

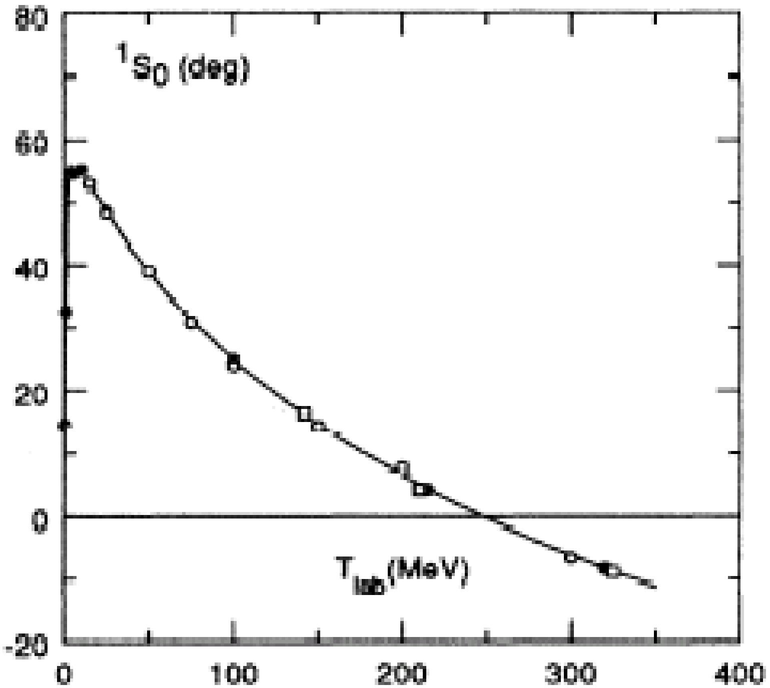
Asymptotic form:

$$u_l(r) \rightarrow \sin\left(kr - \frac{l\pi}{2} + \delta_l\right) \quad (r \rightarrow \infty)$$

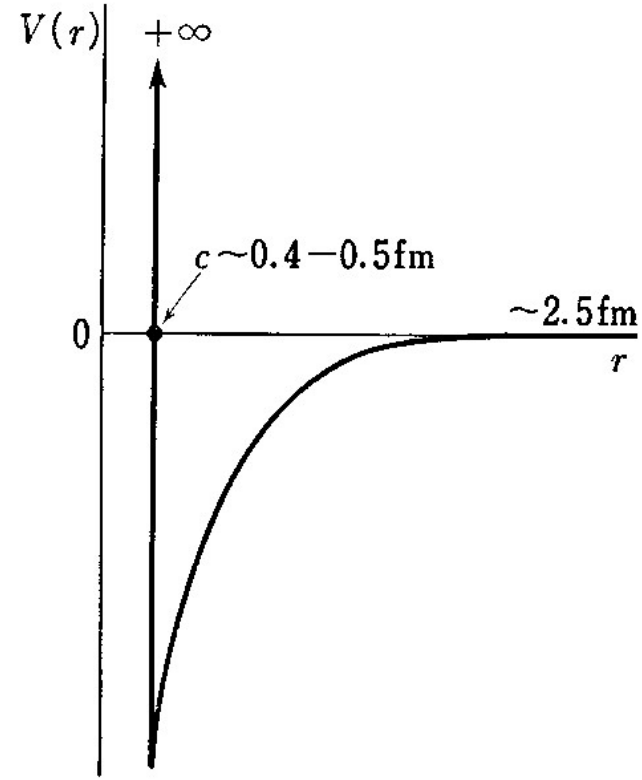


（b） 斥　力　repulsion

$\delta > 0 \rightarrow$ attraction
$\delta < 0 \rightarrow$ repulsion



Phase shift: +ve $\longrightarrow$ -ve
at high energies



Existence of short range
repulsive core

Bruckner's G-matrix

Nucleon-nucleon interaction *in medium*

Nucleon-nucleon interaction with a hard core

⇒ HF method: does not work

← Matrix elements: diverge

…..but the HF picture seems to work in nuclear systems

cf. magic numbers

Solution: a nucleon-nucleon interaction *in medium* (effective interaction) rather than a bare interaction

⇑

Bruckner's G-matrix