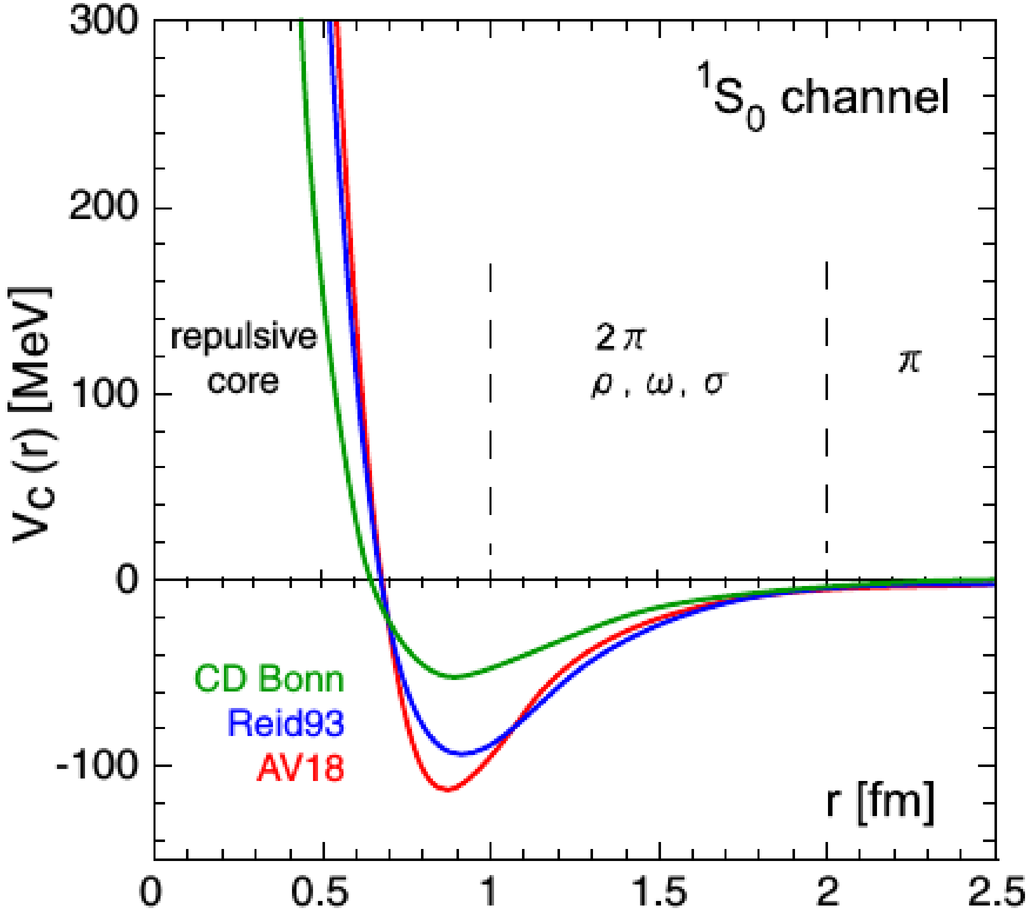
Bare nucleon-nucleon interaction



N. Ishii, S. Aoki, and T. Hatsuda,
PRL99, 022001 (2007)

レポート問題4：

テンソル力に出てくるテンソル演算子

$$S_{12} \; = \; \frac{3(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

が2核子の合成スピン $\boldsymbol{S}=\boldsymbol{s}_1+\boldsymbol{s}_2$ の大きさが1の状態（スピン3重項）にのみしか働かないことを示せ。

Bruckner’s G-matrix

Nucleon-nucleon interaction *in medium*

Nucleon-nucleon interaction with a hard core

⇨ HF method: does not work

← Matrix elements: diverge

…..but the HF picture seems to work in nuclear systems

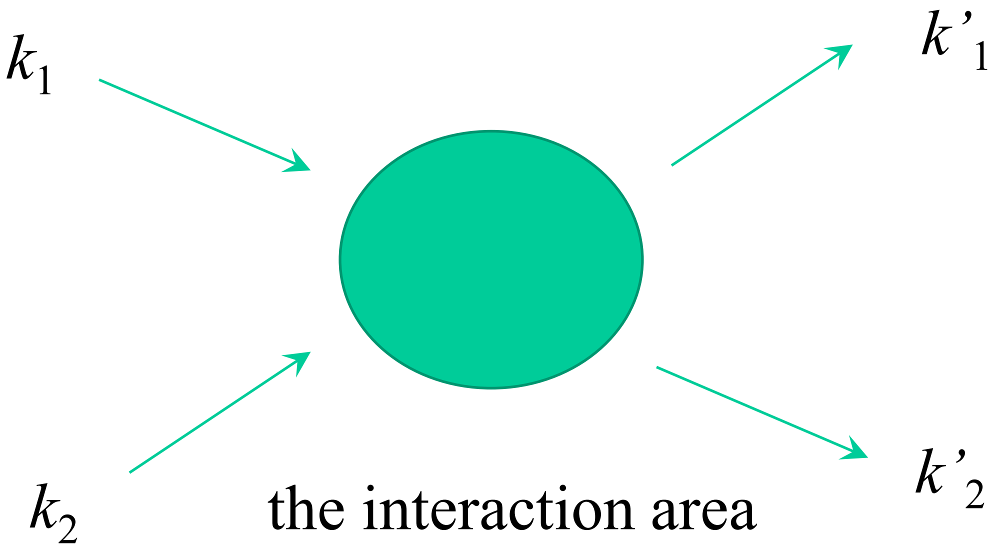
cf. magic numbers

Solution: a nucleon-nucleon interaction *in medium* (effective interaction) rather than a bare interaction

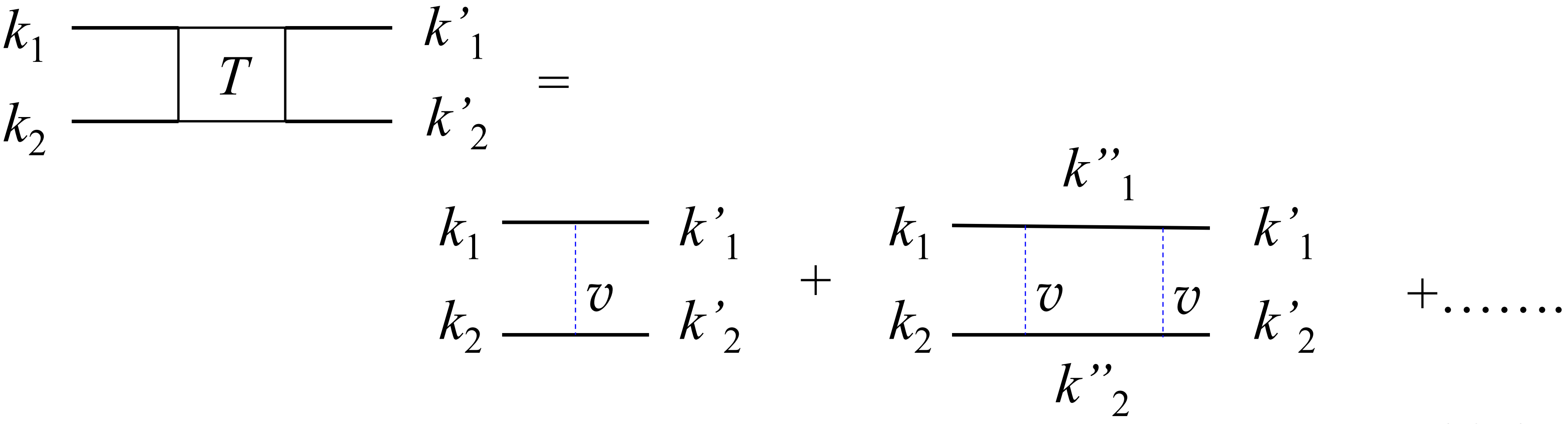
⇧

Bruckner’s G-matrix

➢ two-body (multiple) scattering *in vacuum*



the interaction area

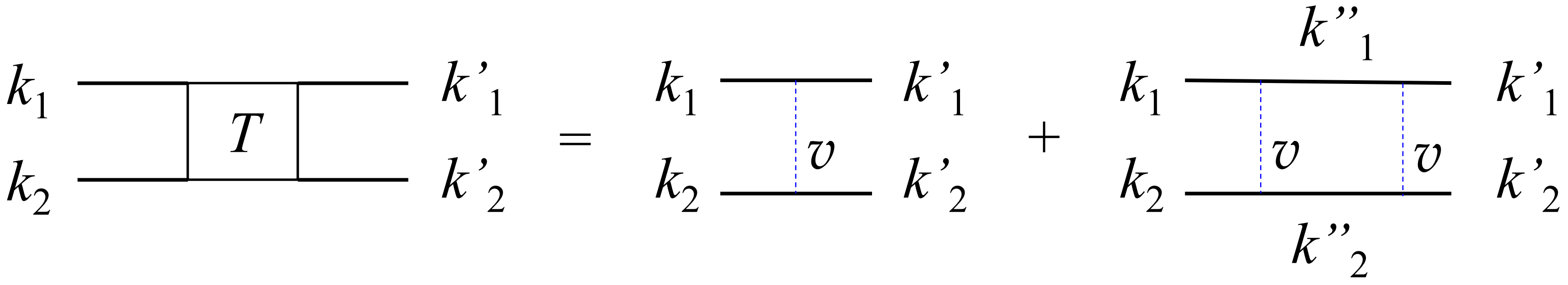


the 1st order

the 2nd order

higher orders

➢ two-body (multiple) scattering *in vacuum*



+…….

Lippmann-Schwinger equation

$$T = v + v\frac{1}{E - H_0}T$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V - E\right)\psi = 0$$

$$\Rightarrow \left(-\frac{\hbar^2}{2m}\nabla^2 - E\right)\psi = -V\psi$$

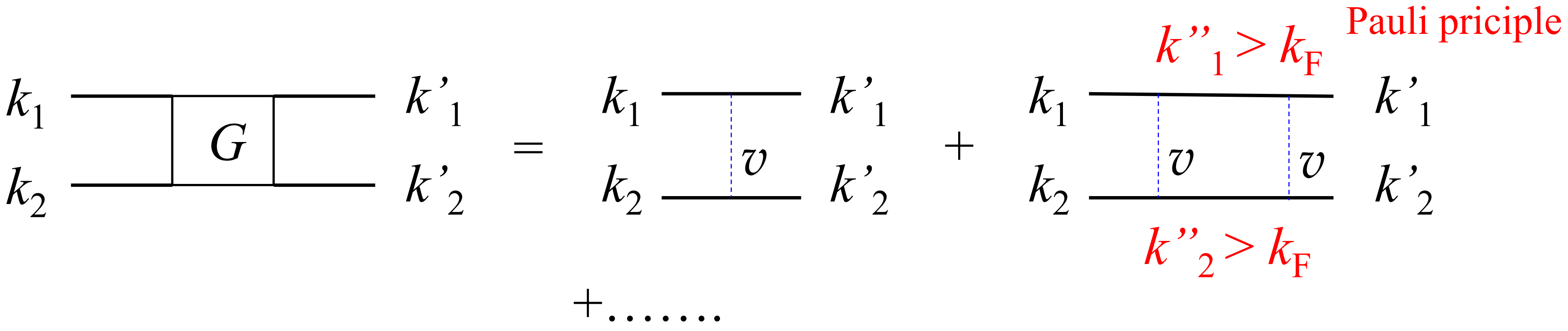
$$\Rightarrow \psi = \phi - \frac{1}{H_0 - E}V\psi \qquad H_0 = -\frac{\hbar^2}{2m}\nabla^2, \quad (H_0 - E)\phi = 0$$

$$\Rightarrow V\psi = V\phi - V\frac{1}{H_0 - E}V\psi \qquad \Rightarrow \qquad T = V - V\frac{1}{H_0 - E}T$$

$$(V\psi = T\phi)$$

核内における核子間相互作用（媒質効果）

- two-body (multiple) scattering *in medium*



Bethe-Goldstone equation

$$G = v + v\frac{Q_F}{E - H_0}G$$

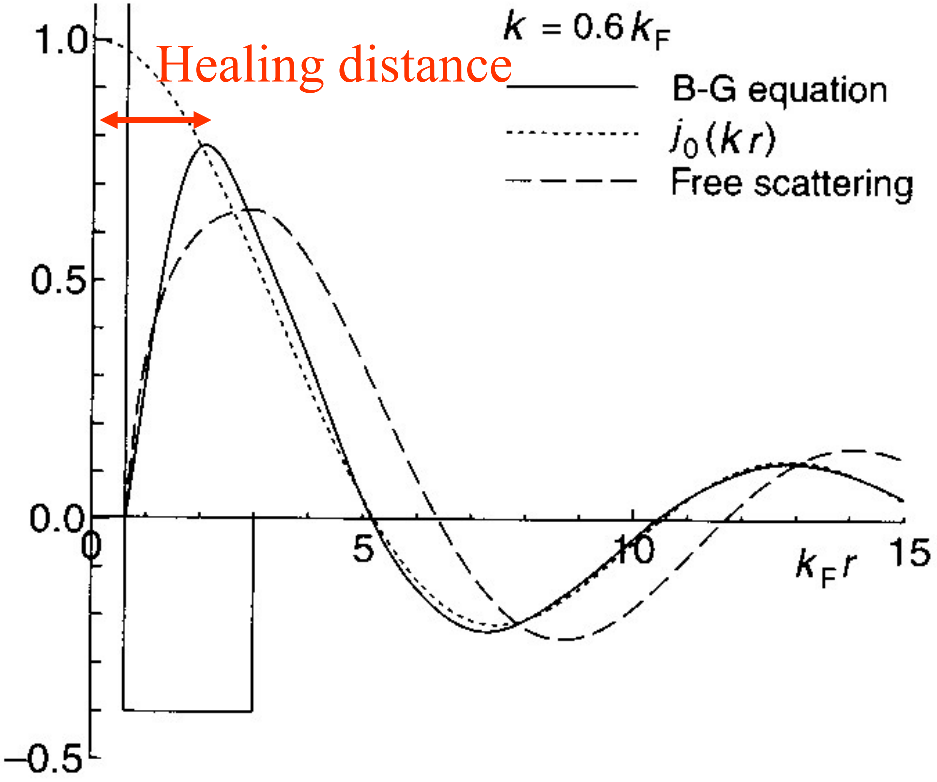
*scattering: suppressed
because intermediate states have to have
$k > k_F$ → independent particle picture

◆Hard core

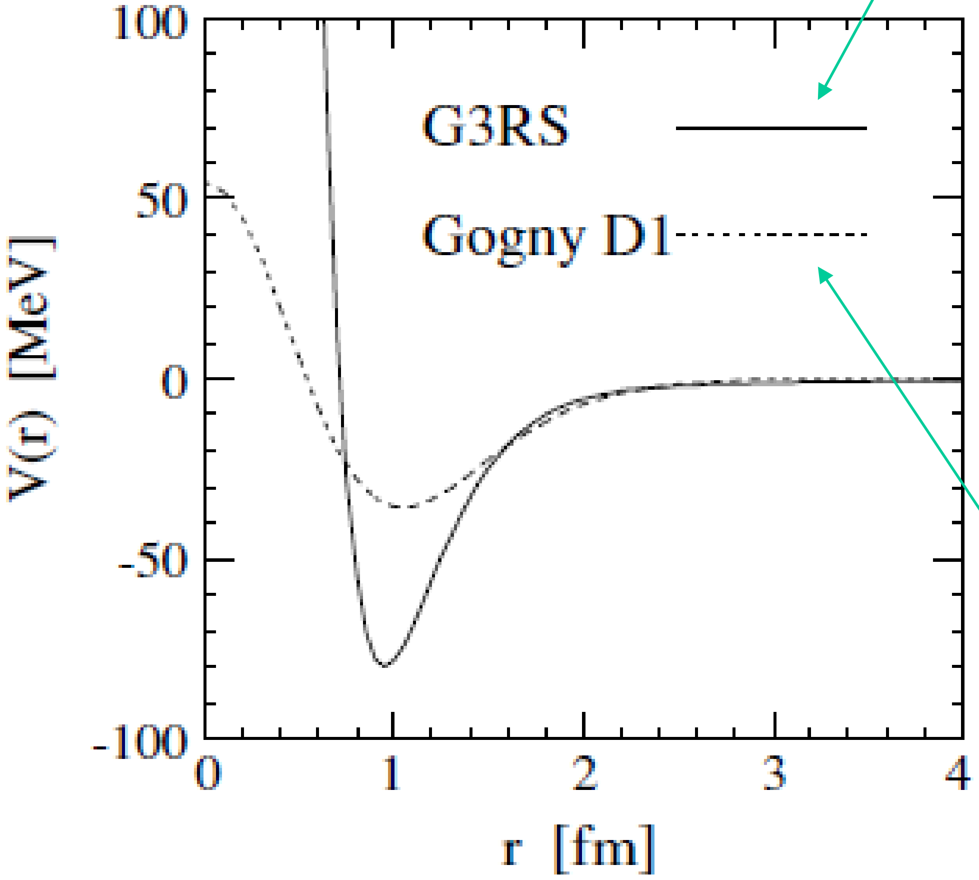
$$G = v + v\frac{Q_F}{E - H_0}G \quad \Longleftrightarrow \quad G = \frac{v}{1 - v\,\frac{Q_F}{E-H_0}}$$

Even if v tends to infinity, G may stay finite.

◆Independent particle motion



→ use G instead of v in mean-field calculations bare interaction



effective
interaction

M. Matsuo, Phys. Rev. C73(‘06)044309

Phenomenological effective interactions

G-matrix

- ab initio
- but, cumbersome to compute (especially for finite nuclei)
- qualitatively good, but quantitatively not successful

⬇

HF calculations with a phenomenological effective interaction

Philosophy: take the functional form of *G*, but determine the parameters phenomenologically

- Skyrme interaction (non-rel., zero range)
- Gogny interaction (non-rel., finite range)
- Relativistic mean-field model (relativistic, “meson exchanges”)

Skyrme interaction density dependent zero-range interaction

$$\boxed{\begin{aligned} v(\boldsymbol{r},\boldsymbol{r}') \;=\; & t_0(1+x_0\widehat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}') \\ & +\frac{1}{2}t_1(1+x_1\widehat{P}_\sigma)(\boldsymbol{k}^2\delta(\boldsymbol{r}-\boldsymbol{r}')+\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}^2) \\ & +t_2(1+x_2\widehat{P}_\sigma)\boldsymbol{k}\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \\ & +\frac{1}{6}t_3(1+x_3\widehat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}')\rho^\alpha((\boldsymbol{r}+\boldsymbol{r}')/2) \\ & +iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\times\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \end{aligned}}$$

$$\boldsymbol{k}=(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)/2i$$

if x_i=0, t_1=t_2=0:

$$v(\boldsymbol{r},\boldsymbol{r}') \;=\; \underbrace{t_0\delta(\boldsymbol{r}-\boldsymbol{r}')}_{\text{short-range attraction}}+\underbrace{\frac{1}{6}t_3\delta(\boldsymbol{r}-\boldsymbol{r}')\rho^\alpha(\boldsymbol{r})}_{\text{repulsion to avoid collapse}}$$

$$\underbrace{+iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\times\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}}_{\text{spin-orbit interaction}}$$

Skyrme interaction density dependent zero-range interaction

$$\boxed{\begin{aligned} v(\boldsymbol{r},\boldsymbol{r}') &= t_0(1+x_0\hat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}') \\ &+\frac{1}{2}t_1(1+x_1\hat{P}_\sigma)(\boldsymbol{k}^2\delta(\boldsymbol{r}-\boldsymbol{r}')+\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}^2) \\ &+t_2(1+x_2\hat{P}_\sigma)\boldsymbol{k}\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \\ &+\frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}')\rho^\alpha((\boldsymbol{r}+\boldsymbol{r}')/2) \\ &+iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\times\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \end{aligned}}$$

$$\boldsymbol{k}=(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)/2i$$

(note) finite range effect $\Longleftrightarrow$ momentum dependence

$$\begin{aligned} \langle\boldsymbol{p}|V|\boldsymbol{p}'\rangle &= \frac{1}{(2\pi\hbar)^3}\int d\boldsymbol{r}\, e^{-i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{r}/\hbar}V(\boldsymbol{r}) \\ &\sim V_0+V_1(\boldsymbol{p}^2+\boldsymbol{p}'^2)+V_2\boldsymbol{p}\boldsymbol{p}'+\cdots \\ &\to V_0\delta(\boldsymbol{r})+V_1(\hat{\boldsymbol{p}}^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\hat{\boldsymbol{p}}^2)+V_2\,\hat{\boldsymbol{p}}\delta(\boldsymbol{r})\hat{\boldsymbol{p}} \end{aligned}$$

Skyrme interaction density dependent zero-range interaction

$$\boxed{\begin{aligned} v(\boldsymbol{r},\boldsymbol{r}') \; = \; & t_0(1+x_0\hat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}') \\ & +\frac{1}{2}t_1(1+x_1\hat{P}_\sigma)(\boldsymbol{k}^2\delta(\boldsymbol{r}-\boldsymbol{r}')+\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}^2) \\ & +t_2(1+x_2\hat{P}_\sigma)\boldsymbol{k}\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \\ & +\frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}')\rho^\alpha((\boldsymbol{r}+\boldsymbol{r}')/2) \\ & +iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\times\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \end{aligned}}$$

$$\boldsymbol{k}=(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)/2i$$

the exchange potential ⟶ local

$$\begin{aligned} 0 \; = \; & \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2+\int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j|\psi_j(\boldsymbol{r}')|^2\right)d\boldsymbol{r}'-\epsilon_i\right]\psi_i(\boldsymbol{r}) \\ & -\int v(\boldsymbol{r}-\boldsymbol{r}')\left(\sum_j\psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\right)d\boldsymbol{r}'\psi_j(\boldsymbol{r}) \end{aligned}$$

Skyrme interactions: 10 adjustable parameters

$$
\begin{aligned}
v(\boldsymbol{r},\boldsymbol{r}') &= t_0(1+x_0\hat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}') \\
&+\frac{1}{2}t_1(1+x_1\hat{P}_\sigma)(\boldsymbol{k}^2\delta(\boldsymbol{r}-\boldsymbol{r}')+\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}^2) \\
&+t_2(1+x_2\hat{P}_\sigma)\boldsymbol{k}\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k} \\
&+\frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\delta(\boldsymbol{r}-\boldsymbol{r}')\rho^\alpha((\boldsymbol{r}+\boldsymbol{r}')/2) \\
&+iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\times\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}
\end{aligned}
$$

A fitting strategy:

B.E. and r_{rms}: ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{90}Zr, ^{208}Pb,…..

Infinite nuclear matter: E/A, ρ_{eq} ,….

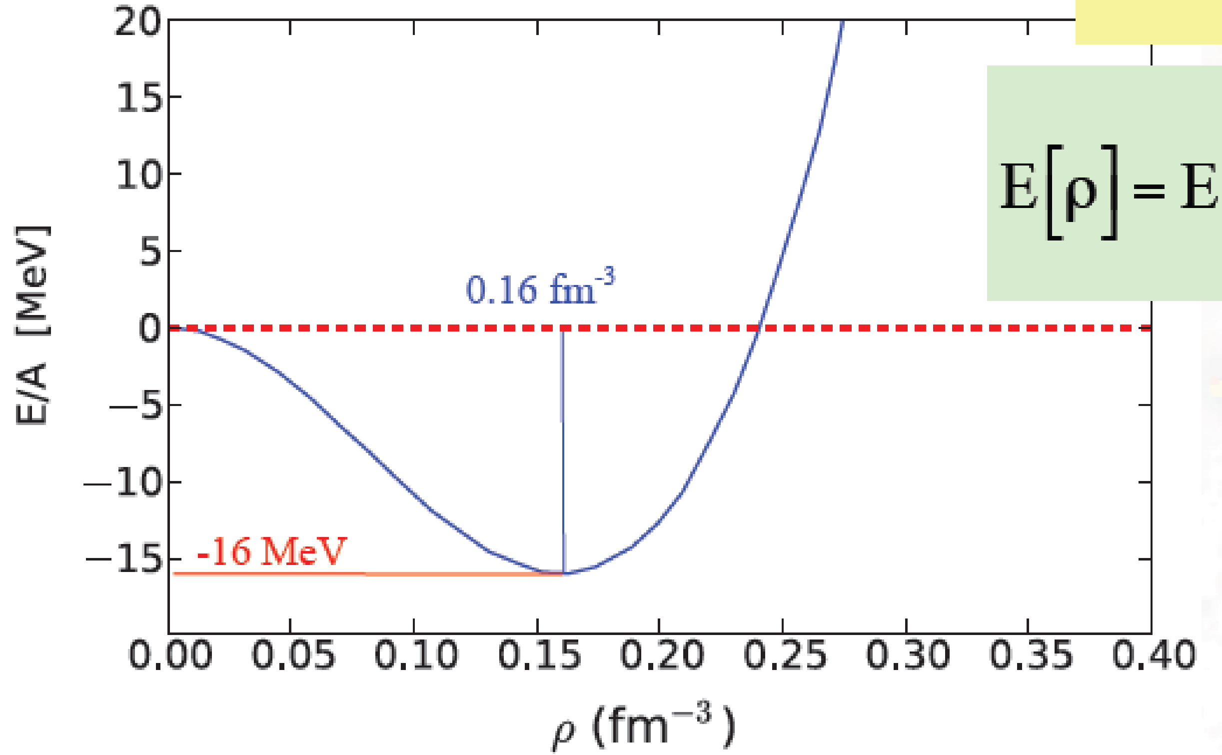
Parameter sets:

SIII, SkM*, SGII, SLy4,…….

EOS of infinite nuclear matter

$$K_\infty = 9\rho^2 \left. \frac{d^2\left[E(\rho)/\rho\right]}{d\rho^2} \right|_{\rho_0}$$

$$E\left[\rho\right] = E\left[\rho_0\right] + \frac{1}{18} K_\infty \left(\frac{\rho - \rho_0}{\rho_0} \right)^2$$



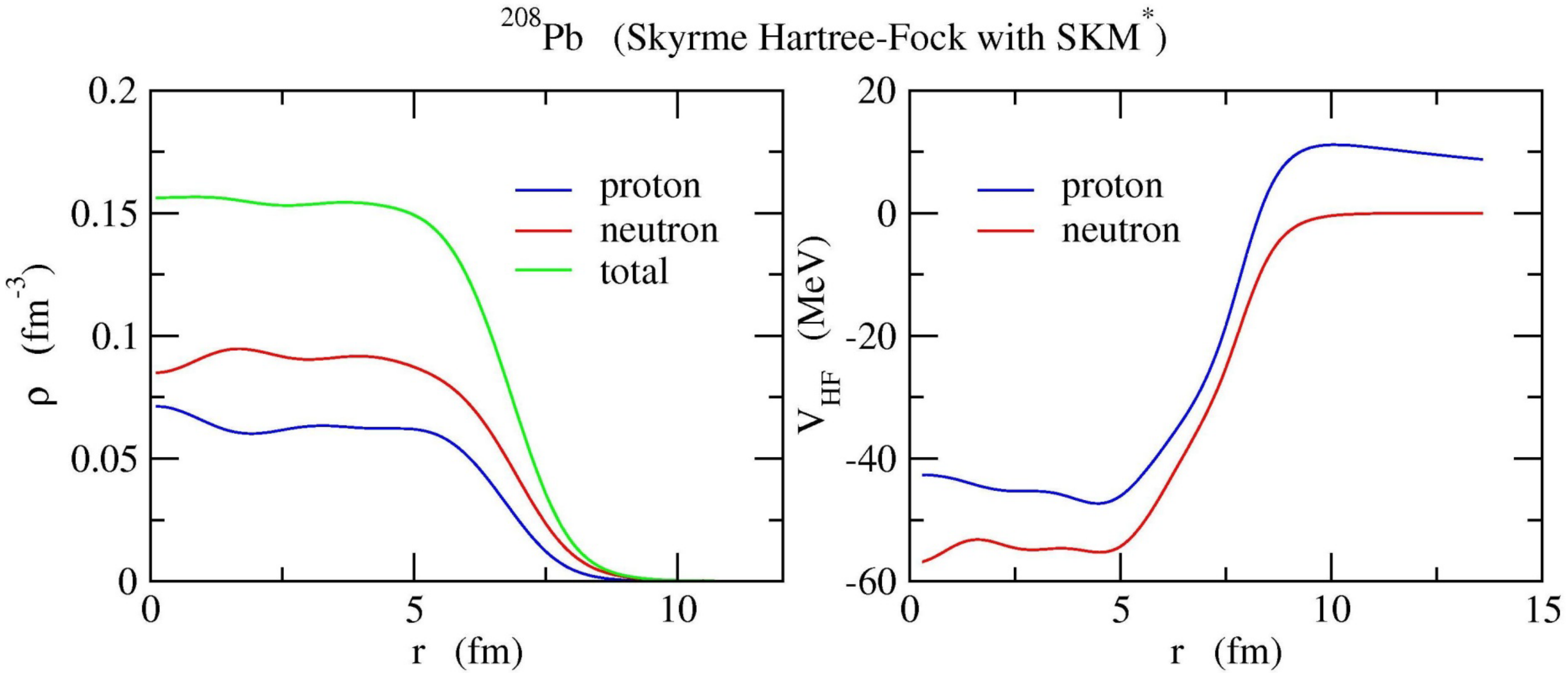
slide: Carlos Bertulani

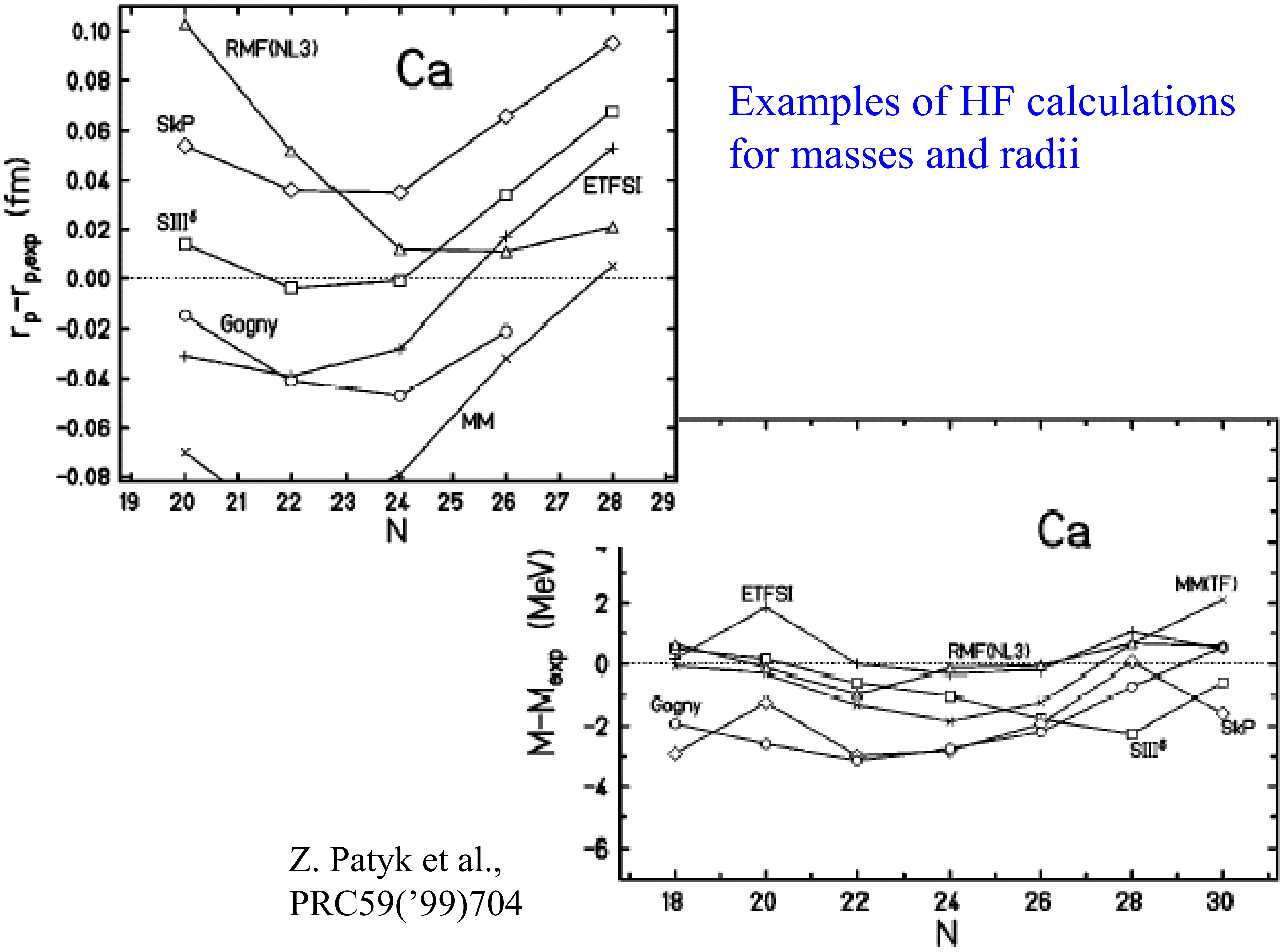
$$\boxed{\begin{aligned}&-\frac{\hbar^2}{2m}\nabla^2\psi_i(\boldsymbol{r})+\int v(\boldsymbol{r},\boldsymbol{r}')\rho_{\mathsf{HF}}(\boldsymbol{r}')d\boldsymbol{r}'\psi_i(\boldsymbol{r})\\&\qquad-\int\rho_{\mathsf{HF}}(\boldsymbol{r},\boldsymbol{r}')v(\boldsymbol{r},\boldsymbol{r}')\psi_i(\boldsymbol{r}')\,d\boldsymbol{r}'=\epsilon_i\,\psi_i(\boldsymbol{r})\end{aligned}}$$

Iteration

V_{HF}: depends on ψ_i ⟵ non-linear problem

Iteration: $\{\psi_i\}\rightarrow\rho_{\mathsf{HF}}\rightarrow V_{\mathsf{HF}}\rightarrow\{\psi_i\}\rightarrow\cdots$

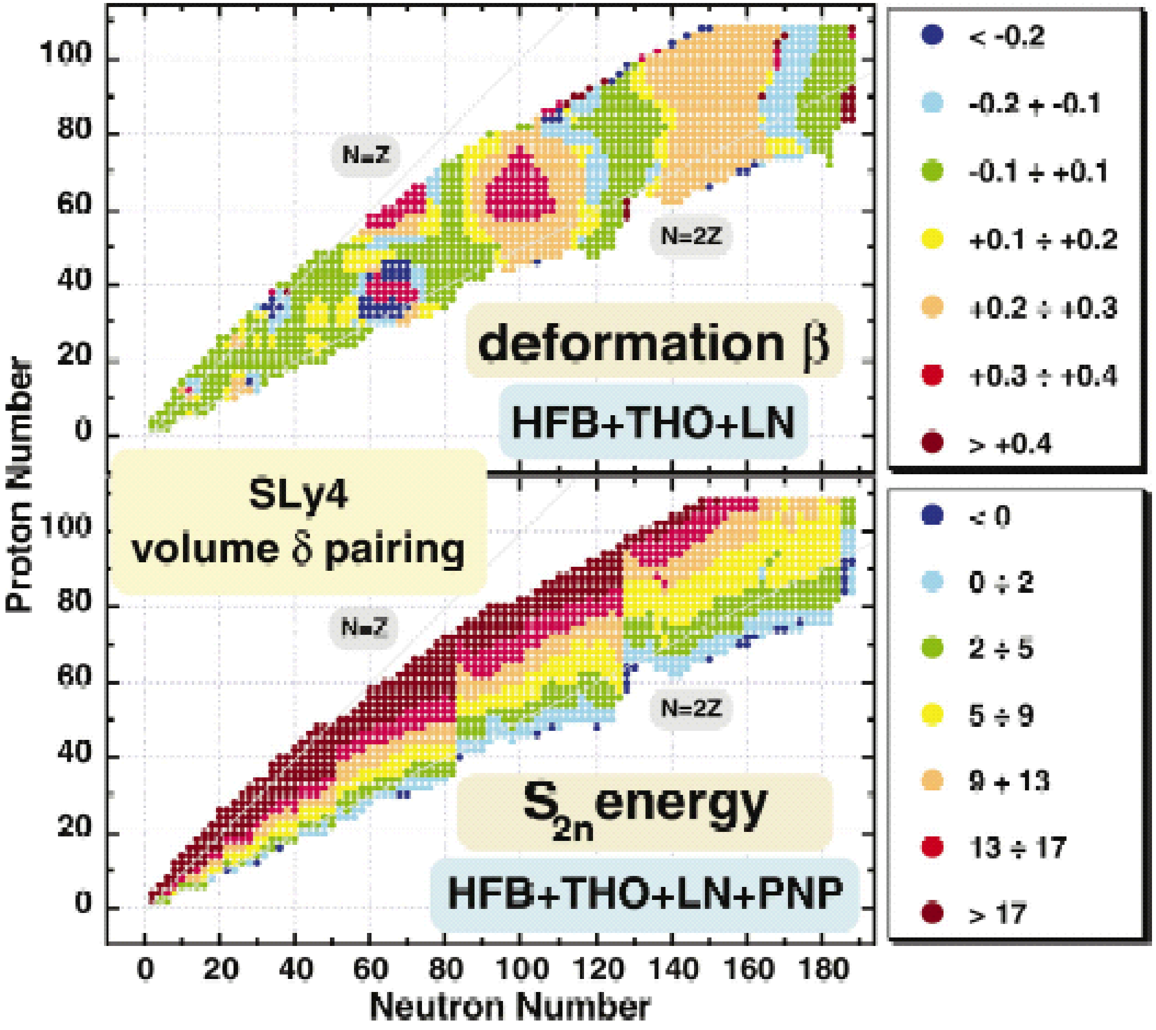




Examples of HF calculations
for masses and radii

Z. Patyk et al.,
PRC59('99)704

deformation and two-neutron separation energy



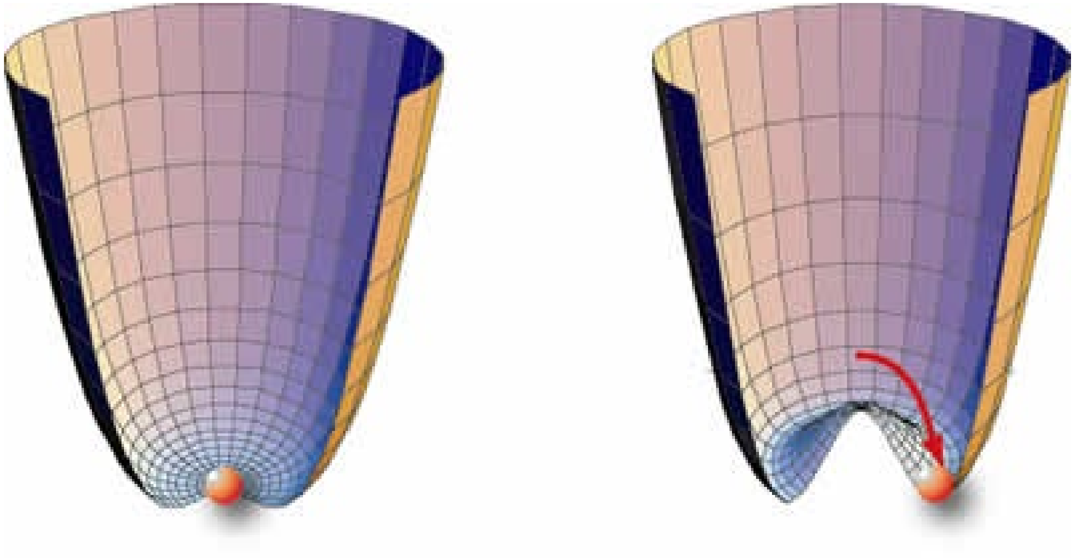
M.V. Stoitsov et al., PRC68('03)054312

Mean-field approximation and deformation

$$H=\sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\nabla_i^2+V_{\rm MF}(\boldsymbol{r}_i)\right)+\cancel{\frac{1}{2}\sum_{i,j}^{A}v(\boldsymbol{r}_i,\boldsymbol{r}_j)-\sum_i V_{\rm MF}(\boldsymbol{r}_i)}$$

⇨ $\Psi_{\rm MF}$: ハミルトニアン H が持っている対称性を持たなくてもいい

“対称性の自発的破れ”



Ψ_{MF} ：ハミルトニアン H が持っている対称性を持たなくてもいい

典型的な例

- ➢ 並進対称性: 原子核のDFTでは常に破れる

$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i - \boldsymbol{r}_j) \rightarrow \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \underline{\underline{V_{\mathrm{MF}}(\boldsymbol{r}_i)}}\right)$$

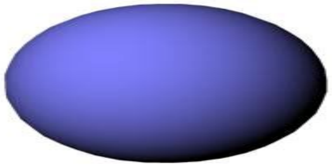
Ψ_{MF} ：ハミルトニアン H が持っている対称性を持たなくてもいい

典型的な例

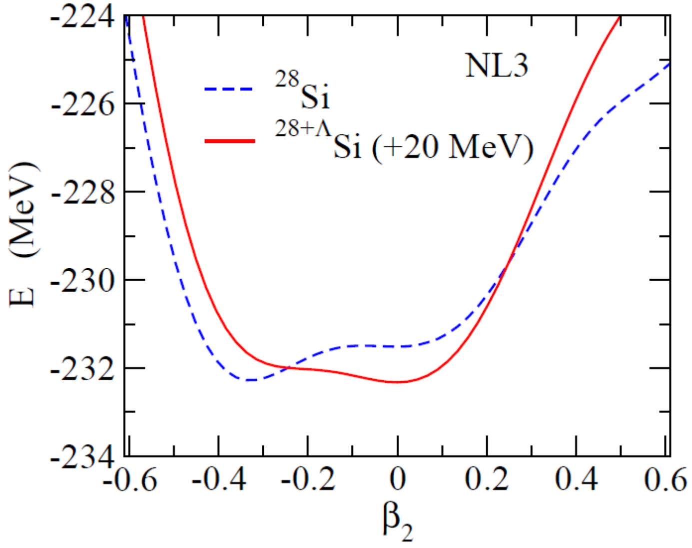
- **並進対称性:** 原子核のDFTでは常に破れる

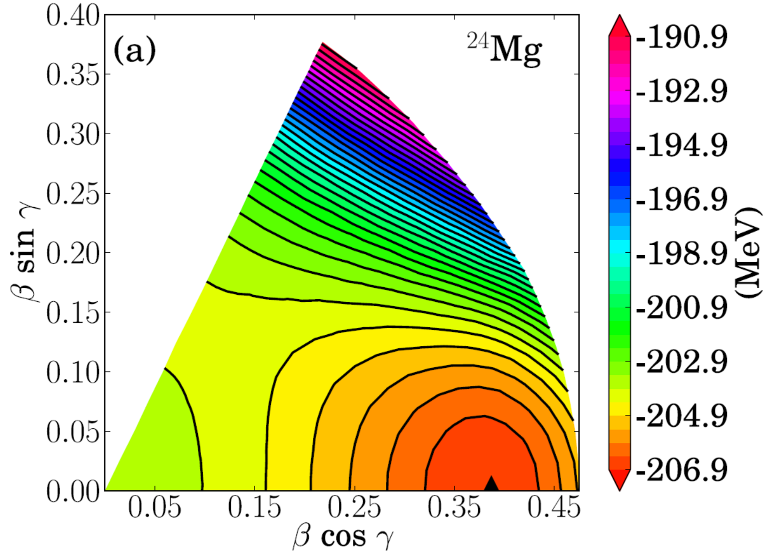
$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\nabla_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i - \boldsymbol{r}_j) \to \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\nabla_i^2 + \underline{\underline{V_{\mathrm{MF}}(\boldsymbol{r}_i)}}\right)$$

- **回転対称性**



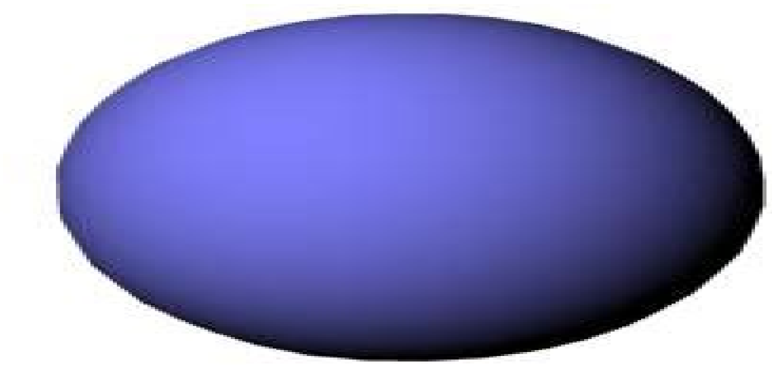
変形した基底状態





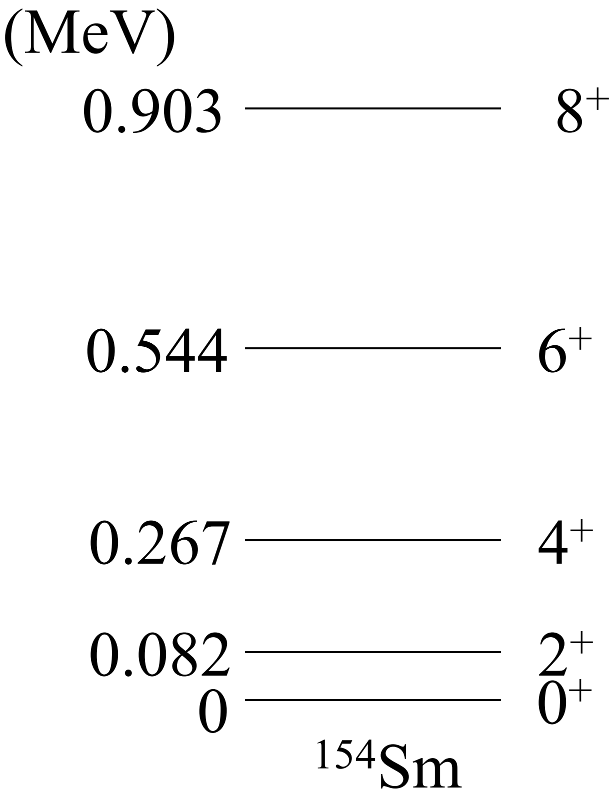
Nuclear Deformation

実験的な証拠



Nuclear Deformation

Excitation spectra of ^{154}Sm



(MeV)

0.903 8^+

0.544 6^+

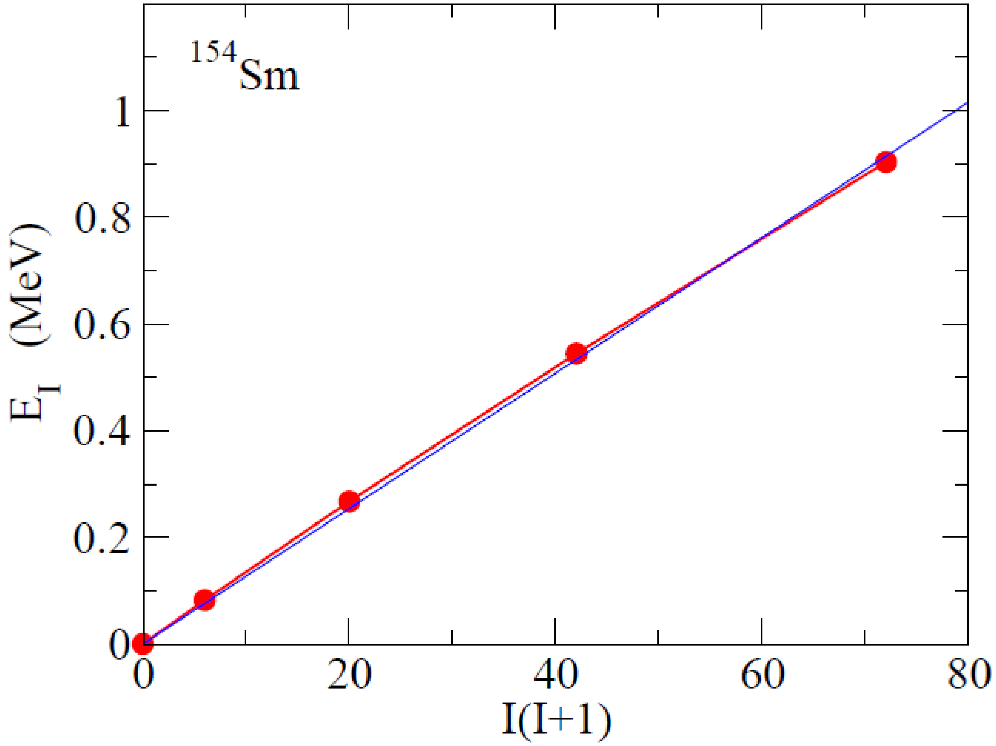
0.267 4^+

0.082 2^+

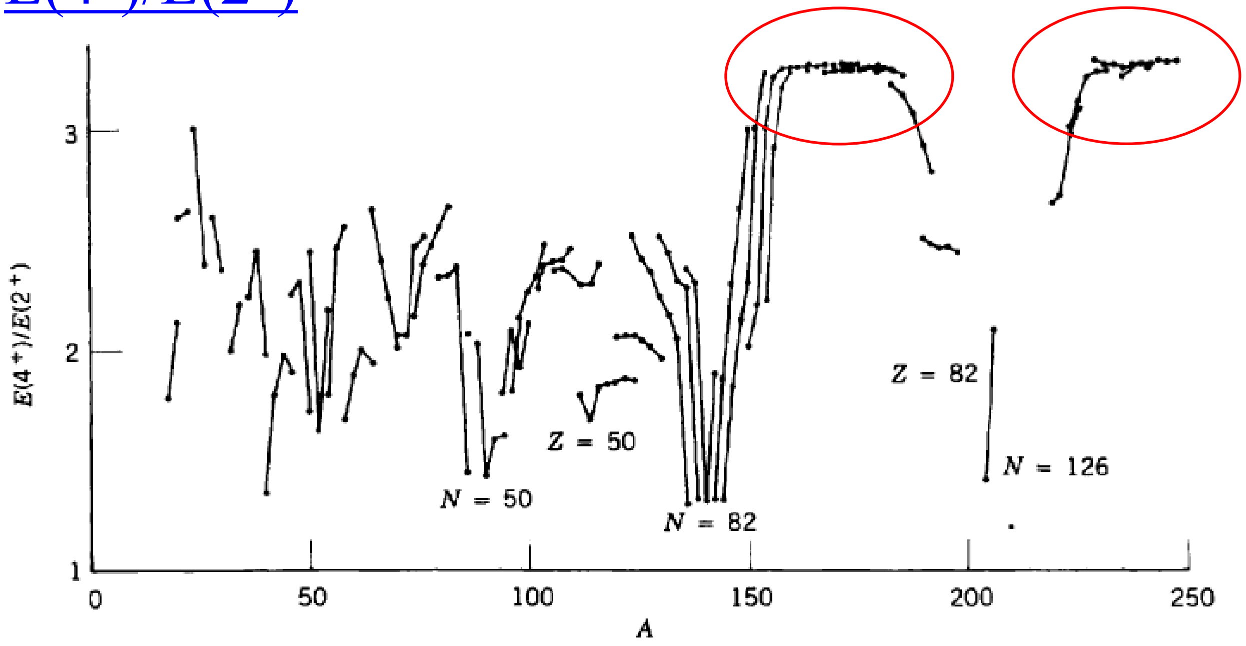
0 0^+

^{154}Sm

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



$E(4^+)/E(2^+)$



deformed nuclei:

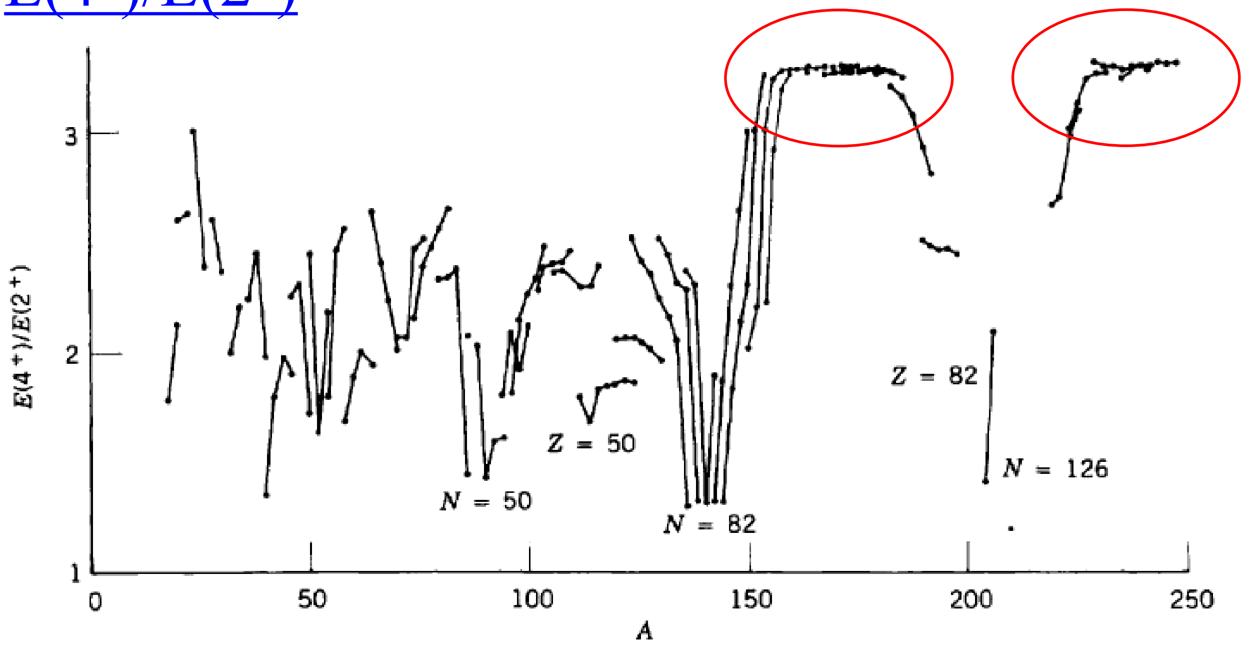
$E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:

$E(4^+)/E(2^+) \sim 2$

K.S. Krane, “Introductory Nuclear Physics”

E(4^+)/E(2^+)

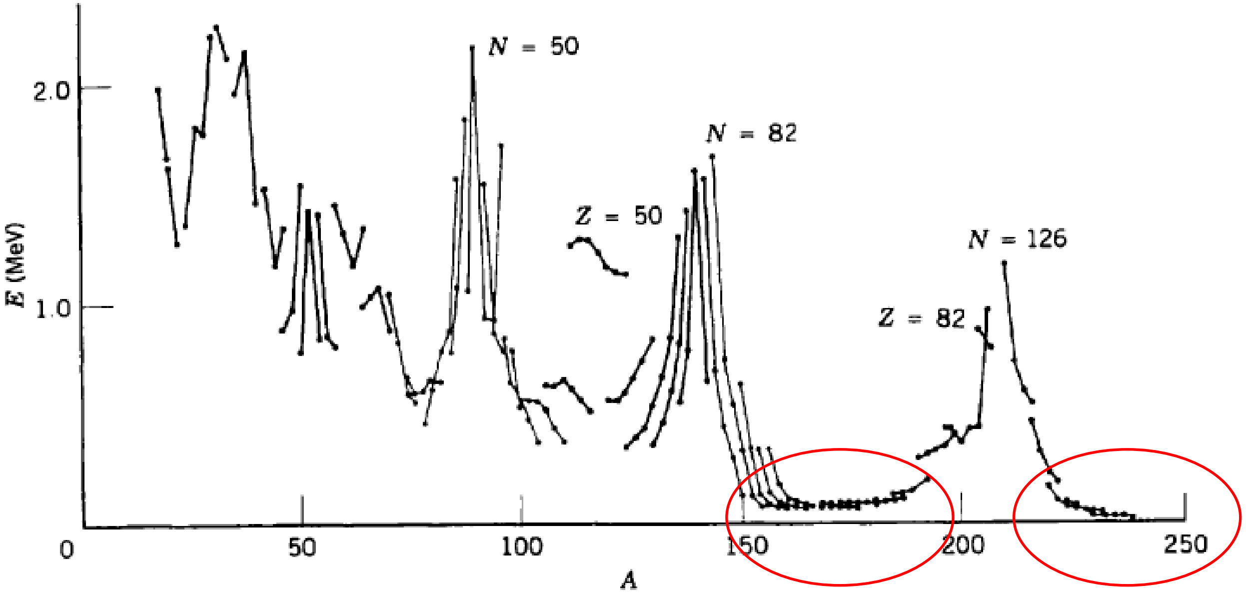


deformed nuclei:
E(4^+)/E(2^+) ~ 3.3

spherical nuclei:
E(4^+)/E(2^+) ~ 2

K.S. Krane. “Introductory Nuclear Physics”

E(2^+)



a small energy
→ spontaneously
symm. breaking

deformed nuclei

$$R(\theta,\phi) = R_0\,[1 + a_{20}Y_{20}(\theta) + a_{22}(Y_{22}(\theta,\phi) + Y_{2-2}(\theta,\phi))]$$

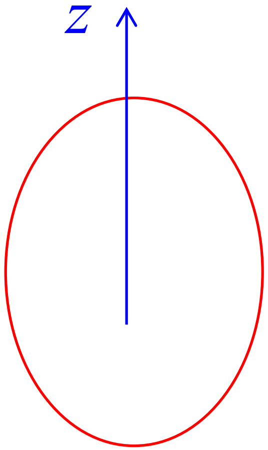
$$\boxed{a_{20} \equiv \beta\cos\gamma, \qquad a_{22} \equiv \frac{1}{\sqrt{2}}\beta\sin\gamma}$$

(γ : triaxiality)

For $\gamma = 0$: $a_{20} = \beta$, $a_{22} = 0$

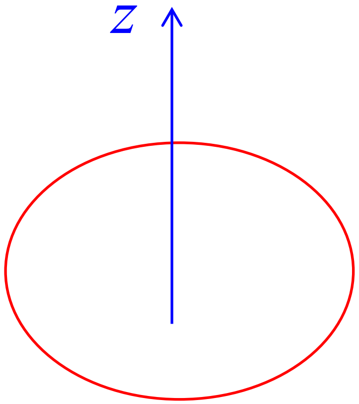
↪ $$R(\theta,\phi) = R_0\,[1 + \beta Y_{20}(\theta)]$$

No ϕ dependence: axial symmetri along z-axis



$\beta > 0$

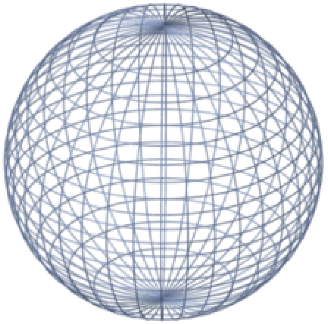
prolate deformation

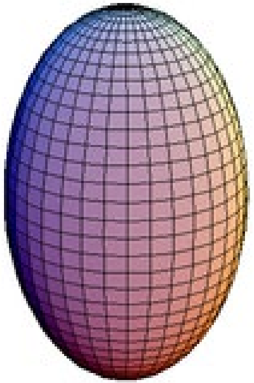


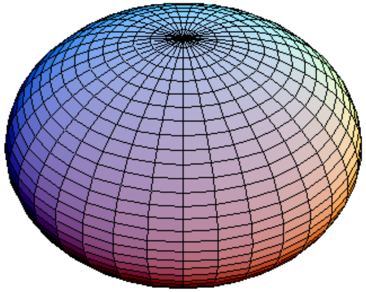
$\beta < 0$

oblate deformation

Nuclear Deformation





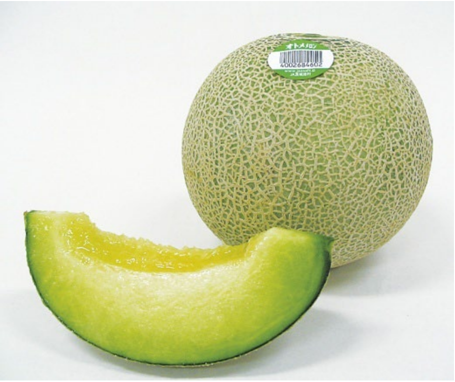


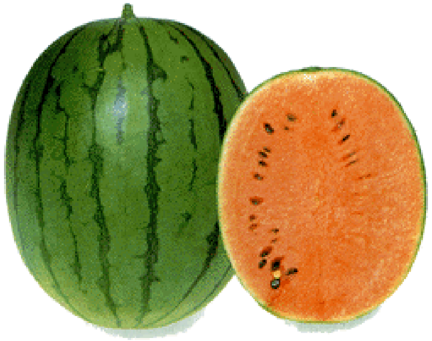
spherical

prolate

oblate

triaxial





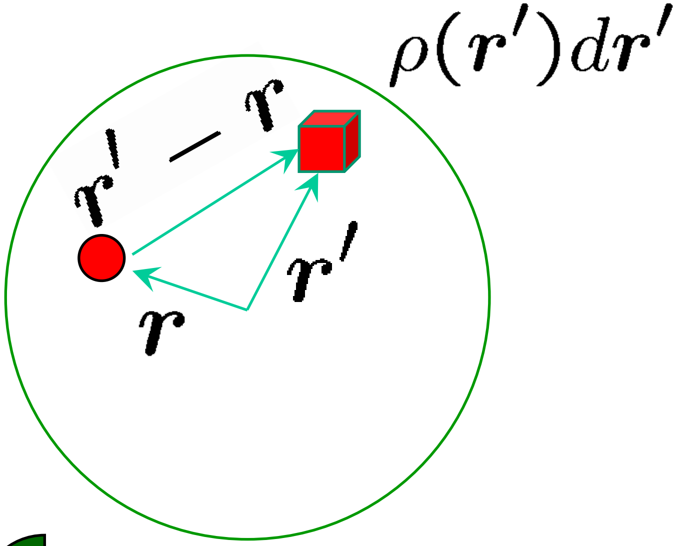
$\beta = 0$

$$\beta > 0 \quad \gamma = 0$$

$$\beta < 0 \quad \gamma = 0$$

$$\beta > 0 \quad 0 < \gamma < \pi/3$$

One-particle motion in a deformed potential

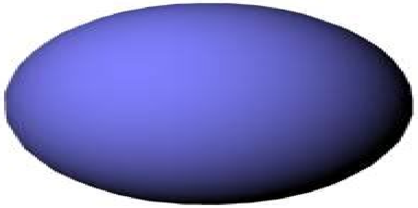


naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

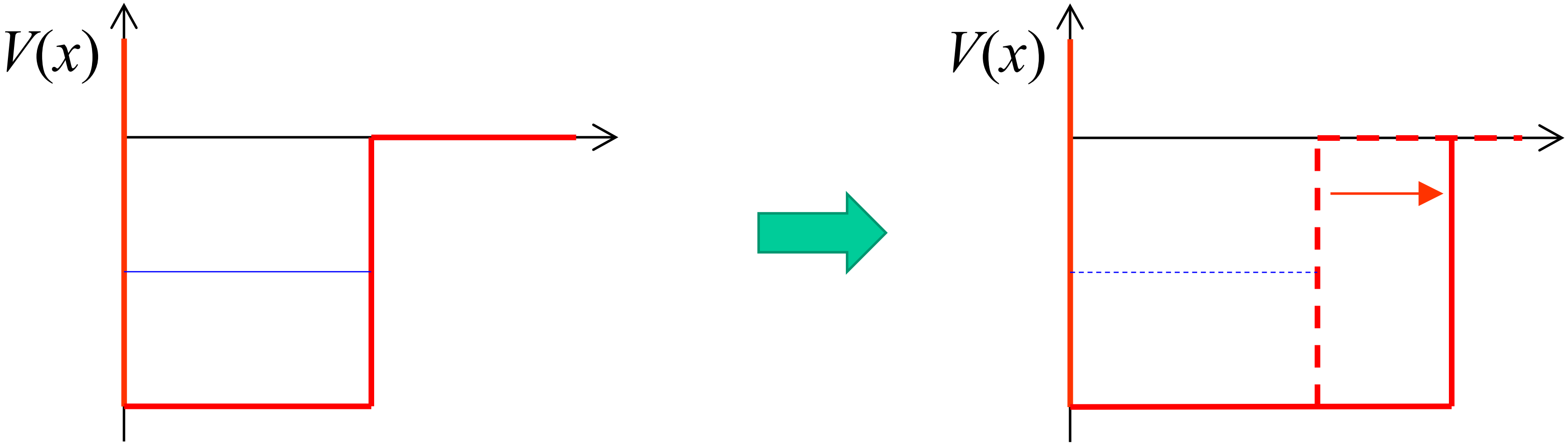
$$\boxed{V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \quad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')}$$

$\rightarrow V(r)$



$\rightarrow V(r,\theta)$

（準備）1次元井戸型ポテンシャル



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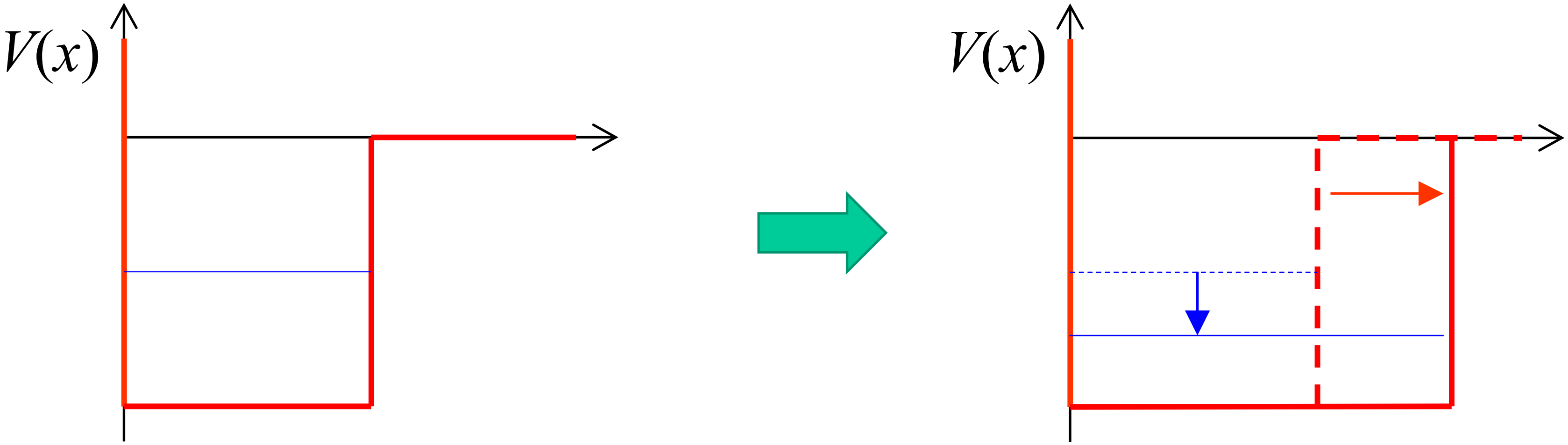

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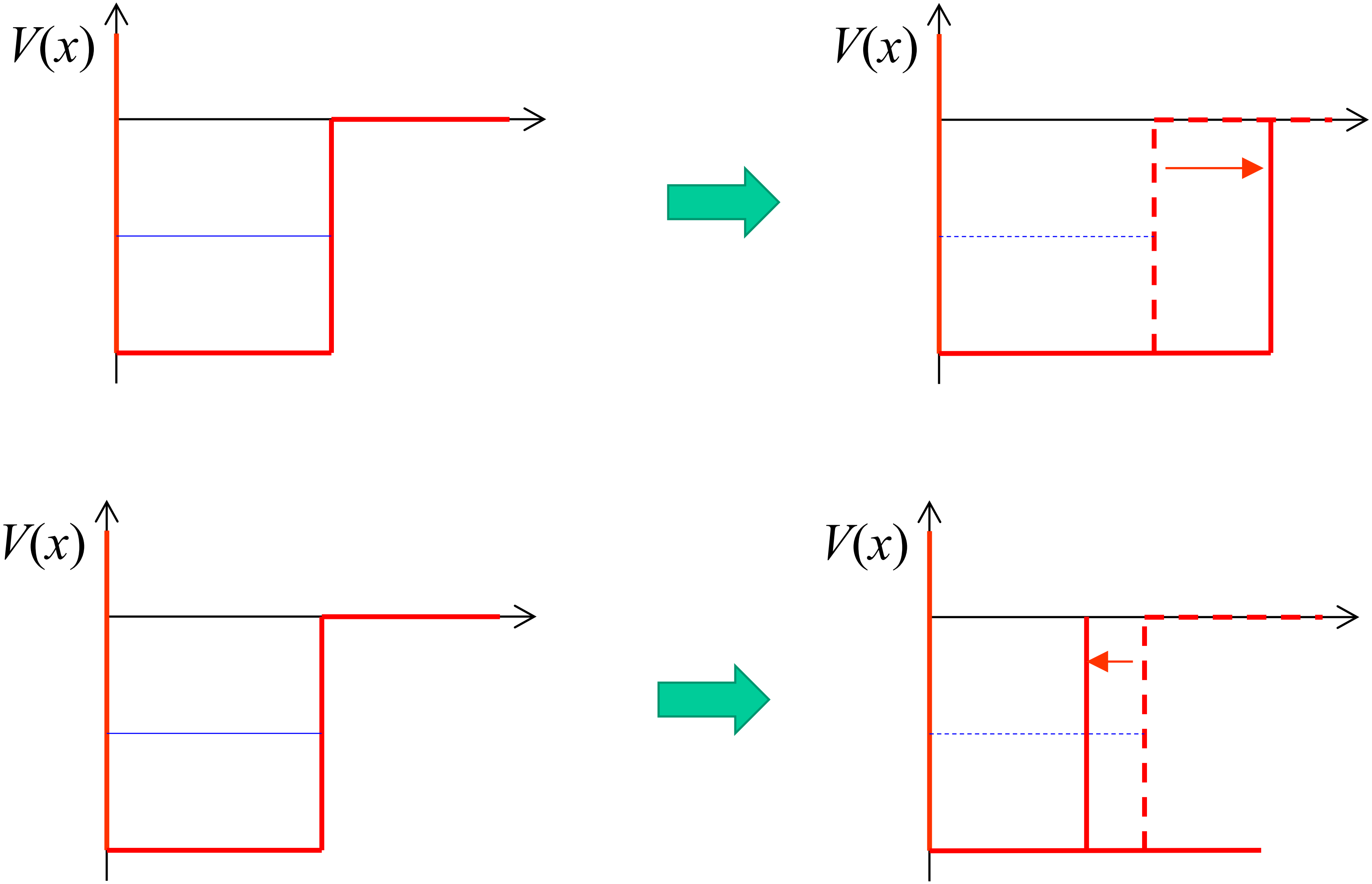
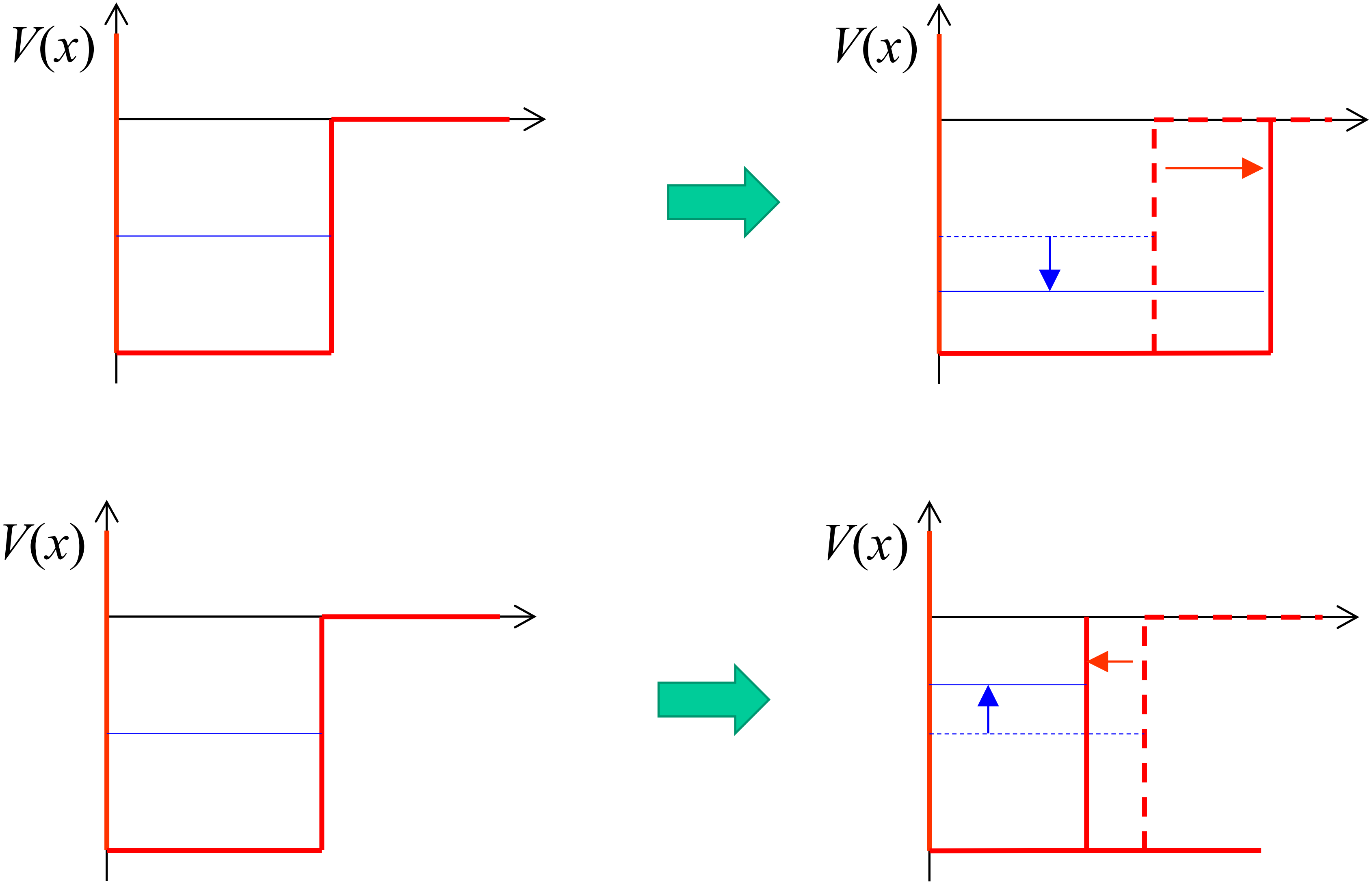
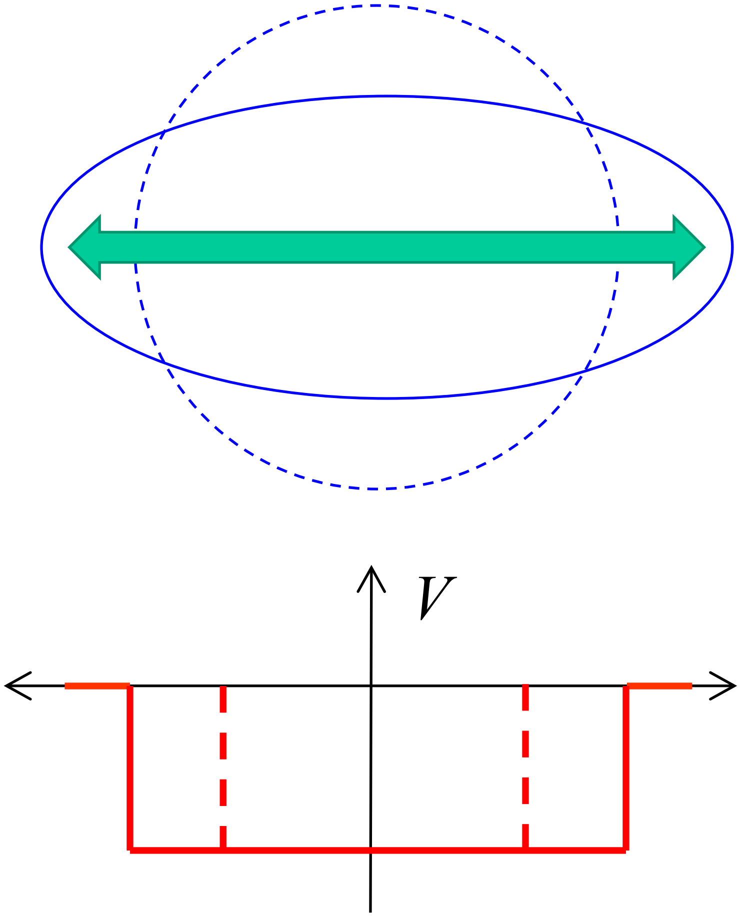


（準備）1次元井戸型ポテンシャル



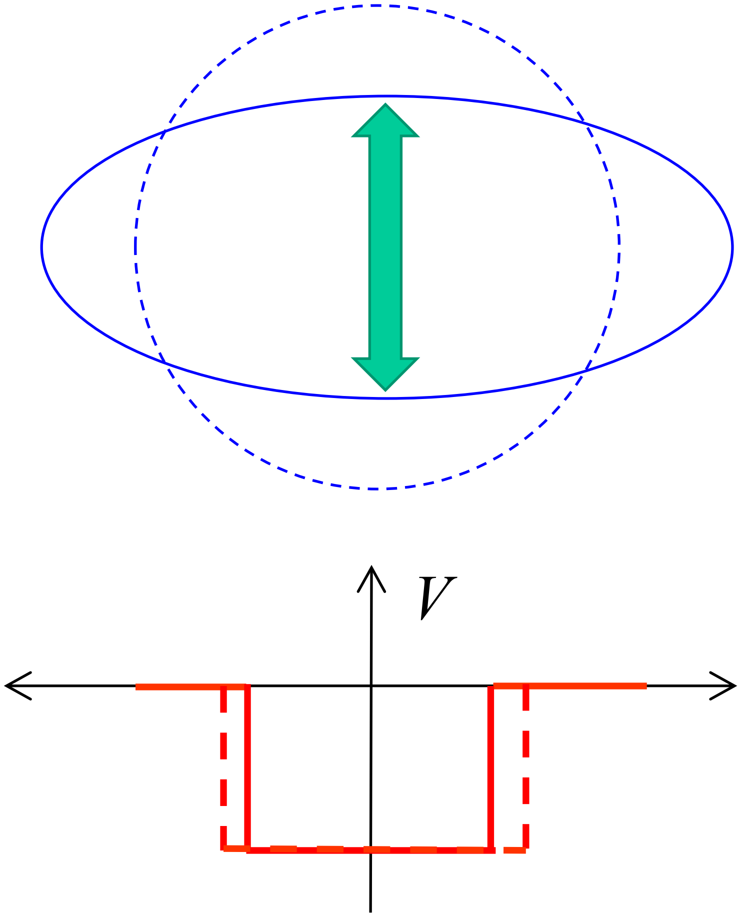
One-particle motion in a deformed potential

長軸に沿った運動



$E \rightarrow$ 小

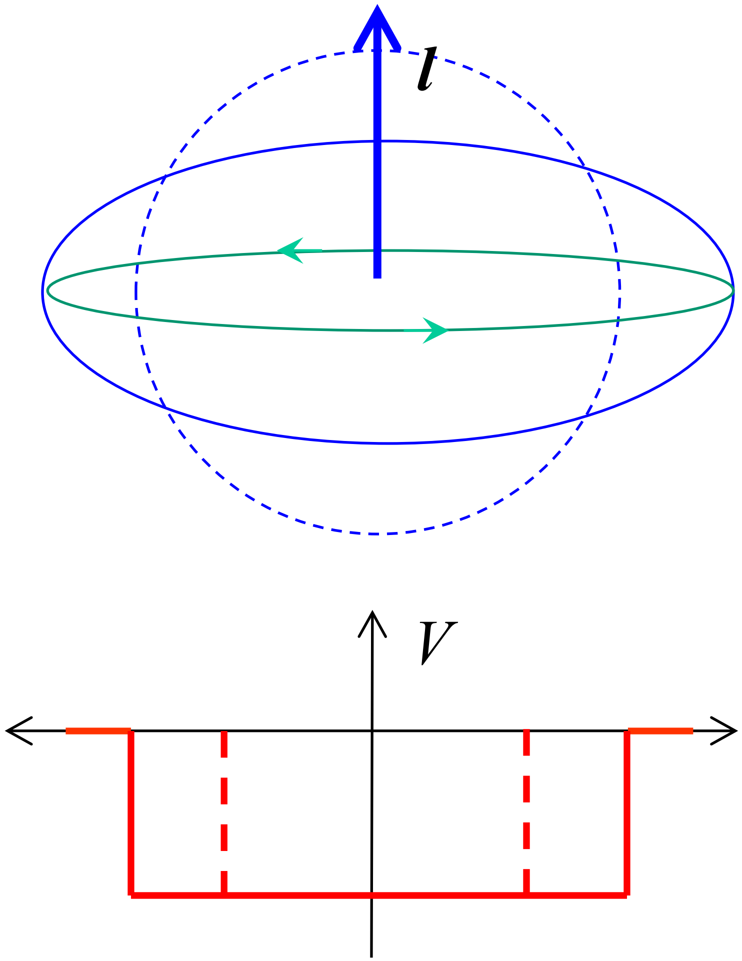
短軸に沿った運動



$E \rightarrow$ 大

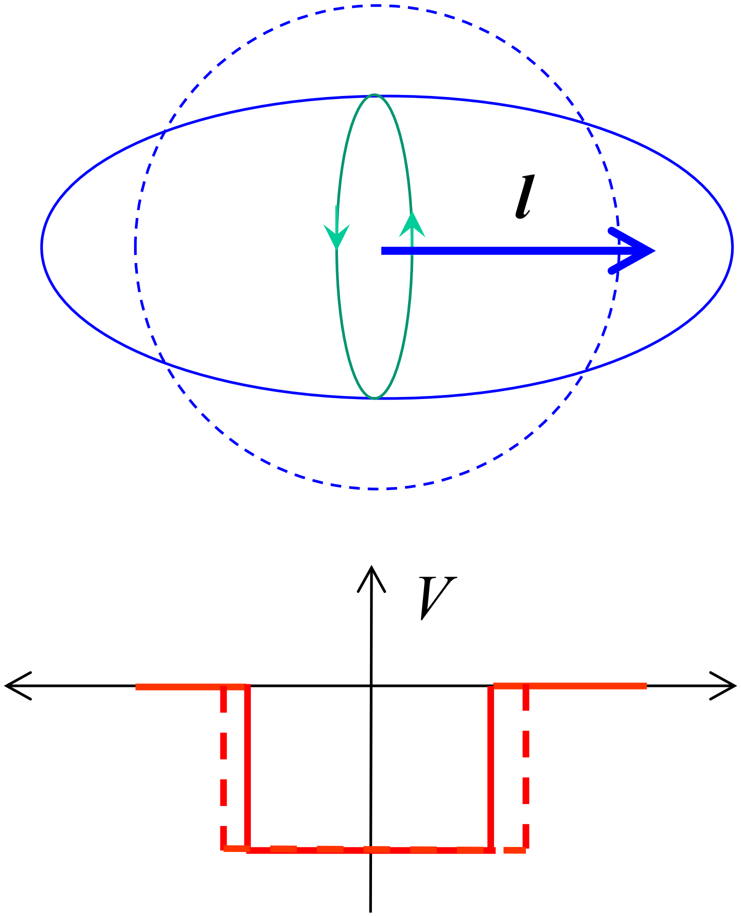
One-particle motion in a deformed potential

長軸に沿った運動



$E \rightarrow$ 小

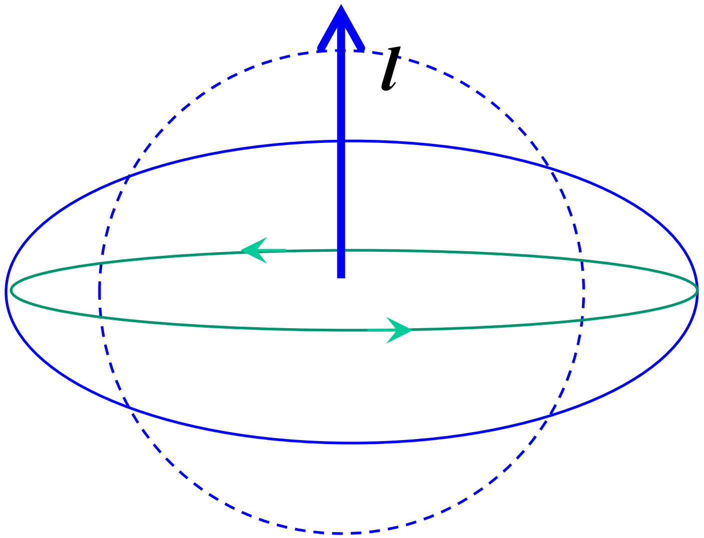
短軸に沿った運動



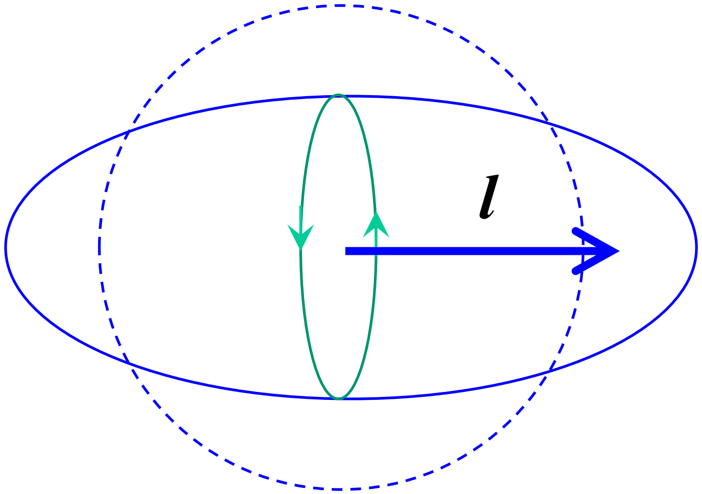
$E \rightarrow$ 大

One-particle motion in a deformed potential

長軸に沿った運動



短軸に沿った運動



→ z軸

軌道が
スプリット