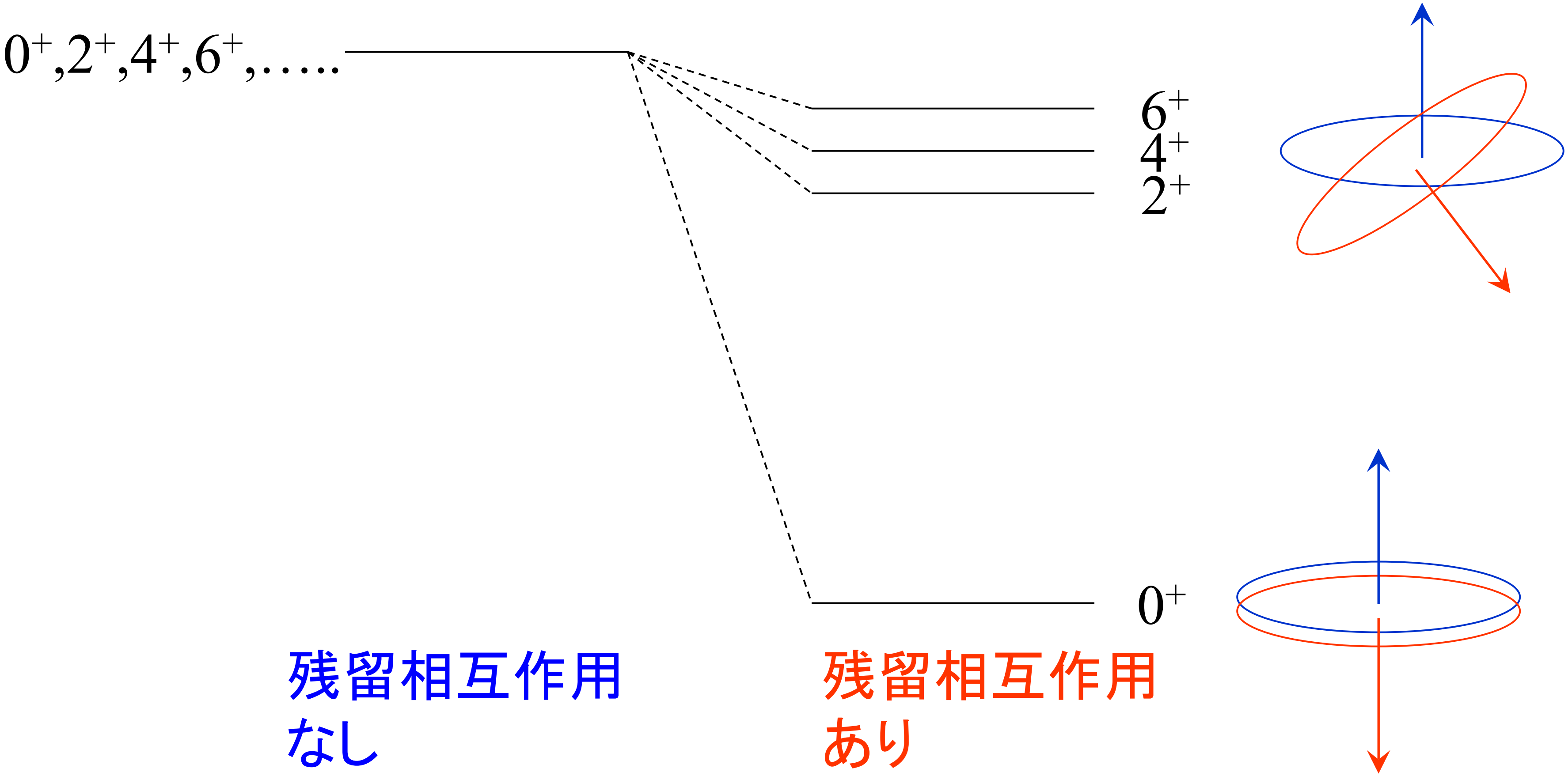
Pairing Correlation（対相関）

残留相互作用の果たす役割

$$
\begin{aligned}
H &= \sum_{i=1}^{A} -\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) \\
&= \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\mathsf{MF}}(\boldsymbol{r}_i)\right) + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) - \sum_{i} V_{\mathsf{MF}}(\boldsymbol{r}_i)
\end{aligned}
$$

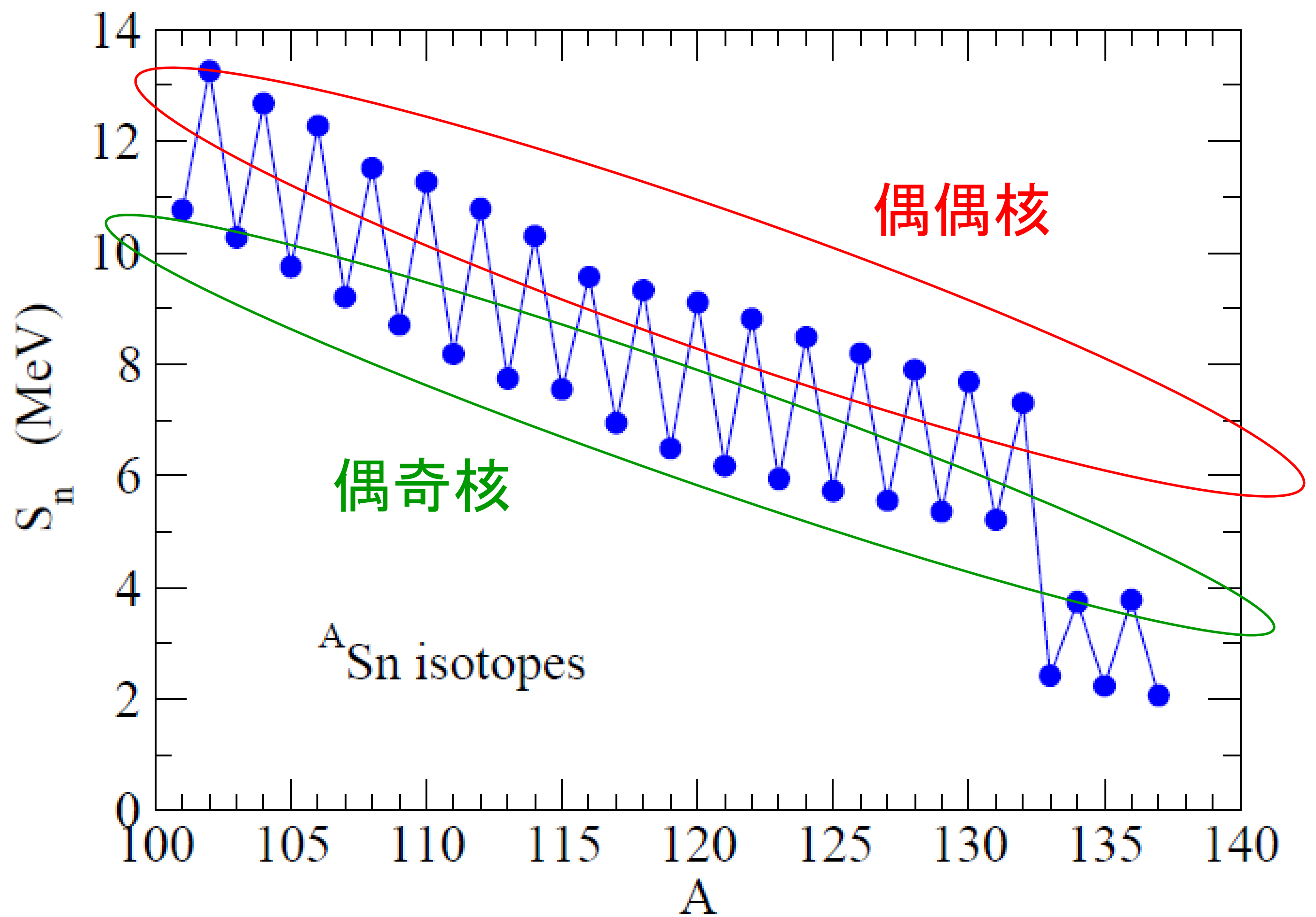


0+,2+,4+,6+,.....
6+
4+
2+
0+
残留相互作用
なし
残留相互作用
あり

対相関エネルギー

偶数個の中性子から1つ中性子を取る方が奇数個から取るより大きなエネルギーが必要：対相関

even-odd staggering



1n separation energy: $S_n\,(A,Z) = B(A,Z) - B(A-1,Z)$

(参考)中性子誘起核分裂

$$n + {}^{235}U \rightarrow {}^{236}U^* \rightarrow \text{fission}$$

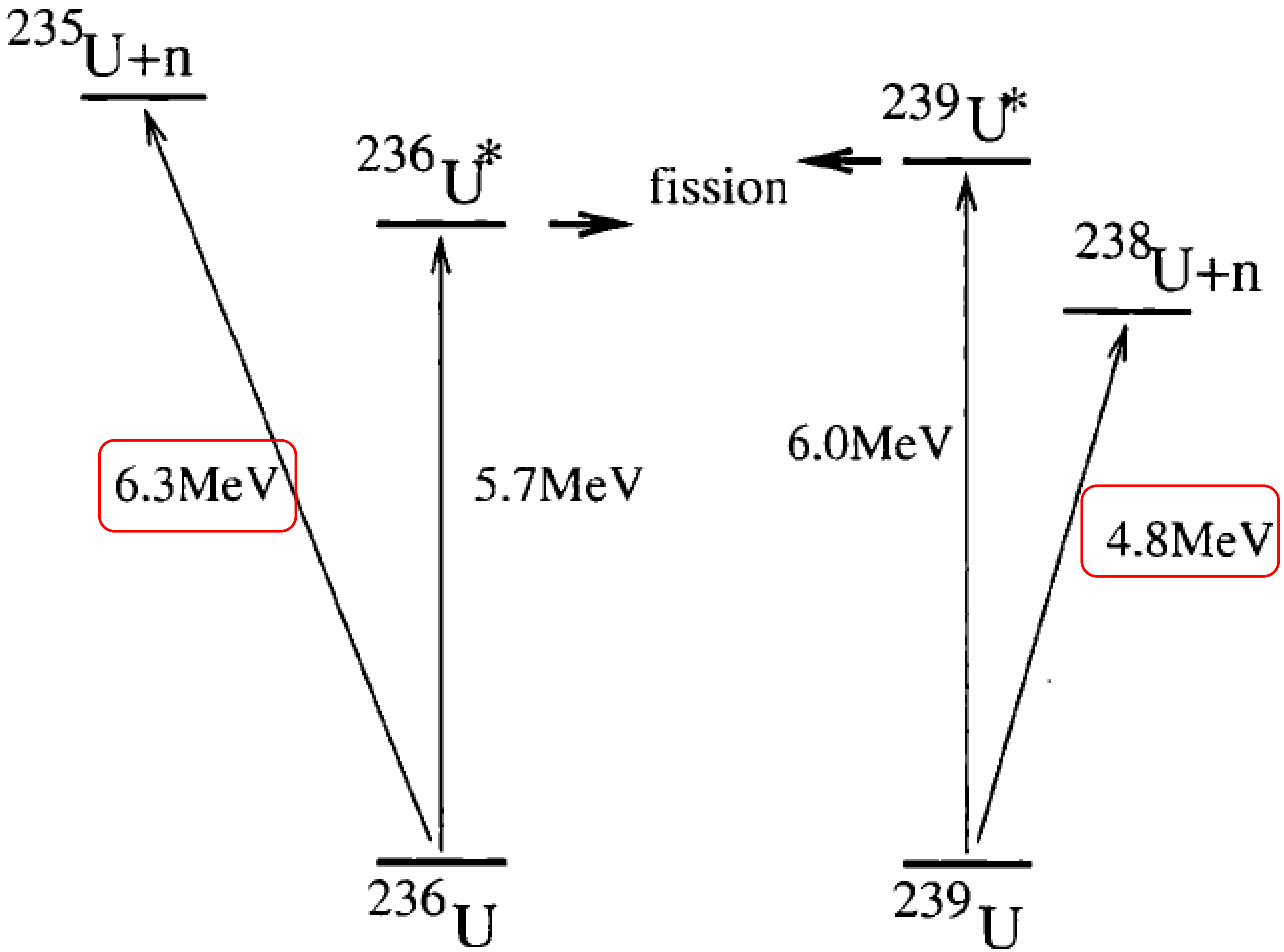
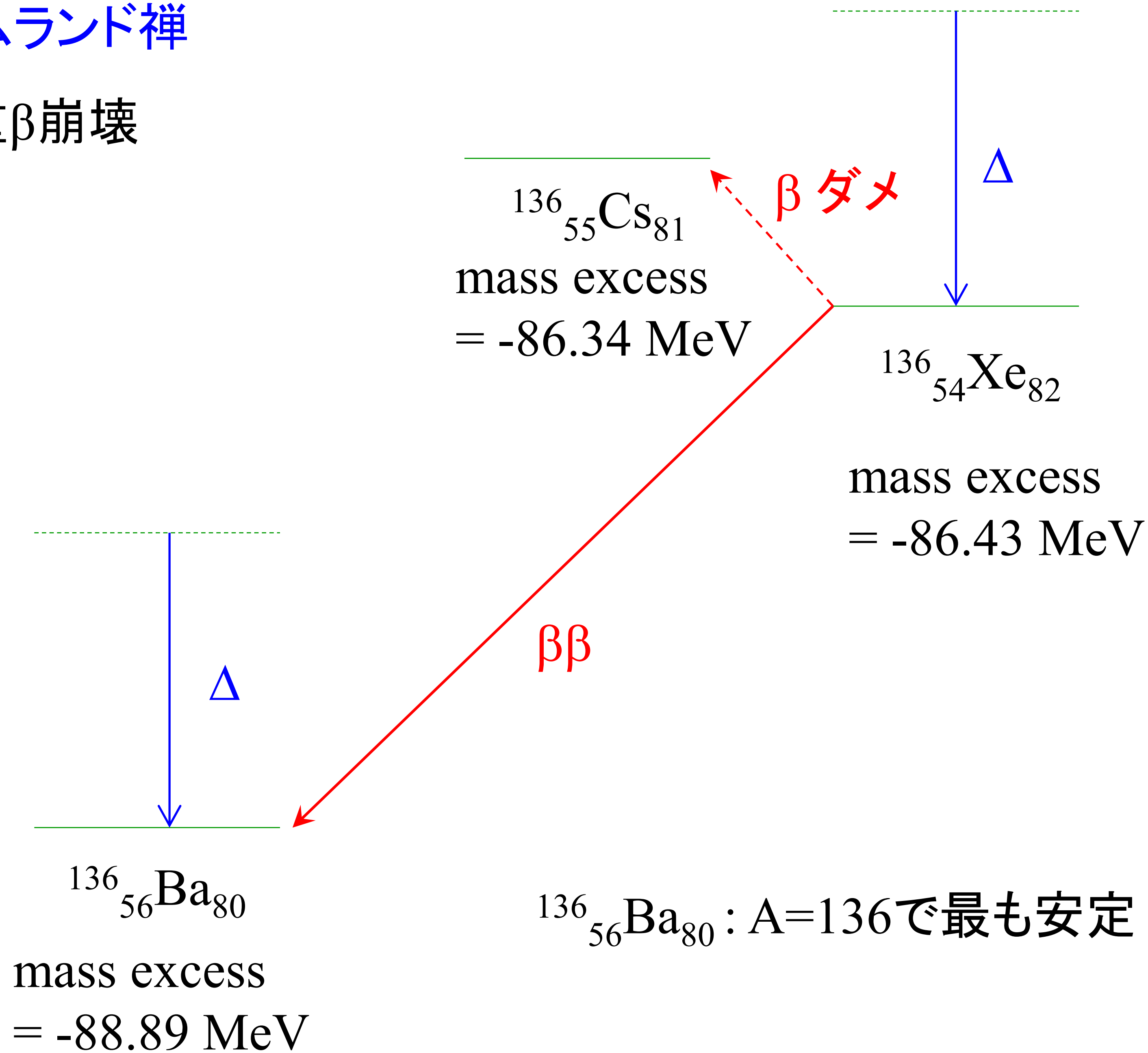


Fig. 6.6. Levels of the systems $A = 236$ and $A = 239$ involved in the fission of ^{236}U and ^{239}U. The addition of a motionless (or thermal) neutron to ^{235}U can lead to the fission of ^{236}U. On the other hand, fission of ^{239}U requires the addition of a neutron of kinetic energy $T_n = 6.0 - 4.8 = 1.2$ MeV.

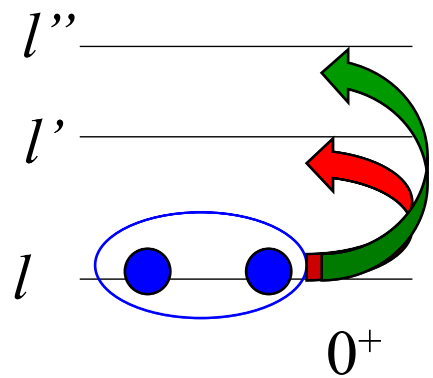
（参考）カムランド禅

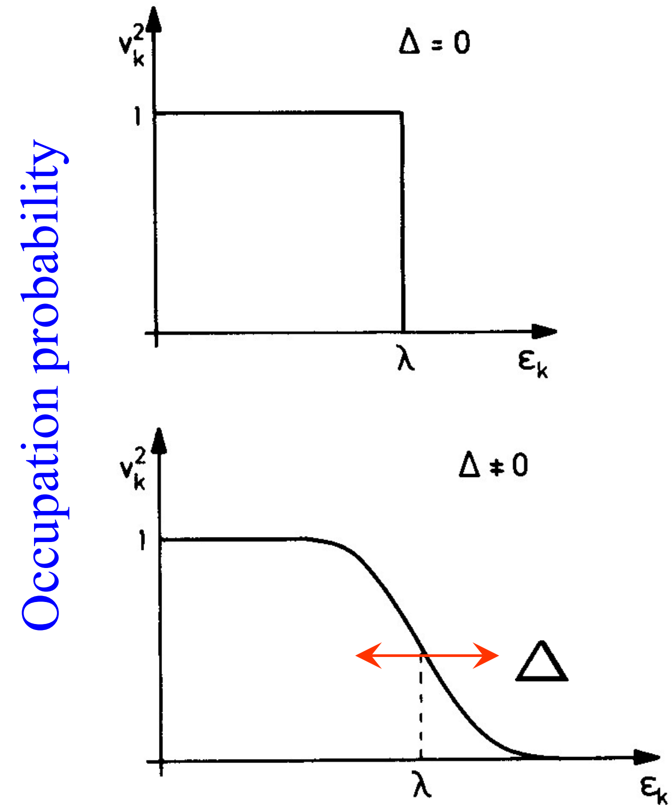
^{136}Xeの2重β崩壊



$^{136}{}_{56}Ba_{80}$: A=136で最も安定

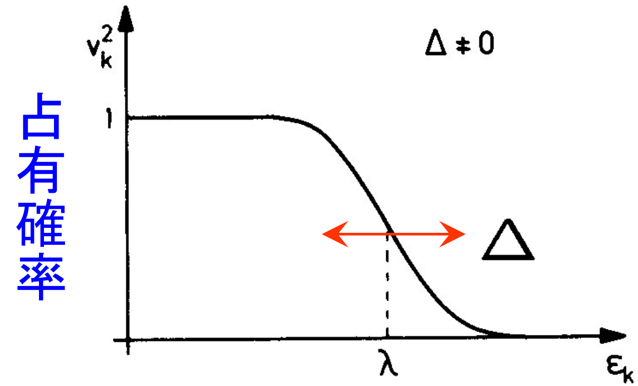
波動関数:



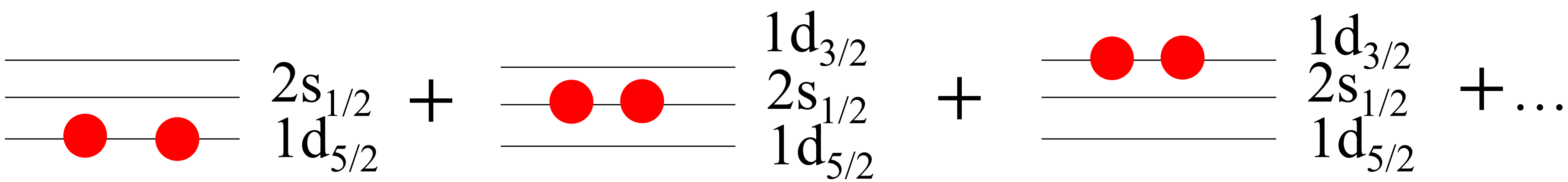


$$|\Psi_{0+}\rangle = |(ll)L=0\rangle + \sum_{l'} \frac{\langle (l'l')L=0|v_{\rm res}|(ll)L=0\rangle}{2\epsilon_l - 2\epsilon_{l'}} |(l'l')L=0\rangle + \cdots$$

波動関数:



$$|\Psi_{\mathrm{g.s.}}\rangle =$$



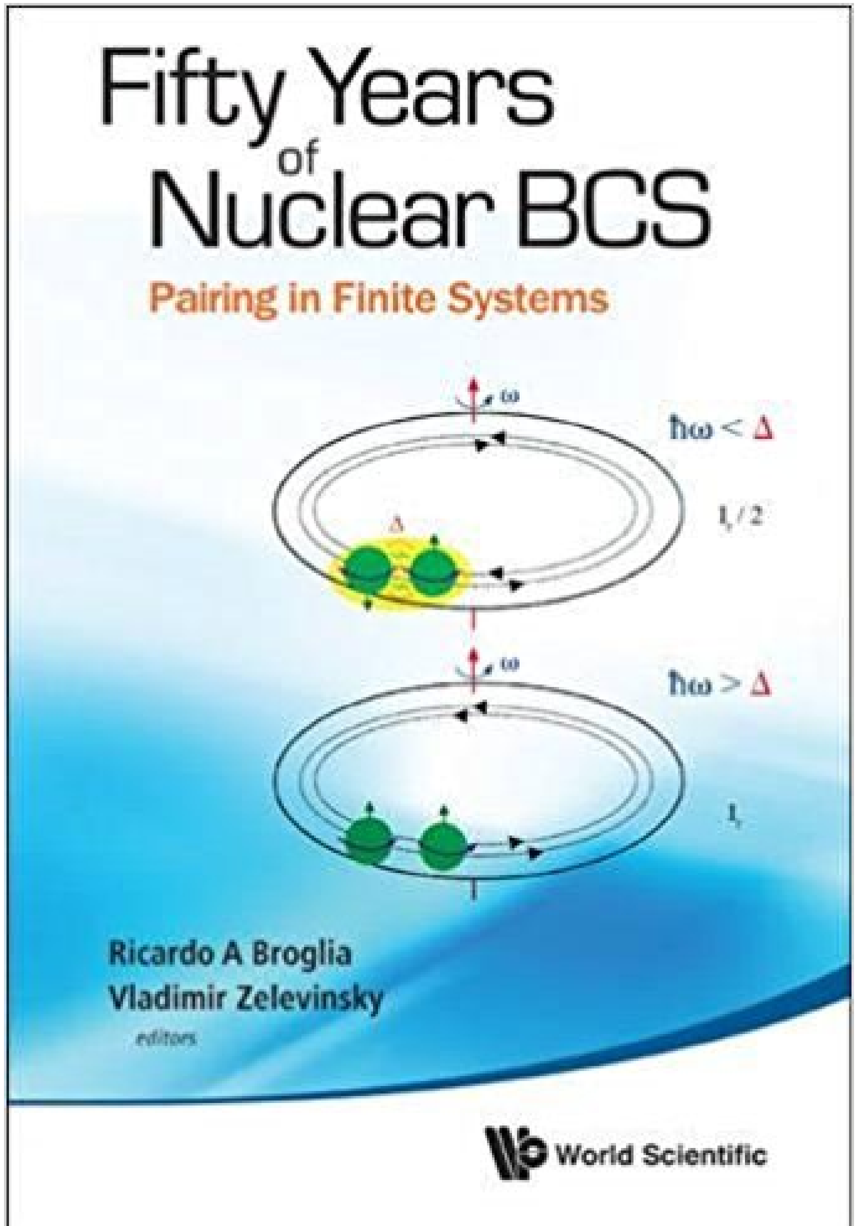
いろいろな配位を混ぜることによって対相関エネルギーを稼ぐ

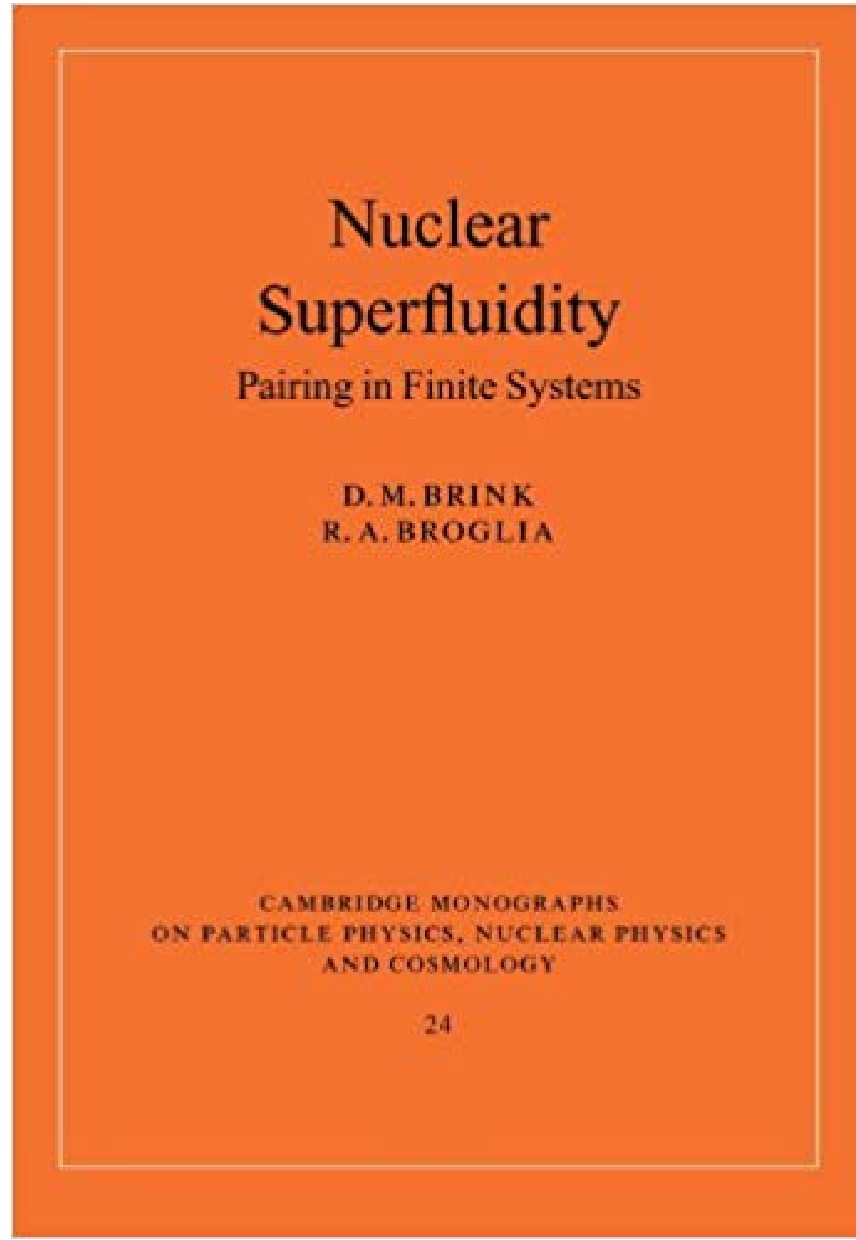
→ 各軌道は部分的にのみ占有されることになる

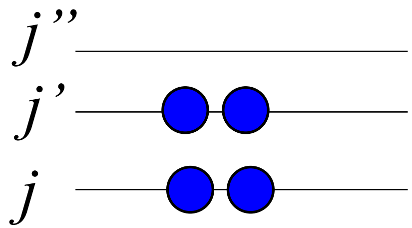
占有確率はエネルギーを最小化するように決定

cf. BCS 理論

超流動状態

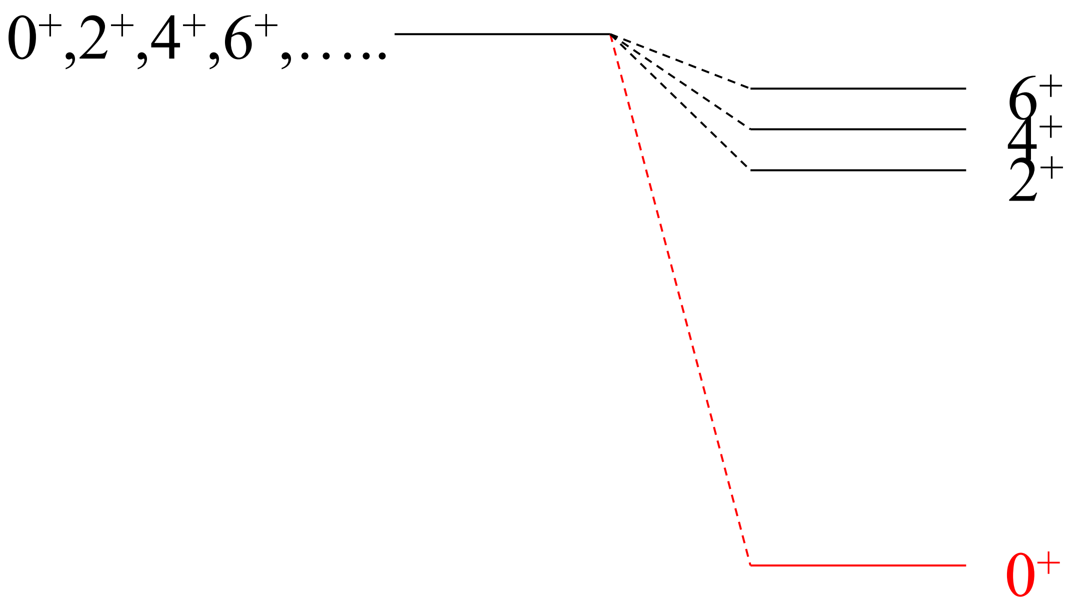

Fifty Years of Nuclear BCS
Pairing in Finite Systems
ħω < Δ
Δ
ħω > Δ
Ricardo A Broglia
Vladimir Zelevinsky
editors
World Scientific


Nuclear Superfluidity
Pairing in Finite Systems
D. M. BRINK
R. A. BROGLIA
CAMBRIDGE MONOGRAPHS
ON PARTICLE PHYSICS, NUCLEAR PHYSICS
AND COSMOLOGY
24



複数個のレベルに
複数個のペアがある問題

$$v_{\rm res}(\boldsymbol{r},\boldsymbol{r}') \quad \sim -g\,\delta(\boldsymbol{r}-\boldsymbol{r}')$$



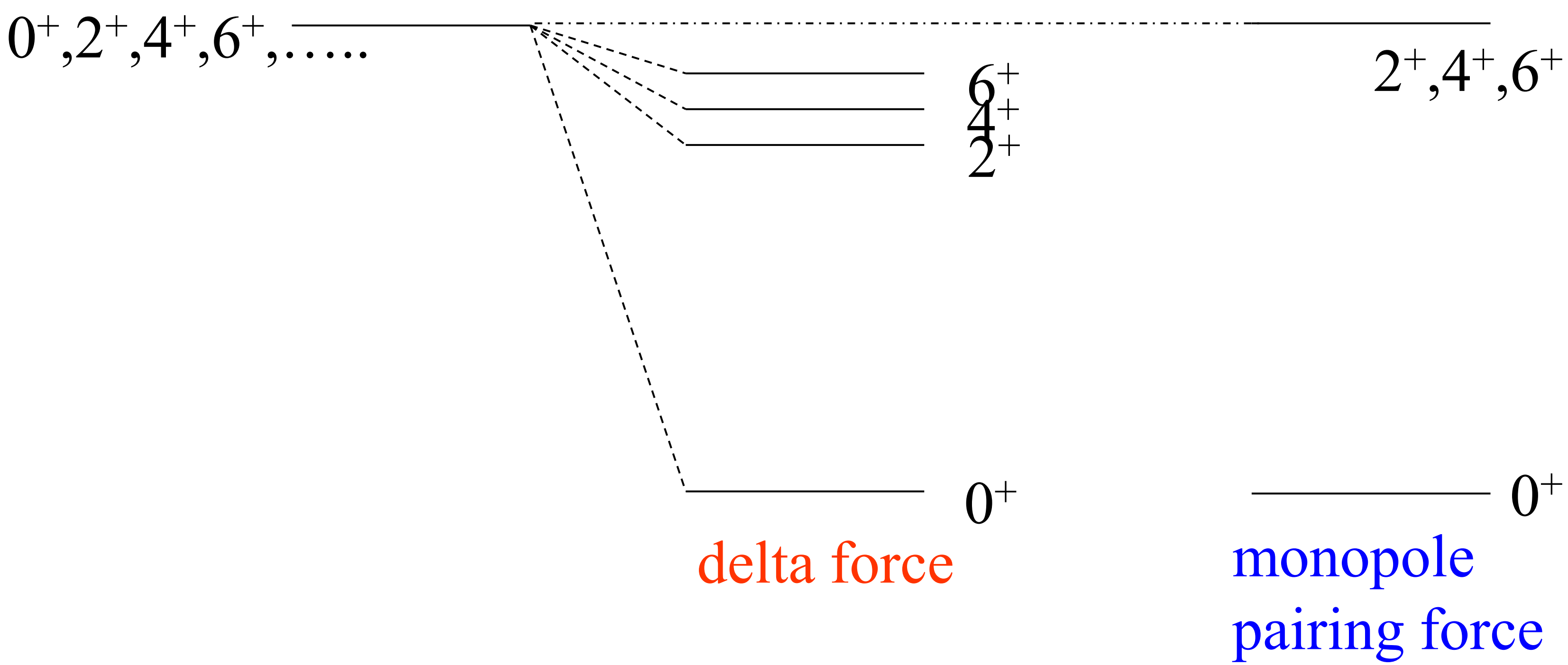
デルタ関数のままでもいいが、説明を簡単にするために
もう少し簡単にした相互作用を導入する。

Simplified pairing interaction

$$V = -G\,P^\dagger P; \qquad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

$\bar{\nu}$: the time reversed state of ν

e.g., $|\nu\rangle = |njlm\rangle, \quad |\bar{\nu}\rangle = |njl-m\rangle$



Simplified pairing interaction

$$V = -G\,P^\dagger P; \qquad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

$\bar{\nu}$: the time reversed state of ν

$$H = \sum_k \epsilon_k (a^\dagger_k a_k + a^\dagger_{\bar{k}} a_{\bar{k}}) - G \left(\sum_{k>0} a^\dagger_k a^\dagger_{\bar{k}}\right) \left(\sum_{k>0} a_{\bar{k}} a_k\right)$$

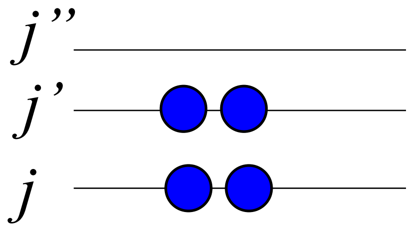
ε_2

ε_1

$$H = \begin{pmatrix} 2\epsilon_1 - G & -G & 0 & 0 \\ -G & 2\epsilon_2 - G & 0 & 0 \\ 0 & 0 & \epsilon_1 + \epsilon_2 & 0 \\ 0 & 0 & 0 & \epsilon_1 + \epsilon_2 \end{pmatrix}$$

$$\rightarrow \quad \Psi_{\rm g.s.} = C_1 \Psi_1 + C_2 \Psi_2$$

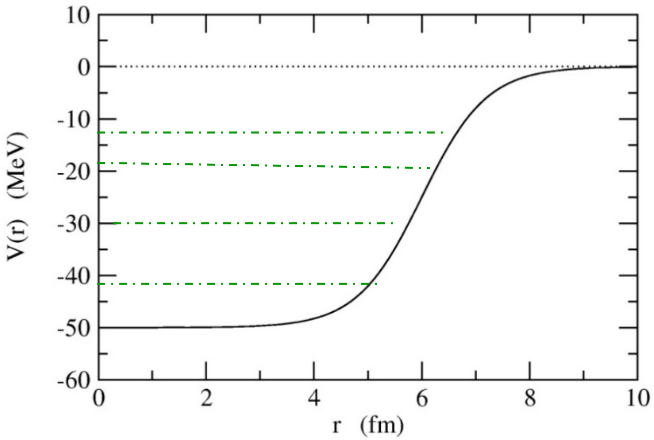
$\varepsilon_1 = \varepsilon_2 = 0$なら $\rightarrow \quad \Psi_{\rm g.s.} = \frac{1}{\sqrt{2}}(\Psi_1 + \Psi_2); \quad E_{\rm g.s.} = -2G$



複数個のレベルに
複数個のペアがある問題

HF+BCS theory

①平均場近似をして核子の感じるポテンシャルを求める
（平均的な振る舞いをまず決める）



$$H = \sum_k \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \left(\sum_{k>0} a_k^\dagger a_{\bar{k}}^\dagger \right) \left(\sum_{k>0} a_{\bar{k}} a_k \right)$$

②各準位の占有確率を決める。
決め方は、残留相互作用も含めてエネルギーが最小になるようにする。

$$H = \sum_{\nu} \epsilon_{\nu}(a_{\nu}^{\dagger}a_{\nu} + a_{\bar{\nu}}^{\dagger}a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a_{\nu}^{\dagger}a_{\bar{\nu}}^{\dagger}\right)\left(\sum_{\nu>0} a_{\bar{\nu}}a_{\nu}\right)$$

2体の相互作用
→ １体近似をする

cf. HF potential

$$V_H(\boldsymbol{r}) = \int v(\boldsymbol{r},\boldsymbol{r}')\rho_{\mathrm{HF}}(\boldsymbol{r}')d\boldsymbol{r}$$

Solve the pairing Hamiltonian

$$H = \sum_{\nu} \epsilon_{\nu} (a_{\nu}^{\dagger} a_{\nu} + a_{\bar{\nu}}^{\dagger} a_{\bar{\nu}}) - G \left(\sum_{\nu > 0} a_{\nu}^{\dagger} a_{\bar{\nu}}^{\dagger} \right) \left(\sum_{\nu > 0} a_{\bar{\nu}} a_{\nu} \right)$$

in the mean-field approximation

- Mean-field approximation:

$$V = -G\, P^{\dagger} P \to -G \left(\langle P^{\dagger} \rangle P + P^{\dagger} \langle P \rangle \right) = -\Delta (P^{\dagger} + P)$$

⟺ particle number violation

we consider $H' = H - \lambda\hat{N}$ instead of H :

$$
\begin{aligned}
H' &= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger\hat{P} \\
&\to \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(\hat{P}^\dagger + \hat{P}) \\
&= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta\sum_{k>0}(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)
\end{aligned}
$$

we consider $H' = H - \lambda\hat{N}$ instead of H :

$$\begin{aligned} H' &= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger\hat{P} \\ &\to \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(\hat{P}^\dagger + \hat{P}) \\ &= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta\sum_{k>0}(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \end{aligned}$$

● Transform *H'* in a form of

$$H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

we consider $H' = H - \lambda\hat{N}$ instead of H :

$$H' = \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger\hat{P}$$
$$\to \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(\hat{P}^\dagger + \hat{P})$$
$$= \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

- Transform H' in a form of

$$H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

→ g.s.: $\alpha_k|BCS\rangle = 0$

1st excited state: $|1_k\rangle = \alpha_k^\dagger|BCS\rangle$ at E_k

…. and so on.

Bogoliubov transformation

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

(Quasi-particle operator)

or $$a_\nu^\dagger = u_\nu \alpha_\nu^\dagger + v_\nu \alpha_{\bar{\nu}}, \qquad a_{\bar{\nu}}^\dagger = u_\nu \alpha_{\bar{\nu}}^\dagger - v_\nu \alpha_\nu$$

(note)

$$\{\alpha_\nu, \alpha_{\nu'}\} = 0, \qquad \{\alpha_\nu, \alpha_{\nu'}^\dagger\} = \delta_{\nu,\nu'}$$

$$\rightarrow \quad u_\nu^2 + v_\nu^2 = 1$$

Bogoliubov transformation

$$\alpha^\dagger_\nu = u_\nu a^\dagger_\nu - v_\nu a_{\bar{\nu}}, \qquad \alpha^\dagger_{\bar{\nu}} = u_\nu a^\dagger_{\bar{\nu}} + v_\nu a_\nu$$

(Quasi-particle operator)

or $$a^\dagger_\nu = u_\nu \alpha^\dagger_\nu + v_\nu \alpha_{\bar{\nu}}, \qquad a^\dagger_{\bar{\nu}} = u_\nu \alpha^\dagger_{\bar{\nu}} - v_\nu \alpha_\nu$$

$$H' = \sum_{k>0} (\epsilon_k - \lambda)(a^\dagger_k a_k + a^\dagger_{\bar{k}} a_{\bar{k}}) - \Delta \sum_{k>0} (a^\dagger_k a^\dagger_{\bar{k}} + a_{\bar{k}} a_k)$$
$$\rightarrow$$

using the quasi-particle operators:

$$
\begin{aligned}
H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\
&= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
&\quad + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta (u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
\end{aligned}
$$

using the quasi-particle operators:

$$
\begin{aligned}
H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\
&= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
&\quad + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
\end{aligned}
$$

if $\quad 2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2) = 0$

then $\quad H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$

with $\quad E_k = (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k$

$$H' = \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$
$$+ \sum_{k>0} [2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)$$

$$\begin{cases} 0 = 2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2) \\ 1 = u_k^2 + v_k^2 \end{cases}$$

$$u_\nu^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$
$$v_\nu^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$

$$H' = \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$
$$+ \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)$$

$$u_\nu^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$
$$v_\nu^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$

$$E_k = (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k$$
$$= \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

$$H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

Ground state wave function:

$$\alpha_k |BCS\rangle = 0$$

$$|BCS\rangle \propto \prod_{\nu>0} \alpha_\nu \alpha_{\bar{\nu}} |0\rangle$$

$$= \prod_{\nu>0} v_\nu \left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right) \Big| 0 \Big\rangle$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right) \Big| 0 \Big\rangle$$

Ground state wave function:

$$\alpha_k |BCS\rangle = 0$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle$$

(note) $\left\langle BCS|a^\dagger_\nu a_\nu|BCS\right\rangle = |v_\nu|^2$: occupation probability

(note)

$$E'_{\rm BCS} = \langle BCS|H'|BCS\rangle \sim 2\sum_{\nu>0}(\epsilon_\nu - \lambda)v_\nu^2 - \frac{\Delta^2}{G}$$

(note) $\left(1 + \frac{v_\nu}{u_\nu} a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle = \exp\left(\frac{v_\nu}{u_\nu} a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle$

$$|\Psi\rangle \propto \exp\left(\sum_{\nu>0}\frac{v_\nu}{u_\nu} a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle$$

(pair condensed wave function)

Gap equation

$$\begin{cases} u_\nu^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{E_\nu}\right) \\ v_\nu^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{E_\nu}\right) \end{cases}$$

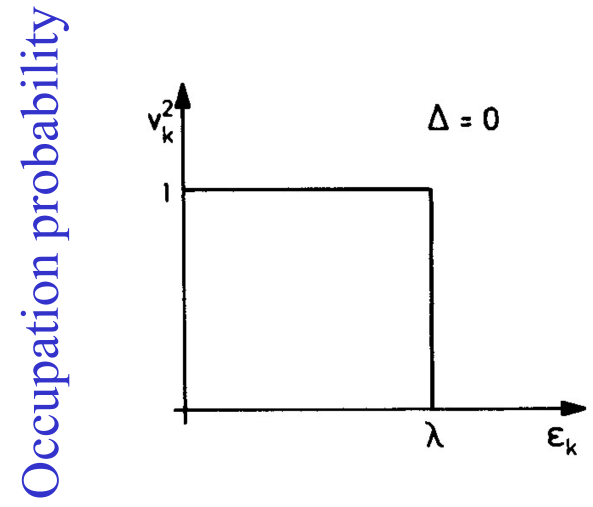
$$E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}$$

$$\begin{aligned} \Delta &= G\langle BCS|\hat{P}|BCS\rangle = G\sum_{\nu>0} u_\nu v_\nu \\ &= \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{E_\nu} \end{aligned}$$

(Gap equation)

$$N = 2\sum_{\nu>0} v_\nu^2 \qquad \leftarrow \lambda$$

$$\Delta = \frac{G}{2} \sum_{\nu>0} \frac{\Delta}{E_\nu}$$



i) Trivial solution: always exists

$$\Delta = 0$$

$$\Delta = G \sum_{\nu>0} u_\nu v_\nu$$

$$\begin{aligned} v_\nu^2 &= 1 \quad (\epsilon_\nu \leq \lambda) \\ &= 0 \quad (\epsilon_\nu > \lambda) \end{aligned}$$

$$|\Psi\rangle = \prod_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger |0\rangle$$

$$\Delta = \frac{G}{2} \sum_{\nu > 0} \frac{\Delta}{E_\nu}$$

i) Trivial solution: always exists

$$\Delta = 0$$

G a/o N $\longrightarrow$ large

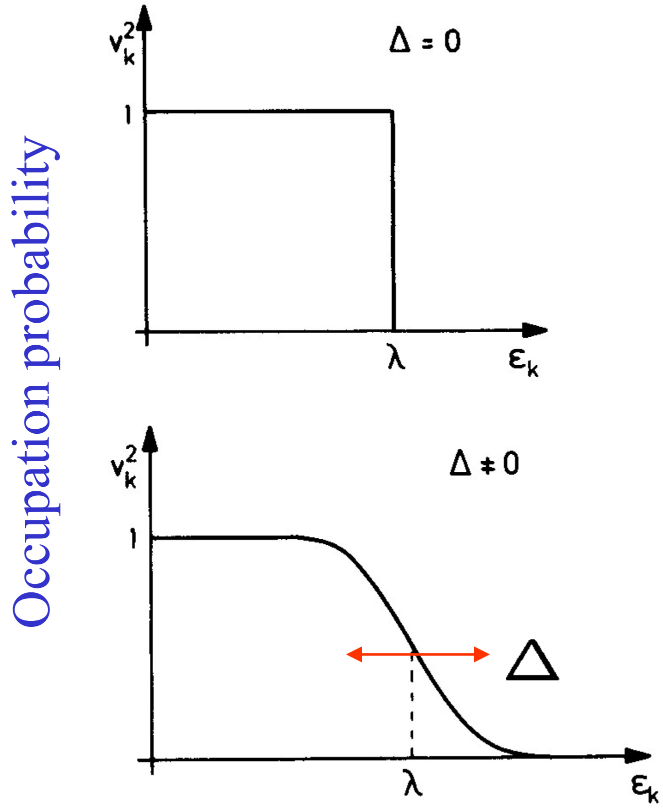
ii) Superfluid solution

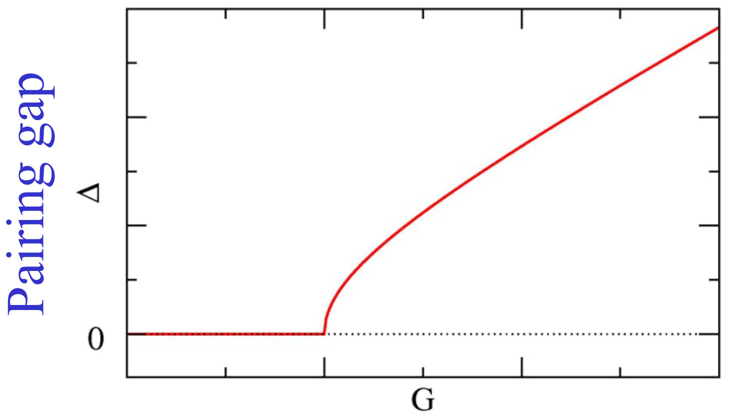
$$\Delta \neq 0$$

$$v_\nu^2 < 1$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu \, a_\nu^\dagger a_{\bar{\nu}}^\dagger \right) \Big| 0 \Big\rangle$$

Number fluctuation





Normal-Superfulid phase transition

Quasi-particle excitations

$$H = \sum_{\nu} \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \left(\sum_{k>0} a_k^\dagger a_{\bar{k}}^\dagger \right) \left(\sum_{k>0} a_{\bar{k}} a_k \right)$$

ハミルトニアンを書き直すと:

$$H \sim E_{BCS} + \sum_k E_k \, \alpha_k^\dagger \alpha_k$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

（ボゴリューボフ変換）

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_k E_k \, \alpha_k^\dagger \alpha_k$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

（ボゴリューボフ変換）

基底状態: $|\mathrm{BCS}\rangle$

1準粒子状態: $\alpha_k^\dagger|\mathrm{BCS}\rangle$ 　　奇核に対応

2準粒子状態: $\alpha_k^\dagger \alpha_{k'}^\dagger|\mathrm{BCS}\rangle$

- N +/- 2 の原子核
- 同じ原子核の励起状態に対応

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_k E_k \alpha_k^\dagger \alpha_k$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

（ボゴリューボフ変換）

基底状態: $|\mathrm{BCS}\rangle$

1準粒子状態: $\alpha_k^\dagger|\mathrm{BCS}\rangle$ 　奇核に対応

2準粒子状態: $\alpha_k^\dagger \alpha_{k'}^\dagger|\mathrm{BCS}\rangle$

- N +/- 2 の原子核
- 同じ原子核の励起状態に対応

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2} \geq \Delta$$

（エネルギー・ギャップ）

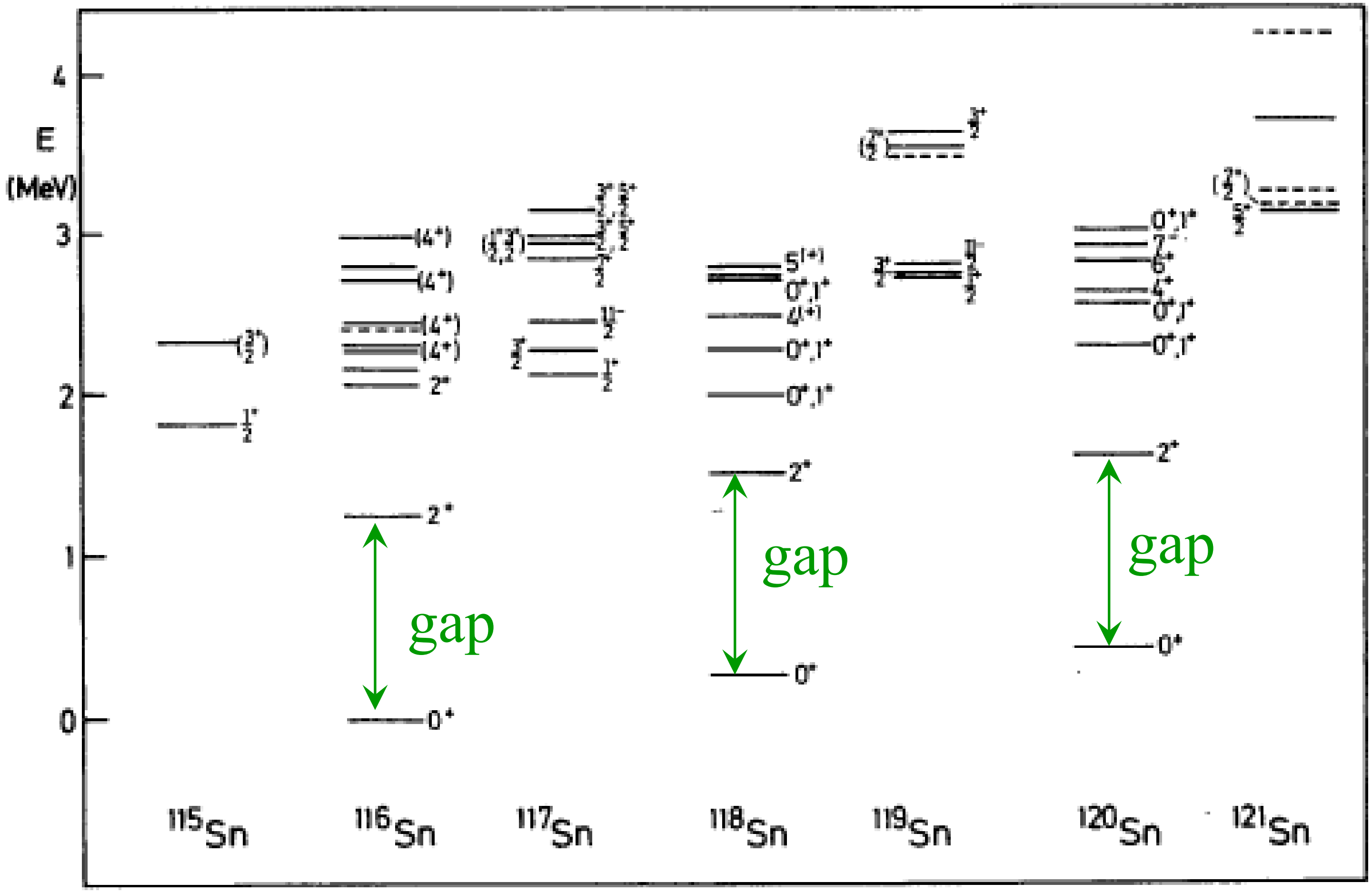


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Ring-Schuck

Effects of pairing on moment of inertia

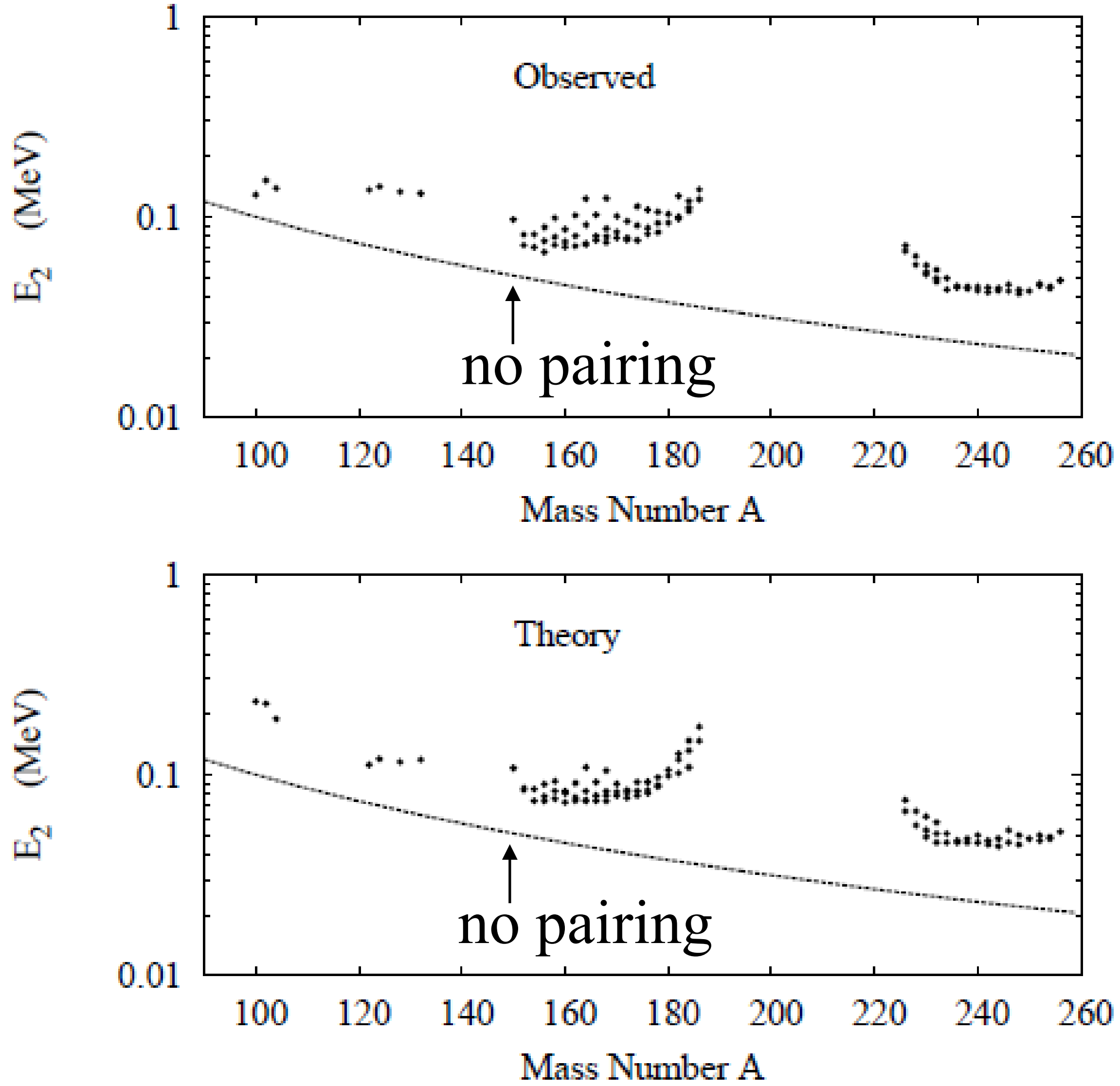


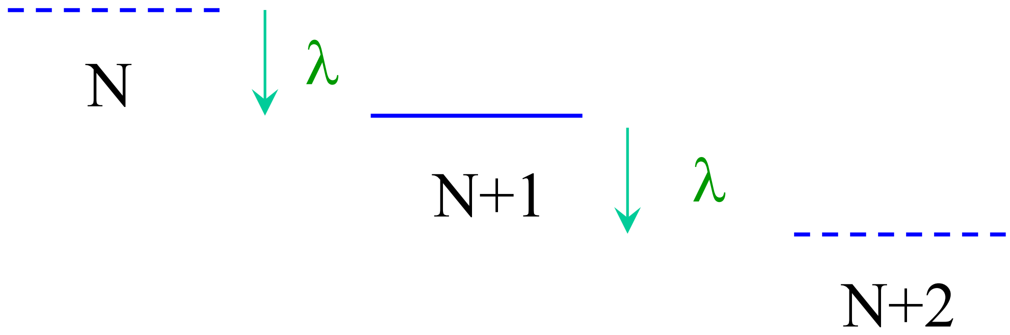
Fig. 9. Excitation energy of the first 2^+ state in deformed nuclei. The line shows the prediction assuming a rigid rotor.

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

G.F. Bertsch, in "Fifty years of nuclear BCS"

Even-odd mass difference and pairing gap

$$E(N+2,Z) = E(N,Z)+2\lambda$$
$$E(N+1,Z) = E(N,Z)+\lambda+\Delta$$

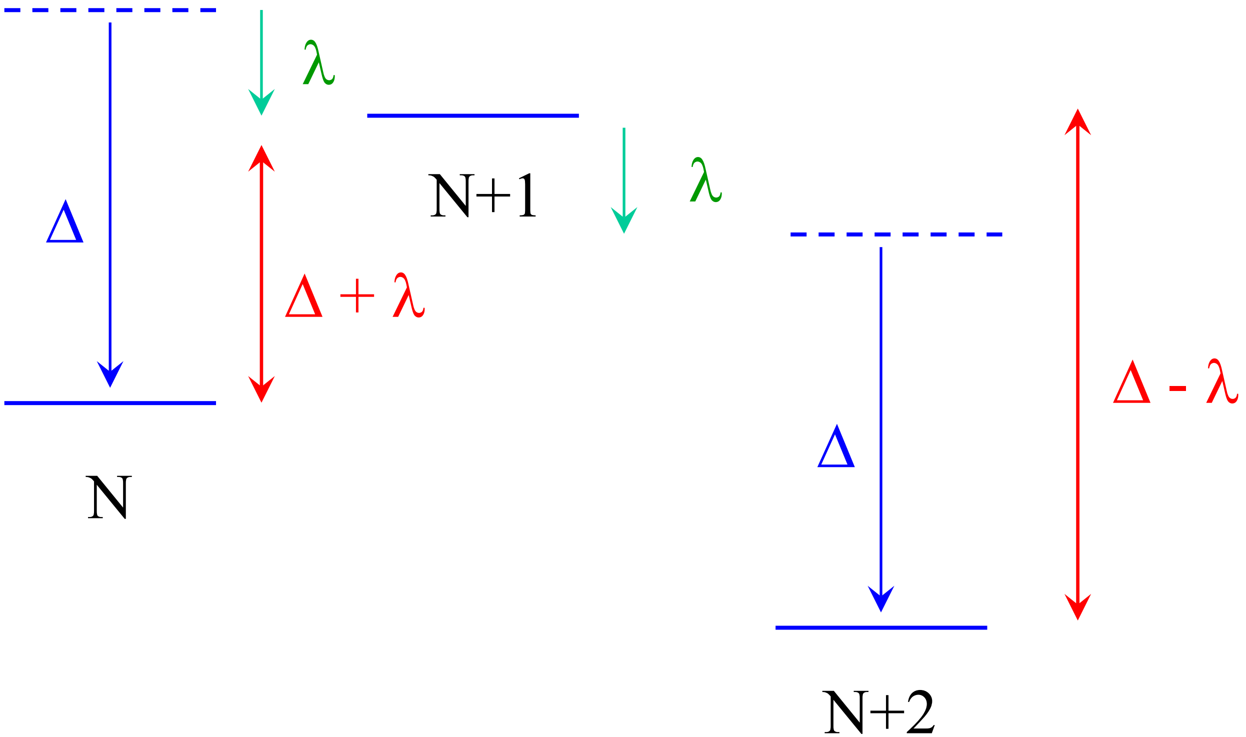


(note) $\lambda < 0$

$$-\Delta_n \sim [E(N+2,Z)-2E(N+1,Z)+E(N,Z)]/2$$

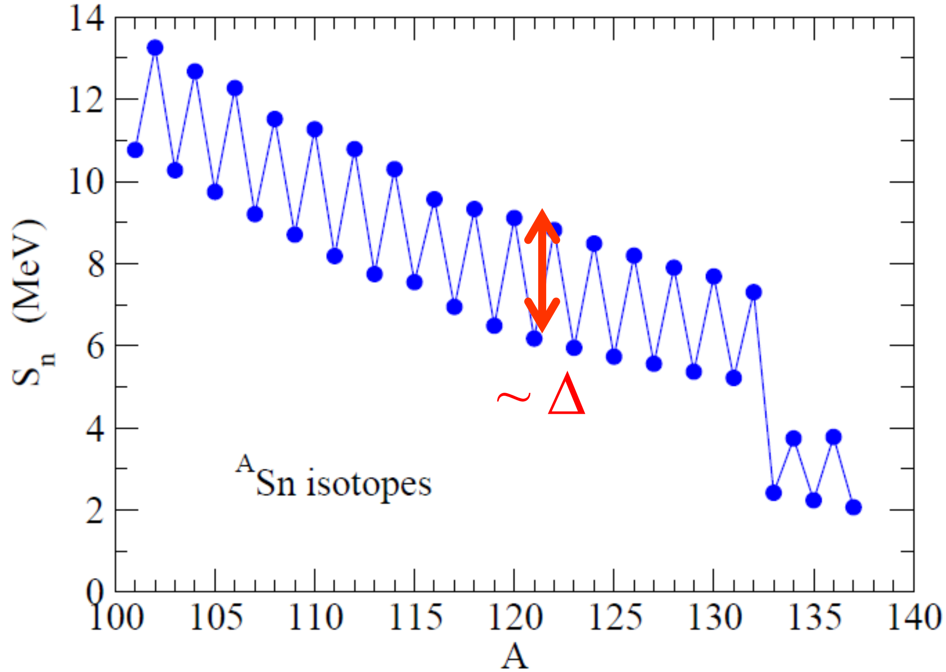
Even-odd mass difference and pairing gap

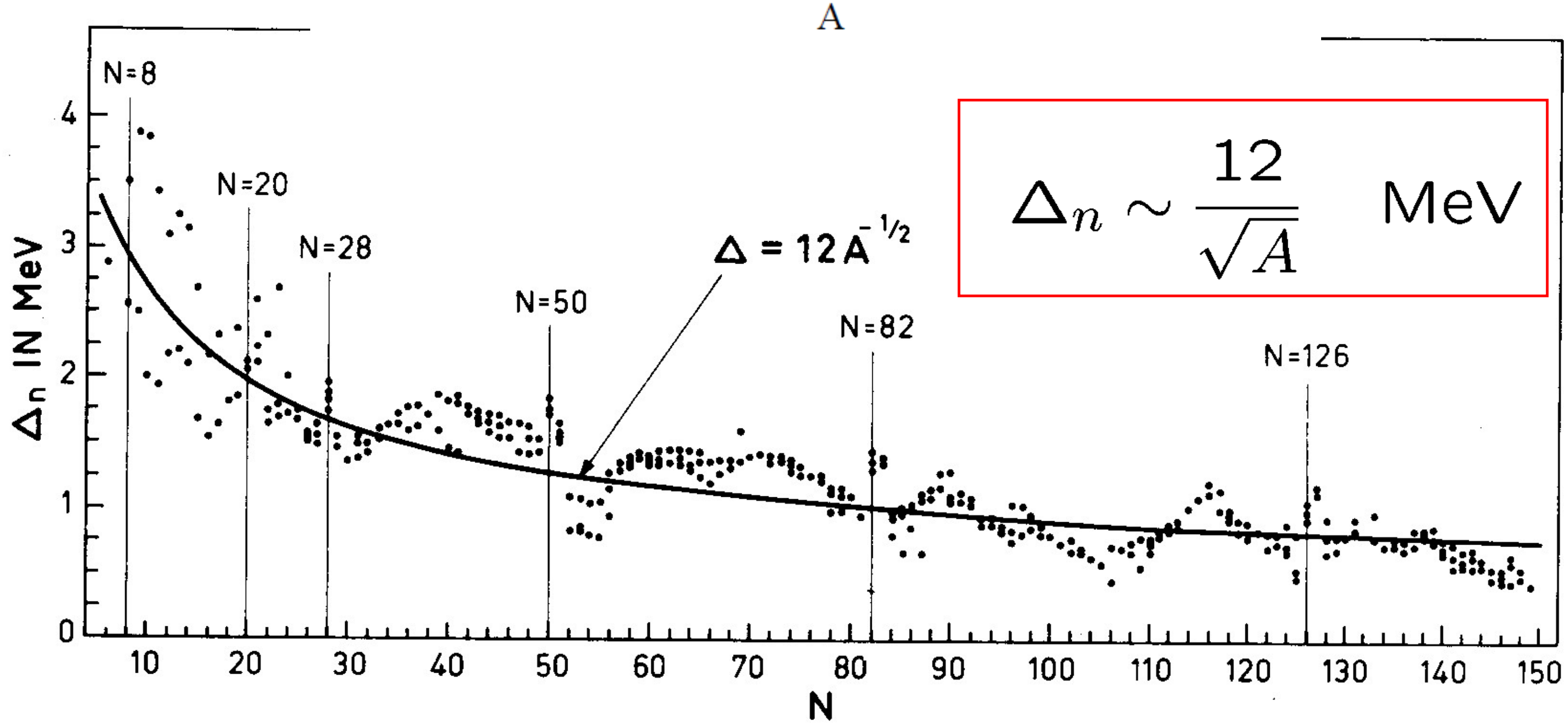
$$E(N+2,Z) = E(N,Z) + 2\lambda$$
$$E(N+1,Z) = E(N,Z) + \lambda + \Delta$$



(note) $\lambda < 0$

$$-\Delta_n \sim [E(N+2,Z) - 2E(N+1,Z) + E(N,Z)]/2$$





$$\Delta_n \sim \frac{12}{\sqrt{A}} \quad \text{MeV}$$

粒子数射影法

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|\,0\Big\rangle$$

様々な粒子数の状態が混ざっている $\quad |BCS\rangle = \sum_{N_k} C_{N_k} |N_k\rangle$

ただし、平均値だけは正しく設定されている:

$$\langle BCS|\hat{N}|BCS\rangle = N$$

粒子数射影: $\quad \hat{P}_N|BCS\rangle = C_N|N\rangle$

$$\hat{P}_N = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i(\hat{N}-N)\phi}$$

Hartree-Fock-Bogoliubov (HFB) Theory

HF+BCS 法: 2ステップ
（まず平均場を求め、次に占有確率）

$$\psi_k(\boldsymbol{r}), u_k, v_k$$

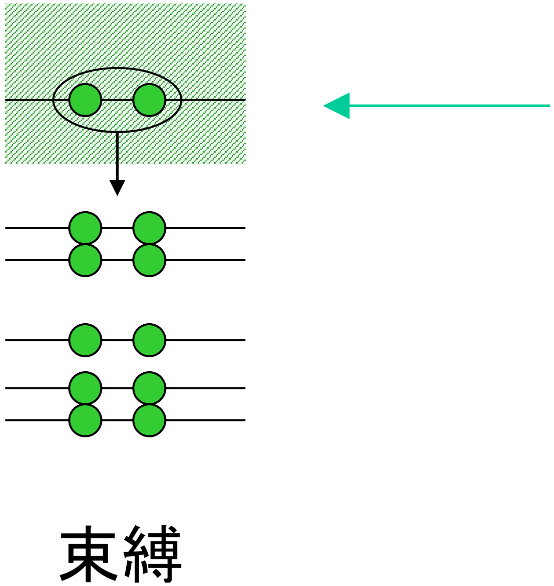
→ 改良：両方同時に行う
Hartree-Fock-Bogoliubov (HFB) theory:

波動関数と占有確率を同時に求める

$$U_k(\boldsymbol{r}), V_k(\boldsymbol{r})$$

cf. weakly bound systems
（ガスの問題）

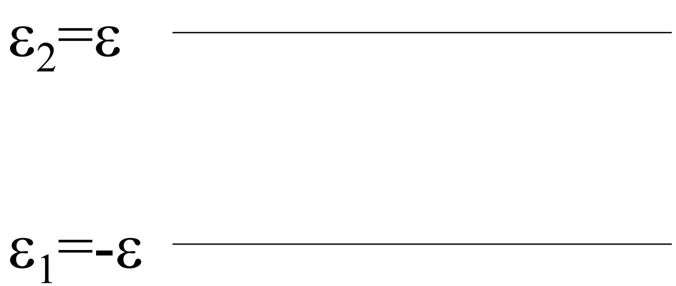
$$\begin{pmatrix} \hat{h}(\boldsymbol{r}) - \lambda & \tilde{\Delta}(\boldsymbol{r}) \\ \tilde{\Delta}(\boldsymbol{r})^* & -\hat{h}(\boldsymbol{r}) + \lambda \end{pmatrix} \begin{pmatrix} U_k(\boldsymbol{r}) \\ V_k(\boldsymbol{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\boldsymbol{r}) \\ V_k(\boldsymbol{r}) \end{pmatrix}$$



BCS法だと散乱状態をそのまま占有させるので、中性子が抜けていく（束縛核のまわりに中性子のガスができる）。

HFB法だと全体で束縛するということがもともと取り入れられているので中性子ガスは発生しない。

レポート問題6



左の図のように、ε と $-\varepsilon$ のエネルギーを持つ2つの準位があるとする。それぞれの準位はともに角運動量 j を持ち、$2j+1$ 重に縮退しているとする。系の粒子数が

$$N = 2j + 1 \equiv 2\Omega$$

で与えられているときにハミルトニアン

$$H = \sum_{\nu} \epsilon_{\nu}(a_{\nu}^{\dagger}a_{\nu} + a_{\bar{\nu}}^{\dagger}a_{\bar{\nu}}) - G\left(\sum_{\nu>0} a_{\nu}^{\dagger}a_{\bar{\nu}}^{\dagger}\right)\left(\sum_{\nu>0} a_{\bar{\nu}}a_{\nu}\right)$$

をBCS近似で解き、それぞれの準位に対する占有確率、およびペアリングギャップ Δ を求めよ。ただし、BCS近似の式は既知のものとして使ってもよい。G の大小によって、場合分けをして答えよ。

（注）対称性から、化学ポテンシャル λ は $\lambda=0$ となる。

（注2）それぞれの準位には、j とその z 成分 m でラベルされる状態が $2j+1$個ある $(m=-j,\ldots,j)$ 。ハミルトニアンのなかで $\nu > 0$ は $m>0$ をとるという意味。すなわち、

$$\sum_{\nu>0} a_{\nu}^{\dagger}a_{\bar{\nu}}^{\dagger} = \sum_{k=1,2}\sum_{m_k>0} a_{j_k m_k}^{\dagger}a_{j_k\ -m_k}^{\dagger}$$