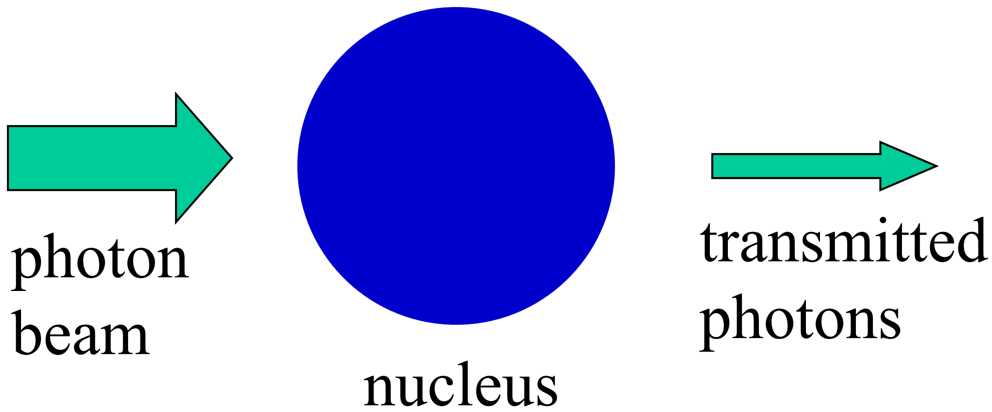
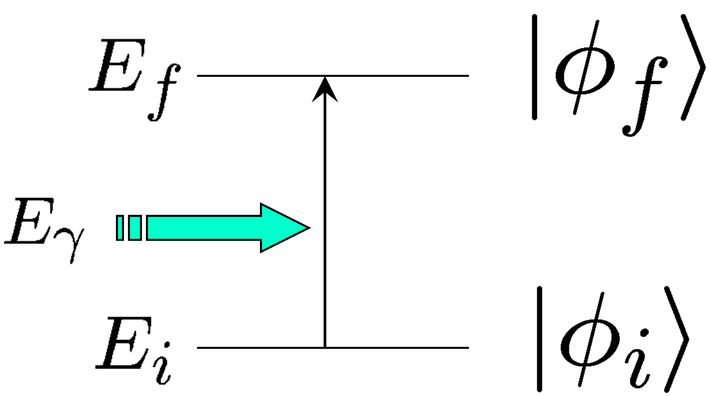
Collective Vibrations

Photo absorption cross section





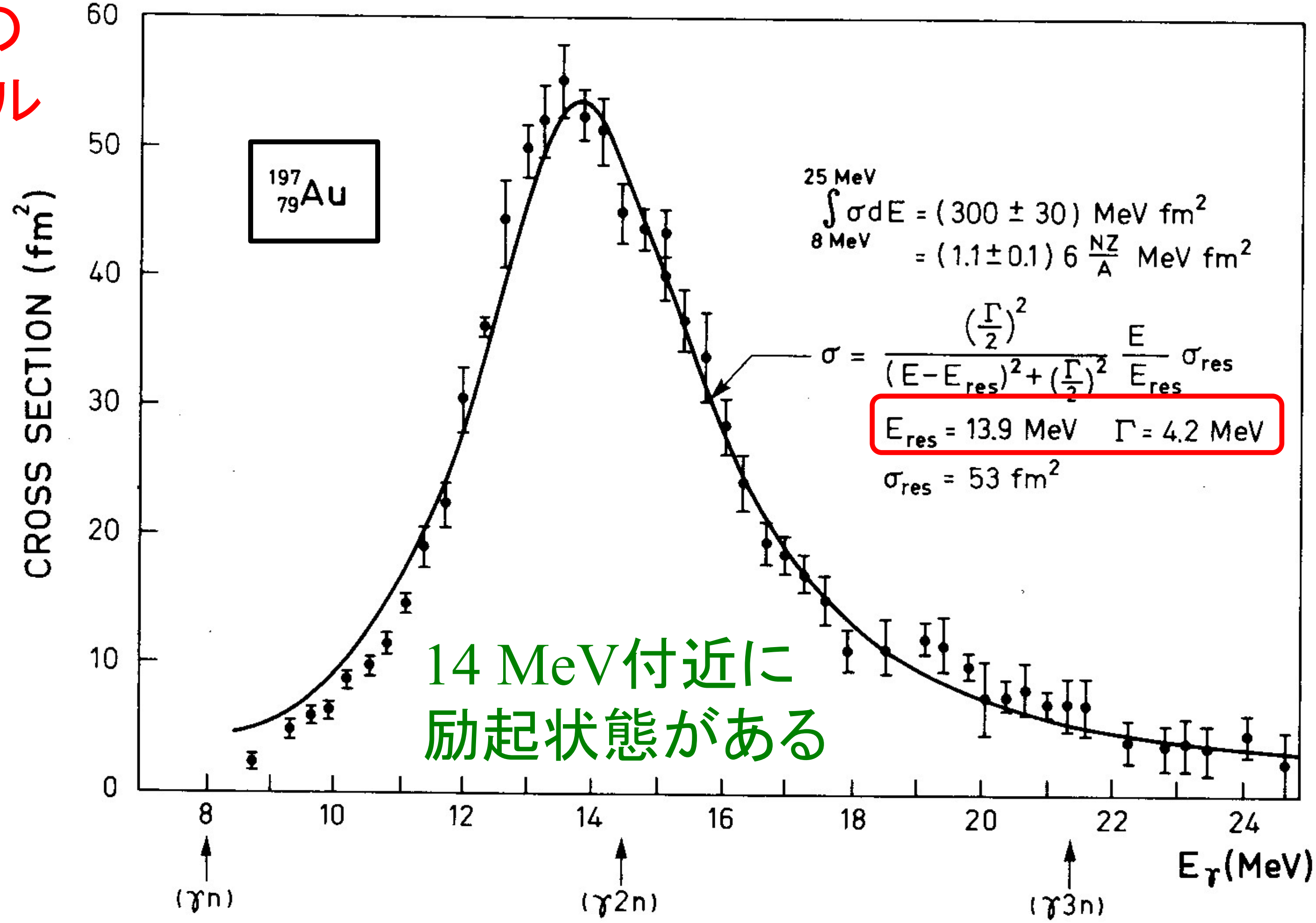
The state is strongly excited when $E_f - E_i = E_\gamma$.

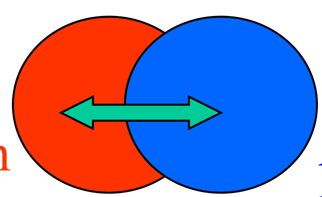
フェルミの黄金律

$$\lambda = \frac{2\pi}{\hbar}|\langle\phi_f|\hat{T}|\phi_i\rangle|^2\delta(E_f - E_i - E_\gamma)$$

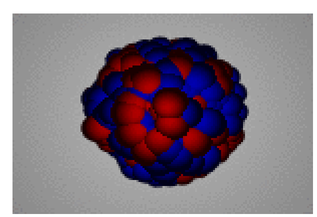
Giant Dipole Resonance (GDR)　巨大双極子共鳴

光吸収のスペクトル





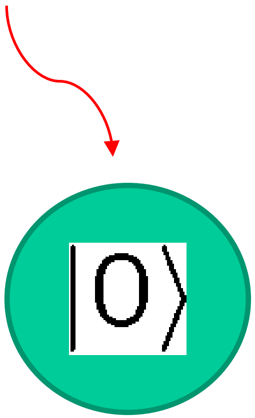
Isovector type

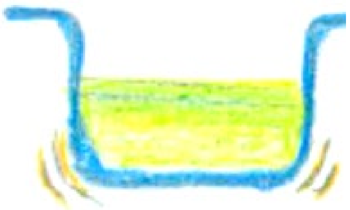


Sum Rule

$$|\psi\rangle = F|0\rangle$$

F（電磁場などの外場）





図：松柳研一氏

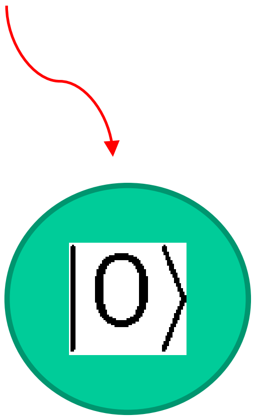
外場をかけて原子核をゆすってみる

Sum Rule

$$|\psi\rangle = F|0\rangle$$

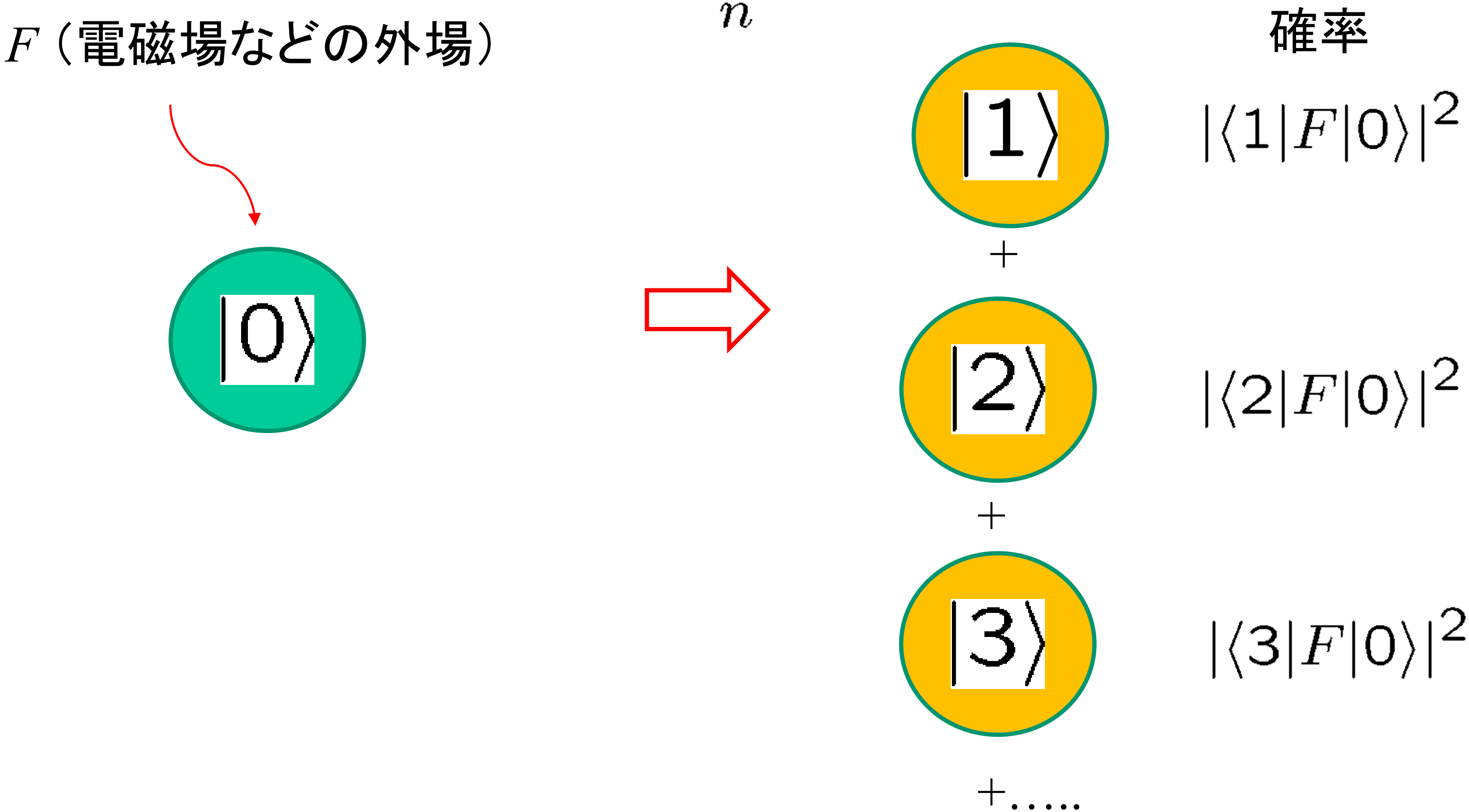
$$= \sum_n |n\rangle\langle n|F|0\rangle$$

F（電磁場などの外場）



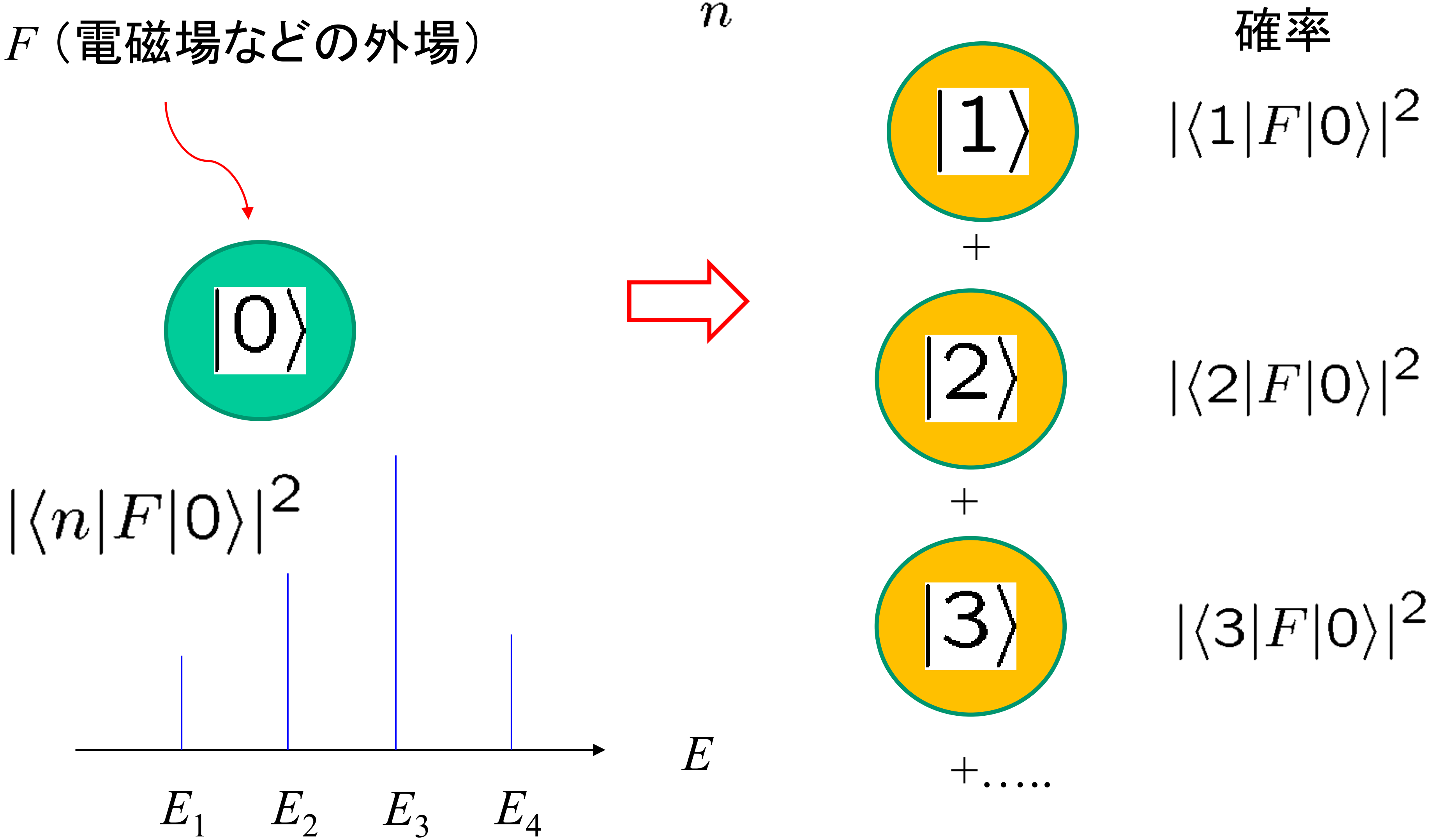
Sum Rule

$$|\psi\rangle = F|0\rangle$$
$$= \sum_n |n\rangle\langle n|F|0\rangle$$



Sum Rule

$$|\psi\rangle = F|0\rangle$$
$$= \sum_n |n\rangle\langle n|F|0\rangle$$

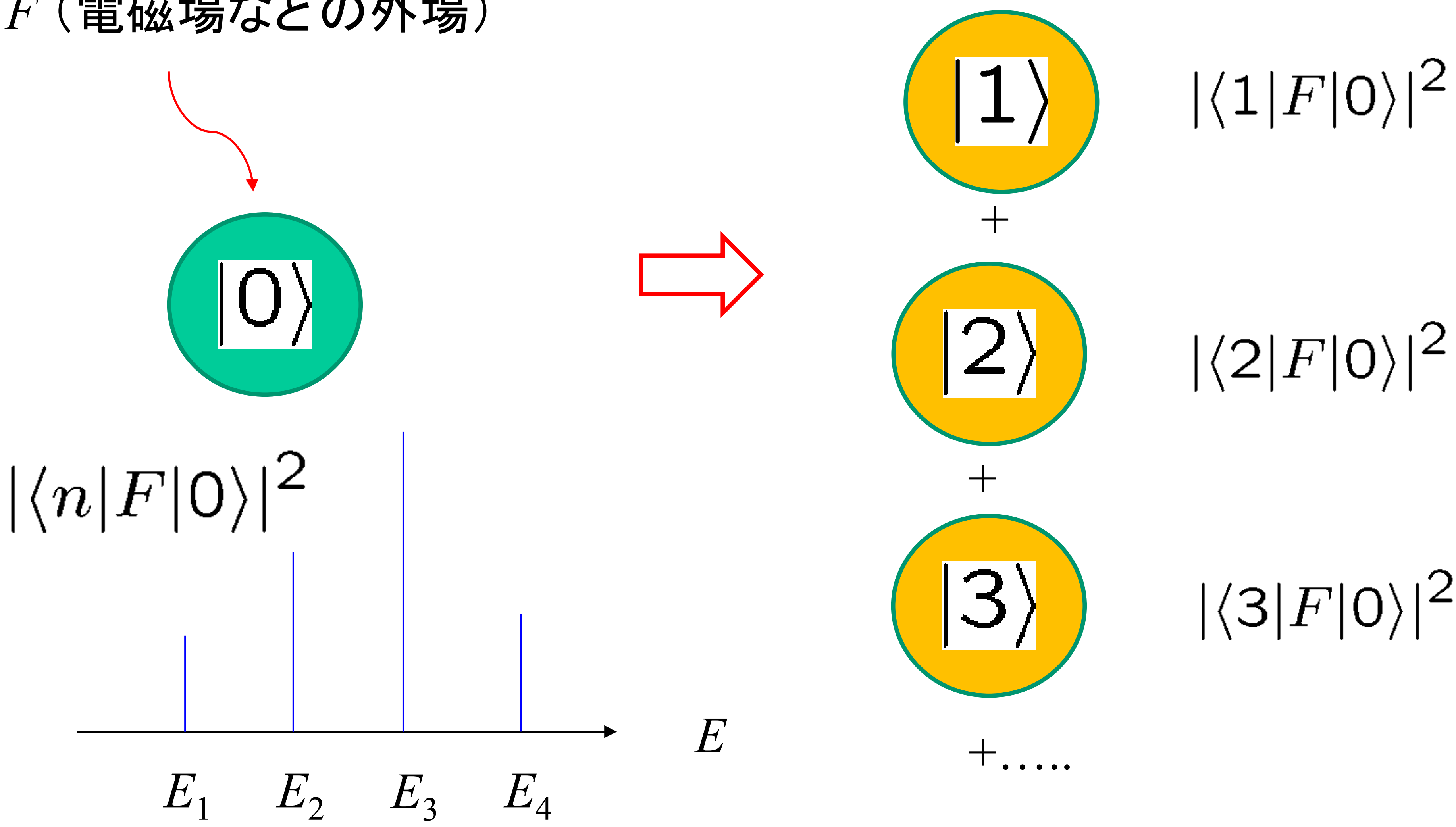


Sum Rule

Strength function:

$$S(E)=\sum_n |\langle n|F|0\rangle|^2\,\delta(E_n-E_0-E)$$

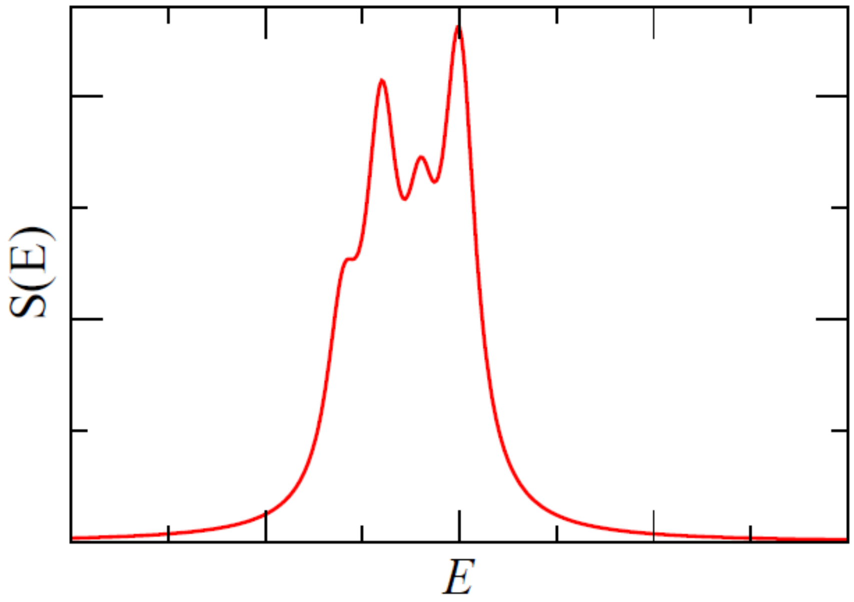
F（電磁場などの外場）



Sum Rule

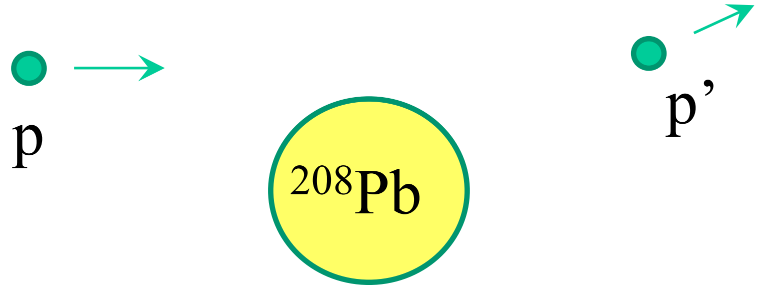
Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

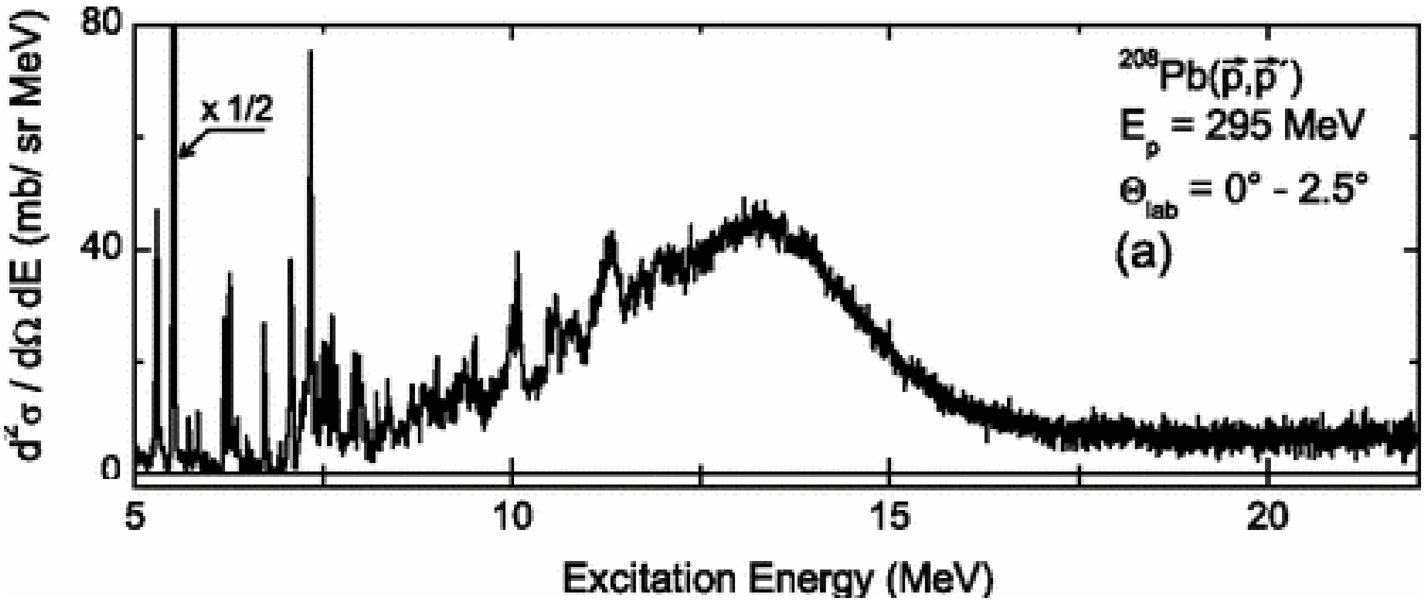


Sum Rule

例えば:



非弾性散乱(^{208}Pb の励起）のスペクトル

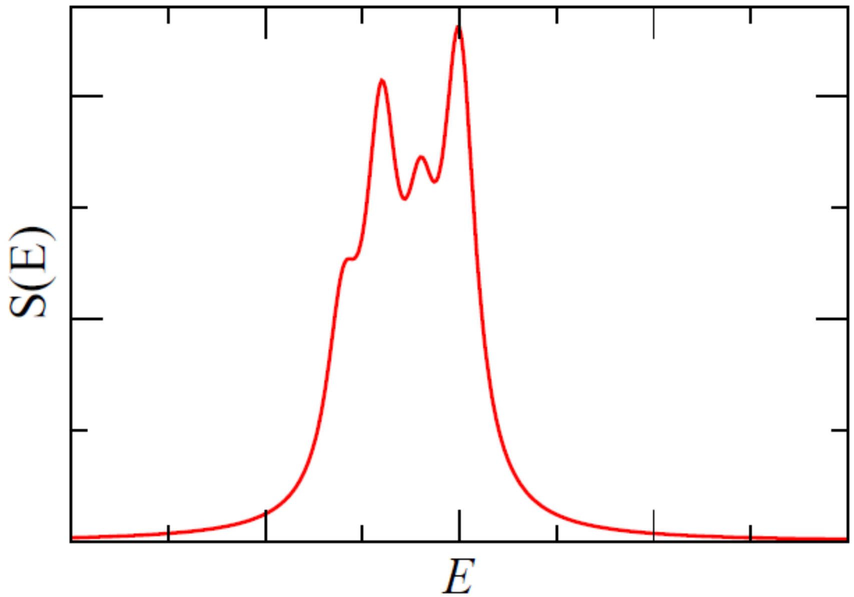


A. Tamii et al., PRL107, 062502 (2011)

Sum Rule

Strength function:

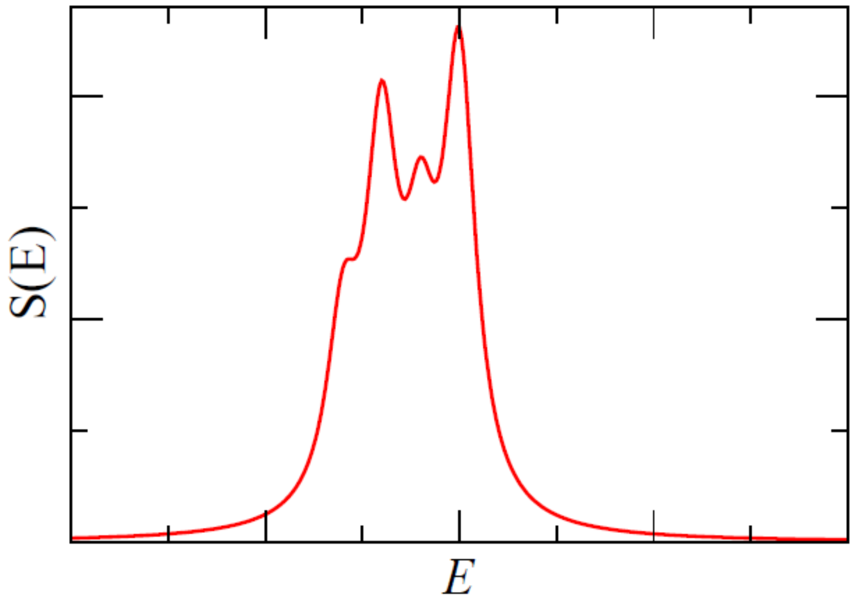
$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$



Sum Rule

Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$



強度関数のモーメント:

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$

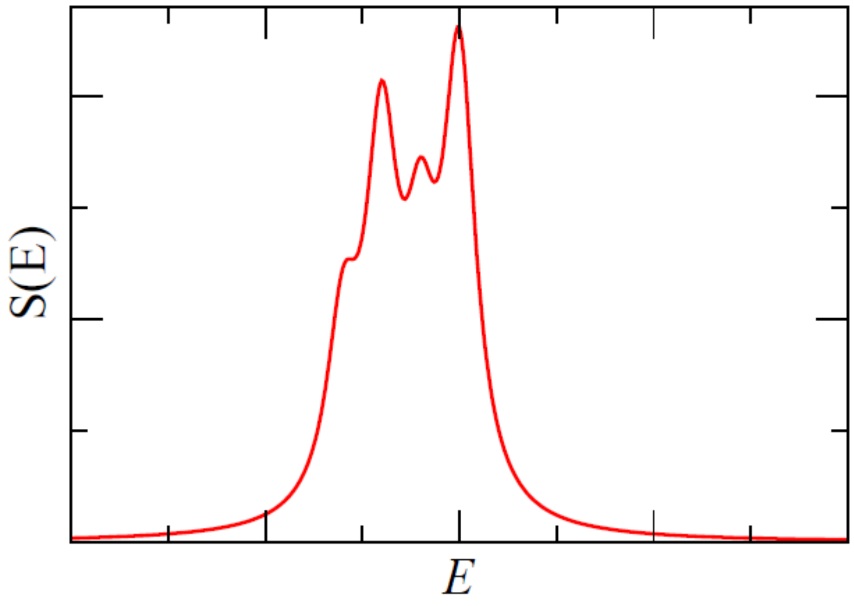
✓ energy weighted sum rule

$$S_1 \equiv \int ES(E)dE = \sum_n (E_n - E_0)|\langle n|F|0\rangle|^2$$

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

F^2の基底状態期待値

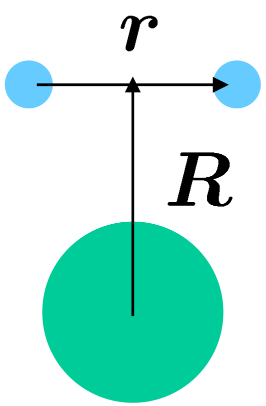


$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

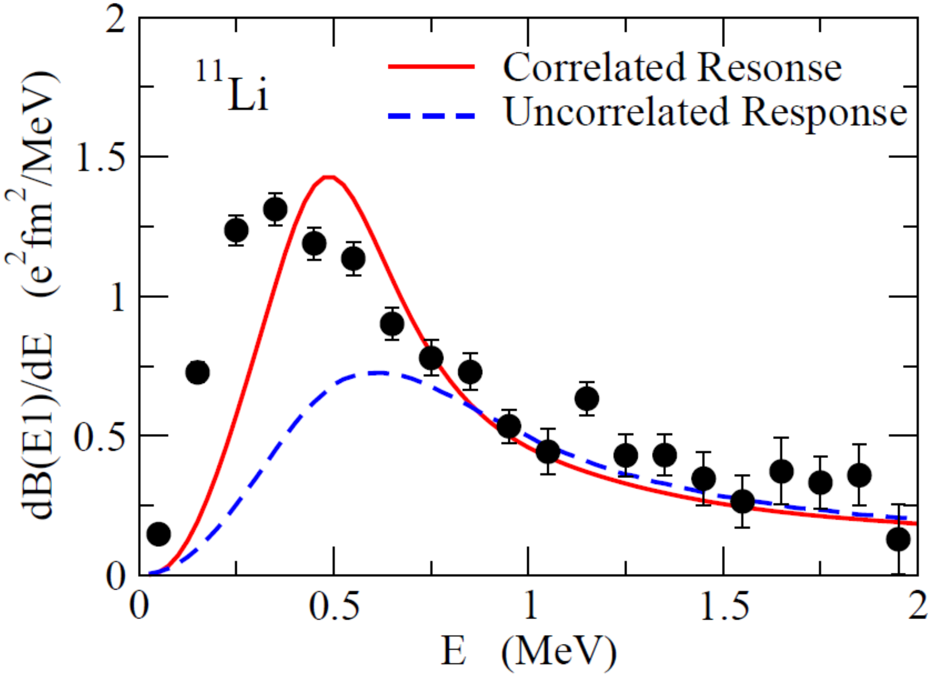
✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

F^2の基底状態期待値



cf. geometry of Borromean nuclei



experimental data:
T. Nakamura et al., PRL96('06)252502

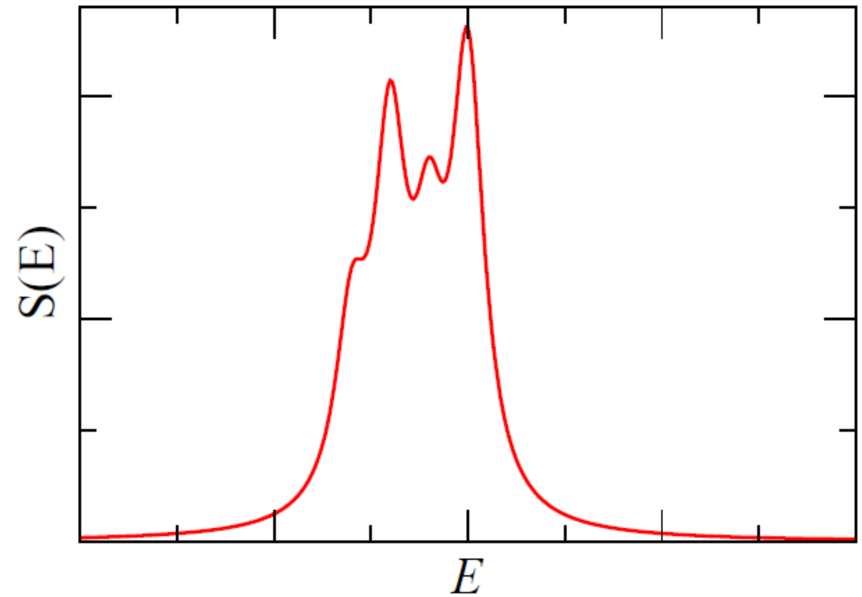
$$B(E1) = \sum_i B(E1; gs \to i)$$
$$= \frac{3}{\pi}\left(\frac{Ze}{A}\right)^2 \langle R^2\rangle$$

$$\Rightarrow \langle\theta_{nn}\rangle = 65.2^{+11.4}_{-13.0} \quad (^{11}\text{Li})$$
$$= 74.5^{+11.2}_{-13.1} \quad (^{6}\text{He})$$

K.H. and H. Sagawa,
PRC76('07)047302

✓ energy weighted sum rule

$$
\begin{aligned}
S_1 &\equiv \int E S(E) dE \\
&= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \\
&= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle
\end{aligned}
$$



$$
S(E) = \sum_\nu |\langle \nu | F | 0 \rangle|^2 \times \delta(E_\nu - E_0 - E)
$$

$$
\begin{aligned}
\frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle &= \frac{1}{2} \langle F(HF - FH) - (HF - FH)F \rangle \\
&= \langle FHF - E_0 F^2 \rangle \\
&= \sum_\nu E_\nu |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \\
&= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2
\end{aligned}
$$

Energy weighted sum rule:

$$\begin{aligned} S_1 &= \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 \\ &= \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle \end{aligned}$$

For $F = F(\boldsymbol{r})$ (local operator)

$$\begin{aligned} [H,F] &= \left[-\frac{\hbar^2}{2m}\nabla^2, F\right] \\ &= -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F\cdot\nabla) \end{aligned}$$

$\Rightarrow$ $$[F,[H,F]] = \frac{\hbar^2}{m}(\nabla F)^2$$

$\Rightarrow$ $$S_1 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

For $F=z$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|z|0\rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

⬆

Model independent

For $F = r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$

$$S_1 = \frac{\lambda(2\lambda+1)\hbar^2}{8\pi m} A\,\langle r^{2\lambda-2}\rangle$$

Photo absorption cross section:

$$\sigma_{\mathrm{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\tilde{z} = \sum_p (z_p - Z_{cm}) = \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n\right)\right\}$$

$$= \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n\right)$$

$$\int \sigma_{\mathrm{abs}}(E_\gamma)dE_\gamma = \frac{4\pi^2 e^2}{\hbar c}\cdot\frac{\hbar^2}{2m}\cdot\frac{NZ}{A}$$

$$= \frac{2\pi^2 e^2 \hbar}{mc}\cdot\frac{NZ}{A}$$

Giant Dipole Resonance (GDR)

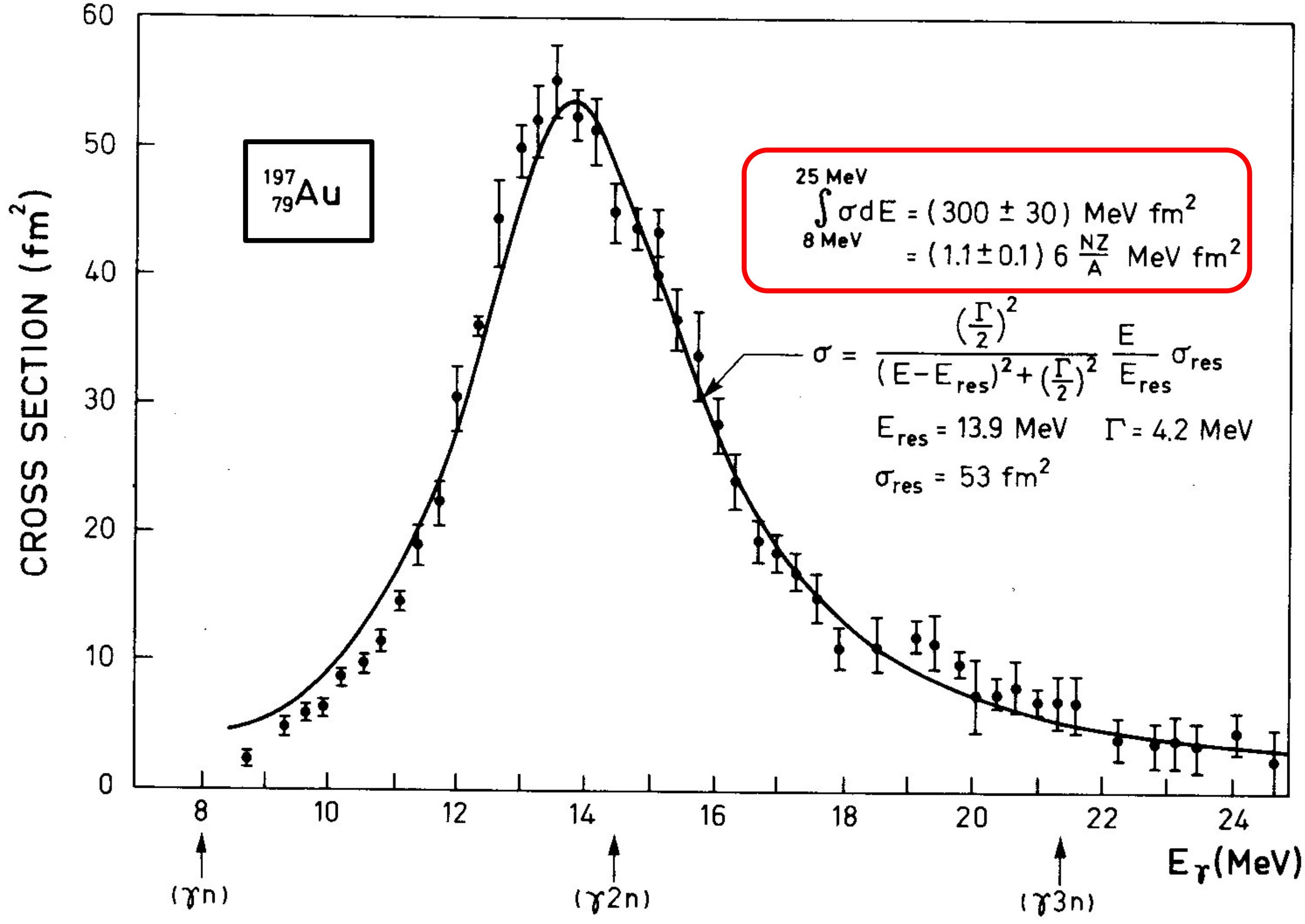


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

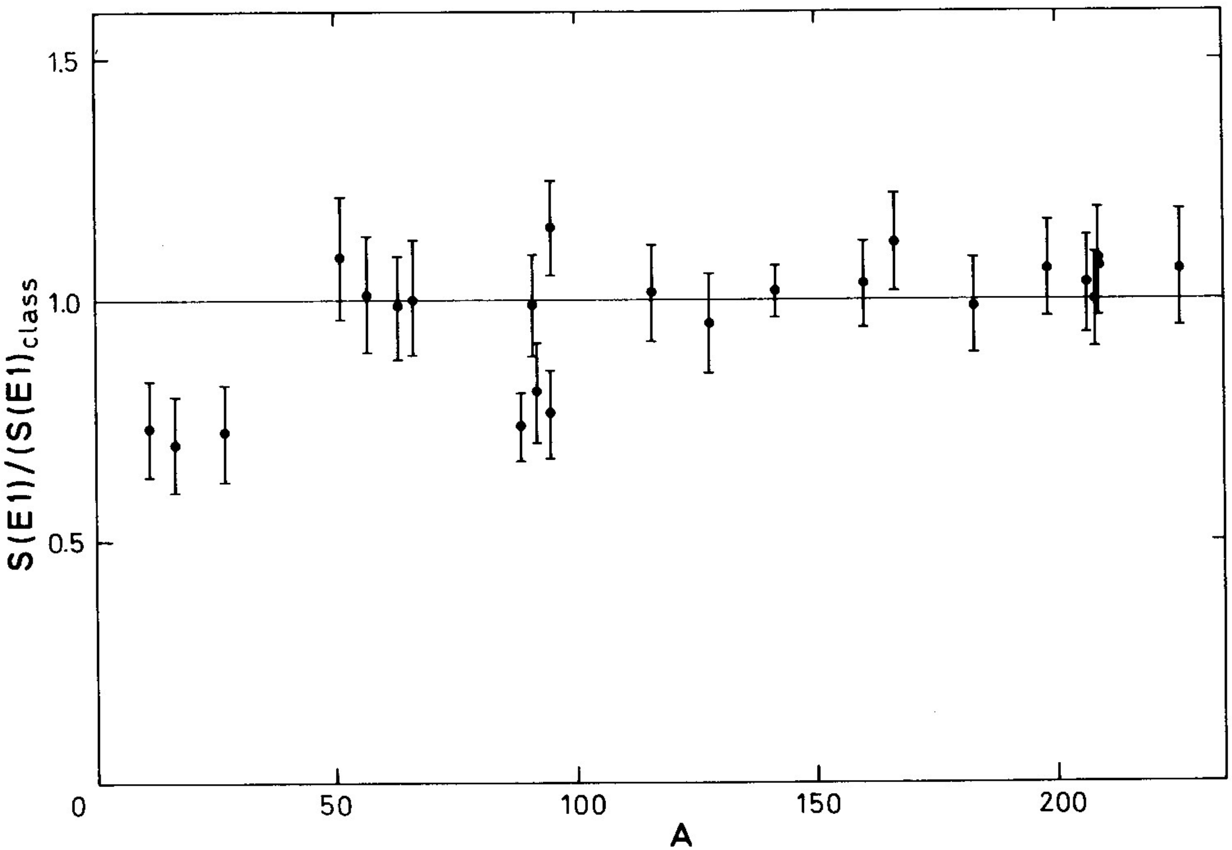


Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

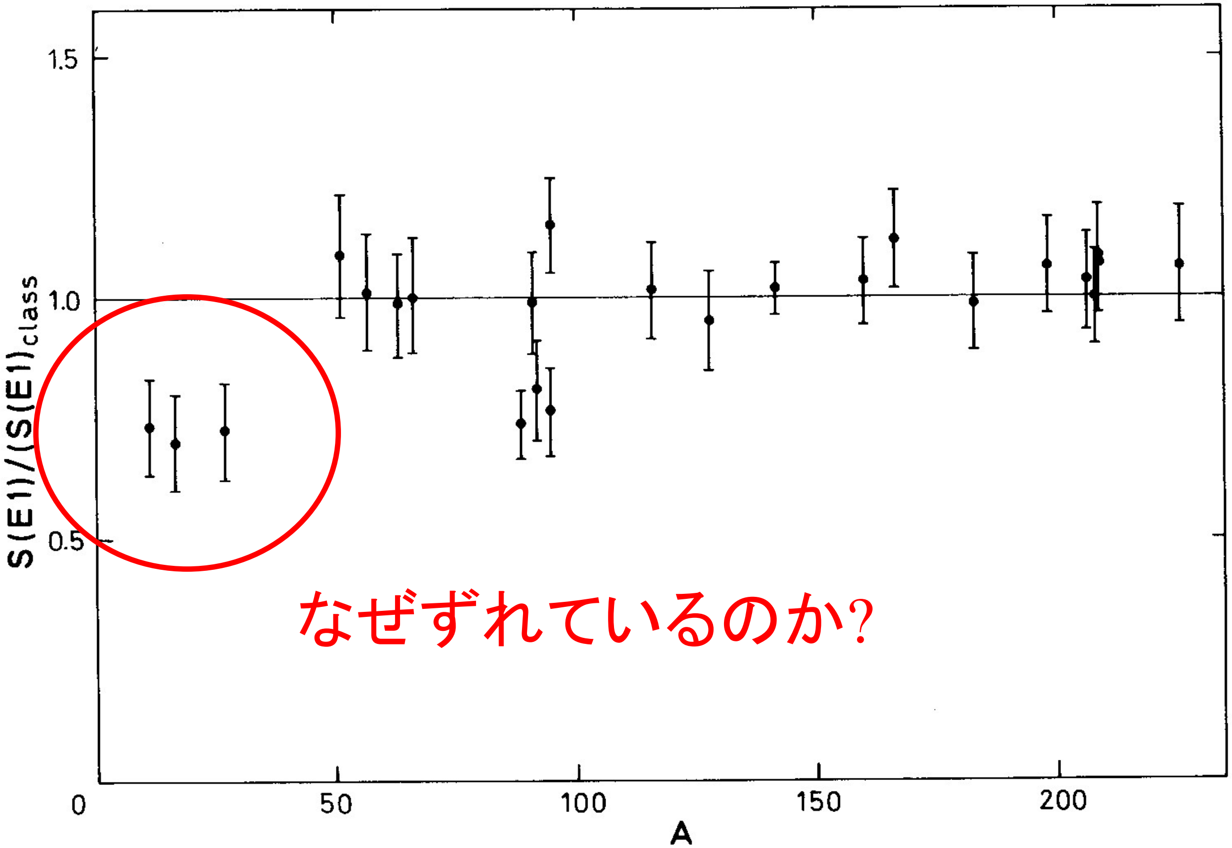
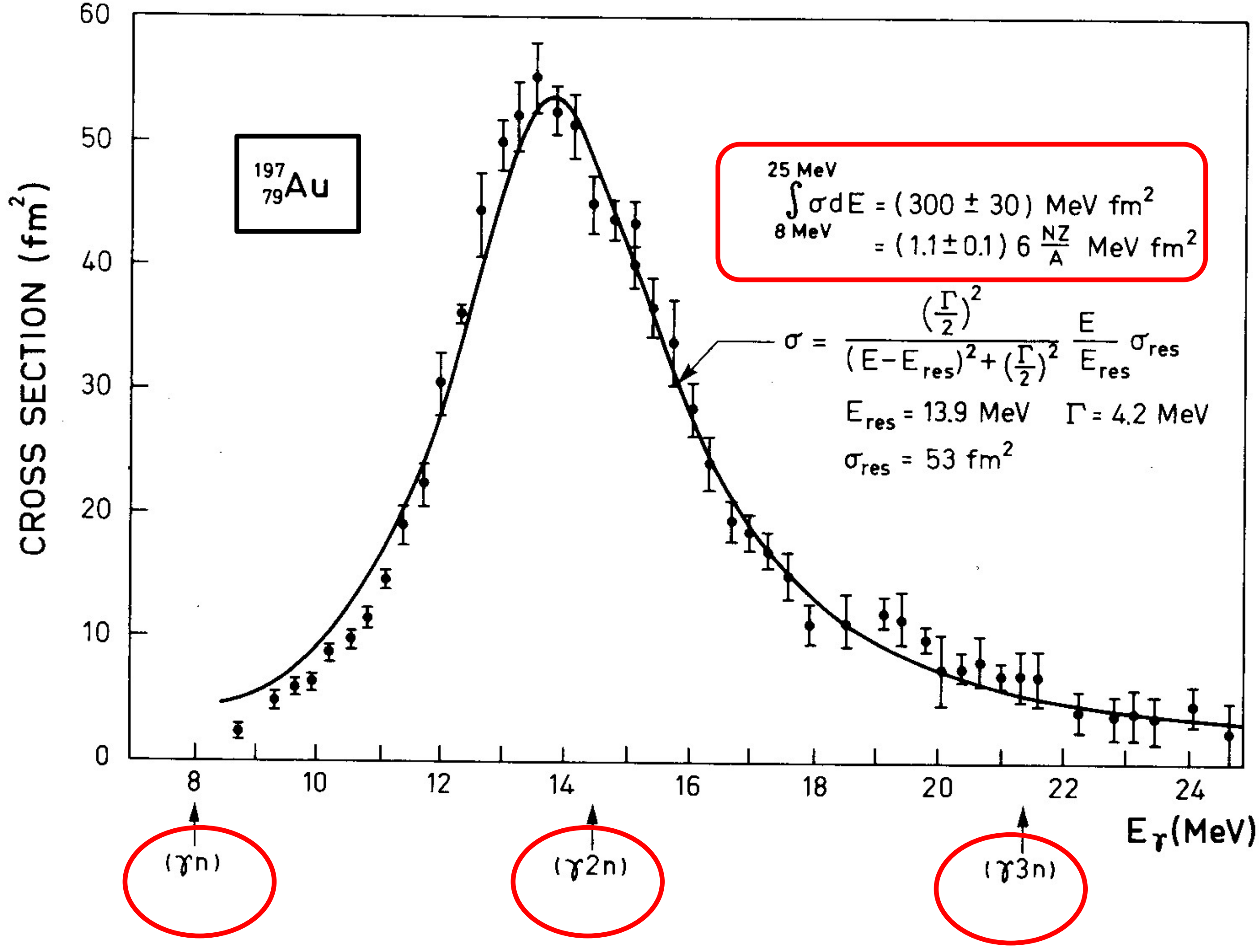


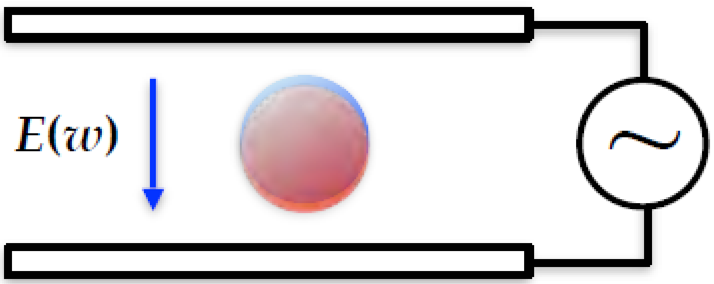
Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

Giant Dipole Resonance (GDR)



光吸収→中性子の放出を測定

分極率と inverse energy weighted sum rule



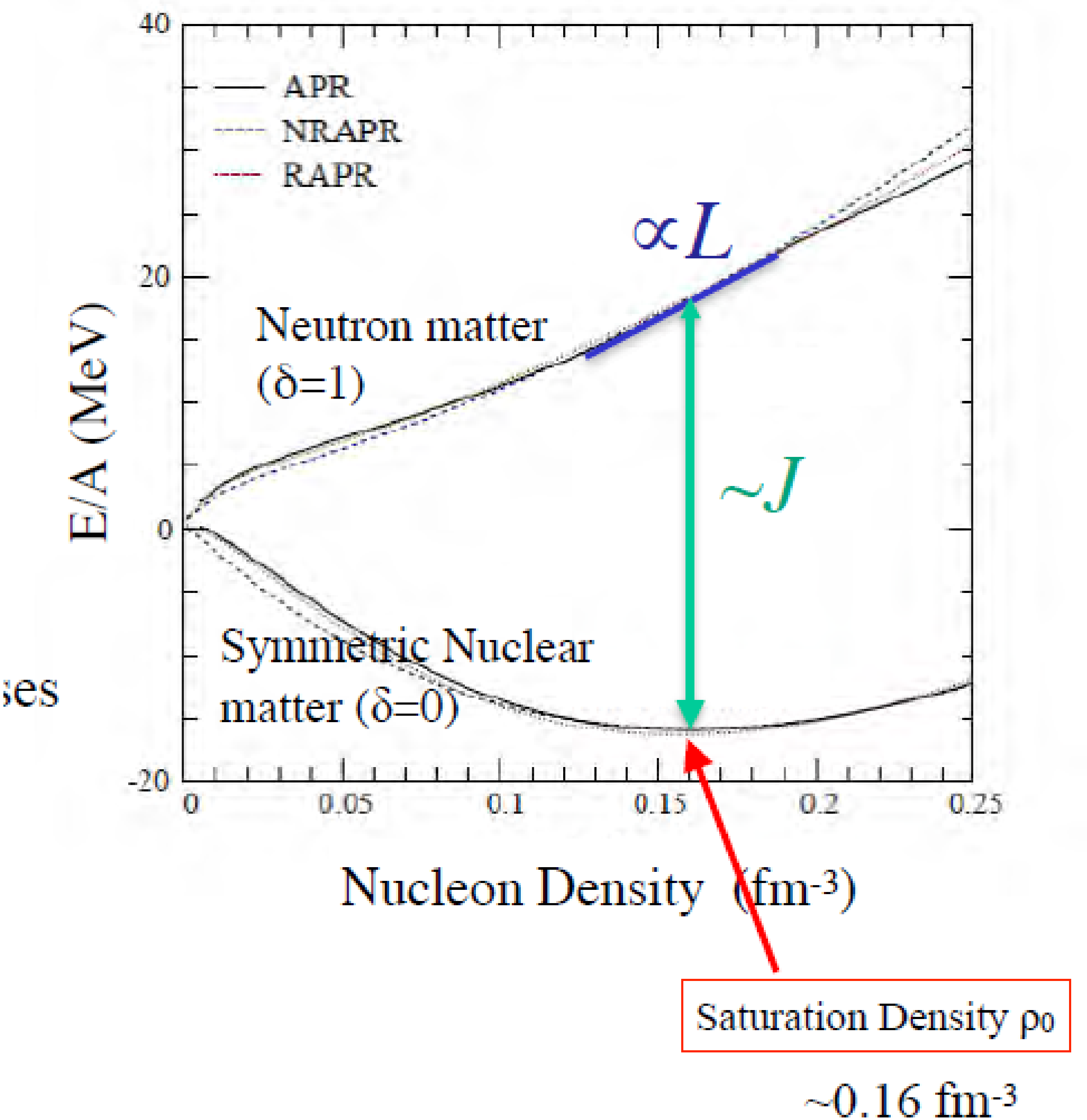
図：民井さん

原子核の静電分極率
→対称エネルギー

$$H = H_0 - \lambda F$$

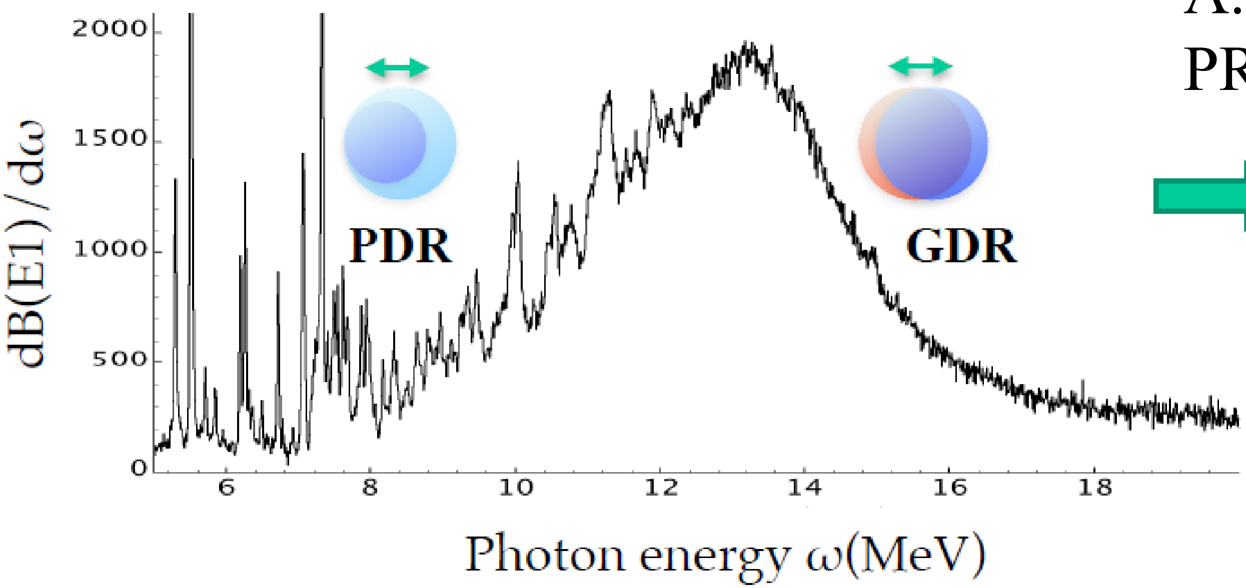
一次の摂動論：$|\tilde{\psi}_0\rangle = |\psi_0\rangle - \lambda \sum_{n>0} \frac{\langle\psi_n|F|\psi_0\rangle}{E_0 - E_n}|\psi_n\rangle$

$$\rightarrow \langle\tilde{\psi}_0|F|\tilde{\psi}_0\rangle = \langle\psi_0|F|\psi_0\rangle + \underbrace{2\sum_{n>0}\frac{|\langle\psi_n|F|\psi_0\rangle|^2}{E_n - E_0}}_{\text{分極率}}\lambda$$



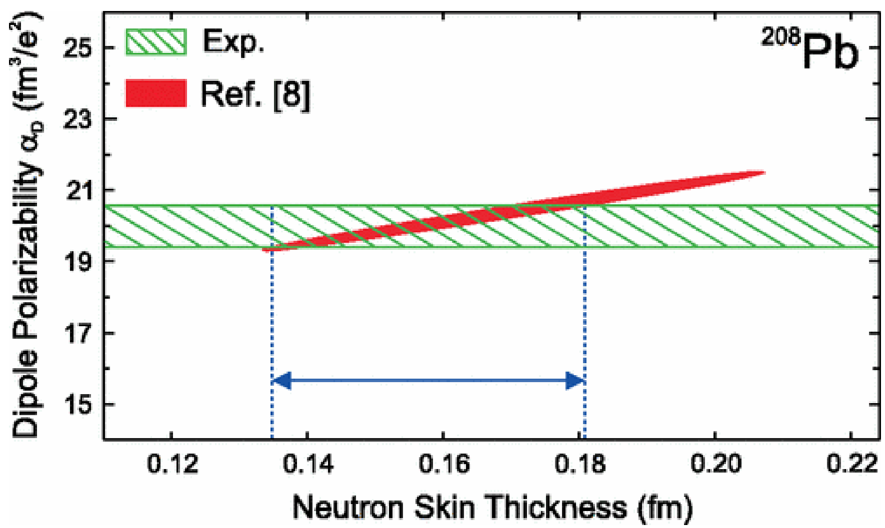
E/A (MeV)
40
20
0
-20
APR
NRAPR
RAPR
∝L
Neutron matter
(δ=1)
~J
Symmetric Nuclear
matter (δ=0)
ses
0
0.05
0.1
0.15
0.2
0.25
Nucleon Density (fm-3)
Saturation Density ρ0
~0.16 fm-3

スライド : 民井さん



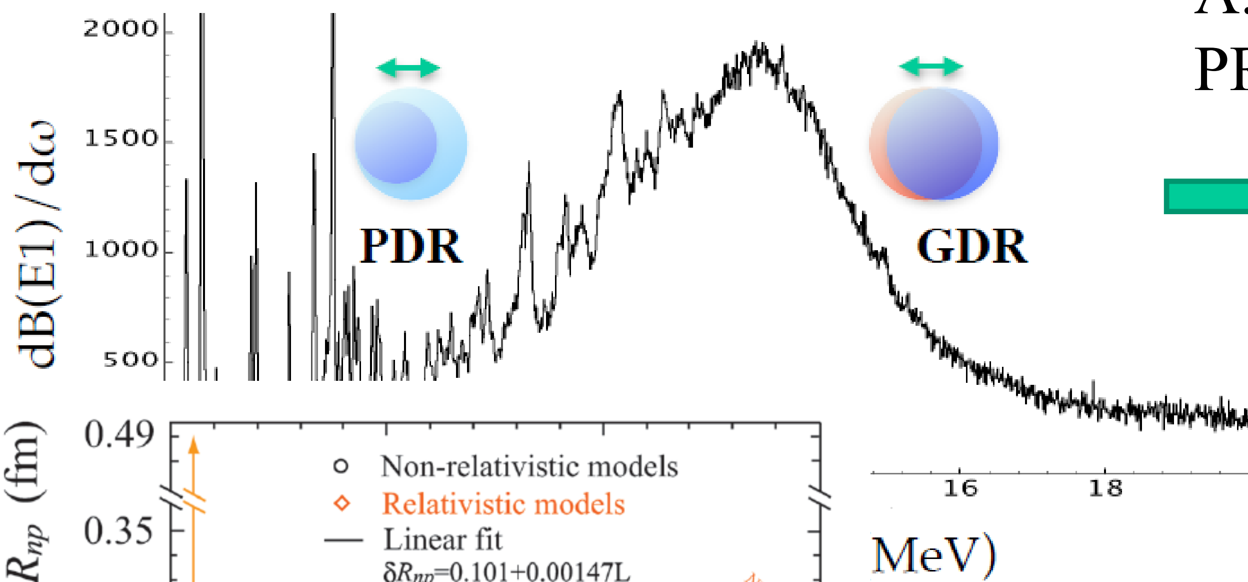
A. Tamii et al., PRL107, 062502 (2011)

$\alpha = 20.1 +/- 0.6 \text{ fm}^3$

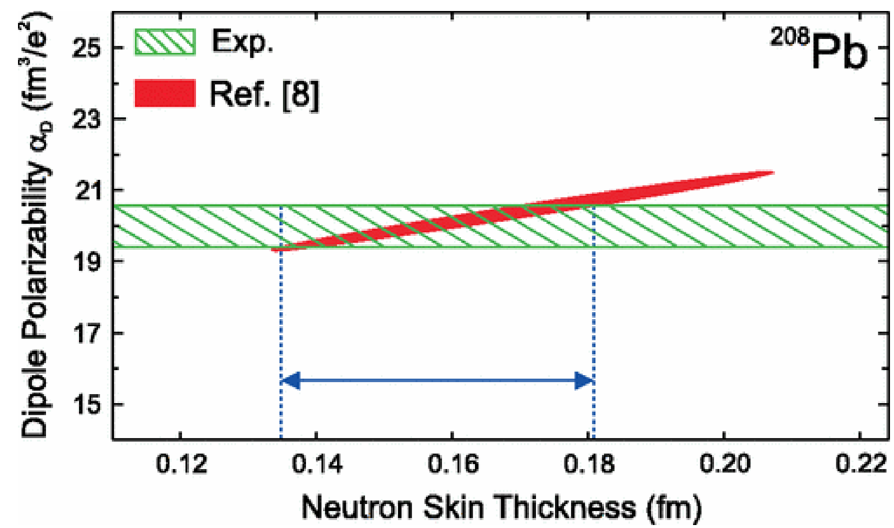


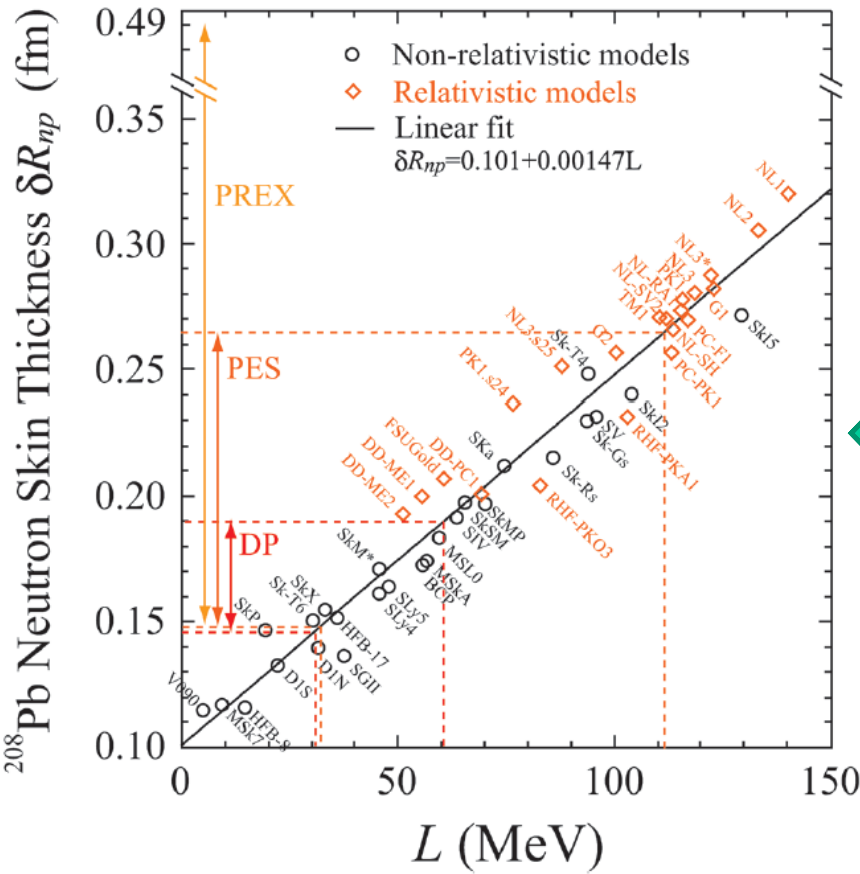
$r_{skin} = 0.156\ ^{+0025}\ _{-0.021}$ fm

A. Tamii et al.,
PRL107, 062502 (2011)



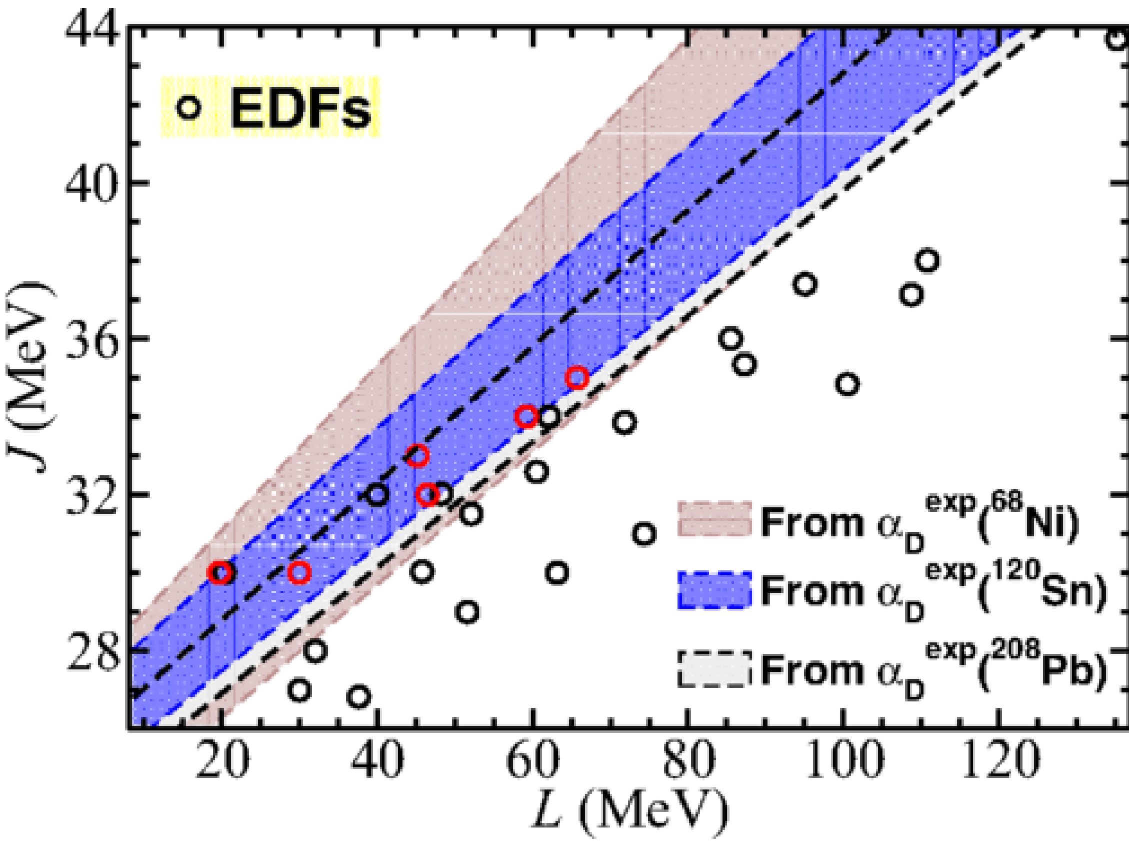
$\alpha = 20.1 +/- 0.6\ \mathrm{fm}^3$

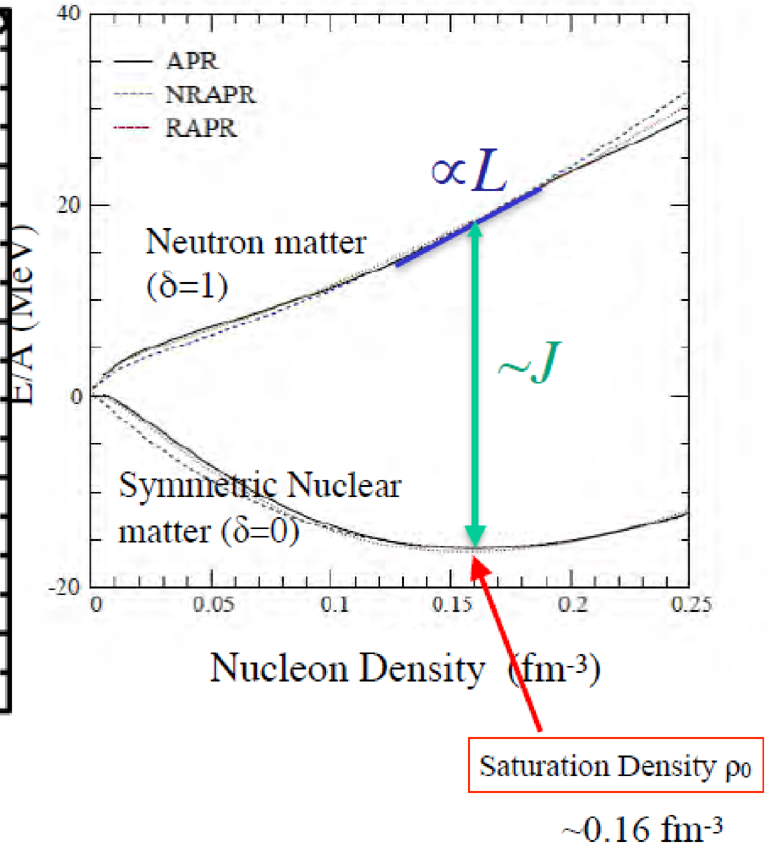




$r_{skin} = 0.156^{+0025}_{-0.021}$ fm

民井、銭廣(日本物理学会誌)



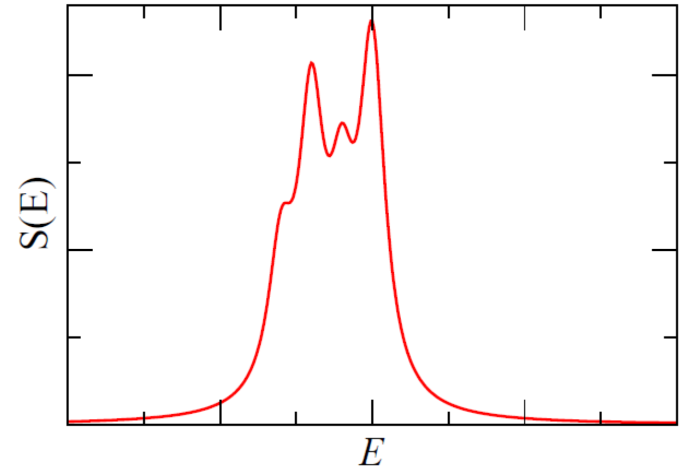


X. Roca-Maza et al.,
PRC92, 064303 (2015)

スライド：民井さん

和則の利点

$$S_0 = \langle 0|F^2|0\rangle$$
$$S_1 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$



和則:
励起状態の(ある種の)情報が基底状態の
性質のみによって表わされる
(励起状態の情報を知っている必要がない)。

- 実験で強度分布が測られた時、測られた範囲外にも強度があるかどうか (missing strength) 判断できる。
- 強度分布を測ることによって原子核の半径などの情報を得られる。
- 実験データや数値計算のチェックになる。
(和則の値よりとても大きくなると何かがおかしい)。

Giant Quadrupole Resonance (GQR)

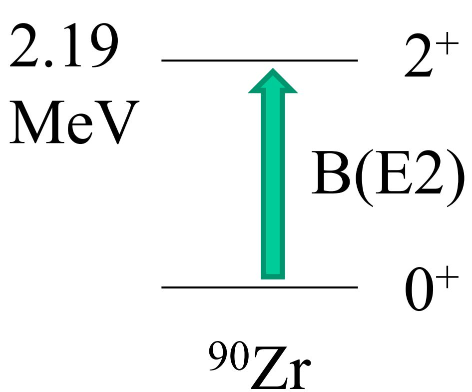
VOLUME 29, NUMBER 16 PHYSICAL REVIEW LETTERS 16 OCTOBER 1972

Giant Multipole Resonances in ^{90}Zr Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in ^{90}Zr was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

GQRの発見以前は、低エネルギー 2^+ 状態のみが知られていた

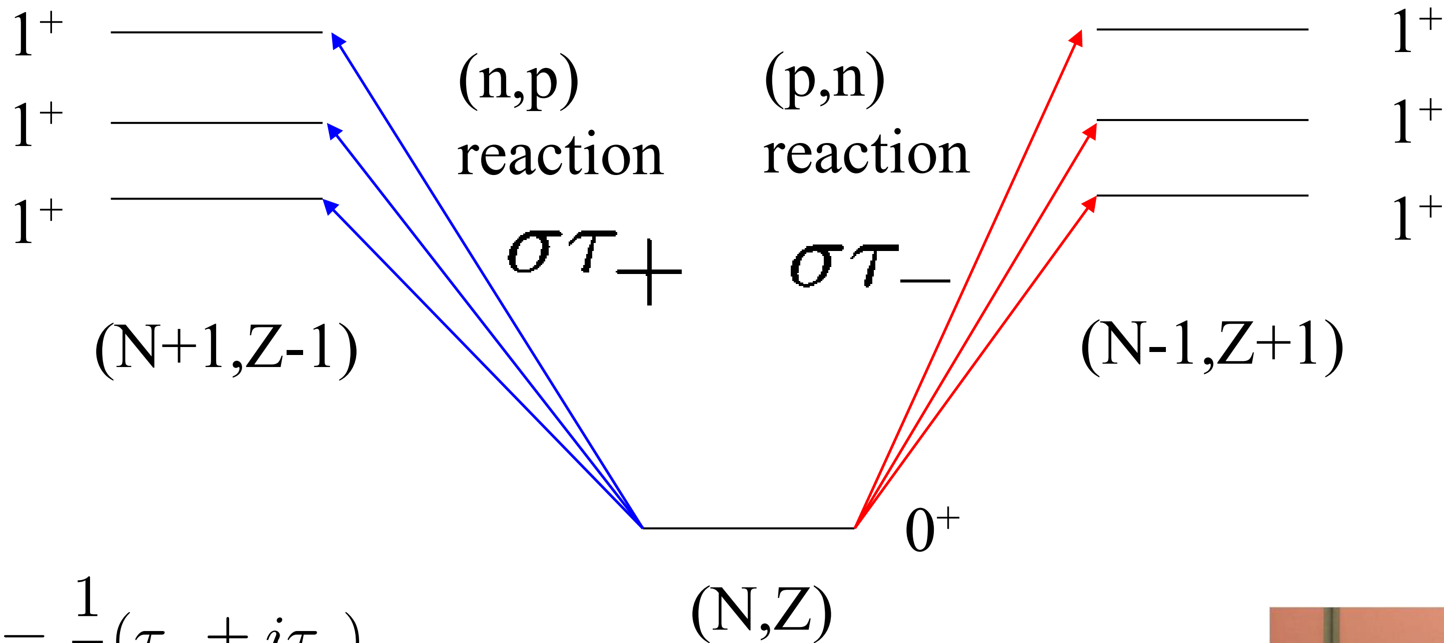


B(E2) = 26.85 W.u. ⟶ EWSR の 2.5% 程度
残りはどこに?

⟶ GQR (~14 MeV) が
発見され解決

Ikeda sum rule

charge exchange reactions: Gamow-Teller transitions



$$\tau_\pm = \frac{1}{2}(\tau_x \pm i\tau_y)$$

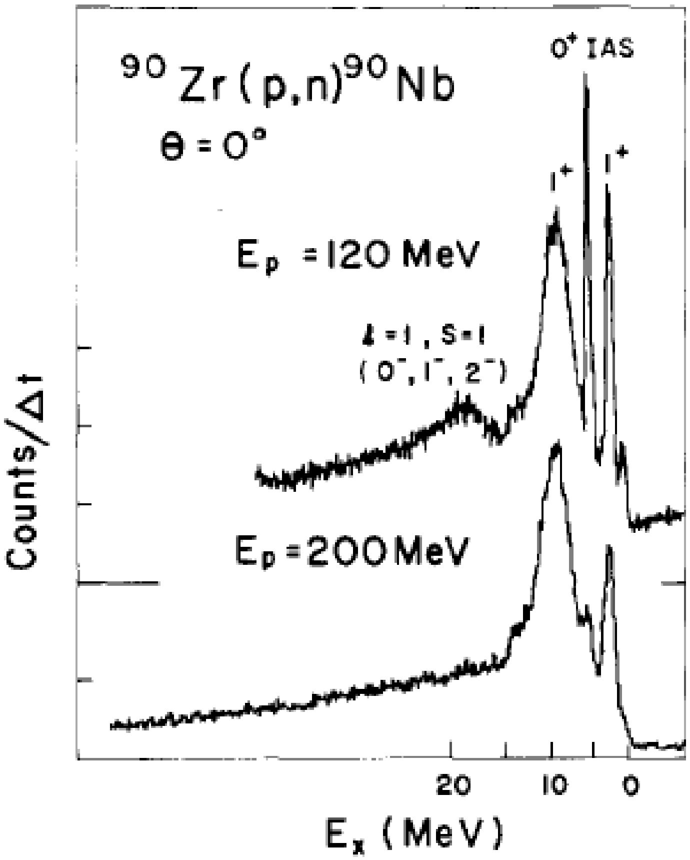
$$\tau_+|p\rangle = |n\rangle, \quad \tau_-|n\rangle = |p\rangle$$

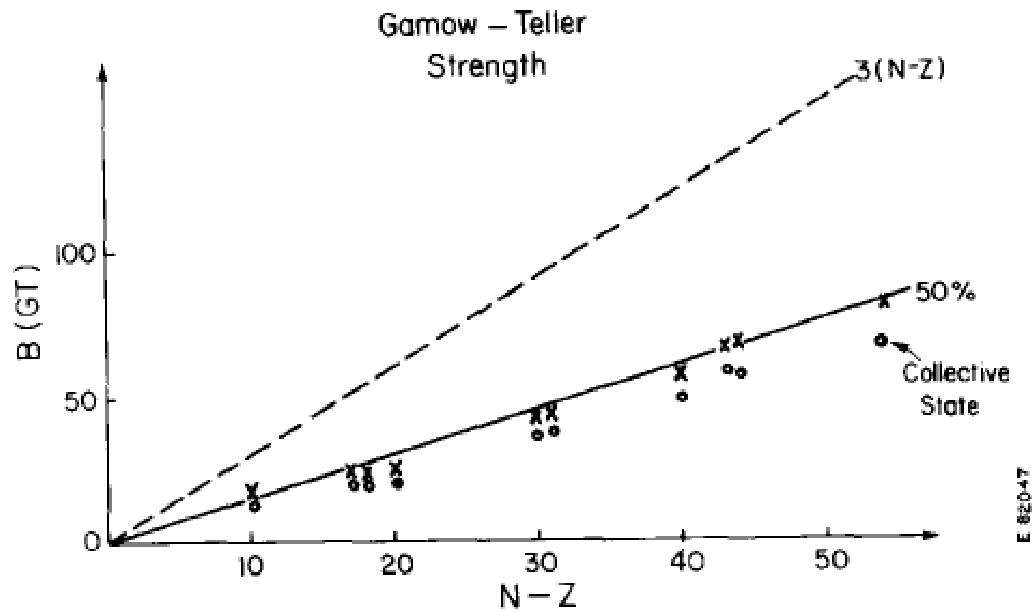
Ikeda sum rule

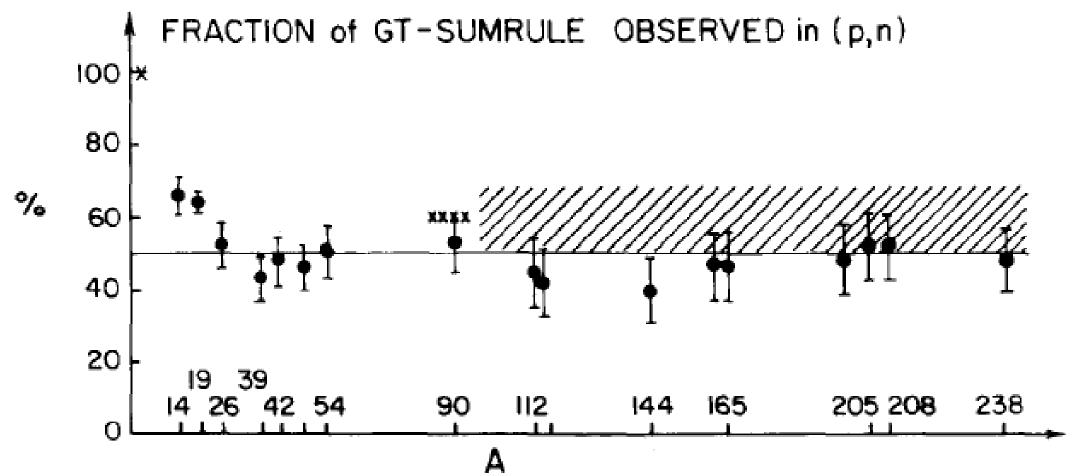
$$S_0(\sigma\tau_-) - S_0(\sigma\tau_+) = 3(N-Z)$$

池田清美氏

the situation before 1997





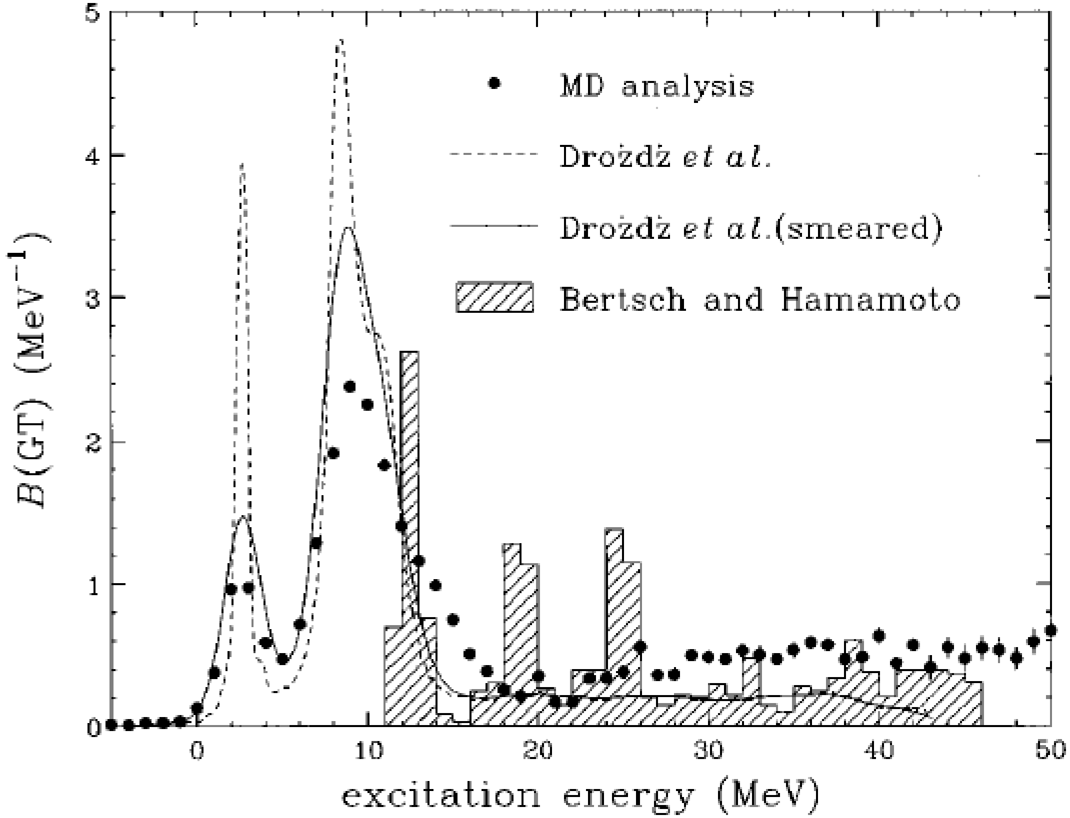


the “quenching problem” of GT strength

quark (Δ resonance)?

C. Gaarde, Nucl. Phys. A396(‘83)127c

^{90}Zr (p,n) ^{90}Nb



T. Wakasa et al., PRC55 ('97) 2909

$S_- - S_+ = 27.0 \pm 1.6 = (90 \pm 5)\%$ of Ikeda sum rule

→ quark contribution: small

レポート問題2（〆切：12月2日（土））

1次元調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

の n 番目の固有状態 $|n\rangle$ を考える(n=0 が基底状態)。

1) 演算子 x^2 に対して、強度関数を求めよ。n が 0 の場合、1 の場合、2以上の場合で場合分けせよ。

2) 演算子 x で遷移できる状態 $|k\rangle$ を全て書き出し(状態 k も調和振動子の固有状態)、遷移確率

$$P_{n\to k} = |\langle k|x|n\rangle|^2$$

を求めよ。

3)演算子 x に対して energy weighted sum rule

$$S_1 = \sum_k (E_k - E_n) P_{n\to k}$$

を計算し、TRK和則が成り立っていることを示せ。

レポート問題3（〆切：12月2日（土））

1) スピン演算子　$\sigma_0 = \sigma_z, \quad \sigma_{\pm 1} = \mp \frac{1}{\sqrt{2}}(\sigma_x \pm i\sigma_y)$

に対し、$\sum_\mu (-1)^\mu \sigma_\mu \sigma_{-\mu}$ を計算せよ。

2) アイソスピン演算子　$\tau_\pm = \frac{1}{2}(\tau_x \pm i\tau_y)$ に対して
交換関係 $[\tau_+, \tau_-]$ を計算せよ。

3) 以下の Ikeda sum rule を証明せよ。

$$(Y_\pm)_\mu \equiv \sum_{i=1}^{A} \tau_\pm(i)\sigma_\mu(i) \quad ; \quad [(Y_\pm)_\mu]^\dagger = (-)^\mu (Y_\mp)_{-\mu}$$

$$\begin{aligned} S_- - S_+ &= \langle 0|Y_-^\dagger Y_-|0\rangle - \langle 0|Y_+^\dagger Y_+|0\rangle \\ &= \sum_\mu \langle 0|(Y_-)_\mu^\dagger (Y_-)_\mu|0\rangle - \langle 0|(Y_+)_\mu^\dagger (Y_+)_\mu|0\rangle \\ &= 3(N-Z) \end{aligned}$$

ここで、|0> は陽子数 Z、中性子数 N, 質量数 A=Z+N を持つ原子核の基底状態である。ただし、$\tau_z|n\rangle = |n\rangle, \quad \tau_z|p\rangle = -|p\rangle$ とする。