集団励起の微視的理論

原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）

集団励起を微視的に理解してみる
（集団励起をミクロに見てみるとどうなっているのか?）

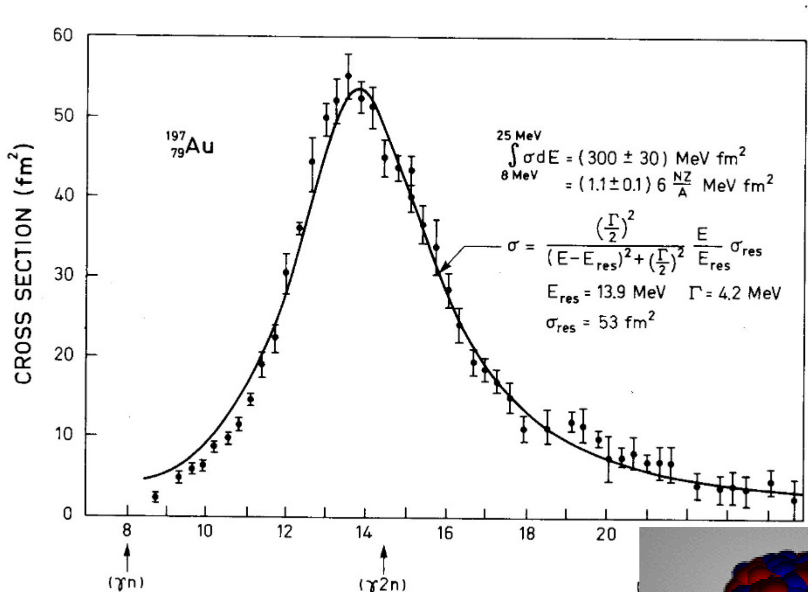
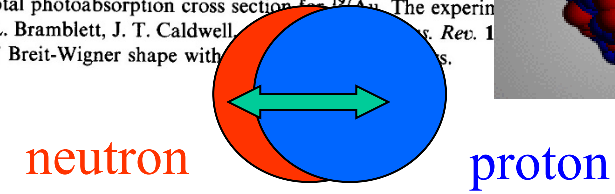
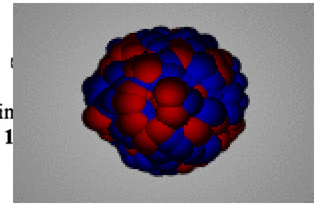


Figure 6-18 Total photoabsorption cross sectio... The experim...
S. C. Fultz, R. L. Bramblett, J. T. Caldwell, ... *Rev.* 1...
solid curve is of Breit-Wigner shape with ...s.

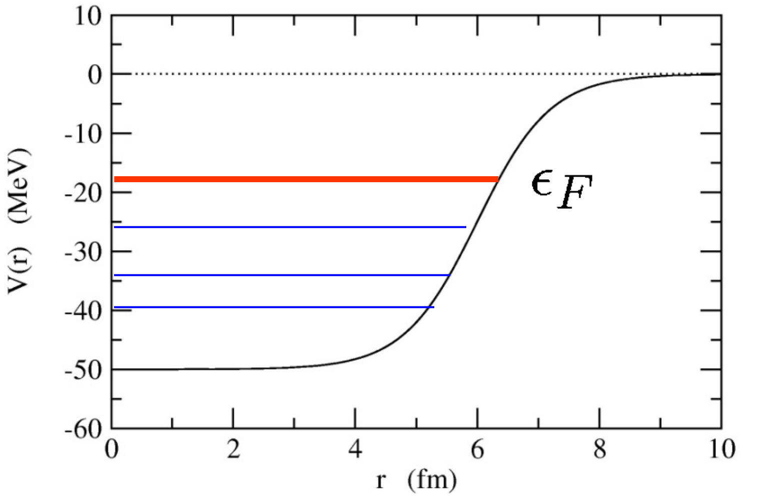




集団励起の例：巨大双極子共鳴

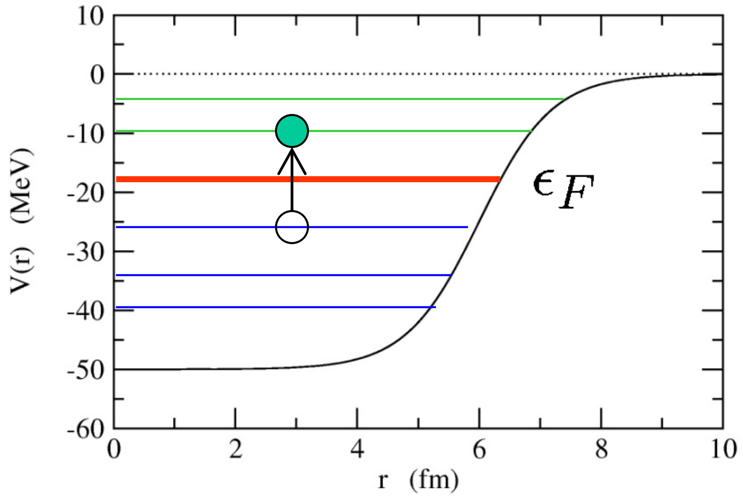
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle = \prod_h a_h^\dagger|0\rangle$$

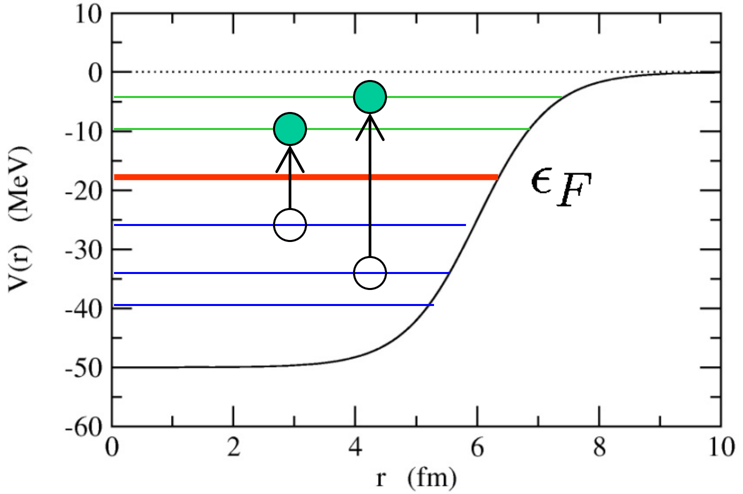
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h|HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'}|HF\rangle$$



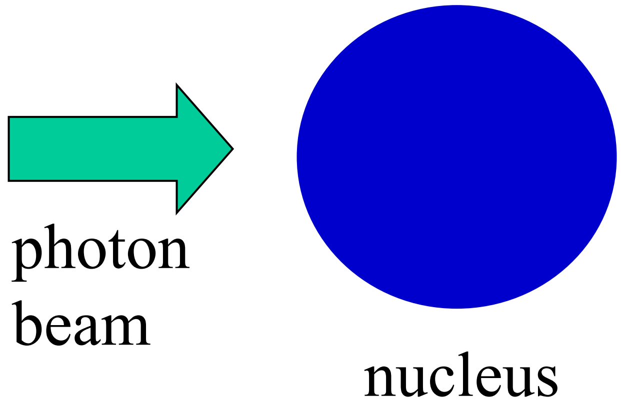
Tamm-Dancoff Approximation

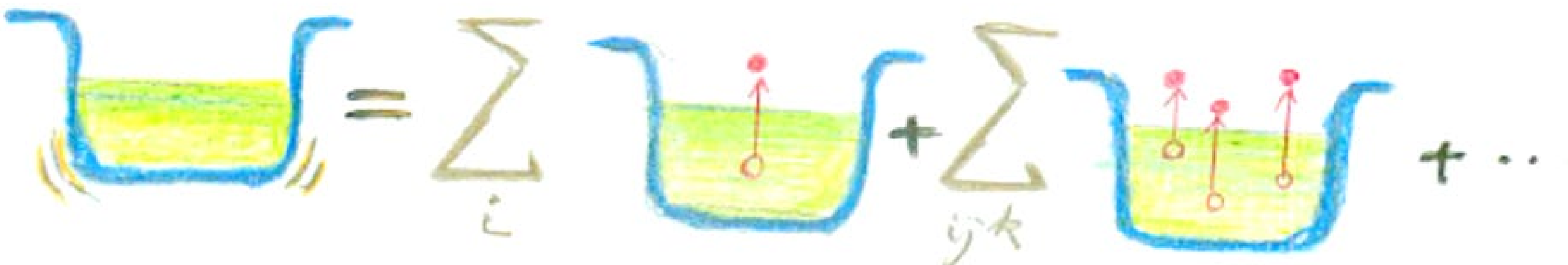
Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$\equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

原子核を外場により揺らしてみると何が起こるのか?





スライド: 松柳研一氏

Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$\equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

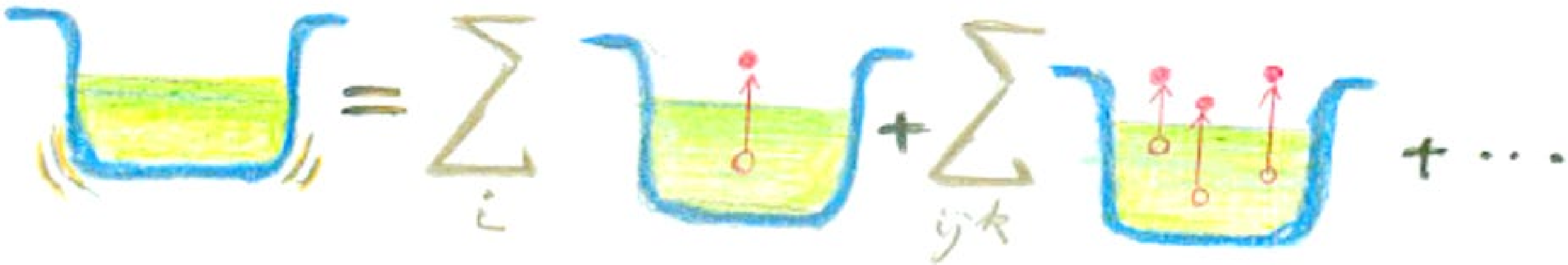
$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

residual interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

Tamm-Dancoff equation;　1p1h の空間でハミルトニアンを対角化

残留相互作用の意味



スライド：松柳研一氏

$$V(\boldsymbol{r}) \sim \int d\boldsymbol{r}'\, v(\boldsymbol{r}, \boldsymbol{r}')\rho(\boldsymbol{r}')$$

vibration: $\rho = \rho_0(\boldsymbol{r}) \to \rho_0(\boldsymbol{r}) + \delta\rho(\boldsymbol{r}, t)$

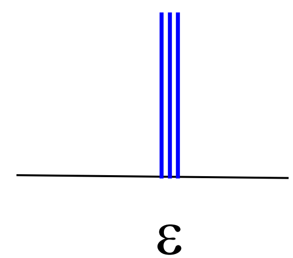
$\delta\rho(\boldsymbol{r}, t)$ → residual interaction

TDA on a schematic model

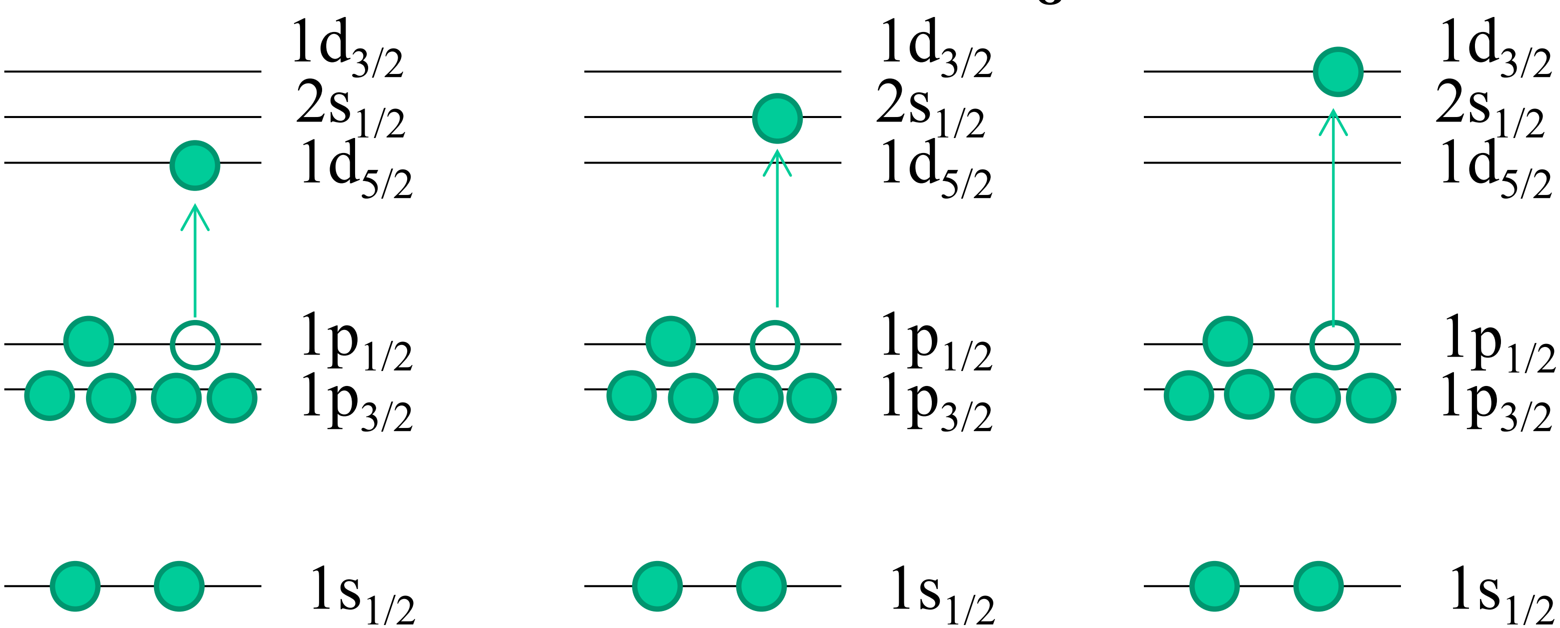
$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:



（例えば）



TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \qquad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization:

$$\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$$

TDA on a schematic model

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\epsilon+3g) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

全ての状態が同位相で寄与
＝コヒーレントな重ね合わせ

他の固有状態:

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

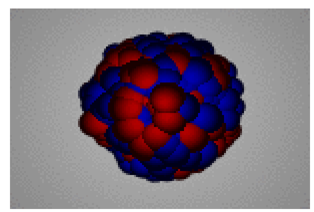
$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

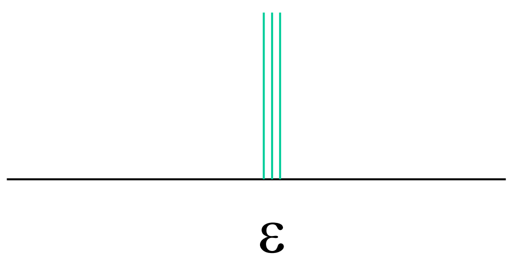
位相がそろっていない

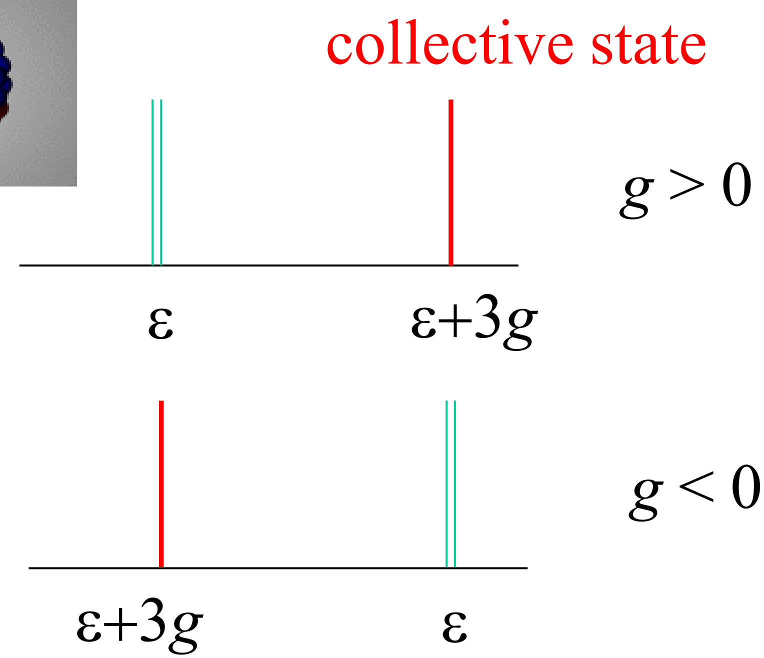
TDA on a schematic model

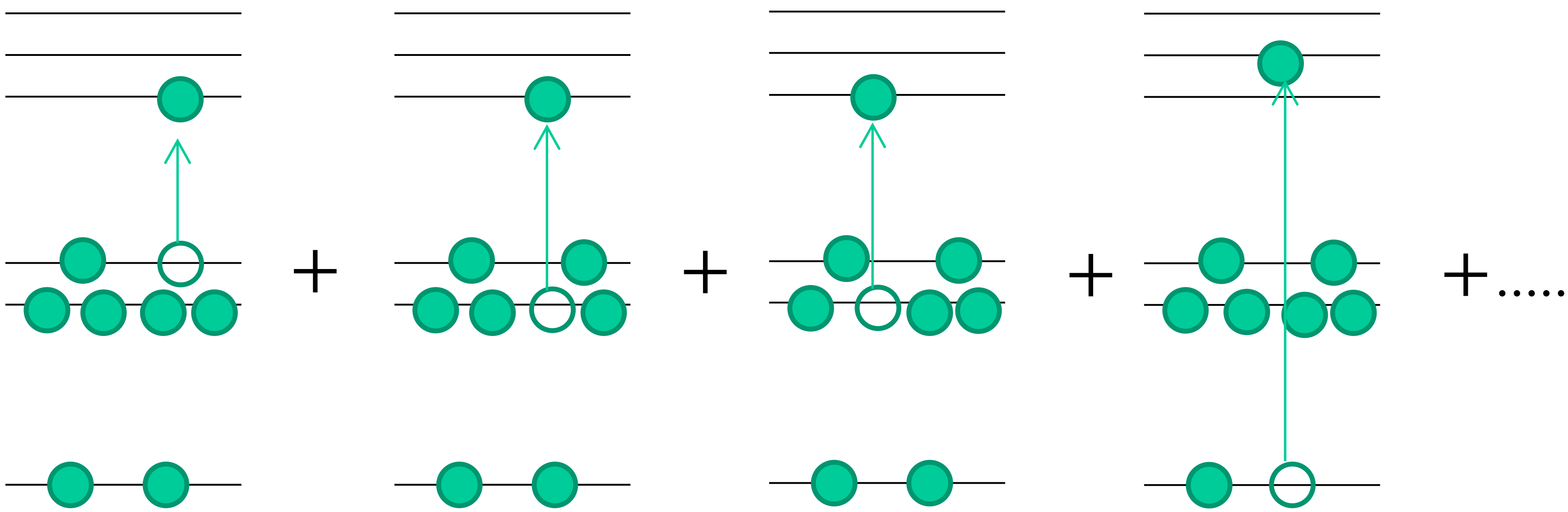
$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization: $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$









複数の粒子・空孔状態をコヒーレントに重ね合わせることによって
多数の核子が励起に関与していることを表現する

$$|\nu\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle \equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

$$\Rightarrow \left| \left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle \right|^2 = \left(\sum_{ph} f_{ph} X_{ph} \right)^2$$

干渉項がすべて
同符号で寄与

TDA on a separable interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\overline{v}|hp'\rangle$$

Separable interaction: $\langle ph'|\overline{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

(separable interaction)

$$H\psi = E\psi; \qquad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \sum_j H_{ij} C_j = E C_i; \qquad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j$ (separable form)

$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$

$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$

$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T \longrightarrow \boxed{\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}}$$

(separable interaction)

$$H\psi = E\psi; \qquad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \quad \sum_j H_{ij} C_j = E C_i; \qquad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j \longrightarrow$

$$\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}$$

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \lambda D_{ph} D^*_{p'h'}$$

$$\longrightarrow \quad \frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

TDA on a schematic model

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

Tamm-Dancoff equation: $\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$(\epsilon_{ph} - E) X_{ph} + \lambda D_{ph} \cdot T = 0 \qquad T \equiv \sum_{ph} D^*_{ph} X_{ph}$$

$$X_{ph} = -\lambda \frac{D_{ph} T}{\epsilon_{ph} - E}$$

$$T = \sum_{ph} D^*_{ph} X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

Graphical solutions $$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

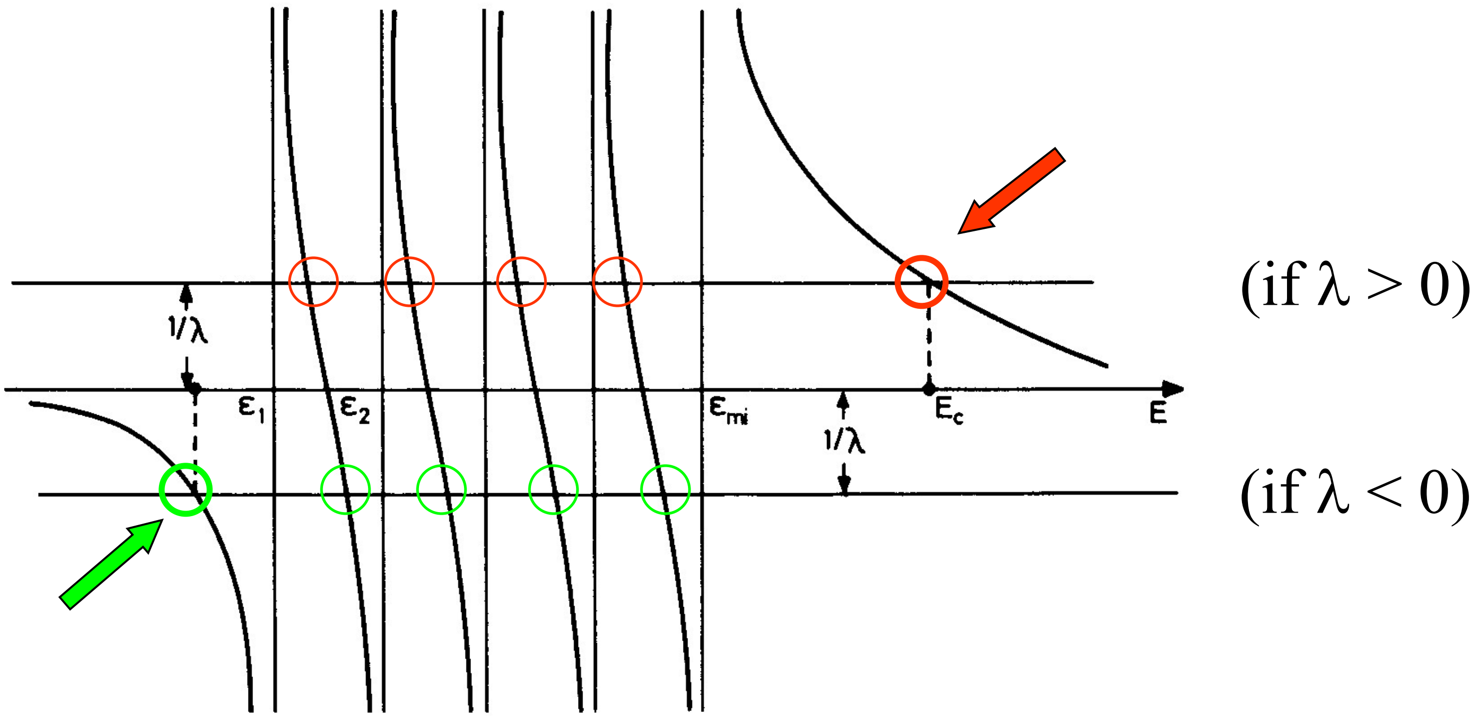


Figure 8.4. Graphical solution of Eq. (8.18).

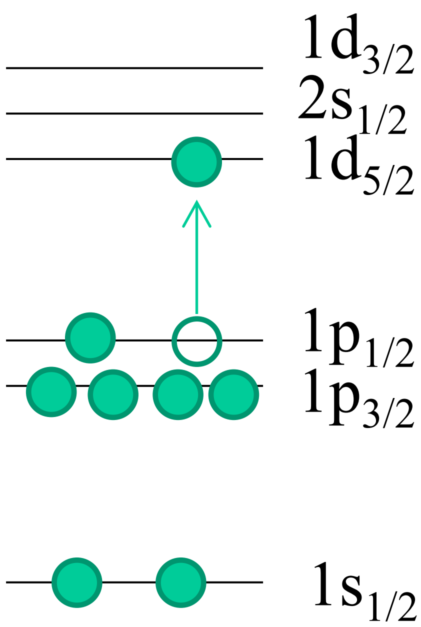
(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon + \lambda \sum_{ph} |D_{ph}|^2, \quad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superpositon of 1p1h states

原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に関与)
- ✓ 集団励起(多くの核子が集団として励起に関与)



一粒子励起の例

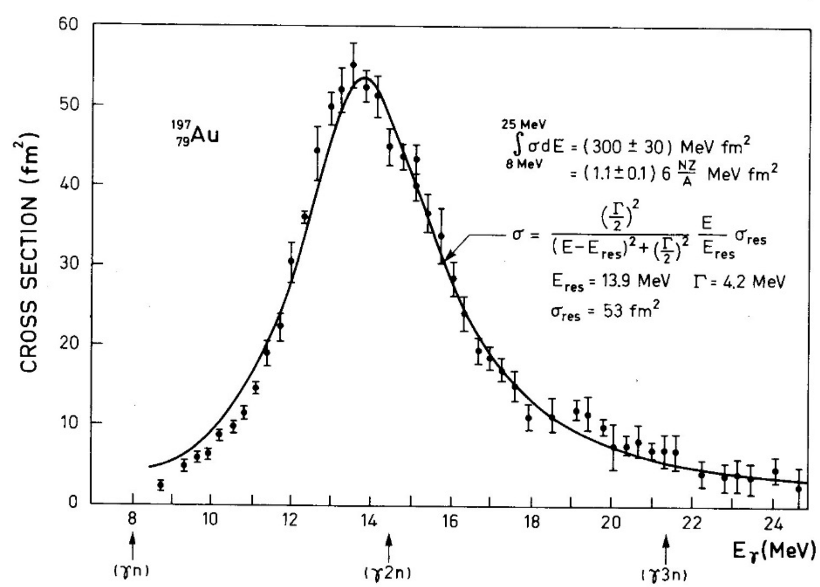
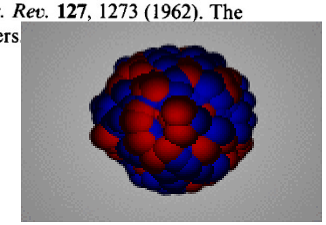
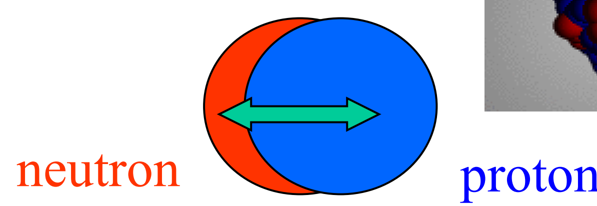


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.





集団励起の例:巨大双極子共鳴

Giant Dipole Resonance (GDR) 巨大双極子共鳴

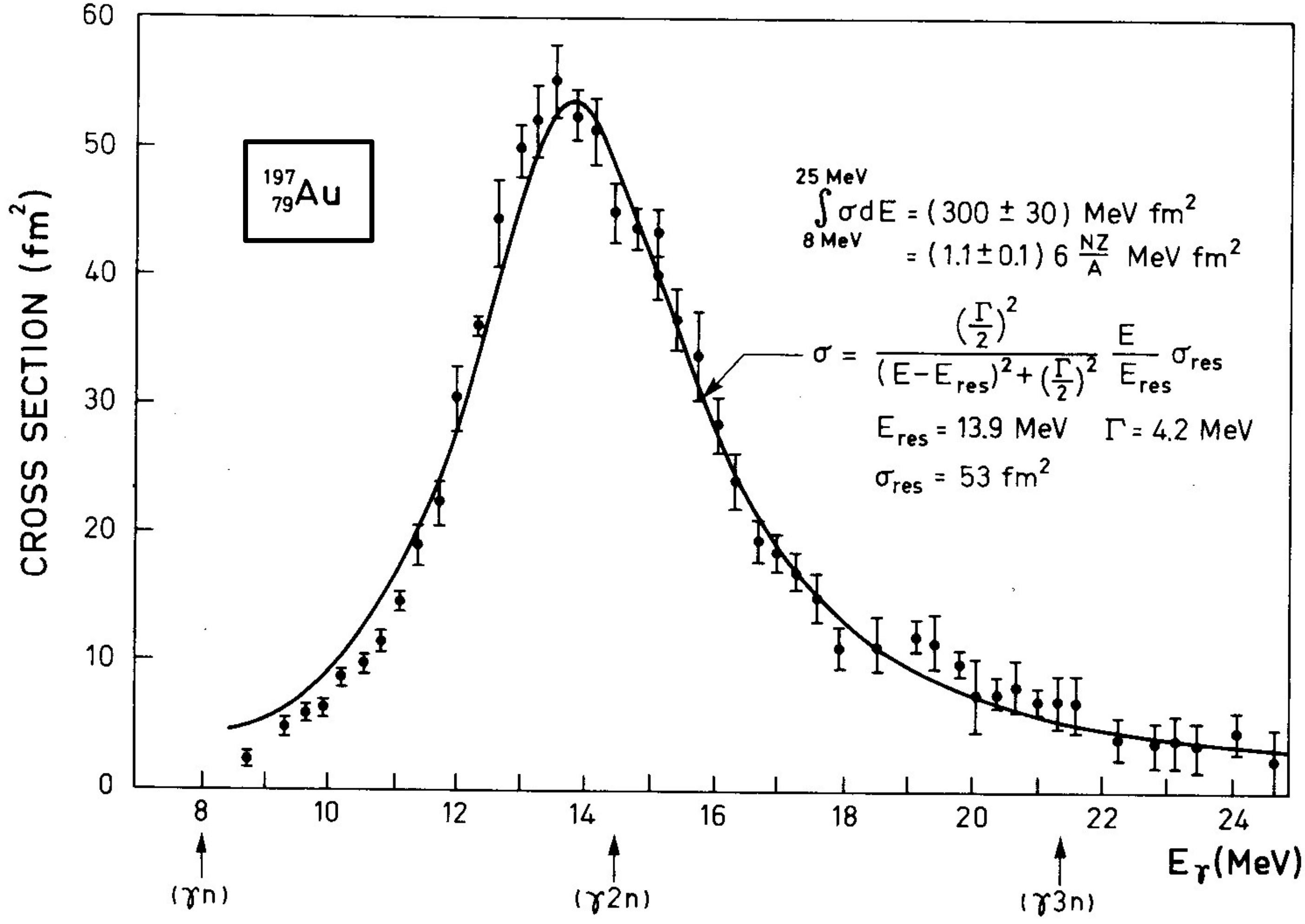


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

cf. $41 \times 197^{-1/3} = 7.05$ MeV $\rightarrow$ 14 MeV

Iso-scalar type modes: $E < \epsilon_{ph} \to \lambda < 0$ (attractive)

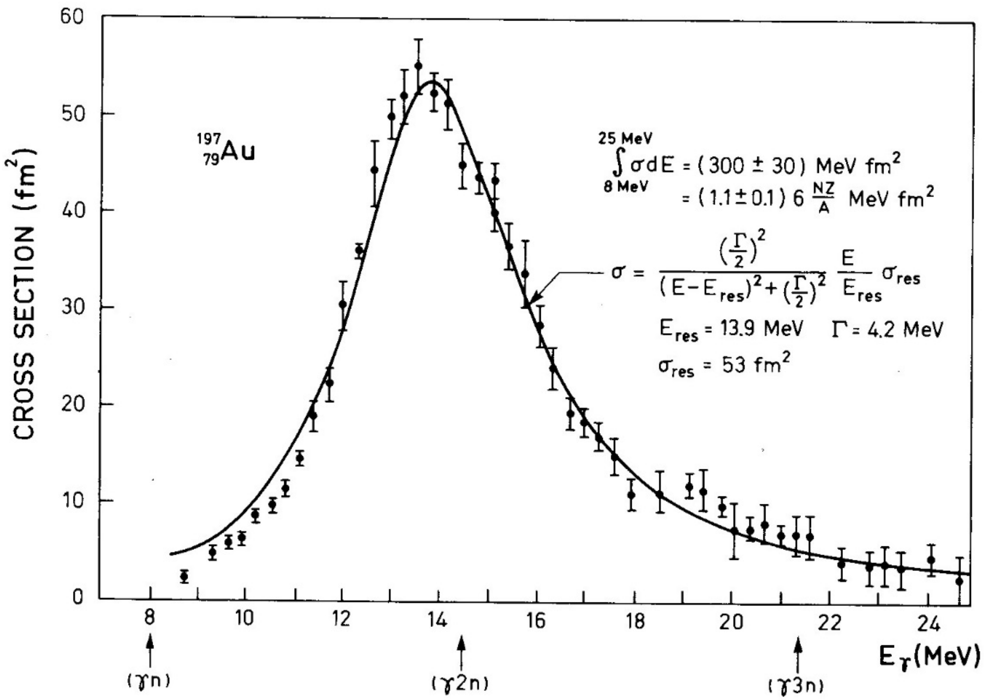
Iso-vector type modes: $E > \epsilon_{ph} \to \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 41A^{-1/3}$

IS GQR: $E \sim 65A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 82A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41A^{-1/3}$ (MeV)



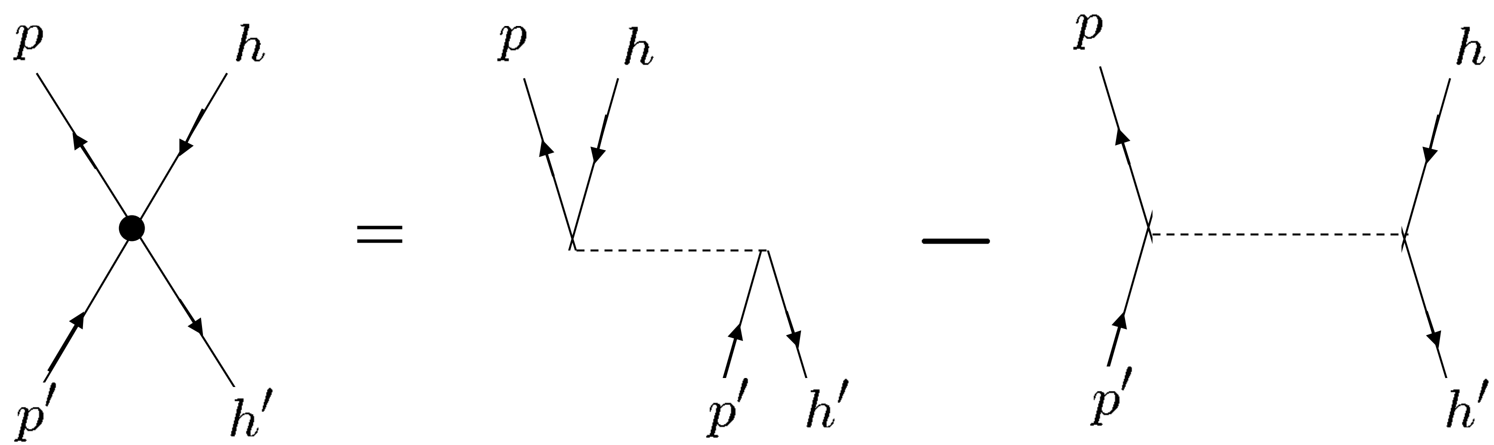
^{197}Au

$$E_{\mathrm{GDR}} = 14 \quad (\mathrm{MeV})$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

$$\sim 7 \quad (\mathrm{MeV})$$

$$\langle ph^{-1}|\bar{v}|p'h'^{-1}\rangle = \langle ph'|\bar{v}|hp'\rangle = \langle ph'|v|hp'\rangle - \langle ph'|v|p'h\rangle$$



Direct term — Exchange term

$$\begin{cases} \langle PP^{-1}|\bar{v}|PP^{-1}\rangle \sim \langle NN^{-1}|\bar{v}|NN^{-1}\rangle = D - E \\ \langle PP^{-1}|\bar{v}|NN^{-1}\rangle = D \quad \text{(no charge exchange)} \end{cases}$$

$$\begin{aligned} \langle IS|\bar{v}|IS\rangle &= 2D - E \sim D \\ \langle IV|\bar{v}|IV\rangle &= -E \sim -D \end{aligned}$$

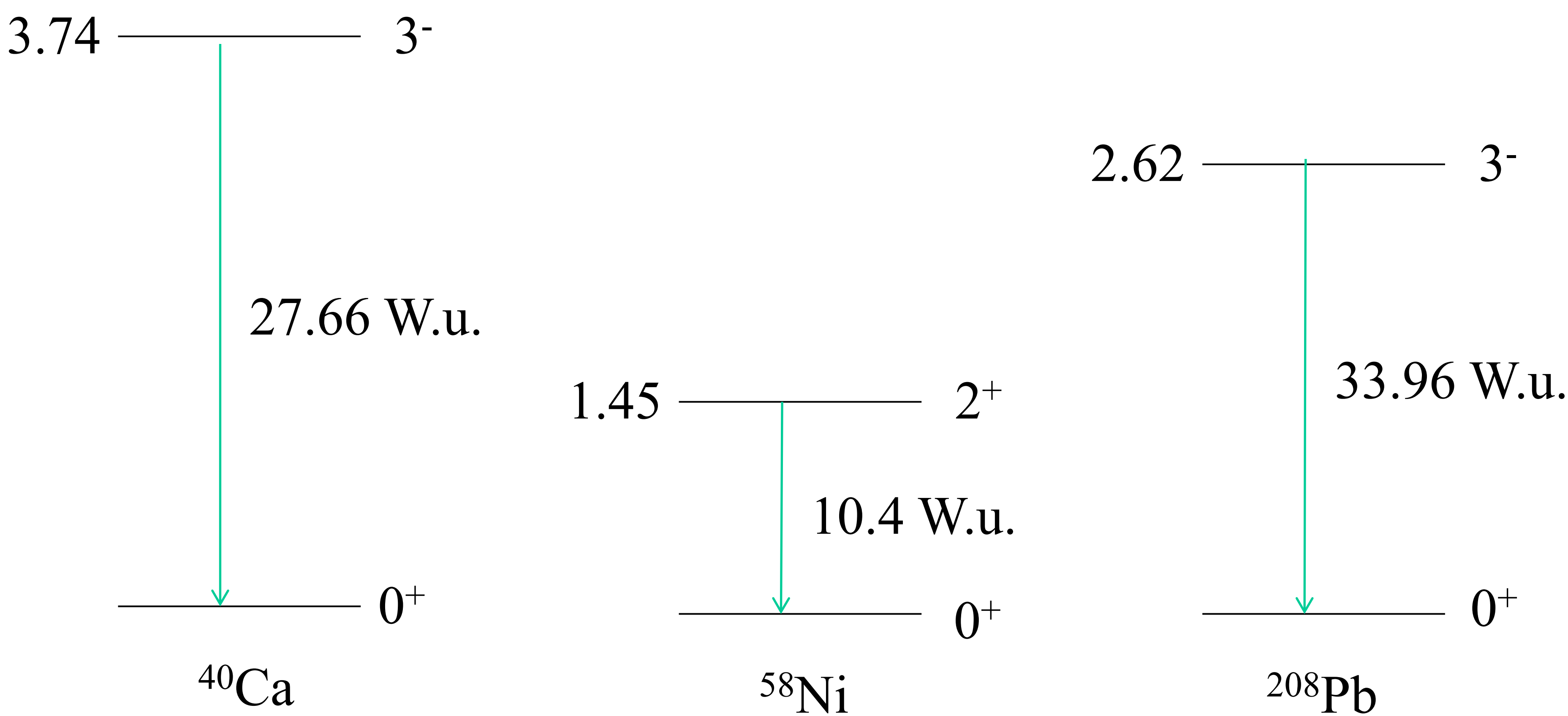
$$\begin{aligned} |IS\rangle &\propto |NN^{-1}\rangle + |PP^{-1}\rangle \\ |IV\rangle &\propto |NN^{-1}\rangle - |PP^{-1}\rangle \end{aligned}$$

どれだけの核子が励起に関与しているのか?

Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \to I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 \qquad (e^2\mathrm{fm}^{2\lambda})$$

exp data:



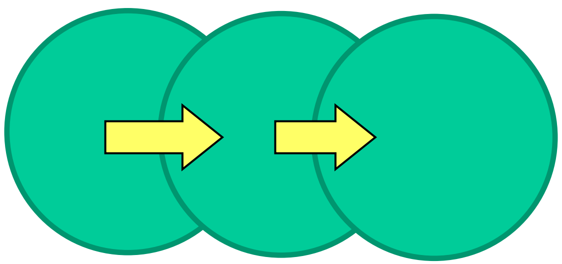
Spurious motion and RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy → zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|0\rangle \qquad \text{(TDA)}$$

→ A better approximation:

the random phase approximation (RPA)

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph} \left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

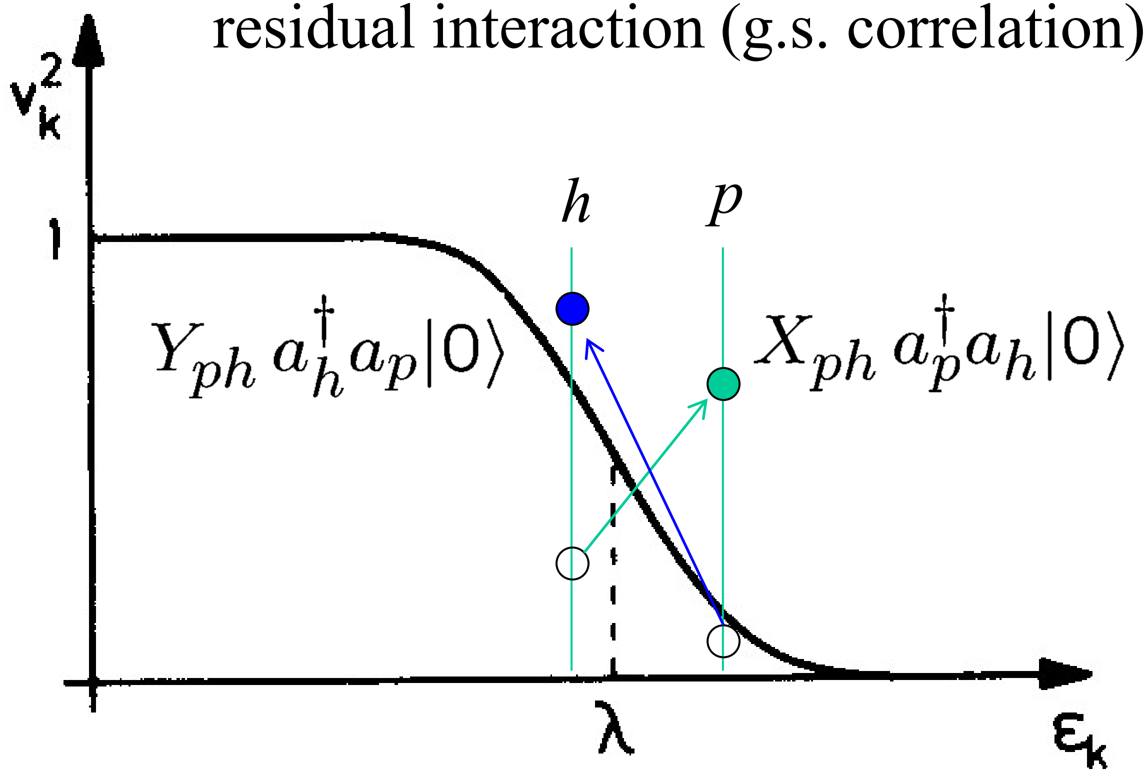
(superposition of 1p1h states)

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\,a_p^\dagger a_h - Y_{ph}\,a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



Random Phase Approximation: Historical note

D. Bohm and D. Pines, Phys. Rev. 92('53)609

The plasma oscillation in an infinite electron gas

$$H = \frac{1}{2m}\sum_i \left(\boldsymbol{p}_i + \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}_i)\right)^2 + \frac{1}{8\pi}\int \boldsymbol{E}(\boldsymbol{x})^2 d\boldsymbol{x} - 2\pi n e^2 \sum_k \frac{1}{k^2}$$

$$\boldsymbol{A}(\boldsymbol{x}) = \sqrt{4\pi c^2}\sum_k q_k \epsilon_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\sum_i \boldsymbol{A}(\boldsymbol{x}_i)^2 = \sum_{ikl} \boldsymbol{\epsilon}_k \cdot \boldsymbol{\epsilon}_l \, q_k q_l \, e^{i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{x}_i}$$

$$\rightarrow \sum_{ik} q_k q_{-k} \qquad (\boldsymbol{k}+\boldsymbol{l} = 0 \text{ only})$$