

集団励起の微視的理論

原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に参与)
- ✓ 集団励起(多くの核子が集団として励起に参与)

集団励起を微視的に理解
してみる
(集団励起をミクロに見て
みるとどうなっているのか?)

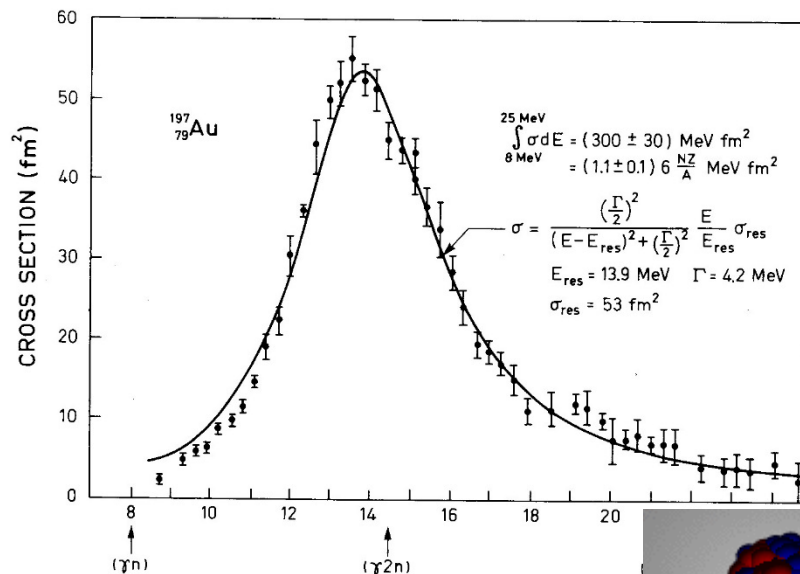
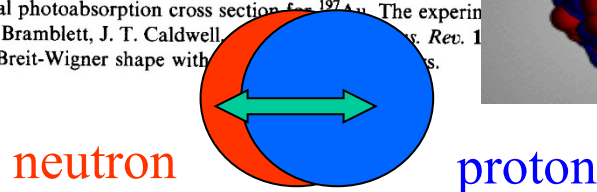
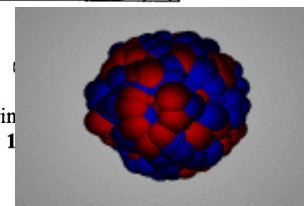
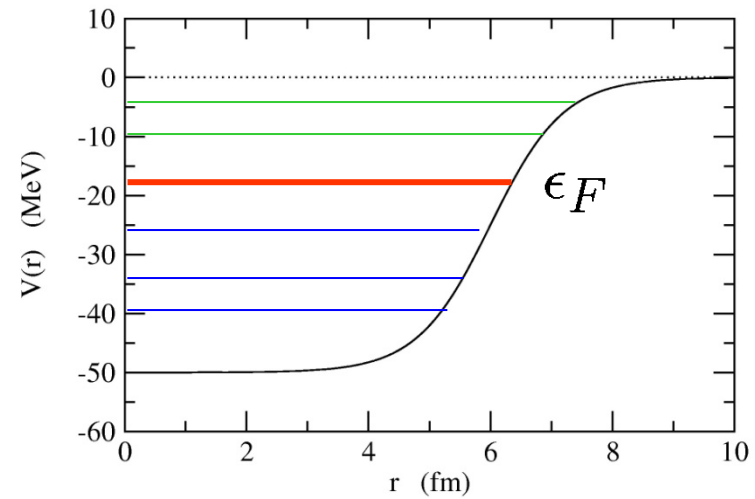


Figure 6-18 Total photoabsorption cross section for ¹⁹⁷Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, *Phys. Rev. C* 10, 1044 (1974). The solid curve is of Breit-Wigner shape with $E_{res} = 13.9$ MeV, $\Gamma = 4.2$ MeV, and $\sigma_{res} = 53$ fm².



集団励起の例: 巨大双極子共鳴

一般に:
$$|\Psi_k\rangle = \sum_i C_i |\Phi(SD)_i\rangle$$



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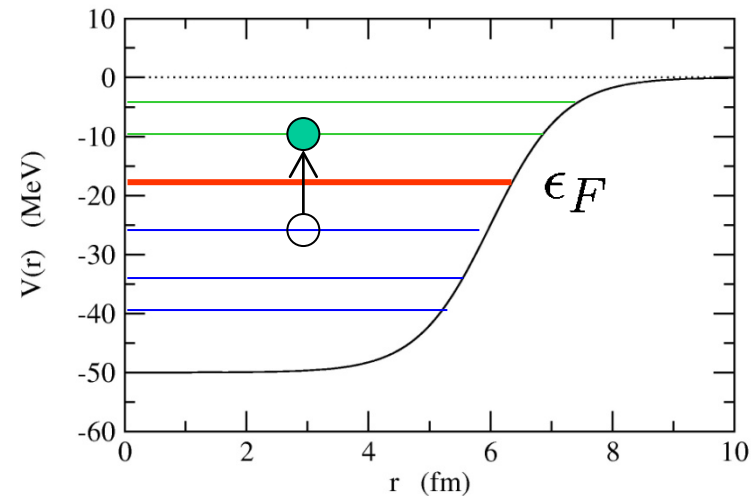
タム・ダンコフ近似

基底状態: $|HF\rangle$

励起状態: $|\nu\rangle = Q_\nu^\dagger |HF\rangle = \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle$

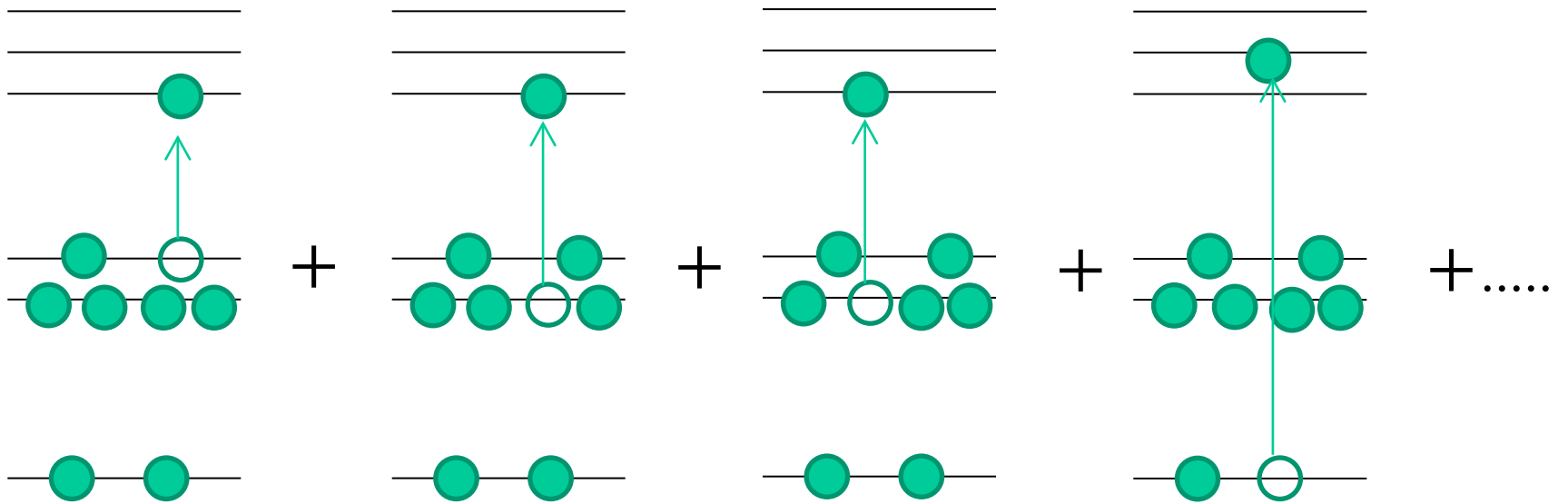
1 particle-1 hole (1p1h) state

$\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$



$a_p^\dagger a_h |HF\rangle$

(1p1h 状態の重ね合わせ)



複数の粒子・空孔状態を**コヒーレント**に重ね合わせることによって
 多数の核子が励起に関与していることを表現する

$$\left| \left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle \right|^2 = \left(\sum_{ph} f_{ph} X_{ph} \right)^2$$

干渉項がすべて
 同符号で寄与

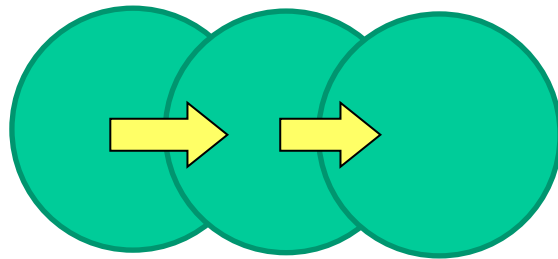
Spurious motion and RPA

Mean-Field Approximation \longleftrightarrow Broken symmetries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

\longrightarrow Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy \rightarrow zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} X_{ph} a_p^\dagger a_h |0\rangle \quad (\text{TDA})$$

→ A better approximation:

the random phase approximation (RPA)

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left(X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

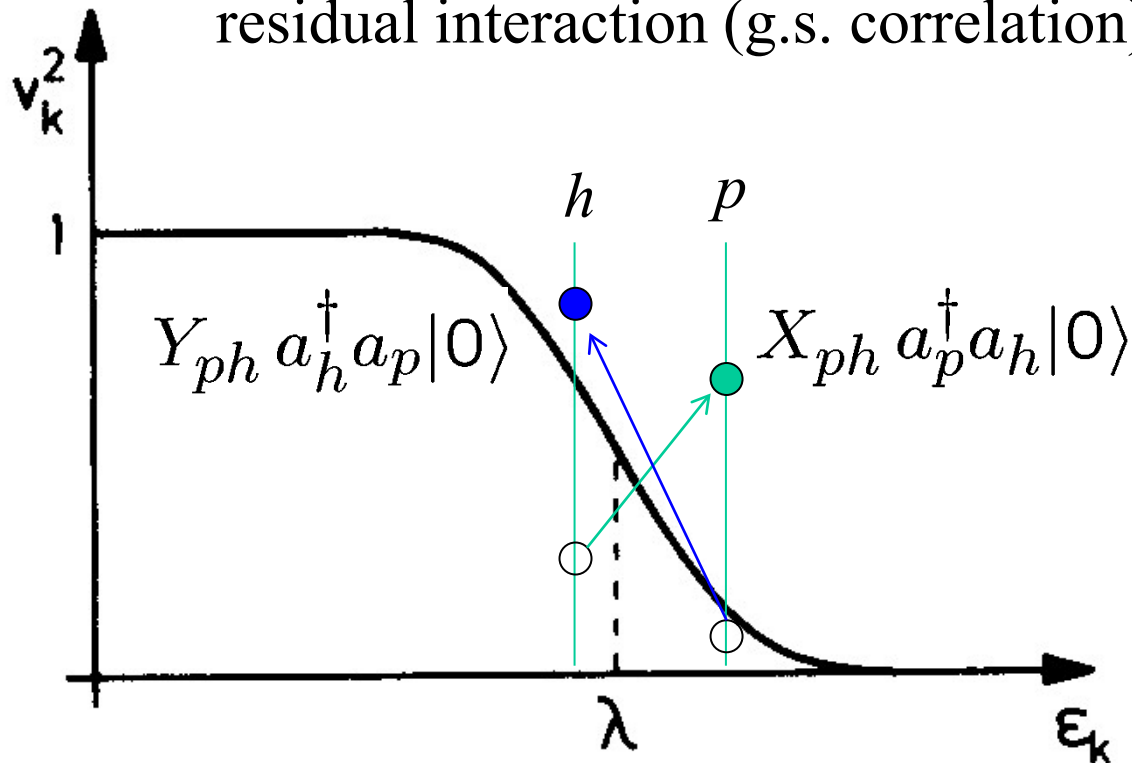
基底状態: $Q_\nu |0\rangle = 0$ で定義。

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left(X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left(X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$



$$\langle HF | [\delta Q, [H, Q_\nu^\dagger]] | HF \rangle = E_\nu \langle HF | [\delta Q, Q_\nu^\dagger] | HF \rangle$$

→ coupled equations for X and Y

$$\langle HF | [\delta Q, [H, Q_\nu^\dagger]] | HF \rangle = E_\nu \langle HF | [\delta Q, Q_\nu^\dagger] | HF \rangle$$

$$Q_\nu^\dagger = \sum_{ph} X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \quad \delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

RPA equation:

$$\sum_{p'h'} A_{ph,p'h'} X_{p'h'} + B_{ph,p'h'} Y_{p'h'} = E_\nu X_{ph}$$

$$\sum_{p'h'} B_{ph,p'h'}^* X_{p'h'} + A_{ph,p'h'}^* Y_{p'h'} = -E_\nu Y_{ph}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$B_{ph,p'h'} = \langle pp' | \bar{v} | hh' \rangle$$

or

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

Spurious motion in RPA

Mean-Field Approximation \longleftrightarrow Broken symmetries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

\longrightarrow Zero mode (Nambu-Goldstone mode)

$$[H, \hat{O}] = 0$$

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$



\hat{O} is a solution of RPA with $E=0$

$$Q^\dagger = \hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

(note) $Q_{\text{TDA}}^\dagger = \sum_{ph} O_{ph} a_p^\dagger a_h \longrightarrow [H, Q_{\text{TDA}}^\dagger] \neq 0$

Spurious motion in RPA

Mean-Field Approximation \longleftrightarrow Broken symmetries

- Center of mass localization (single center)
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Restoration of broken symmetries

\longrightarrow Zero mode (Nambu-Goldstone mode)

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$



if $[H, \hat{O}] = 0$

Then \hat{O} is a solution of RPA with $E=0$



The physical solutions are completely separated out from the spurious modes.

他のRPAの定式化

- 線形応答理論

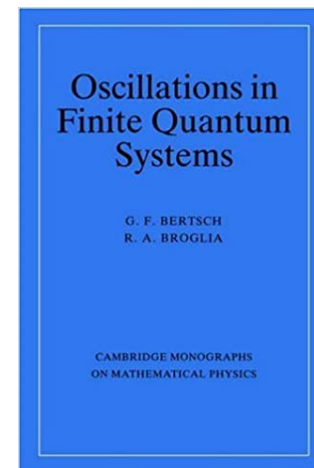
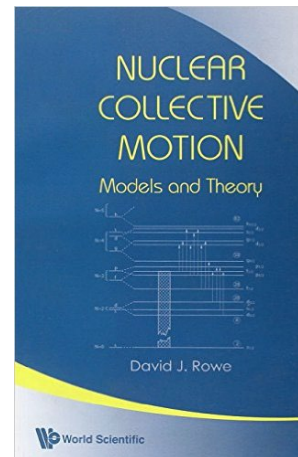
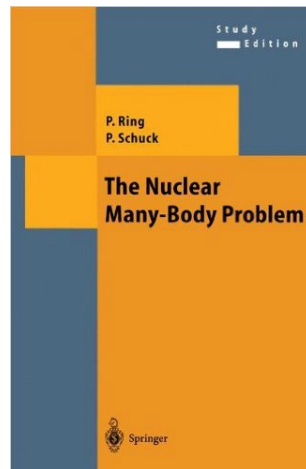


外場で原子核を揺すった時に、
原子核がどのように応答するか摂動論
を使って議論する
→固有モードを見つける

- 時間に依存するハートリー・フォック(TDHF)方程式を線形化

$$i\hbar\dot{\rho}(t) = [h[\rho], \rho] \quad \longleftarrow \quad \rho(t) = \rho_0 + \delta\rho(t)$$

詳しくは:



Comparison between Skyrme-(Q)RPA calculation and exp. data

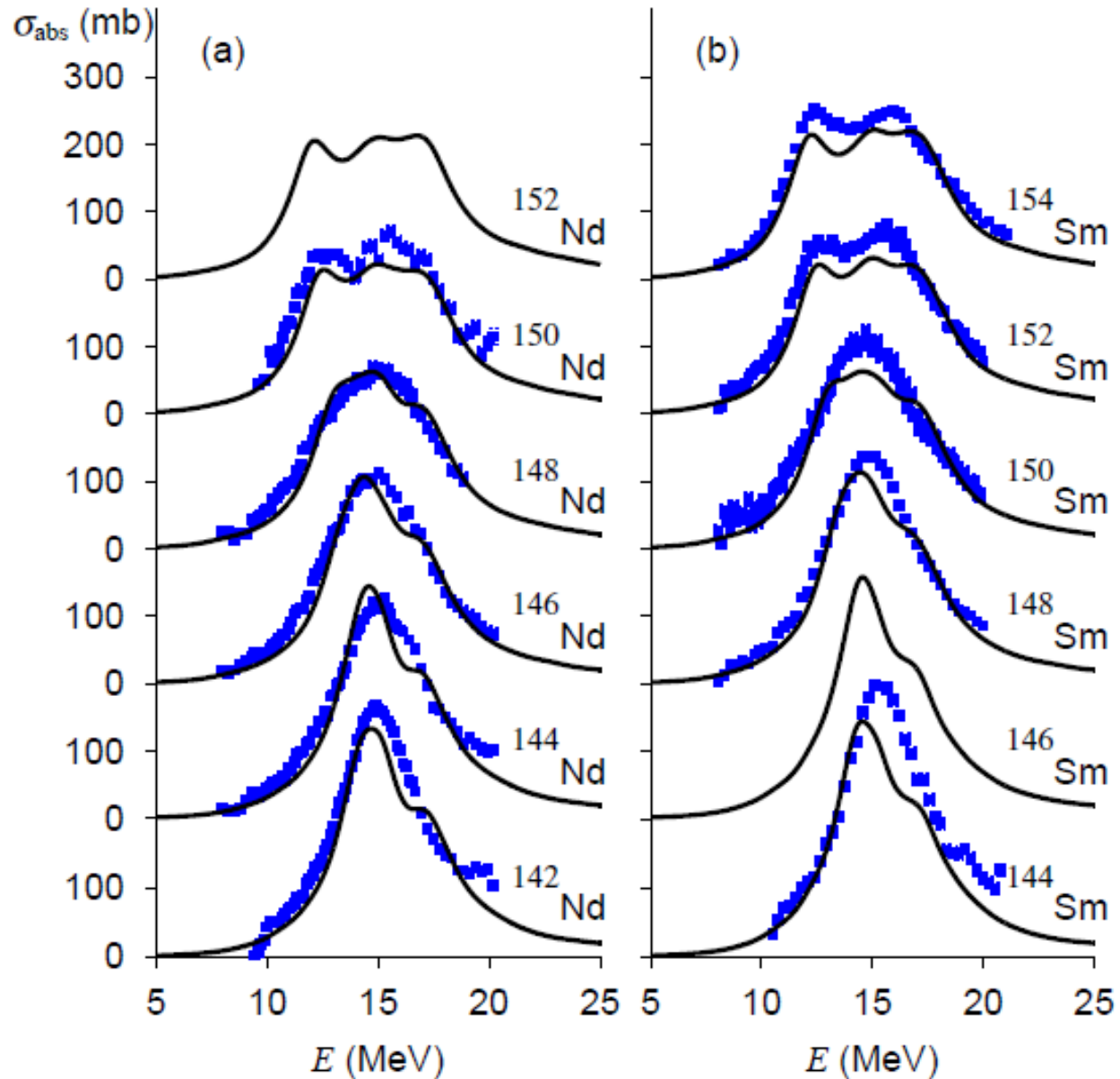
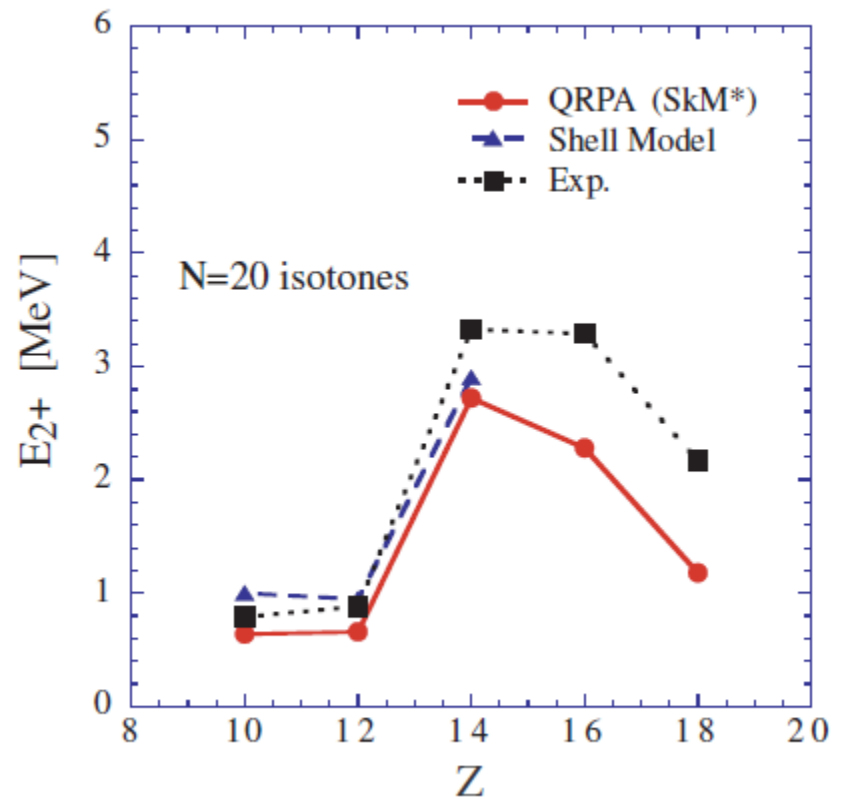
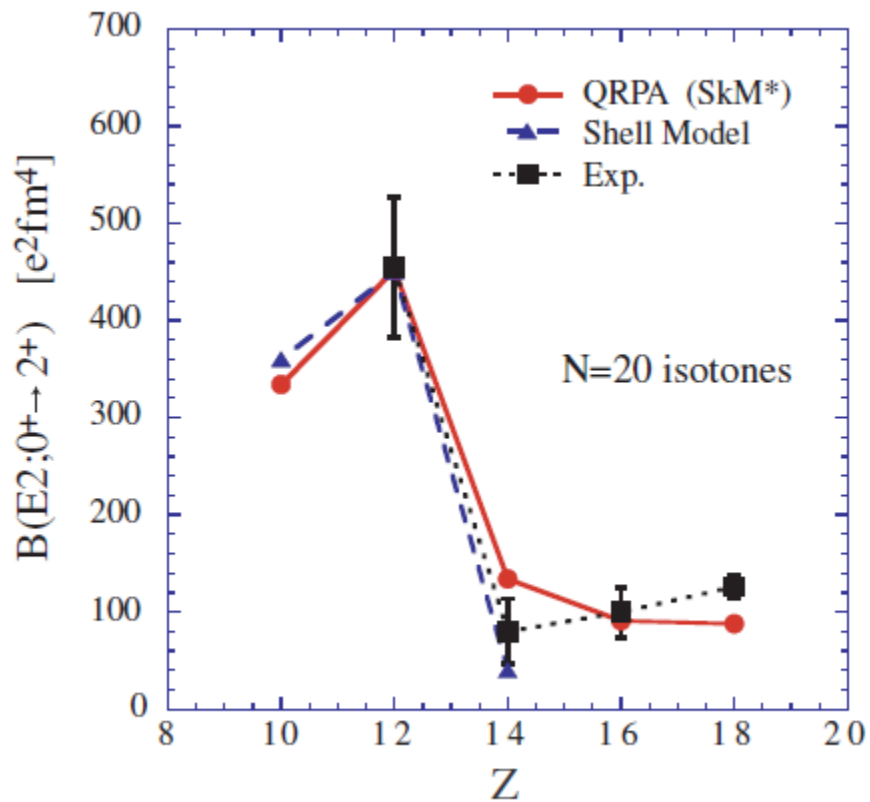


photo-absorption
cross section
(GDR)



K. Yoshida
and T. Nakatsukasa,
PRC83('11)021304



M. Yamagami and Nguyen Van Giai, PRC69 ('04) 034301

RPA on a schematic model

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$B_{ph,p'h'} = \langle pp' | \bar{v} | hh' \rangle$$

Separable interaction:

$$\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$$

$$\langle pp' | \bar{v} | hh' \rangle = \lambda D_{ph} D_{p'h'}$$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation:

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

RPA on a schematic model

$$\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$$

$$\langle pp' | \bar{v} | hh' \rangle = \lambda D_{ph} D_{p'h'}$$

Separable interaction:

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(RPA dispersion relation)

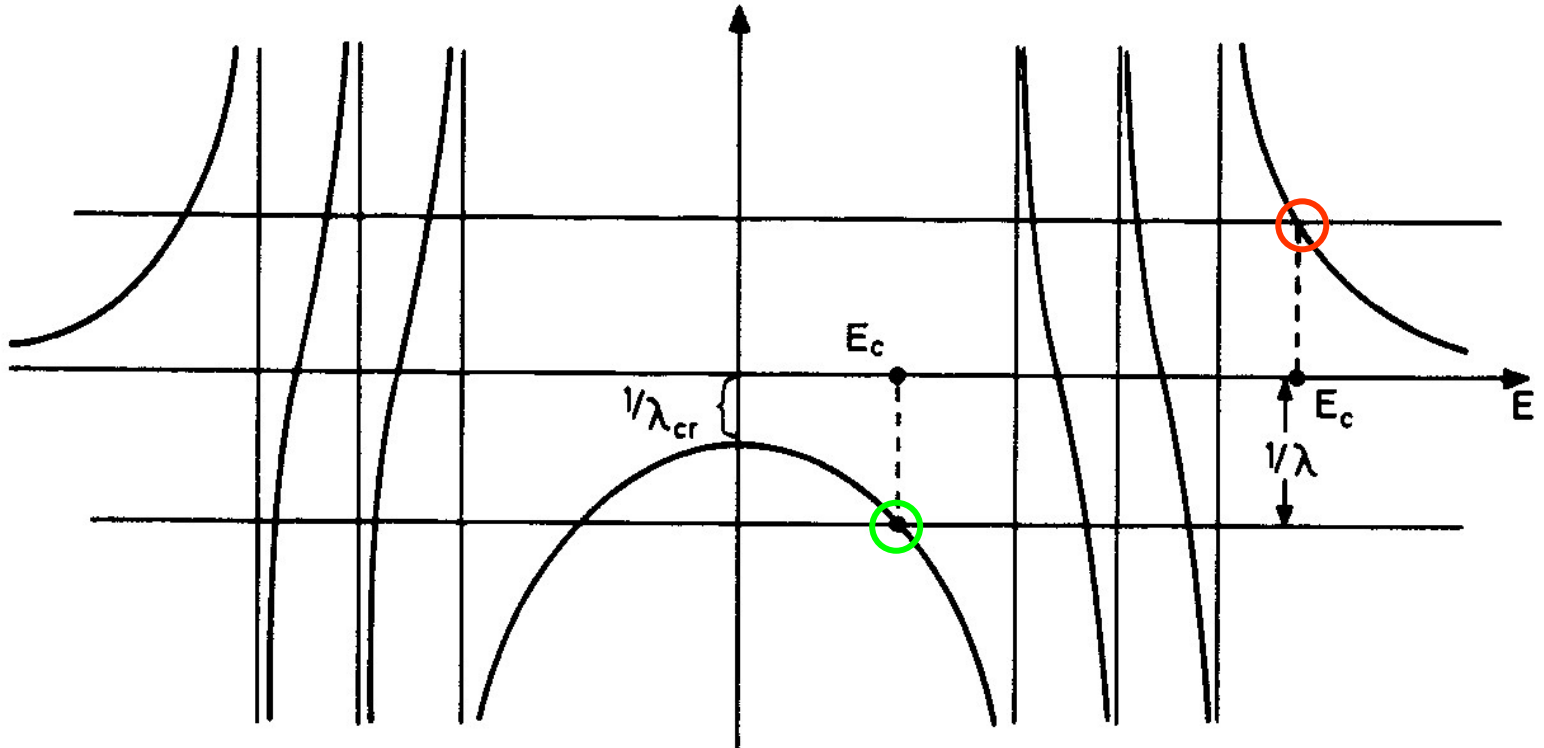



Figure 8.11. Graphical solution of the dispersion relation (8.135).

RPA on a schematic model

$$\begin{aligned}\langle ph' | \bar{v} | hp' \rangle &= \lambda D_{ph} D_{p'h'}^* \\ \langle pp' | \bar{v} | hh' \rangle &= \lambda D_{ph} D_{p'h'}\end{aligned}$$


Separable interaction:


$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

$\epsilon_{ph} = \epsilon$ のとき、

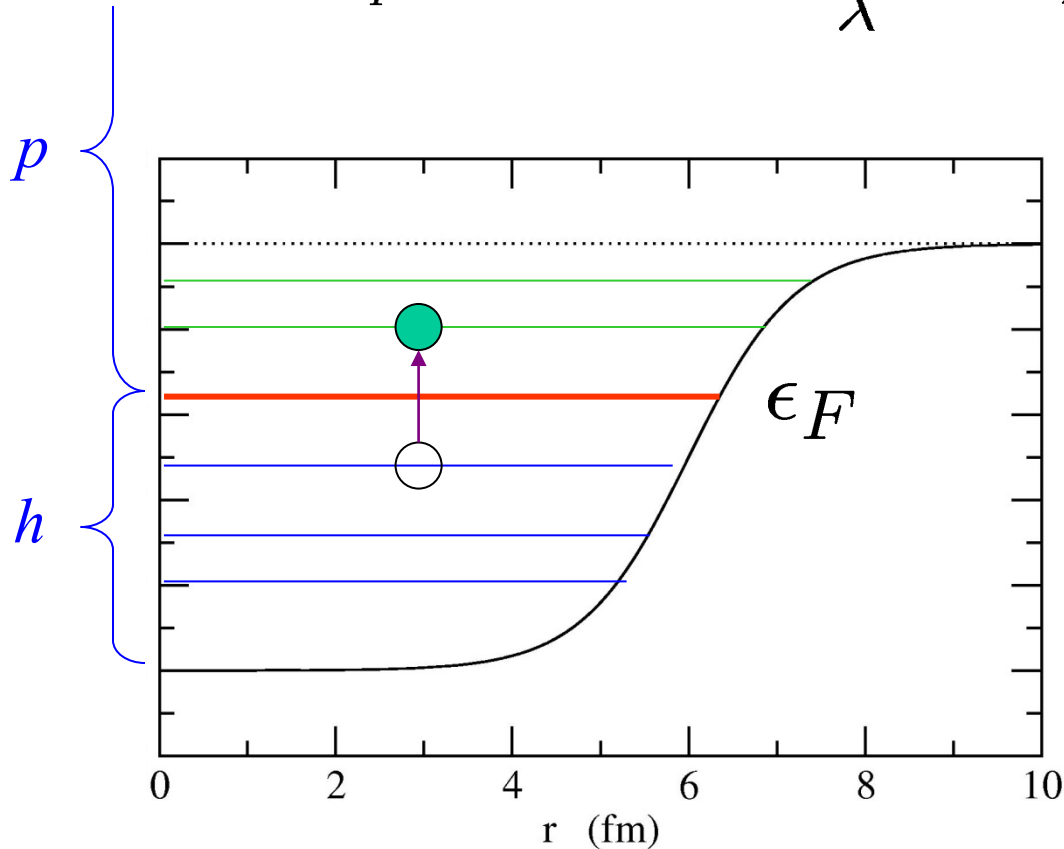
$$\frac{1}{\lambda} = \sum_{ph} |D_{ph}|^2 \left(\frac{1}{E - \epsilon} - \frac{1}{E + \epsilon} \right) = \sum_{ph} |D_{ph}|^2 \frac{2\epsilon}{E^2 - \epsilon^2}$$


$$E^2 = \epsilon^2 + 2\epsilon\lambda \sum_{ph} |D_{ph}|^2$$

λ が負(引力)だと、どこかで $E^2 < 0$ となる

Continuum Excitations

$$\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^* \longrightarrow \frac{1}{\lambda} = - \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$

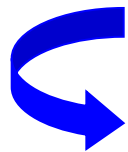


h : all the occupied (bound) states

p : the bound excited states + continuum states

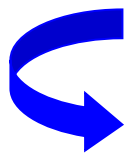
$$\frac{1}{\lambda} = - \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E} = - \sum_{ph} \langle \phi_h | D^\dagger | \phi_p \rangle \frac{1}{\epsilon_p - \epsilon_h - E} \langle \phi_p | D | \phi_h \rangle$$

(note) $\hat{h}\phi_p = \epsilon_p\phi_p$



$$\frac{1}{\lambda} = - \sum_{ph} \langle \phi_h | D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} | \phi_p \rangle \langle \phi_p | D | \phi_h \rangle$$

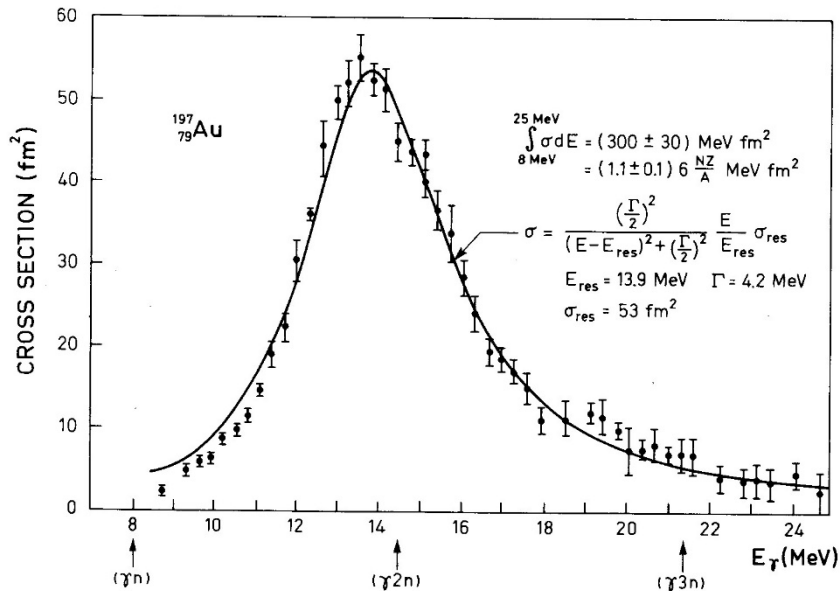
$$1 = \sum_i |\phi_i\rangle\langle\phi_i| = \sum_p |\phi_p\rangle\langle\phi_p| + \sum_h |\phi_h\rangle\langle\phi_h|$$



$$\frac{1}{\lambda} = - \sum_h \langle \phi_h | D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} \left[1 - \sum_{h'} |\phi_{h'}\rangle\langle\phi_{h'}| \right] D | \phi_h \rangle$$

particle 状態の和がなくなった→連続状態もすべて自動的に入る

巨大共鳴の幅



i) 連続状態との結合 (粒子放出)

escape width Γ^\uparrow

continuum RPA

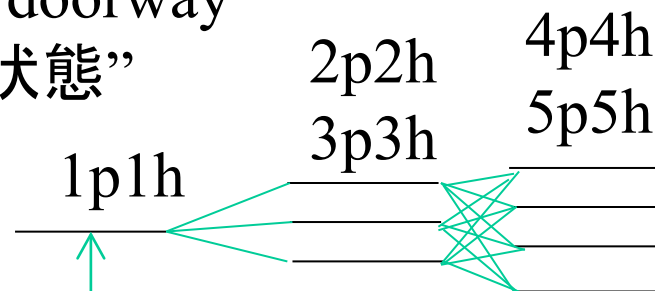
ii) より複雑な状態との結合

spreading width Γ^\downarrow

軽い核を除き
幅の主成分

1体演算子

“doorway
状態”



レポート問題4 (×切: 12月2日(土))

1. 分離型相互作用

$$\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$$
$$\langle pp' | \bar{v} | hh' \rangle = \lambda D_{ph} D_{p'h'}$$

の場合にRPA方程式を解き、RPA dispersion relation を導け。

2. RPA のA 行列、B行列が実行列のとき、 $X+Y$ の従う方程式を導け。

3. RPA のA 行列、B行列が

$$A = \begin{pmatrix} \epsilon & g \\ g & \epsilon \end{pmatrix}, \quad B = \begin{pmatrix} 0 & g \\ g & 0 \end{pmatrix}$$

で与えられているときに RPA方程式を解き、(正の)固有値2つを求めよ。

原子核反応論

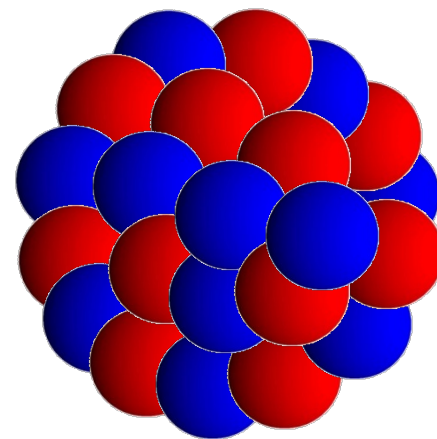
□ 原子核物理:核子多体系としての原子核の振る舞い

← 核子間相互作用から理解する

➤ 静的な振る舞い:原子核構造論

- ✓ 基底状態の性質
(質量、大きさ、形など)
- ✓ 励起状態の性質

➤ ダイナミクス:原子核反応論



原子核反応論

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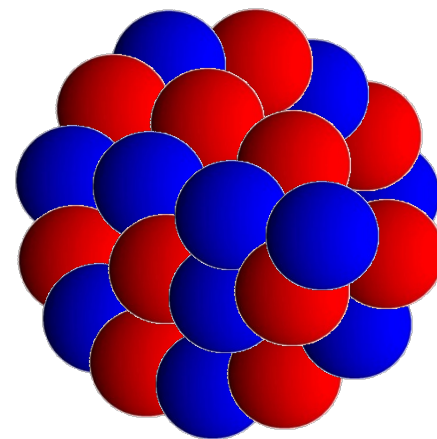
- ✓ 基底状態の性質
(質量、大きさ、形など)
- ✓ 励起状態の性質

➤ ダイナミクス:原子核反応論

原子核は複合粒子

- ✓ 豊富な反応様式

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応



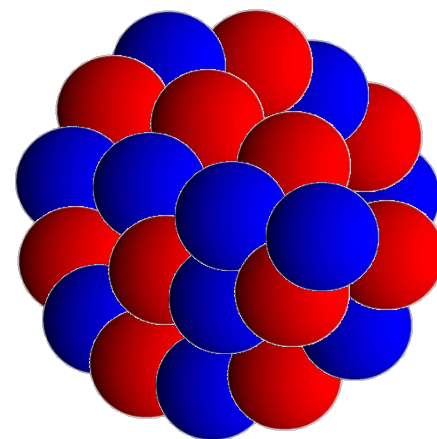
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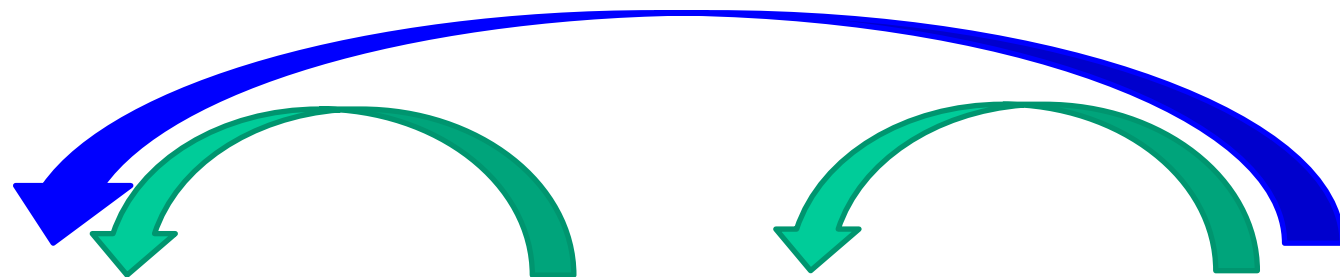
➤ ダイナミクス: 原子核反応論

原子核は複合粒子

- ✓ 豊富な反応様式
- ✓ 核構造と核反応の織り成す様々なインタープレイ

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応

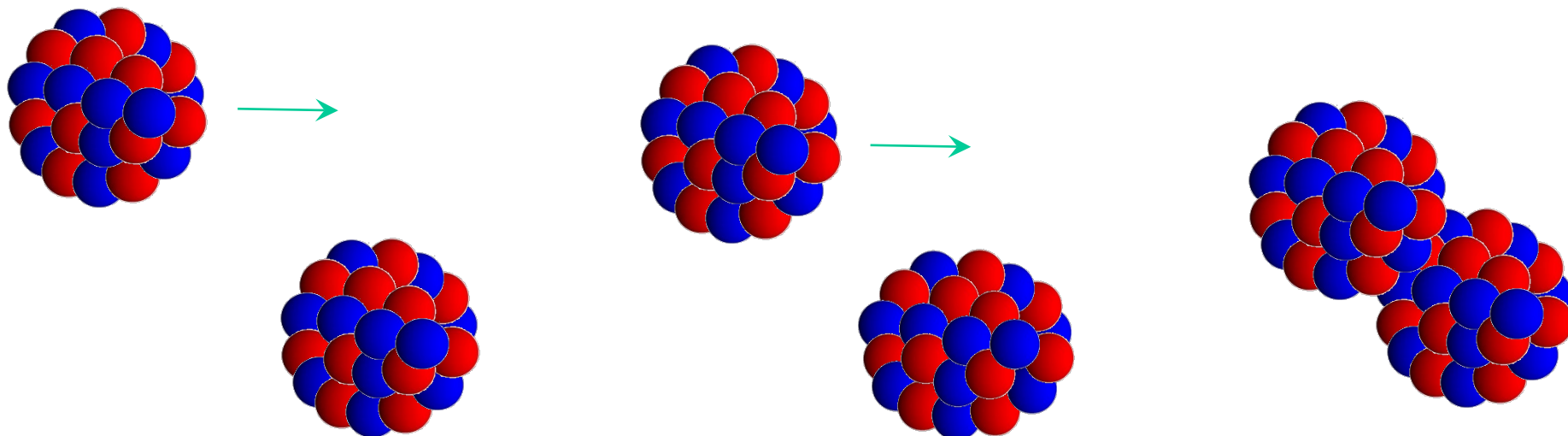
量子多体系のダイナミクス(原子核反応)



弾性散乱

非弾性散乱

核融合



これらを統一的に記述するのが
核反応論の醍醐味

