

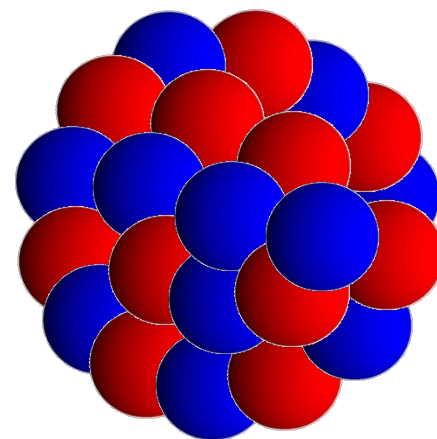
原子核反応論

□ 原子核物理: 核子多体系としての原子核の振る舞い

← 核子間相互作用から理解する

➤ 静的な振る舞い: 原子核構造論

- ✓ 基底状態の性質
(質量、大きさ、形など)
- ✓ 励起状態の性質



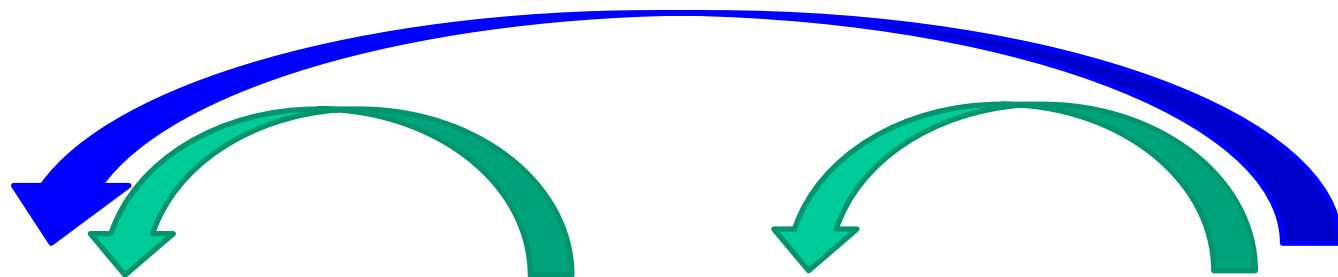
➤ ダイナミクス: 原子核反応論

原子核は複合粒子

- ✓ 豊富な反応様式
- ✓ 核構造と核反応の織り成す様々なインタープレイ

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応

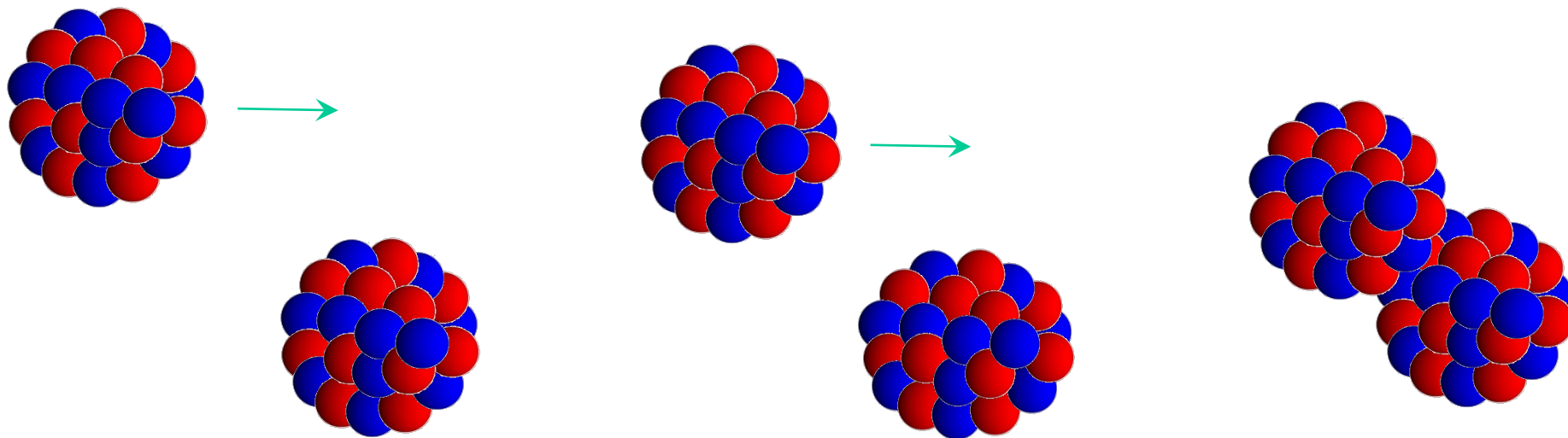
量子多体系のダイナミクス(原子核反応)



弾性散乱

非弾性散乱

核融合

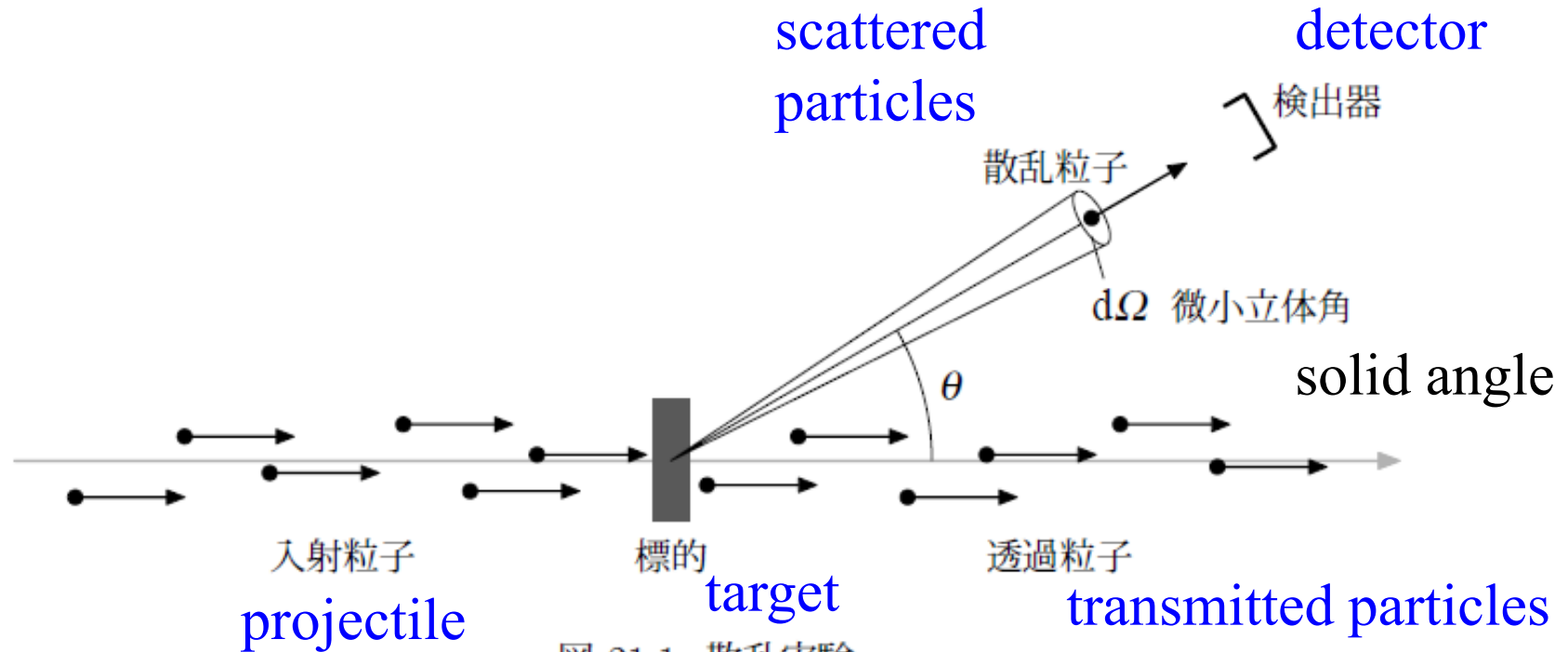


これらを統一的に記述するのが
核反応理論



Nuclear Reactions

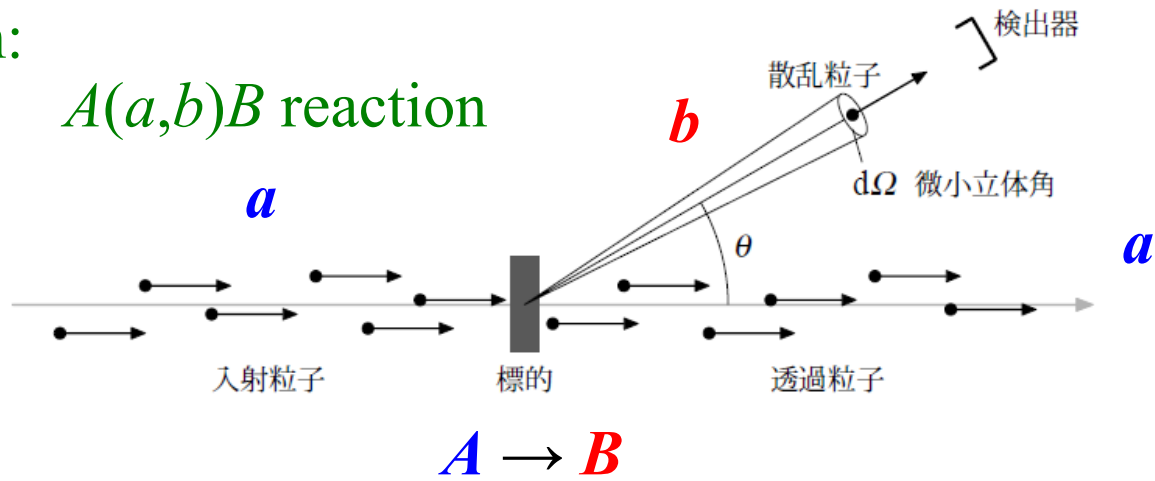
Shape, interaction, and excitation structures of nuclei ← scattering expt.
cf. Experiment by Rutherford (α scatt.)



http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

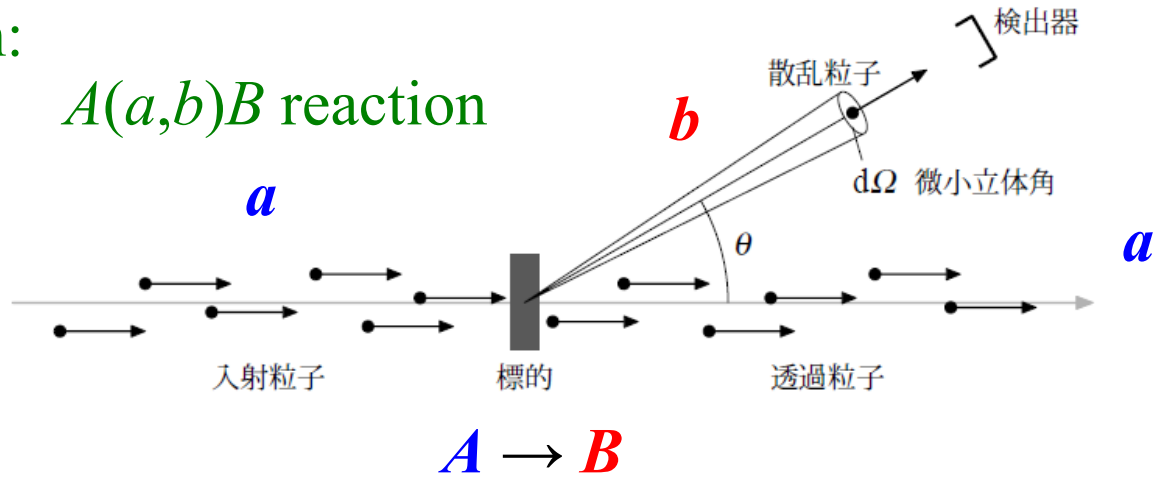
notation:

$A(a,b)B$ reaction

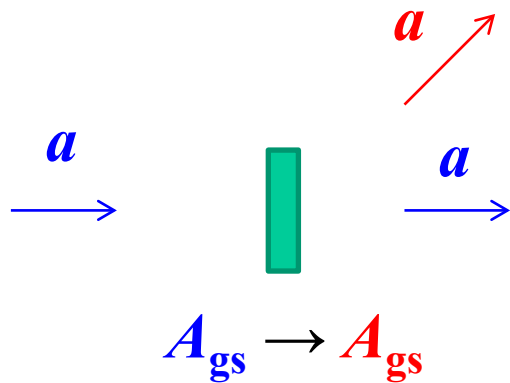


notation:

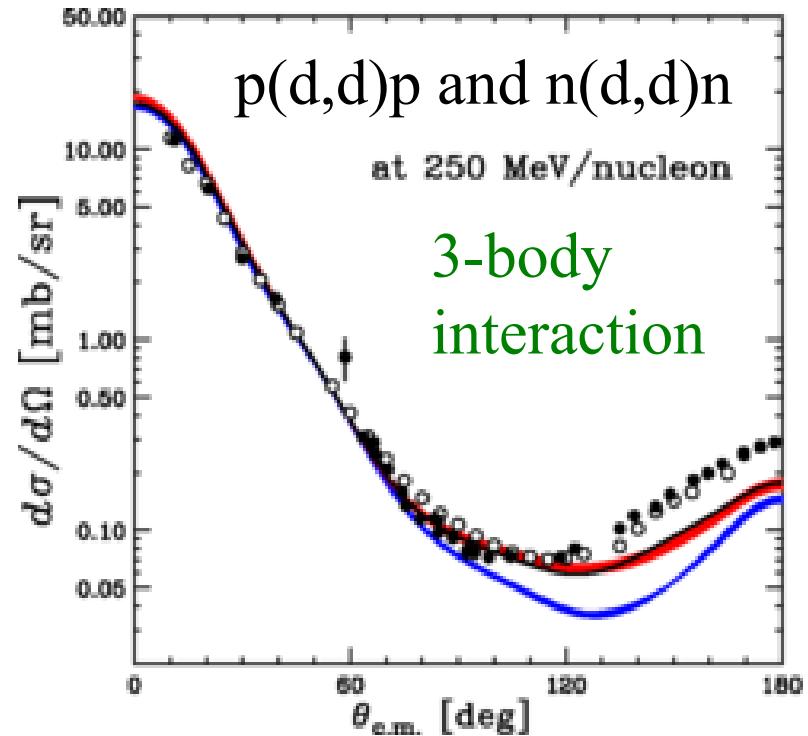
$A(a,b)B$ reaction

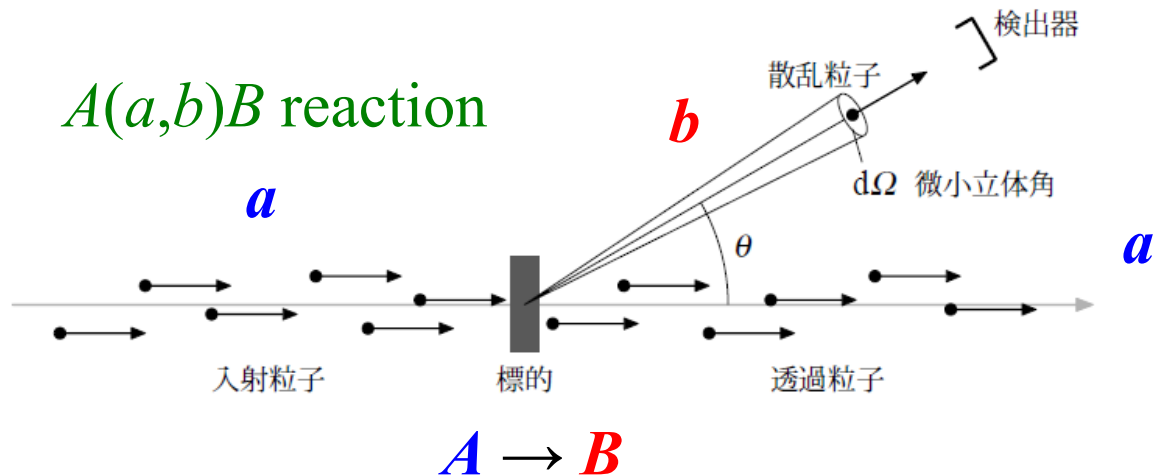


✓ elastic scattering

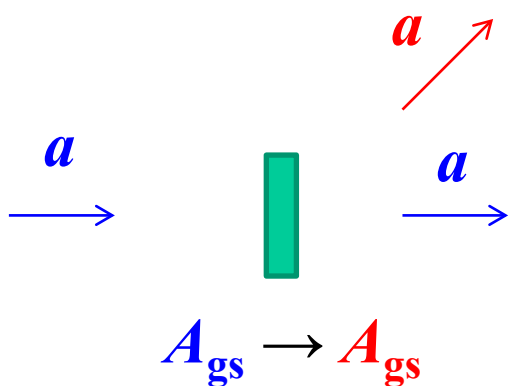


fundamental interaction
between a and A



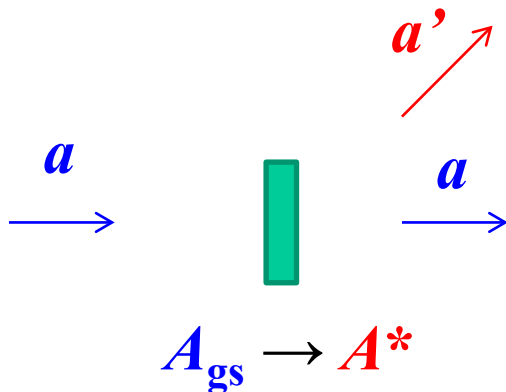


✓ elastic scattering

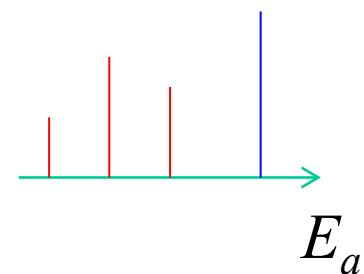


fundamental interaction
between a and A

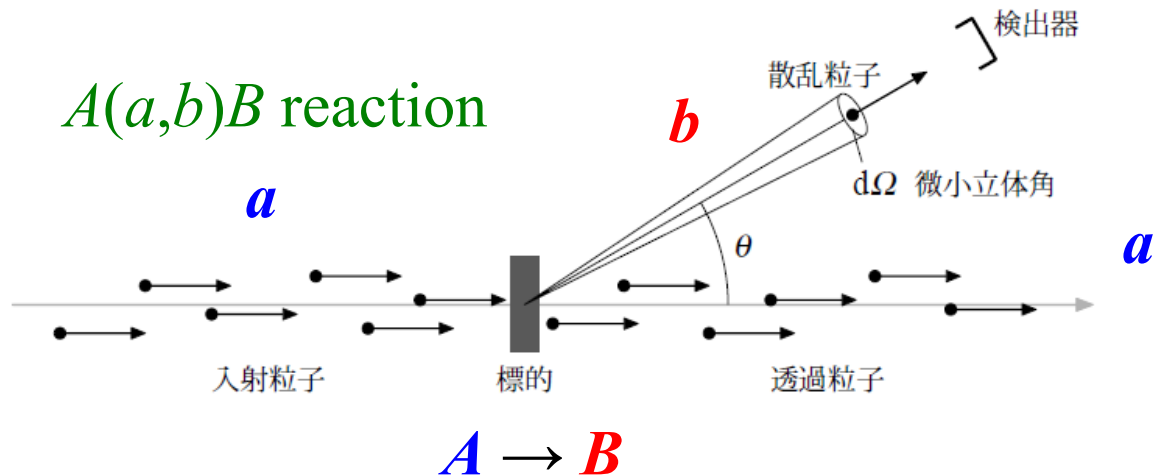
✓ inelastic scattering



excitation spectrum
of a nucleus A

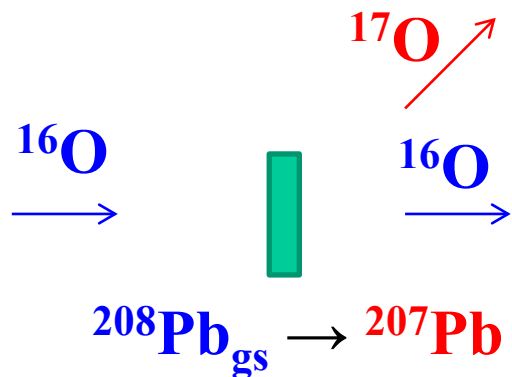


励起状態の
✓ エネルギー
✓ 角運動量



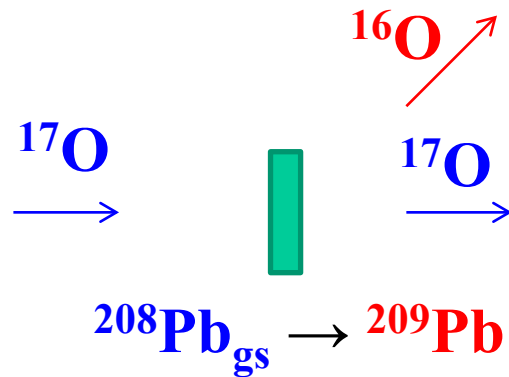
transfer reactions

✓ transfer reaction
(pick-up reaction)



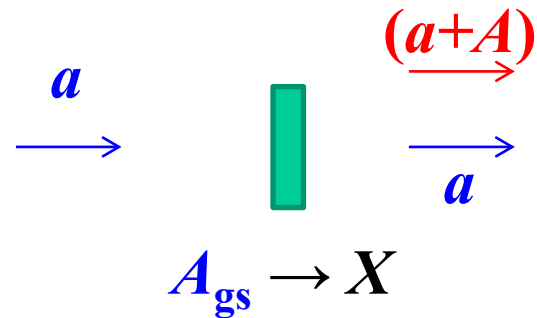
level schem of ^{207}Pb

✓ transfer reaction
(stripping reaction)



level schem of ^{209}Pb

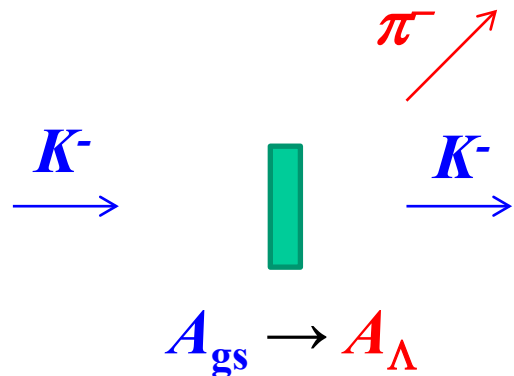
✓ fusion reaction



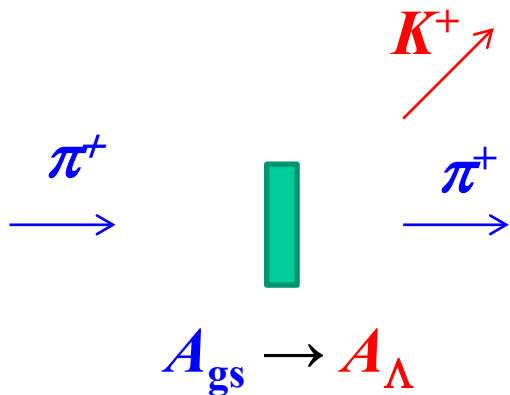
- interaction between a and A
- structure of a and A
- 未知核の生成

hypernucleus production reactions

✓ (K^-,π^-) reaction

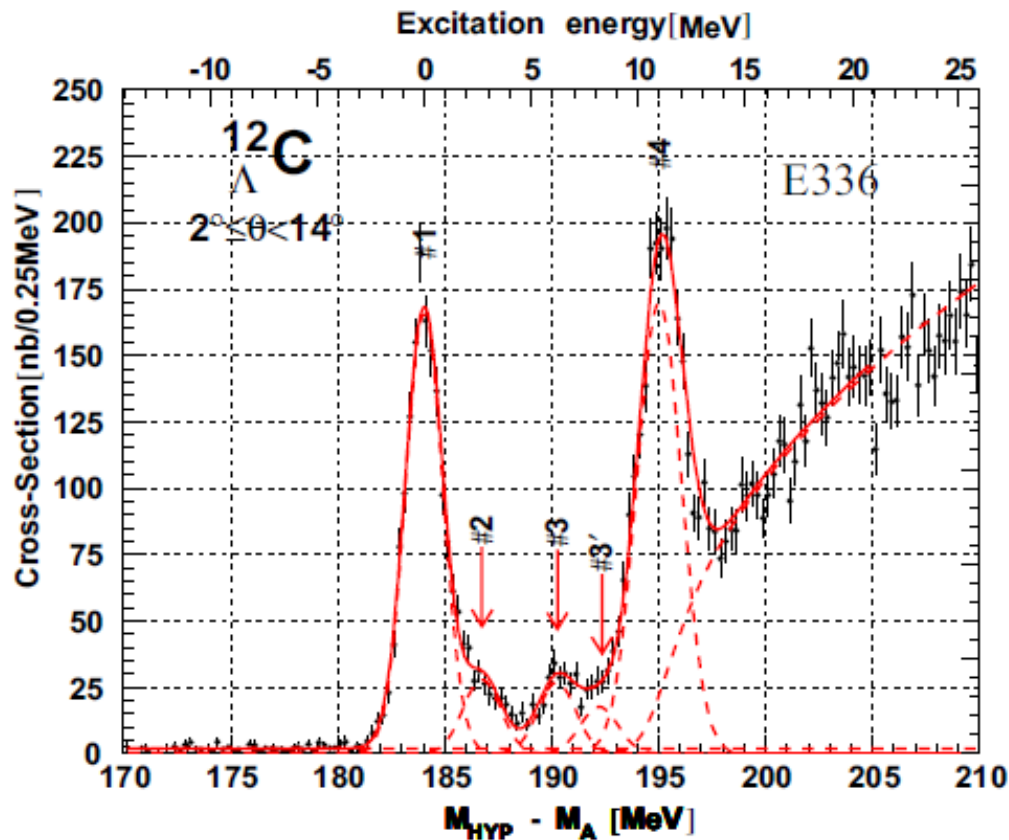


✓ (π^+,K^+) reaction



excitation spectrum
of a hypernucleus A_{Λ}

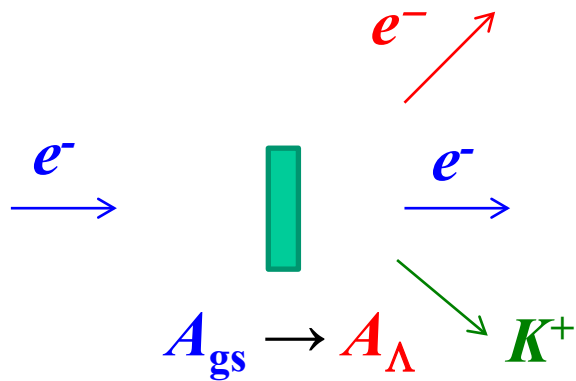
$^{12}\text{C} (\pi^+,K^+) ^{12}_{\Lambda}\text{C}$ reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 ('06)564

“reaction spectroscopy”

✓(e,e'K⁺) reaction

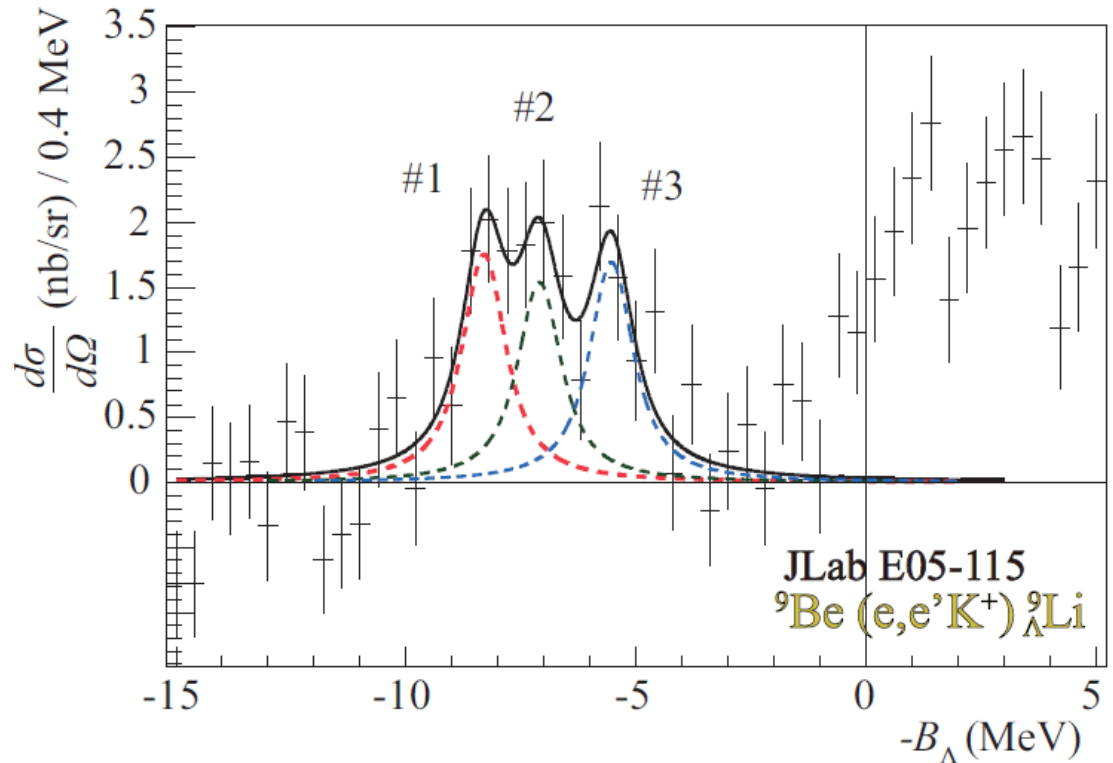


S.N. Nakamura et al.,
PRL110('13)012502

T. Gogami,
Ph.D. Thesis (Tohoku U.)
2014

K.N. Suzuki, T. Gogami et al.,
PTEP2022 (2022) 013D01

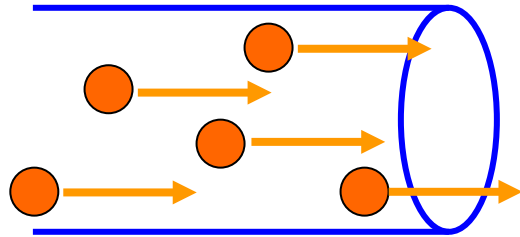
${}^9\text{Be}(e, e' K^+) {}^9_\Lambda\text{Li}$



T. Gogami et al., PRC103('21)L041301

${}^3\text{He}(e, e' K^+) nn\Lambda$

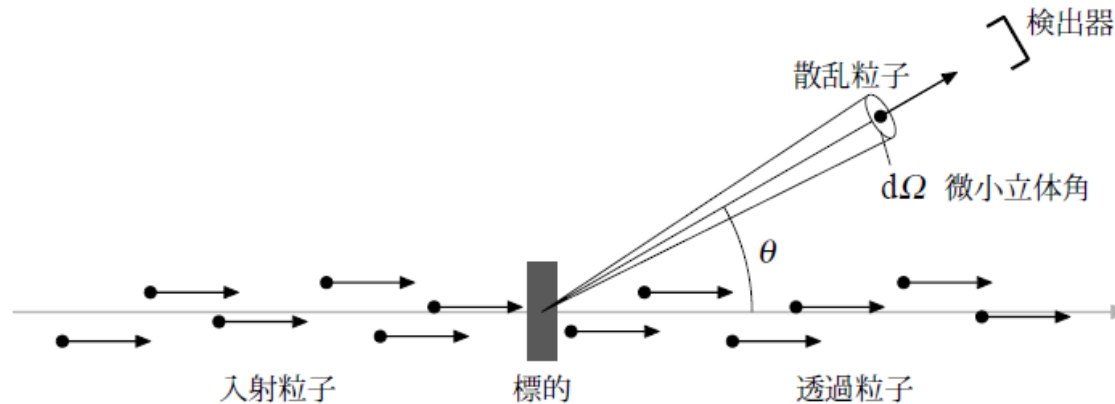
Cross sections



incident beam

flux = the number of particles
crossing unit area
per unit time

$$j = \rho_P \cdot v$$

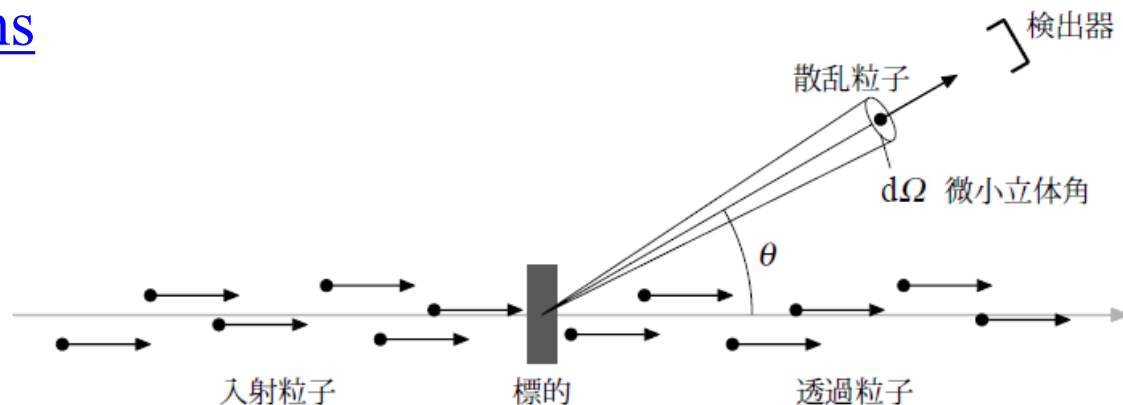


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow R = N_T \cdot \sigma \cdot j$$

← cross section

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

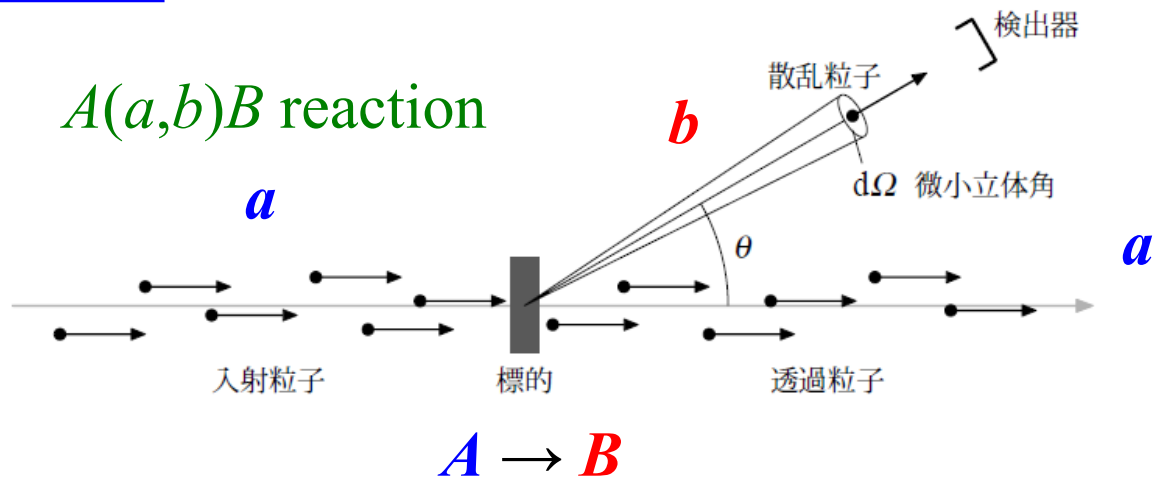
$$\longrightarrow R = N_T \cdot \sigma \cdot j \quad \leftarrow \text{cross section}$$

differential cross sections (angular distribution)

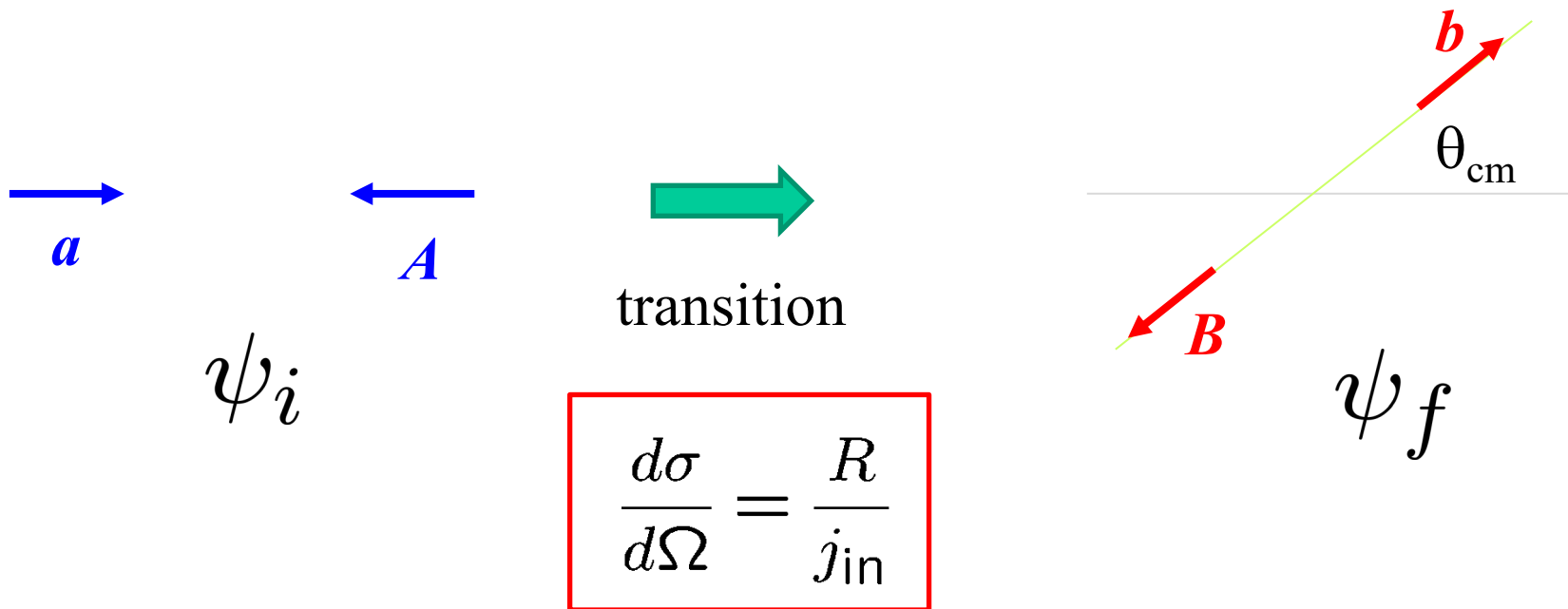
$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega \quad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm² = 100 fm² (1 mb = 10^{-3} b = 0.1 fm²)

断面積 (量子力学)



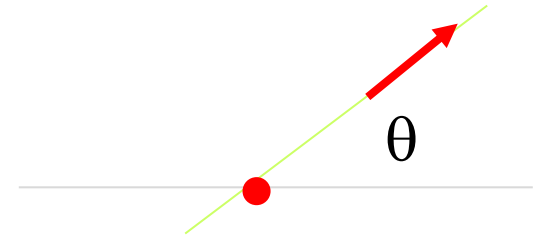
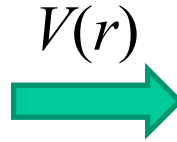
center of mass frame



Born approximation

$$\psi_f(\mathbf{r}) = e^{i\mathbf{p}_f \cdot \mathbf{r} / \hbar}$$

$$\psi_i(\mathbf{r}) = e^{i\mathbf{p}_i \cdot \mathbf{r} / \hbar}$$



$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \underline{V(r)} - E \right) \psi(\mathbf{r}) = 0$$

perturbation

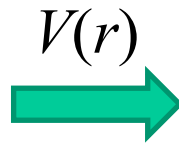
transition rate for elastic scattering:

$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}_f}{(2\pi\hbar)^3} |\langle \psi_f | V | \psi_i \rangle|^2 \delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2 \hbar^4} \int d\Omega |\tilde{V}(\mathbf{q})|^2 \end{aligned}$$

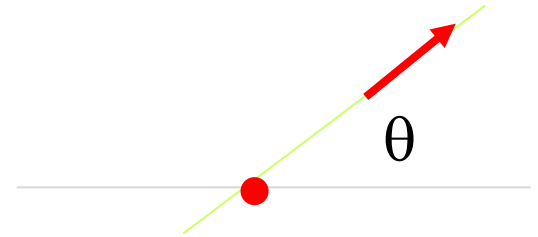
$$\tilde{V}(\mathbf{q}) = \int d\mathbf{r} e^{i(\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{r} / \hbar} V(r) \equiv \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} V(r)$$

Born approximation

$$\psi_i(\mathbf{r}) = e^{i\mathbf{p}_i \cdot \mathbf{r} / \hbar}$$



$$\psi_f(\mathbf{r}) = e^{i\mathbf{p}_f \cdot \mathbf{r} / \hbar}$$



$$W_{fi} = \frac{\mu p_i}{4\pi^2 \hbar^4} \int d\Omega |\tilde{V}(\mathbf{q})|^2$$

$$\tilde{V}(\mathbf{q}) = \int d\mathbf{r} e^{i(\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{r} / \hbar} V(r) \equiv \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} V(r)$$

momentum transfer

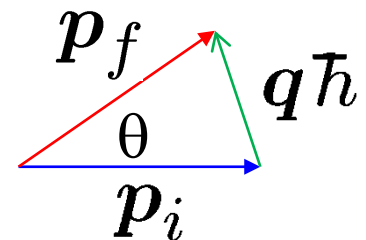


incident flux: $j_{\text{inc}} = \rho_i v = p_i / \mu$



$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \frac{\mu^2}{4\pi^2 \hbar^4} |\tilde{V}(\mathbf{q})|^2$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin \frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\mathbf{r}' \frac{\rho_{\text{ch}}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

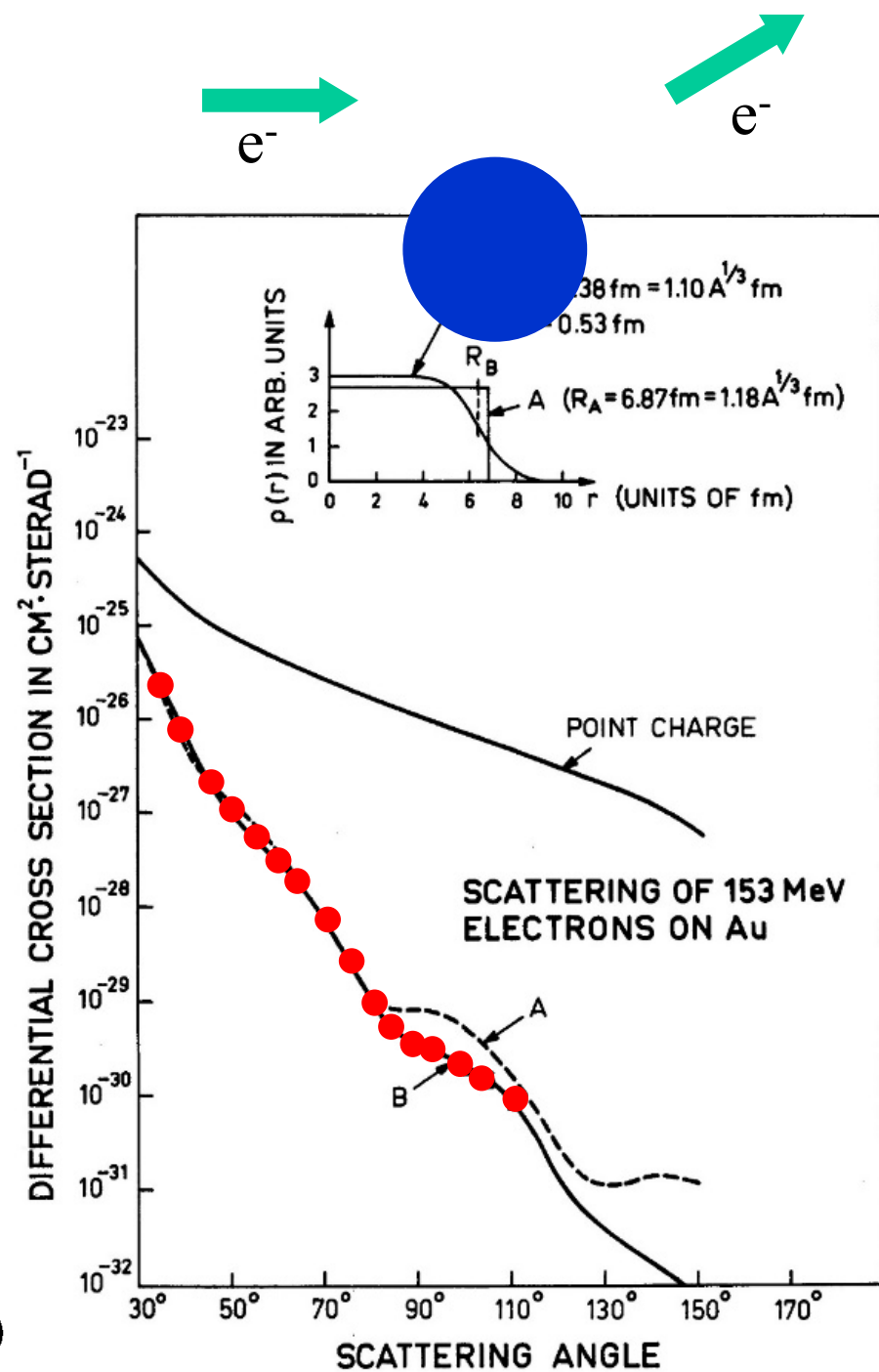
$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{e^4}{(4E \sin^2 \theta/2)^2} |F(\mathbf{q})|^2 \\ &= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega} \right) |F(\mathbf{q})|^2 \end{aligned}$$

Form factor

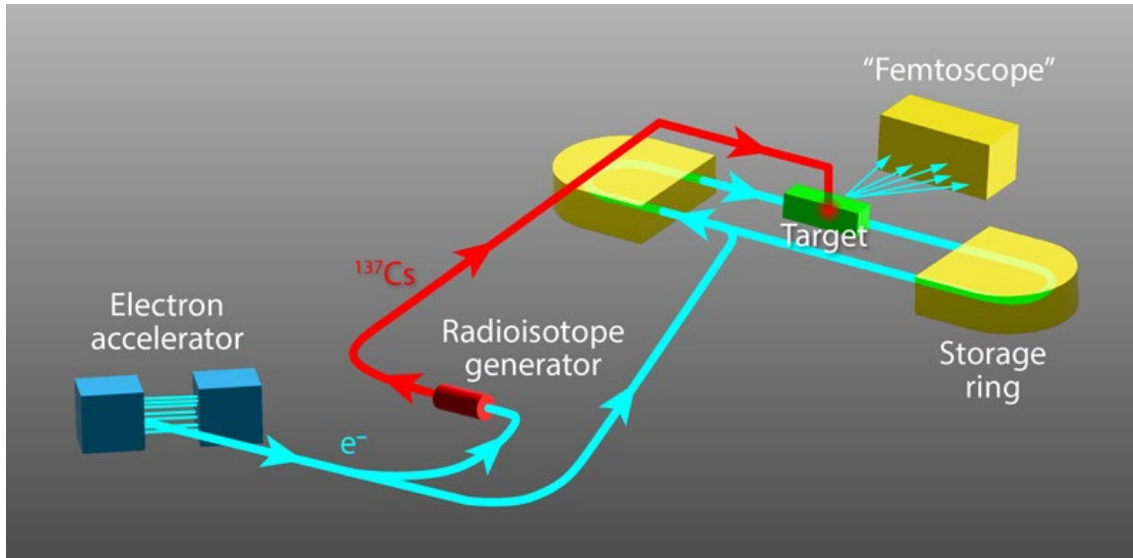
$$F(\mathbf{q}) = \int e^{-i\mathbf{q}\cdot\mathbf{r}} \rho_{\text{ch}}(\mathbf{r}) d\mathbf{r}$$

* relativistic correction:

$$\begin{aligned} \frac{d\sigma_{\text{Ruth}}}{d\Omega} &\rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega} \\ &= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2 \frac{\theta}{2} \right) \\ &\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2 \frac{\theta}{2} \quad (v \rightarrow c) \end{aligned}$$

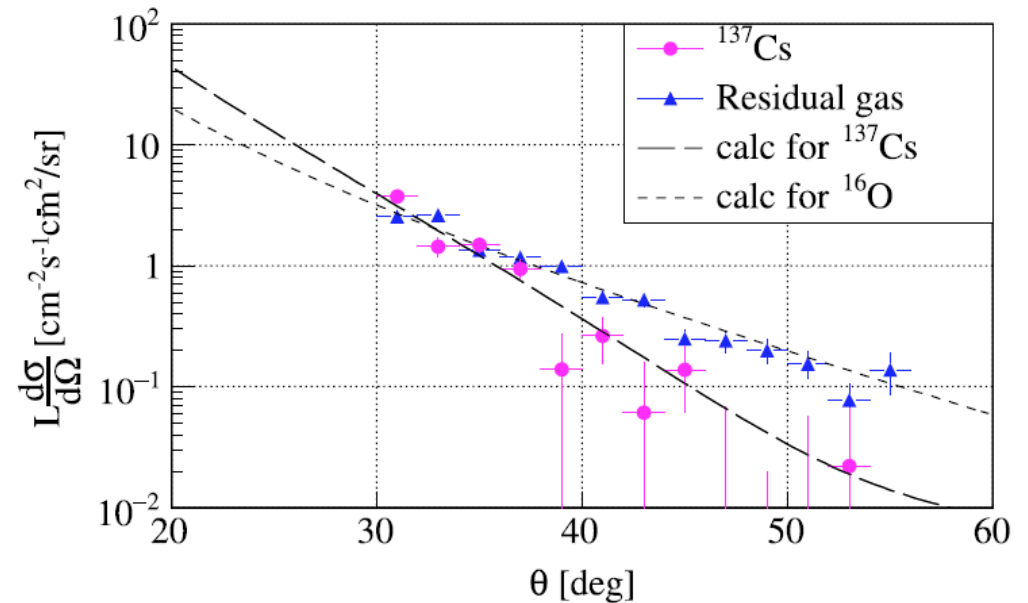


cf. electron scattering off unstable nuclei (SCRIT)



^{137}Cs ($T_{1/2}=30.08$ y)

K. Tsukada et al.,
PRL131, 092502 (2023)



レポート問題5(×切:12月2日(土))

電子と原子核の相互作用が

$$V(\mathbf{r}) = -e^2 \int d\mathbf{r}' \frac{\rho_{\text{ch}}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

で与えられているとする。ここで、 ρ_{ch} は原子核の電荷密度で

$$\int d\mathbf{r} \rho_{\text{ch}}(\mathbf{r}) = Z$$

と規格化されているとする。ボルン近似を用いて弾性散乱の断面積を求め、

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E \sin^2 \theta/2} \right)^2 |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \int e^{-i\mathbf{q}\cdot\mathbf{r}} \rho_{\text{ch}}(\mathbf{r}) d\mathbf{r}$$

となることを示せ。

レポート問題6 (×切: 12月2日(土))

q が小さいところで電子散乱の形状因子 $F(q)$ を決めることにより、原子核の荷電半径

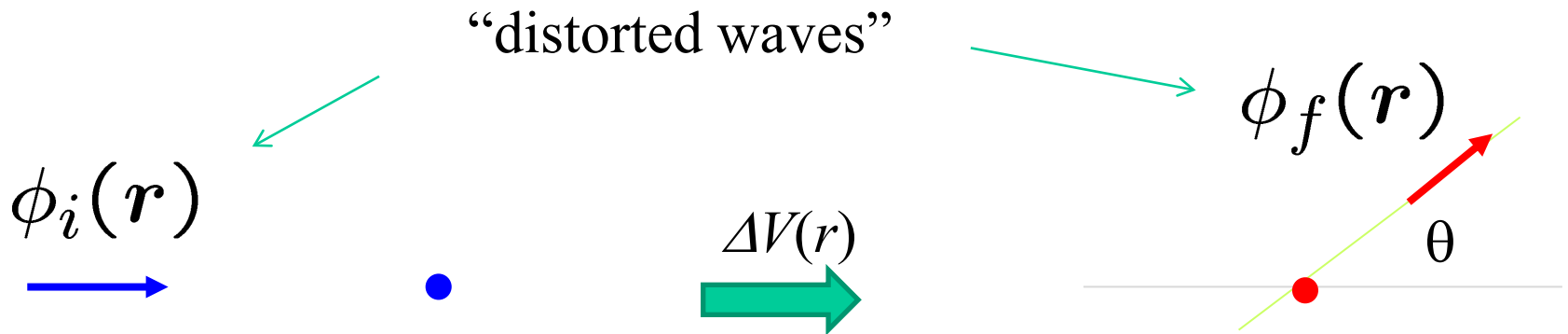
$$\langle r^2 \rangle = \frac{1}{Z} \int dr r^2 \rho_{\text{ch}}(r)$$

を求められることを示せ。

Distorted Wave Born approximation (DWBA)

$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \underbrace{V(r)}_{\text{perturbation}} - E \right) \psi(\mathbf{r}) = 0$$

→
$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \underbrace{V_0(r)}_{\text{unperturbed}} + \underbrace{V(r) - V_0(r)}_{\text{perturbation}} - E \right) \psi(\mathbf{r}) = 0$$

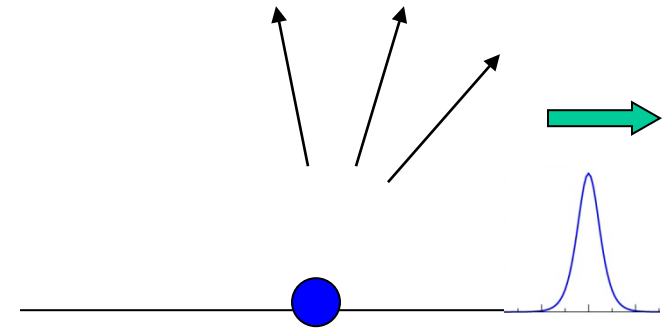
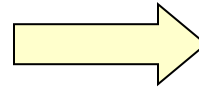


- ✓ inelastic scattering
- ✓ transfer reactions

How to choose $V_0(r)$? : Optical model

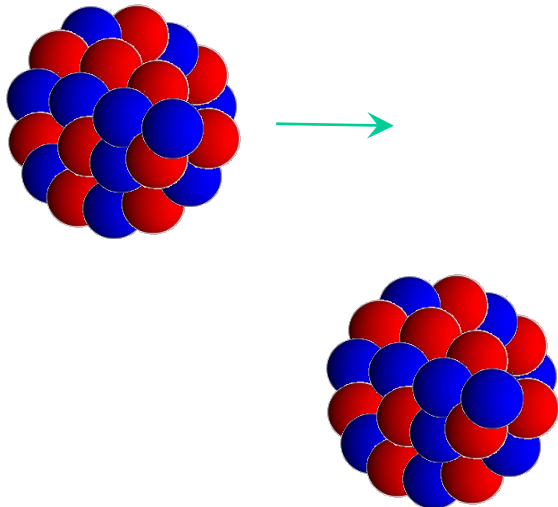
Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)

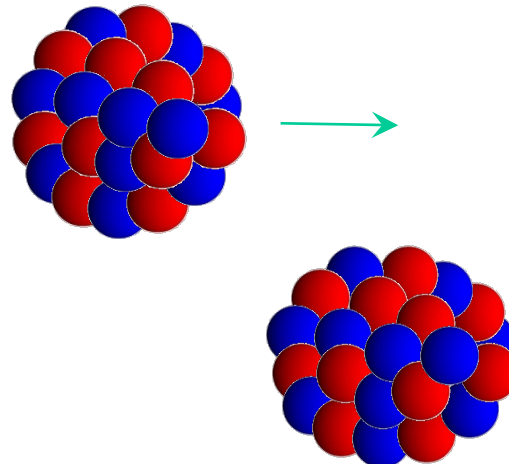


Loss of incident flux
(absorption)

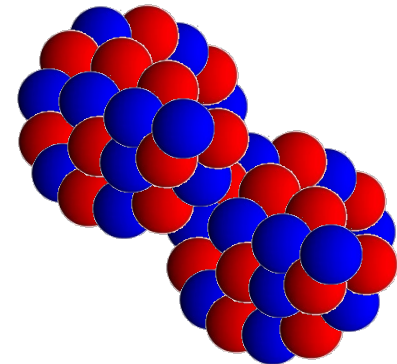
弹性散乱



非弹性散乱



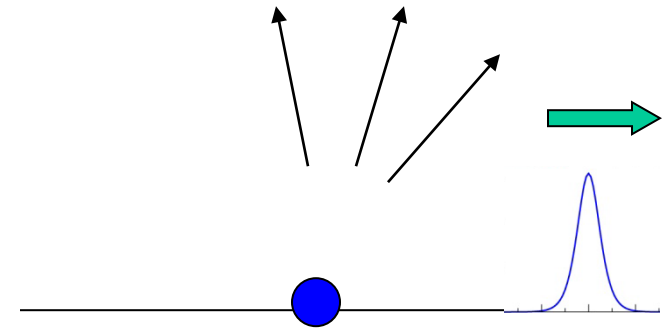
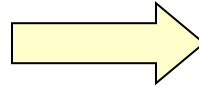
核融合



How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux
(absorption)

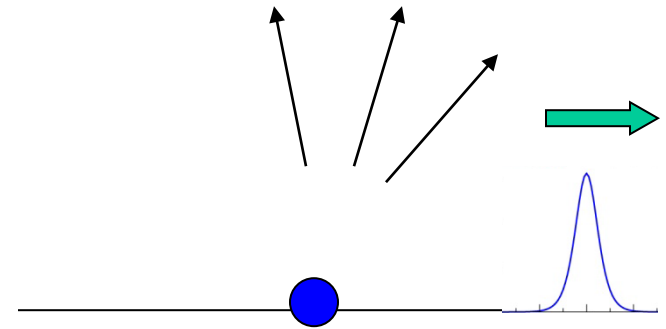
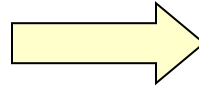
Optical potential

$$V_{\text{opt}}(\mathbf{r}) = V(\mathbf{r}) - iW(\mathbf{r}) \quad (W > 0)$$

How to choose $V_0(r)$? : Optical model

Reaction processes

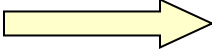
- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux
(absorption)

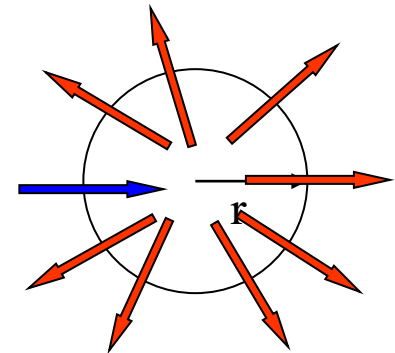
Optical potential

$$V_{\text{opt}}(\mathbf{r}) = V(\mathbf{r}) - iW(\mathbf{r}) \quad (W > 0)$$


$$\nabla \cdot \mathbf{j} = \dots = -\frac{2}{\hbar} W |\psi|^2$$

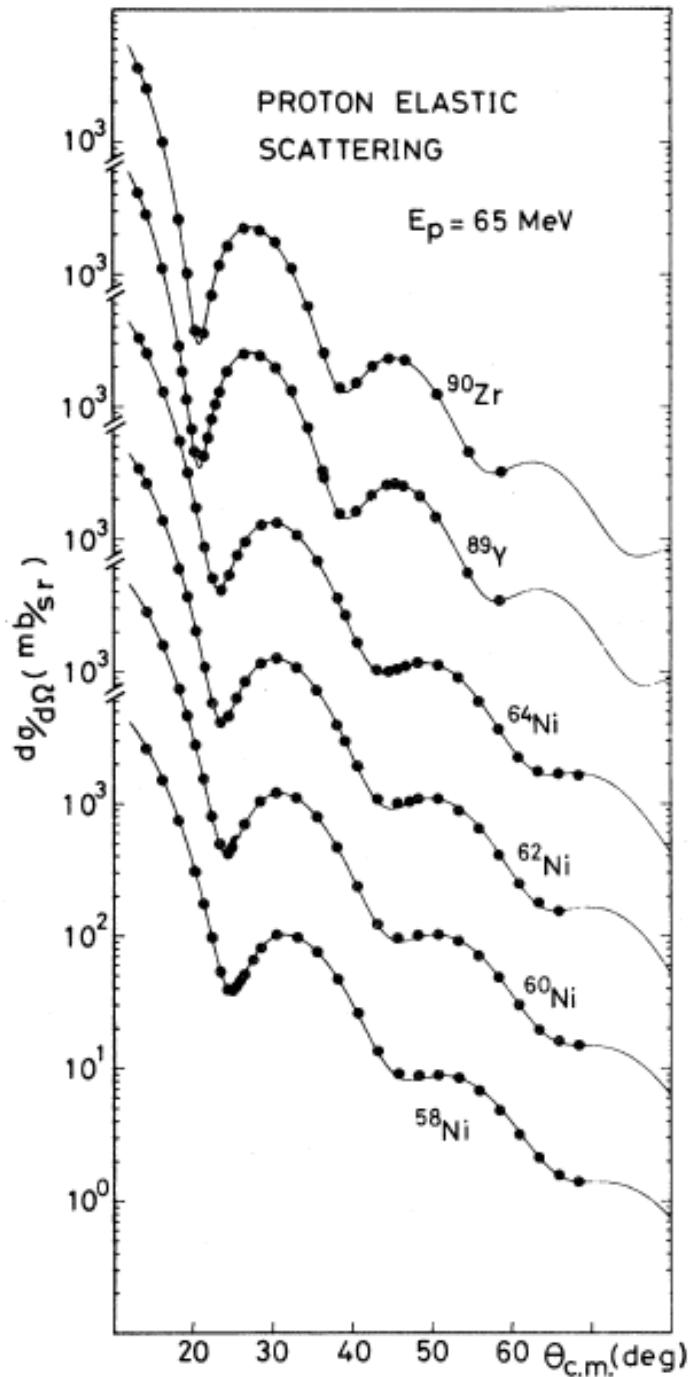
(note) Gauss's theorem

$$\int_S \mathbf{j} \cdot \mathbf{n} dS = \int_V \nabla \cdot \mathbf{j} dV$$



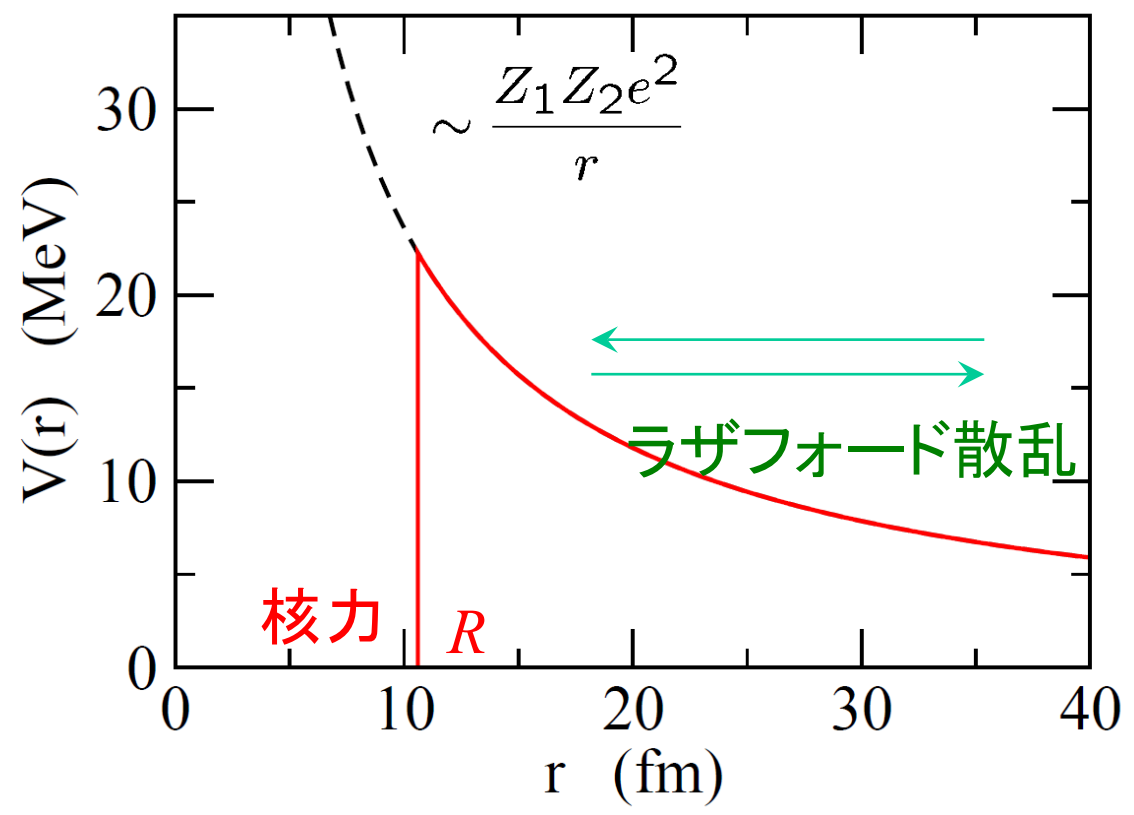
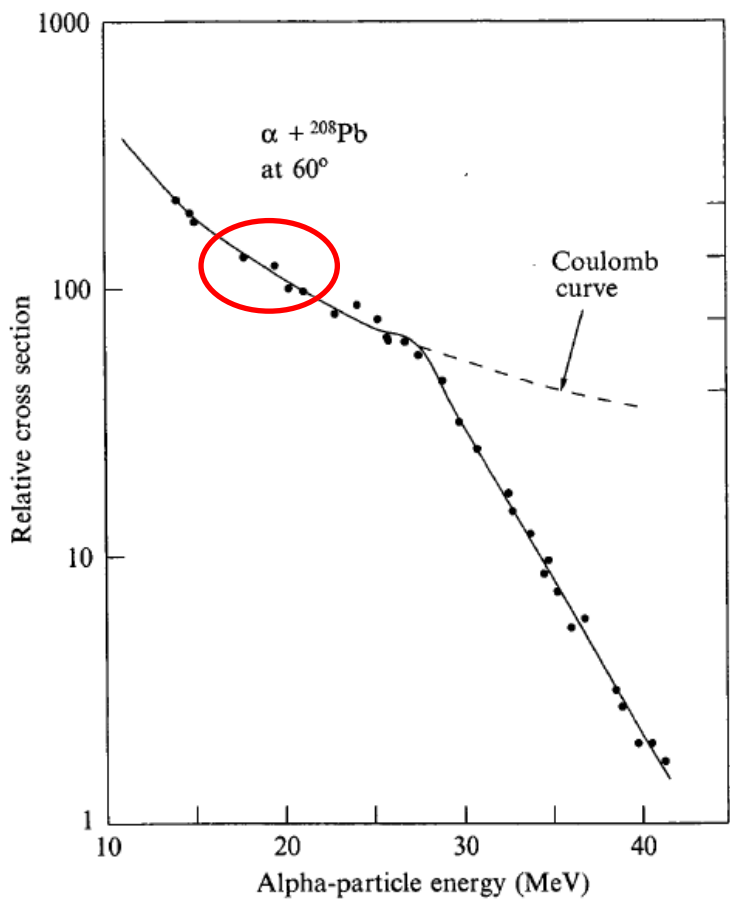
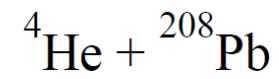
$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\text{opt}}(r) - E \right) \psi(\mathbf{r}) = 0$$

Woods-Saxon + volume & surface
imaginary parts

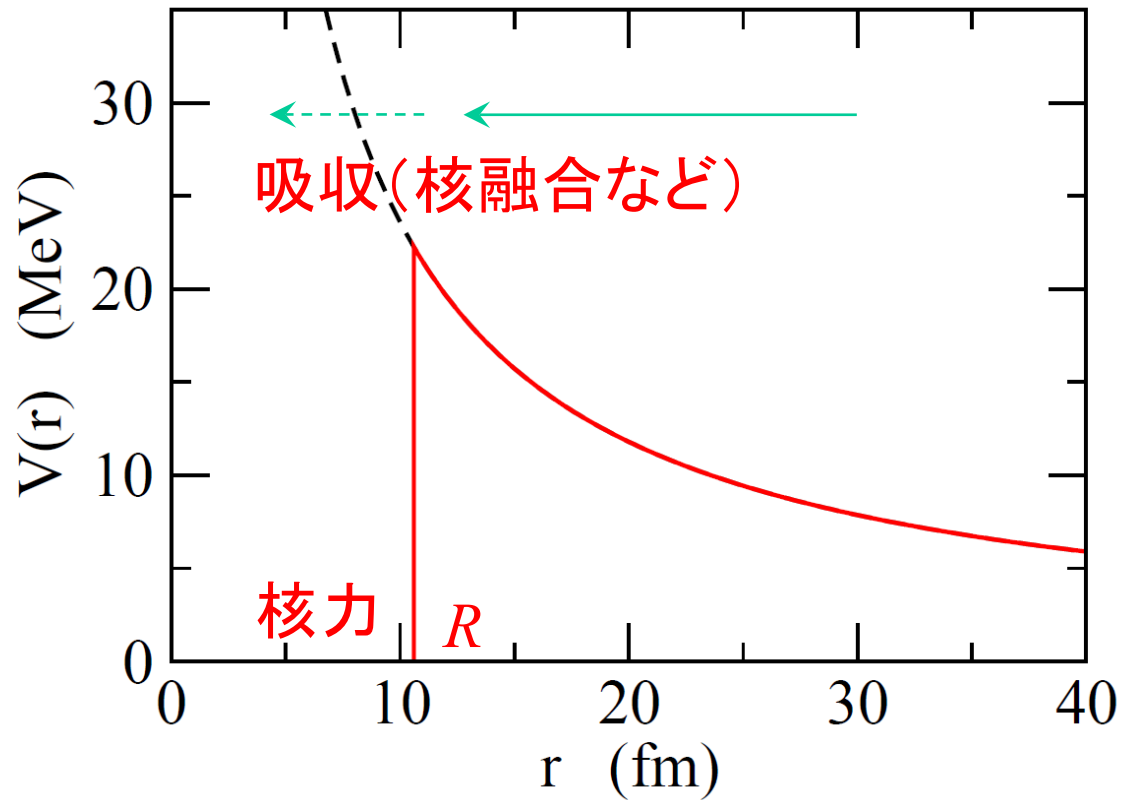
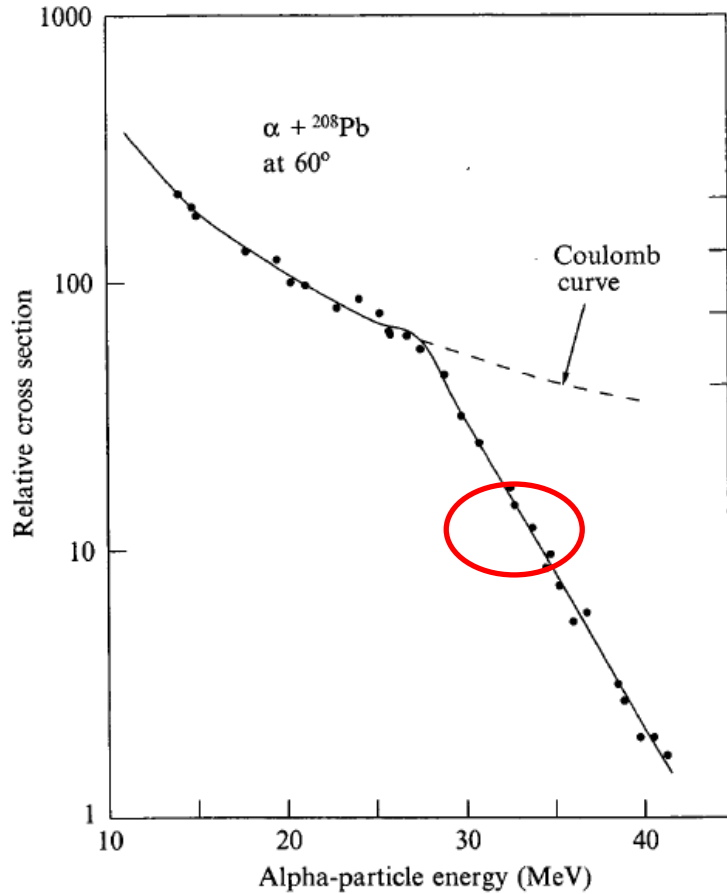
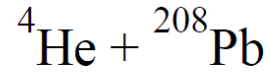


H. Sakaguchi et al.,
PRC26 (1982) 944

原子核の吸収から原子核の大きさを見る



原子核の吸収から原子核の大きさを見る



$$\rightarrow R \sim 1.41 A^{1/3} + 2.11 \text{ fm}$$

(核力のレンジや密度分布のテールの効果により
実際の半径はもう少し小さい)

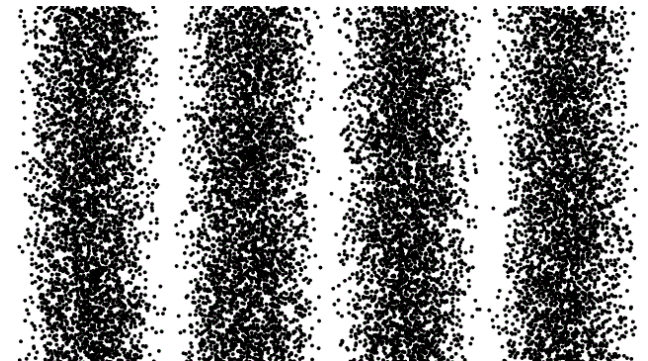
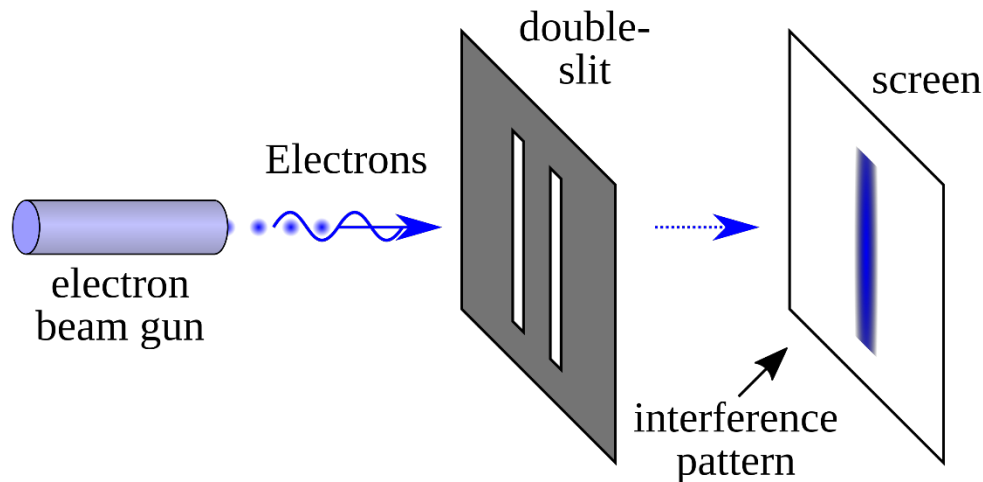
核反応は宝の山: 核反応に見られる量子性

a superposition principle

$$\psi = \alpha\psi_1 + \beta\psi_2$$

$$\rightarrow |\psi|^2 = |\alpha\psi_1|^2 + |\beta\psi_2|^2 + \underbrace{(\alpha\psi_1)^*(\beta\psi_2) + (\alpha\psi_1)(\beta\psi_2)^*}_{\text{interference}}$$

when two processes are in principle indistinguishable
→ take square after adding two amplitudes

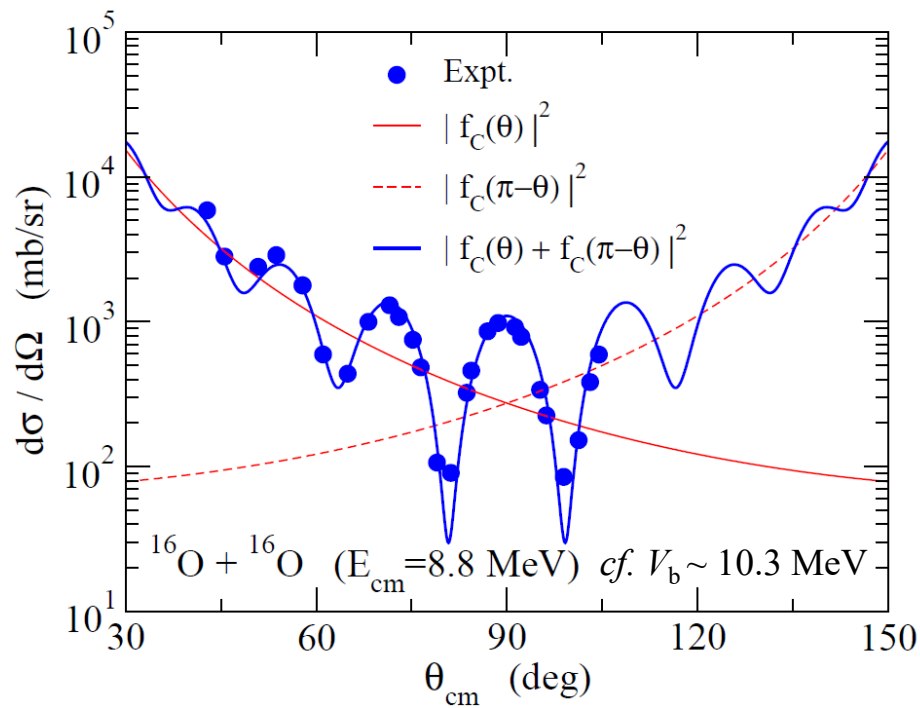


Wikipedia

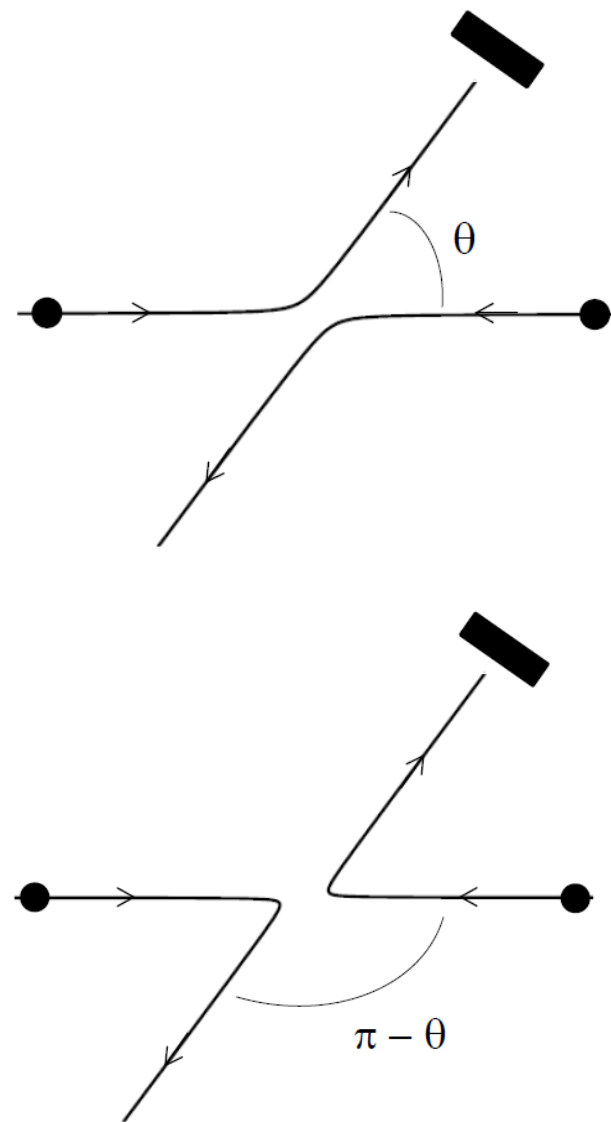
核反応は宝の山: 核反応に見られる量子性

例) モット散乱: 同種粒子の散乱

$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi - \theta)|^2$$

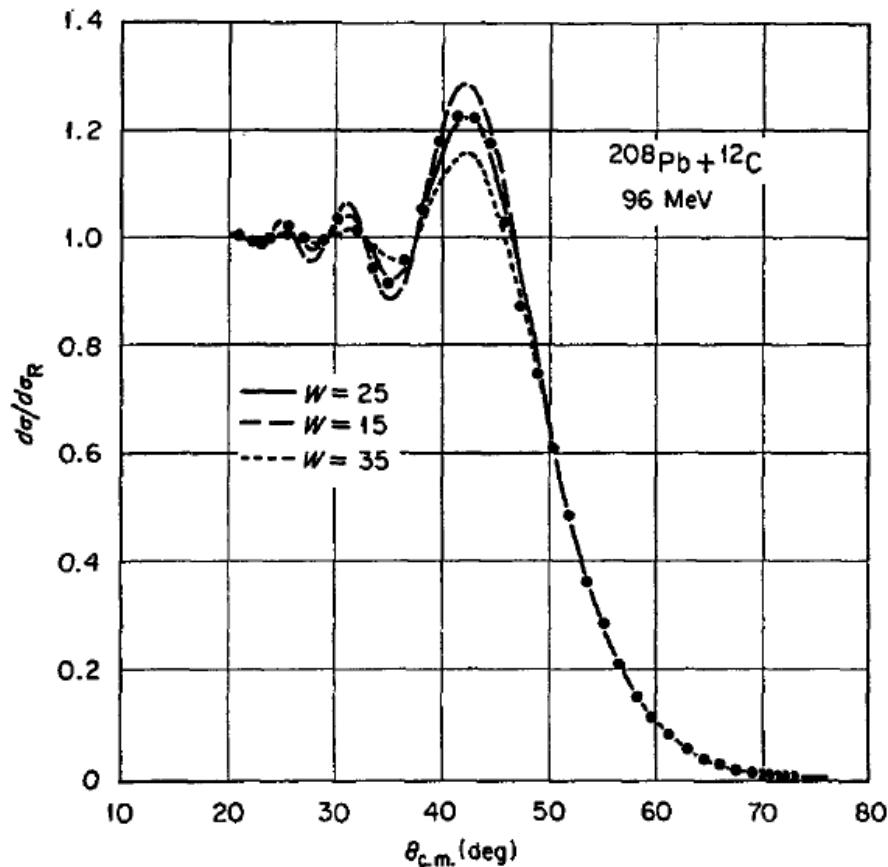


expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878



➤ Coulomb-Nuclear interference

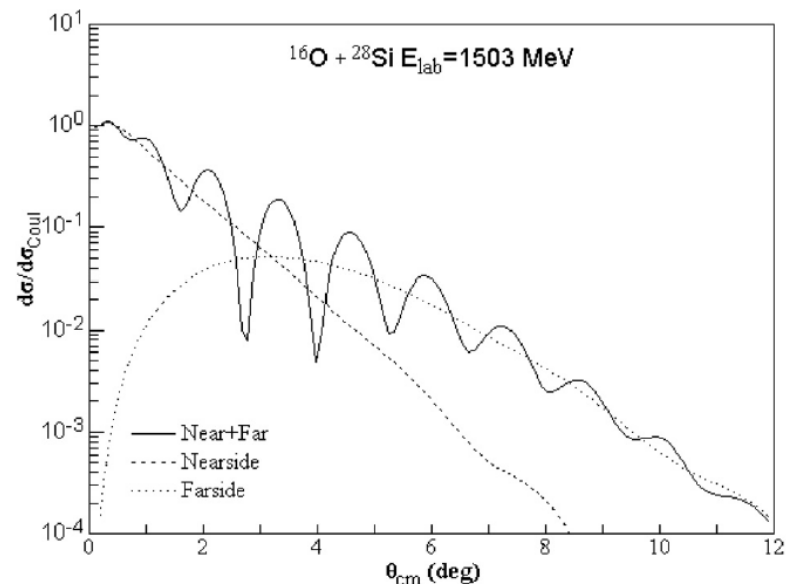
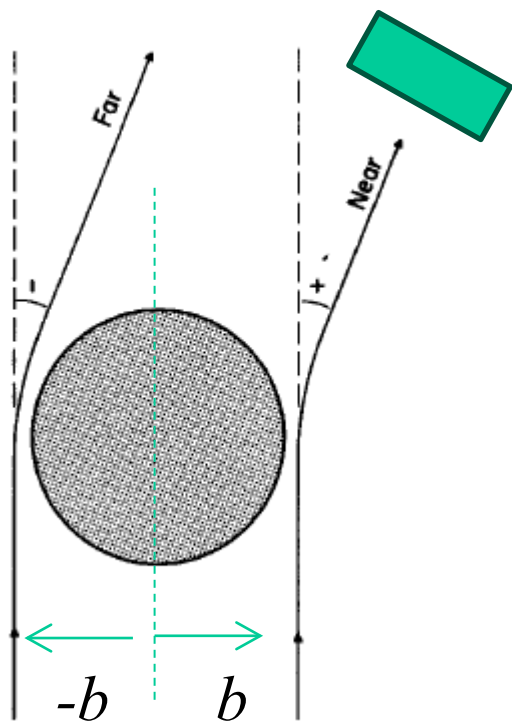
$$f(\theta) = f_C(\theta) + f_N(\theta) \rightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$



J.B. Ball et al.,
NPA252 ('75) 208

a special case: Fresnel oscillations ($S_l = 0$ ($l < l_g$); $S_l = e^{2i\sigma_l}$ ($l > l_g$))

➤ near side - far side interference



M.H. Cha,
Comp. Phys. Comm. 176 ('07) 318

R.C. Fuller, PRC12('75)1561

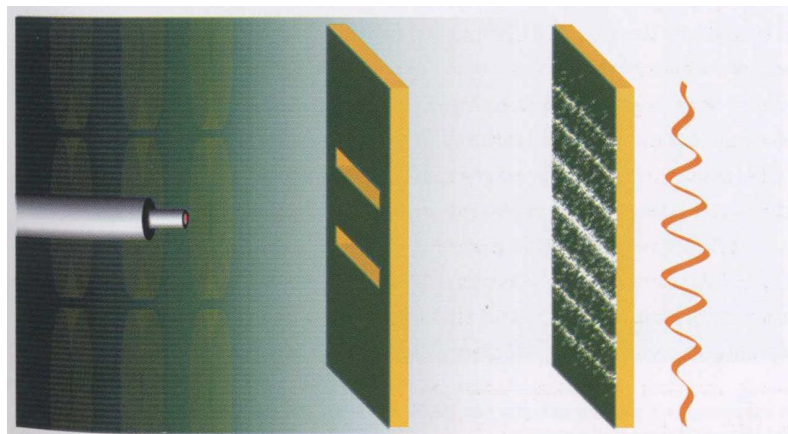
N. Rowley and C. Marty,

NPA266('76)494

M.S. Hussein and K.W. McVoy,

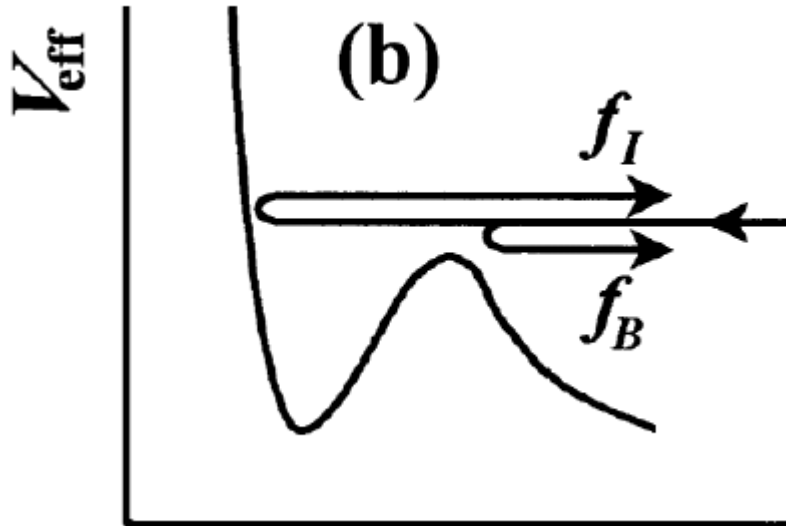
Prog. in Part. and Nucl. Phys.

12 ('84)103

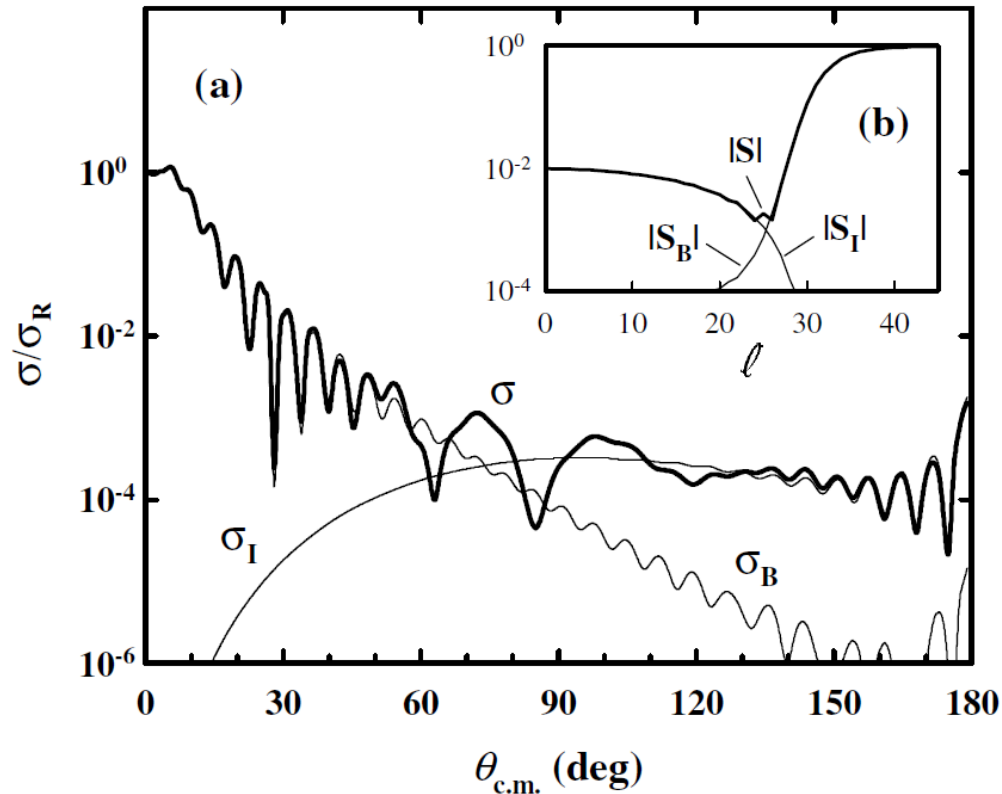


➤ barrier wave – internal wave interference

cf. D.M. Brink and N. Takigawa, NPA279 ('77) 159



$^{16}\text{O}+^{16}\text{O}$ at 124 MeV



David M Brink

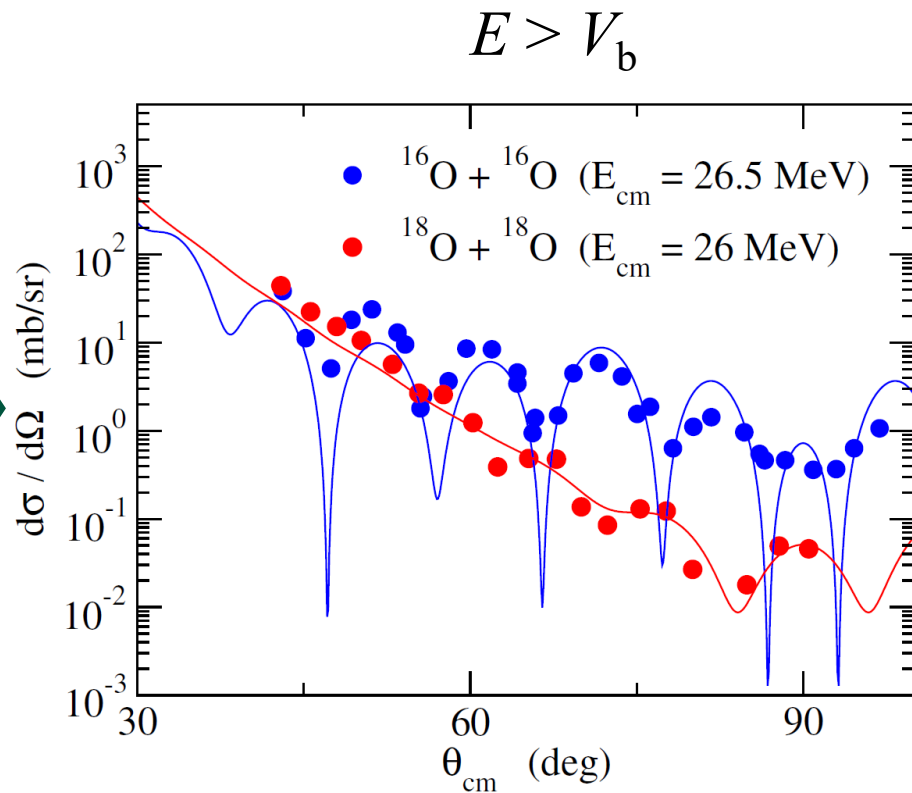
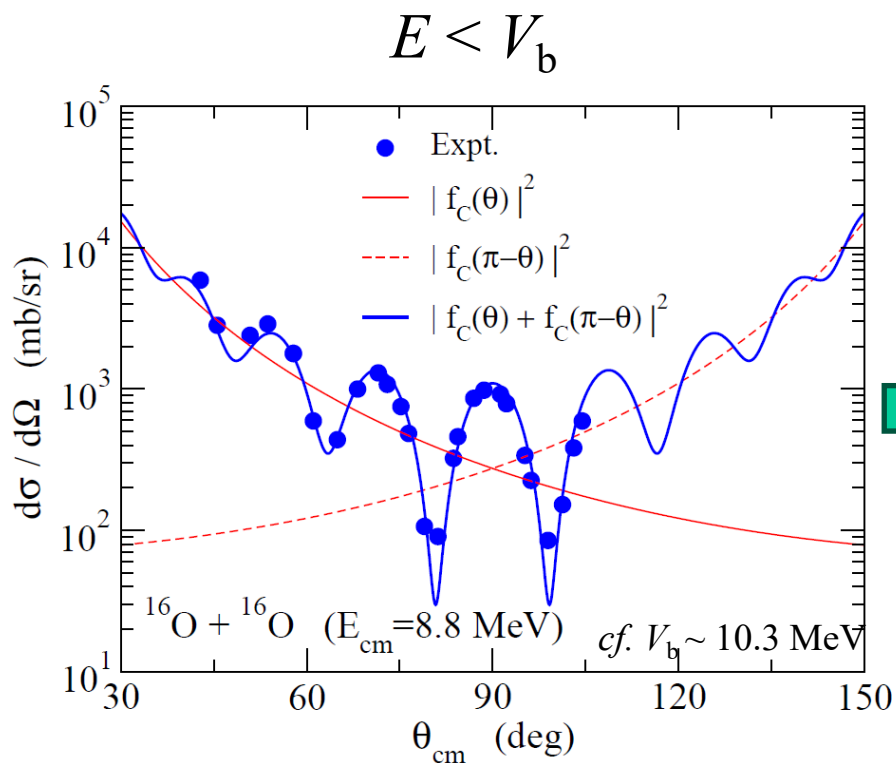


F. Michel et al., PRL85 ('00) 1823

anomalous large angle scattering

核反応は宝の山: 核反応に見られる量子性

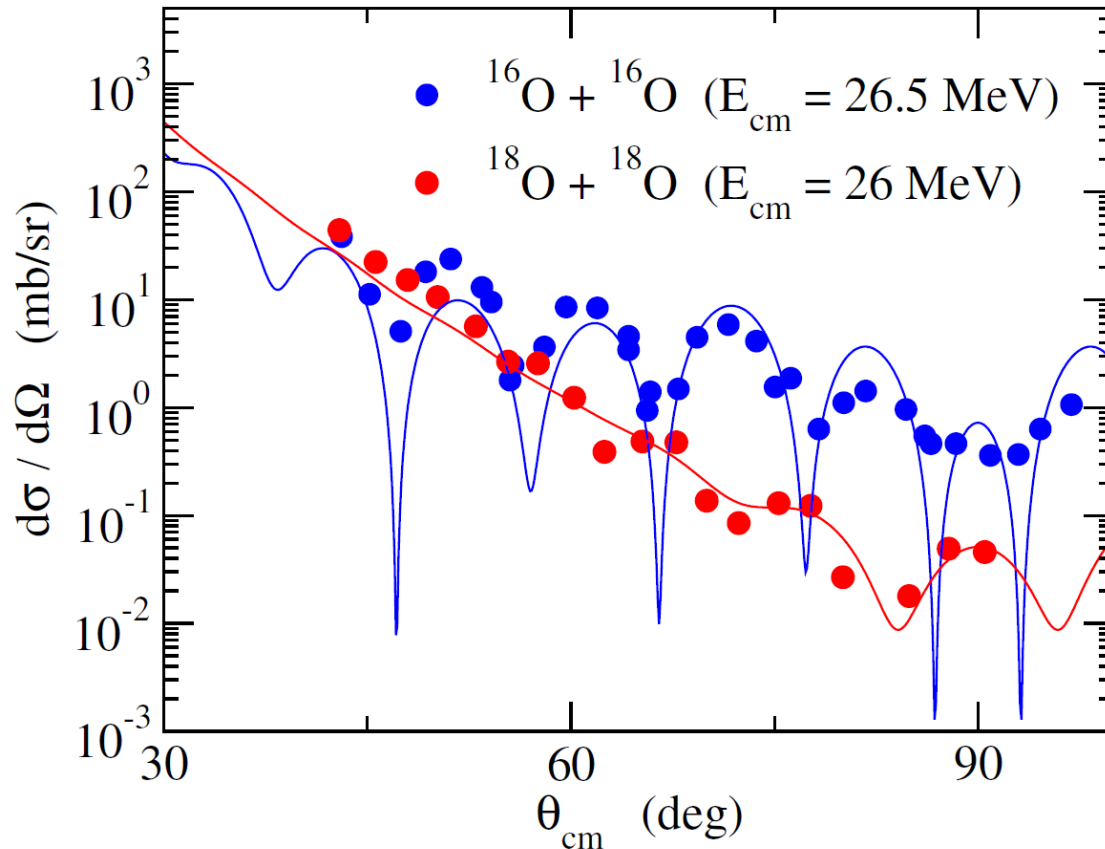
$^{16}\text{O}+^{16}\text{O}$ system



expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878

核反応は宝の山: 核反応に見られる量子性

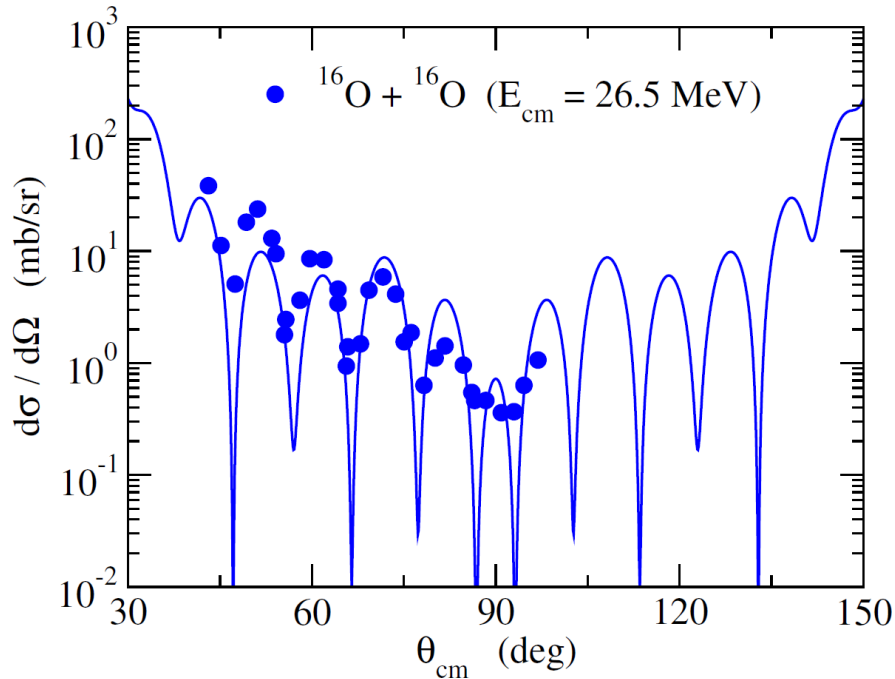
$^{16}\text{O}+^{16}\text{O}$ と $^{18}\text{O}+^{18}\text{O}$ の比較 ($E_{\text{cm}} \sim 2.5 V_b$)



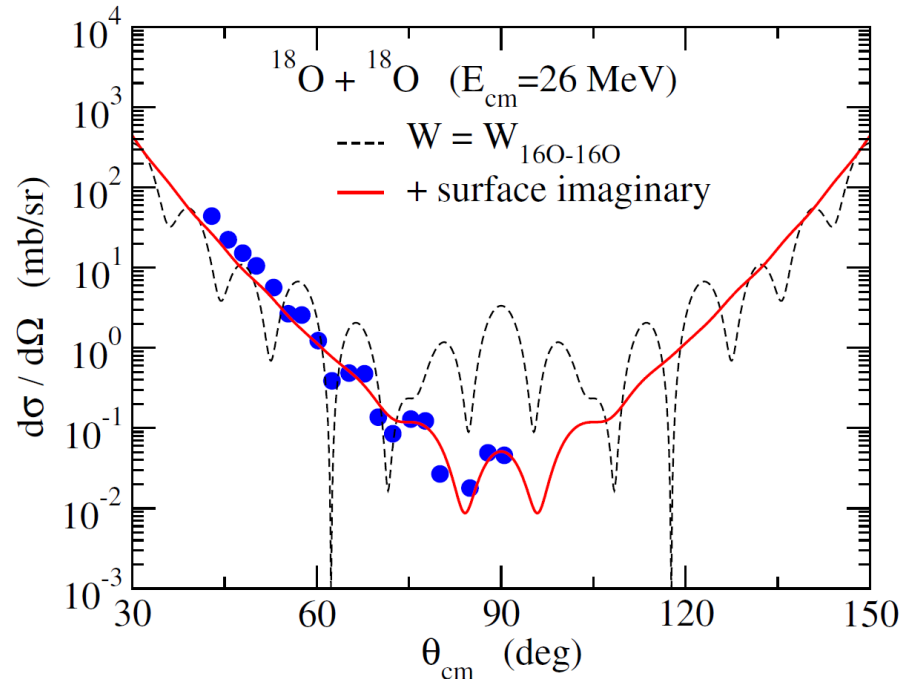
$^{18}\text{O}+^{18}\text{O}$: 干渉パターンが大分弱くなっている

$^{18}\text{O} = ^{16}\text{O} + 2n \rightarrow$ 非弾性チャンネルとの強い結合

光学ポテンシャル計算



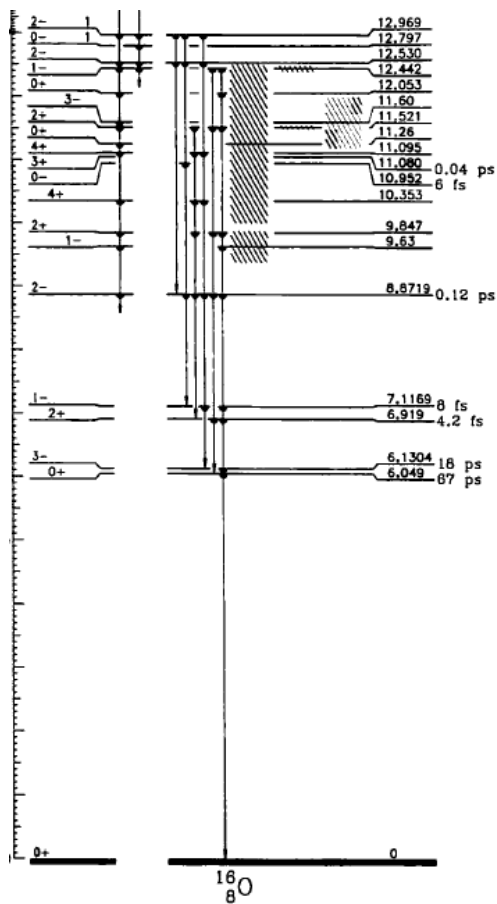
深い WS^2 型ポテンシャル
によるフィット



同じポテンシャルで $^{18}\text{O}+^{18}\text{O}$
はフィットできず

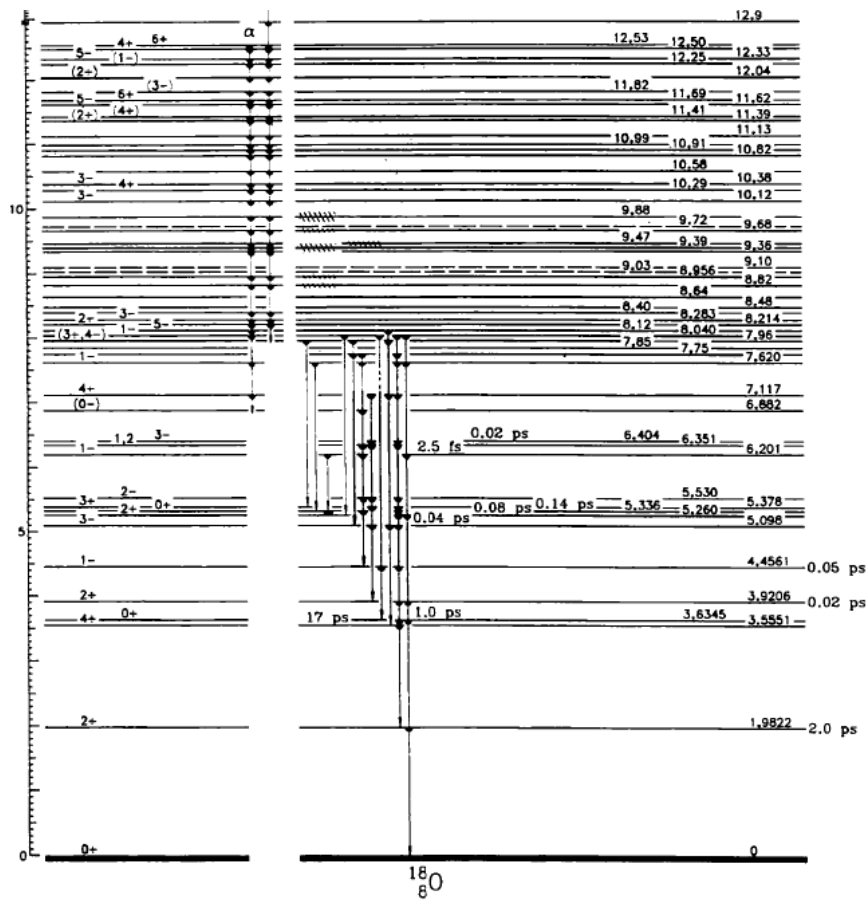
→吸収を強くする必要あり
(ここでは表面型吸収
ポテンシャルを導入)

Spectra up to $E^* = 13$ MeV



^{16}O

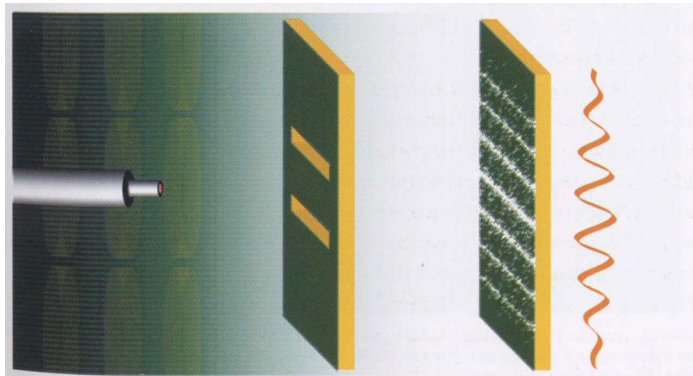
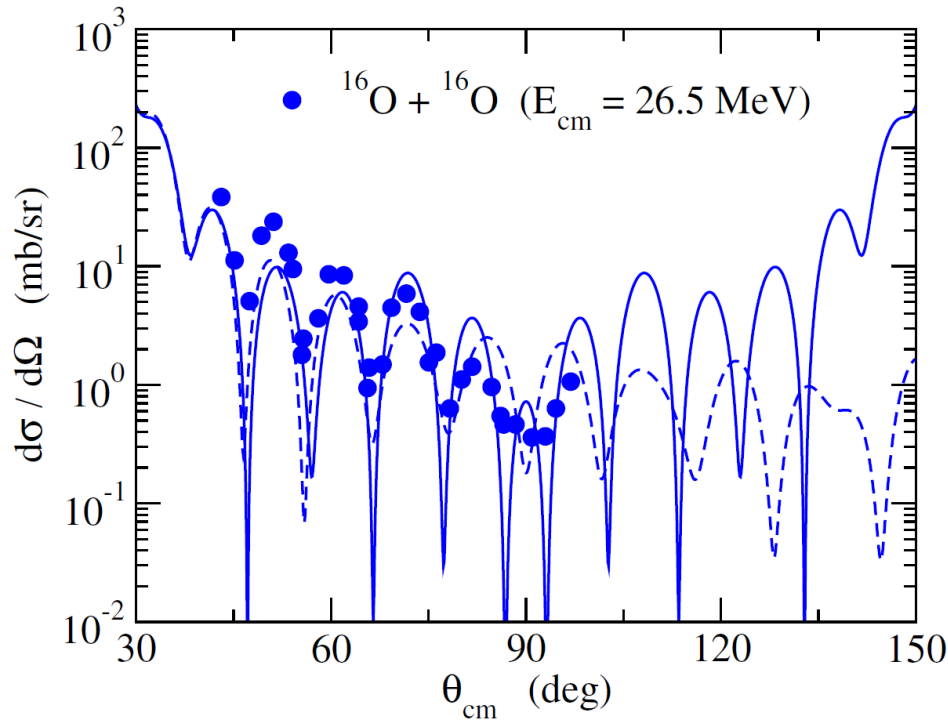
20 levels



^{18}O

56 levels

干渉の起源

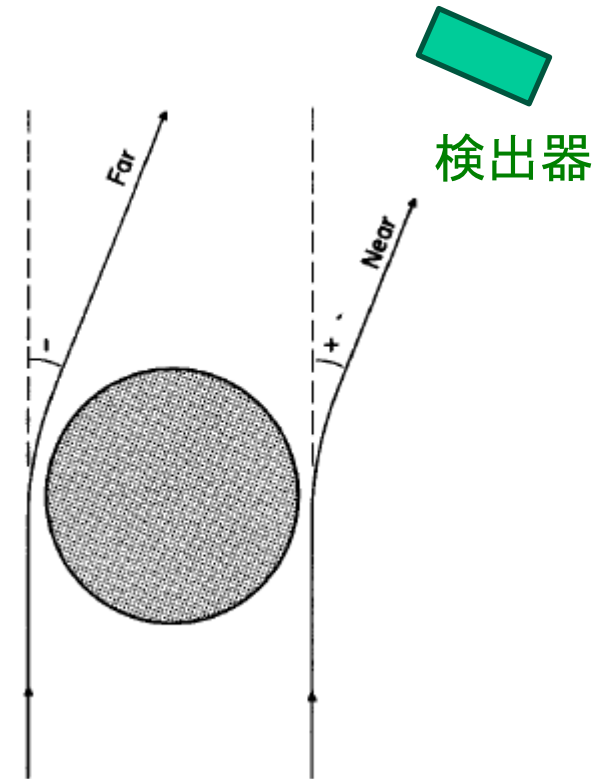


cf. 二重スリット

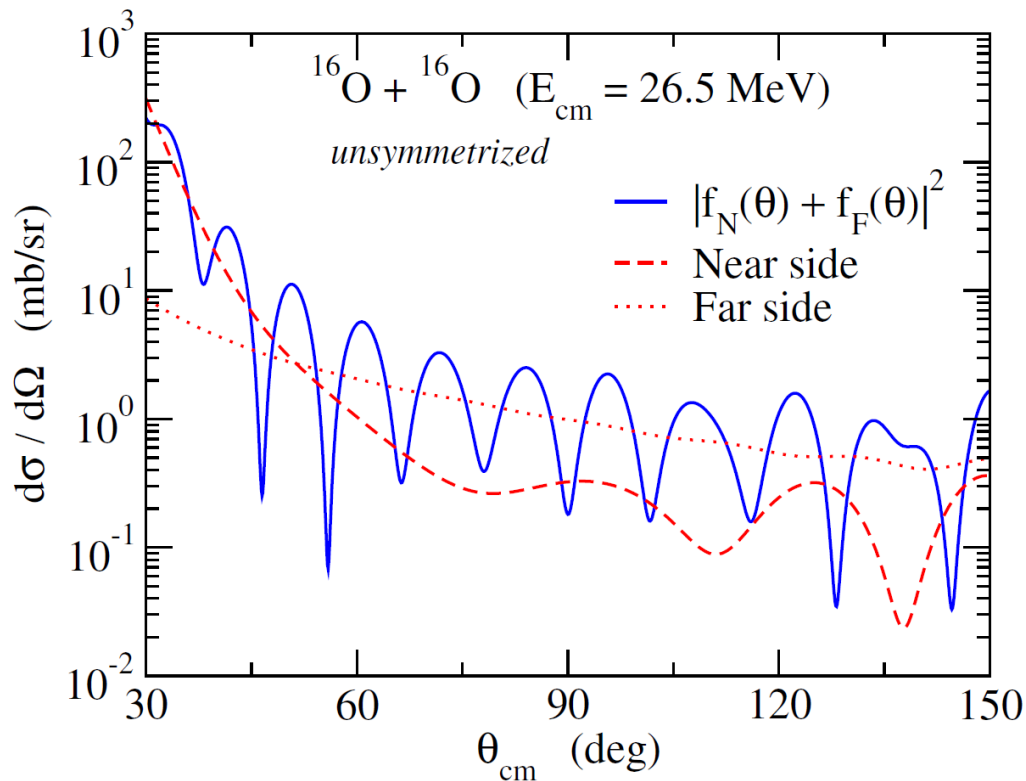
対称化しなくても強い振動



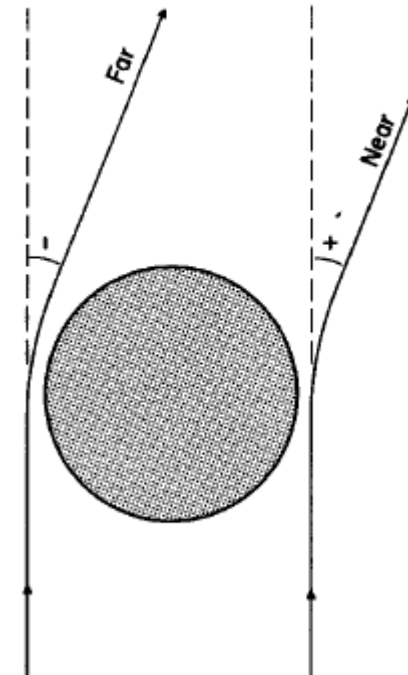
- ✓ 対称化による干渉はマイナー
- ✓ near-sideとfar-sideの干渉が主



M.S. Hussein and K.W. McVoy,
PPNP 12 ('84)103.



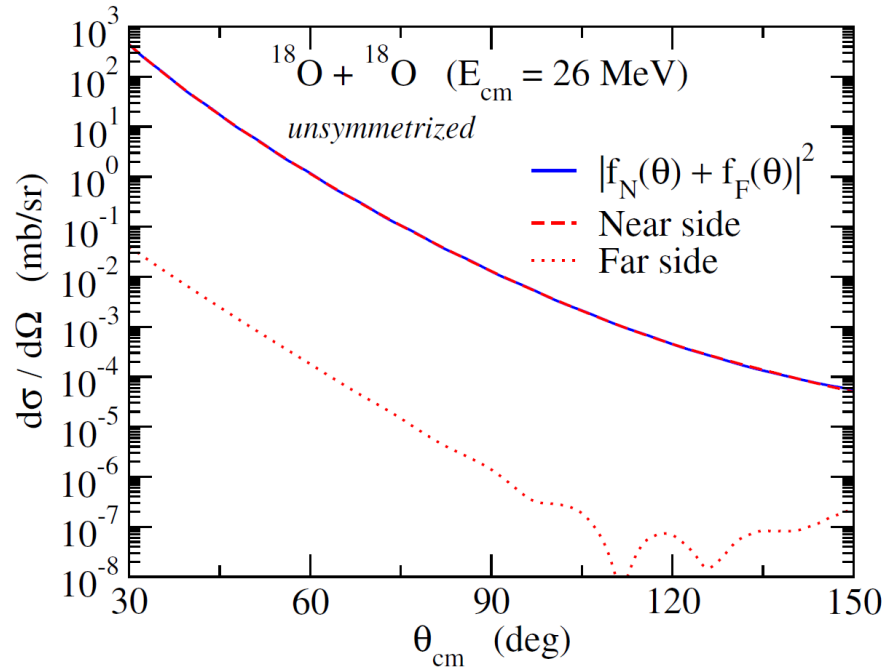
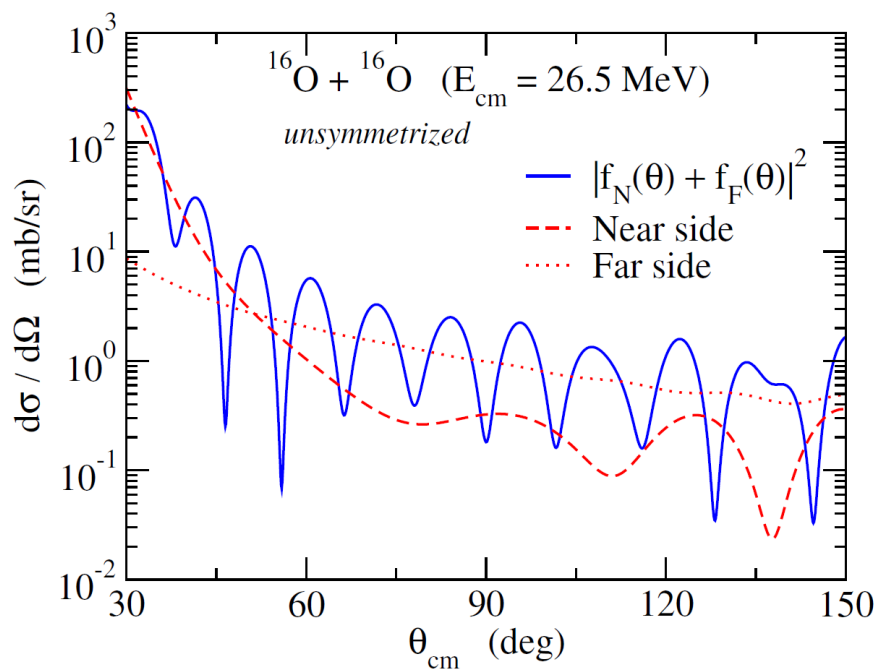
near side-far side interference



$$P_l(\cos \theta) \rightarrow \frac{1}{2} \left[P_l(\cos \theta) \mp i \frac{2}{\pi} Q_l(\cos \theta) \right]$$

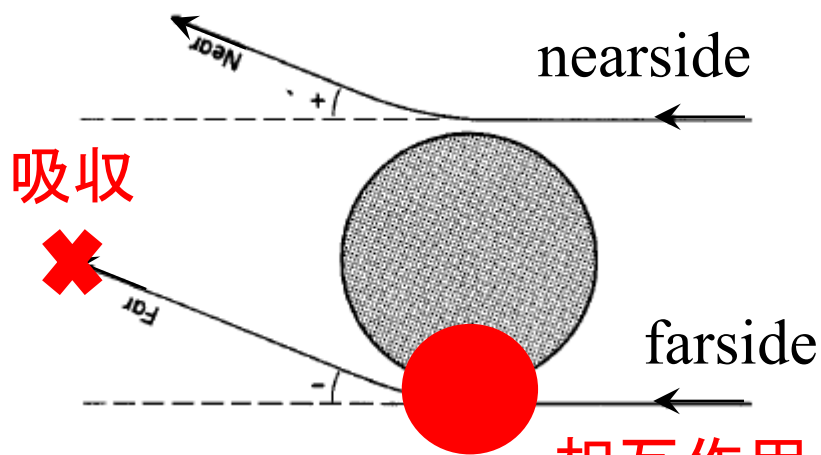
↗ N
↘ F

Q_l : 第2種ルジャンドル関数



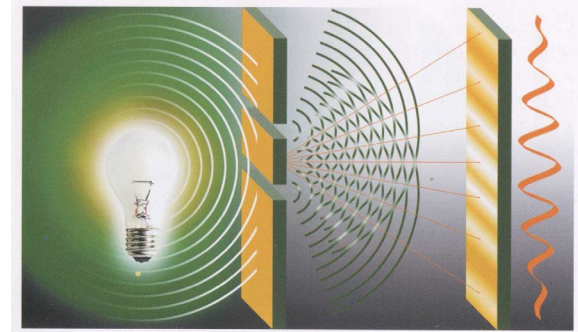
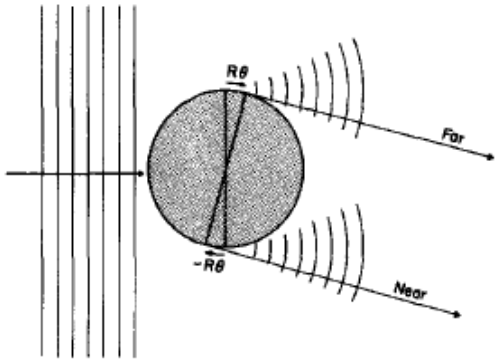
強い吸収のため、 $^{18}\text{O}+^{18}\text{O}$ では far-side 成分が大きく減衰
 → 干渉パターンがほとんど消えている

cf. 一重スリット



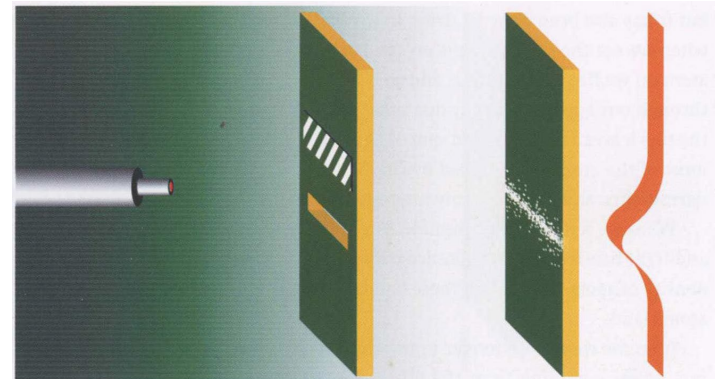
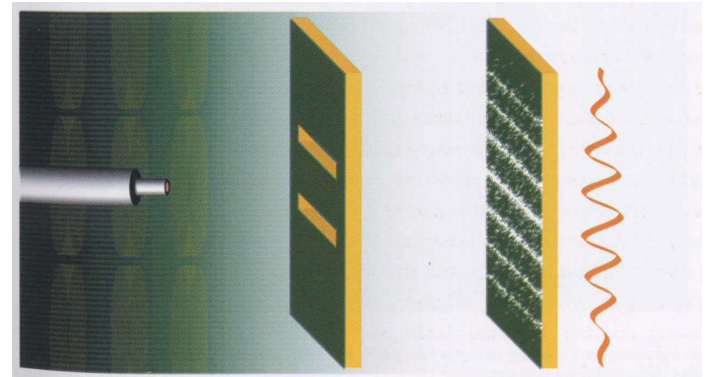
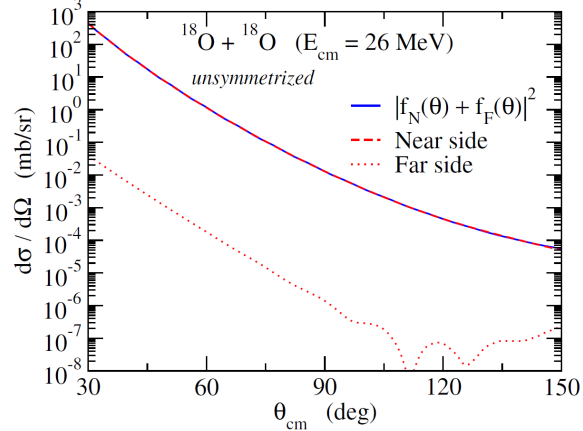
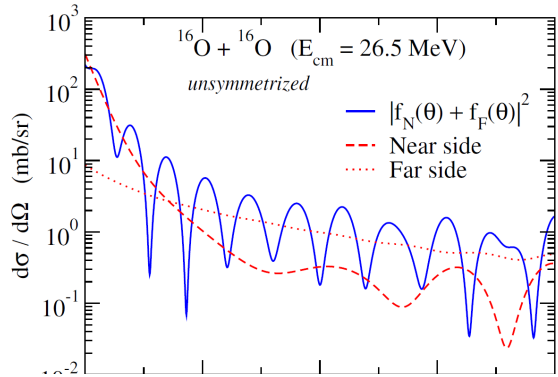
相互作用→デコヒーレンス

analogy to the double slit problem



M.S. Hussein and K.W. McVoy,
 Prog. in Part. and Nucl. Phys. 12 ('84)103

J. Al-Khalili, "Quantum"

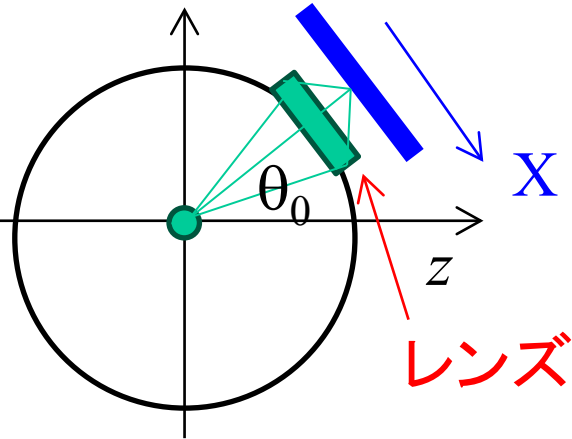


核反応のイメージング

K. Hagino and T. Yoda,
arXiv: 2309.10261

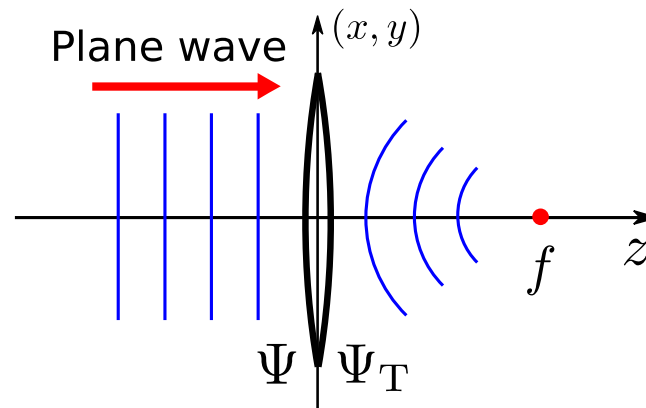
(x, y) スクリーン 点Pにレンズをおき、スクリーン上に「集光」する

散乱振幅のフーリエ変換



$$\Phi(X, Y) \propto \int_{\theta_0 - \Delta\theta}^{\theta_0 + \Delta\theta} \sin \theta d\theta \int_{\varphi_0 - \Delta\varphi}^{\varphi_0 + \Delta\varphi} d\varphi \times e^{ik((\theta - \theta_0)X + (\varphi - \varphi_0)Y)} f(\theta, \varphi)$$

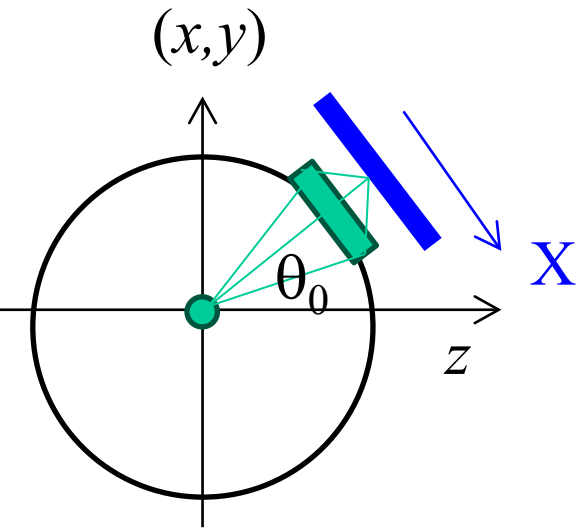
$$I(X, Y) = |\Phi(X, Y)|^2$$



K. Hashimoto et al., PRD101, 066018 (2020)

核反応のイメージング

K. Hagino and T. Yoda,
arXiv: 2309.10261

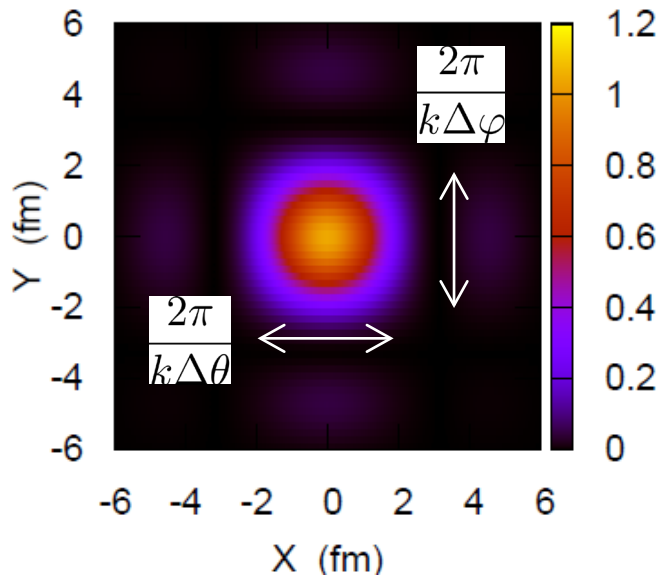


$$\Phi(X, Y) \propto \int_{\theta_0 - \Delta\theta}^{\theta_0 + \Delta\theta} \sin \theta d\theta \int_{\varphi_0 - \Delta\varphi}^{\varphi_0 + \Delta\varphi} d\varphi$$

$$\times e^{ik((\theta - \theta_0)X + (\varphi - \varphi_0)Y)} f(\theta, \varphi)$$

$$I(X, Y) = |\Phi(X, Y)|^2$$

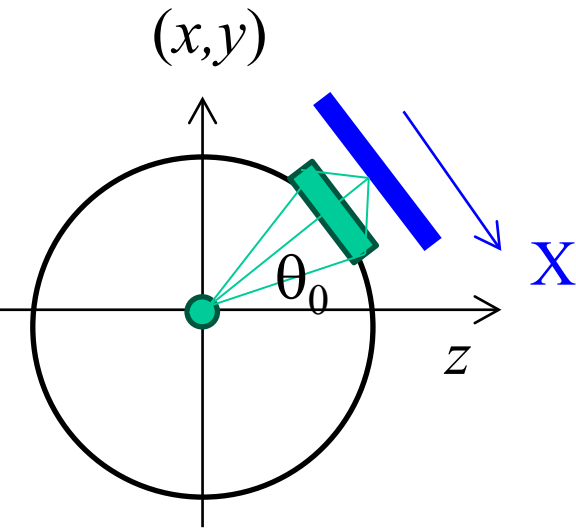
$f(\theta, \varphi) = \text{const.}$ ならば....



$$\int_{\varphi_0 - \Delta\varphi}^{\varphi_0 + \Delta\varphi} d\varphi e^{ik(\varphi - \varphi_0)Y} = 2\Delta\varphi \frac{\sin(kY \Delta\varphi)}{kY \Delta\varphi}$$

核反応のイメージング

K. Hagino and T. Yoda,
arXiv: 2309.10261

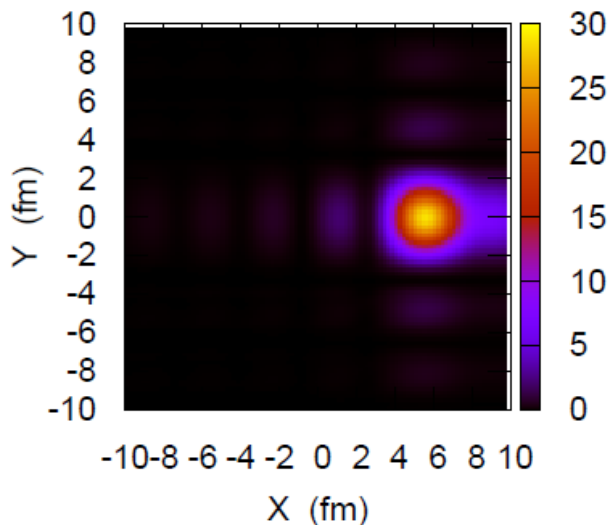


$$\Phi(X, Y) \propto \int_{\theta_0 - \Delta\theta}^{\theta_0 + \Delta\theta} \sin \theta d\theta \int_{\varphi_0 - \Delta\varphi}^{\varphi_0 + \Delta\varphi} d\varphi$$

$$\times e^{ik((\theta - \theta_0)X + (\varphi - \varphi_0)Y)} f(\theta, \varphi)$$

$$I(X, Y) = |\Phi(X, Y)|^2$$

ラザフォード散乱 $f(\theta, \phi) = f_c(\theta, \phi)$ ならば....



$^{16}\text{O} + ^{16}\text{O}$ at $E_{\text{cm}} = 8.8 \text{ MeV}$

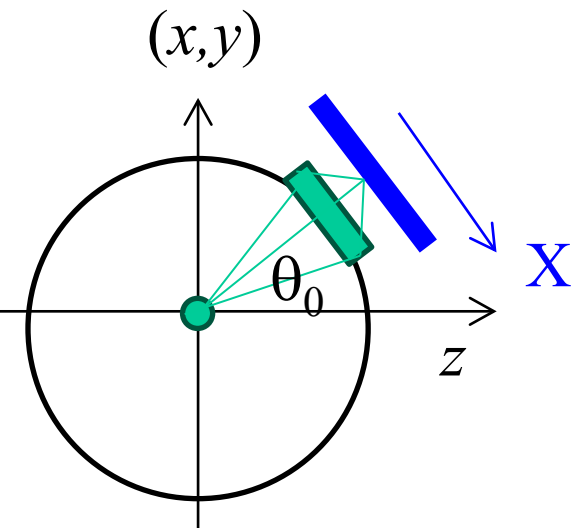
$$\alpha = k$$

$$\theta_0 = 90 \text{ deg.}$$

$$\Delta\theta = \Delta\phi = 30 \text{ deg.}$$



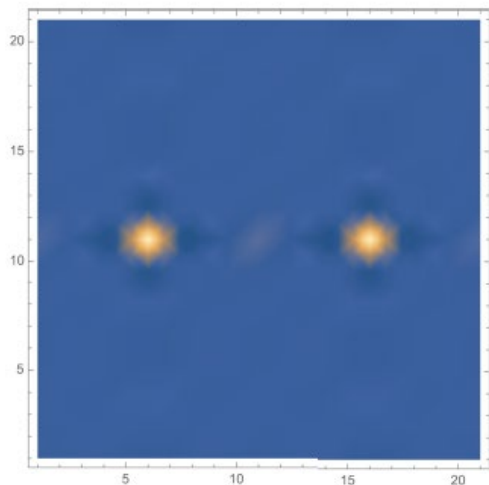
$$b_{\text{cl}} = 5.24 \text{ fm} \sim X_{\text{peak}}$$



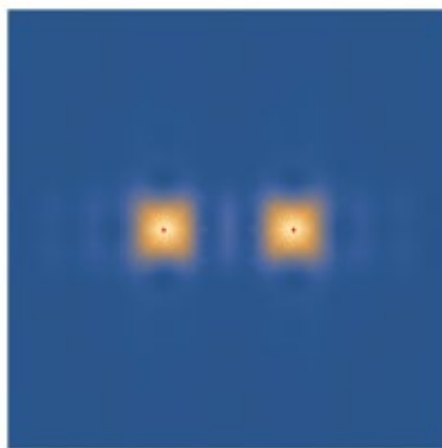
散乱振幅のフーリエ変換

$$\Phi(X, Y) \propto \int_{\theta_0 - \Delta\theta}^{\theta_0 + \Delta\theta} \sin \theta d\theta \int_{\varphi_0 - \Delta\varphi}^{\varphi_0 + \Delta\varphi} d\varphi \times e^{ik((\theta - \theta_0)X + (\varphi - \varphi_0)Y)} f(\theta, \varphi)$$

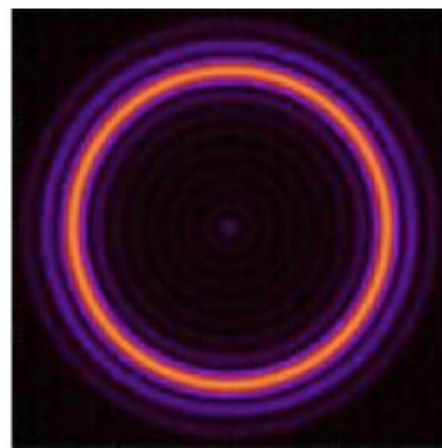
$$I(X, Y) = |\Phi(X, Y)|^2$$



二重スリットの問題



弦の散乱



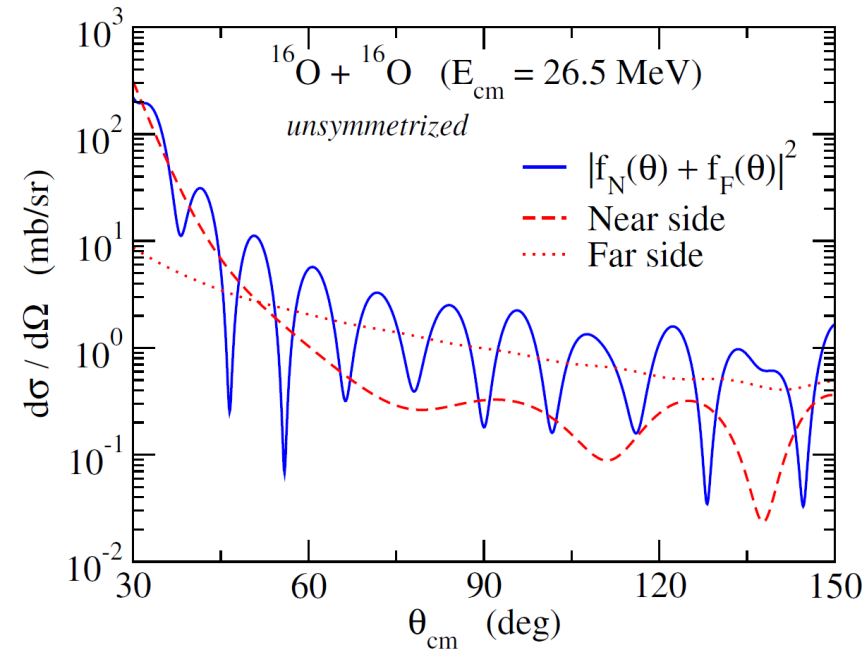
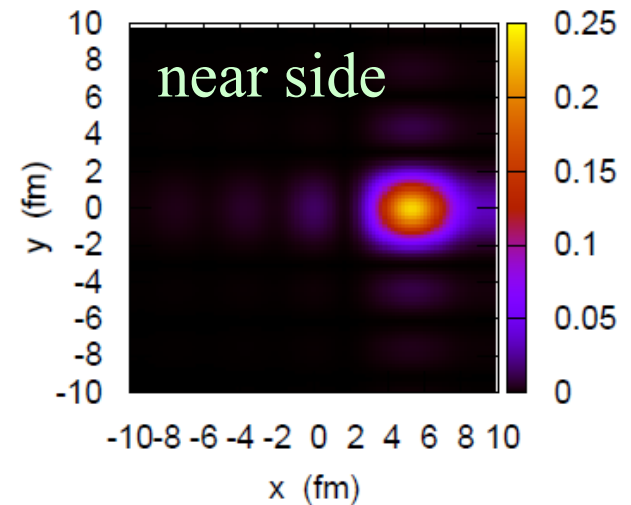
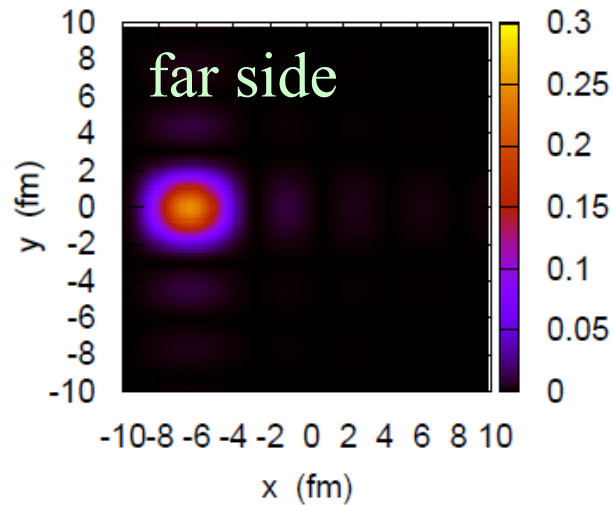
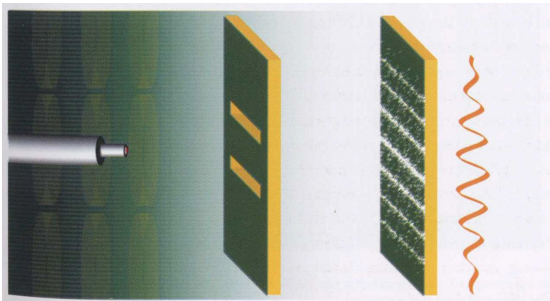
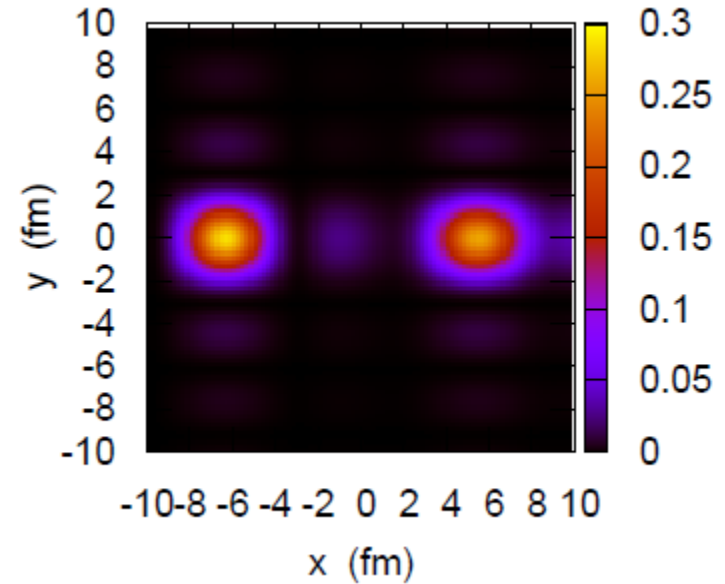
AdS/CFTによるブラックホールのイメージング

K. Hashimoto, Y. Matsuo, and T. Yoda, PTEP2023, 043B04 (2023)

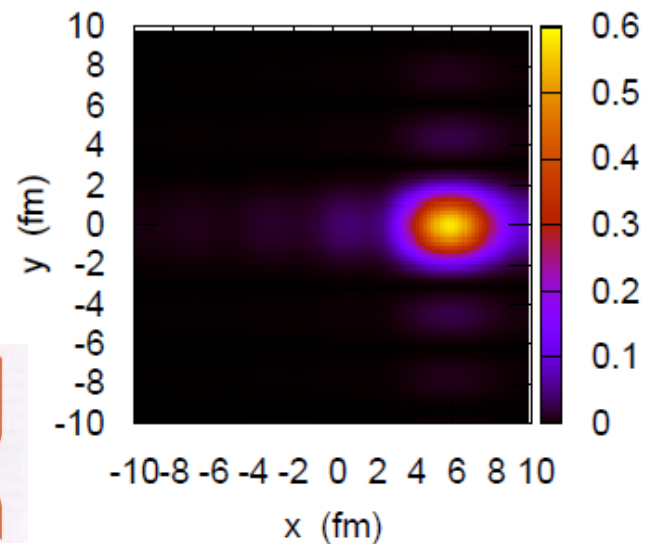
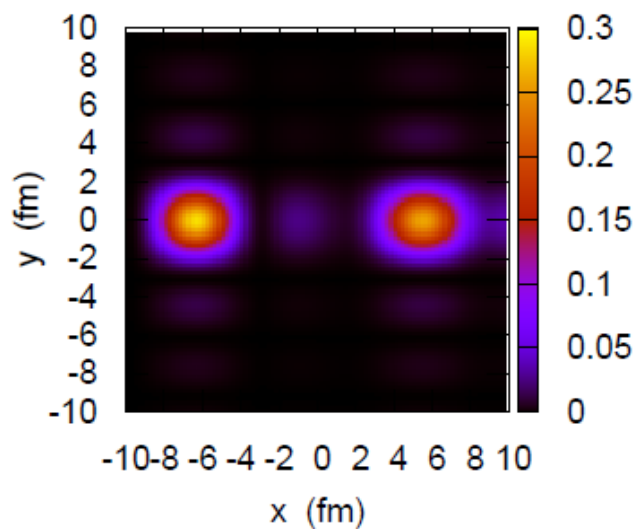
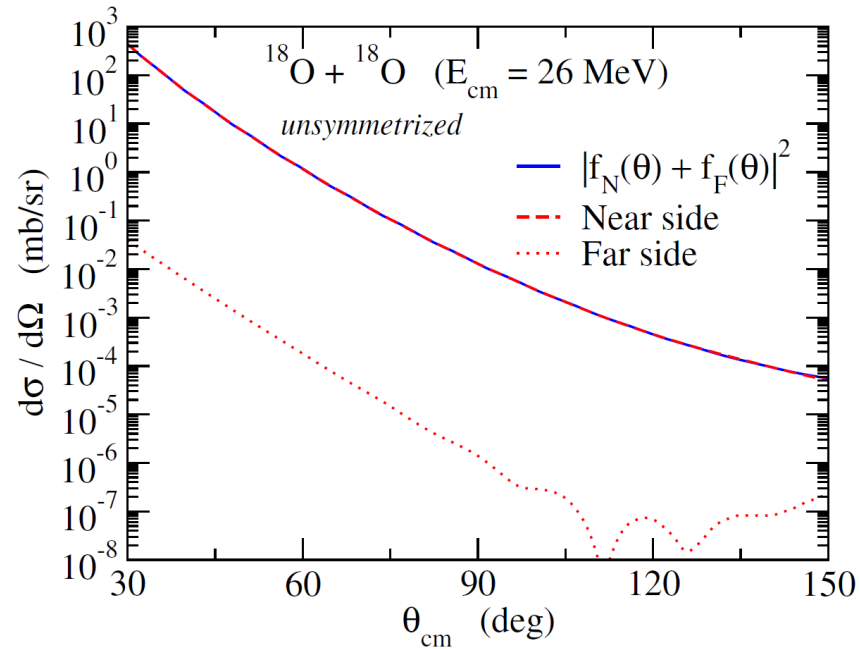
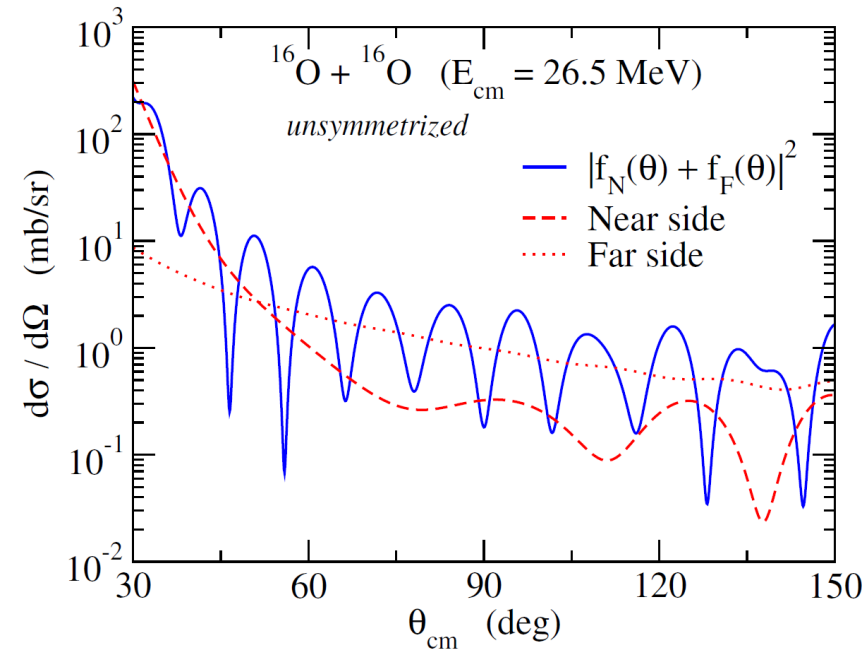
K. Hashimoto, S. Kinoshita, and K. Murata, PRL123, 031602 (2019)

PRD101, 066018 (2020)

核反応のイメージング


 $\theta_0 = 55$ deg., $\Delta\theta = 15$ deg.


核反応のイメージング

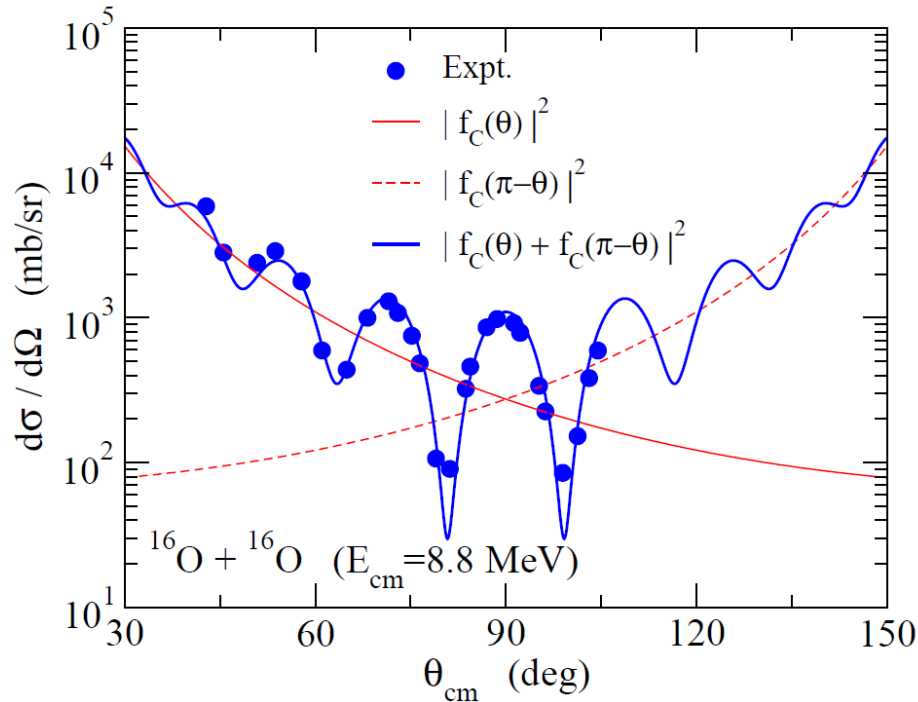


Imaging of nuclear reactions

$$\Phi(X, Y) \propto \int_{\theta_0 - \Delta\theta}^{\theta_0 + \Delta\theta} \sin \theta d\theta \int_{\varphi_0 - \Delta\varphi}^{\varphi_0 + \Delta\varphi} d\varphi e^{i\alpha((\theta - \theta_0)X + (\varphi - \varphi_0)Y)} f(\theta, \varphi)$$

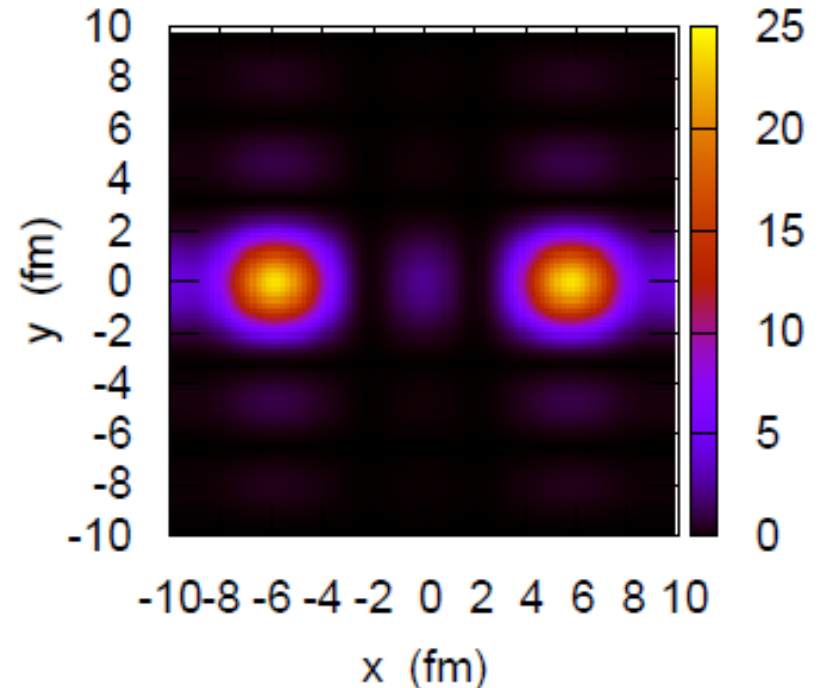
Imaging of Mott scattering

$$I(X, Y) = |\Phi(X, Y)|^2$$



$$\alpha = k$$

$$\theta_0 = 90 \text{ deg.}, \Delta\theta = \Delta\phi = 30 \text{ deg.}$$



(note) for $\theta_0 = 90 \text{ deg.}$,

$$\Phi_\theta(X, Y) = \Phi_{\pi - \theta}(-X, Y)$$