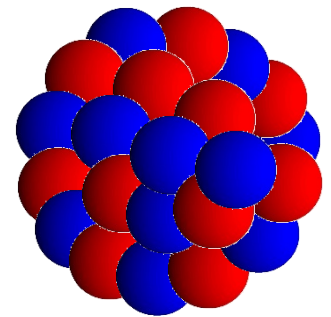


原子核基礎論B

原子核理論研究室(物2) 萩野浩一

シラバス(後半)

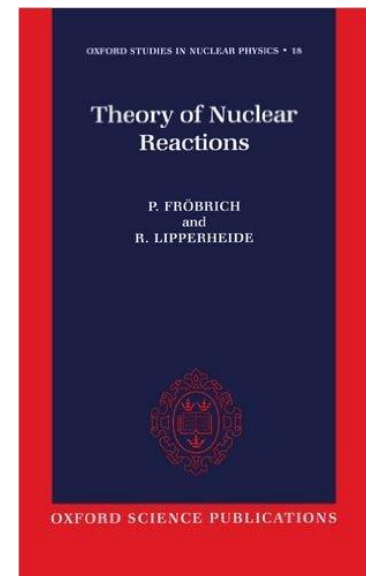
1. 原子核の集団運動(2コマ)
2. 集団運動の微視的理解(2コマ)
3. 原子核反応論基礎(1コマ)
4. 非束縛核の物理: 共鳴状態について(1コマ)
5. 超重元素の物理(1コマ)



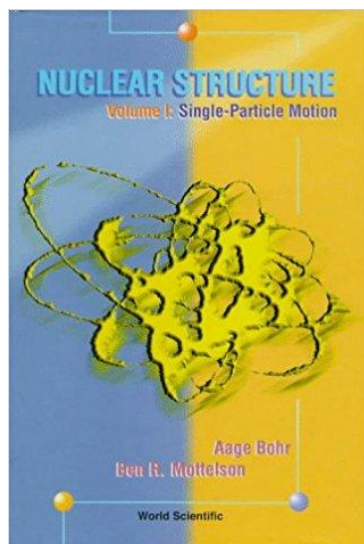
主に原子核の励起状態と原子核反応

成績: レポート80%、出席など20%
質問: 講義中にも適宜聞いて下さい。

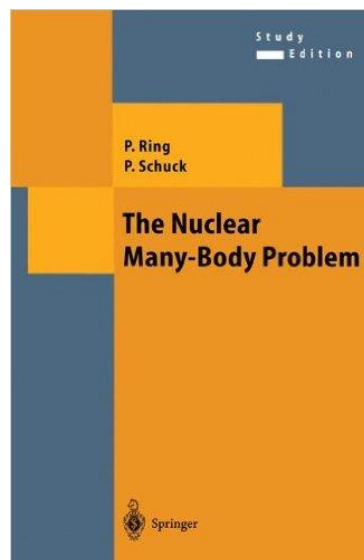
参考書



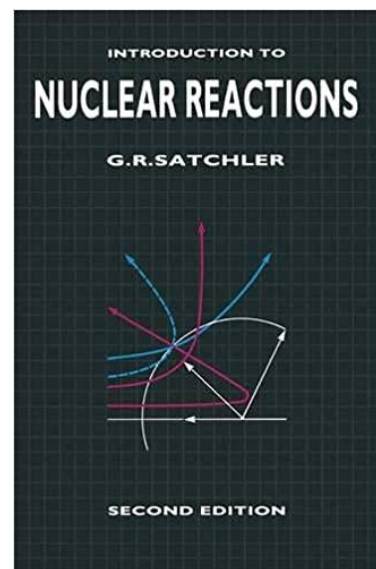
Frobrich-Lipperheide



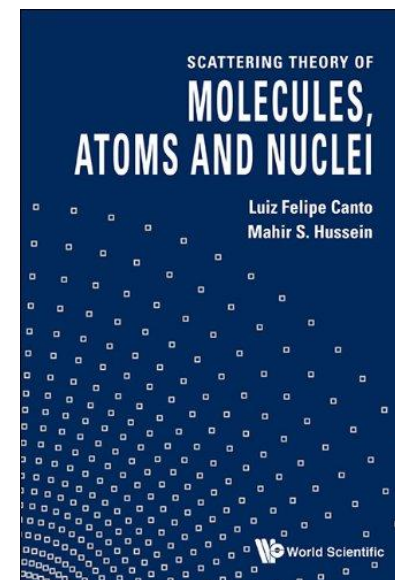
Bohr-Mottelson



Ring-Schuck



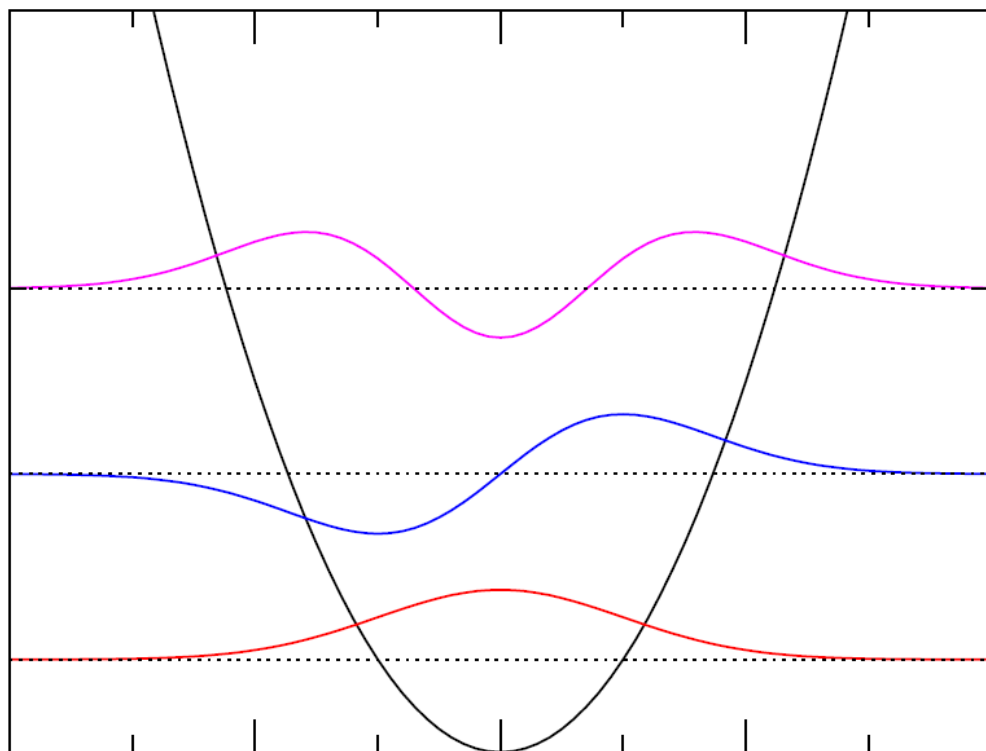
Satchler



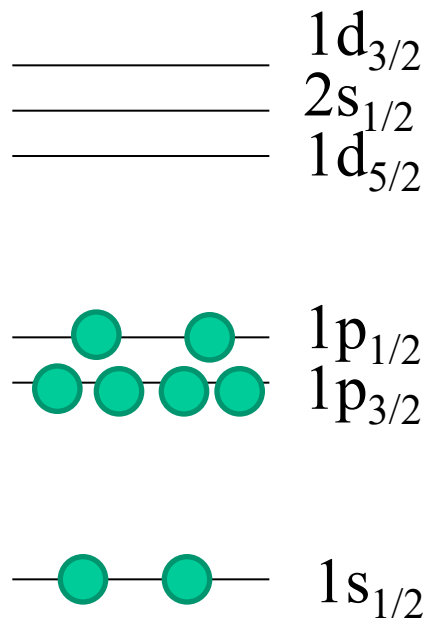
Canto-Hussein

励起状態

ポテンシャル中の1粒子の場合

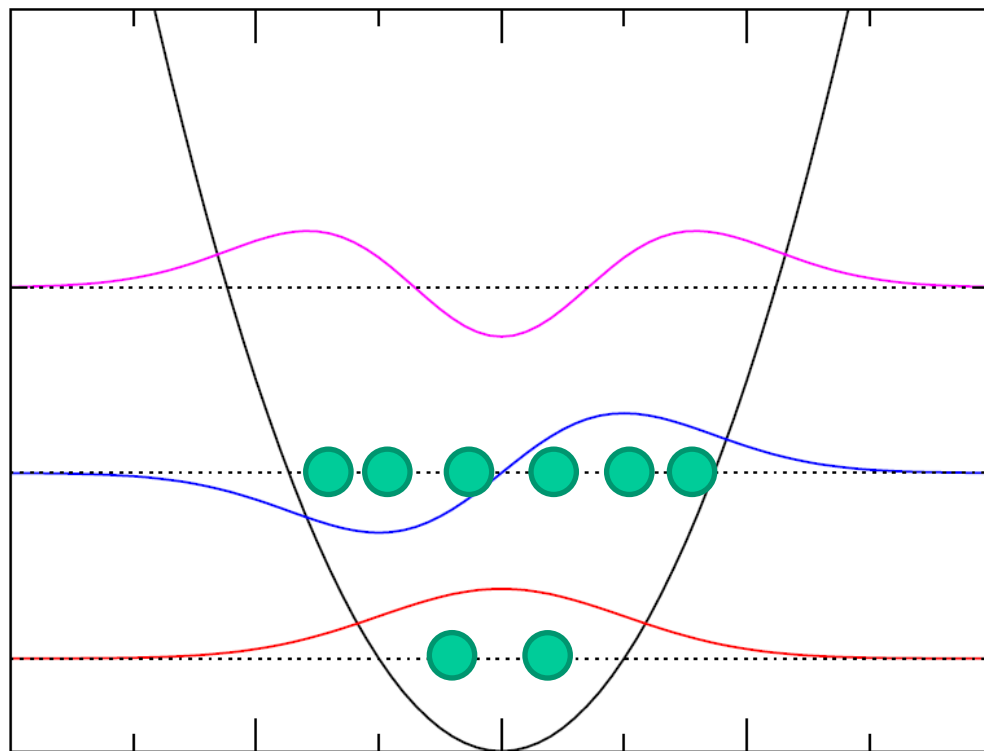


原子核の励起状態



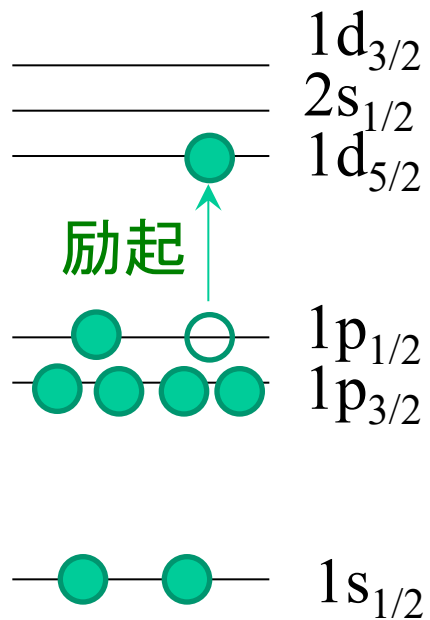
基底状態

多体系の場合

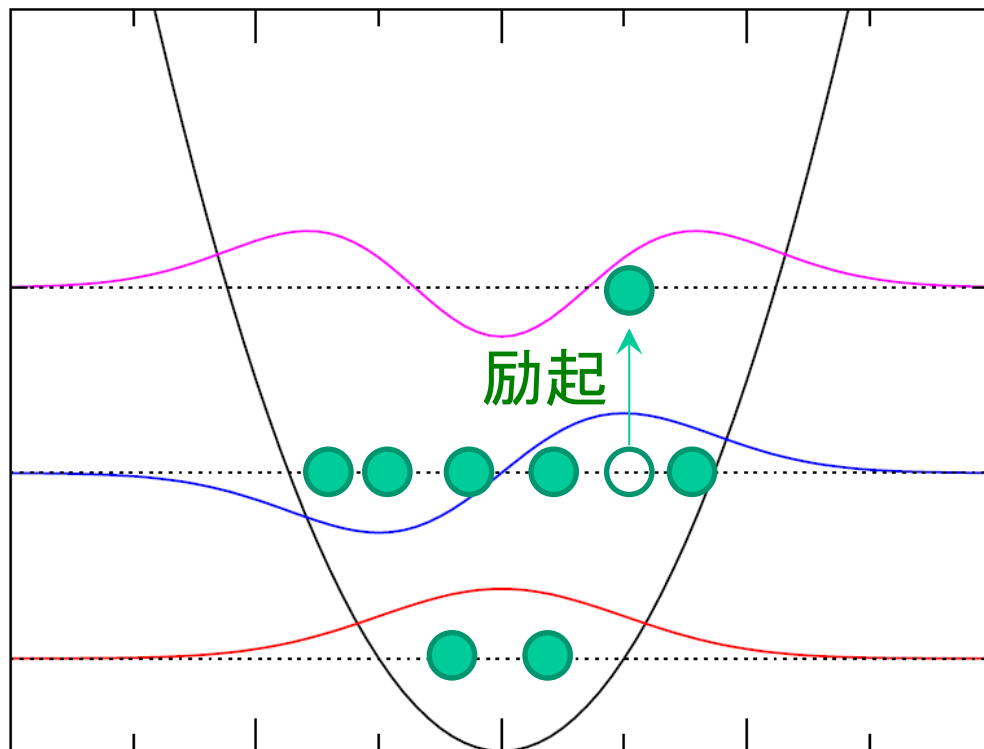


基底状態

原子核の励起状態

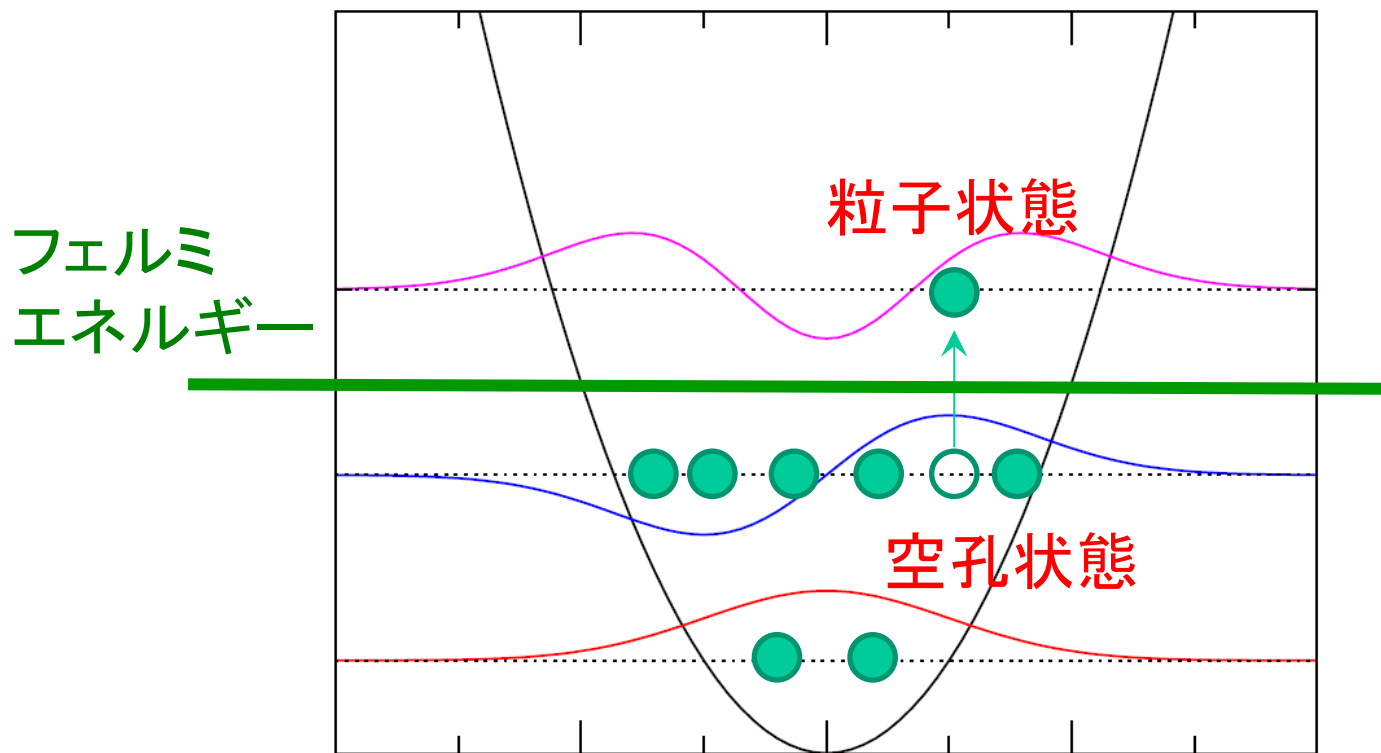


多体系の場合



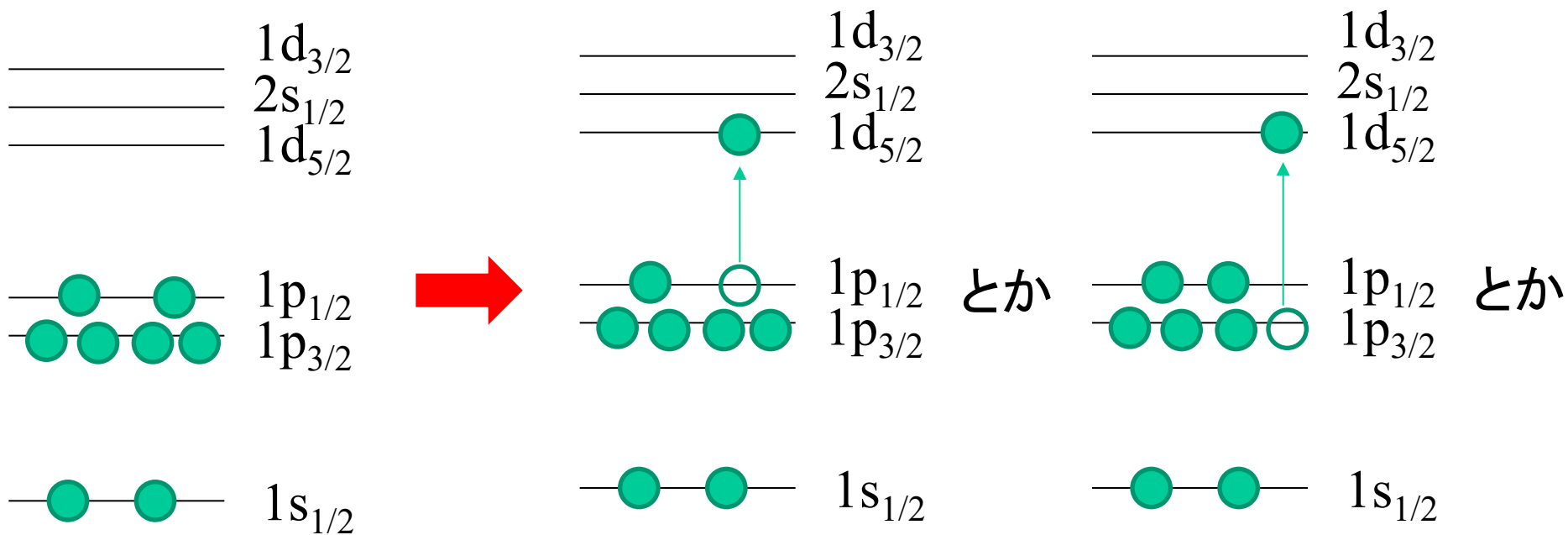
原子核の励起状態

多体系の場合



1粒子1空孔(1p1h)状態

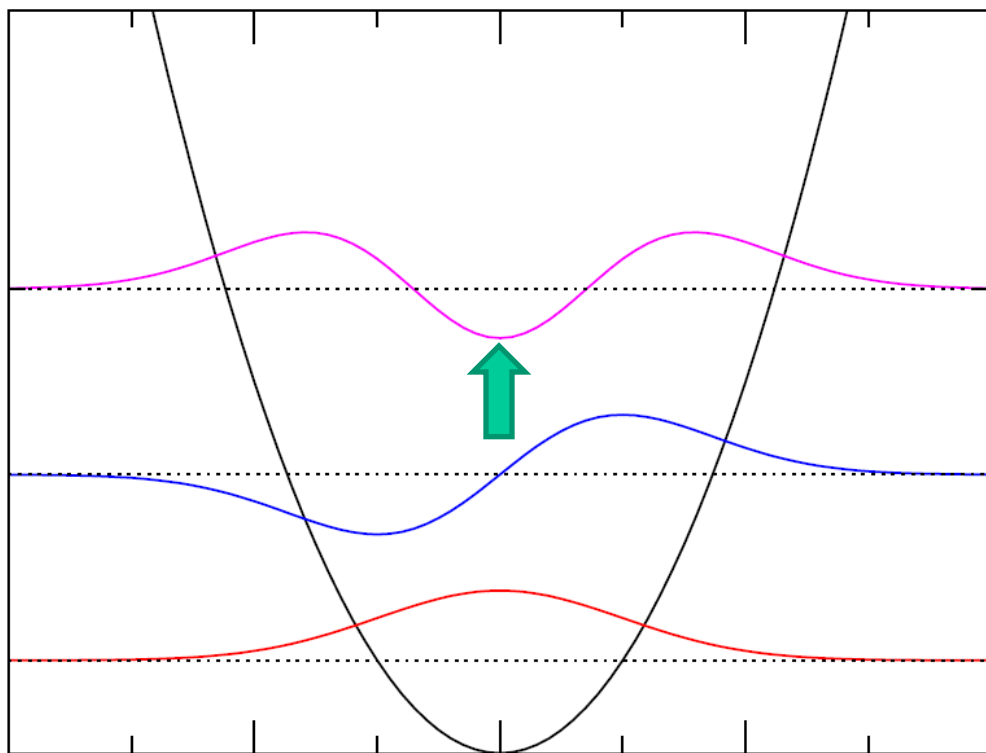
原子核の励起状態



基底状態

1粒子1空孔(1p1h)状態

粒子2個あげる励起や、複数個あげる励起も。
2p2h 状態、np-nh状態。



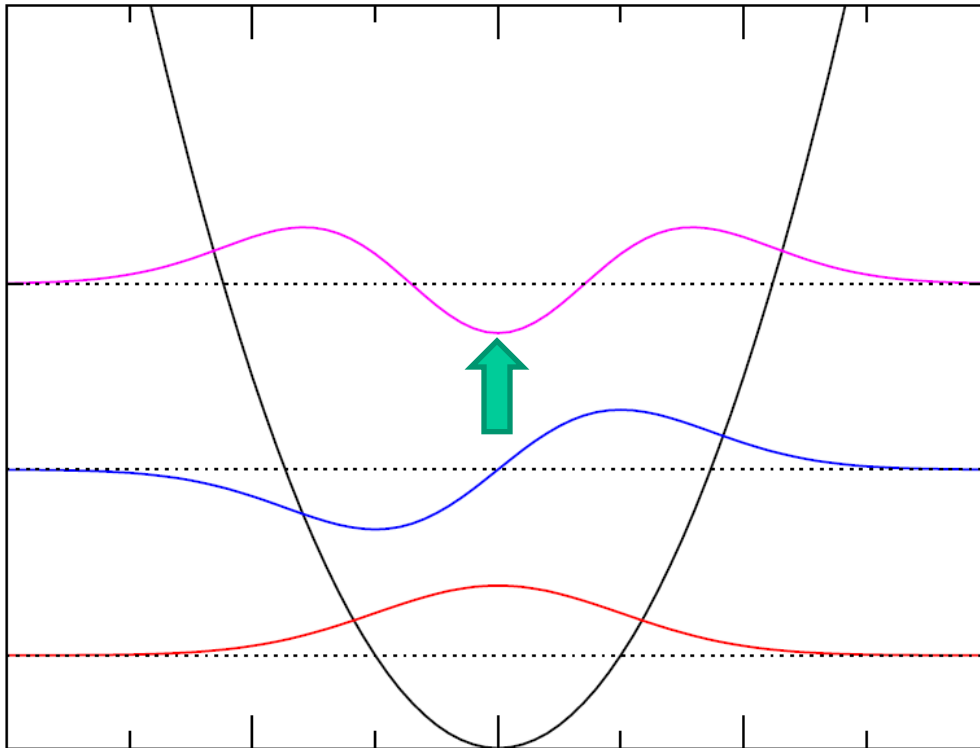
原子核では

$$\hbar\omega \sim 41 A^{-1/3} \quad (\text{MeV})$$

$$\leftarrow R \sim 1.2 A^{1/3} \quad (\text{fm})$$

$A = 16$ だと 16.27 MeV

cf. 実際に、 ^{16}O の 16.2 MeV に 1^- 状態、
 15.4 MeV , 15.8 MeV に 3^- 状態



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cf. 実際に、 ^{16}O の16.2 MeV
に 1- 状態、
15.4 MeV, 15.8 MeV に
3- 状態

.....でも実際にはこのようには理解できない励起状態
も多数存在する(集団励起)

Giant Dipole Resonance (GDR) 巨大双極子共鳴

光吸収の
スペクトル

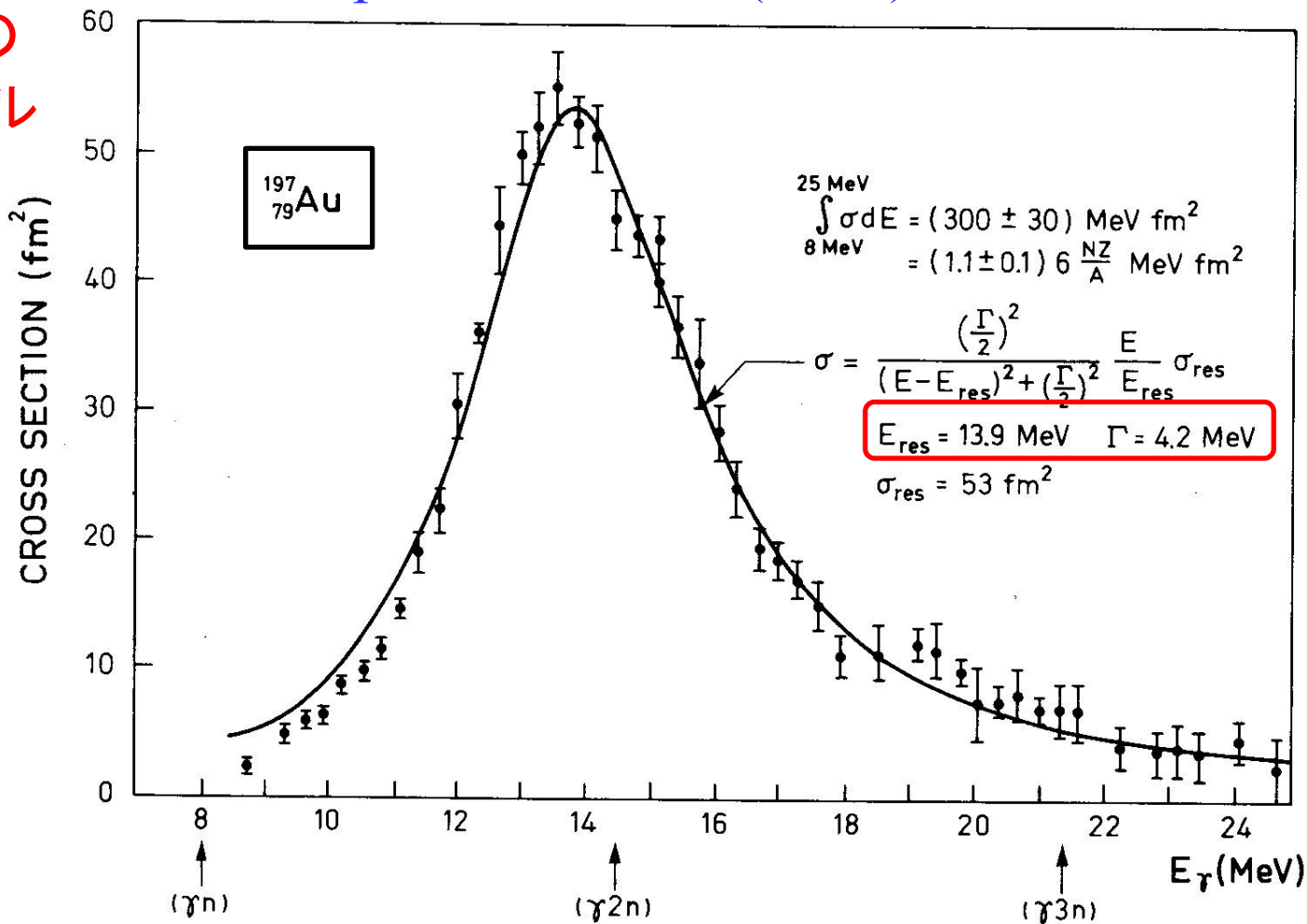


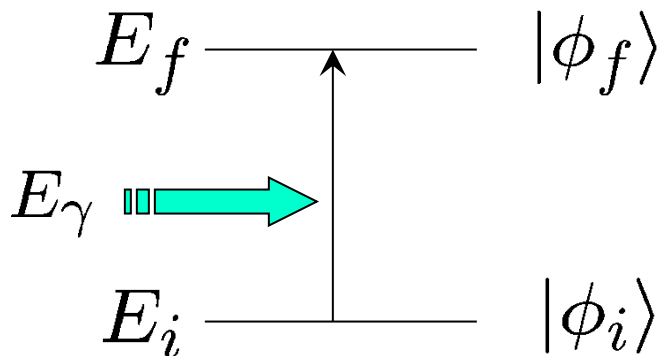
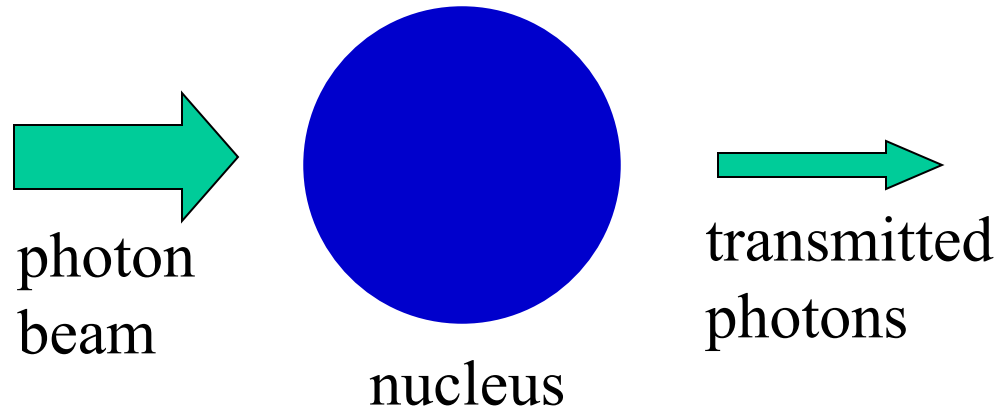
Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

cf. $41 \times 197^{-1/3} = 7.05 \text{ MeV}$

Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section



The state is strongly excited when $E_f - E_i = E_\gamma$.

フェルミの黄金律

$$\lambda = \frac{2\pi}{\hbar} |\langle \phi_f | \hat{T} | \phi_i \rangle|^2 \delta(E_f - E_i - E_\gamma)$$

Giant Dipole Resonance (GDR) 巨大双極子共鳴

光吸収の
スペクトル

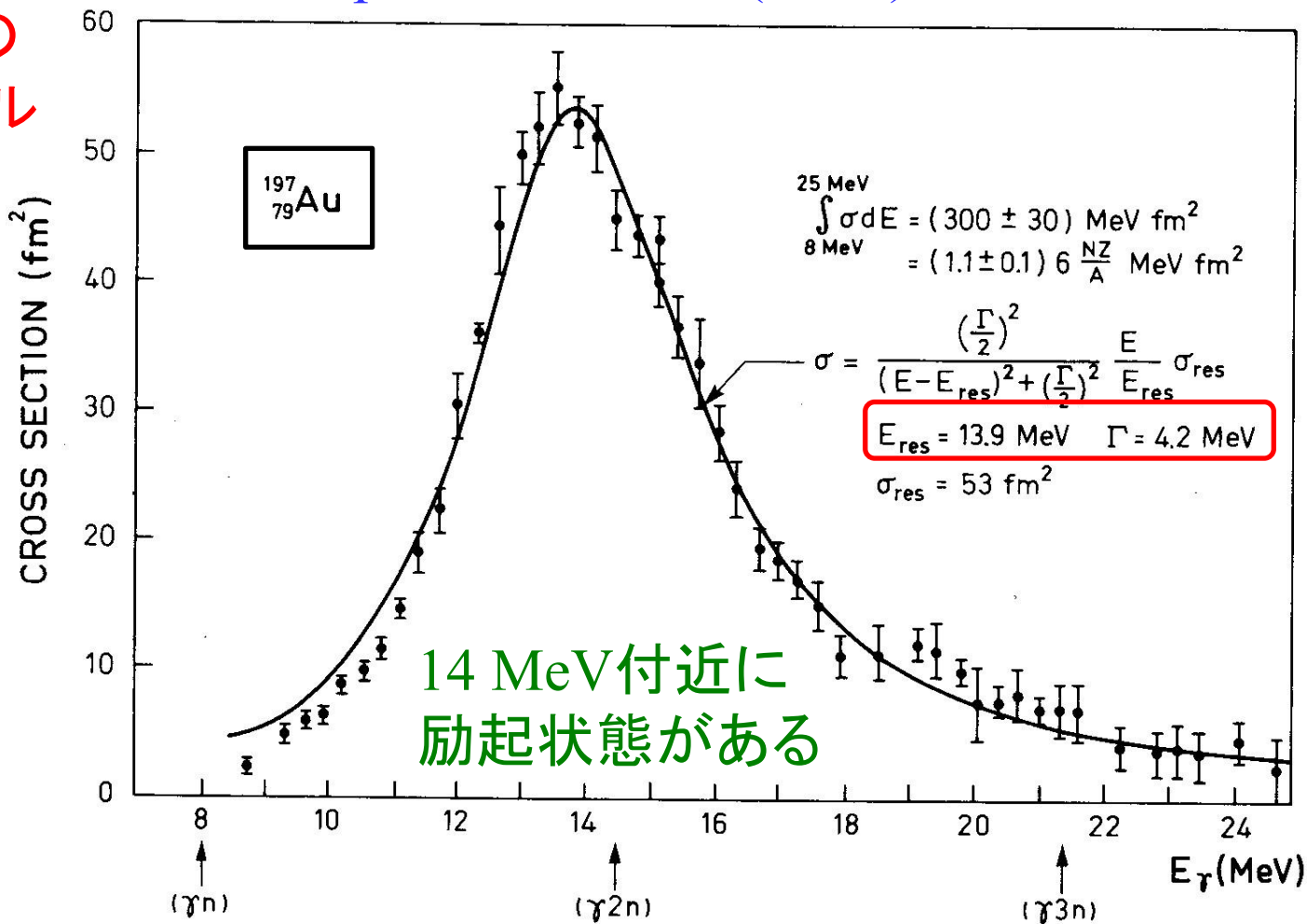


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cf. $41 \times 197^{-1/3} = 7.05 \text{ MeV}$

Remarks

i) Photon interaction \longleftrightarrow dipole excitation

$$H_{\text{int}} = \frac{1}{2m} \frac{e}{c} (\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p})$$

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\alpha=1,2} \sqrt{\frac{2\pi c^2 \hbar}{\omega V}} (a_{\mathbf{k}\alpha} \boldsymbol{\epsilon}_{\alpha} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}} t} + h.c.)$$

$$e^{i\mathbf{k} \cdot \mathbf{r}} \sim 1 \quad (\text{dipole approximation})$$

$E_{\gamma} \sim 10 \text{ MeV}$, $R \sim 5 \text{ fm}$ のときに、 kR はどのくらいになるか?

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$$k = \frac{E_{\gamma}}{\hbar c} \sim \frac{10}{200} \text{ (1/fm)} \rightarrow kR \sim 50/200 = 0.25$$

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$$\sigma_{\text{abs}}(E_{\gamma}) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_{\gamma} - E_f + E_i)$$

$$\tilde{z} = \sum_p (z_p - Z_{cm})$$

Remarks

i) Photon interaction \longleftrightarrow dipole excitation

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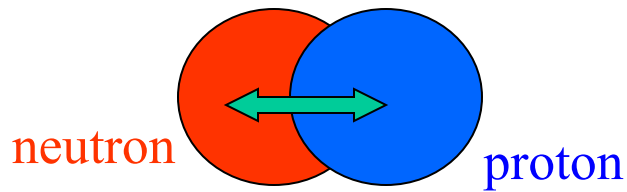
$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\alpha=1,2} \sqrt{\frac{2\pi c^2 \hbar}{\omega V}} (a_{\mathbf{k}\alpha} \boldsymbol{\epsilon}_{\alpha} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}} t} + h.c.)$$

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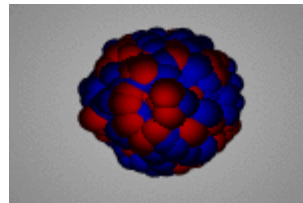


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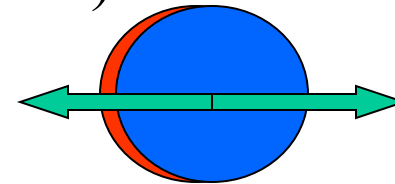
ii) Isospin



Isovector type



(note)



Isoscalar dipole motion

\longleftrightarrow c.m. motion (to the first order)

$$\tilde{z} = \sum_p (z_p - Z_{cm})$$


Remarks

i) Photon interaction \longleftrightarrow dipole excitation

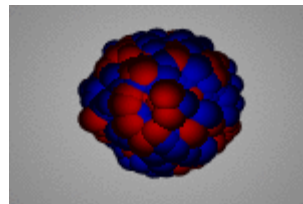
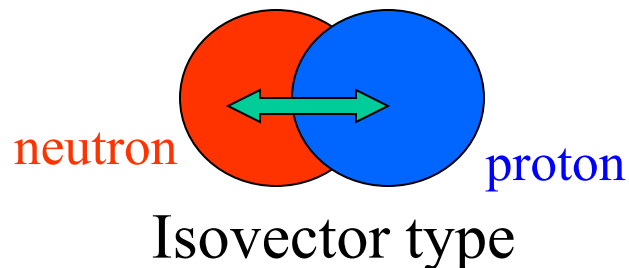
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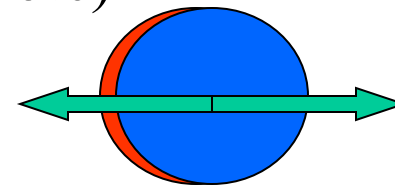

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ii) Isospin



(note)

$$\tilde{z} = \sum_p (z_p - Z_{\text{cm}})$$



iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

Giant Dipole Resonance (GDR) 巨大双極子共鳴

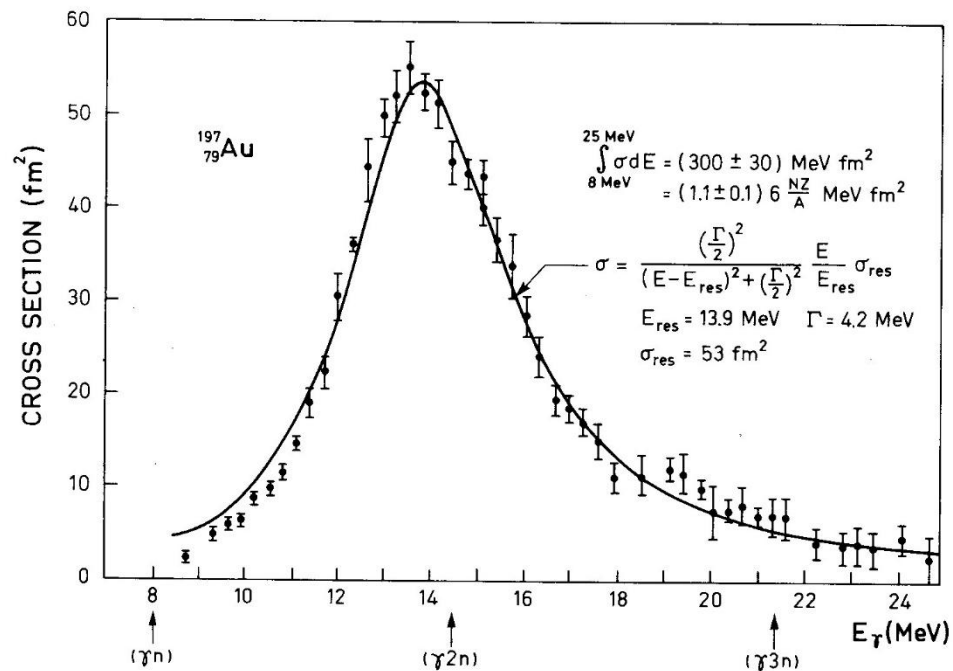
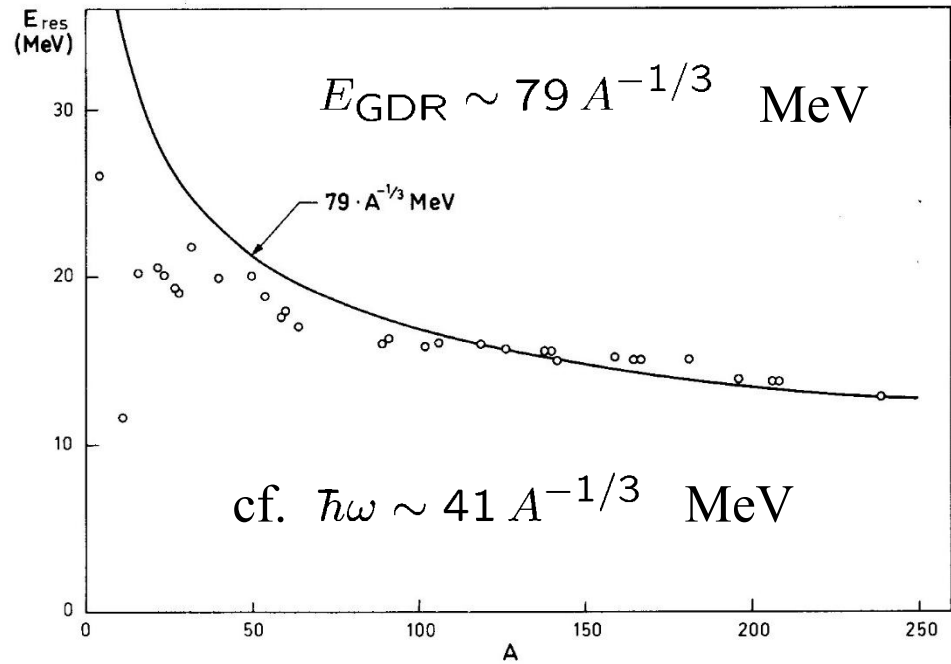
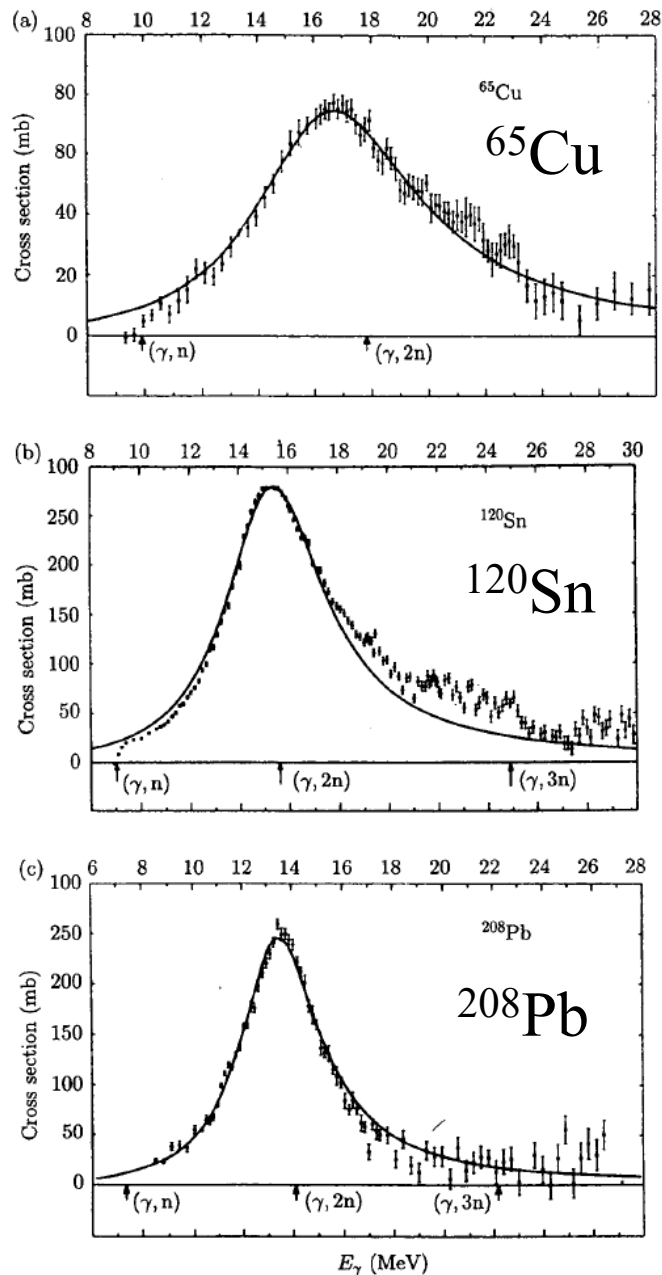


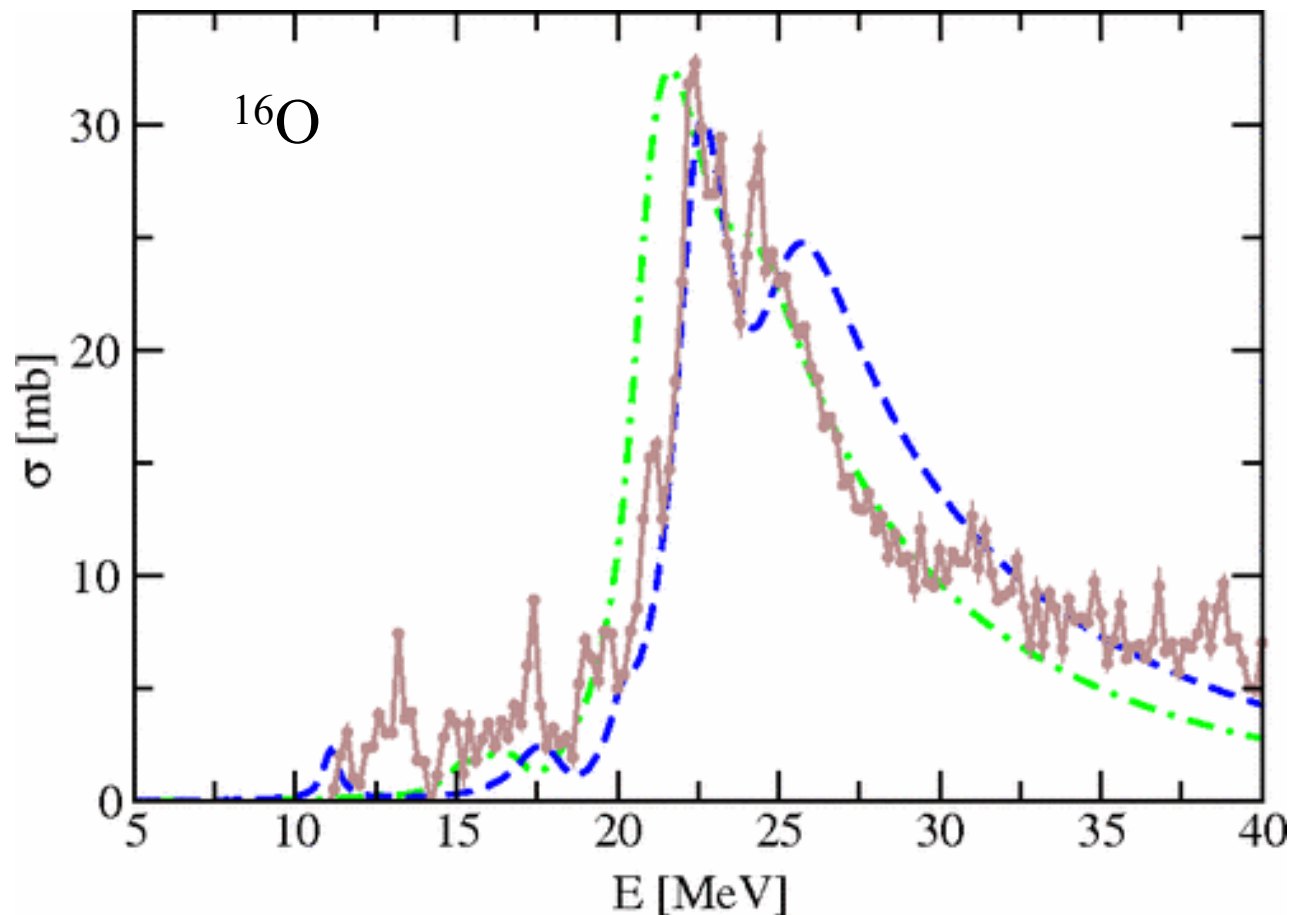
Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



Bohr-Mottelson
 “Nuclear Structure vol. II”

M.N. Harakeh and A. van der Woude,
 “Giant Resonances”

FIG. 1.2. The photo-neutron cross section $\sigma(\gamma, n)$ as a function of the photon energy for the three nuclei ^{208}Pb , ^{120}Sn and ^{65}Cu . Note that for these nuclei $\sigma(\gamma, n) \approx \sigma_{\text{abs}}(\gamma)$. From reference (BER75).



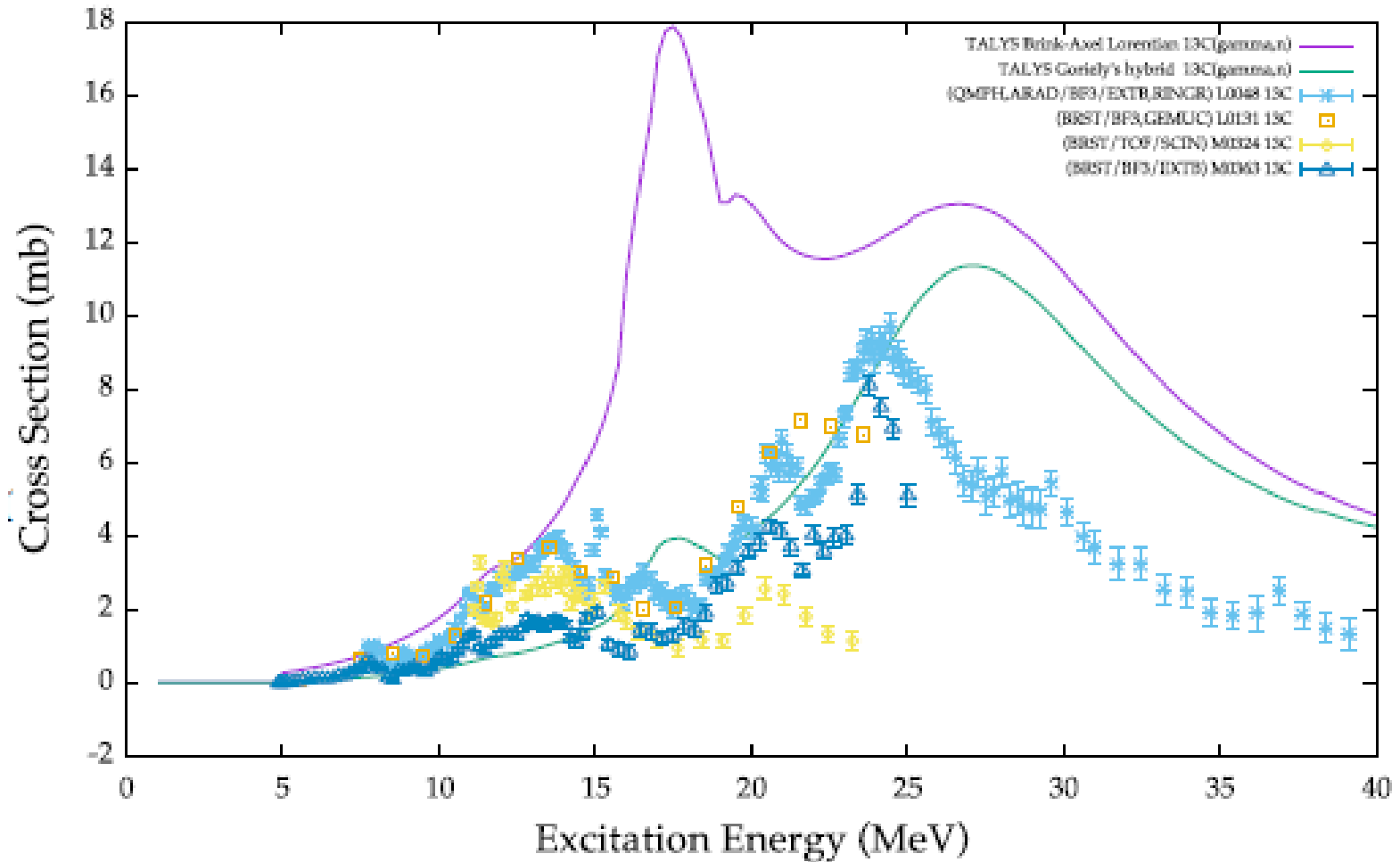
実験データ: 茶色

N. Lyutorovich et al., Phys. Rev. Lett. 109 (2012) 092502

cf. $41 \times 16^{-1/3} = 16.27 \text{ MeV}$

ただし、軽い核 (Fe, Ni 程度以下) のデータはまだ少ない
→ PANDRA プロジェクト (RCNP 民井さん) 2022年～

$^{13}\text{C}(\gamma, n)$

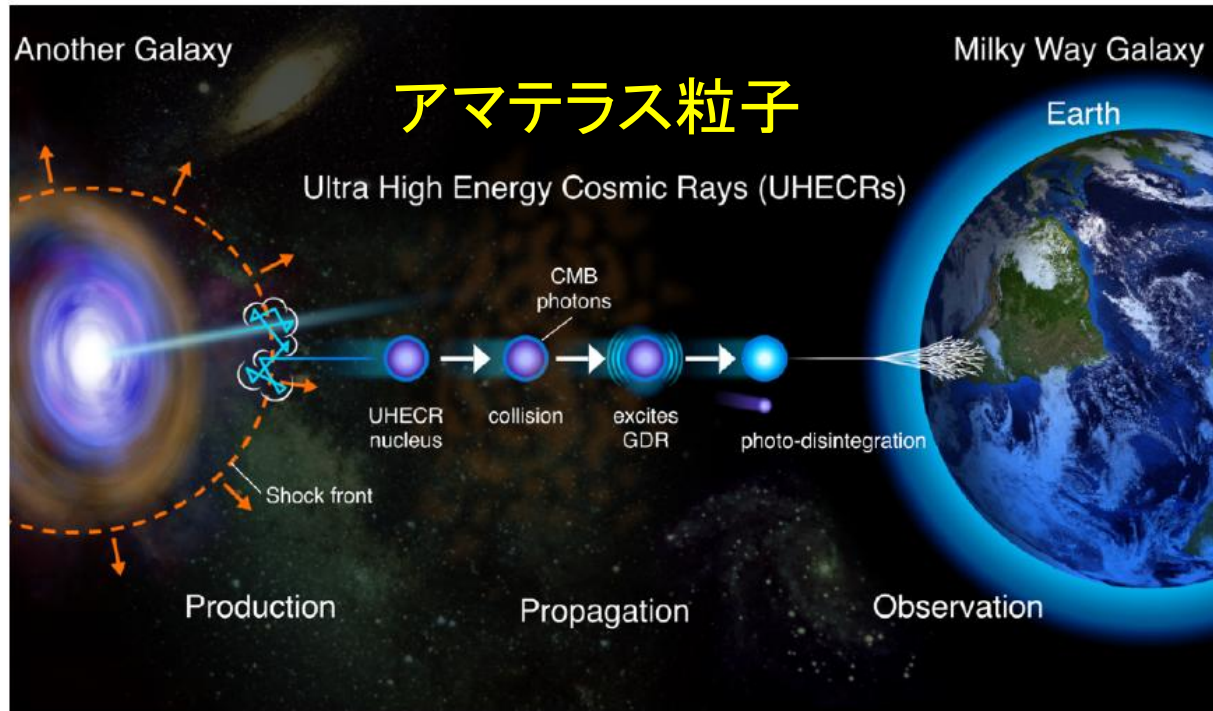


スライド: 民井さん

- 実験データ同士の矛盾
- 軽い核の理論計算

PANDORA Project

Photo-Absorption of Nuclei and Decay Observation for Reactions in Astrophysics



スライド: 民井さん

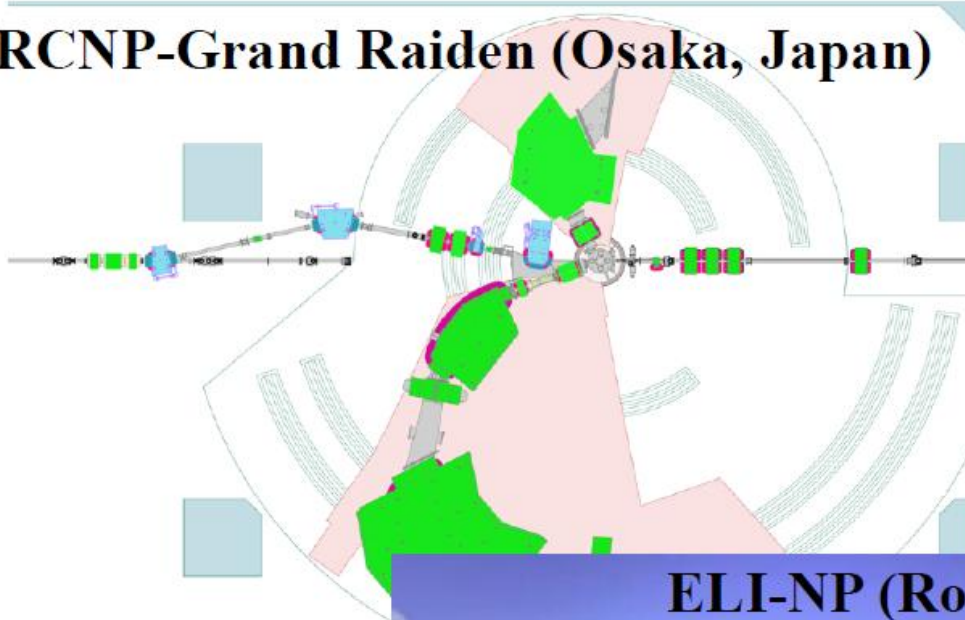
高エネルギー宇宙線の観測で、原子核のGDRが大きな不定性
の原因になっている

PANDORA Project

Photo-Absorption of Nuclei and Decay Observation for Reactions in Astrophysics

Joint project among three experimental facilities with nuclear theories and astrophysical simulations

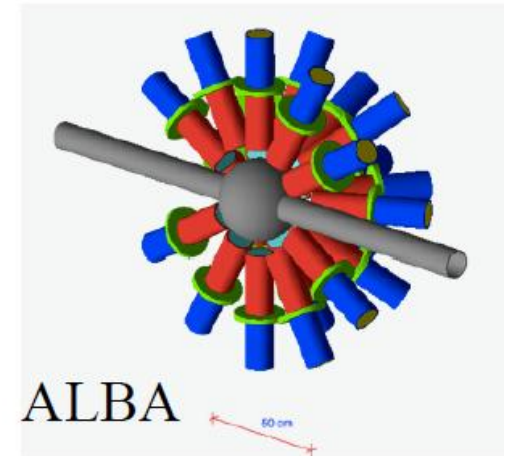
RCNP-Grand Raiden (Osaka, Japan)



ELI-NP (Romania)

complementary
experimental
techniques

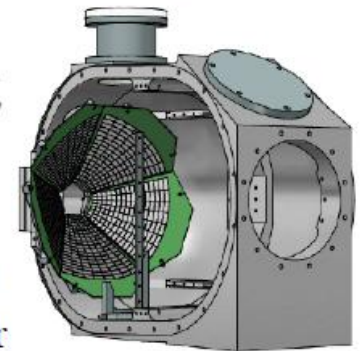
iThemba LABS South Africa

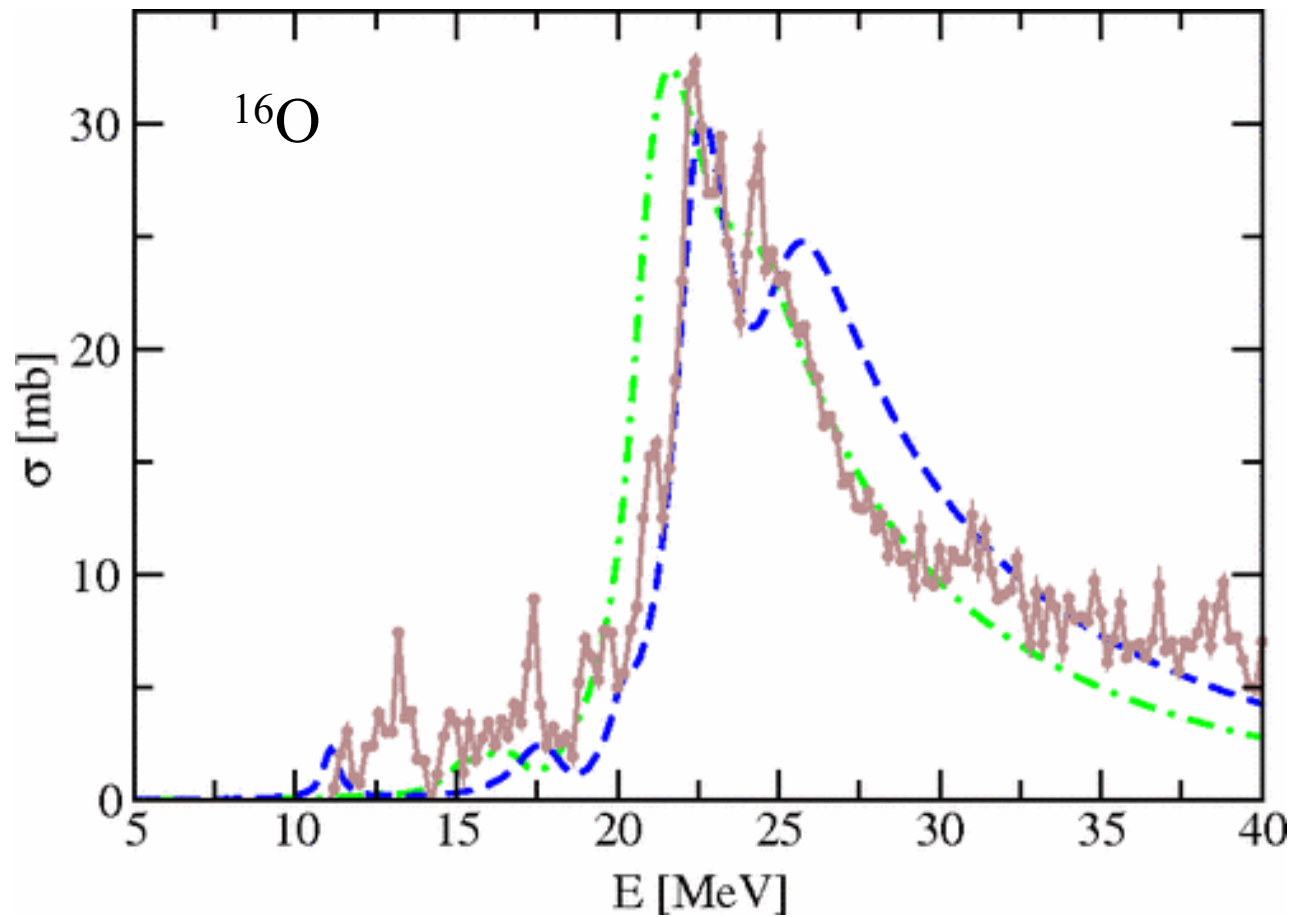


ALBA

CAKE

decay
charge
particle
detector
array



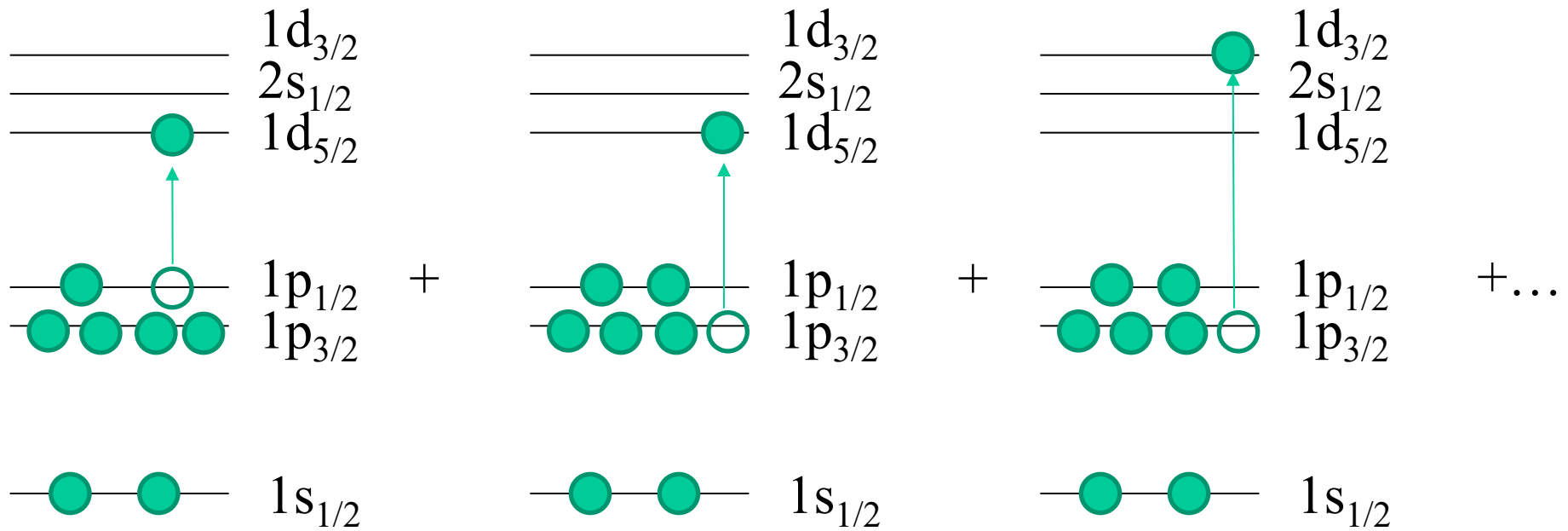


実験データ: 茶色

N. Lyutorovich et al., Phys. Rev. Lett. 109 (2012) 092502

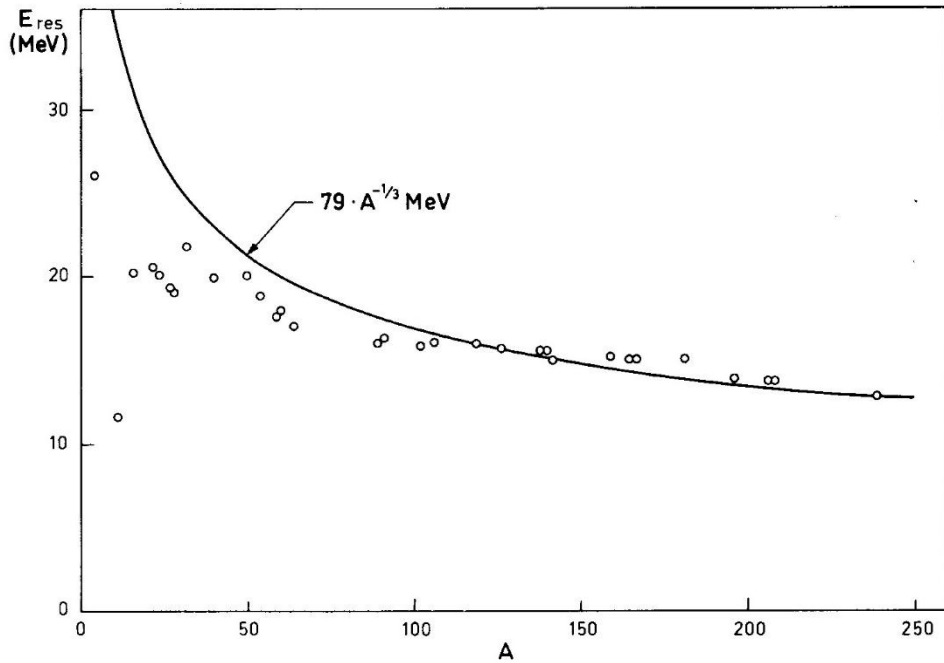
cf. $41 \times 16^{-1/3} = 16.27 \text{ MeV}$

何故励起エネルギーが大きくなるのか？

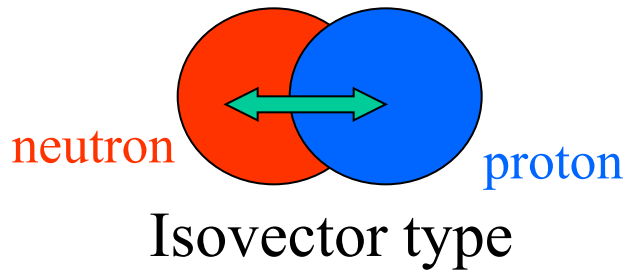


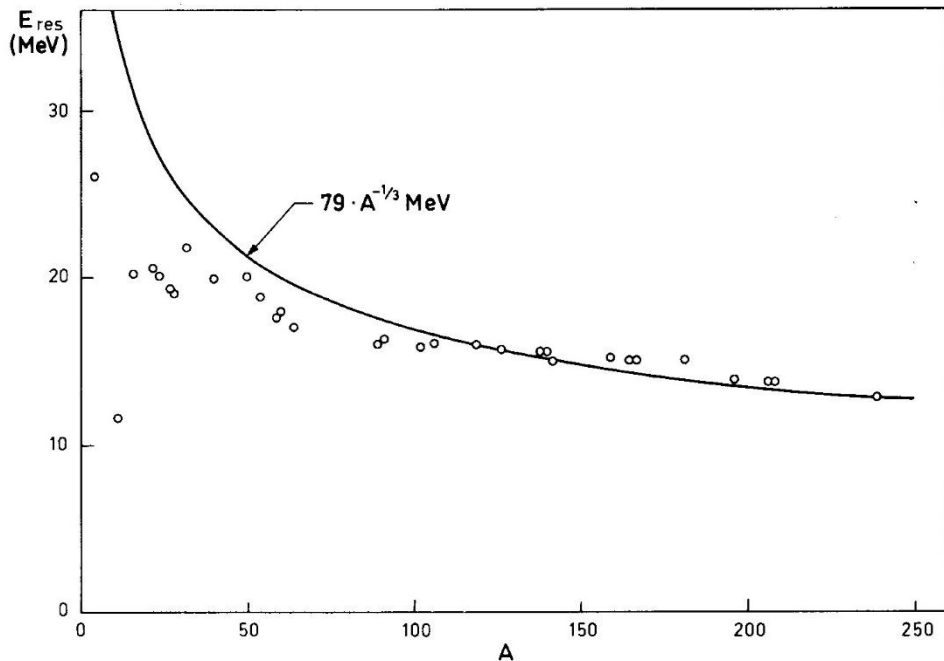
様々な励起状態がコヒーレントに重ね合わさることにより
 「集団的」になる。→(次回もう少し詳しく)

残留相互作用が大きな役割

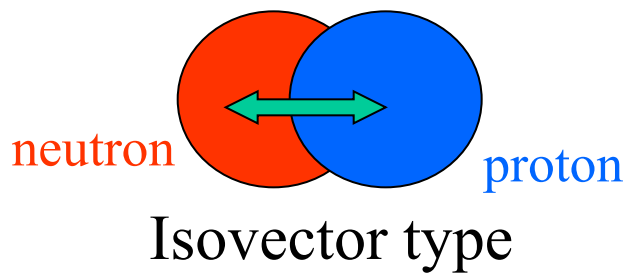


Bohr-Mottelson
“Nuclear Structure vol. II”

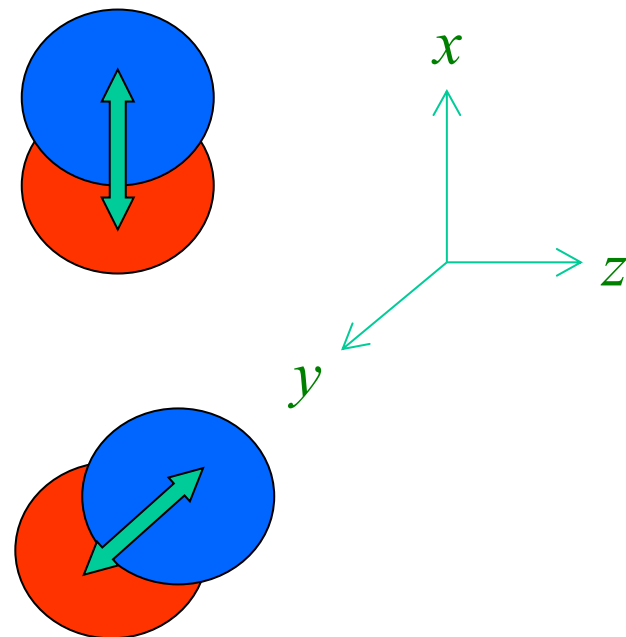


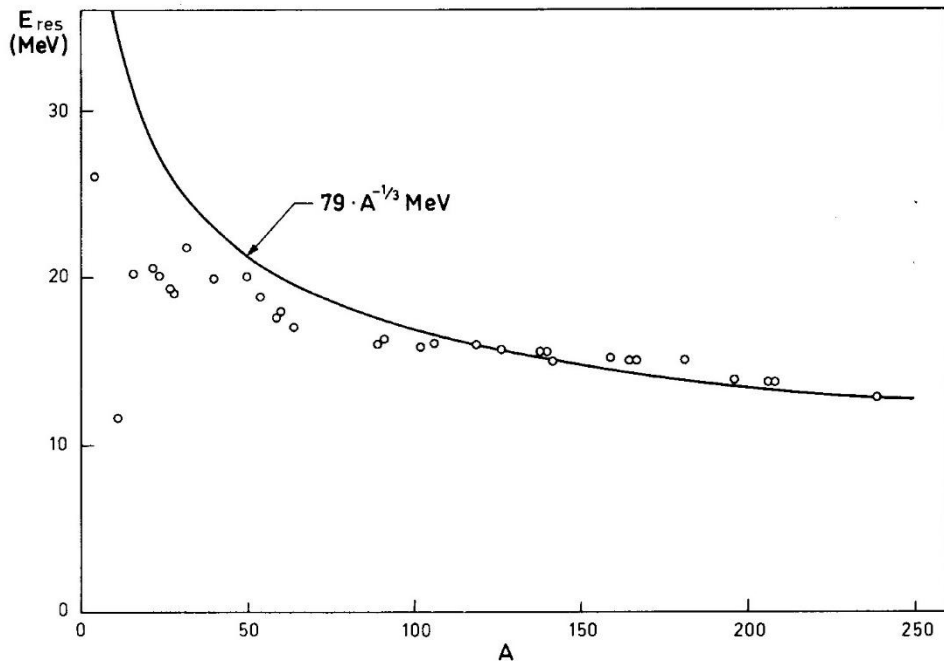


Bohr-Mottelson
 “Nuclear Structure vol. II”

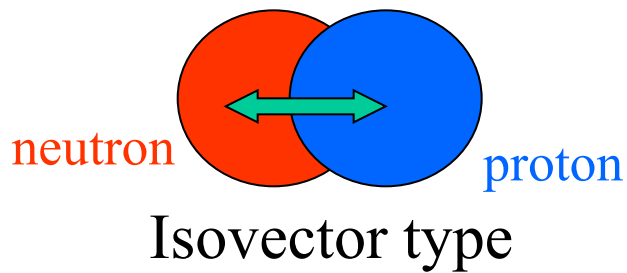


$I=1$ なので3つのモード





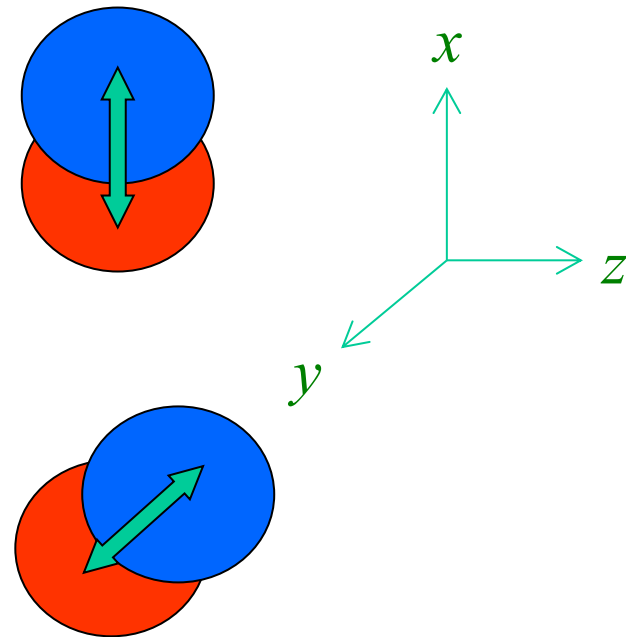
Bohr-Mottelson
 “Nuclear Structure vol. II”



$$E_{GDR} \propto A^{-1/3}$$

$$\propto 1/R$$

$I=1$ なので3つのモード



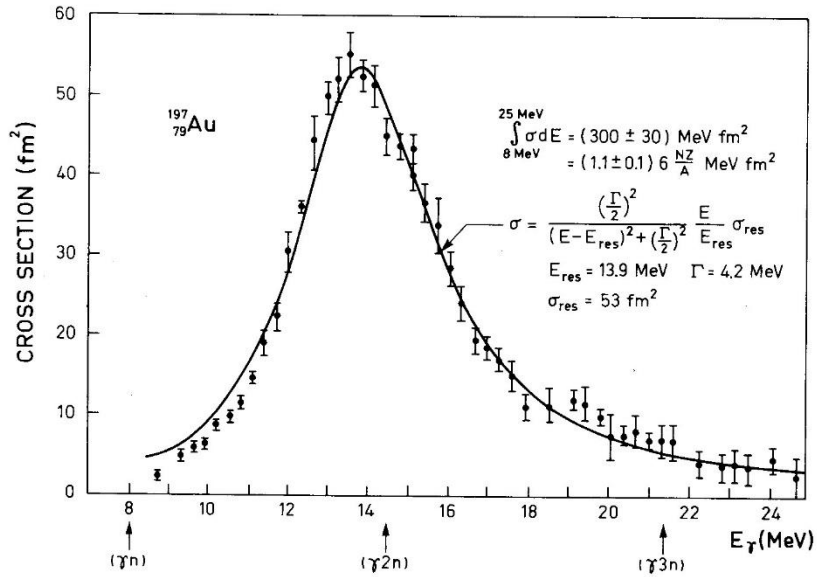
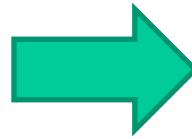
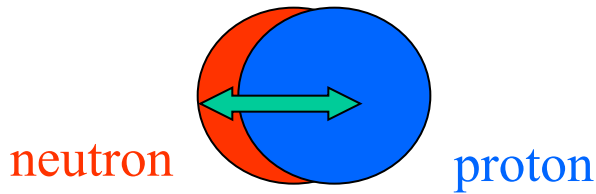


Figure 6-18 Total photoabsorption cross section for ^{197}Au . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

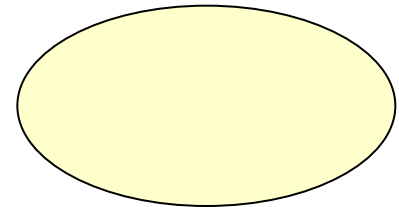


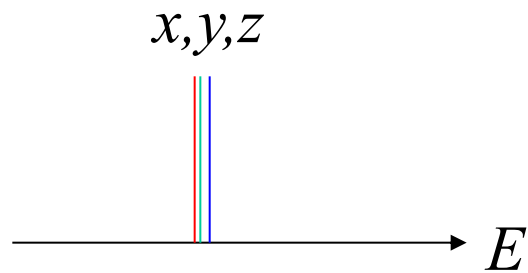
?



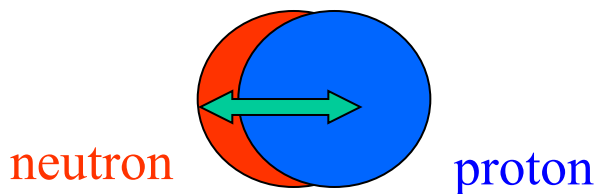
$$E_{\text{GDR}} \propto 1/R$$

deformed nucleus



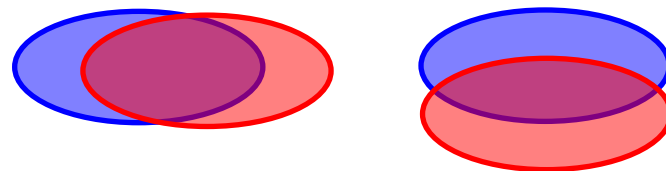
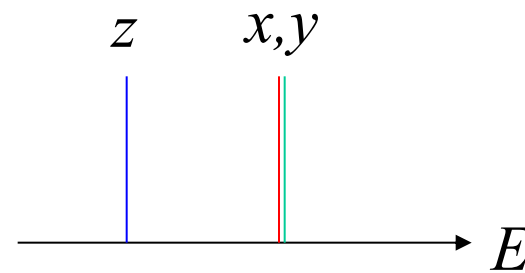
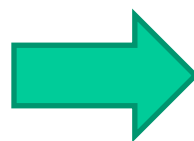


spherical nucleus

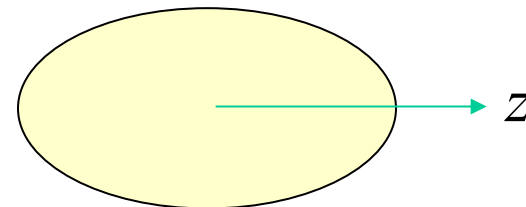


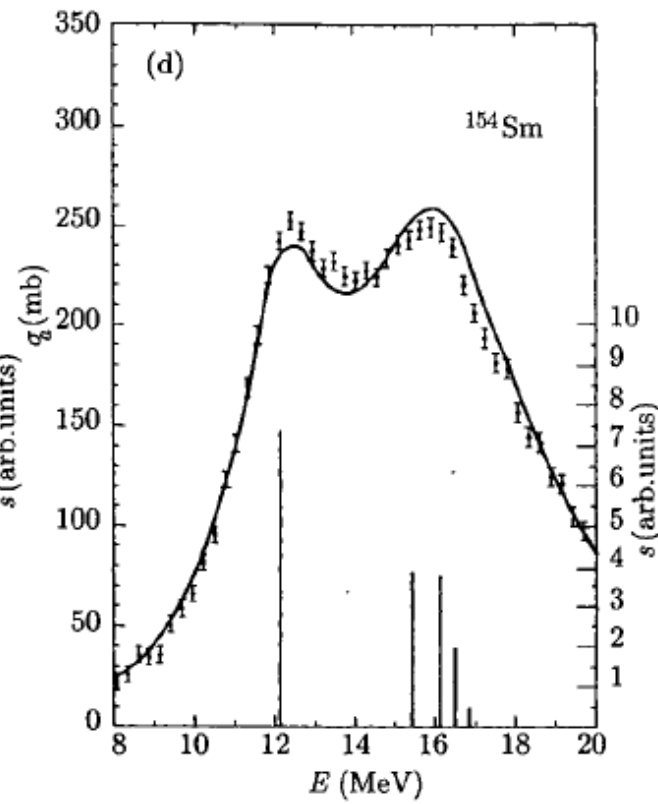
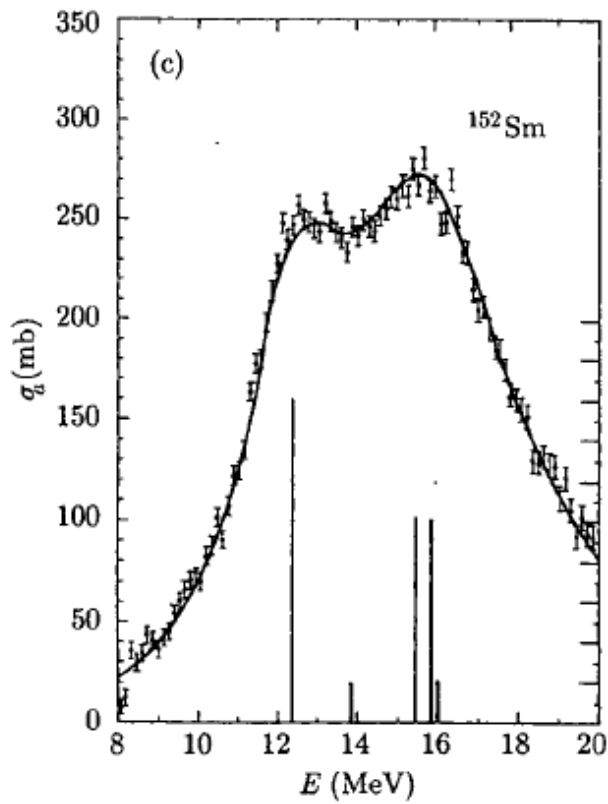
$$E_{\text{GDR}} \propto 1/R$$

(prolate deformation)



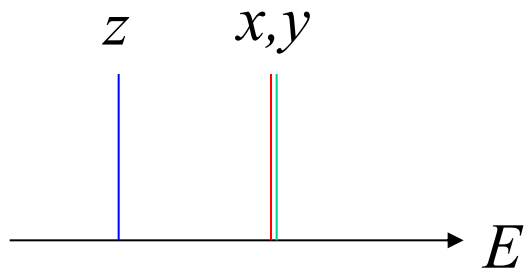
deformed nucleus



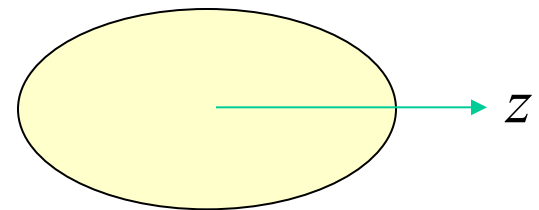


M.N. Harakeh and
A. van der Woude,
“Giant Resonances”

(prolate deformation)



deformed nucleus



Deformation effect

$$E_{\text{GDR}} \sim A^{-1/3} \sim 1/R$$

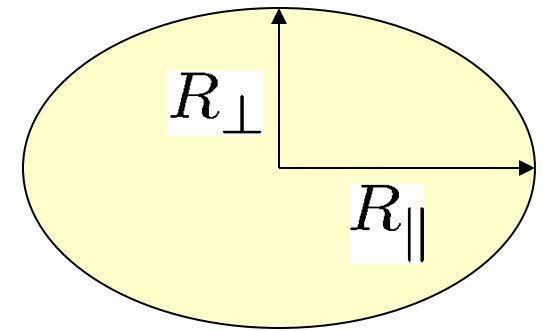
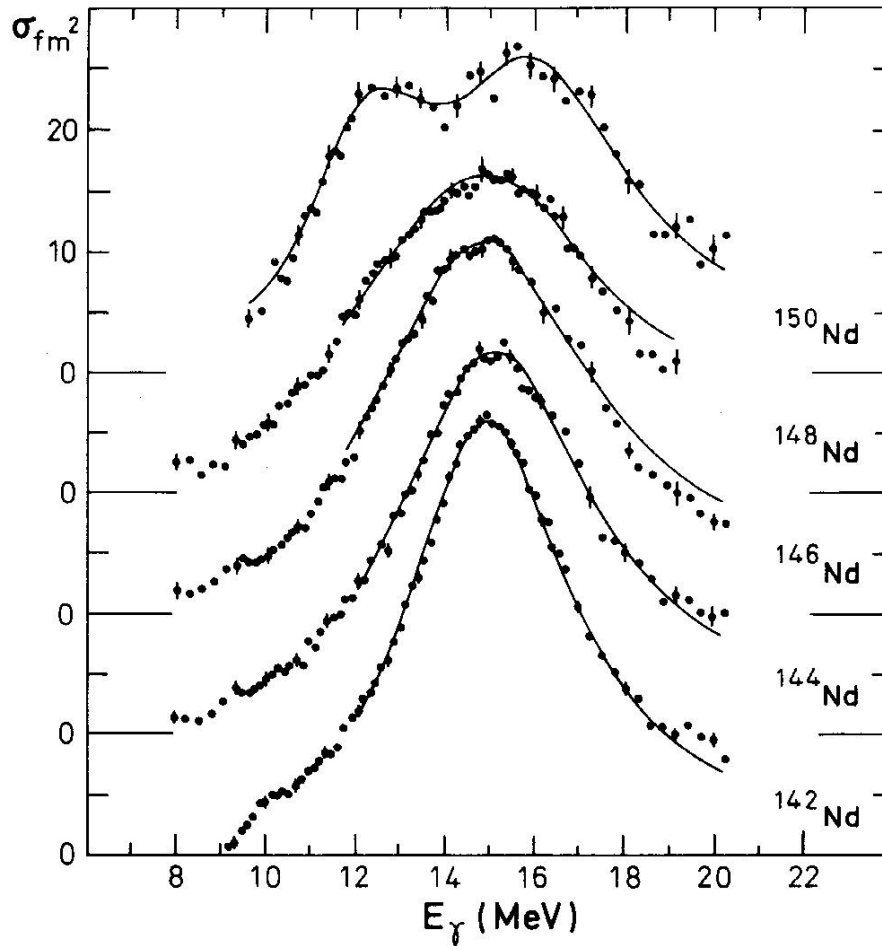
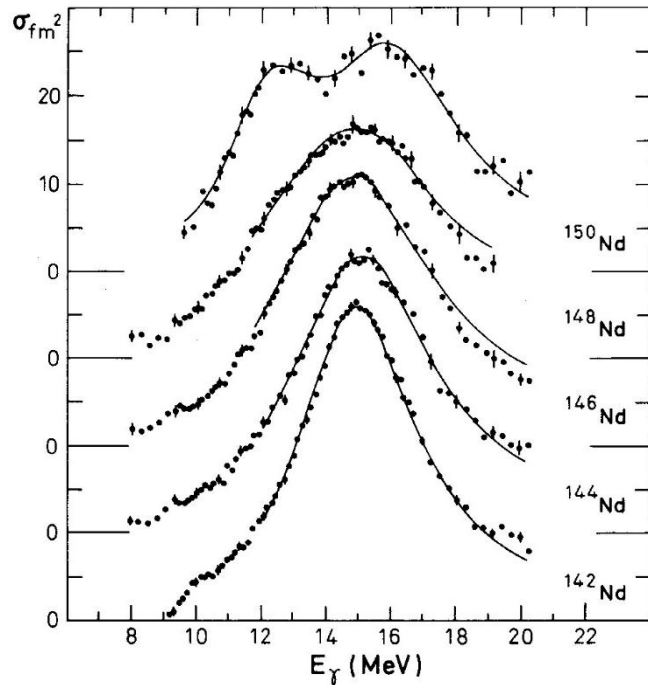


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssièrre, *Nuclear Phys. A172*, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.

レポート問題2(×切:1月27日(火)23:55)



左に示す ^{150}Nd 核の光吸収断面積(巨大双極子共鳴:GDR)の実験データから ^{150}Nd 核の変形度を見積もってみよう。

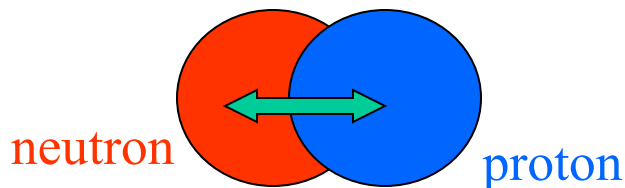
- (i) GDRのエネルギーはおおよそ $E \sim 79 A^{-1/3}$ MeV となることが知られている。原子核の半径を $R_0 = r_0 A^{1/3}$ とすると、これは $E \sim 79 r_0 / R_0$ MeV となることを示している。左の図の ^{150}Nd のデータは 12 MeV と 16 MeV にピークを持つが、 $r_0 = 1.1$ fm としてそれぞれのエネルギーに相当する半径 R_0 の大きさを求めよ。(この場合、 $R_0 = r_0 A^{1/3}$ にはならないことに注意せよ。)

- (ii) (i) で求めた2つの半径(小さい方から R_1, R_2 とする)が ^{150}Nd の変形に起因すると仮定する。このとき、半径が

$$R(\theta) = R_0(1 + \beta Y_{20}(\theta)); \quad R_0 = r_0 A^{1/3}$$
で与えられるとし、 $R_1 = R(\theta = \pi/2)$, $R_2 = R(\theta = 0)$ と考え、二つの半径の差 $R_2 - R_1$ の値から変形度 β を求めよ。

Giant Dipole Resonances

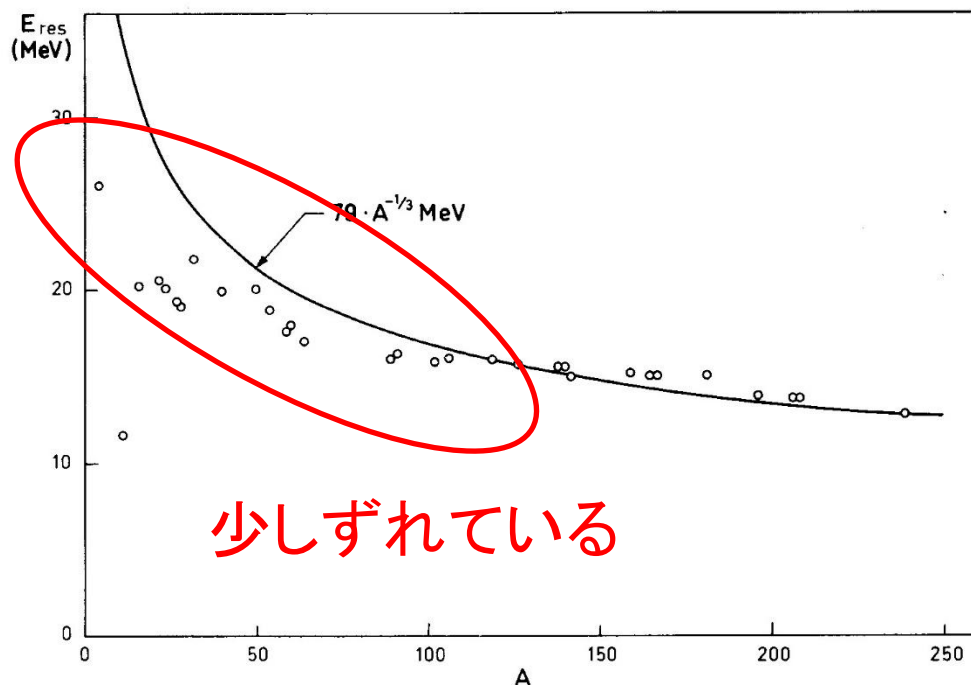
● Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

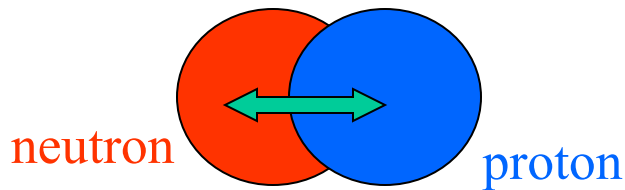
→ $E_{\text{GDR}} \sim A^{-1/6}$

→ Inconsistent with expt.
(except for light nuclei)



Giant Dipole Resonances

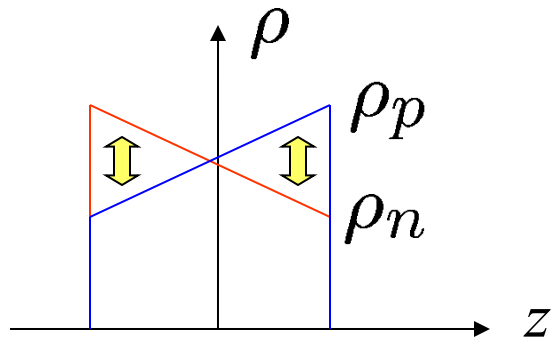
• Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow E_{\text{GDR}} \sim A^{-1/6}$$

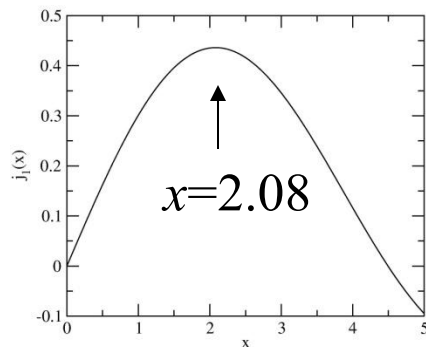
• Steinwedel-Jensen type



$$\hat{Q} = j_1(kr) Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow E_{\text{GDR}} \sim A^{-1/3}$$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x \cos x) / x^2$$

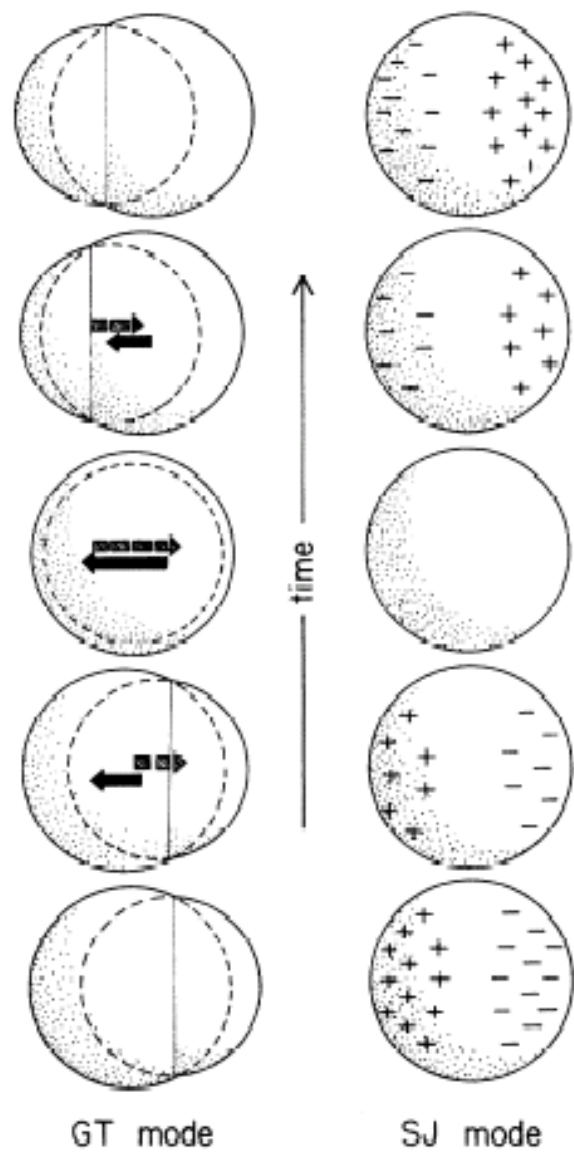


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.

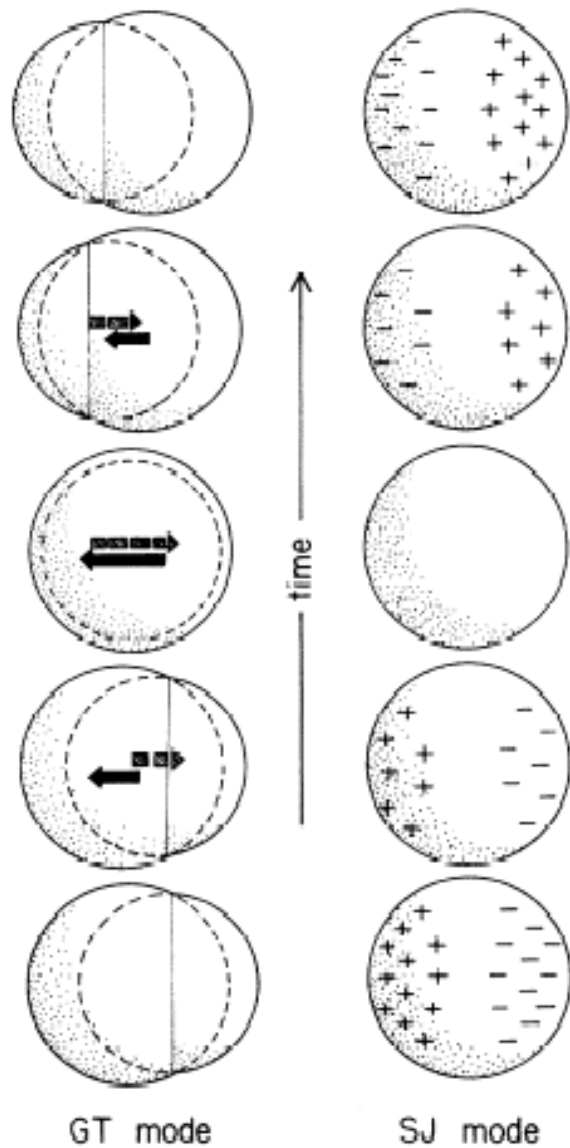
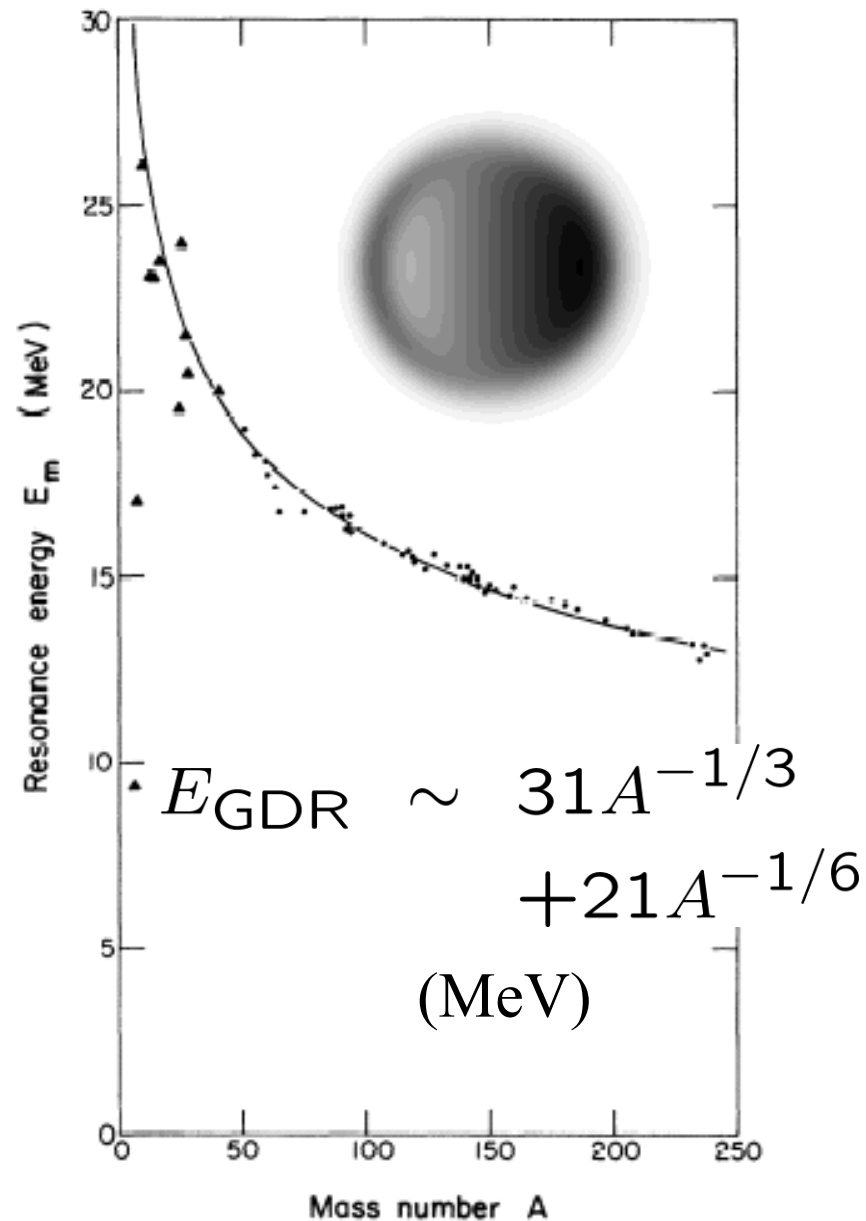
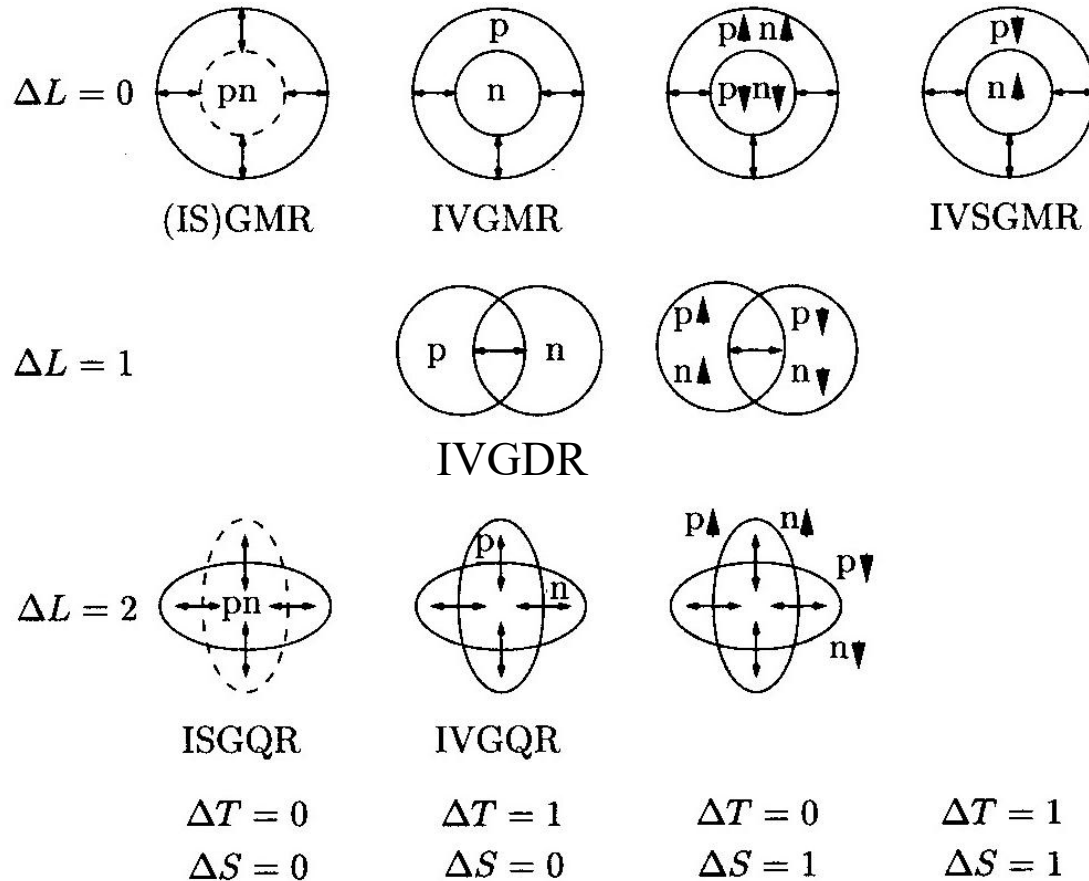


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.



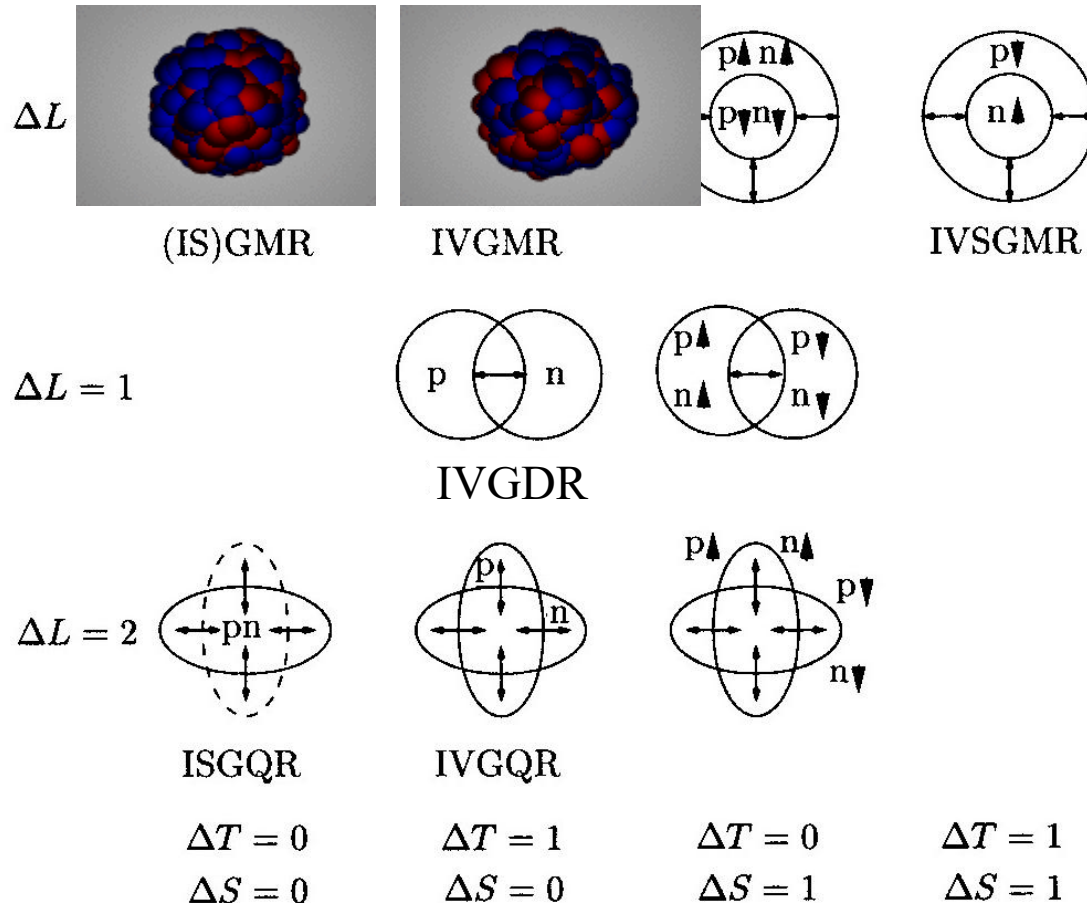
ii) Inelastic scattering

(e,e') , (p,p') , (α,α') , Heavy-ion \longrightarrow Higher multipolarities



ii) Inelastic scattering

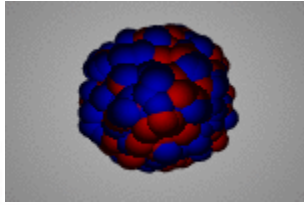
(e,e') , (p,p') , (α,α') , Heavy-ion \longrightarrow Higher multipolarities



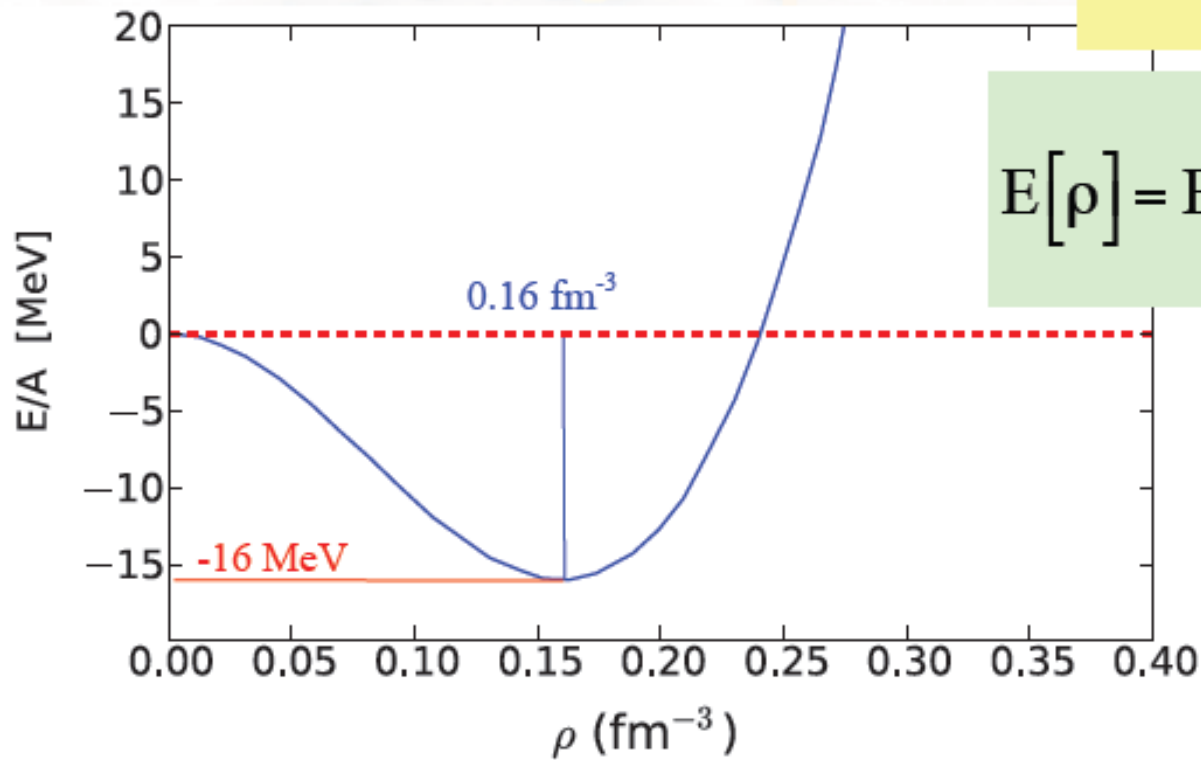
movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

EOS of infinite nuclear matter



原子核がどのくらい硬いのか

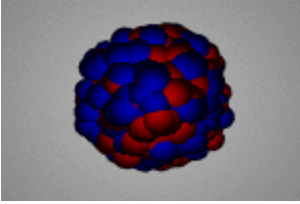


$$K_{\infty} = 9\rho^2 \left. \frac{d^2[E(\rho)/\rho]}{d\rho^2} \right|_{\rho_0}$$

$$E[\rho] = E[\rho_0] + \frac{1}{18} K_{\infty} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2$$

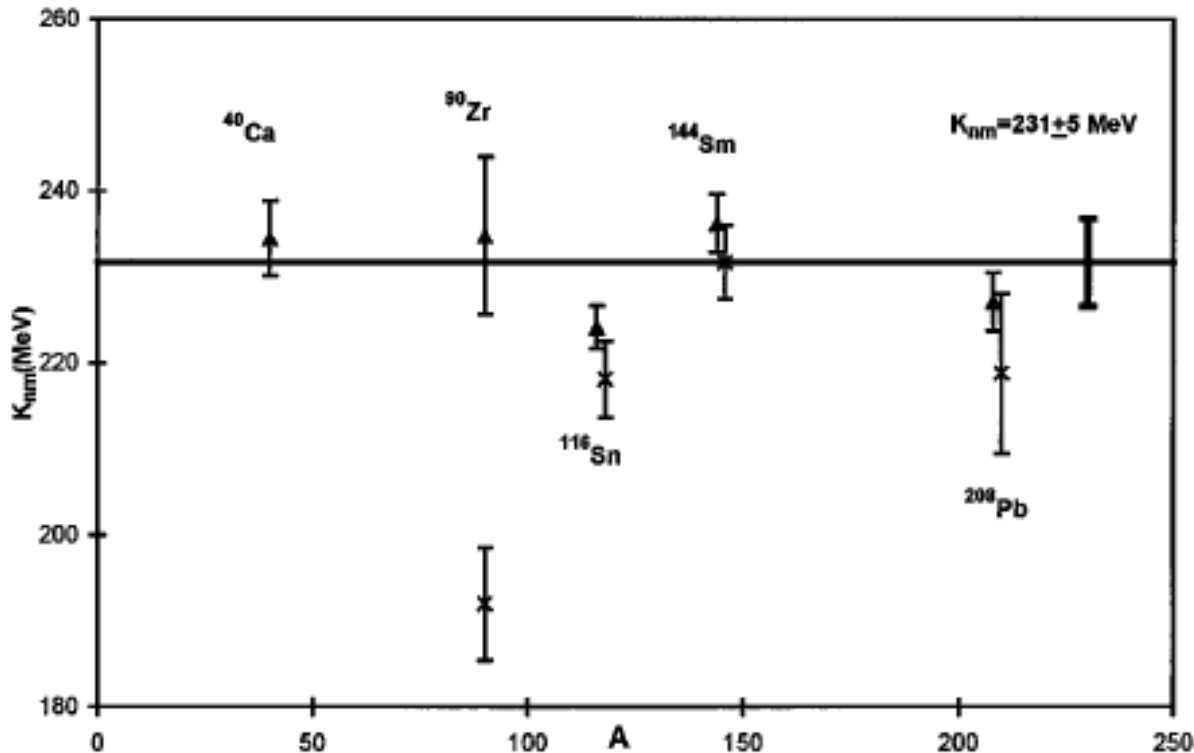
cf. 中性子星の大きさ
や重さ(MR曲線)

Isoscalar giant monopole resonances (breathing mode)



$$E_{\text{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m \langle r^2 \rangle}}$$

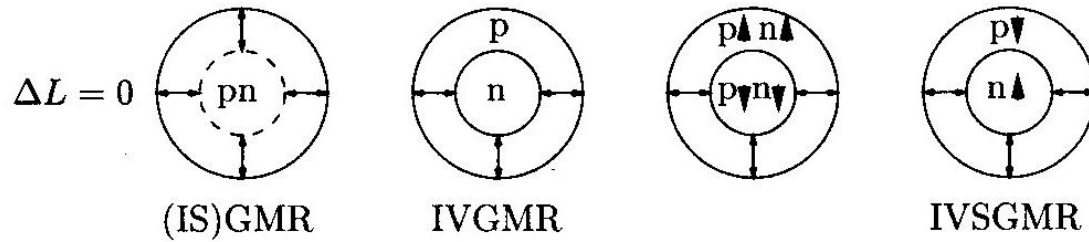
J.P. Blaizot,
Phys. Rep. 64 ('80) 171



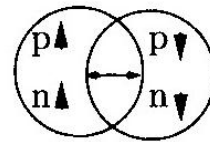
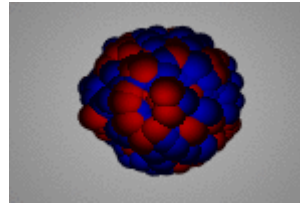
$K \sim 231 \pm 5 \text{ MeV}$

ii) Inelastic scattering

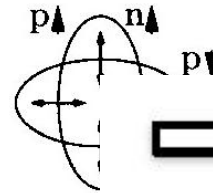
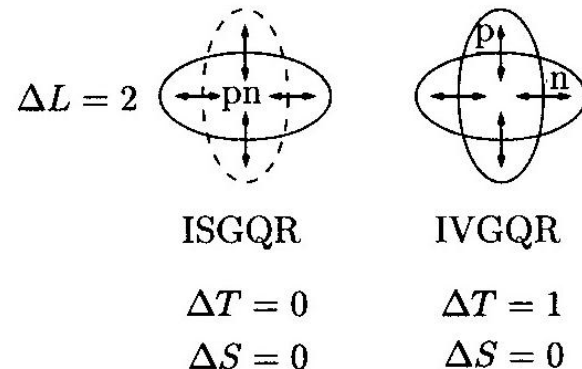
(e,e'), (p,p'), (α,α'), Heavy-ion \longrightarrow Higher multipolarities



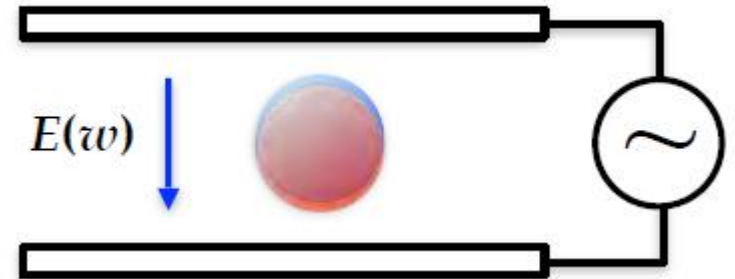
$\Delta L = 1$



原子核の
静電分極率
 \rightarrow 対称エネルギー



ΔT
 ΔS



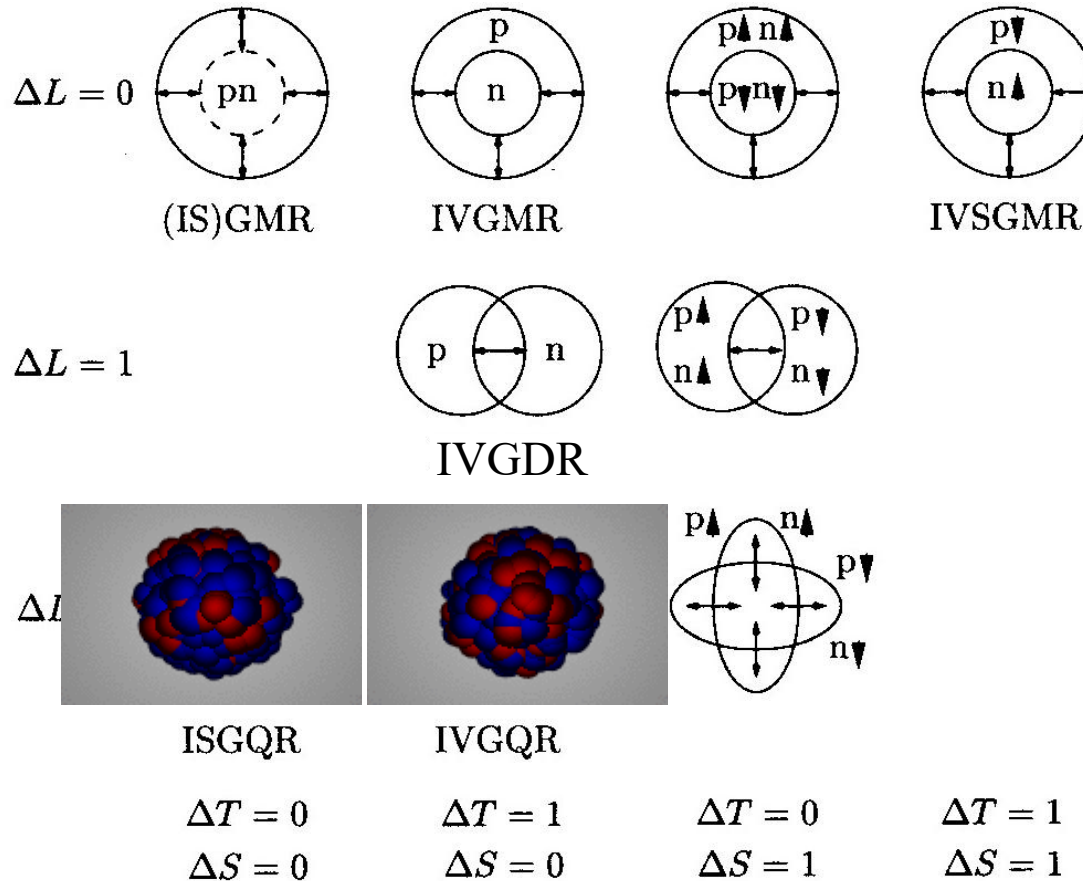
図：民井さん

movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

ii) Inelastic scattering

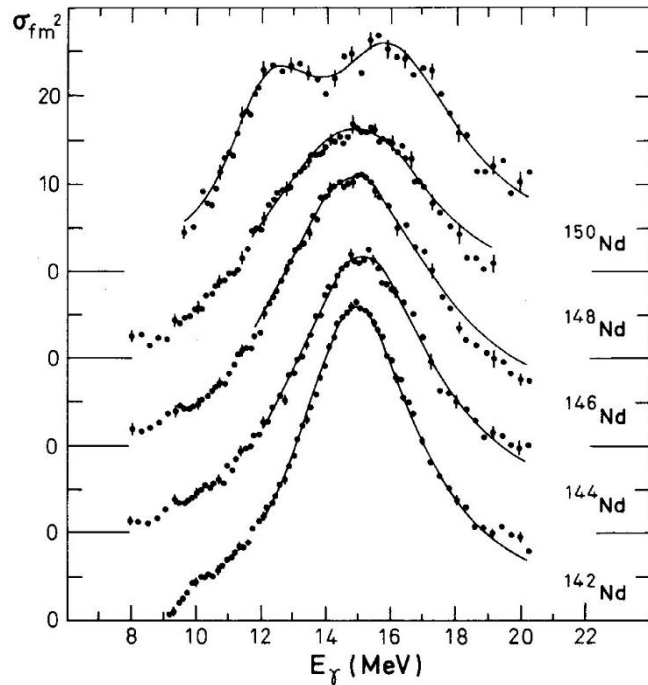
(e,e'), (p,p'), (α,α'), Heavy-ion \longrightarrow Higher multipolarities



(note) $\Delta L = 2 \longrightarrow \Delta N = 2$ Giant Resonance (GQR)

$\Delta N = 0$ Low-lying state

レポート問題2 (×切: 1月27日(火)23:55)



左に示す ^{150}Nd 核の光吸収断面積(巨大双極子共鳴: GDR)の実験データから ^{150}Nd 核の変形度を見積もってみよう。

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