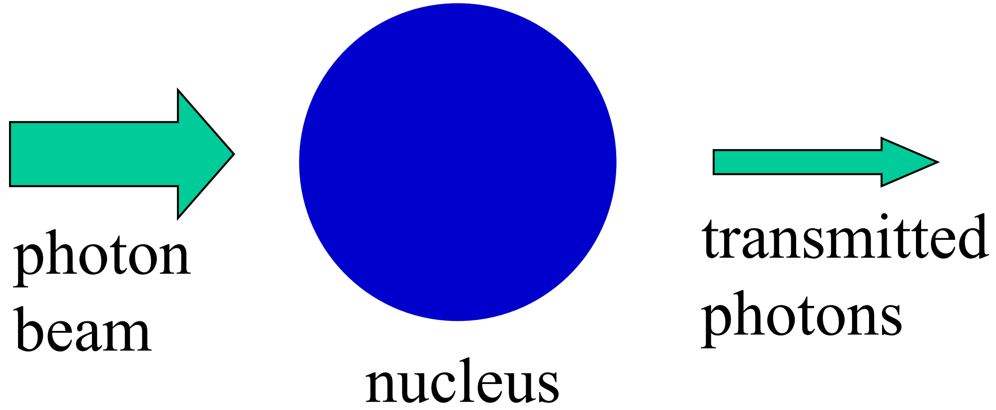
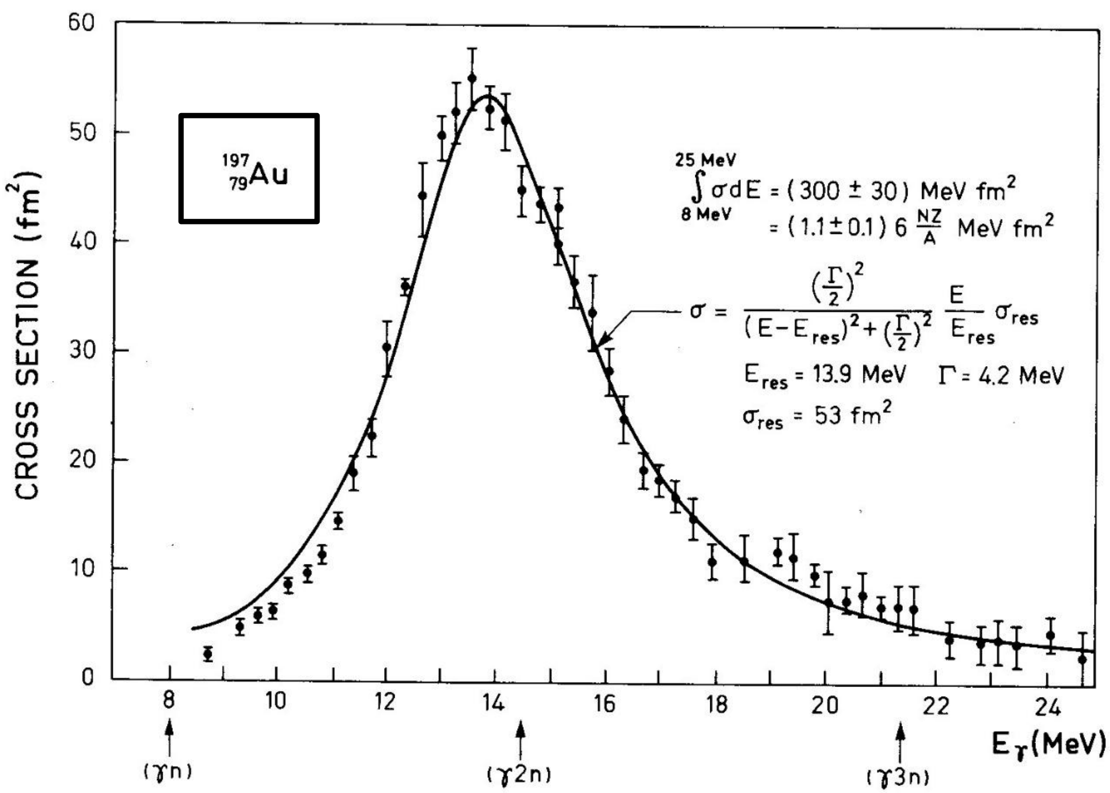
# Collective Vibrations

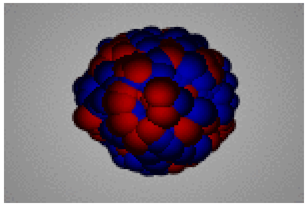
光吸収断面積

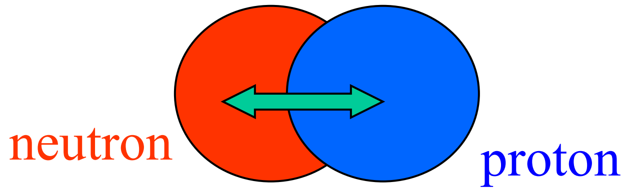



## Giant Dipole Resonance (GDR)　巨大双極子共鳴




14 MeV付近に
励起状態がある



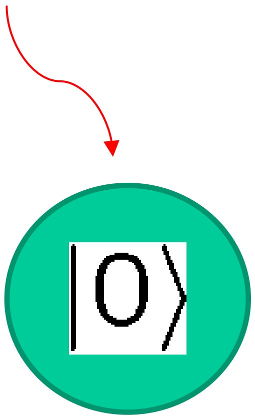


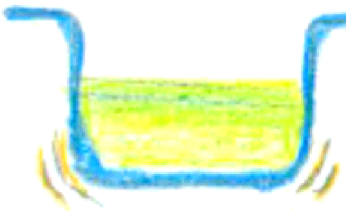

Isovector type

# Sum Rule

$$|\psi\rangle = F|0\rangle$$

$F$（電磁場などの外場）



図：松柳研一氏

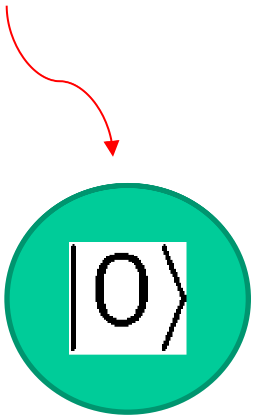
外場をかけて原子核をゆすってみる

# Sum Rule

$$|\psi\rangle = F|0\rangle$$

$$= \sum_n |n\rangle\langle n|F|0\rangle$$

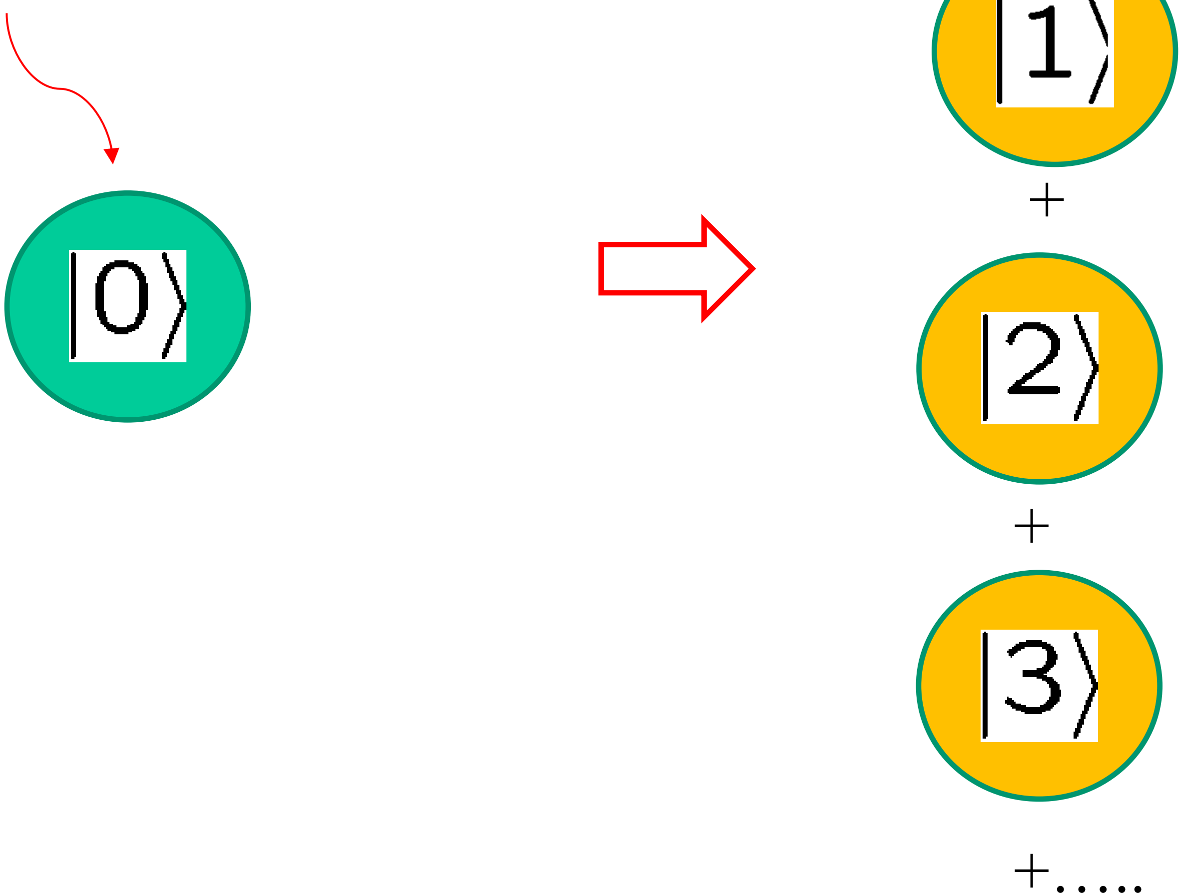
$F$（電磁場などの外場）



# Sum Rule

$$|\psi\rangle = F|0\rangle$$
$$= \sum_n |n\rangle\langle n|F|0\rangle$$

$F$（電磁場などの外場）




確率

$|\langle 1|F|0\rangle|^2$

$|\langle 2|F|0\rangle|^2$

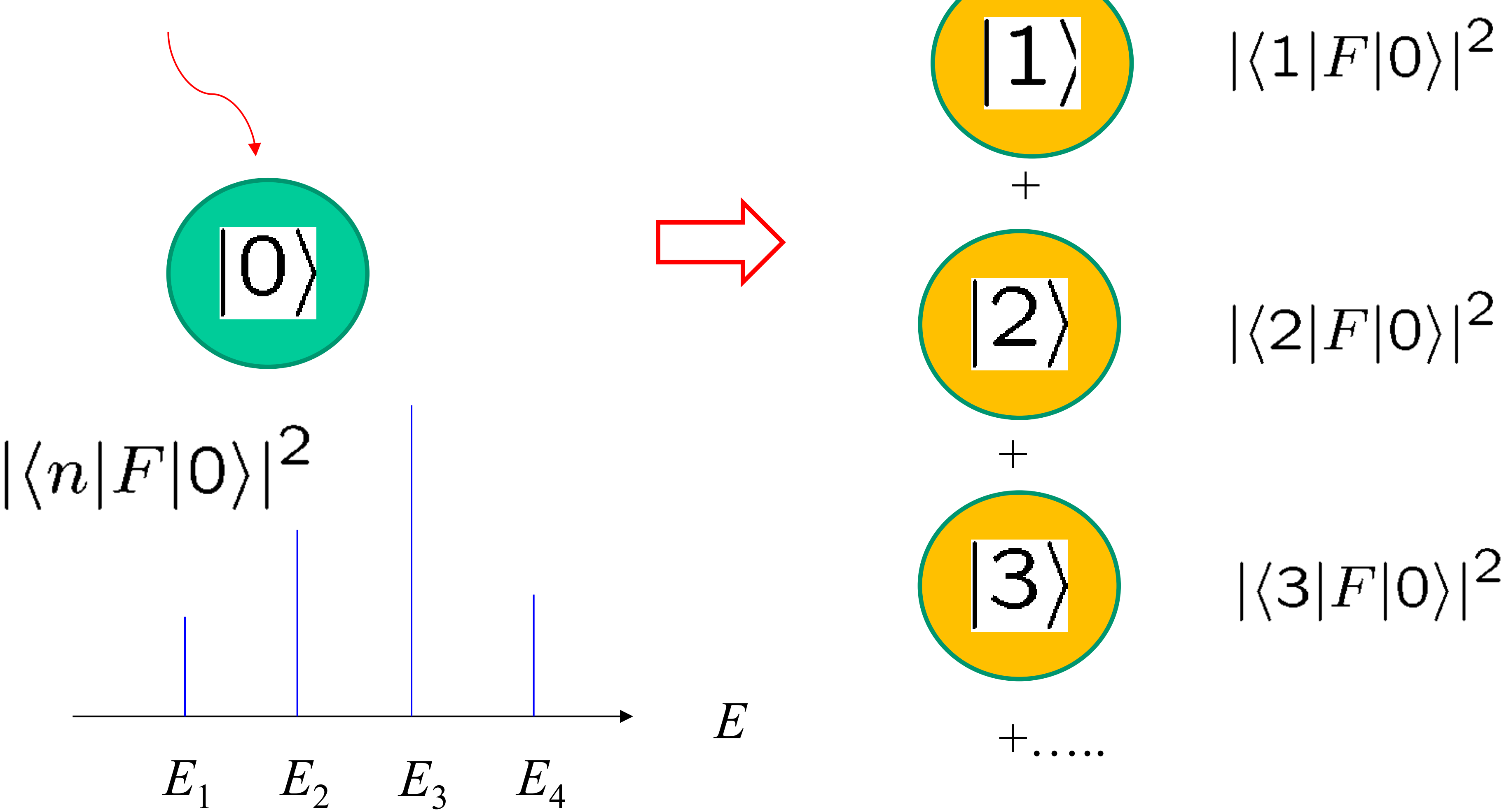
$|\langle 3|F|0\rangle|^2$

# Sum Rule

Strength function
（強度関数）:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \, \delta(E_n - E_0 - E)$$

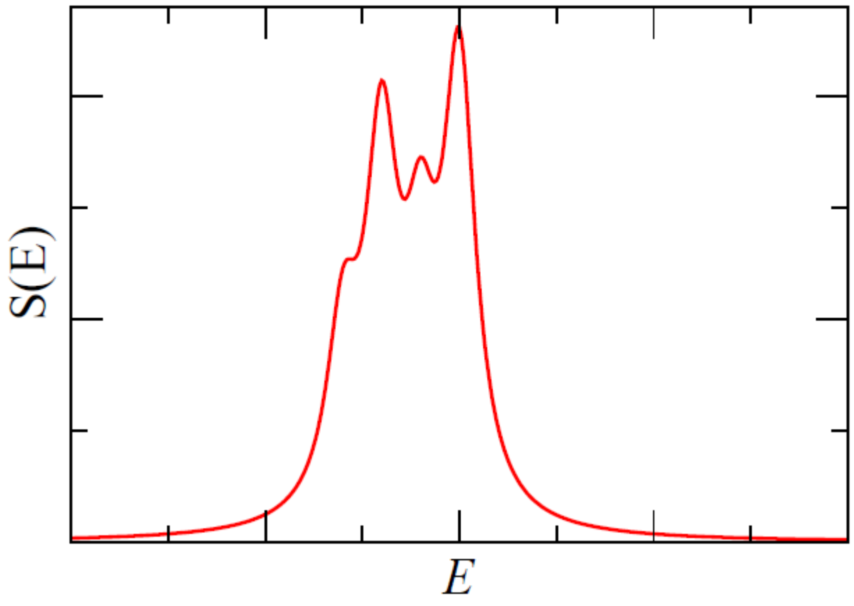
$F$（電磁場などの外場）



# Sum Rule

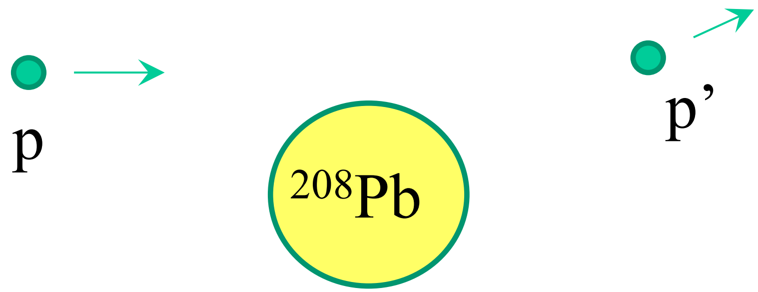
Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

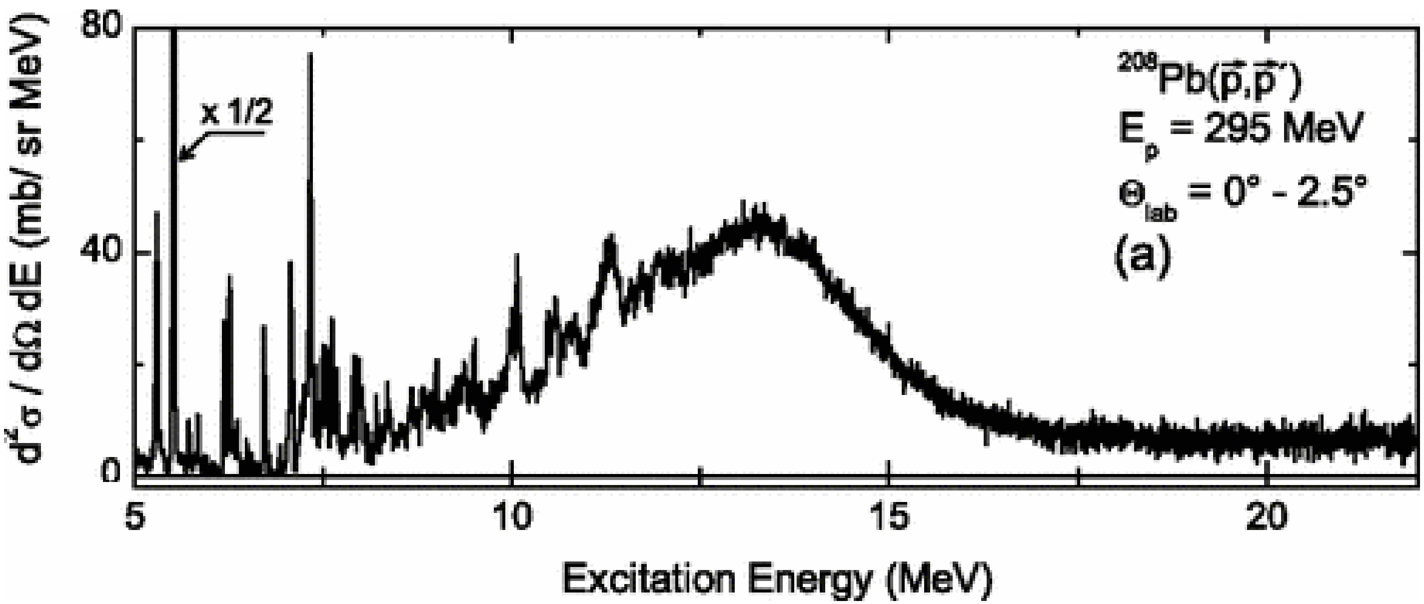


# Sum Rule

例えば:




非弾性散乱($^{208}$Pb の励起)のスペクトル

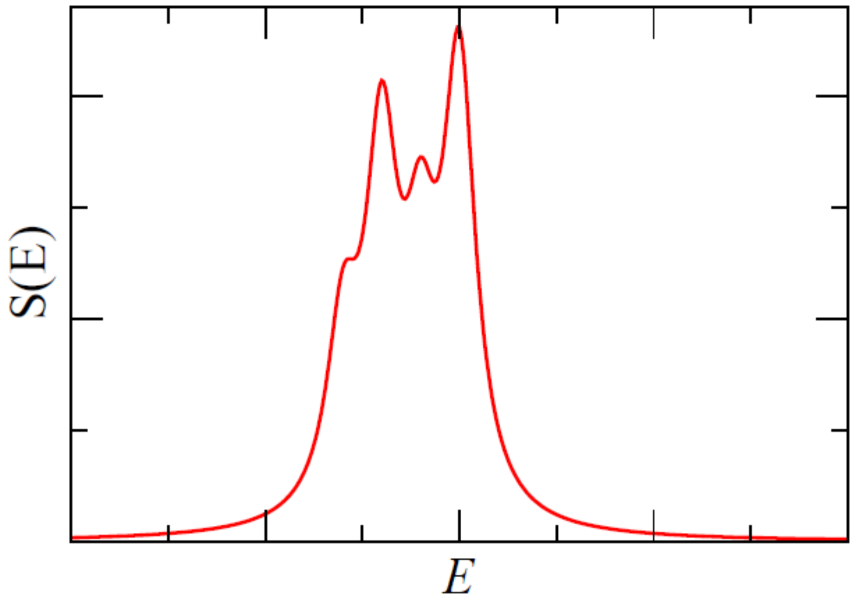



A. Tamii et al., PRL107, 062502 (2011)

# Sum Rule

Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$




強度関数のモーメント:

$$S_k \equiv \int E^k S(E) dE$$

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$

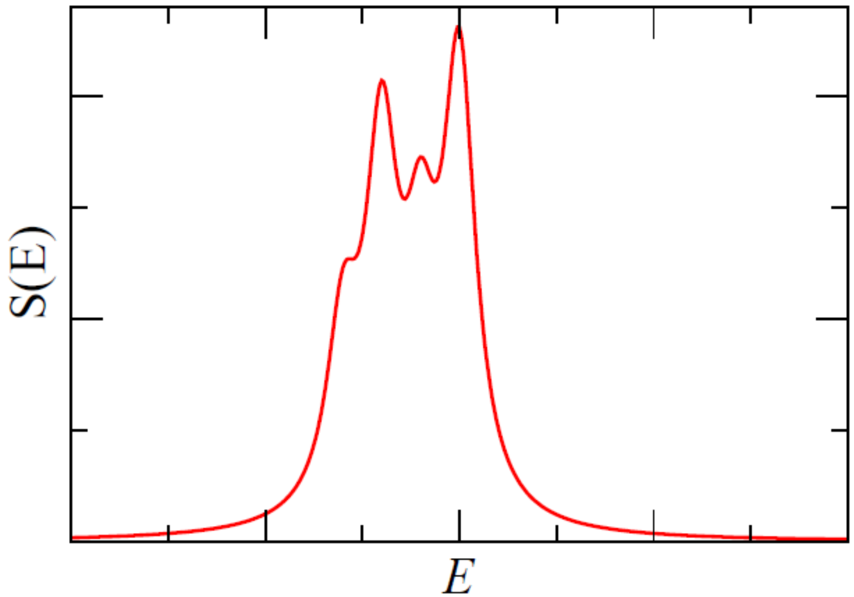
✓ energy weighted sum rule

$$S_1 \equiv \int ES(E)dE = \sum_n (E_n - E_0)|\langle n|F|0\rangle|^2$$

## ✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

$F^2$の基底状態期待値

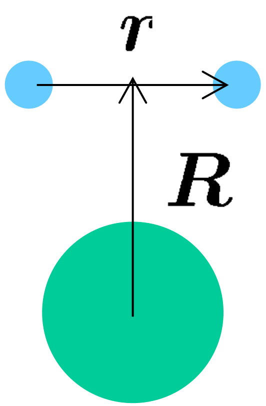



$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

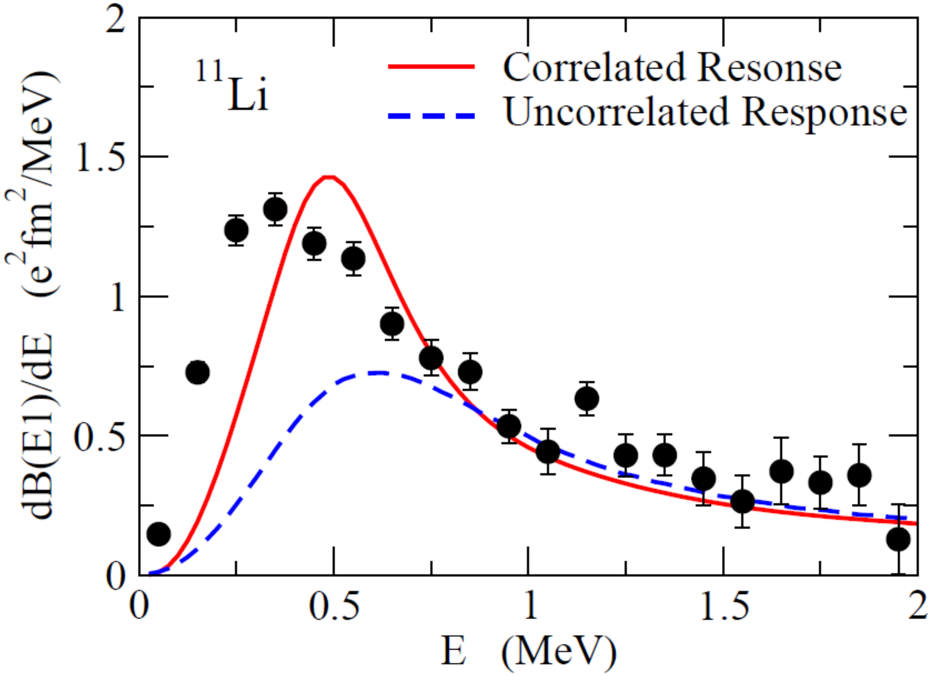
## ✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

$F^2$の基底状態期待値




cf. geometry of Borromean nuclei




experimental data:
T. Nakamura et al., PRL96('06)252502

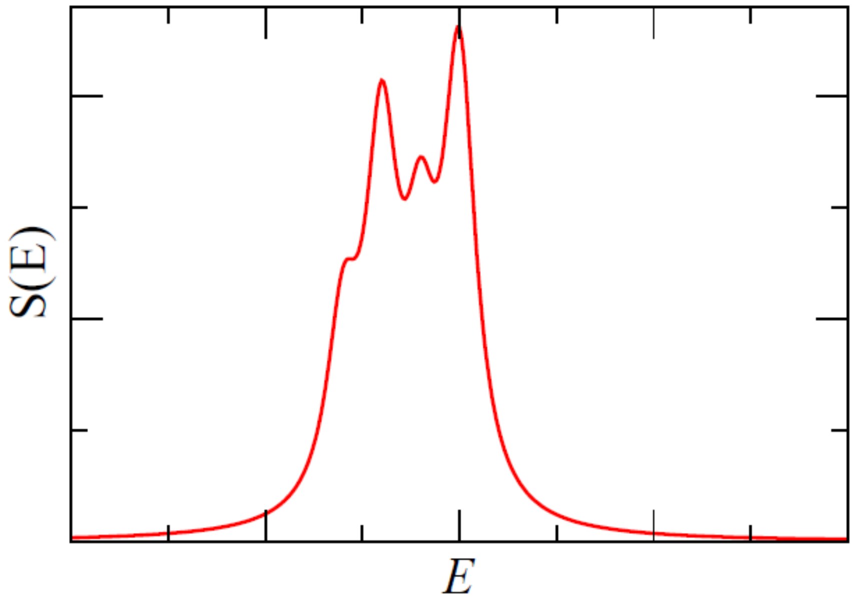
$$B(E1) = \sum_i B(E1; gs \to i)$$
$$= \frac{3}{\pi}\left(\frac{Ze}{A}\right)^2 \langle R^2\rangle$$

⇨ $\langle\theta_{nn}\rangle = 65.2^{+11.4}_{-13.0}$ ($^{11}$Li)
$= 74.5^{+11.2}_{-13.1}$ ($^{6}$He)

K.H. and H. Sagawa,
PRC76('07)047302

## ✓ energy weighted sum rule

$$S_1 \equiv \int E S(E) dE$$
$$= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2$$
$$= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle$$




$$S(E) = \sum_\nu |\langle \nu | F | 0 \rangle|^2 \times \delta(E_\nu - E_0 - E)$$

$$\frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle = \frac{1}{2} \langle F(HF - FH) - (HF - FH)F \rangle$$
$$= \langle FHF - E_0 F^2 \rangle$$
$$= \sum_\nu E_\nu |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle$$
$$= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2$$

Energy weighted sum rule:

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$

For $F = F(\boldsymbol{r})$ (local operator)

$$[H,F] = \left[-\frac{\hbar^2}{2m}\nabla^2, F\right] = -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F\cdot\nabla)$$

⇨ $$[F,[H,F]] = \frac{\hbar^2}{m}(\nabla F)^2$$

⇨ $$S_1 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

For $F=z$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|z|0\rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

Model independent

$$\sigma_{\rm abs}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\rightarrow \int \sigma_{\rm abs}(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2\hbar}{mc}\cdot\frac{NZ}{A}$$

# レポート問題3（〆切：1月27日（火）23:55）

１次の摂動論を用いると、エネルギー $E_i$ にある原子核の状態 $\phi_i$ がエネルギー $E_\gamma$ のフォトンを吸収してエネルギー $E_f$ にある状態 $\phi_f$ に遷移するときの断面積は以下で与えられる。

$$\sigma_{\mathrm{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

ここで、tilde $z$ は、重心から測った陽子の $z$ 座標の和で、

$$\tilde{z} = \sum_p (z_p - Z_{cm}) \ = \ \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n\right)\right\} = \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n\right)$$

で与えられる。$N$, $Z$ はそれぞれ原子核の中性子数、陽子数である。TRK和則を用いて、

$$\int \sigma_{\mathrm{abs}}(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2 \hbar}{mc}\cdot\frac{NZ}{A}$$

となることを示せ。

# Giant Dipole Resonance (GDR)

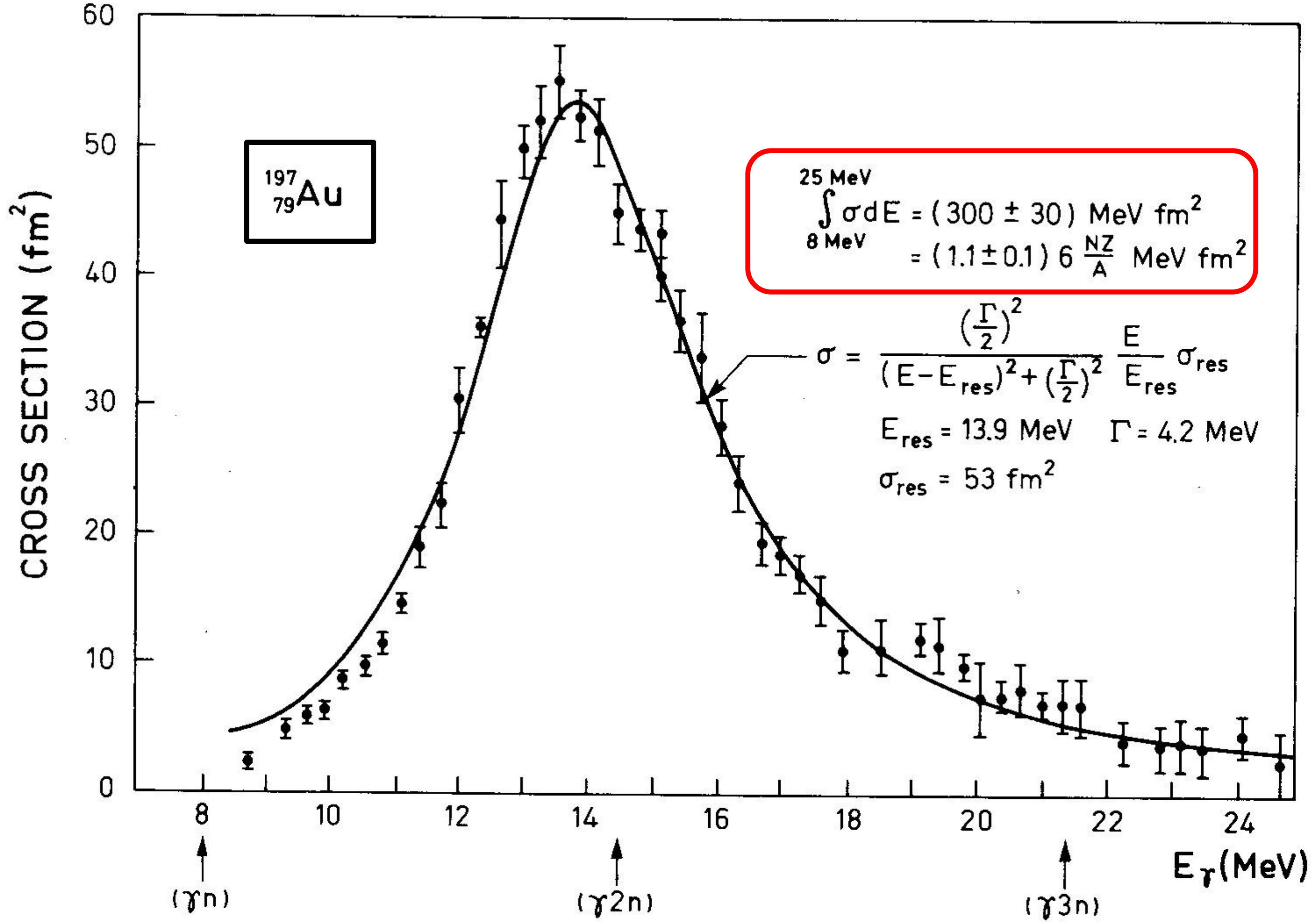



**Figure 6-18** Total photoabsorption cross section for $^{197}$Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

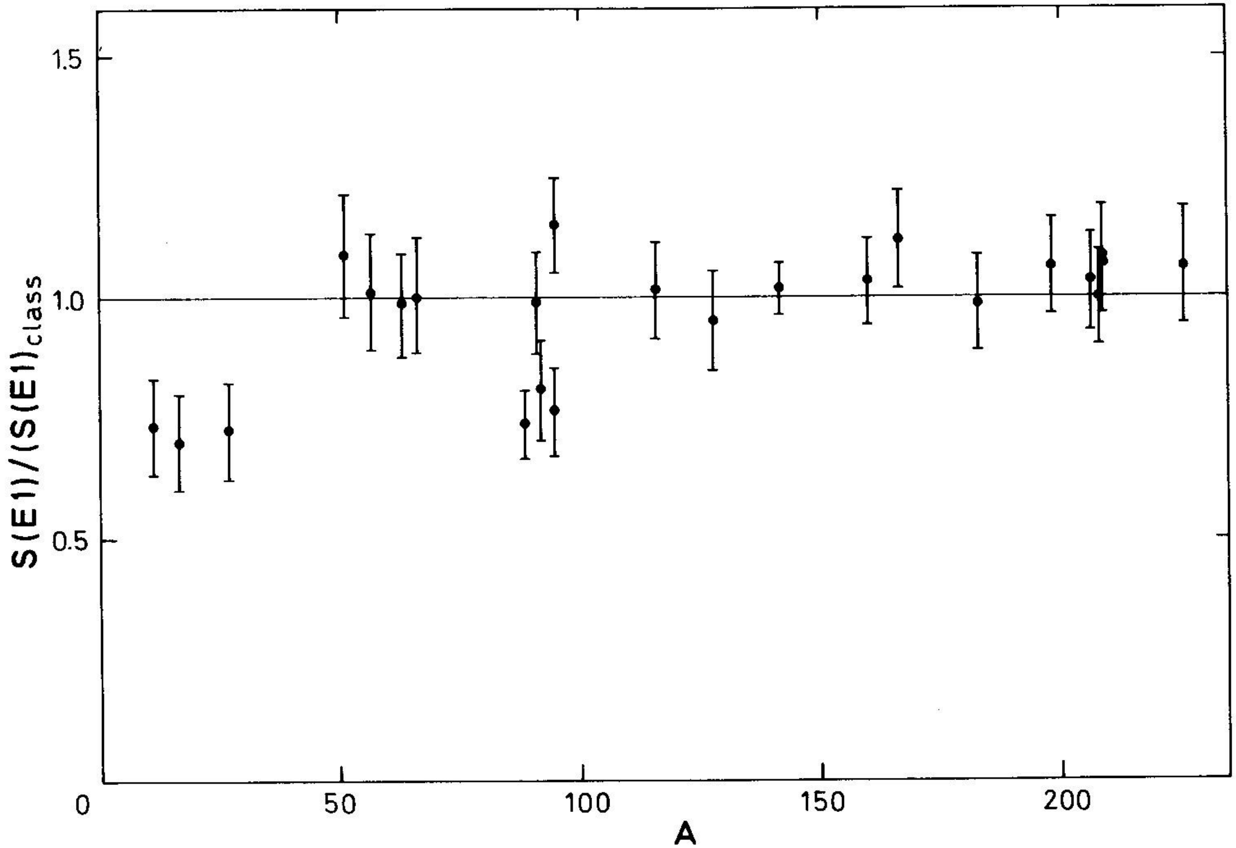



**Figure 6-20** Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic $\gamma$ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of ($\gamma$p) processes must be included and the data are from: $^{12}$C and $^{27}$Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); $^{16}$O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

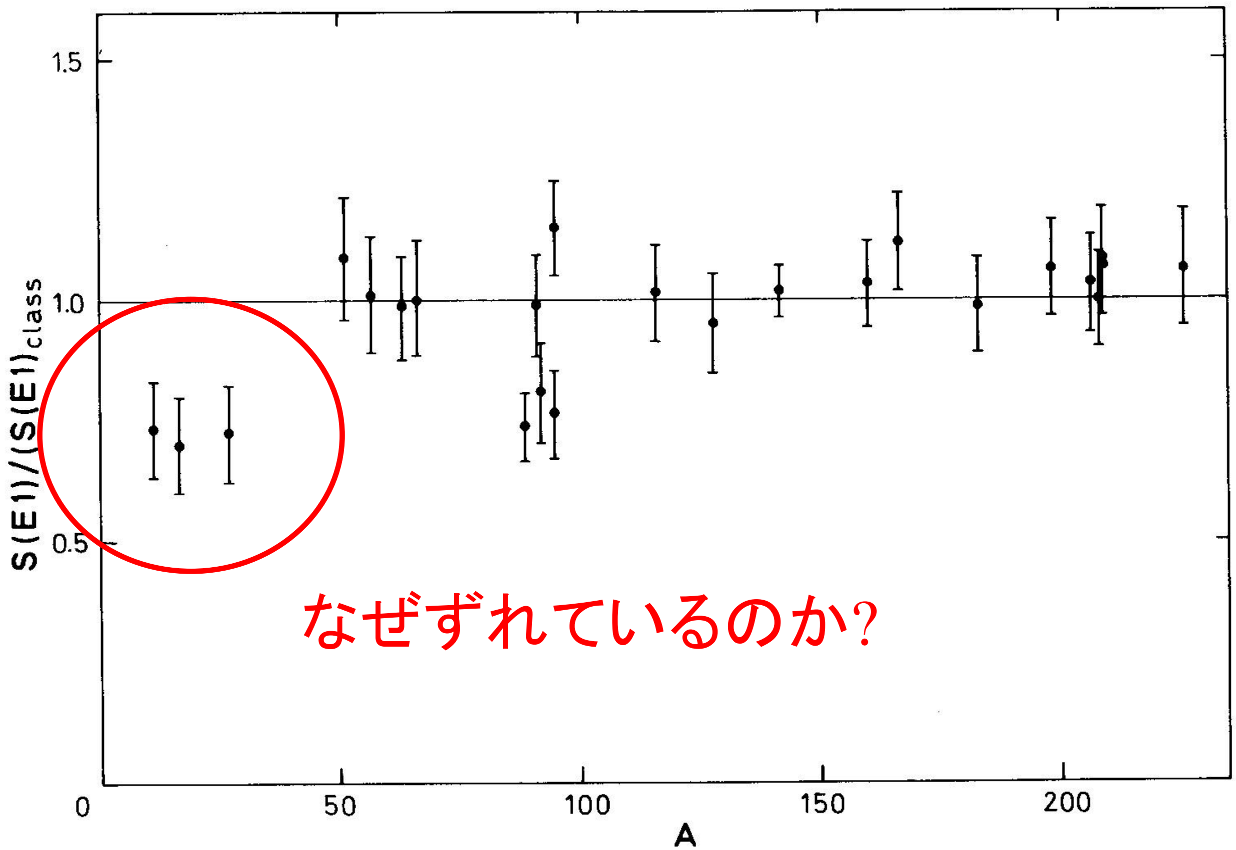
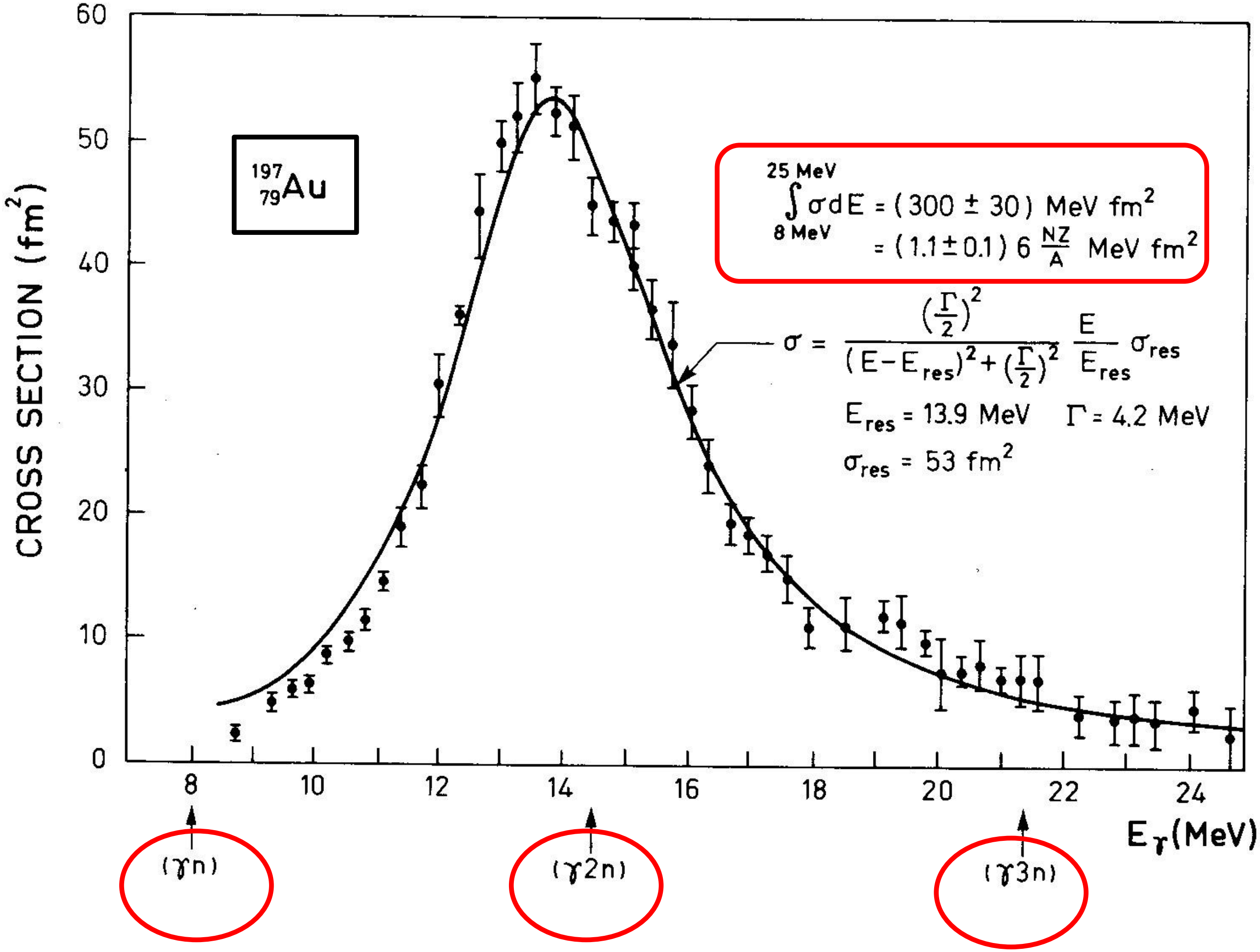


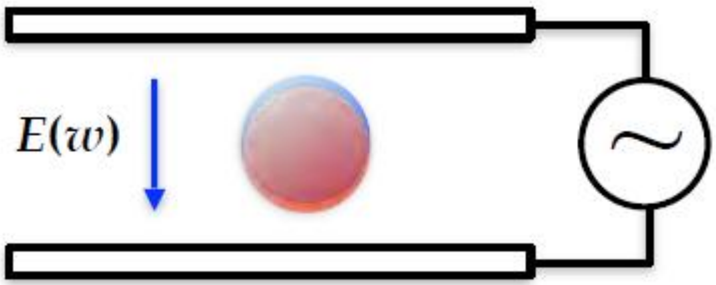

**Figure 6-20** Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic $\gamma$ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of $(\gamma p)$ processes must be included and the data are from: $^{12}C$ and $^{27}Al$ (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); $^{16}O$ (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

# Giant Dipole Resonance (GDR)




光吸収→中性子の放出を測定

# 分極率と inverse energy weighted sum rule



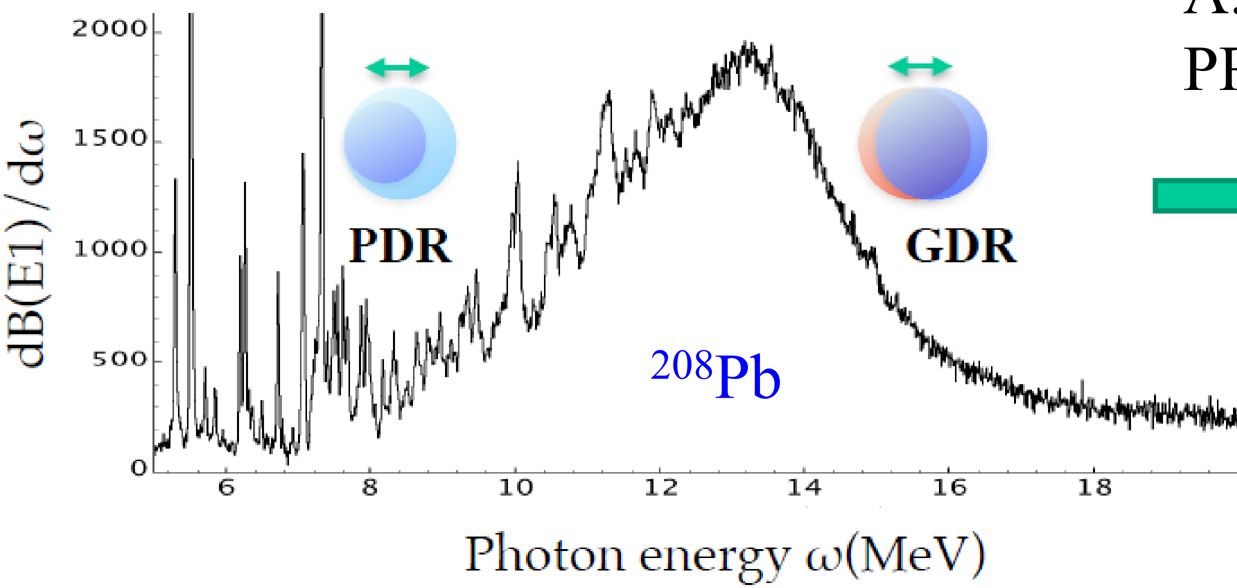

原子核の静電分極率
→対称エネルギー

図：民井さん

$$H = H_0 - \lambda F$$

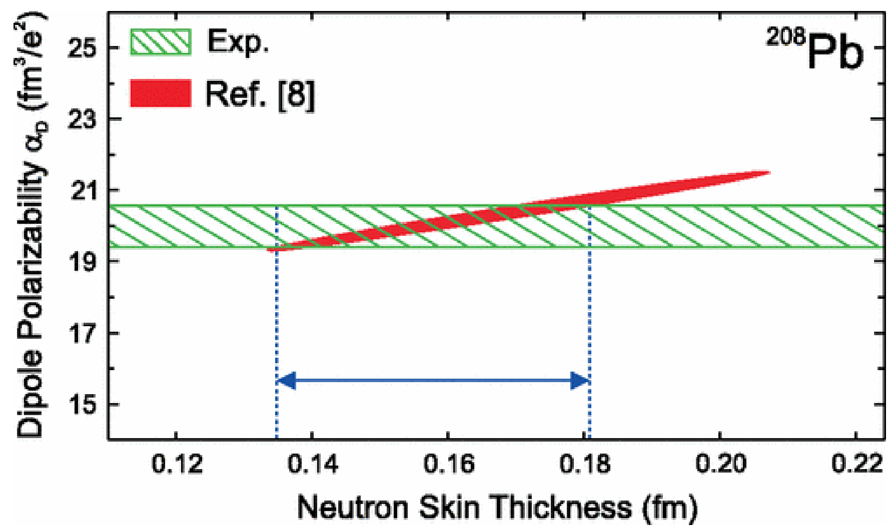
一次の摂動論： $|\tilde{\psi}_0\rangle = |\psi_0\rangle - \lambda \sum_{n>0} \frac{\langle\psi_n|F|\psi_0\rangle}{E_0 - E_n} |\psi_n\rangle$

$$\rightarrow \langle\tilde{\psi}_0|F|\tilde{\psi}_0\rangle = \langle\psi_0|F|\psi_0\rangle + \underbrace{2 \sum_{n>0} \frac{|\langle\psi_n|F|\psi_0\rangle|^2}{E_n - E_0}}_{\text{分極率}} \lambda$$



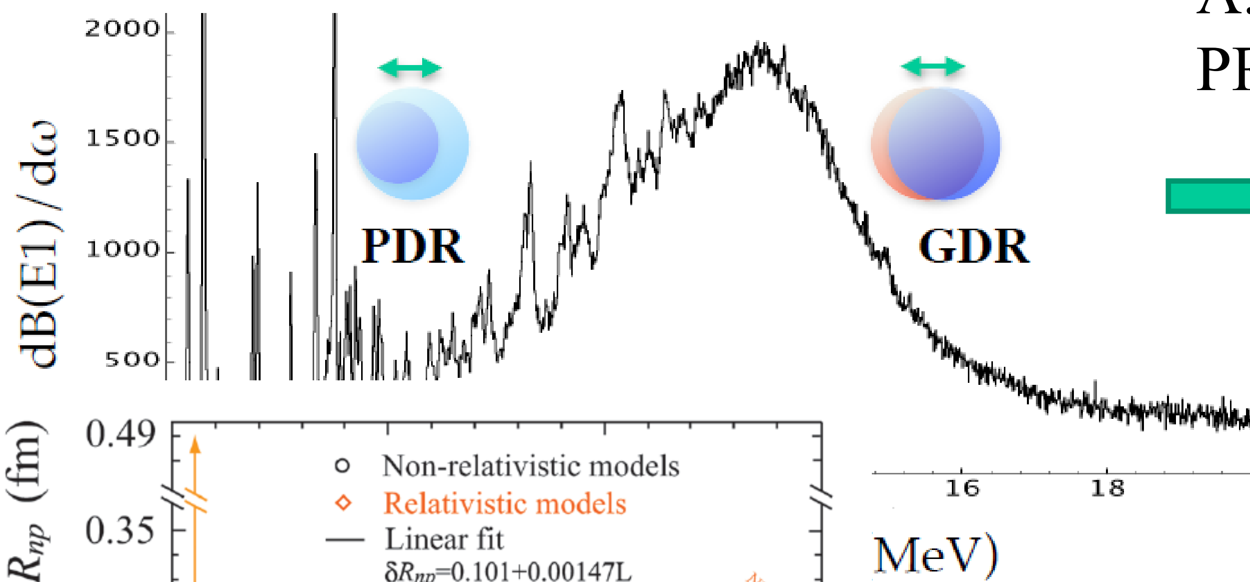

A. Tamii et al., PRL107, 062502 (2011)

$\alpha$=20.1+/− 0.6 fm$^3$

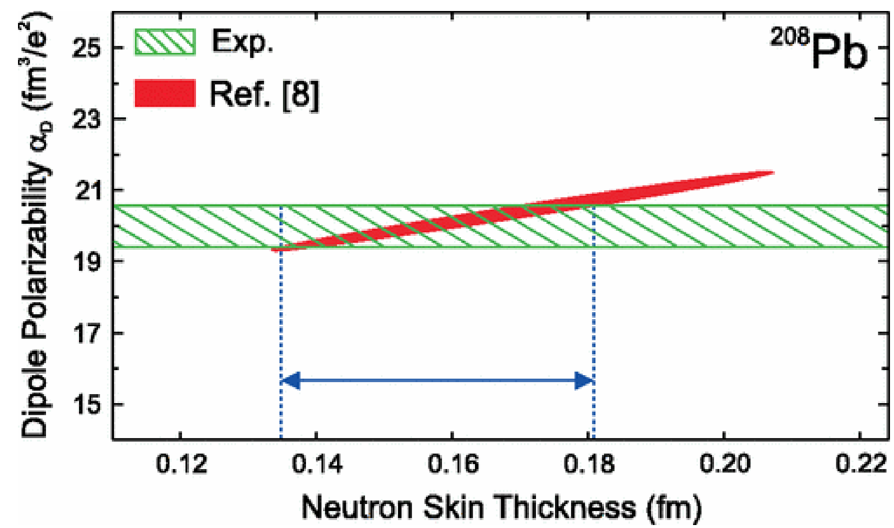


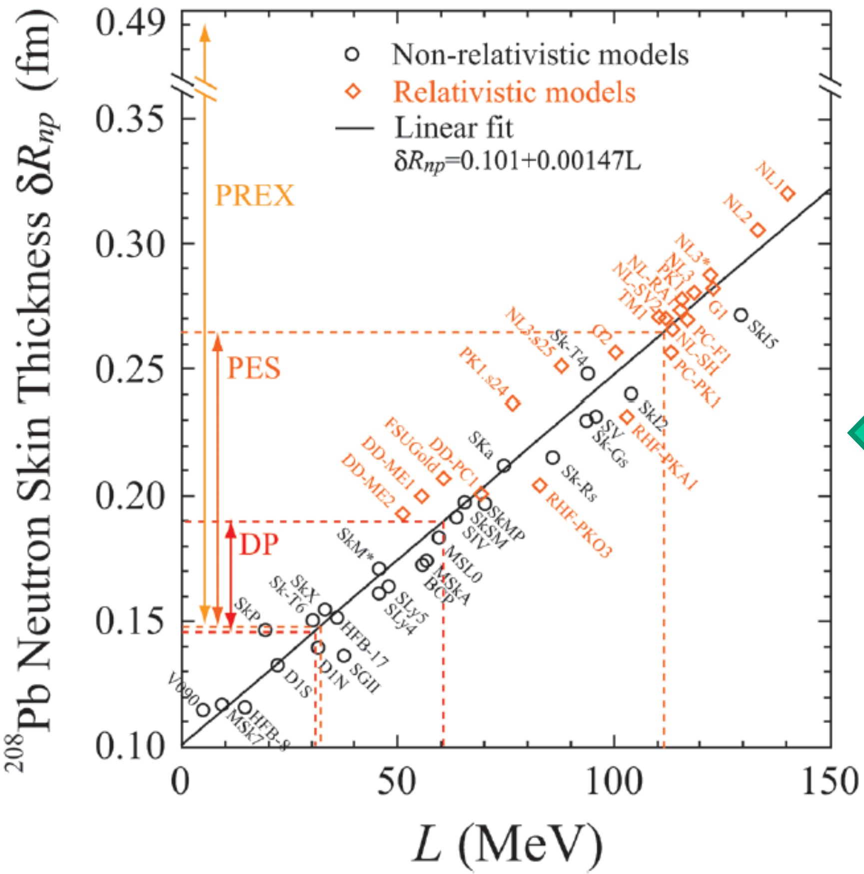

$r_{skin} = 0.156\ ^{+0.025}\ _{-0.021}$ fm

A. Tamii et al.,
PRL107, 062502 (2011)



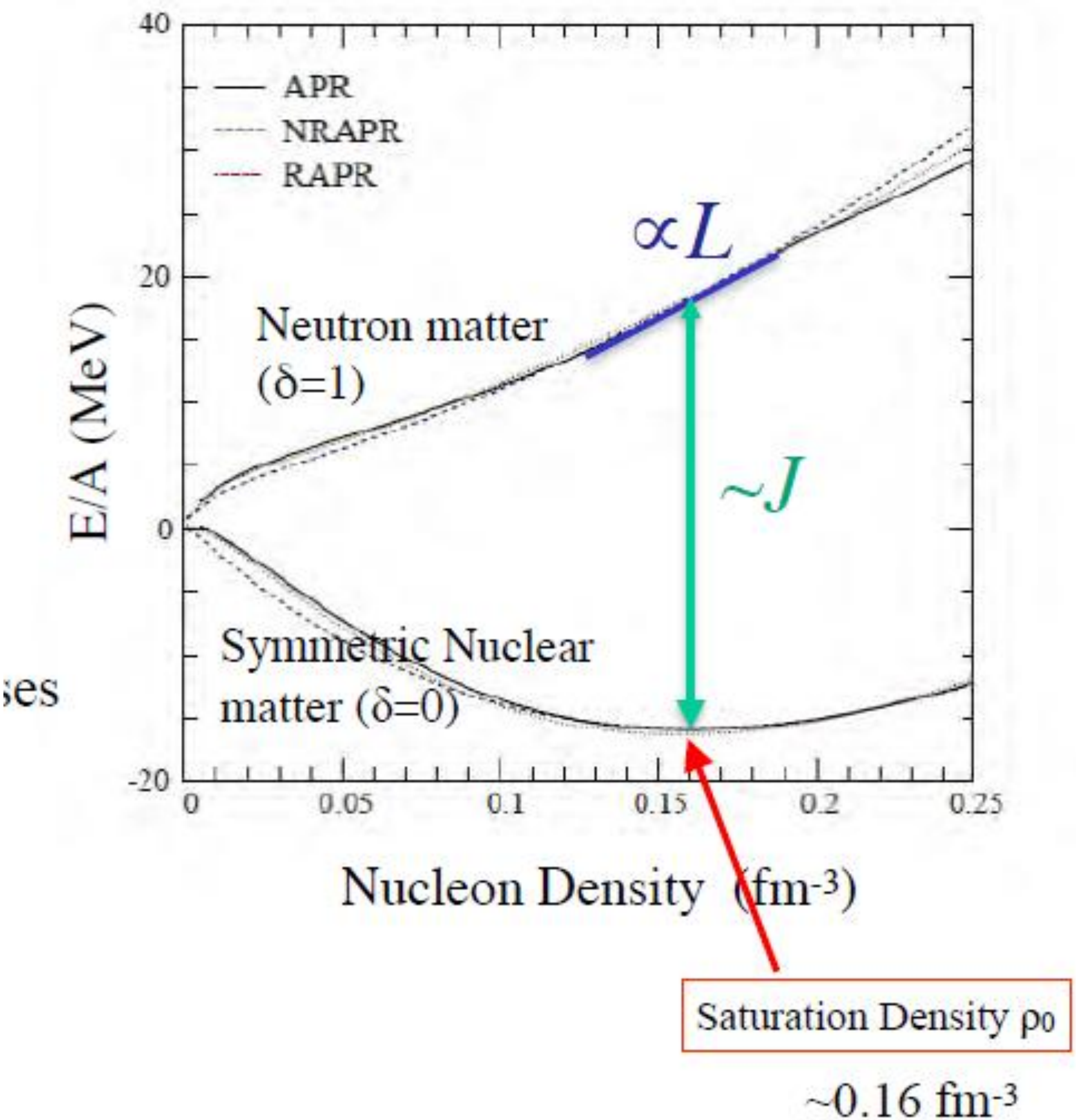

$\alpha$=20.1+/− 0.6 fm$^3$




$r_{skin} = 0.156\ ^{+0025}_{-0.021}$ fm

民井、銭廣（日本物理学会誌）

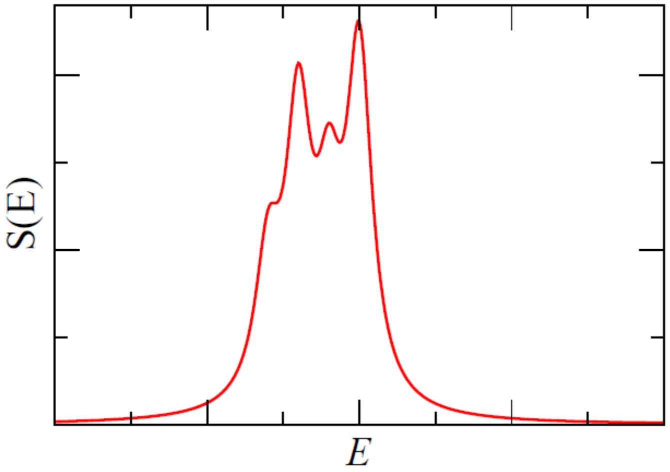


40
20
0
-20
0
0.05
0.1
0.15
0.2
0.25
APR
NRAPR
RAPR
E/A (MeV)
Neutron matter
(δ=1)
$\propto L$
$\sim J$
Symmetric Nuclear
matter (δ=0)
ses
Nucleon Density (fm$^{-3}$)
Saturation Density $\rho_0$
~0.16 fm$^{-3}$


スライド: 民井さん

# 和則の利点

$$S_0 = \langle 0|F^2|0\rangle$$
$$S_1 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$

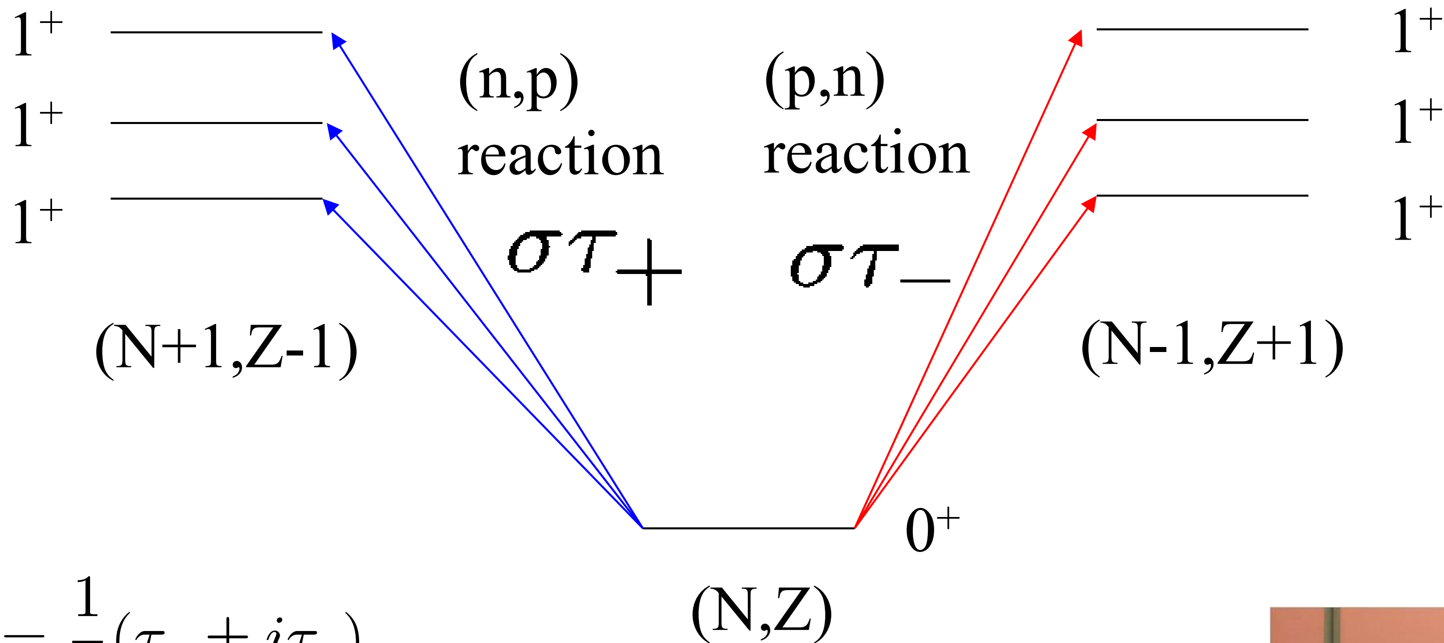



和則：
励起状態の（ある種の）情報が基底状態の性質のみによって書ける（励起状態の情報を知っている必要がない）。

- 実験で強度分布が測られた時、測られた範囲外にも強度があるかどうか (missing strength) 判断できる。
- 強度分布を測ることによって原子核の半径などの情報を得られる。
- 実験データや数値計算のチェックになる。（和則の値よりとても大きくなると何かがおかしい）。

# 池田和則 (Ikeda sum rule)

charge exchange reactions: Gamow-Teller transitions




$$\tau_\pm = \frac{1}{2}(\tau_x \pm i\tau_y)$$

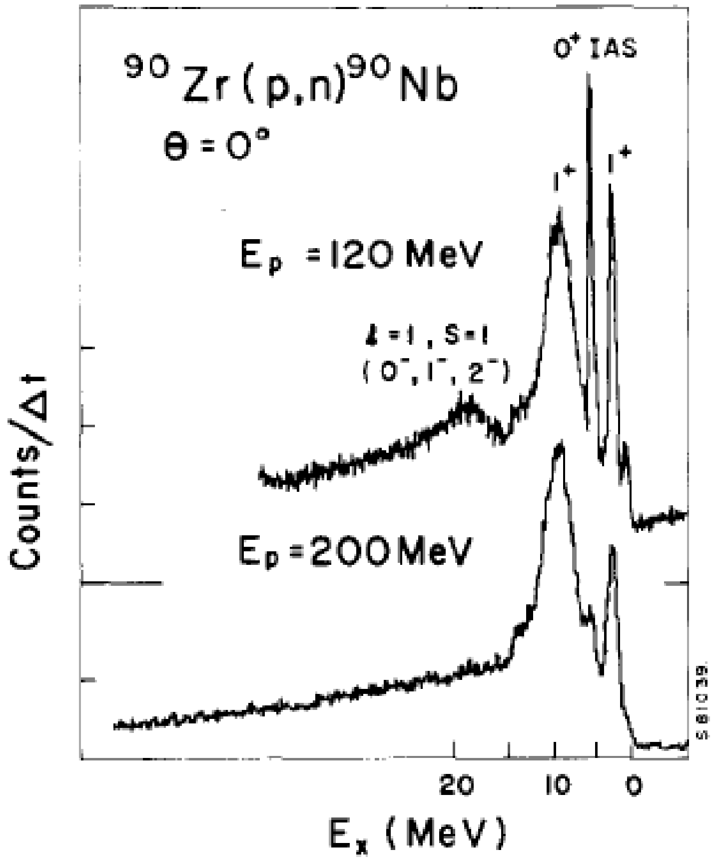
$$\tau_+|p\rangle = |n\rangle, \quad \tau_-|n\rangle = |p\rangle$$

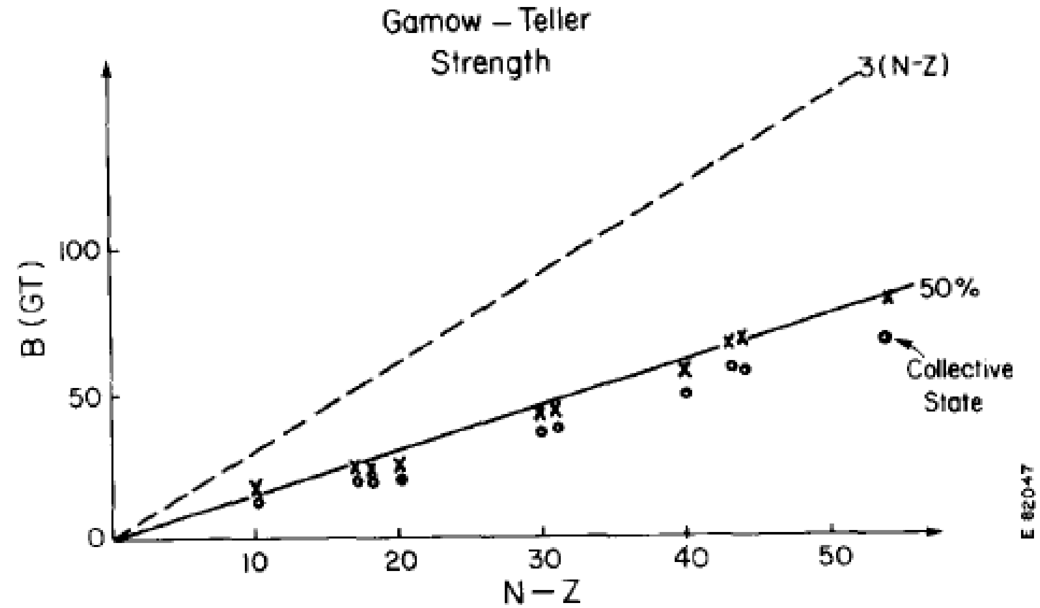
Ikeda sum rule

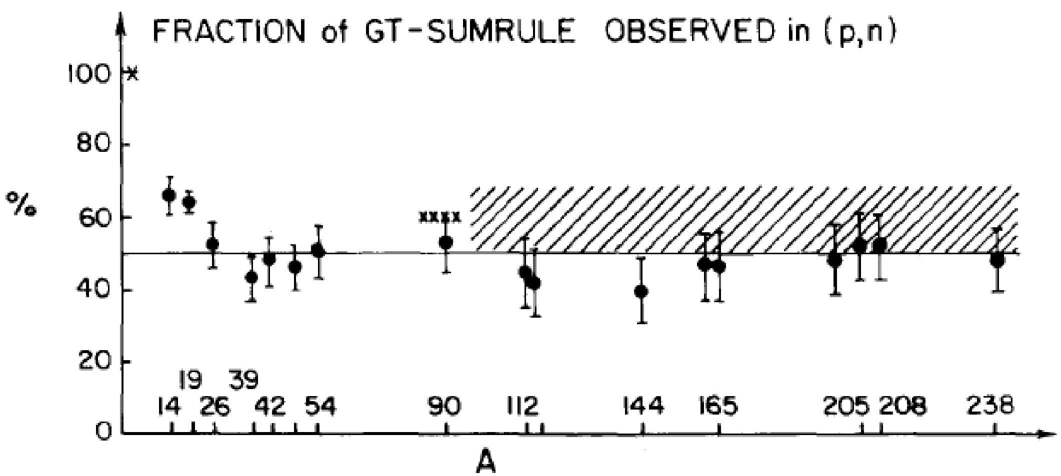
$$S_0(\sigma\tau_-) - S_0(\sigma\tau_+) = 3(N-Z)$$

池田清美氏

# the situation before 1997



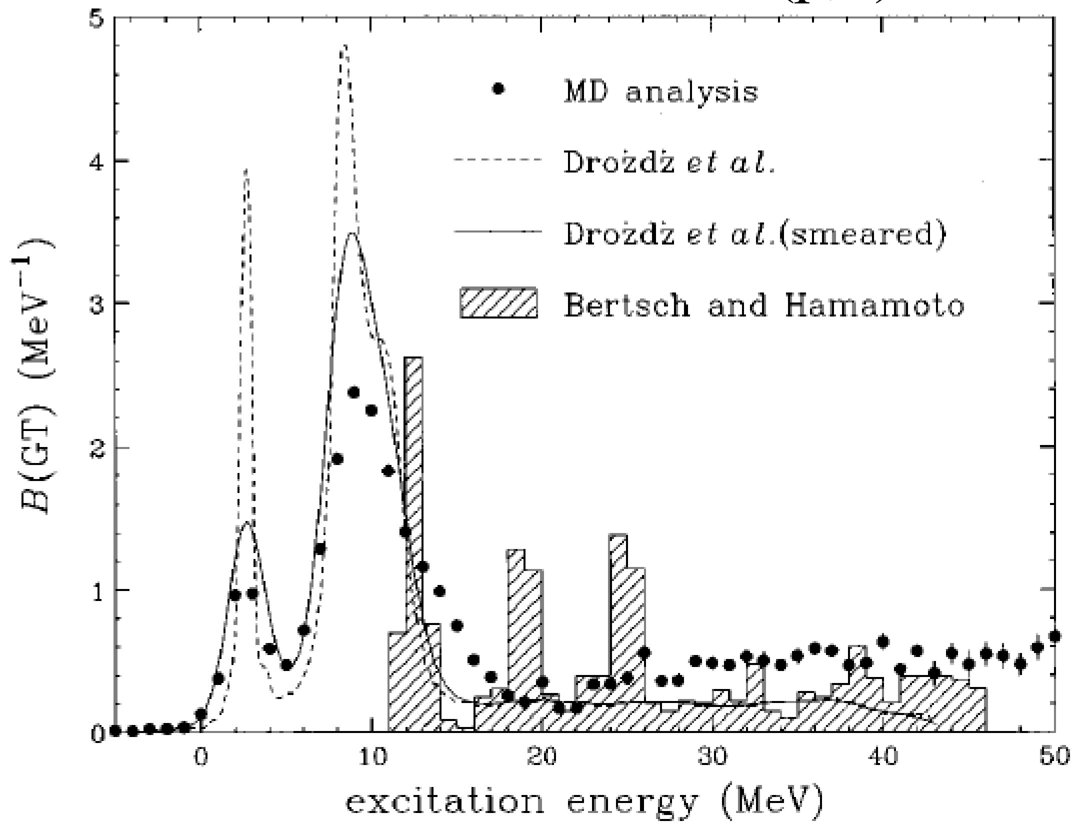




the "quenching problem"
of GT strength

quark (Δ resonance)?

C. Gaarde, Nucl. Phys. A396('83)127c

$^{90}Zr$ (p,n) $^{90}Nb$




T. Wakasa et al., PRC55 (‘97) 2909

$S_- - S_+ = 27.0 \pm 1.6 = (90 \pm 5)\%$ of Ikeda sum rule

→ quark contribution: small

## レポート問題4（〆切：1月27日（火）23:55）

1次元調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

の $n$ 番目の固有状態 $|n\rangle$ を考える($n$=0 が基底状態）。

1) 演算子 $x^2$ に対して、強度関数を求めよ。$n$ が 0 の場合、1 の場合、2以上の場合で場合分けせよ。

2) 演算子 $x$ で遷移できる状態 $|k\rangle$ を全て書き出し（状態 $k$ も調和振動子の固有状態）、遷移確率

$$P_{n\to k} = |\langle k|x|n\rangle|^2$$

を求めよ。

3)演算子 $x$ に対して energy weighted sum rule

$$S_1 = \sum_k (E_k - E_n) P_{n\to k}$$

を計算し、TRK和則が成り立っていることを示せ。

# 集団励起の微視的理論

## 原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）

集団励起を微視的に理解してみる
（集団励起をミクロに見てみるとどうなっているのか?）

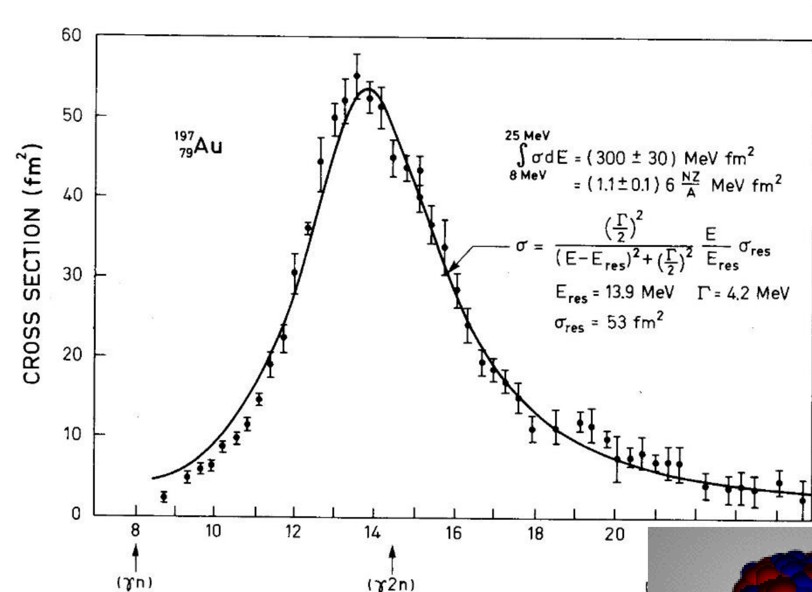
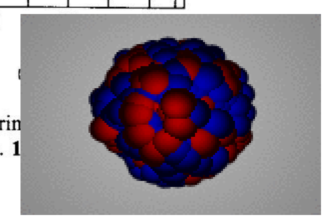
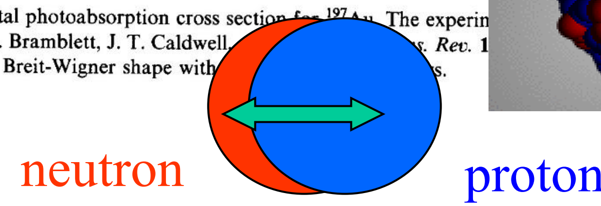



**Figure 6-18** Total photoabsorption cross section for $^{197}$Au. The experin… S. C. Fultz, R. L. Bramblett, J. T. Caldwell… *Rev.* 1… solid curve is of Breit-Wigner shape with…

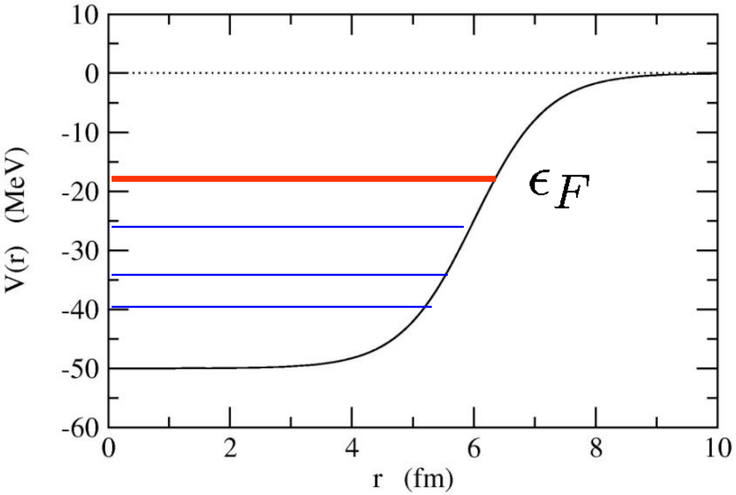





集団励起の例：巨大双極子共鳴

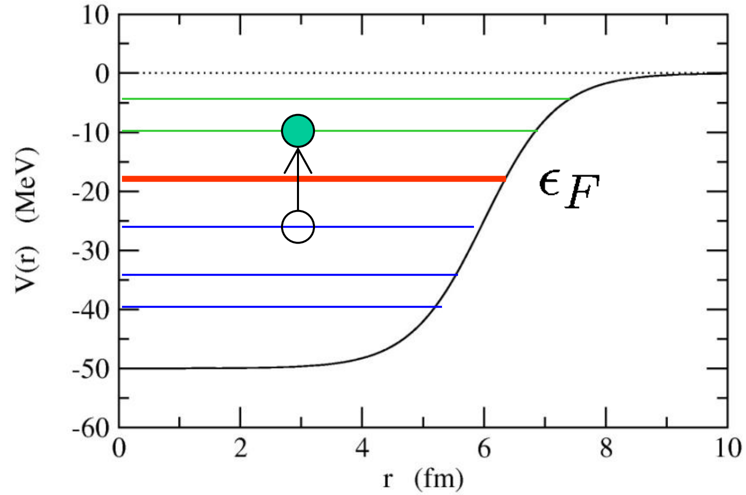
# Particle-Hole excitations

## Hartree-Fock state




$$|HF\rangle = \prod_h a_h^\dagger |0\rangle$$

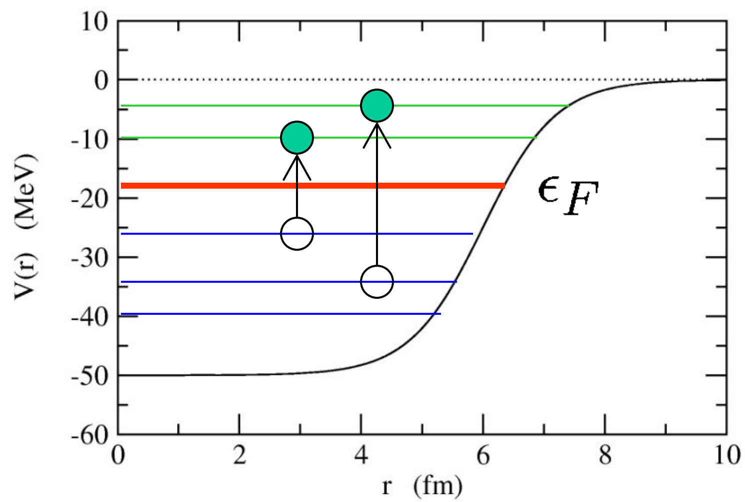
## 1 particle-1 hole (1p1h) state




$$a_p^\dagger a_h |HF\rangle$$

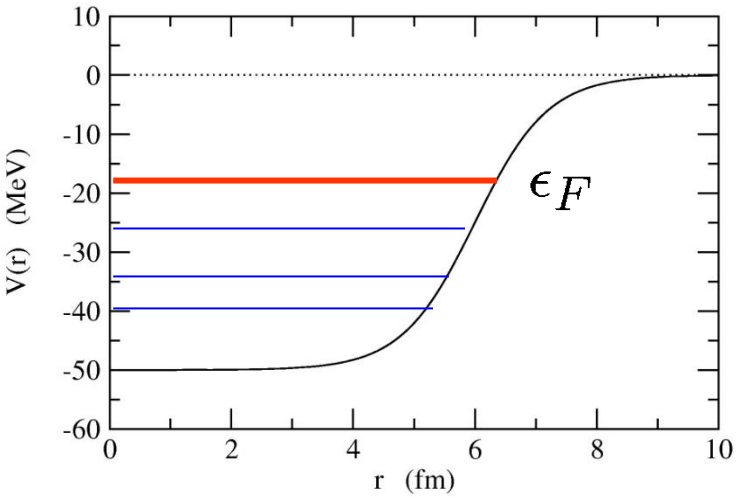
## 2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$



# Tamm-Dancoff Approximation

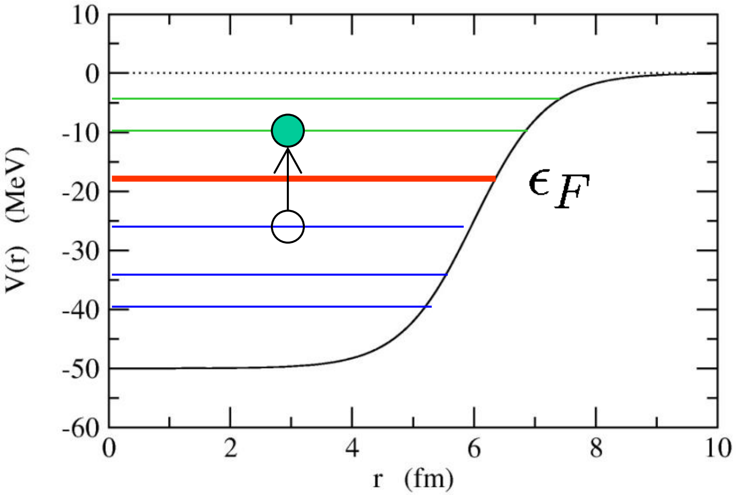
基底状態: $|HF\rangle = \prod_h a_h^\dagger |0\rangle$




励起状態:

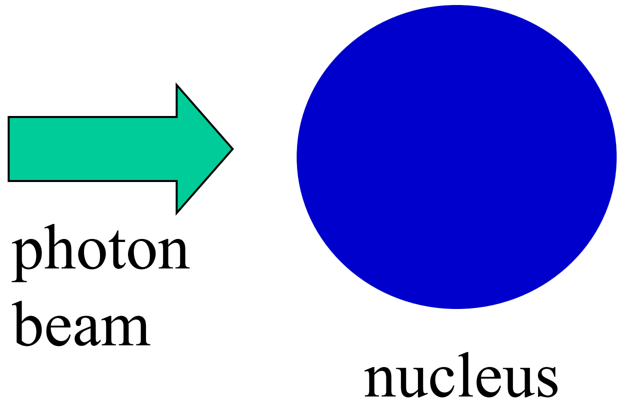
$$|\nu\rangle = Q_\nu^\dagger |HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle$$

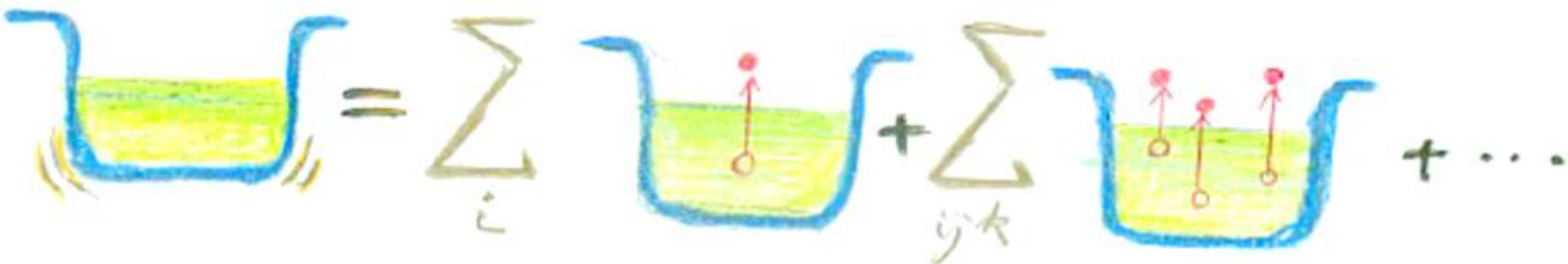
$$\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$




(1p1h 状態の重ね合わせ
:2p2h以上は寄与しないと仮定)

# 原子核を外場により揺らしてみると何が起こるのか?



スライド：松柳研一氏

# Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$\equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

$$\sum_{p'h'} H_{ph,p'h'}X_{p'h'} = E_\nu X_{ph}$$

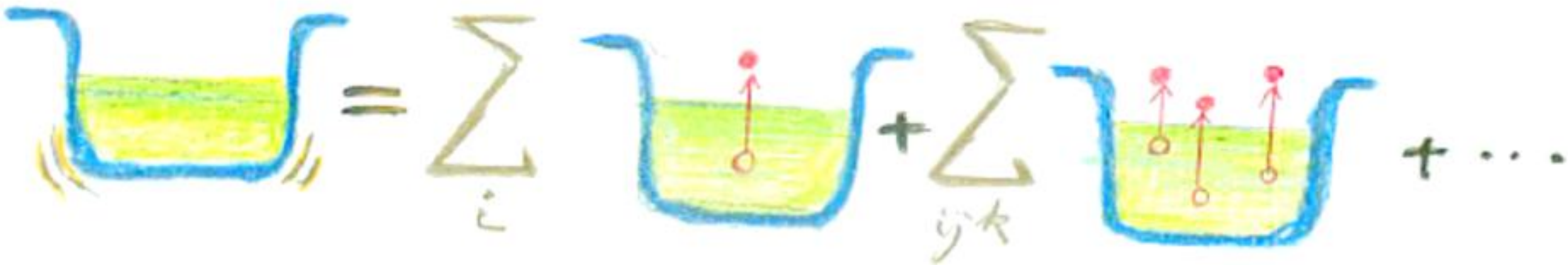
$$H_{ph,p'h'} = \langle ph^{-1}|H|p'h'^{-1}\rangle$$

$$= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

residual interaction

Tamm-Dancoff equation； 1p1h の空間でハミルトニアンを対角化

# 残留相互作用の意味




スライド：松柳研一氏

$$V(\boldsymbol{r}) \sim \int d\boldsymbol{r}'\, v(\boldsymbol{r}, \boldsymbol{r}')\rho(\boldsymbol{r}')$$

vibration: $\rho = \rho_0(\boldsymbol{r}) \to \rho_0(\boldsymbol{r}) + \delta\rho(\boldsymbol{r}, t)$

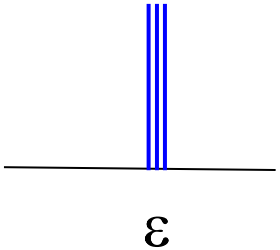
residual interaction

# TDA on a schematic model

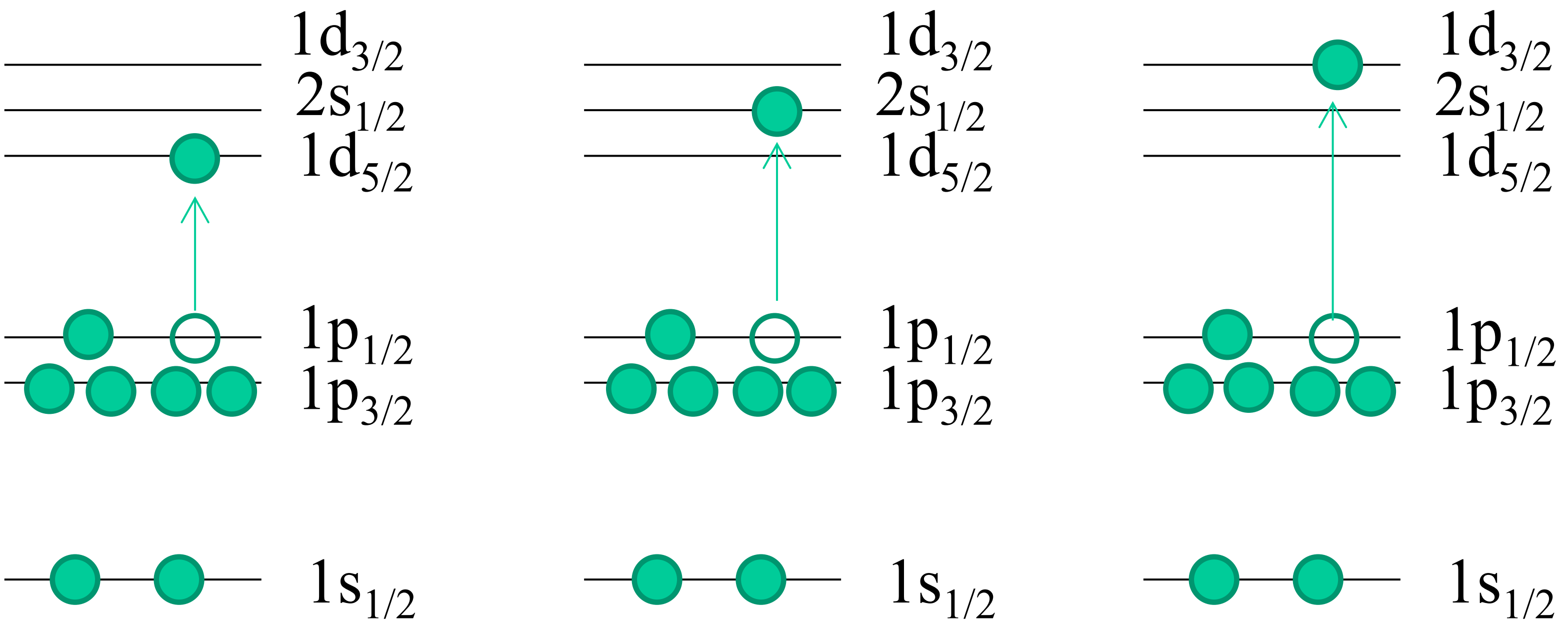
$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:




（例えば）



# TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \qquad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization:

$$\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$$

# TDA on a schematic model

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\epsilon+3g) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

全ての状態が同位相で寄与
＝コヒーレントな重ね合わせ

他の固有状態:

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

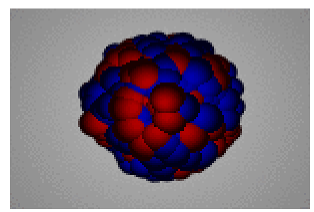
$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

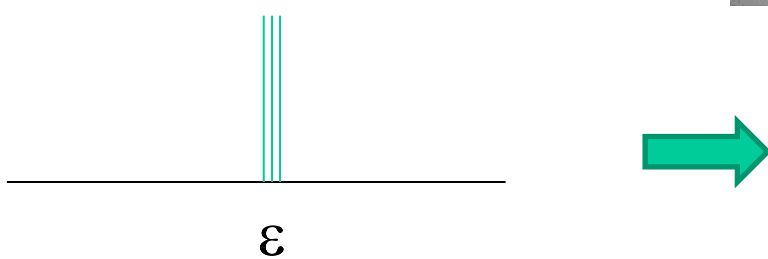
位相がそろっていない

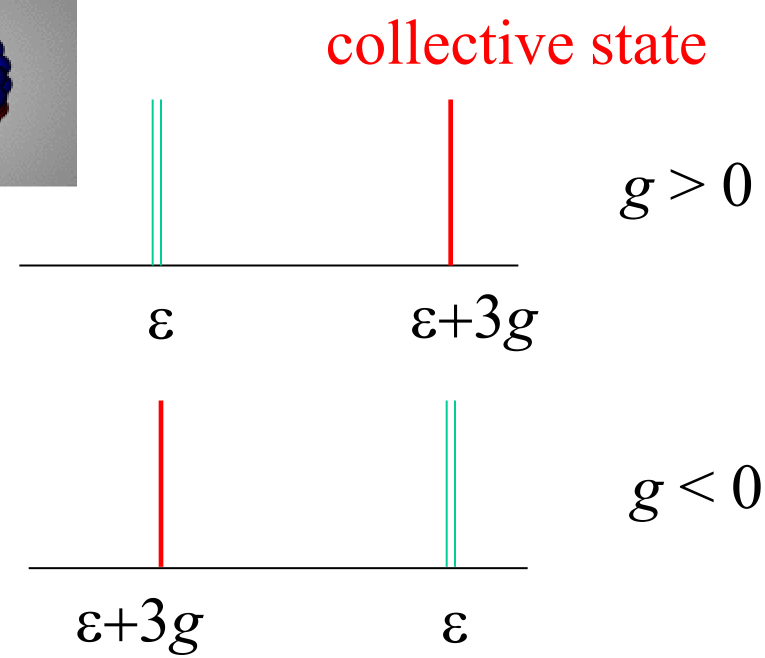
# TDA on a schematic model

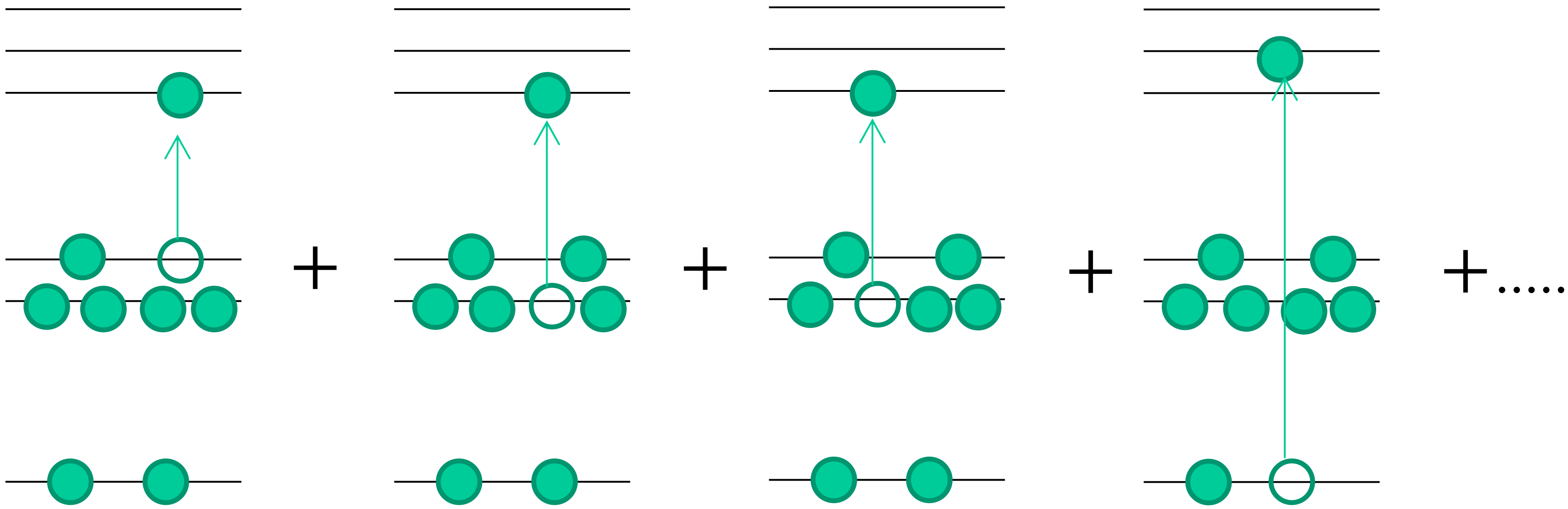
$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization: $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$







複数の粒子・空孔状態をコヒーレントに重ね合わせることによって
多数の核子が励起に関与していることを表現する

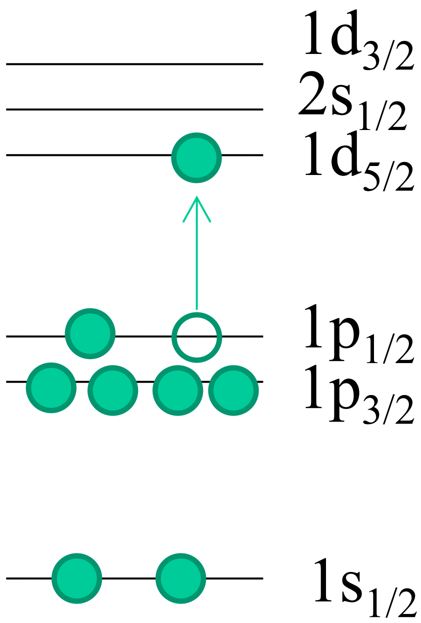
$$|\nu\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle \equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

$$\Rightarrow \quad \left| \left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle \right|^2 = \left( \sum_{ph} f_{ph} X_{ph} \right)^2$$

干渉項がすべて
同符号で寄与

# 原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）




一粒子励起の例

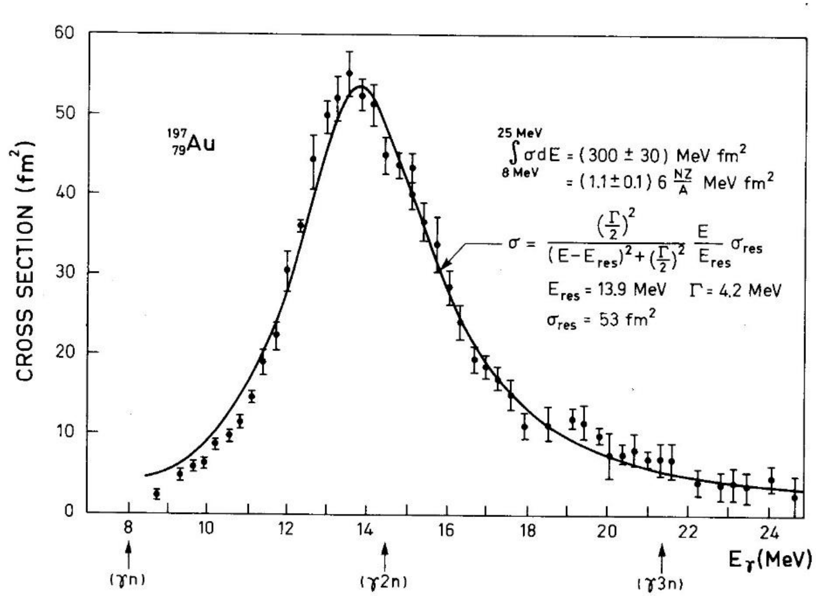
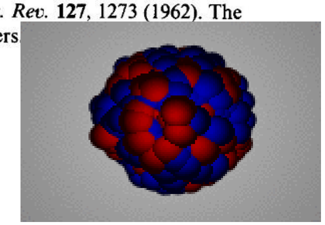
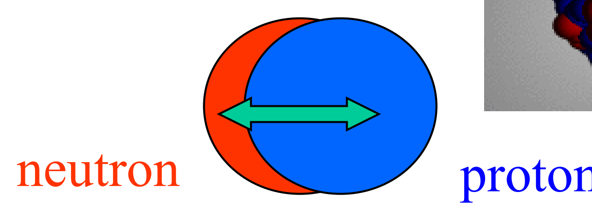



**Figure 6-18** Total photoabsorption cross section for $^{197}$Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.






集団励起の例：巨大双極子共鳴

# Giant Dipole Resonance (GDR) 巨大双極子共鳴

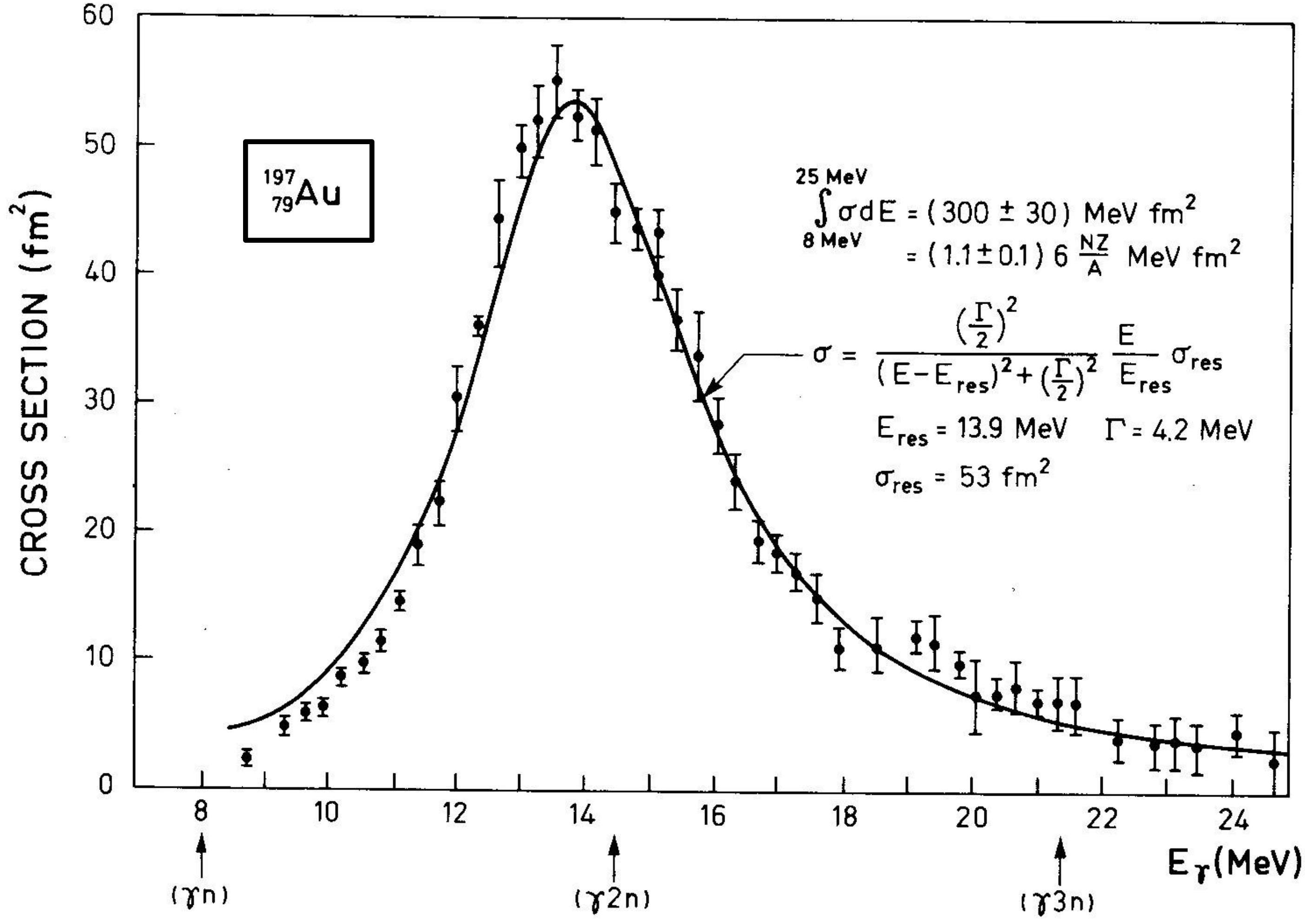



**Figure 6-18** Total photoabsorption cross section for $^{197}$Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

cf. $41 \times 197^{-1/3} = 7.05$ MeV → 14 MeV

Iso-scalar type modes: $E < \epsilon_{ph} \to \lambda < 0$ (attractive)

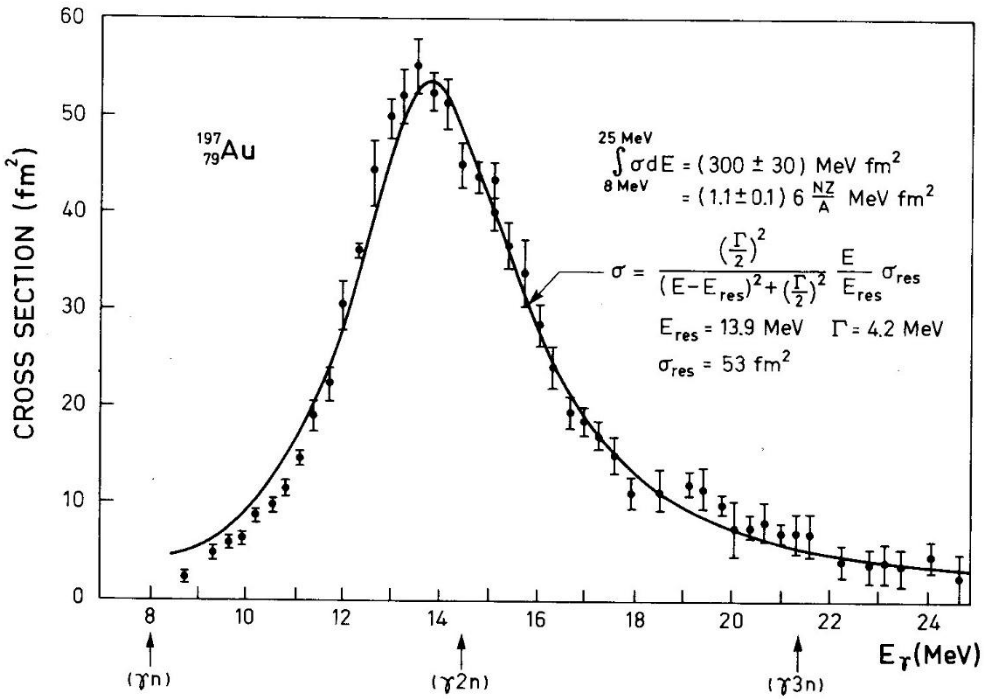
Iso-vector type modes: $E > \epsilon_{ph} \to \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 41A^{-1/3}$

IS GQR: $E \sim 65A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 82A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41A^{-1/3}$ (MeV)




$^{197}$Au

$$E_{\mathrm{GDR}} = 14 \text{ (MeV)}$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3} \sim 7 \text{ (MeV)}$$

# どれだけの核子が励起に関与しているのか?

Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \to I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 \qquad (e^2\mathrm{fm}^{2\lambda})$$

exp data: