§. リップマン－シュウィンガー方程式

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V\right)\psi = E\psi$$

$$\rightarrow \quad \underbrace{\left(-\frac{\hbar^2}{2\mu}\nabla^2\right.}_{\hat{H}_0} \left. - E\right)\psi = -V\psi$$

形式解

$$\boxed{\psi = \phi - \frac{1}{\hat{H}_0 - E - i\eta} V\psi}$$

リップマン－シュウィンガー方程式

$$(\hat{H}_0 - E)\phi = 0$$

正の微小量
（波動関数の境界条件を与える。）

・グリーン関数

$$\hat{G}_0^{(+)} = \frac{1}{\hat{H}_0 - E - i\eta}$$

座標表示では

$$G_0^{(+)}(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r} | \frac{1}{\hat{H}_0 - \underbrace{E}_{\frac{k^2\hbar^2}{2\mu}} - i\eta} | \mathbf{r}' \rangle$$

$$= \int d\mathbf{k}' \langle \mathbf{r} | \mathbf{k}' \rangle \cdot \frac{1}{\frac{k'^2\hbar^2}{2\mu} - \frac{k^2\hbar^2}{2\mu} - i\eta} \langle \mathbf{k}' | \mathbf{r}' \rangle$$

$$\int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} = \delta(\mathbf{r}-\mathbf{r}')$$

$$= \int \frac{d\mathbf{k}'}{(2\pi)^3} e^{i\mathbf{k}'\cdot\mathbf{r}} \cdot \frac{2\mu}{\hbar^2} \frac{1}{k'^2 - k^2 - i\eta'} e^{-i\mathbf{k}'\cdot\mathbf{r}'}$$

$$\left(\eta' = \frac{2\mu}{\hbar^2}\eta\right)$$

$$= \frac{2\mu}{\hbar^2} \cdot \frac{1}{(2\pi)^3} \int k'^2 dk' d\hat{\mathbf{k}}' \, e^{ik's\cos\theta} \frac{1}{k'^2 - k^2 - i\eta'}$$

$$(\mathbf{s} \equiv \mathbf{r} - \mathbf{r}')$$

$$= \frac{1}{(2\pi)^3} \cdot \frac{2\mu}{\hbar^2} \int_0^\infty k'^2 dk' \cdot \underbrace{2\pi \int_{-1}^{1} d(\cos\theta) \frac{e^{ik's\cos\theta}}{k'^2 - k^2 - i\eta'}}_{= \frac{2\pi}{k'^2 - k^2 - i\eta'} \frac{1}{ik's} \left(e^{ik's} - e^{-ik's}\right)}$$

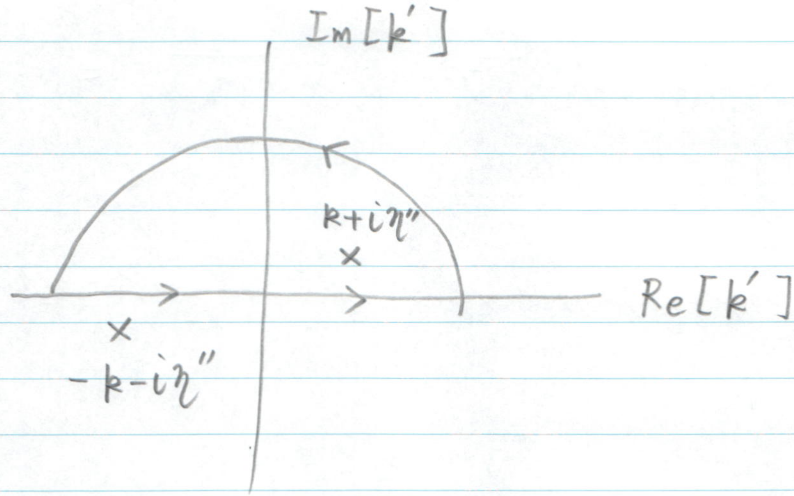
$$= \frac{1}{i} \cdot \frac{1}{(2\pi)^2} \cdot \frac{2\mu}{\hbar^2} \underbrace{\int_0^\infty dk' \frac{k' dk'}{k'^2 - k^2 - i\eta'} \left(\frac{e^{ik's}}{s} - \frac{e^{-ik's}}{s}\right)}$$

$$= \int_{-\infty}^{\infty} dk' \frac{k' dk'}{k'^2 - k^2 - i\eta'} \cdot \frac{e^{ik's}}{s}$$

$$= \int_{-\infty}^{\infty} dk' \frac{k' dk'}{(k' + k + i\eta'')(k' - k - i\eta'')} \cdot \frac{e^{ik's}}{s}$$

$$\left(\eta'' = \frac{\eta'}{2k}\right)$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} dk' \left(\frac{1}{k' + k + i\eta''} + \frac{1}{k' - k - i\eta''}\right) \times \frac{e^{ik's}}{s}$$



⇓

$$G^{(+)}(\mathbf{r},\mathbf{r}') = \frac{1}{\cancel{i}}\frac{1}{(2\pi)^2}\cdot\frac{2\mu}{\hbar^2}\cdot\frac{1}{2}\cdot 2\pi \cancel{i}\,\frac{e^{iks}}{s}$$

$$= \frac{\mu}{2\pi\hbar^2}\cdot\frac{e^{iks}}{s}$$

$$= \frac{2\mu}{\hbar^2}\cdot\frac{1}{4\pi}\,\frac{e^{+ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$ (外向き波)

$$\psi(\mathbf{r}) = \phi(\mathbf{r}) - \int d\mathbf{r}'\, G^{(+)}(\mathbf{r}, \mathbf{r}')\, V(\mathbf{r}')\, \psi(\mathbf{r}')$$

$$= e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{2\mu}{\hbar^2} \cdot \frac{1}{4\pi} \int d\mathbf{r}'\, \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\, V(\mathbf{r}')\, \psi(\mathbf{r}')$$

(note) $r \to \infty$ では

$$k|\mathbf{r}-\mathbf{r}'| = k\sqrt{r^2 - 2\mathbf{r}\cdot\mathbf{r}' + r'^2}$$

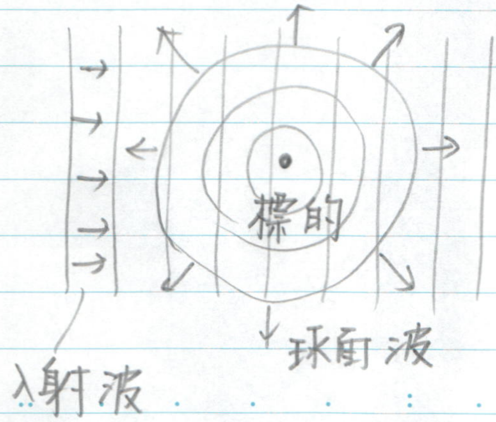
$$\sim kr - \underbrace{\left(k\frac{\mathbf{r}}{r}\right)}_{=\mathbf{k}'} \cdot \mathbf{r}'$$

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} \underbrace{- \frac{\mu}{2\pi\hbar^2} \int d\mathbf{r}'\, e^{-i\mathbf{k}'\cdot\mathbf{r}'}\, V(\mathbf{r}')\, \psi(\mathbf{r}')}_{= f(\theta)} \cdot \frac{e^{ikr}}{r}$$

$|\mathbf{r}-\mathbf{r}'| \sim r$

$f(\theta)$ 散乱振幅

$$= \underbrace{e^{i\mathbf{k}\cdot\mathbf{r}}}_{\text{入射波}} + \underbrace{f(\theta)\cdot\frac{e^{ikr}}{r}}_{\text{散乱波(球面波)}}$$



§. 散乱振幅と散乱断面積

散乱波 $\psi_{sc}(r) = f(\theta)\cdot\dfrac{e^{ikr}}{r}$ に対するフラックス

$$j_{sc} = \frac{\hbar}{2i\mu}\left(\psi_{sc}^{*}\nabla\psi_{sc} - \psi_{sc}\nabla\psi_{sc}^{*}\right)$$

を計算する。

(note) $\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$ $\quad r = \sqrt{x^2+y^2+z^2}$

$\downarrow$

$$\frac{\partial r}{\partial x} = \frac{x}{r}$$

$$z = r\cos\theta \quad\rightarrow\quad \overset{\frac{\partial z}{\partial x}}{0} = \frac{\partial r}{\partial x}\cos\theta - r\sin\theta\cdot\frac{\partial\theta}{\partial x}$$

$$\updownarrow\quad \frac{\partial\theta}{\partial x} = \frac{x\cos\theta}{r^2\sin\theta} = \frac{1}{r}\cos\theta\cos\varphi$$

$$y = r\sin\theta\sin\varphi \quad\rightarrow\quad 0 = \frac{x}{r}\sin\theta\sin\varphi + r\cos\theta\cdot\frac{1}{r}\cos\theta\cos\varphi\times\sin\varphi + r\sin\theta\cos\varphi\cdot\frac{\partial\varphi}{\partial x}$$

$$= \sin^2\theta\,\underline{\sin\varphi\cos\varphi} + \cos^2\theta\,\underline{\cos\varphi\sin\varphi} + r\sin\theta\cos\varphi\frac{\partial\varphi}{\partial x}$$

$$= \sin\varphi\cos\varphi + r\sin\theta\cos\varphi\frac{\partial\varphi}{\partial x}$$

$$\updownarrow\quad \frac{\partial\varphi}{\partial x} = -\frac{\sin\varphi}{r\sin\theta}$$