

§. リッポフマン-シュウィンガー-方程式

$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + V\right) \psi = E \psi$$

$$\rightarrow \underbrace{\left(-\frac{\hbar^2}{2\mu} \nabla^2 - E\right)}_{\hat{H}_0} \psi = -V \psi$$

形式解

$$\psi = \phi - \frac{1}{\hat{H}_0 - E - i\eta} V \psi$$

リッポフマン-シュウィンガー-方程式

$$(\hat{H}_0 - E) \phi = 0$$

正の微小量
(波動関数の境界条件を与える。)

・ グリーン関数

$$\hat{G}_0^{(+)} = \frac{1}{\hat{H}_0 - E - i\eta}$$

座標表示では

$$G_0^{(+)}(r, r') = \langle r | \frac{1}{\hat{H}_0 - E - i\eta} | r' \rangle$$

||| $\frac{k^2 \hbar^2}{2\mu}$

$$= \int d\mathbf{k}' \langle r | \mathbf{k}' \rangle \cdot \frac{1}{\frac{k'^2 \hbar^2}{2\mu} - \frac{k^2 \hbar^2}{2\mu} - i\eta} \langle \mathbf{k}' | r' \rangle$$

$$\int \frac{dk}{(2\pi)^3} e^{ik \cdot (r-r')} = \delta(r-r')$$

No.

$$= \int \frac{dk'}{(2\pi)^3} e^{ik' \cdot r} \cdot \frac{2M}{\hbar^2} \frac{1}{k'^2 - k^2 - i\eta'} e^{-ik' \cdot r'}$$

($\eta' = \frac{2M}{\hbar^2} \eta$)

$$= \frac{2M}{\hbar^2} \cdot \frac{1}{(2\pi)^3} \int k'^2 dk' d\hat{k}' e^{ik' \cdot s \cos\theta} \frac{1}{k'^2 - k^2 - i\eta'}$$

($s \equiv r - r'$)

$$= \frac{1}{(2\pi)^3} \cdot \frac{2M}{\hbar^2} \int_0^\infty k'^2 dk' \cdot 2\pi \int_{-1}^1 d(\cos\theta) \frac{e^{ik' s \cos\theta}}{k'^2 - k^2 - i\eta'}$$

||

$$\frac{2\pi}{k'^2 - k^2 - i\eta'} \frac{1}{ik's} (e^{ik's} - e^{-ik's})$$

$$= \frac{1}{i} \cdot \frac{1}{(2\pi)^2} \cdot \frac{2M}{\hbar^2} \int_0^\infty dk' \frac{k' dk'}{k'^2 - k^2 - i\eta'} \left(\frac{e^{ik's}}{s} - \frac{e^{-ik's}}{s} \right)$$

||

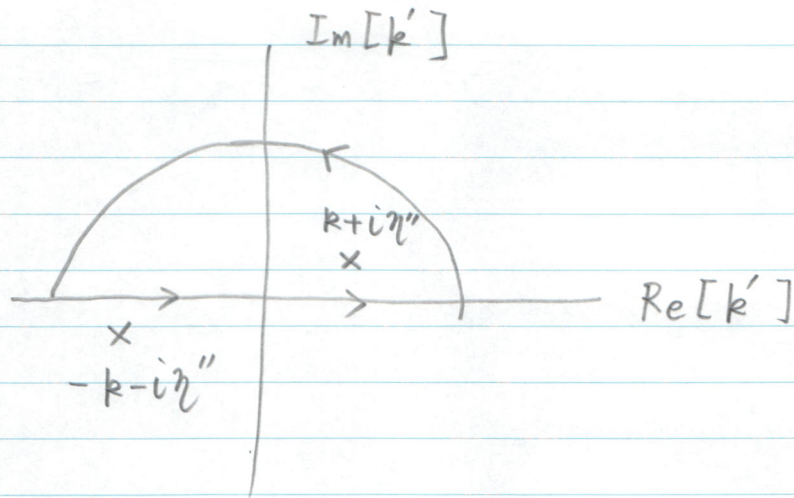
$$\int_{-\infty}^\infty dk' \frac{k' dk'}{k'^2 - k^2 - i\eta'} \cdot \frac{e^{ik's}}{s}$$

||

$$\int_{-\infty}^\infty dk' \frac{k' dk'}{(k' + k + i\eta'')(k' - k - i\eta'')} \cdot \frac{e^{ik's}}{s}$$

($\eta'' = \frac{\eta'}{2k}$)

$$\frac{1}{2} \int_{-\infty}^\infty dk' \left(\frac{1}{k' + k + i\eta''} + \frac{1}{k' - k - i\eta''} \right) \times \frac{e^{ik's}}{s}$$



$$\begin{aligned}
 \Downarrow \\
 G^{(+)}(r, r') &= \frac{1}{i} \cdot \frac{1}{(2\pi)^2} \cdot \frac{2\mu}{\hbar^2} \cdot \frac{1}{2} \cdot 2\pi i \cdot \frac{e^{iks}}{s} \\
 &= \frac{\mu}{2\pi\hbar^2} \cdot \frac{e^{iks}}{s} \\
 &= \frac{2\mu}{\hbar^2} \cdot \frac{1}{4\pi} \frac{e^{+ik|r-r'|}}{|r-r'|} \quad (\text{外向き波})
 \end{aligned}$$

↓

$$\begin{aligned} \psi(r) &= \phi(r) - \int dr' G^{(+)}(r, r') V(r') \psi(r') \\ &= e^{ik \cdot r} - \frac{2\mu}{\hbar^2} \cdot \frac{1}{4\pi} \int dr' \frac{e^{ik|r-r'|}}{|r-r'|} V(r') \psi(r') \end{aligned}$$

(note) $r \rightarrow \infty$ 近似

$$k|r-r'| = k\sqrt{r^2 - 2r \cdot r' + r'^2}$$

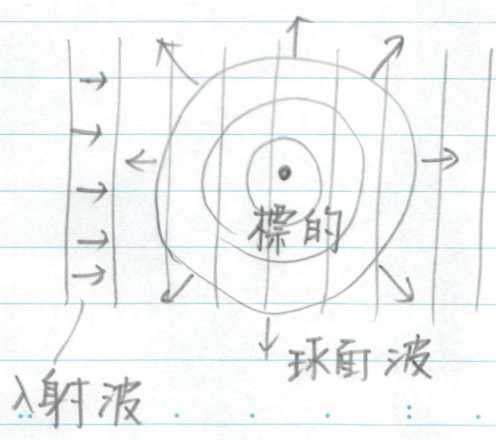
$$\sim kr - \underbrace{\left(k \frac{r}{r} \cdot r'\right)}_{\substack{|| \\ k'}}$$

↓

$$\psi(r) = e^{ik \cdot r} - \underbrace{\frac{\mu}{2\pi\hbar^2} \int dr' e^{-ik' \cdot r'} V(r') \psi(r')}_{||} \cdot \underbrace{\frac{e^{ikr}}{r}}_{\substack{\uparrow \\ |r-r'| \sim r}}$$

$f(\theta)$ 散乱振幅

$$= \underbrace{e^{ik \cdot r}}_{\text{入射波}} + \underbrace{f(\theta) \cdot \frac{e^{ikr}}{r}}_{\text{散乱波 (球面波)}}$$



§. 散乱振幅と散乱断面積

散乱波 $\psi_{sc}(r) = f(\theta) \cdot \frac{e^{ikr}}{r}$ に対応する 77ページ

$$j_{sc} = \frac{\hbar}{2i\mu} (\psi_{sc}^* \nabla \psi_{sc} - \psi_{sc} \nabla \psi_{sc}^*)$$

を計算する。

(note)
$$\begin{cases} x = r \sin \theta \cos \varphi \\ y = r \sin \theta \sin \varphi \\ z = r \cos \theta \end{cases}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

↓

$$\frac{\partial r}{\partial x} = \frac{x}{r}$$

$$\frac{\partial z}{\partial x}$$

$$z = r \cos \theta \rightarrow 0 = \frac{\partial r}{\partial x} \cos \theta - r \sin \theta \cdot \frac{\partial \theta}{\partial x}$$

$$\downarrow \frac{\partial \theta}{\partial x} = \frac{x \cos \theta}{r^2 \sin \theta} = \frac{1}{r} \cos \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi \rightarrow 0 = \frac{\partial r}{\partial x} \sin \theta \sin \varphi + r \cos \theta \cdot \frac{1}{r} \cos \theta \cos \varphi \times \sin \varphi + r \sin \theta \cos \varphi \cdot \frac{\partial \varphi}{\partial x}$$

$$= \sin^2 \theta \sin \varphi \cos \varphi + \cos^2 \theta \cos \varphi \sin \varphi + r \sin \theta \cos \varphi \frac{\partial \varphi}{\partial x}$$

$$= \sin \varphi \cos \varphi + r \sin \theta \cos \varphi \frac{\partial \varphi}{\partial x}$$

$$\downarrow \frac{\partial \varphi}{\partial x} = - \frac{\sin \varphi}{r \sin \theta}$$

$$\begin{aligned} \downarrow \quad \partial_x &= \frac{\partial r}{\partial x} \partial_r + \frac{\partial \theta}{\partial x} \partial_\theta + \frac{\partial \varphi}{\partial x} \partial_\varphi \\ &= \sin\theta \cos\varphi \partial_r + \frac{1}{r} \cos\theta \cos\varphi \partial_\theta - \frac{\sin\varphi}{r \sin\theta} \partial_\varphi \end{aligned}$$

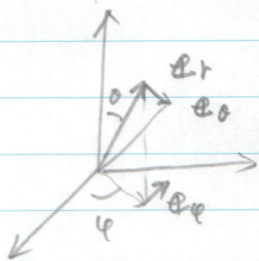
同様に

$$\partial_y = \sin\theta \sin\varphi \partial_r + \frac{1}{r} \cos\theta \sin\varphi \partial_\theta + \frac{\cos\varphi}{r \sin\theta} \partial_\varphi$$

$$\partial_z = \cos\theta \partial_r - \frac{\sin\theta}{r} \partial_\theta$$

$$\begin{aligned} \downarrow \quad \nabla &= \left[\sin\theta \cos\varphi \partial_r + \frac{1}{r} \cos\theta \cos\varphi \partial_\theta - \frac{\sin\varphi}{r \sin\theta} \partial_\varphi \right] \mathbf{e}_x \\ &+ \left[\sin\theta \sin\varphi \partial_r + \frac{1}{r} \cos\theta \sin\varphi \partial_\theta + \frac{\cos\varphi}{r \sin\theta} \partial_\varphi \right] \mathbf{e}_y \\ &+ \left[\cos\theta \partial_r - \frac{1}{r} \sin\theta \partial_\theta \right] \mathbf{e}_z \end{aligned}$$

(note)



$$\begin{cases} \mathbf{e}_r = \sin\theta \cos\varphi \mathbf{e}_x + \sin\theta \sin\varphi \mathbf{e}_y + \cos\theta \mathbf{e}_z \\ \mathbf{e}_\theta = \cos\theta \cos\varphi \mathbf{e}_x + \cos\theta \sin\varphi \mathbf{e}_y - \sin\theta \mathbf{e}_z \\ \mathbf{e}_\varphi = -\sin\varphi \mathbf{e}_x + \cos\varphi \mathbf{e}_y \end{cases}$$

$$(note) \quad \nabla \cdot \mathbf{e}_r = \partial_r, \quad \nabla \cdot \mathbf{e}_\theta = \frac{1}{r} \partial_\theta, \quad \nabla \cdot \mathbf{e}_\varphi = \frac{1}{r \sin\theta} \partial_\varphi$$

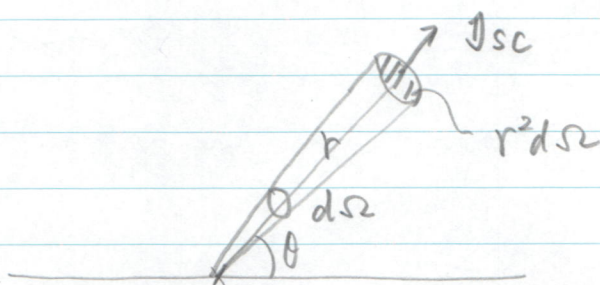
$$\downarrow \quad \boxed{\nabla = \partial_r \mathbf{e}_r + \frac{1}{r} \partial_\theta \mathbf{e}_\theta + \frac{1}{r \sin\theta} \partial_\varphi \mathbf{e}_\varphi}$$

$$\begin{aligned} \Downarrow \quad j_{sc} &= \frac{\hbar}{2i\mu} \left[f^*(\theta) \frac{e^{-ikr}}{r} (\partial_r \partial_r + \partial_\theta \frac{1}{r} \partial_\theta) f(\theta) \frac{e^{ikr}}{r} - c.c. \right] \\ &= \frac{\hbar}{2i\mu} \left[f^*(\theta) \cdot \frac{e^{-ikr}}{r} \left\{ f(\theta) \left(\frac{ik}{r} e^{ikr} - \frac{1}{r^2} e^{ikr} \right) \partial_r \right. \right. \\ &\quad \left. \left. + \frac{e^{ikr}}{r^2} f'(\theta) \partial_\theta \right\} - c.c. \right] \end{aligned}$$

$$\sim \frac{\hbar}{2i\mu} \cdot 2ik \frac{|f(\theta)|^2}{r^2} \partial_r \quad (r \rightarrow \infty)$$

$$= \frac{k\hbar}{\mu} \cdot \frac{|f(\theta)|^2}{r^2} \partial_r$$

検出器
↑
非常な遠い
 $\frac{1}{r} \gg \frac{1}{r^2}$



単位時間、立体角 $d\Omega$ に散乱される粒子数

$$= r^2 d\Omega \cdot j_{sc} \cdot \partial_r = \frac{k\hbar}{\mu} |f(\theta)|^2 d\Omega$$

$$\Downarrow \quad \boxed{\frac{d\sigma}{d\Omega} = \frac{1}{j_{in}} \cdot \frac{k\hbar}{\mu} |f(\theta)|^2 = |f(\theta)|^2}$$

• ボール近似

$$\psi = \phi - \frac{1}{H_0 - E - i\eta} V \psi$$

$$\sim \phi - \frac{1}{H_0 - E - i\eta} V \phi$$

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2} \int d\mathbf{r}' e^{-i\mathbf{k}' \cdot \mathbf{r}'} V(\mathbf{r}') \phi(\mathbf{r}')$$

$$= -\frac{\mu}{2\pi\hbar^2} \int d\mathbf{r}' e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}'} V(\mathbf{r}')$$

$$= -\frac{\mu}{2\pi\hbar^2} \tilde{V}(\vec{\theta})$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{\mu^2}{4\pi^2\hbar^4} |\tilde{V}(\vec{\theta})|^2$$

← フェルミの黄金則によるものと一致