2. 時間に依存する摂動論

2.1. 時間に依存する結合チャンネル方程式

時間を含む Schrödinger eq. : $(i\hbar\,\partial_t - H_0)\,\psi(t) = 0$

suppose $\quad H_0\,\phi_n = \varepsilon_n\,\phi_n$

and $\quad$ at $\quad t=0 \quad \psi(t) = \phi_n$

$\downarrow$

$$\psi(t) = e^{-i\varepsilon_n t/\hbar}\,\phi_n \qquad (\text{定常解})$$

> 問題：（時間に依存する）ポテンシャル $V(t)$ が加わった時，系はどのように時間発展するか？

て

$$\big(i\hbar\,\partial_t - H_0 - V(t)\big)\,\psi(t) = 0 \qquad \text{with} \quad \psi(t=0) = \phi_n$$

を解く

$$\psi(t) = \sum_m c_m(t)\, e^{-i\varepsilon_m t/\hbar}\,\phi_m \qquad \text{と展開}$$

$\downarrow \qquad c_m(t=0) = \delta_{n,m}$

$$i\hbar\dot{\psi} = i\hbar\sum_m \dot{c}_m\, e^{-i\varepsilon_m t/\hbar}\,\phi_m + i\hbar\sum_m c_m\left(\frac{-i\varepsilon_m}{\hbar}\right)e^{-i\varepsilon_m t/\hbar}$$

$$= i\hbar\sum_m \dot{c}_m\, e^{-i\varepsilon_m t/\hbar}\,\phi_m + \sum_m \varepsilon_m\, c_m\, e^{-i\varepsilon_m t/\hbar}\,\phi_m$$

$$H_0\psi = \sum_m c_m\, e^{-i\varepsilon_m t/\hbar}\, H_0\,\phi_m$$

$$= \sum_m \varepsilon_m\, c_m\, e^{-i\varepsilon_m t/\hbar}\,\phi_m$$

$$\rightarrow \left(i\hbar \sum_m \dot{c}_m e^{-i\varepsilon_m t/\hbar} \phi_m - V \sum_m c_m e^{-i\varepsilon_m t/\hbar} \phi_m \right) = 0$$

$\langle \phi_k |$ →

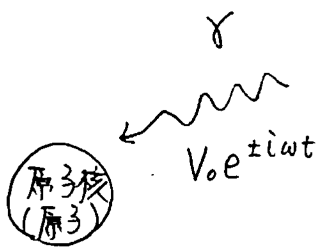
$$i\hbar \dot{c}_k e^{-i\varepsilon_k t/\hbar} - \sum_m c_m \langle \phi_k | V | \phi_m \rangle e^{-i\varepsilon_m t/\hbar} = 0$$

$$\rightarrow \boxed{i\hbar \dot{c}_k = \sum_m c_m e^{i(\varepsilon_k - \varepsilon_m)t/\hbar} \langle \phi_k | V | \phi_m \rangle}$$

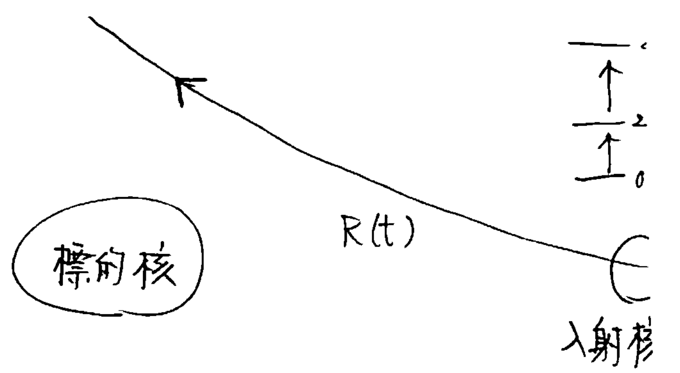
(時間に依存する結合チャンネル方程式

$C_k(0) = \delta_{k,n}$

例)



基底状態,



$$V(t) \propto Z_p Z_T e^2 \frac{R_T^2}{R(t)^3} \cdot x$$

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$$

ニュートリノ振動

$\nu_e \leftrightarrow \nu_\mu$

$$i\hbar \partial_t \begin{pmatrix} c_e \\ c_\mu \end{pmatrix} = \begin{pmatrix} V_{ee} & V_{e\mu} \\ V_{\mu e} & V_{\mu\mu} \end{pmatrix} \begin{pmatrix} c_e \\ c_\mu \end{pmatrix}$$

$$i\hbar\,\dot{c}_k(t) = \sum_m c_m(t)\, e^{i\varepsilon_{km} t/\hbar}\, V_{km}(t)$$

$\downarrow$

$$c_k(t) = \delta_{k,n} + \frac{1}{i\hbar}\int_0^t dt' \sum_m \underbrace{c_m(t')}_{\displaystyle \delta_{m,n} + \frac{1}{i\hbar}\int_0^{t'} dt'' \sum_{m'} c_{m'}(t'')\, e^{i\varepsilon_{mm'} t''/\hbar} \times V_{mm'}(t'')} e^{i\varepsilon_{km} t'/\hbar}\, V_{km}(t')$$

$$= \delta_{k,n} + \frac{1}{i\hbar}\int_0^t dt'\, e^{i\varepsilon_{kn} t'/\hbar}\, V_{kn}(t')$$

$$+ \left(\frac{1}{i\hbar}\right)^2 \int_0^t dt' \int_0^{t'} dt'' \sum_{m,m'} e^{i\varepsilon_{km} t'/\hbar}\, e^{i\varepsilon_{mm'} t''/\hbar}\, V_{km}(t')\, V_{mm'}(t'') \times c_{m'}(t'')$$

2.2. 時間に依存する摂動論

$$i\hbar\,\dot{c}_k = \sum_m c_m\, e^{i(\varepsilon_k-\varepsilon_m)t/\hbar} \langle \phi_k | \lambda V | \phi_m \rangle$$

$$c_k(t) = c_k^{(0)} + \lambda c_k^{(1)} + \lambda^2 c_k^{(2)} + \cdots$$

$$c_k^{(0)}(t) = c_k(0) = \delta_{k,n}$$

$$i\hbar\left(\dot{c}_k^{(0)} + \lambda \dot{c}_k^{(1)} + \lambda^2 \dot{c}_k^{(2)} + \cdots\right)$$

$$= \sum_m \left(c_m^{(0)} + \lambda c_m^{(1)} + \lambda^2 c_m^{(2)} + \cdots\right) e^{i\varepsilon_{km} t/\hbar} \times \langle \phi_k | \lambda V | \phi_m$$

$\varepsilon_k - \varepsilon_m$

$$O(\lambda^0):\quad i\hbar\,\dot{c}_k^{(0)} = 0 \quad \rightarrow \quad c_k^{(0)} = \text{const.} = \delta_{k,n}$$

$$O(\lambda^1):\quad i\hbar\,\dot{c}_k^{(1)} = \sum_m c_m^{(0)} e^{i\varepsilon_{km}t/\hbar} \langle \phi_k | V | \phi_m \rangle$$

$$= e^{i\varepsilon_{kn}t/\hbar} \langle \phi_k | V | \phi_n \rangle$$

$$\rightarrow \quad \boxed{c_k^{(1)} = \frac{1}{i\hbar} \int_0^t e^{i\varepsilon_{kn}t'/\hbar} V_{kn}(t')\, dt'}$$

$$O(\lambda^2): \quad i\hbar \dot{C}_k^{(2)} = \sum_m C_m^{(1)} e^{i\varepsilon_{km}t/\hbar} V_{km}(t)$$

$$\rightsquigarrow \boxed{\begin{aligned} C_k^{(2)} &= \frac{1}{i\hbar} \sum_m \int_0^t dt' \, C_m^{(1)}(t') e^{i\varepsilon_{km}t'/\hbar} V_{km}(t') \\ &= -\frac{1}{\hbar^2} \sum_m \int_0^t dt' \, e^{i\varepsilon_{km}t'/\hbar} V_{km}(t') \\ &\quad \times \int_0^{t'} dt'' \, e^{i\varepsilon_{mn}t''/\hbar} V_{mn}(t'') \end{aligned}}$$

at time t, $C_k(t) \neq 0$ for $k \neq n$.

$\longleftrightarrow$ 摂動を加えた後で系の状態を観測すれば

最初の状態 n と異なる状態 k に系が存在する確率がある

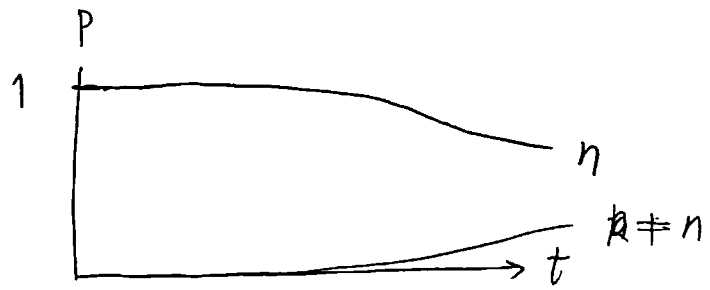
「摂動により $n \rightarrow k$ へ遷移した」

(note) $\psi(t) = \sum_m C_m(t) e^{-i\varepsilon_m t/\hbar} \phi_m$

$\downarrow$

$$P_k(t) = |\langle \phi_k | \psi(t) \rangle|^2 = |C_k(t)|^2$$

(遷移確率)



$$P_k(t) = |C_k(t)|^2$$

$$= \left(C_k^{(0)} + \lambda C_k^{(1)} + \lambda^2 C_k^{(2)} + \cdots\right)^* \left(C_k^{(0)} + \lambda C_k^{(1)} + \lambda^2 C_k^{(2)} + \cdots\right)$$

$$= |C_k^{(0)}|^2 + \lambda \left(C_k^{(1)*} C_k^{(0)} + C_k^{(0)*} C_k^{(1)}\right) + \lambda^2 \left(C_k^{(0)*} C_k^{(2)} + C_k^{(1)*} C_k^{(1)} + C_k^{(2)*} C_k^{(0)}\right) + \cdots$$

[

$$= \delta_{k,n} + \lambda\, \delta_{kn} \left\{ \cancel{-\frac{1}{i\hbar}\int_0^t e^{i\varepsilon_{kn}t'/\hbar} V_{kn}(t')\,dt'} \cancel{+ \frac{1}{i\hbar}\int_0^t e^{i\varepsilon_{kn}t'/\hbar} V_{kn}(t')\,dt'} \right\}$$

$$+ \lambda^2 \Big\{ -\delta_{k,n} \cdot \frac{1}{\hbar^2} \sum_m \int_0^t dt'\, e^{i\varepsilon_{km}t'/\hbar} V_{km}(t') \int_0^{t'} dt''\, e^{i\varepsilon_{mn}t''/\hbar} V_{mn}(t'')$$

$$- \delta_{k,n} \cdot \frac{1}{\hbar^2} \sum_m \int_0^t dt'\, e^{-i\varepsilon_{km}t'/\hbar} \underbrace{V_{km}^*(t')}_{=V_{mk}(t')} \int_0^{t'} dt''\, e^{-i\varepsilon_{mn}t''/\hbar} V_{mn}^*(t'')$$

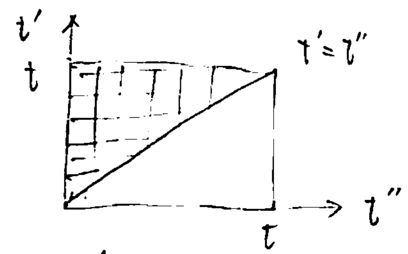
$$+ \frac{1}{\hbar^2} \left| \int_0^t dt'\, e^{i\varepsilon_{kn}t'/\hbar} V_{kn}(t') \right|^2 \Big\}$$

[

$$= \delta_{k,n} + \lambda^2 \Big\{ \frac{1}{\hbar^2} \left| \int_0^t dt'\, e^{i\varepsilon_{kn}t'/\hbar} V_{kn}(t') \right|^2$$

$$- \delta_{k,n} \frac{1}{\hbar^2} \sum_m \int_0^t dt'\, e^{i\varepsilon_{nm}t'/\hbar} V_{nm}(t') \int_0^{t'} dt''\, e^{i\varepsilon_{mn}t''/\hbar} V_{mn}(t'')$$

$$- \delta_{k,n} \frac{1}{\hbar^2} \sum_m \int_0^t dt'\, e^{-i\varepsilon_{nm}t'/\hbar} V_{mn}(t') \int_0^{t'} dt''\, e^{-i\varepsilon_{mn}t''/\hbar} V_{nm}(t'')$$



$$\int_0^t dt' \int_0^{t'} dt'' = \int_0^t dt'' \int_{t''}^t dt'$$

$$= \delta_{k,n} + \lambda^2 \left\{ \frac{1}{\hbar^2} \left| \int_0^t dt' \, e^{i\varepsilon_{kn} t'/\hbar} V_{kn}(t') \right|^2 \right.$$

$$- \delta_{k,n} \cdot \frac{1}{\hbar^2} \sum_m \int_0^t dt' \, e^{-i\varepsilon_{mn} t'/\hbar} V_{mn}^*(t') \int_0^{t'} dt'' \, e^{i\varepsilon_{mn} t''/\hbar} V_{mn}(t'')$$

$$- \delta_{k,n} \cdot \frac{1}{\hbar^2} \sum_m \int_0^t dt'' \, e^{-i\varepsilon_{mn} t''/\hbar} V_{mn}^*(t'') \int_{t''}^{t} dt' \, e^{i\varepsilon_{mn} t'/\hbar} V_{mn}(t')$$

$$= \delta_{k,n} + \lambda^2 \left\{ \frac{1}{\hbar^2} \left| \int_0^t dt' \, e^{i\varepsilon_{kn} t'/\hbar} V_{kn}(t') \right|^2 \right.$$

$$- \delta_{k,n} \cdot \frac{1}{\hbar^2} \sum_m \int_0^t dt' \, e^{-i\varepsilon_{mn} t'/\hbar} V_{mn}^*(t') \cdot \int_0^t dt' \, e^{i\varepsilon_{mn} t'/\hbar} V_{mn}(t'$$

$$= \delta_{k,n} + \lambda^2 \left\{ \frac{1}{\hbar^2} \left| \int_0^t dt' \, e^{i\varepsilon_{kn} t'/\hbar} V_{kn}(t') \right|^2 \right.$$

$$\left. - \delta_{k,n} \cdot \frac{1}{\hbar^2} \sum_m \left| \int_0^t dt' \, e^{i\varepsilon_{mn} t'/\hbar} V_{mn}(t') \right|^2 \right\}$$

$\Downarrow$

$$P_k(t) = \frac{1}{\hbar^2} \lambda^2 \left| \int_0^t dt' \, e^{i\varepsilon_{kn} t'/\hbar} V_{kn}(t') \right|^2 \quad \text{for } k \neq n$$

$$= 1 - \frac{\lambda^2}{\hbar^2} \sum_{m \neq n} \left| \int_0^t dt' \, e^{i\varepsilon_{mn} t'/\hbar} V_{mn}(t') \right|^2 = 1 - \sum_{m \neq n} P_m \quad \text{for } k$$

$$|\lambda c_k^{(1)}|^2$$

○ 別の導出法：相互作用表示

$$i\hbar\,\partial_t |\psi(t)\rangle = (H_0 + V(t))|\psi(t)\rangle$$

<u>define</u> $|\tilde{\psi}(t)\rangle = e^{iH_0t/\hbar}|\psi(t)\rangle$

$\downarrow$

$$i\hbar\,\partial_t |\tilde{\psi}(t)\rangle = -H_0\,e^{iH_0t/\hbar}|\psi(t)\rangle + e^{iH_0t/\hbar}\cdot i\hbar\partial_t|\psi$$

$$= e^{iH_0t/\hbar}\,V(t)\,|\psi(t)\rangle$$

$$= \underbrace{e^{iH_0t/\hbar}\,V(t)\,e^{-iH_0t/\hbar}}_{\tilde{V}(t)}\;\underbrace{e^{iH_0t/\hbar}|\psi(t)\rangle}_{|\tilde{\psi}(t)\rangle}$$

<u>define</u>

$$|\tilde{\psi}(t)\rangle = \tilde{U}(t)|\tilde{\psi}(0)\rangle$$

$\downarrow$

$$\underbrace{[\,i\hbar\,\dot{\tilde{U}} - \tilde{V}\,\tilde{U}\,]}_{0}|\tilde{\psi}(0)\rangle = 0 \quad \text{with} \quad \tilde{U}(0) = 1.$$

$$i\hbar \dot{\tilde{U}} = \tilde{V}\tilde{U} \qquad \text{with} \quad \tilde{U}(0) = 1$$

$$\downarrow \quad \tilde{U} = 1 + \frac{1}{i\hbar}\int_0^t dt' \, \tilde{V}(t') \underbrace{\tilde{U}(t')}_{\displaystyle \shortparallel \atop \displaystyle 1 + \frac{1}{i\hbar}\int_0^{t'} dt'' \, \tilde{V}(t'')\tilde{U}(t'')}$$

$$= 1 + \frac{1}{i\hbar}\int_0^t dt' \, \tilde{V}(t') - \frac{1}{\hbar^2}\int_0^t dt' \, \tilde{V}(t') \int_0^{t'} dt'' \, \tilde{V}(t'')\tilde{U}(t'')$$

$$|\tilde{\Psi}(0)\rangle = |\Psi(0)\rangle = |\phi_n\rangle$$

$$P_k(t) = |\langle \phi_k | \Psi(t)\rangle|^2$$

$$= |\underbrace{\langle \phi_k | e^{-iH_0 t/\hbar}}_{\displaystyle \downarrow \atop \displaystyle \langle \phi_k | e^{-i\varepsilon_k t/\hbar}} \underbrace{e^{iH_0 t/\hbar} |\Psi(t)\rangle}_{\displaystyle \downarrow \atop \displaystyle |\tilde{\Psi}(t)\rangle}|^2$$

$$= |\langle \phi_k | \tilde{\Psi}(t)\rangle|^2$$

$$= \left| \frac{1}{i\hbar}\int_0^t dt' \underbrace{\langle \phi_k | \tilde{V}(t') | \phi_n\rangle}_{\displaystyle \shortparallel \atop \displaystyle e^{i\varepsilon_{kn}t'/\hbar} V_{kn}(t')} \right|^2 \qquad (k \neq n)$$

Supplement 15-B

The Interaction Picture

For the discussion of systems involving only two or three levels, it is particularly convenient to use a description of the time evolution of the system that lies between the Schrödinger picture and the Heisenberg picture, both of which were discussed in Chapter 6. Let us start with the Schrödinger equation, which reads

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}H|\psi(t)\rangle = -\frac{i}{\hbar}(H_0 + H_1)|\psi(t)\rangle \tag{15B-1}$$

We can write this in the form

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle \tag{15B-2}$$

where

$$\frac{d}{dt}U(t) = -\frac{i}{\hbar}(H_0 + H_1)U(t) \tag{15B-3}$$

The initial condition is $U(0) = 1$.

The procedure calls for the definition of a new state vector $|\psi_I(t)\rangle$ defined by

$$|\psi_I(t)\rangle = e^{iH_0t/\hbar}|\psi(t)\rangle \tag{15B-4}$$

It follows that

$$\begin{aligned}\frac{d}{dt}|\psi_I(t)\rangle &= \frac{i}{\hbar}H_0|\psi_I(t)\rangle + e^{iH_0t/\hbar}\left(-\frac{i}{\hbar}\right)(H_0 + H_1)|\psi(t)\rangle \\ &= e^{iH_0t/\hbar}\left(-\frac{i}{\hbar}\right)H_1e^{-iH_0t/\hbar}|\psi_I(t)\rangle\end{aligned}$$

If we now define

$$V(t) = e^{iH_0t/\hbar}H_1e^{-iH_0t/\hbar} \tag{15B-5}$$

we end up with the equation

$$\frac{d}{dt}|\psi_I(t)\rangle = \left(-\frac{i}{\hbar}\right)V(t)|\psi_I(t)\rangle \tag{15B-6}$$

Solving this equation is not trivial, and in general the best one can do is to find a solution in terms of a power series in $V(t)$. The formal procedure for solving this in a way that incorporates the initial condition

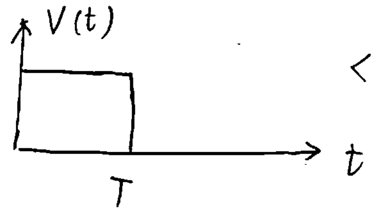
$$|\psi_I(0)\rangle = |\phi\rangle \tag{15B-7}$$

is to write

$$|\psi_I(t)\rangle = U_I(t)|\phi\rangle \tag{15B-8}$$

2.3. 時間を含まない摂動による遷移

$$V(t) = V \qquad (0 \le t \le T)$$



$$\langle \phi_k | V(t) | \phi_n \rangle = V_{kn} \quad (0 \le t \le$$

$$\int_0^T e^{i\varepsilon_{kn} t'/\hbar} V_{kn}(t') dt' = V_{kn} \frac{\hbar}{i\varepsilon_{kn}} \left(e^{i\varepsilon_{kn} T/\hbar} - 1 \right) \quad (\varepsilon_{kn} \neq 0$$

$$= V_{kn} T \qquad (\varepsilon_{kn} = 0)$$

$$\downarrow$$

$$C_k^{(1)} = -\frac{V_{kn}}{\varepsilon_{kn}} \left(e^{i\varepsilon_{kn} T/\hbar} - 1 \right)$$

$$C_k^{(2)} = -\frac{1}{\hbar^2} \sum_m V_{km} V_{mn} \int_0^T dt' e^{i\varepsilon_{km} t'/\hbar} \underbrace{\int_0^{t'} dt'' e^{i\varepsilon_{mn} t''/\hbar}}_{= \frac{\hbar}{i\varepsilon_{mn}} \left(e^{i\varepsilon_{mn} t'/\hbar} - 1 \right.}$$

$$= -\frac{1}{\hbar^2} \sum_m V_{km} V_{mn} \cdot \frac{\hbar}{i\varepsilon_{mn}} \left\{ \frac{\hbar}{i\varepsilon_{kn}} \left(e^{i\varepsilon_{kn} T/\hbar} - 1 \right) - \frac{\hbar}{i\varepsilon_{km}} \times \left(e^{i\varepsilon_{km} T/\hbar} - 1 \right. \right.$$

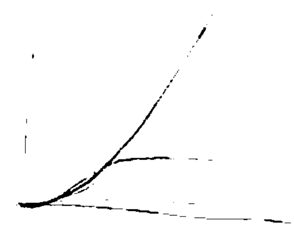
$$= \sum_m \frac{V_{km} V_{mn}}{\varepsilon_{mn}} \left\{ \frac{e^{i\varepsilon_{kn} T/\hbar} - 1}{\varepsilon_{kn}} - \frac{e^{i\varepsilon_{km} T/\hbar} - 1}{\varepsilon_{km}} \right\}$$

$$P_k(T) = \frac{\lambda^2}{\hbar^2} |V_{kn}|^2 \cdot \left| \frac{\hbar}{i\,\varepsilon_{kn}} \left(e^{i\varepsilon_{kn}T/\hbar} - 1\right) \right|^2 \qquad (\text{if } \varepsilon_{kn} \neq 0$$

$$= \frac{\lambda^2}{\varepsilon_{kn}^2} |V_{kn}|^2 \cdot \left| e^{i\varepsilon_{kn}T/2\hbar} \left(e^{i\varepsilon_{kn}T/2\hbar} - e^{-i\varepsilon_{kn}T/2\hbar}\right) \right|^2$$

$$= \frac{4\lambda^2}{\varepsilon_{kn}^2} |V_{kn}|^2 \sin^2\left(\frac{\varepsilon_{kn}T}{2\hbar}\right)$$

$$P_k(T) = \lambda^2 |V_{kn}|^2 T^2 \qquad (\text{if } \varepsilon_{kn} = 0)$$



○ 収束の条件：

$$c_k^{(1)} = -\frac{V_{kn}}{\varepsilon_{kn}} \left(e^{i\varepsilon_{kn}T/\hbar} - 1\right)$$

$\downarrow$

$$\left| \lambda \frac{V_{kn}}{\varepsilon_{kn}} \underline{\left(e^{i\varepsilon_{kn}T/\hbar} - 1\right)} \right| \ll 1$$

(note) $\quad \left| e^{i\varepsilon_{kn}T/\hbar} - 1 \right|^2 = 4\sin^2\left(\frac{\varepsilon_{kn}T}{2\hbar}\right)$

$\downarrow$

· $|\lambda V_{kn}| \ll |\varepsilon_{kn}|$

or

· $|\varepsilon_{kn} T/\hbar| \ll 1$

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(note) $\int_{-\infty}^{\infty} dx \frac{1}{x^2} \sin^2 x = \pi$

2.4 周期的な摂動による遷移

$V(t) = V e^{\mp i\omega t}$

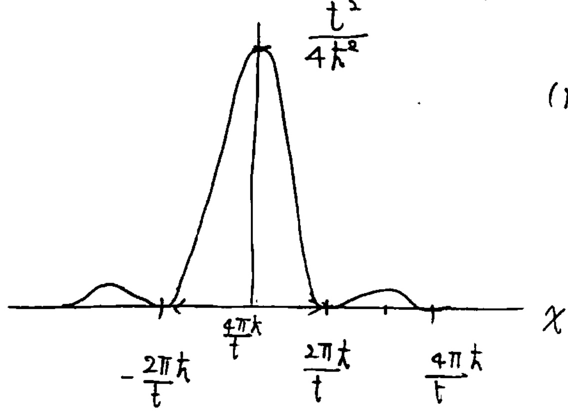
$\downarrow$ $\int_0^t dt' e^{i\varepsilon_{kn} t'/\hbar} V_{kn}(t') dt' = V_{kn} \int_0^t dt' e^{i(\varepsilon_{kn} \mp \hbar\omega) t'/\hbar}$

$= V_{kn} \frac{\hbar}{i(\varepsilon_{kn} \mp \hbar\omega)} \left(\underbrace{e^{i(\varepsilon_{kn} \mp \hbar\omega) t/\hbar} - 1}_{= 2i e^{\frac{i\Delta t}{2\hbar}} \frac{e^{i\frac{\Delta t}{2\hbar}} - e^{-i\frac{\Delta t}{2\hbar}}}{2i}} \right)$

$\downarrow$ $P_k(t) = \frac{4\lambda^2}{\Delta^2} |V_{kn}|^2 \sin^2\left(\frac{\Delta \cdot t}{2\hbar}\right)$

$\Delta = \varepsilon_k - \varepsilon_n \mp \hbar\omega$

$f(x) = \frac{1}{x^2} \sin^2\left(\frac{xt}{2\hbar}\right) \xrightarrow[t\to\infty]{} \frac{\pi t}{2\hbar} \delta(x)$

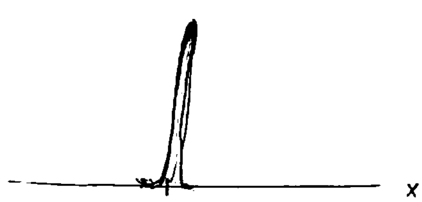


(note) $\int_{-\infty}^{\infty} dx \frac{1}{x^2} \sin^2\left(\frac{xt}{2\hbar}\right)$

$= \frac{t}{2\hbar} \int_{-\infty}^{\infty} dy \frac{1}{y^2} \sin^2 y$

$= \frac{\pi t}{2\hbar}$

$\downarrow$



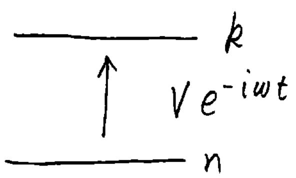
(note)

$\Delta E \cdot \Delta t \sim \hbar$

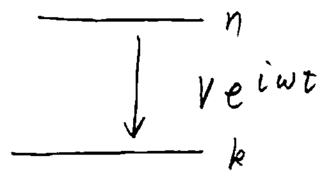
$\downarrow$ $\boxed{P_k(t) \longrightarrow \frac{2\pi}{\hbar} t \, \lambda^2 |V_{kn}|^2 \delta(\varepsilon_k - \varepsilon_n \mp \hbar\omega)}$

単位時間当たりの遷移確率：

$$\boxed{T_k = P_k/t = \frac{2\pi}{\hbar}\lambda^2 |V_{kn}|^2 \delta(\varepsilon_k - \varepsilon_n \mp \hbar\omega)}$$



$\varepsilon_k = \varepsilon_n + \hbar\omega$



$\varepsilon_k = \varepsilon_n - \hbar\omega$