。摂動論のまとめ

1）時間に依存しない場合

$$H = H_0 + V$$

$$H_0 \,|\phi_n\rangle = \varepsilon_n \,|\phi_n\rangle$$

$$H \psi_n = E_n \psi_n$$

$$E_n = \varepsilon_n + \langle \phi_n | V | \phi_n \rangle + \sum_{l \neq n} \frac{|\langle \phi_n | V | \phi_l \rangle|^2}{\varepsilon_n - \varepsilon_0} + \cdots$$

$$\psi_n = \phi_n + \sum_{l \neq n} \frac{\langle \phi_l | V | \phi_n \rangle}{\varepsilon_n - \varepsilon_l} \,|\phi_l\rangle + \cdots$$

2）時間に依存する場合

$$\left[i\hbar\, \partial_t - H_0 - V(t) \right] \psi(t) = 0$$

$$\psi(t) = \sum_m C_m(t)\, e^{-i\varepsilon_m t/\hbar}\, \phi_m$$

$$C_m(t) = \delta_{m,n} + \frac{1}{i\hbar} \int_0^t e^{i(\varepsilon_m - \varepsilon_n)t'/\hbar} \times \langle \phi_m | V(t') | \phi_n \rangle \, dt'$$

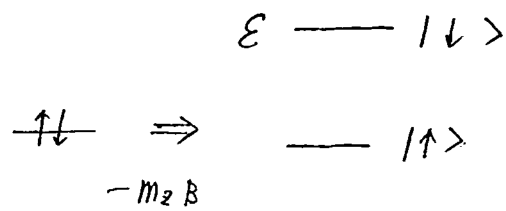
遷移確率：$P_k(t) = |\langle \phi_k | \psi(t) \rangle|^2 = |C_k(t)|^2$

$$= \begin{cases} \dfrac{1}{\hbar^2} \left| \displaystyle\int_0^t e^{i\varepsilon_{kn} t'/\hbar}\, V_{kn}(t')\, dt' \right|^2 & (k \neq n) \\ 1 - \displaystyle\sum_{m \neq n} P_k(t) & (k = n) \end{cases}$$

4.6. 二準位問題

4.6.1. 時間に依存しないハミルトニアン

$$H = \begin{pmatrix} -\frac{\varepsilon}{2} & V \\ V & \frac{\varepsilon}{2} \end{pmatrix}$$



(note)

$$H = -\frac{1}{2}\varepsilon\,\sigma_z + V\,\sigma_x$$

$V=0$ のとき

$$H = \begin{pmatrix} -\frac{\varepsilon}{2} & 0 \\ 0 & \frac{\varepsilon}{2} \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad E_0 = -\frac{\varepsilon}{2}$$

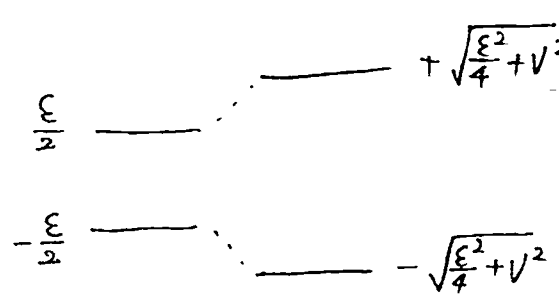
$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad E_1 = \frac{\varepsilon}{2}$$

$V \neq 0$ のとき：

対角化

$$\det\begin{pmatrix} -\frac{\varepsilon}{2} - \lambda & V \\ V & \frac{\varepsilon}{2} - \lambda \end{pmatrix} = \lambda^2 - \frac{\varepsilon^2}{4} - V^2 = 0$$

$$\leadsto \quad \lambda_\pm = \pm\sqrt{\frac{\varepsilon^2}{4} + V^2}$$



波動関数:

$$\begin{pmatrix} -\frac{\varepsilon}{2} & V \\ V & \frac{\varepsilon}{2} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\frac{\varepsilon}{2}\alpha + V\beta \\ V\alpha + \frac{\varepsilon}{2}\beta \end{pmatrix} = \lambda \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\rightsquigarrow \quad \alpha = \frac{\lambda - \frac{\varepsilon}{2}}{\sqrt{(\lambda - \frac{\varepsilon}{2})^2 + V^2}}, \quad \beta = \frac{V}{\sqrt{(\lambda - \frac{\varepsilon}{2})^2 + V^2}}$$

(note)

$$\begin{pmatrix} -\frac{\varepsilon}{2} & V \\ V & \frac{\varepsilon}{2} \end{pmatrix} \begin{pmatrix} \lambda - \frac{\varepsilon}{2} \\ V \end{pmatrix} = \begin{pmatrix} -\frac{\varepsilon}{2}\lambda + \overbrace{\frac{\varepsilon^2}{4}}^{=\lambda^2 - V^2} + V^2 \\ \lambda V \cancel{- \frac{\varepsilon}{2}V} \cancel{+ \frac{\varepsilon}{2}V} \end{pmatrix}$$

$$= \lambda \begin{pmatrix} \lambda - \frac{\varepsilon}{2} \\ V \end{pmatrix}$$

○摂動論で解く

$$H = \underbrace{\begin{pmatrix} -\frac{\varepsilon}{2} & 0 \\ 0 & \frac{\varepsilon}{2} \end{pmatrix}}_{\text{非摂動ハミルトニアン}} + \underbrace{\begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}}_{\text{摂動項}}$$

$V=0$ のとき

$$|0^{(0)}\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad E_0^{(0)} = -\frac{\varepsilon}{2}$$

$$|1^{(0)}\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad E_1^{(0)} = \frac{\varepsilon}{2}$$

$V \neq 0$

$$E_n = E_n^{(0)} + \langle \phi_n | V | \phi_n \rangle + \sum_{k \neq n} \frac{|\langle \phi_k | V | \phi_n \rangle|^2}{E_n^{(0)} - E_k^{(0)}} + \cdots$$

$$|\tilde{\phi}_n\rangle = |\phi_n\rangle + \sum_{k \neq n} \frac{\langle \phi_k | V | \phi_n \rangle}{E_n^{(0)} - E_k^{(0)}} |\phi_k\rangle + \cdots$$

$\downarrow$

$$E_0 = E_0^{(0)} + \langle 0|V|0\rangle + \frac{|\langle 0|V|1\rangle|^2}{E_0^{(0)} - E_1^{(0)}} + \cdots$$

$$E_1 = E_1^{(0)} + \langle 1|V|1\rangle + \frac{|\langle 1|V|0\rangle|^2}{E_1^{(0)} - E_0^{(0)}} + \cdots$$

(note)

$$\langle 0|V|0\rangle = (1 \quad 0)\begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= (1 \quad 0)\begin{pmatrix} 0 \\ V \end{pmatrix} = 0$$

$$\langle 1|V|1\rangle = 0$$

$$\langle 0|V|1\rangle = (1 \quad 0)\begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (1 \quad 0)\begin{pmatrix} V \\ 0 \end{pmatrix} = V$$

$$= \langle 1|V|0\rangle$$

$\downarrow$

$$E_0 = -\frac{\varepsilon}{2} + \frac{V^2}{-\frac{\varepsilon}{2} - \frac{\varepsilon}{2}} + \cdots = -\frac{\varepsilon}{2} - \frac{V^2}{\varepsilon}$$

$$E_1 = +\frac{\varepsilon}{2} + \frac{V^2}{\frac{\varepsilon}{2} + \frac{\varepsilon}{2}} = \frac{\varepsilon}{2} + \frac{V^2}{\varepsilon}$$

(note) $E = \pm\sqrt{\frac{\varepsilon^2}{4} + V^2} = \pm\sqrt{\frac{\varepsilon^2}{4}\left(1 + \frac{4V^2}{\varepsilon^2}\right)}$

$$\sim \pm\frac{\varepsilon}{2}\left(1 + \frac{2V^2}{\varepsilon^2}\right) = \pm\left(\frac{\varepsilon}{2} + \frac{V^2}{\varepsilon}\right).$$

波動関数：

$$|\tilde{0}\rangle = |0\rangle + \frac{\langle 1|V|0\rangle}{-\frac{\varepsilon}{2}-\frac{\varepsilon}{2}}|1\rangle + \cdots$$

$$= \begin{pmatrix} 1 \\ -\frac{V}{\varepsilon} \end{pmatrix}$$

$$|\tilde{1}\rangle = |1\rangle + \frac{\langle 0|V|1\rangle}{\frac{\varepsilon}{2}+\frac{\varepsilon}{2}}|0\rangle = \begin{pmatrix} \frac{V}{\varepsilon} \\ 1 \end{pmatrix}$$

(note) exact w.f. $\propto \begin{pmatrix} \lambda - \frac{\varepsilon}{2} \\ V \end{pmatrix}$

$$\lambda - \frac{\varepsilon}{2} \sim \pm\left(\frac{\varepsilon}{2} + \frac{V^2}{\varepsilon}\right) - \frac{\varepsilon}{2}$$

$$= \begin{cases} \frac{\varepsilon}{2} + \frac{V^2}{\varepsilon} - \frac{\varepsilon}{2} = \frac{V^2}{\varepsilon} \\ -\frac{\varepsilon}{2} - \frac{V^2}{\varepsilon} - \frac{\varepsilon}{2} = -\varepsilon - \frac{V^2}{\varepsilon} \sim -\varepsilon \end{cases}$$

$$\begin{pmatrix} -\varepsilon \\ V \end{pmatrix}, \quad \begin{pmatrix} \frac{V^2}{\varepsilon} \\ V \end{pmatrix}$$

。変分法で解く

assume $\quad |\tilde{0}\rangle = \dfrac{1}{\sqrt{1+\alpha^2}} \begin{pmatrix} 1 \\ \alpha \end{pmatrix}$

$\downarrow$

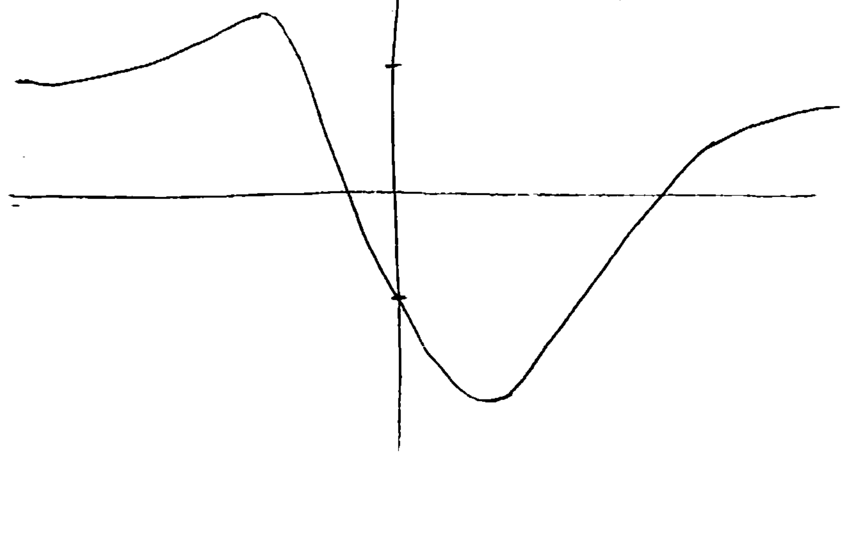
$$f(\alpha) \equiv \frac{\langle \tilde{0} | H | \tilde{0} \rangle}{\langle \tilde{0} | \tilde{0} \rangle} = \frac{1}{(1+\alpha^2)} \begin{pmatrix} 1 & \alpha \end{pmatrix} \underbrace{\begin{pmatrix} -\frac{\varepsilon}{2} & V \\ V & \frac{\varepsilon}{2} \end{pmatrix} \begin{pmatrix} 1 \\ \alpha \end{pmatrix}}_{= \begin{pmatrix} -\frac{\varepsilon}{2} + V\alpha \\ V + \frac{\varepsilon}{2}\alpha \end{pmatrix}}$$

$$= \frac{1}{1+\alpha^2} \left(-\frac{\varepsilon}{2} + V\alpha + V\alpha + \frac{\varepsilon}{2}\alpha^2 \right)$$

$$= \frac{1}{1+\alpha^2} \left(\frac{\varepsilon}{2}\alpha^2 + 2V\alpha - \frac{\varepsilon}{2} \right)$$

$\alpha \to \pm\infty : \quad f(\alpha) \Rightarrow \dfrac{1}{\alpha^2} \cdot \dfrac{\varepsilon\alpha^2}{2} = \dfrac{\varepsilon}{2}$

$\alpha \to 0 \quad : \quad f(\alpha) \to \dfrac{1}{1} \cdot \left(-\dfrac{\varepsilon}{2}\right) = -\dfrac{\varepsilon}{2}$



極小点：

$$0 = f'(\alpha) = -\frac{2\alpha}{(1+\alpha^2)^2}\left(\frac{\varepsilon}{2}\alpha^2 + 2V\alpha - \frac{\varepsilon}{2}\right) + \frac{1}{1+\alpha^2}(\varepsilon\alpha + 2V)$$

$$= \frac{1}{(1+\alpha^2)^2}\left\{\varepsilon\alpha + 2V + \cancel{\varepsilon\alpha^3} + \underline{2V\alpha^2} - \cancel{\varepsilon\alpha^3} - \underline{4V\alpha^2} + \varepsilon\alpha\right\}$$

$$= \frac{1}{(1+\alpha^2)^2}\left(-2V\alpha^2 + 2\varepsilon\alpha + 2V\right)$$

$$= \frac{1}{(1+\alpha^2)^2}\cdot(-2V)\left(\alpha^2 - \frac{\varepsilon}{V}\alpha - 1\right)$$

$$\Updownarrow$$

$$\alpha = \frac{1}{2}\left\{\frac{\varepsilon}{V} \pm \sqrt{\frac{\varepsilon^2}{V^2} + 4}\right\} = \frac{1}{2V}\left(\varepsilon \pm \sqrt{\varepsilon^2 + 4V^2}\right)$$

$$f''(\alpha) = -\frac{2\cdot 2\alpha}{(1+\alpha^2)^3}(-2V)\left(\alpha^2 - \frac{\varepsilon}{V}\alpha - 1\right) - \frac{2V}{(1+\alpha^2)^2}\left(2\alpha - \frac{\varepsilon}{V}\right)$$

$$= \frac{-2V}{(1+\alpha^2)^3}\left(\cancel{-4\alpha^3} + \cancel{\frac{4\varepsilon}{V}\alpha^2} + 4\alpha + 2\alpha - \frac{\varepsilon}{V} + \cancel{2\alpha^3} - \cancel{\frac{\varepsilon}{V}\alpha^2}\right)$$

$$= \frac{-2V}{(1+\alpha^2)^3}\left(-2\alpha^3 + \frac{3\varepsilon}{V}\alpha^2 + 6\alpha - \frac{\varepsilon}{V}\right)$$

$V \to$ small

$$\alpha \sim \frac{1}{2V}\left(\varepsilon \pm \varepsilon\sqrt{1+\frac{4V^2}{\varepsilon^2}}\right) \simeq \frac{1}{2V}\left\{\varepsilon \pm \varepsilon\left(1+\frac{2V^2}{\varepsilon^2}\right)\right\}$$

$$= \begin{cases} \frac{1}{2V}\left(2\varepsilon + \frac{2V^2}{\varepsilon^2}\right) \sim \frac{\varepsilon}{V} \\ \frac{1}{2V}\left(\varepsilon - \varepsilon - \frac{2V^2}{\varepsilon}\right) = -\frac{V}{\varepsilon} \end{cases}$$

$$-2\alpha^3 + \frac{3\varepsilon}{V}\alpha^2 + 6\alpha - \frac{\varepsilon}{V}$$

$$= \begin{cases} -2\left(\frac{\varepsilon}{V}\right)^3 + \frac{3\varepsilon}{V}\left(\frac{\varepsilon}{V}\right)^2 + 6\left(\frac{\varepsilon}{V}\right) - \frac{\varepsilon}{V} \sim \frac{\varepsilon^3}{V^3} \\ -2\left(\frac{-V}{\varepsilon}\right)^3 + \frac{3\varepsilon}{V}\left(-\frac{V}{\varepsilon}\right)^2 - 6\cdot\frac{V}{\varepsilon} - \frac{\varepsilon}{V} \sim -\frac{\varepsilon}{V} \end{cases}$$

$\Downarrow$

$$\alpha = \begin{cases} \frac{1}{2V}\left(\varepsilon - \sqrt{\varepsilon^2+4V^2}\right) & \text{for } \frac{\varepsilon}{V} > 0 \\ \frac{1}{2V}\left(\varepsilon + \sqrt{\varepsilon^2+4V^2}\right) & \text{for } \frac{\varepsilon}{V} < 0 \end{cases}$$

$\frac{\varepsilon}{V} > 0$ のとき:

$$\frac{\varepsilon}{2}\alpha^2 + 2V\alpha - \frac{\varepsilon}{2} = \frac{\varepsilon}{2}\cdot\frac{1}{4V^2}\left(\varepsilon^2 + \varepsilon^2 + 4V^2 - 2\varepsilon\sqrt{\varepsilon^2+4V^2}\right)$$

$$+ \varepsilon - \sqrt{\varepsilon^2+4V^2} - \frac{\varepsilon}{2}$$

$$= \frac{\varepsilon^3}{4V^2} - \frac{\varepsilon^2}{4V^2}\sqrt{\varepsilon^2+4V^2} + \varepsilon - \sqrt{\varepsilon^2+4V^2}$$

$$= \varepsilon\left(1+\frac{\varepsilon^2}{4V^2}\right) - \sqrt{\varepsilon^2+4V^2}\left(1+\frac{\varepsilon^2}{4V^2}\right)$$

$$= \left(\varepsilon - \sqrt{\varepsilon^2+4V^2}\right)\left(1+\frac{\varepsilon^2}{4V^2}\right)$$

$$1+\alpha^2 = 1 + \frac{1}{4V^2}\left(\varepsilon^2 + \varepsilon^2 + 4V^2 - 2\varepsilon\sqrt{\varepsilon^2+4V^2}\right)$$

$$= 2 + \frac{2\varepsilon^2}{4V^2} - \frac{\varepsilon}{2V^2}\sqrt{\varepsilon^2+4V^2}$$

$$= 2\left(1+\frac{\varepsilon^2}{4V^2}\right) - \frac{\varepsilon}{2V^2}\sqrt{\varepsilon^2+4V^2}$$

$$= 2\cdot\sqrt{1+\frac{\varepsilon^2}{4V^2}}\left(\sqrt{1+\frac{\varepsilon^2}{4V^2}} - \frac{\varepsilon}{2V}\right) = \frac{1}{V}\sqrt{1+\frac{\varepsilon^2}{4V^2}}\left(\sqrt{\varepsilon^2+4V^2} - \varepsilon\right)$$

$$f(\alpha) = \frac{\left(1+\frac{\varepsilon^2}{4V^2}\right)\left(\cancel{\varepsilon^2 - \varepsilon^2} - 4V^2\right)}{\frac{1}{V}\sqrt{1+\frac{\varepsilon^2}{4V^2}}\left(\cancel{\varepsilon^2} + 4V^2 \cancel{- \varepsilon^2}\right)} = -V\cdot\sqrt{1+\frac{\varepsilon^2}{4V^2}}$$

$$= -\sqrt{\frac{\varepsilon^2}{4} + V^2} = E_{exact}$$

波動関数：

$$\begin{pmatrix} 2V \\ \varepsilon - \sqrt{\varepsilon^2+4V^2} \end{pmatrix} \propto \begin{pmatrix} \dfrac{2V^2}{\varepsilon - \sqrt{\varepsilon^2+4V^2}} \\ V \end{pmatrix}$$

(note) $$\frac{2V^2}{\varepsilon - \sqrt{\varepsilon^2+4V^2}} = 2V^2 \cdot \frac{\varepsilon + \sqrt{\varepsilon^2+4V^2}}{\varepsilon^2 - \varepsilon^2 - 4V^2}$$

$$= -\frac{\varepsilon}{2} - \frac{1}{2}\sqrt{\varepsilon^2+4V^2}$$

$$= -\frac{\varepsilon}{2} - \sqrt{V^2+\frac{\varepsilon^2}{4}}$$: exact wf.

$\frac{\varepsilon}{V} < 0$ のとき

$$\frac{\varepsilon}{2}\alpha^2 + 2V\alpha - \frac{\varepsilon}{2} = \frac{\varepsilon}{2}\cdot\frac{1}{4V^2}\left(\varepsilon^2 + \varepsilon^2 + 4V^2 + 2\varepsilon\sqrt{\varepsilon^2+4V^2}\right)$$

$$+ \varepsilon + \sqrt{\varepsilon^2+4V^2} - \frac{\varepsilon}{2}$$

$$= \frac{\varepsilon^3}{4V^2} + \frac{\varepsilon^2}{4V^2}\sqrt{\varepsilon^2+4V^2} + \varepsilon + \sqrt{\varepsilon^2+4V^2}$$

$$= \varepsilon\left(1+\frac{\varepsilon^2}{4V^2}\right) + \sqrt{\varepsilon^2+4V^2}\left(1+\frac{\varepsilon^2}{4V^2}\right)$$

$$= \left(\varepsilon + \sqrt{\varepsilon^2+4V^2}\right)\left(1+\frac{\varepsilon^2}{4V^2}\right)$$

$$1+\alpha^2 = 1 + \frac{1}{4V^2}\left(\varepsilon^2 + \varepsilon^2 + 4V^2 + 2\varepsilon\sqrt{\varepsilon^2+4V^2}\right)$$

$$= 2\left(1+\frac{\varepsilon^2}{4V^2}\right) + \frac{\varepsilon}{2V^2}\sqrt{\varepsilon^2+4V^2}$$

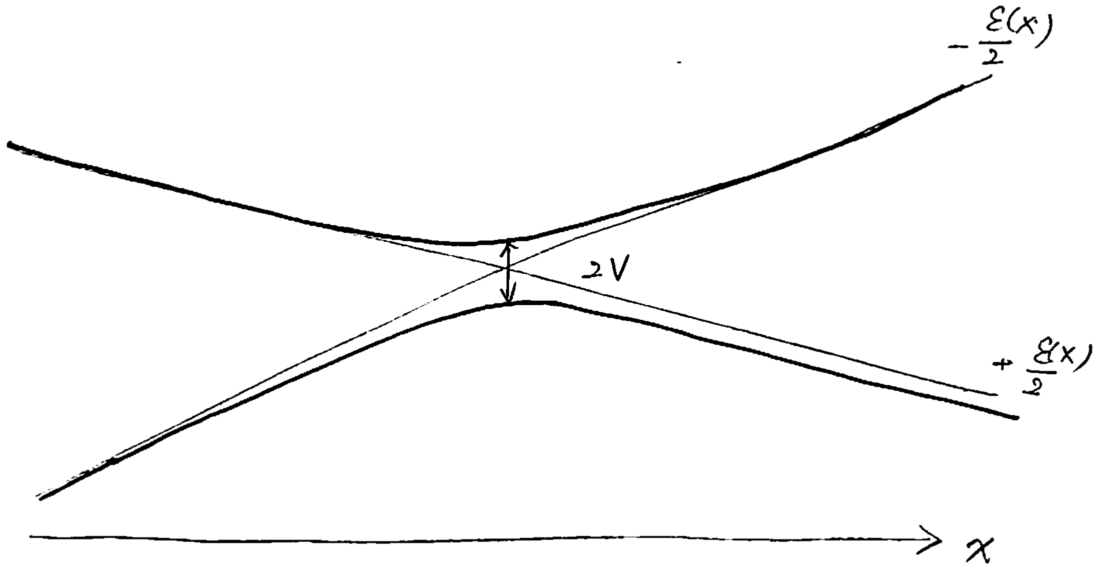
$$= \frac{1}{V}\sqrt{1+\frac{\varepsilon^2}{4V^2}}\left(\sqrt{\varepsilon^2+4V^2} + \varepsilon\right)$$

$\Downarrow$

$$f(\alpha) = V\cdot\sqrt{1+\frac{\varepsilon^2}{4V^2}} = E_{exact}$$

∘ avoided crossing

$$\begin{pmatrix} -\frac{\varepsilon}{2} & V \\ V & \frac{\varepsilon}{2} \end{pmatrix} \longrightarrow \lambda_{\pm} = \pm\sqrt{\frac{\varepsilon^2}{4}+V^2}$$



"avoided crossing"
"level repulsion"

cf. Landau - Zener 公式

$$P = \exp\left[-\frac{2\pi V^2}{\hbar\,|\dot{x}|\,\left|\frac{d}{dx}\left(\varepsilon_1(x)-\varepsilon_2(x)\right)\right|}\right]$$

4.6.2 時間に依存するハミルトニアン

$$H = \begin{pmatrix} -\frac{\varepsilon}{2} & Ve^{i\omega t} \\ Ve^{-i\omega t} & \frac{\varepsilon}{2} \end{pmatrix}$$

$$i\hbar\dot{\phi} = H\phi$$

$$\phi(t) = e^{-i\left(-\frac{\varepsilon}{2}\right)t/\hbar}\, c_0(t)\begin{pmatrix}1\\0\end{pmatrix} + e^{-\frac{i\varepsilon}{2}t/\hbar}\, c_1(t)\begin{pmatrix}0\\1\end{pmatrix}$$

$\downarrow$

$$i\hbar\dot{\phi} = -\frac{\varepsilon}{2}\, e^{\frac{i\varepsilon}{2}t/\hbar}\, c_0\begin{pmatrix}1\\0\end{pmatrix} + i\hbar\, e^{\frac{i\varepsilon t}{2}/\hbar}\, \dot{c}_0\begin{pmatrix}1\\0\end{pmatrix}$$
$$+\frac{\varepsilon}{2}\, e^{\frac{-i\varepsilon t}{2}/\hbar}\, c_0\begin{pmatrix}0\\1\end{pmatrix} + i\hbar\, e^{-\frac{i\varepsilon t}{2}/\hbar}\, \dot{c}_1\begin{pmatrix}0\\1\end{pmatrix}$$

$$H\phi = e^{\frac{i\varepsilon t}{2}/\hbar}\, c_0\begin{pmatrix}-\frac{\varepsilon}{2}\\ Ve^{-i\omega t}\end{pmatrix} + e^{-\frac{i\varepsilon t}{2}/\hbar}\, c_1\begin{pmatrix}Ve^{i\omega t}\\ \frac{\varepsilon}{2}\end{pmatrix}$$

$$\downarrow \begin{cases} i\hbar\dot{c}_0 = V e^{i\omega t}\, e^{-i\varepsilon t/\hbar}\, c_1 \\ i\hbar\dot{c}_1 = V e^{-i\omega t}\, e^{i\varepsilon t/\hbar}\, c_0 \end{cases}$$

$$c_0(0) = 1$$
$$c_1(0) = 0$$

$$\Downarrow$$

$$i\hbar\ddot{c}_1 = i\frac{\varepsilon-\hbar\omega}{\hbar}\underbrace{e^{i(\varepsilon-\hbar\omega)t/\hbar}Vc_0}_{i\hbar\dot{c}_1} + Ve^{i(\varepsilon-\hbar\omega)t/\hbar}\underbrace{\dot{c}_0}_{\frac{1}{i\hbar}Ve^{i\omega t}e^{-i\varepsilon t/\hbar}c_1}$$

$$= -(\varepsilon-\hbar\omega)\dot{c}_1 + \frac{V^2}{i\hbar}c_1$$

$$\boxed{\ddot{c}_1 = \frac{i}{\hbar}(\varepsilon-\hbar\omega)\dot{c}_1 - \frac{V^2}{\hbar^2}c_1}$$

assume

$$c_1(t) = Ae^{\alpha t}$$

$$\downarrow$$

$$\alpha^2 = \frac{i}{\hbar}(\varepsilon-\hbar\omega)\cdot\alpha - \frac{V^2}{\hbar^2}$$

$$\Downarrow \quad \alpha_\pm = \frac{1}{2}\left\{\frac{i}{\hbar}(\varepsilon-\hbar\omega) \pm \sqrt{\frac{-1}{\hbar^2}(\varepsilon-\hbar\omega)^2 - \frac{4V^2}{\hbar^2}}\right\}$$

$$= \frac{i}{2}\left\{\frac{1}{\hbar}(\varepsilon-\hbar\omega) \pm \sqrt{\frac{1}{\hbar^2}(\varepsilon-\hbar\omega)^2 + \frac{4V^2}{\hbar^2}}\right\}$$

$$\equiv i(\beta \pm \gamma)$$

$c_1(0) = 0$

$$c_1(t) = A\left\{e^{i(\beta+\gamma)t} - e^{i(\beta-\gamma)t}\right\}$$

$$= \underline{A}\,e^{i\beta t}\cdot 2i\sin\gamma t.$$

(note)
$$C_0(t) = i\hbar\, \dot{C}_1 \cdot \frac{1}{V}\, e^{i\omega t} e^{-i\varepsilon t/\hbar}$$
$$= \frac{i\hbar}{V} e^{i\omega t} e^{-i\varepsilon t/\hbar} \cdot 2iA \left\{ i\beta\, e^{i\beta t} \sin\gamma t + \gamma e^{i\beta t} \cos\gamma t \right\}$$

$\Downarrow$
$$1 = C_0(0) = \frac{i\hbar}{V} \cdot 2iA \cdot \gamma$$

$\Downarrow$
$$A = -\frac{V}{2\hbar} \cdot \frac{1}{\gamma} = -\frac{V}{2\hbar} \cdot \frac{1}{\frac{1}{2}\sqrt{\frac{1}{\hbar^2}(\varepsilon - \hbar\omega)^2 + \frac{4V^2}{\hbar^2}}}$$
$$= -\frac{V}{\sqrt{(\varepsilon - \hbar\omega)^2 + 4V^2}}$$

遷移確率:

$$P_1(t) = |C_1(t)|^2 = 4A^2 \sin^2 \gamma t$$
$$= \frac{4V^2}{(\varepsilon - \hbar\omega)^2 + 4V^2} \sin^2 \left\{ \frac{1}{2}\sqrt{\left(\frac{\varepsilon - \hbar\omega}{\hbar}\right)^2 + \frac{4V^2}{\hbar^2}}\; t \right\}$$

$$P_0(t) = 1 - P_1(t)$$

(Rabi の公式)

◦ 摂動論で解く

$$\phi(t) = e^{i\frac{\varepsilon t}{2}/\hbar} c_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-i\frac{\varepsilon t}{2}/\hbar} c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$c_m^{(0)}(t) = \delta_{m,n}$$

$$c_m^{(1)}(t) = \frac{1}{i\hbar}\int_0^t e^{i\varepsilon_{mn}t'/\hbar} V_{mn}(t')dt'$$

$\downarrow$

$$c_1(t) \sim \frac{1}{i\hbar}\int_0^t e^{i\varepsilon t'/\hbar} V e^{-i\omega t'} dt'$$

$$= \frac{V}{i\hbar} \cdot \frac{\hbar}{i(\varepsilon - \hbar\omega)} \left(e^{i(\varepsilon-\hbar\omega)t/\hbar} - 1 \right)$$

$$= -\frac{V}{\varepsilon - \hbar\omega} e^{i(\varepsilon-\hbar\omega)t/2\hbar} \cdot 2i \sin\left(\frac{\varepsilon - \hbar\omega}{2\hbar} t\right)$$

$$P_1(t) \sim \frac{4V^2}{(\varepsilon - \hbar\omega)^2} \sin^2\left(\frac{\varepsilon - \hbar\omega}{2\hbar} t\right)$$

・ $\varepsilon \neq \hbar\omega$ のとき

$$\frac{4V^2}{(\varepsilon-\hbar\omega)^2+4V^2} \sim \frac{4V^2}{(\varepsilon-\hbar\omega)^2}$$

$$\sqrt{\left(\frac{\varepsilon-\hbar\omega}{\hbar}\right)^2+\frac{4V^2}{\hbar^2}} \sim \frac{\varepsilon-\hbar\omega}{\hbar}$$

$$\Downarrow \quad P_1^{exact}(t) \sim P_1(t)$$

(note) long t のふるまい:

$$\frac{1}{x^2}\sin^2\left(\frac{xt}{2\hbar}\right) \to \frac{\pi t}{2\hbar}\,\delta(x)$$

$$P_1(t) \sim \frac{2\pi}{\hbar}\, t\, V^2\, \delta(\varepsilon-\hbar\omega)$$

・ $\varepsilon = \hbar\omega$ のとき

$$P_1^{(V)}(t) \sim \frac{V^2}{\hbar^2}\, t^2$$

$$P_1^{(exact)}(t) = \sin^2\left(\frac{Vt}{\hbar}\right) \sim \frac{V^2 t^2}{\hbar^2}$$

small t

量子ゼノン効果

$t=0$ に $|0\rangle$ に状態を用意．

⤳ 時間 t の間に N 回観測を行い，
状態が $|0\rangle$ にあるか $|1\rangle$ にあるか確認，

$|0\rangle$ にあれば 時計が リセットされる

↓

$$P_0(t) \sim \left(1-\frac{V^2}{\hbar^2}\left(\frac{t}{N}\right)^2\right)^N$$

$$\sim 1-\frac{V^2}{\hbar^2}\frac{t^2}{N}$$

$N \to \infty \qquad P_0(t) \to 1.$

$\leftrightarrow$ 何回も観測を行なうと遷移が行らない．

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