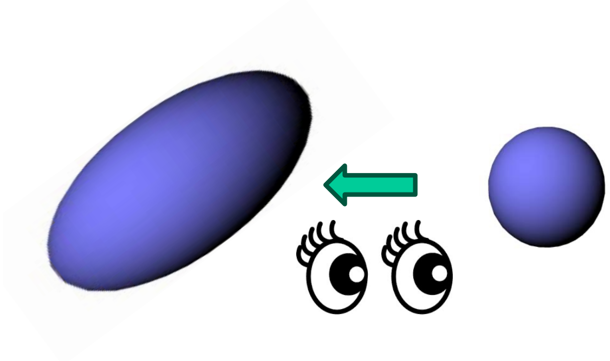
Taking a snapshot of a nucleus
: an intersection between low-energy and relativistic H.I. collisions



Kouichi Hagino

Kyoto University, Kyoto, Japan

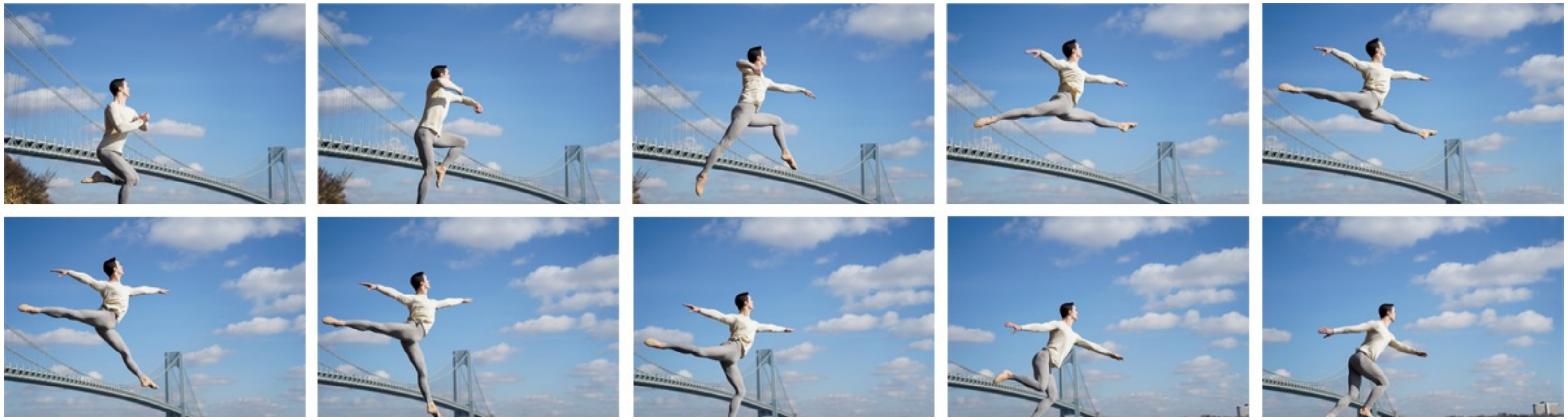
1. Introduction
2. Low-energy Nuclear Reactions: overview
3. Role of deformation in sub-barrier fusion reactions
4. A short comment on relativistic heavy-ion collisions
5. Summary

New Frontiers in Nuclear Physics and Nuclear Astrophysics (NNPA2025), TARLA, Ankara, Turky, 2025.9.1-5

Snapshots

taking snapshots of a "slow" motion with a high-speed camera

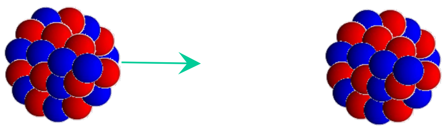
$$\tau_{\mathrm{camera}} \ll \tau_{\mathrm{motion}}$$



https://www.sony.jp/ichigan/products/ILCE-7M3/feature_3.html

(photos with a Sony camera α7III)

⇒ taking snapshots of a nucleus with a "fast" nuclear reaction

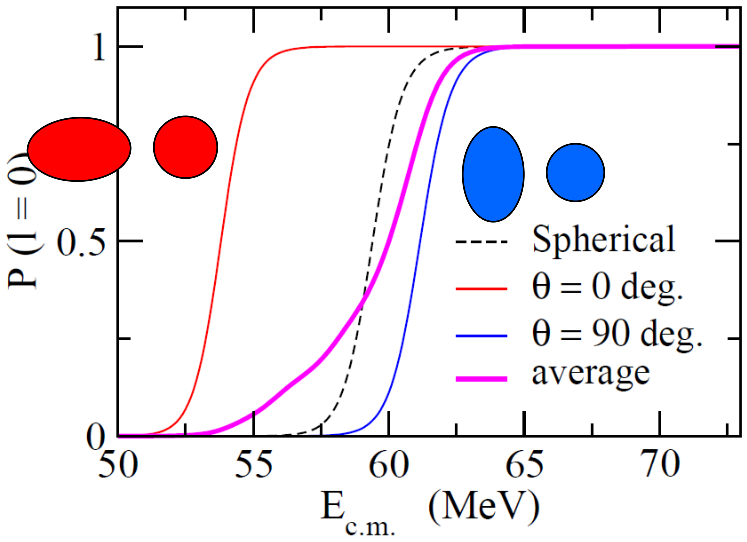


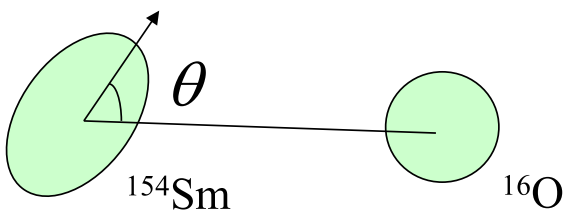
$$\tau_{\mathrm{reaction}} \ll \tau_{\mathrm{nucleus}}$$

Snapshots

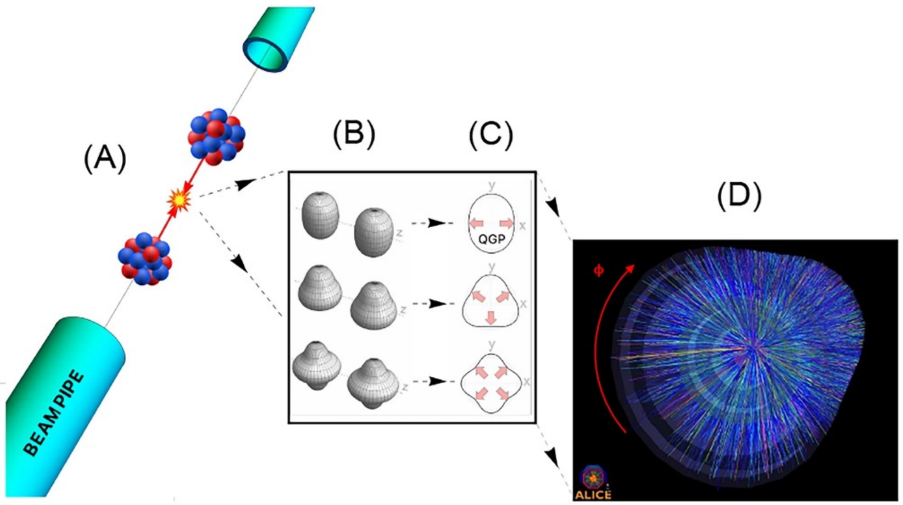
taking a snapshot of a nucleus with a "fast" nuclear reaction

low-energy H.I. fusion reactions of a deformed nucleus





relativistic H.I. collisions with a deformed nucleus

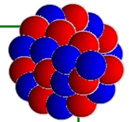


J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

increasing interests in recent years

Introduction: low-energy nuclear reactions

nucleus: a composite system

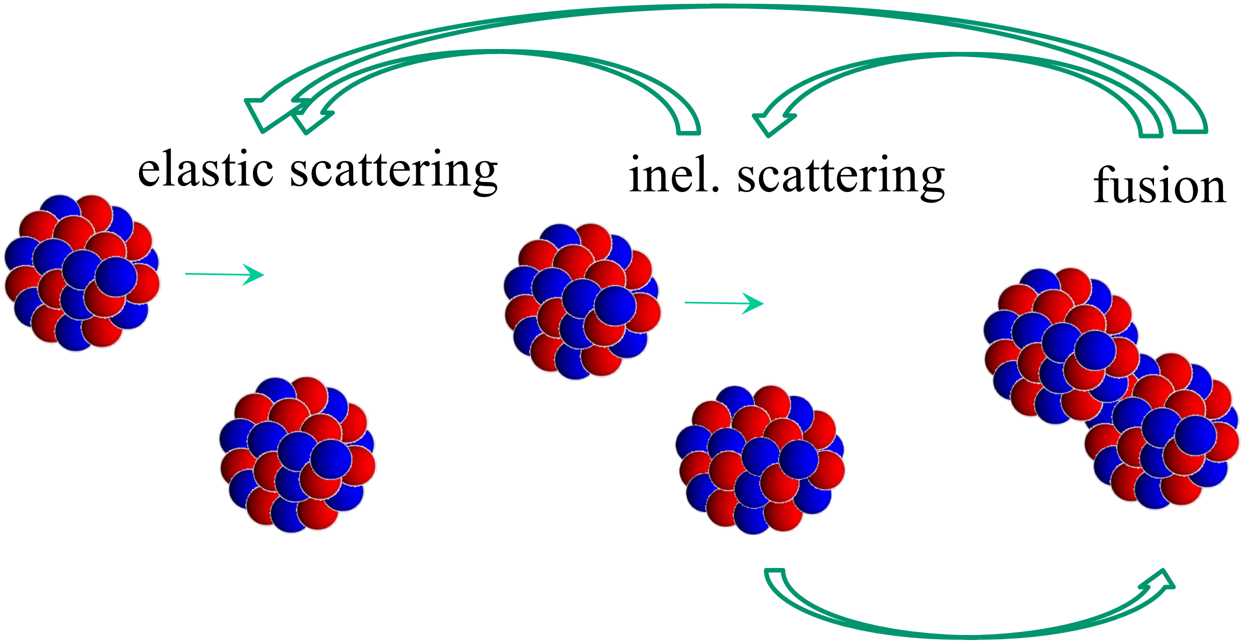


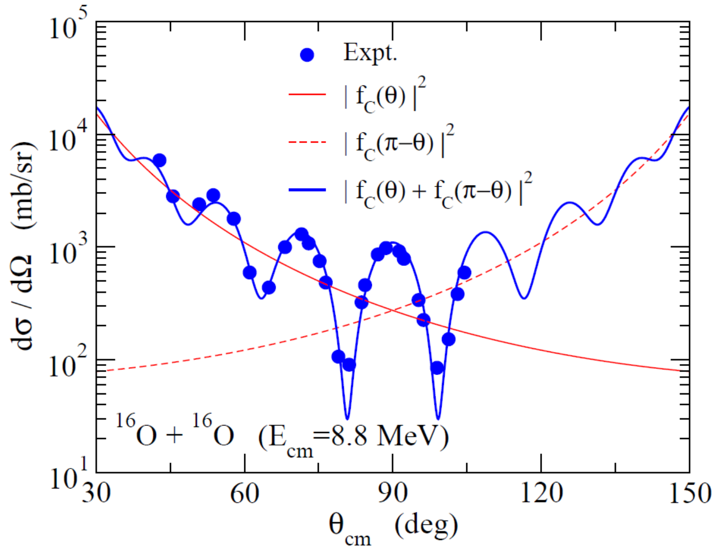
✓ various sort of reactions

✓ an interplay between nuclear structure and reaction

shapes, excitations, …..

- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions





Andrea Vitturi (1949-2024)

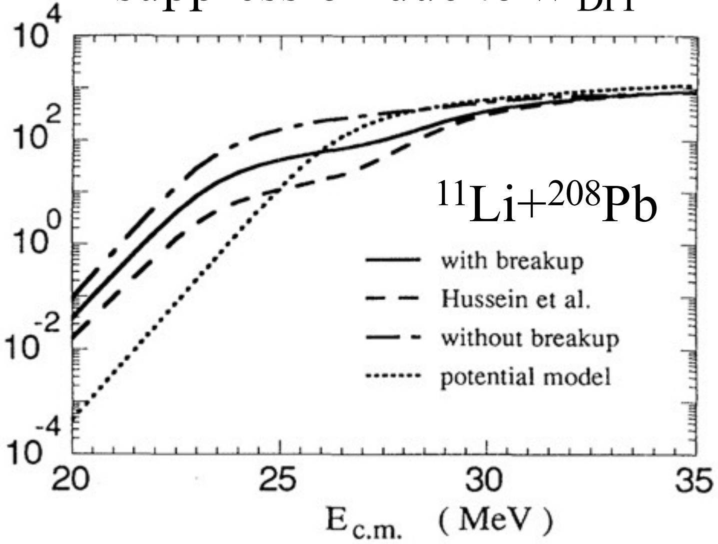
Andrea was a world expert of heavy-ion reactions.

- many papers with Carlos Dasso (Dasso-Vitturi)
- I met Andrea for the first time in summer, 1994.
  - I was a master course student.
  - My supervisor, Noboru Takigawa, took me with him to a month visit to David Brink at Trento.
  - At the time, we traveled to Padova and Catania.

"Inelastic scattering"
S. Landowne and A. Vitturi,
"Treatise of Heavy-Ion Science"
Vol. 1 (1984).

Role of breakup in subbarrier fusion reactions

suppression due to W_{DPP}



M.S. Hussein, M.P. Pato, L.F. Canto, and R. Donangelo, PRC46, 377 (1992).
N. Takigawa, M. Kuratani, and H. Sagawa, PRC47, R2470 (1993).

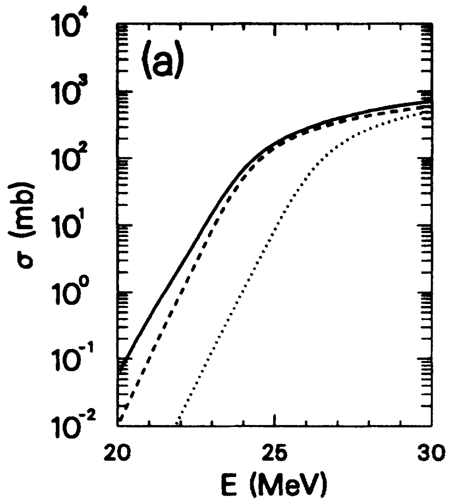
Does the presence of ^{11}Li breakup channels reduce the cross section for fusion processes?

C. H. Dasso[1,2] and A. Vitturi[3]
[1]*The Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark*
[2]*Sektion Physik der Universität München, D-85748, Garching, Germany*
[3]*University of Padova and Istituto Nazionale di Fisica Nucleare, Padova, Italy*

(Received 10 December 1993)

Both V_{DPP} and W_{DPP} should be taken into account

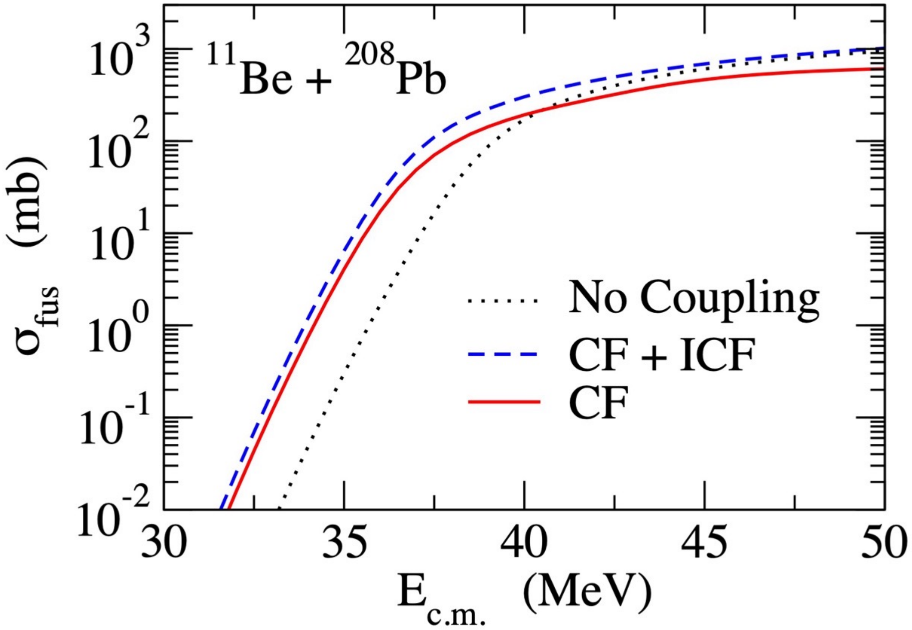
→ enhancement of fusion cross sections
(a schematic 2-channel problem)



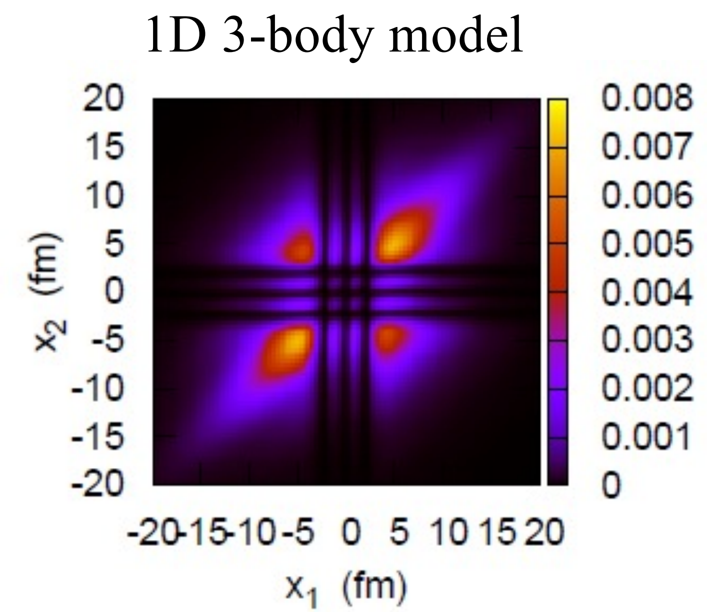
C.H. Dasso and A. Vitturi, Phys. Rev. C50, R12 (1994).

Role of breakup in subbarrier fusion reactions

- ✓ in 1998-2000, I was a post-doc at INT, Seattle.
- ✓ Andrea visited Seattle to attend a program of INT
- ✓ We discussed about fusion of unstable nuclei, and later Andrea invited me to Padova for a month (Lorenzo Fortunato was a student at that time).



K. Hagino, A. Vitturi, C.H. Dasso, and S.M. Lenzi, Phys. Rev. C61, 037602 (2000).



K.Hagino, A. Vitturi, F. Perez-Bernal, and H. Sagawa, J. of Phys. G38 ('11) 015015

NNPA2018 (the 1st NNPA symposium)
May 28-June 1, 2018, Antalya, Turkey

Washiyama

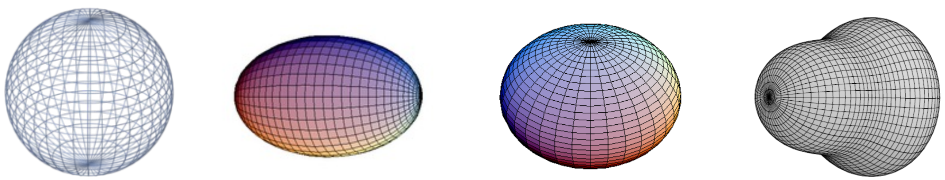
Lucia and Andrea

Sub-barrier fusion reactions and quantum tunneling

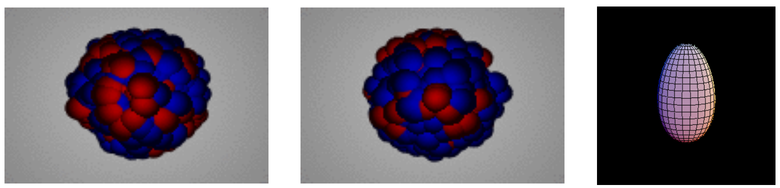
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



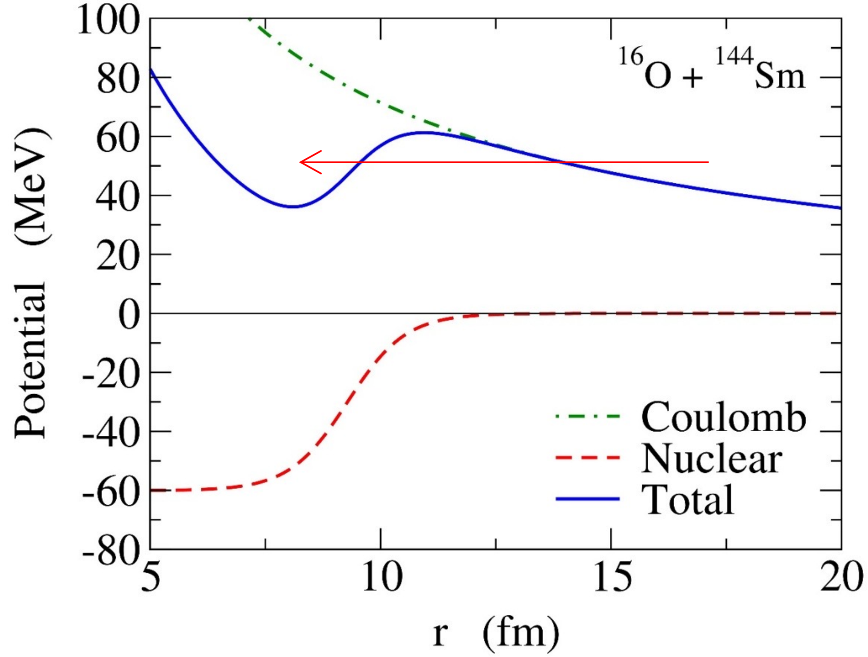
- several surface vibrations



several modes and adiabaticities

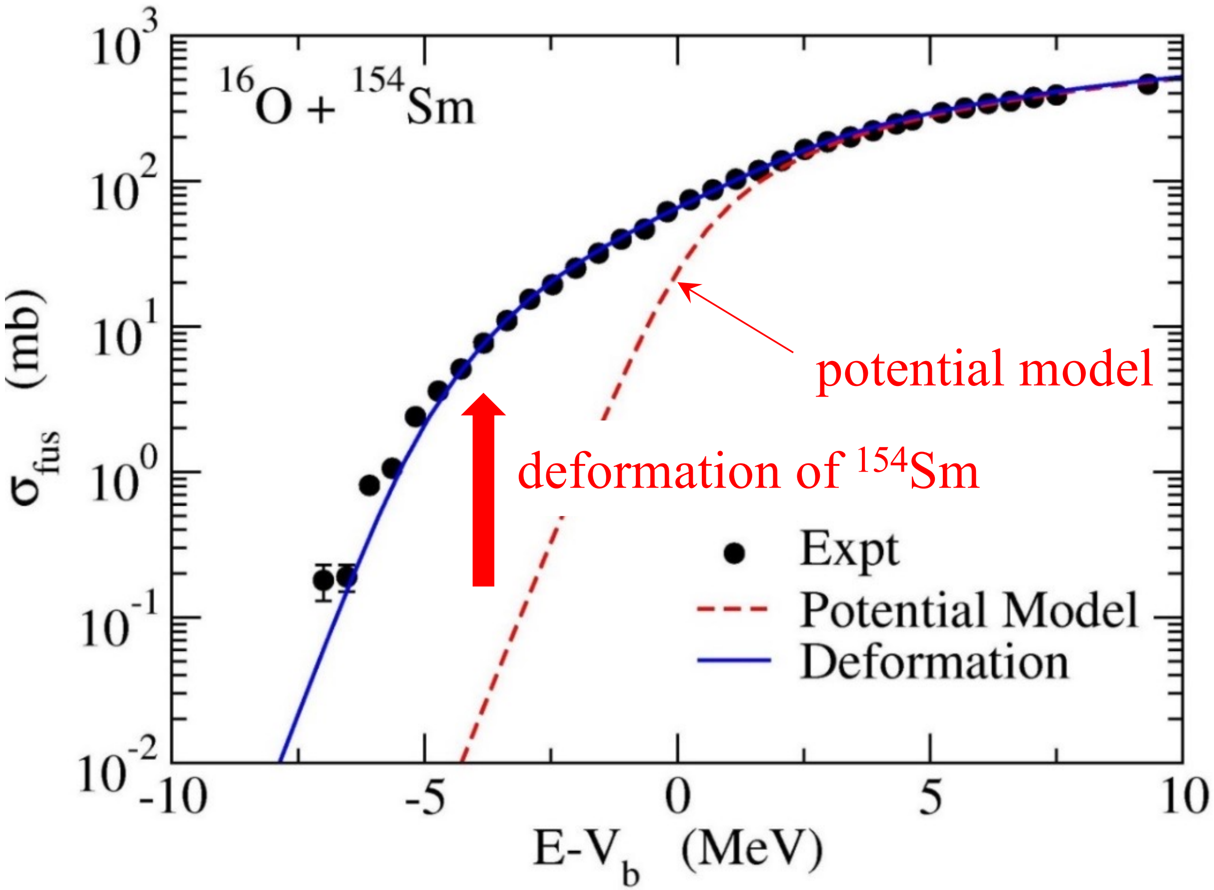
- several types of nucleon transfers

Sub-barrier fusion reactions



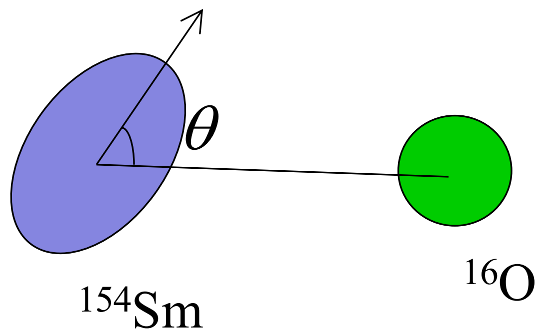
Tunneling probabilities: the exponential E dependence
→ nuclear structure effects are amplified

Sub-barrier fusion reactions and quantum tunneling



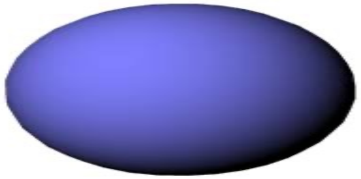
the orientation-average formula

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$



K. H. and N. Takigawa, Prog. Theo. Phys.128 ('12)1061.

Effects of nuclear deformation on fusion



^{154}Sm

(MeV)

0.903	8^+
0.544	6^+
0.267	4^+
0.082	2^+
0	0^+

^{154}Sm

rotational spectrum

a small rotational energy

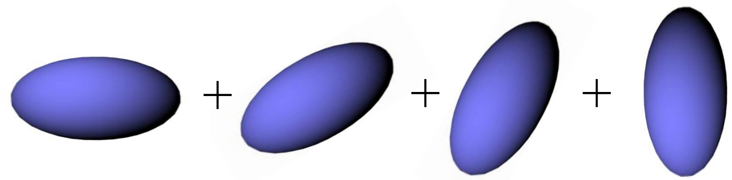
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→ a large moment of inertia J

→ rotation: a slow deg. of freedom

$$E_{\rm rot} \sim E_{2^+} = 82 \text{ keV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\Psi_{0^+} =$



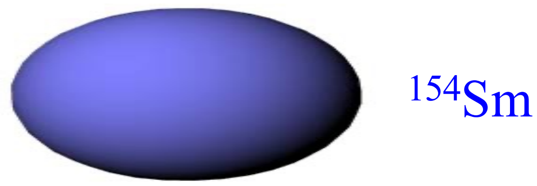
→ a spherical state in the lab. system

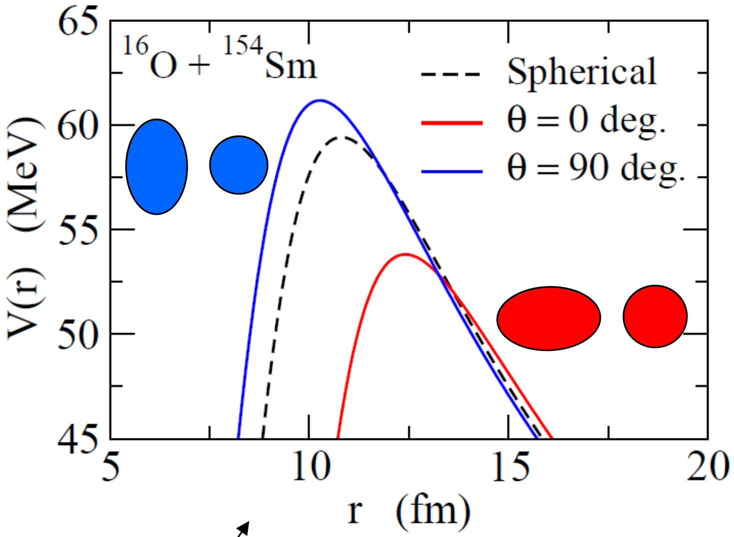
fix the orientation angle to calculate
the fusion probability

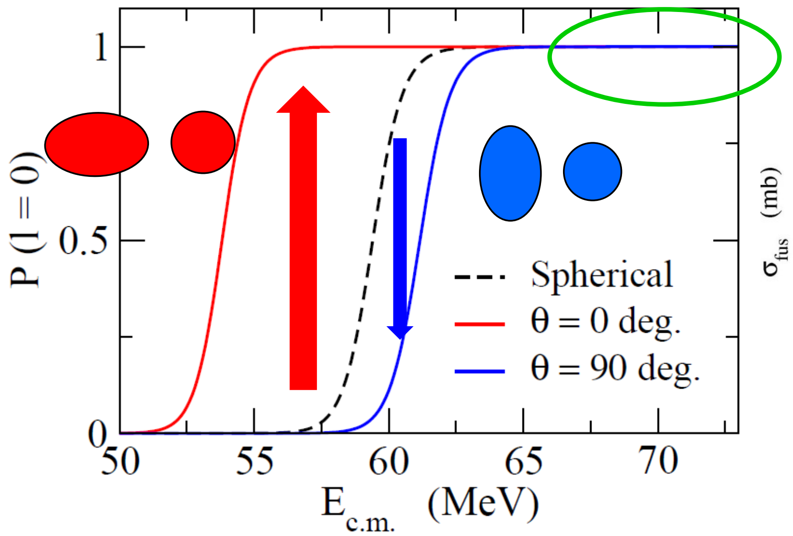
"a snapshot of a rotating nucleus"

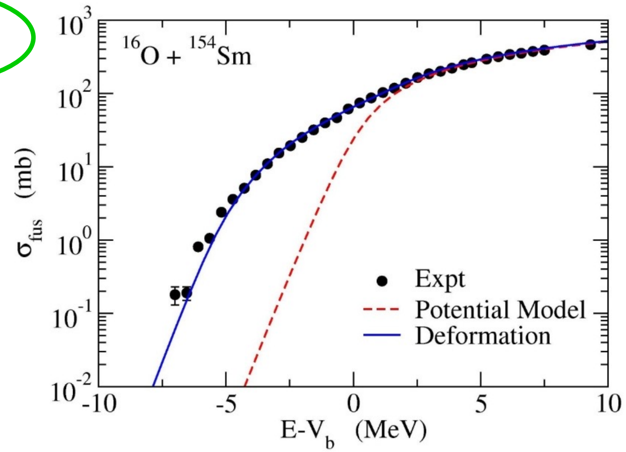
Effects of nuclear deformation on fusion

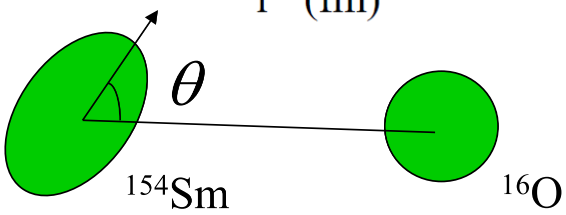
^{154}Sm : a typical deformed nucleus











$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\mathrm{fus}}(E;\theta)$$

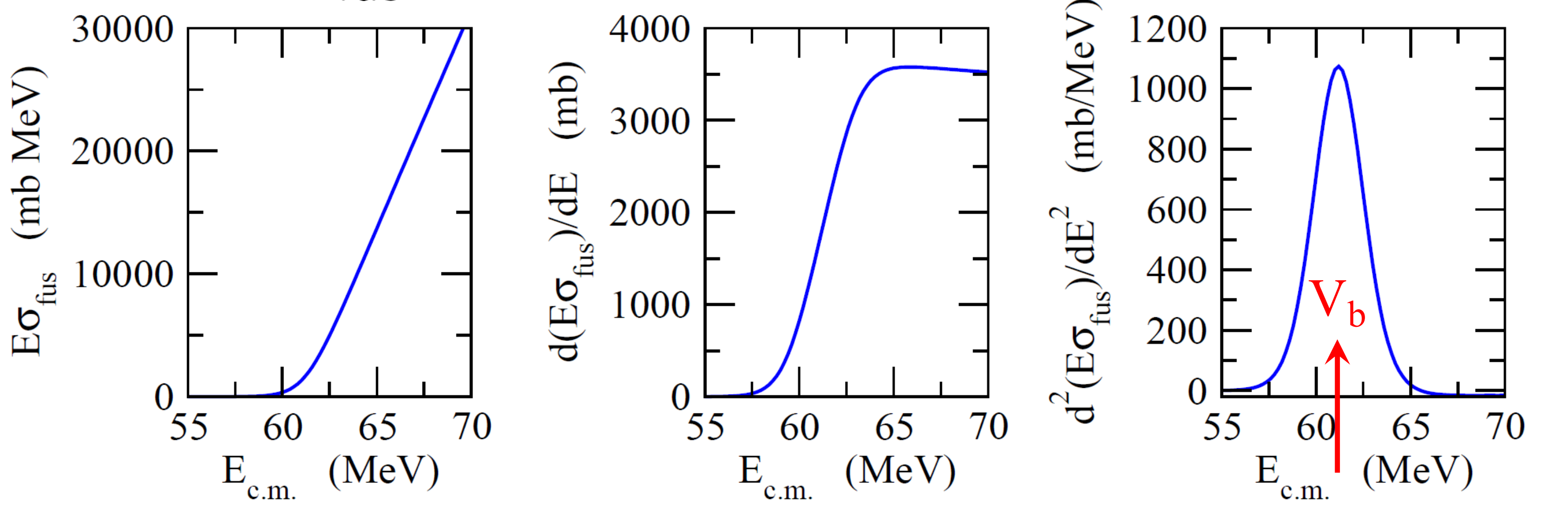
Fusion: strong interplay between nuclear structure and reaction

Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

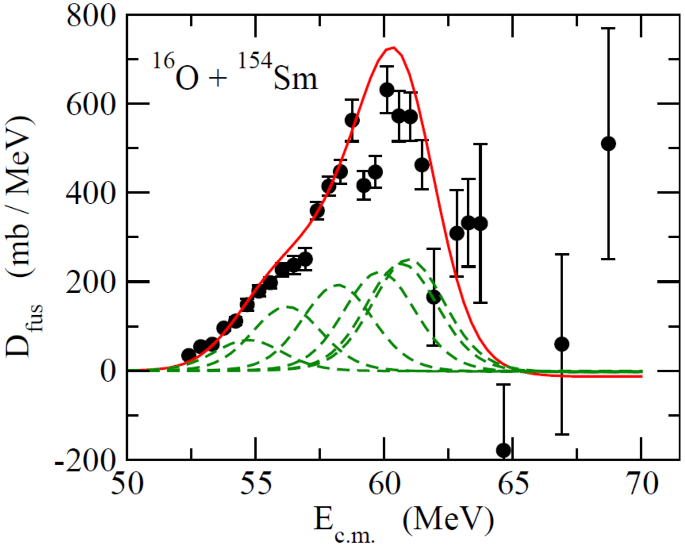
N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

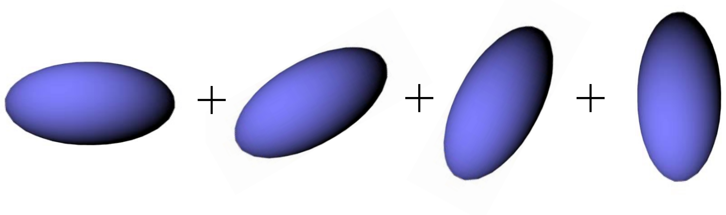
$E\sigma_{\text{fus}}$ $\quad$ $\frac{d(E\sigma_{\text{fus}})}{dE} \sim \pi R_b^2 P_{l=0}(E)$ $\quad$ $\frac{d^2(E\sigma_{\text{fus}})}{dE^2} \sim \pi R_b^2 \cdot dP_{l=0}/dE$

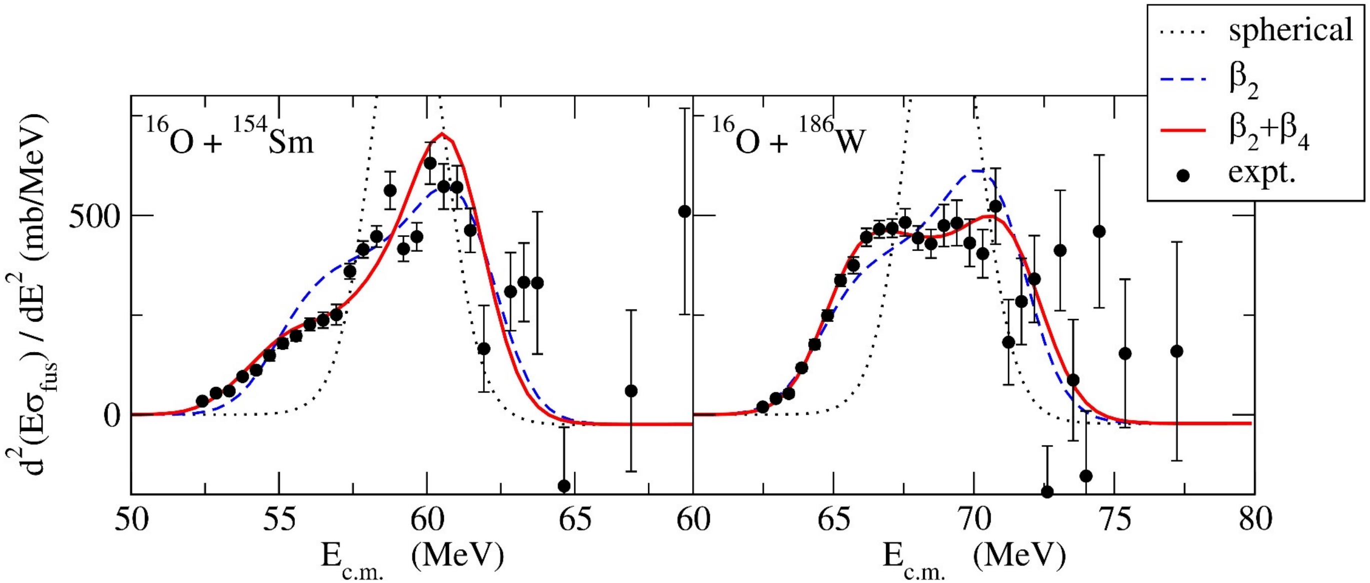


K.H. and N. Takigawa, PTP128 ('12) 1061

Fusion barrier distribution



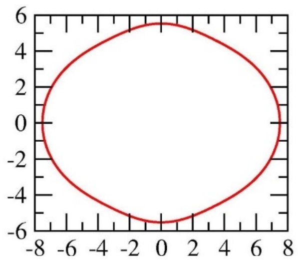


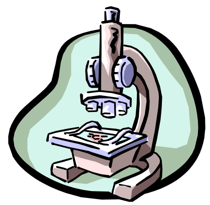


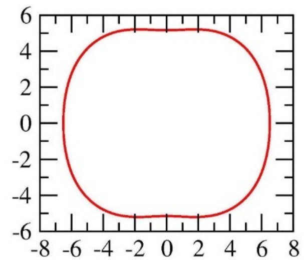
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$\beta_2 = 0.33$ $\beta_2 = 0.29$

$\beta_4 = +0.05$ $\beta_4 = -0.03$







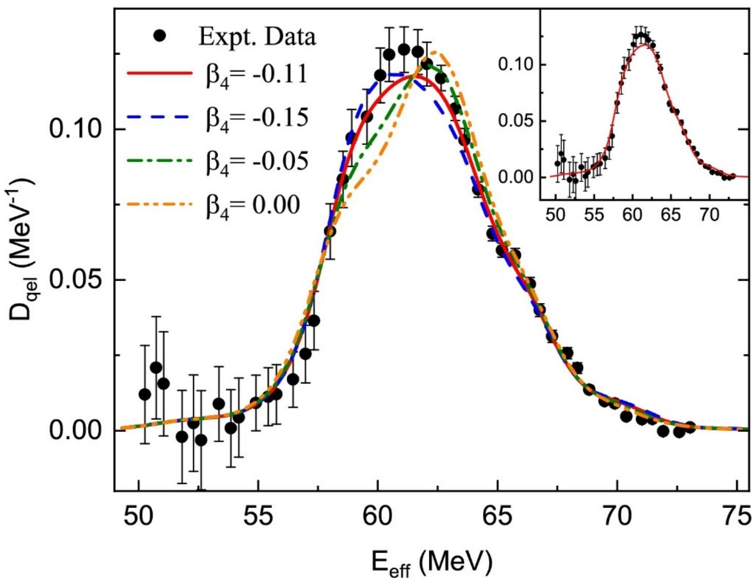
sensitive to the sign of β_4!

Fusion as a quantum tunneling microscope for nuclei

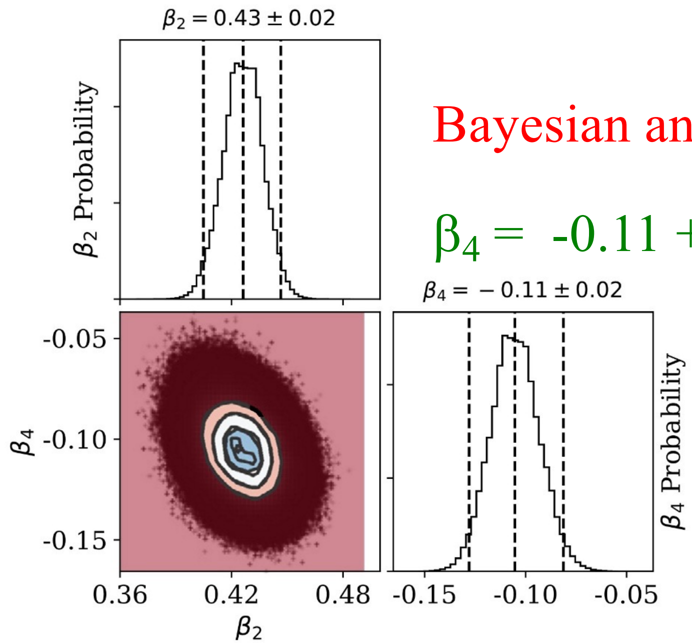
Determination of β_4 of ^{24}Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

^{24}Mg + ^{90}Zr



$\beta_2 = 0.43$ +/- 0.02



Bayesian analysis

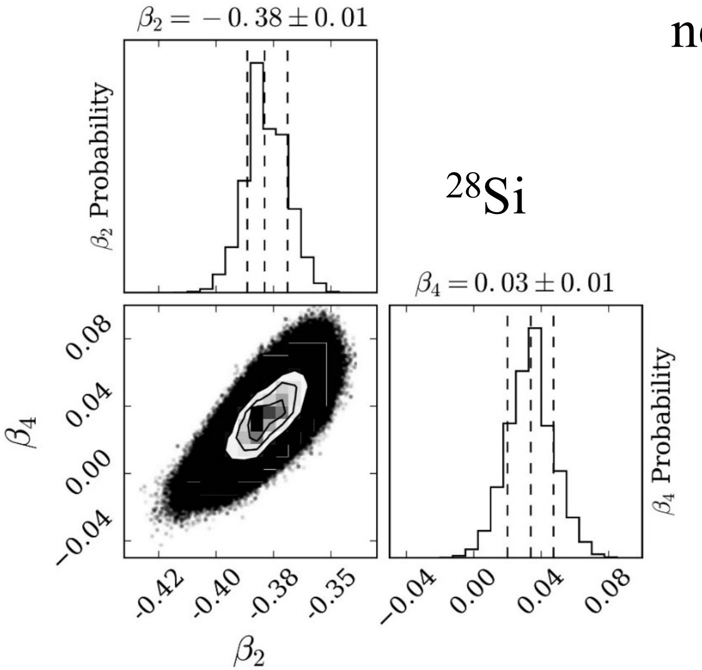
$\beta_4 = -0.11$ +/- 0.02

high precision determination of β_4
→ for the first time

cf. (p,p'): $\beta_4 = -0.05$ +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

Emulator for multi-channel scattering



^{28}Si

needs to repeat many calculations with different (β_2,β_4)

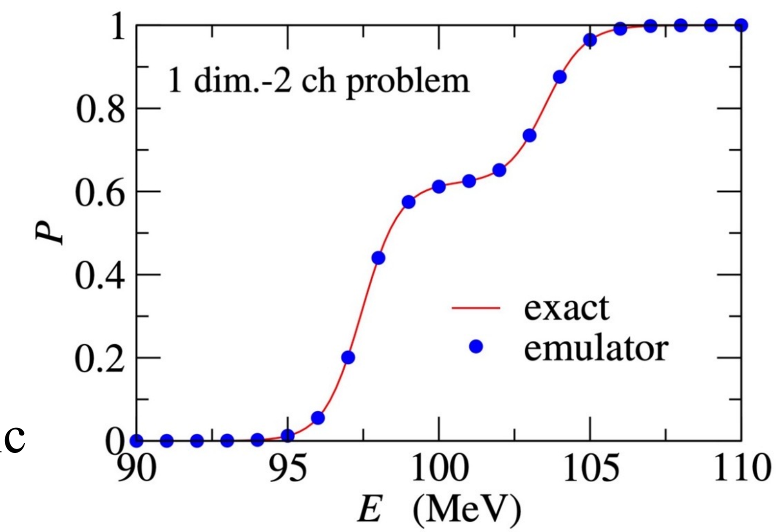
→ an emulator to speed-up the calculations based on the eigenvector continuation

$$\Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i)$$

K. Hagino, Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, Phys. Rev. C112, 024618 (2025).

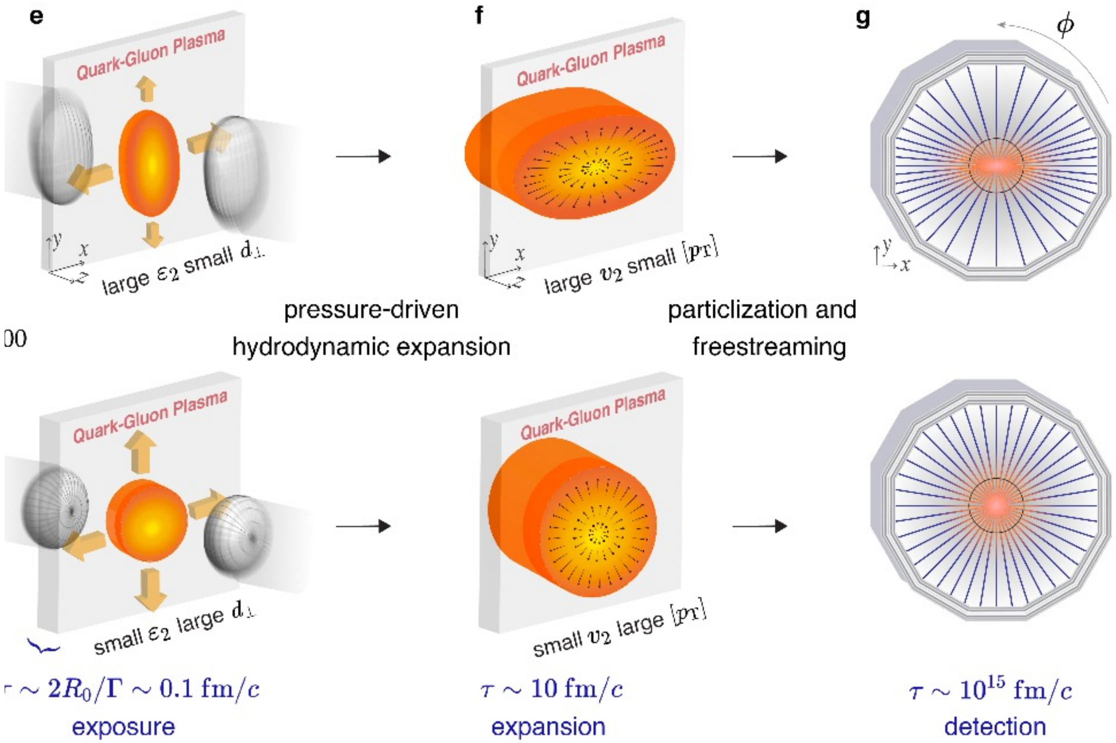
Y.K. Gupta, V.B. Katariya, G.K. Prajapati, K.Hagino et al., PLB845, 138120 (2023).

in future:
applications to realistic systems



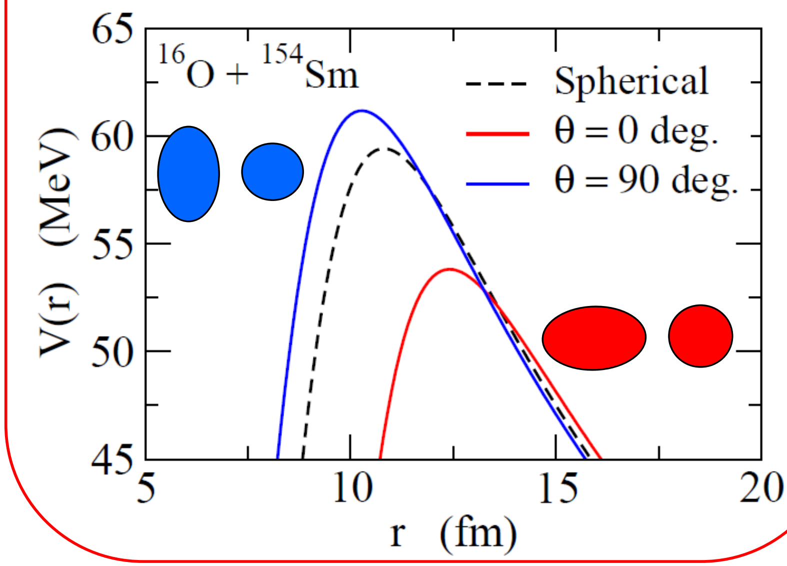
Probing nuclear shapes in Relativistic Heavy-Ion collisions

the initial shape of QGP **expansion** **detection**



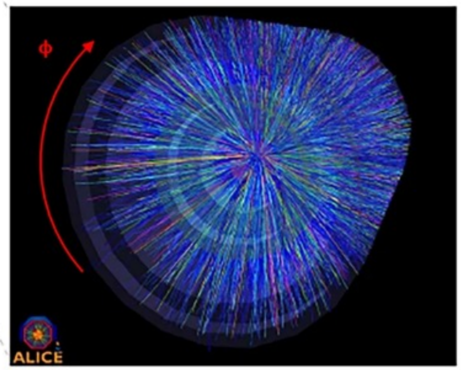
M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

large similarities to
low-energy H.I. fusion reactions
of deformed nuclei



→ am intersection of
High *E* and **Low *E*** HI collisions

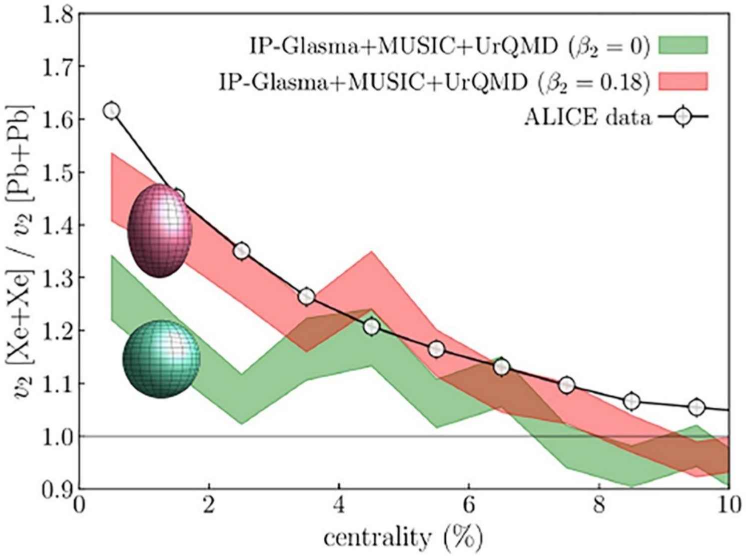
Probing nuclear shapes in Rel. H.I. collisions



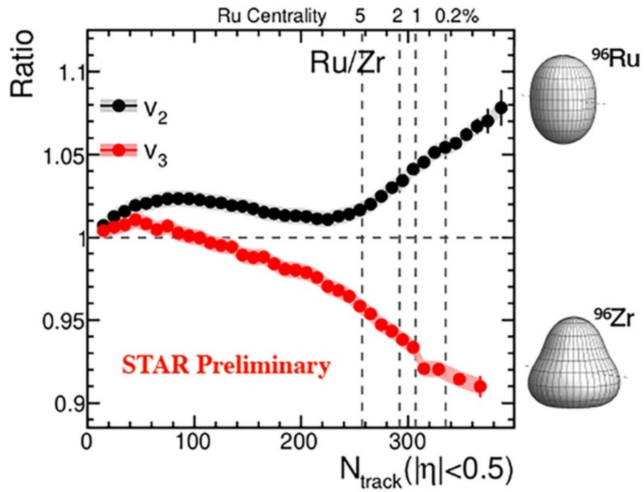
flow:
the final N-distribution

$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos n(\phi - \Psi_n)\right]$$

elliptic flow ν_2



the ratio of ^{129}Xe+^{129}Xe to ^{208}Pb+^{208}Pb
→ quadrupole deformation of ^{129}Xe



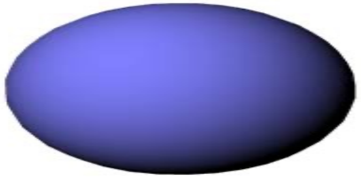
the ratio of ^{96}Ru+^{96}Ru to ^{96}Zr+^{96}Zr
→ octupole deformation of ^{96}Zr

other examples:
- ✓ γ deformation
- ✓ α cluster

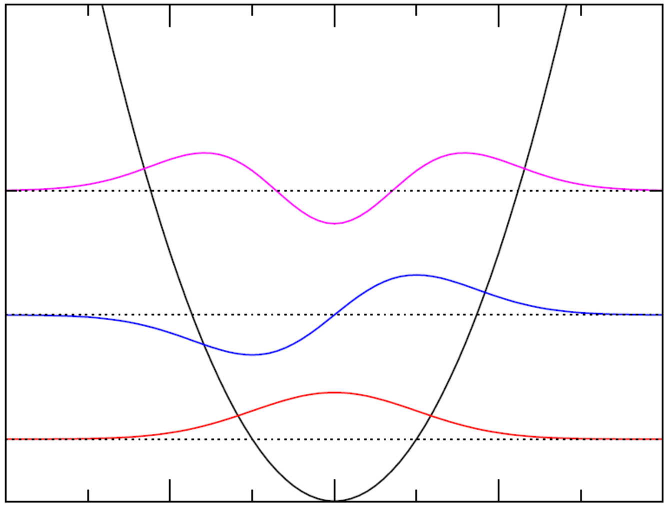
J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

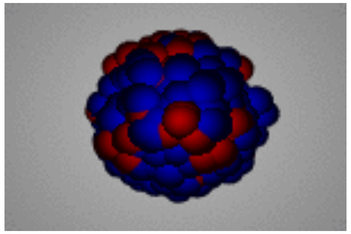
Probing nuclear shapes in Relativistic Heavy-Ion collisions

So far, the focus has been mainly on a static deformation of a deformed nucleus



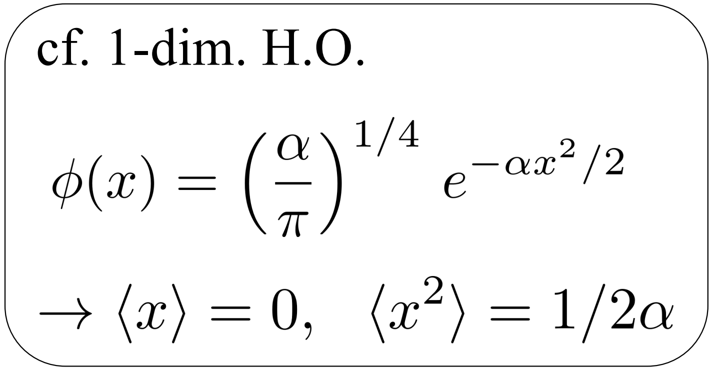
There also exist several dynamical deformations of a spherical nucleus





$\langle \beta \rangle = 0$

but fluctuates around β=0
(zero-point motion)

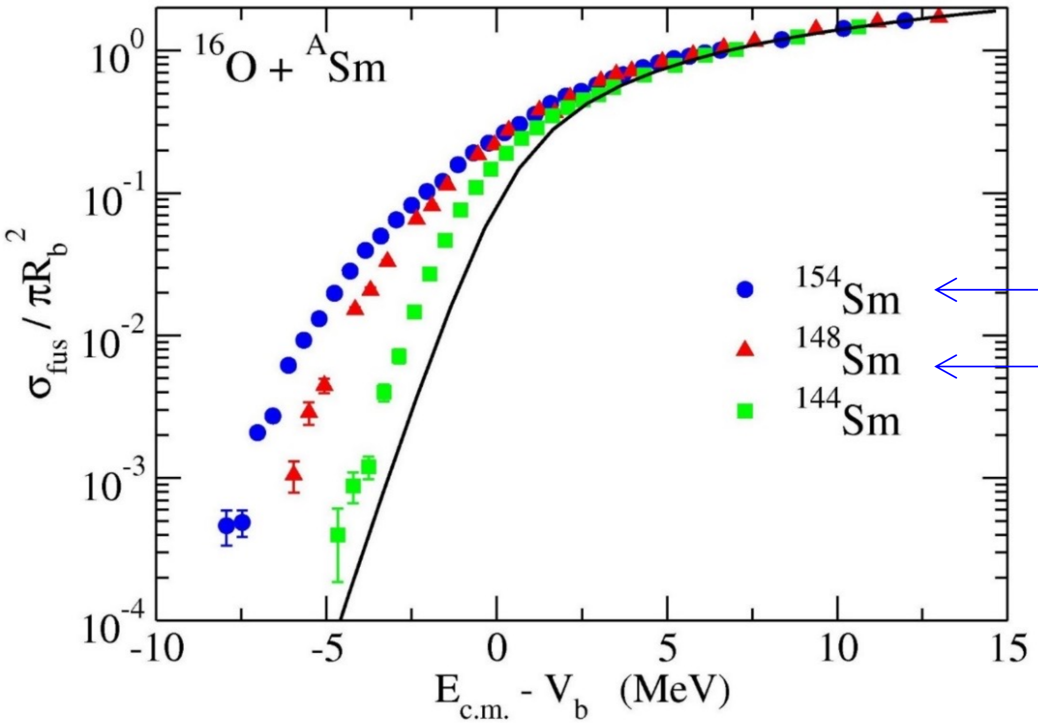


cf. 1-dim. H.O.

$$\phi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

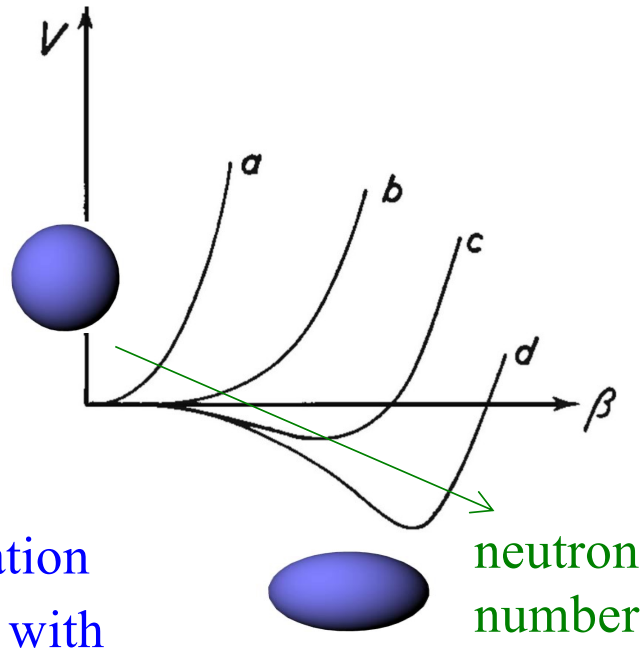
$$\rightarrow \langle x \rangle = 0, \quad \langle x^2 \rangle = 1/2\alpha$$

Surface vibrations of a spherical nucleus can still significantly affect H.I. sub-barrier fusion reactions



static deformation

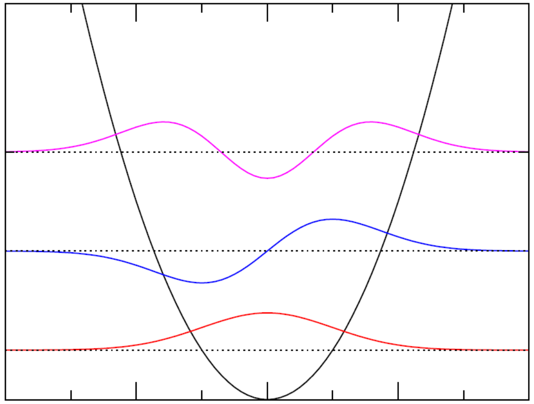
spherical, but with vibration

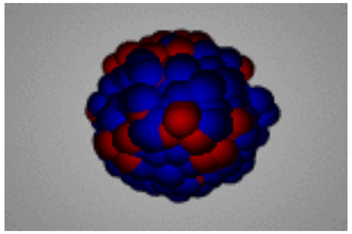


neutron number

the situation may be the same in Relativistic HIC as well

Probing nuclear shapes in Rel. H.I. collisions





$\langle \beta \rangle = 0$

but fluctuates around β=0

In most of the cases, the vibrational motion is not slow for fusion:

$$E_{\rm vib} \sim 2 \text{ MeV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

→ but this can be very slow in rel. H.I. collisions!

the adiabatic approximation for vibrations:

H. Esbensen, Nucl. Phys. A352, 147 (1981)

FUSION AND ZERO-POINT MOTIONS

H. ESBENSEN

Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 14 July 1980

$$\sigma_{\rm fus}(E) \sim \int d\beta \, w(\beta) \sigma_0(E;\beta)$$

$$w(\beta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\beta^2/2\sigma^2}$$

$$\sqrt{\langle \beta_\lambda^2 \rangle} = \frac{4\pi}{3ZR^\lambda}\sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

Probing nuclear shapes in Rel. H.I. collisions

Very recent preprint: D. Xu et al., arXiv: 2504.19644

A "breathing" octupole ^{208}Pb nucleus: resolving the elliptical-to-triangular azimuthal anisotropy puzzle in ultracentral relativistic heavy ion collisions

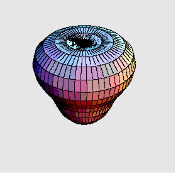
Duoduo Xu,[1] Shujun Zhao,[1] Hao-jie Xu,[2,3,*] Wenbin Zhao,[4,5,†] Huichao Song,[1,6,7,‡] and Fuqiang Wang[8,§]

octupole vibration of ^{208}Pb

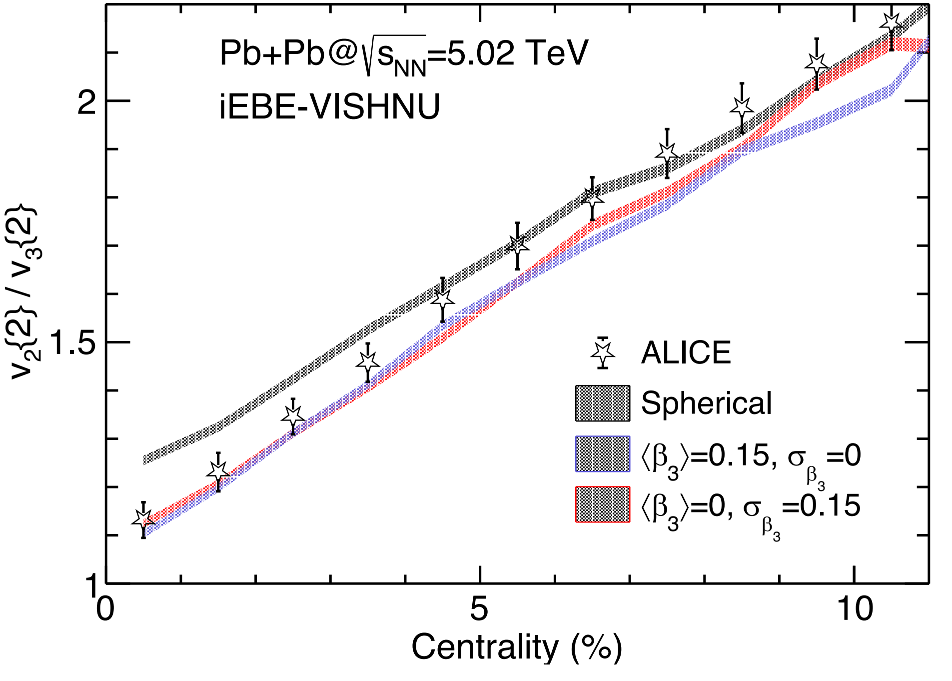
2.6 MeV — 3^-

E3

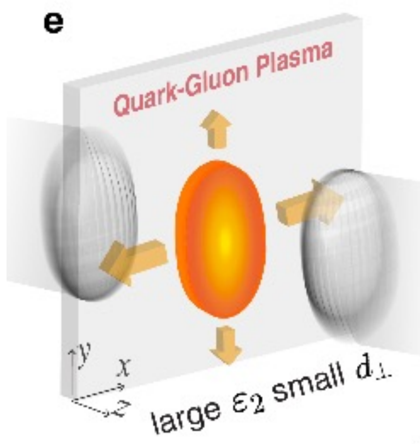
0^+



https://web-docs.gsi.de/~wolle /TELEKOLLEG /KERN/index-s.html



Probing nuclear shapes in Relativistic H.I. collisions



eccentricity parameter: deformation parameter of $\rho_z(x,y) \equiv \int_{-\infty}^{\infty} dz\, \rho(\boldsymbol{r})$

$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\int d\boldsymbol{r}\, r_\perp^2\, e^{2i\phi}\rho(\boldsymbol{r},\{\alpha_{2\mu}\})}{\int d\boldsymbol{r}\, r_\perp^2\, \rho(\boldsymbol{r},\{\alpha_{2\mu}\})} = -\frac{\langle (x-iy)^2 \rangle}{\langle x^2+y^2 \rangle}$$

deformed Woods-Saxon density

$$\rho(\boldsymbol{r},\{\alpha_{\lambda\mu}\}) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a}}; \qquad R(\theta,\phi) = R_0\left(1+\sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})\right)$$

surface vibration

$$H = \frac{1}{2}\sum_{\lambda,\mu}\left(B_\lambda|\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda|\alpha_{\lambda\mu}|^2\right)$$

$$\langle|\epsilon_n|^2\rangle \propto \int \left(\prod_{\lambda,\mu} d\alpha_{\lambda\mu}\, e^{-\alpha^2_{\lambda\mu}/2\sigma^2_\lambda}\right)|\epsilon_n(\{\alpha_{\lambda\mu}\})|^2$$

static deformation (axial symmetry)

$$\alpha_{\lambda\mu} = \beta_\lambda D^\lambda_{0\mu}(\Omega)$$

$$\langle|\epsilon_n|^2\rangle = \int \frac{d\Omega}{8\pi^2}\,|\epsilon_n(\Omega)|^2$$

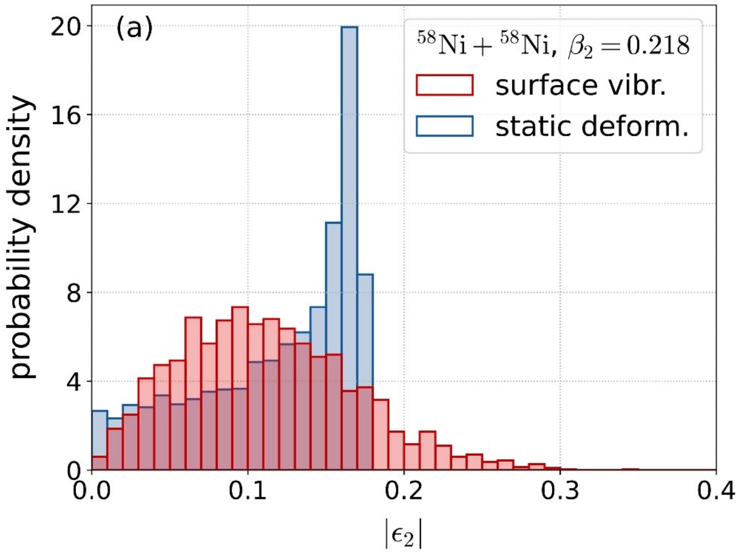
Probing nuclear shapes in Relativistic H.I. collisions

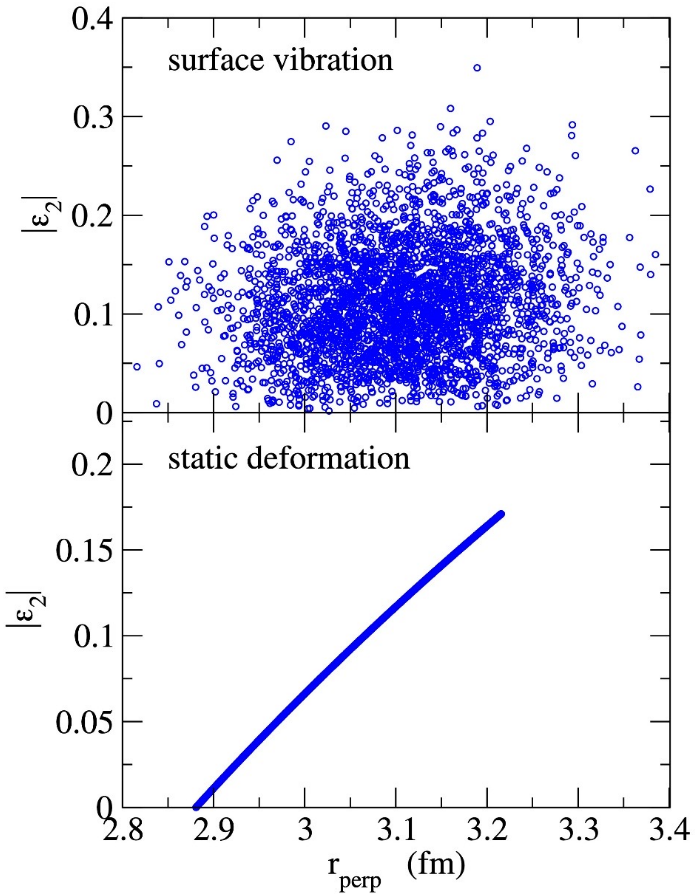
^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

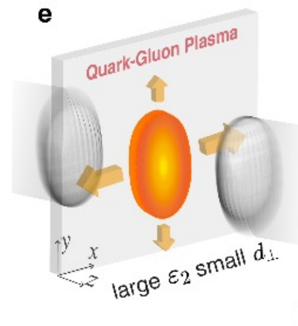
$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\langle (x-iy)^2 \rangle}{\langle x^2+y^2 \rangle}$$

$$\begin{cases} \langle |\epsilon_n|^2 \rangle \propto \int \left(\prod_{\lambda,\mu} d\alpha_{\lambda\mu}\, e^{-\alpha_{\lambda\mu}^2/2\sigma_\lambda^2} \right) |\epsilon_n(\{\alpha_{\lambda\mu}\})|^2 & \text{(vib.)} \\ \langle |\epsilon_n|^2 \rangle = \int \frac{d\Omega}{8\pi^2} |\epsilon_n(\Omega)|^2 & \text{(static def.)} \end{cases}$$

Monte Carlo sampling (3000 samples)







$$\rho_z(x,y) \equiv \int_{-\infty}^{\infty} dz\, \rho(\boldsymbol{r})$$

K. Hagino and M. Kitazawa, arXiv: 2508.05125

Probing nuclear shapes in Relativistic H.I. collisions

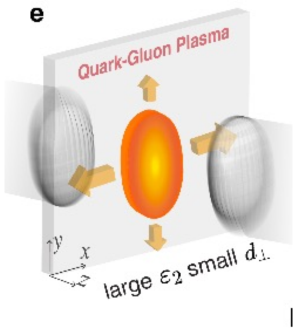
^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

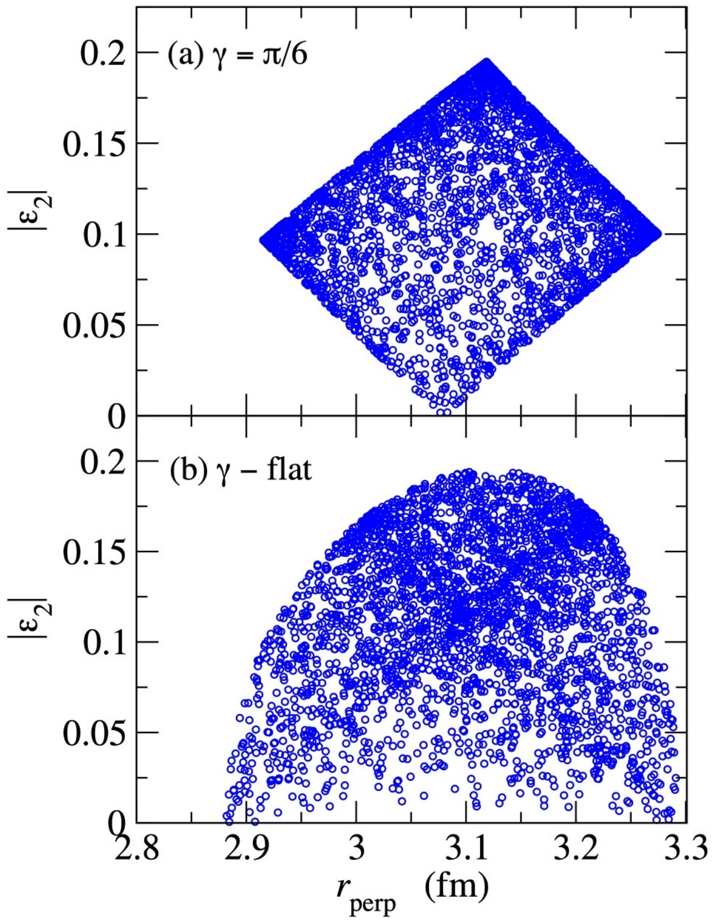
triaxiality

$$\alpha_{2\mu} = D^2_{0\mu}(\Omega)\beta_2 \cos\gamma + \frac{1}{\sqrt{2}}\left(D^2_{2\mu}(\Omega) + D^2_{-2\mu}(\Omega)\right)\beta_2 \sin\gamma$$

$$\langle|\epsilon_n|^2\rangle = \int \frac{d\Omega}{8\pi^2}\,|\epsilon_n(\Omega)|^2$$

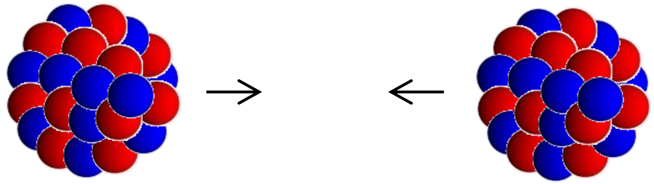
$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\langle(x-iy)^2\rangle}{\langle x^2+y^2\rangle}$$





K. Hagino and M. Kitazawa, arXiv: 2508.05125

Summary

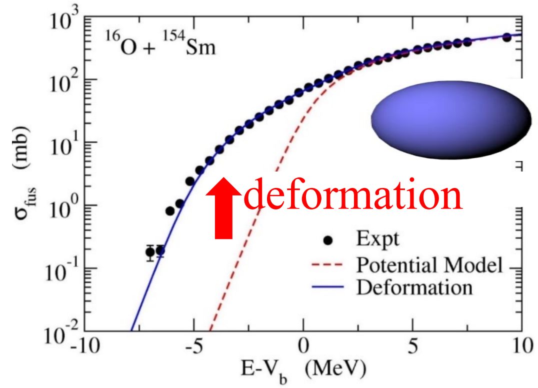


Heavy-ion fusion reactions around the Coulomb barrier

✓Strong interplay between nuclear structure and reaction
✓Quantum tunneling with various intrinsic degrees of freedom
✓Role of deformation in sub-barrier enhancement

→ a snapshot of the rotational motion

↓ amplified

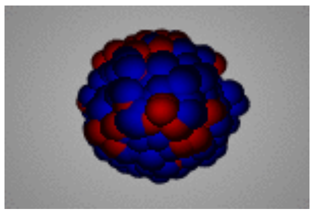


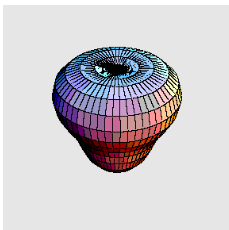
✓ Similarities between low-*E* H.I. fusion and Relativistic H.I. Collisions

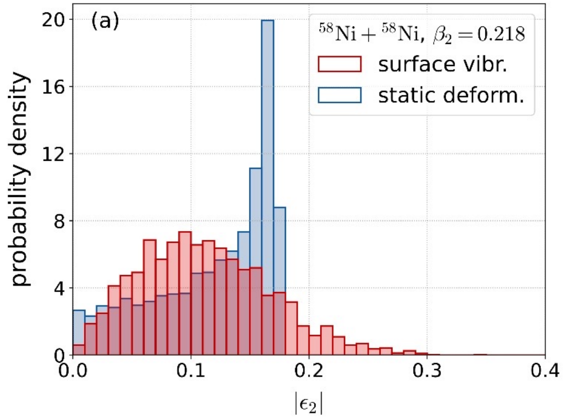
→ a snapshot of a nucleus

A tool to probe nuclear deformations

→ surface vibrations of a spherical nucleus

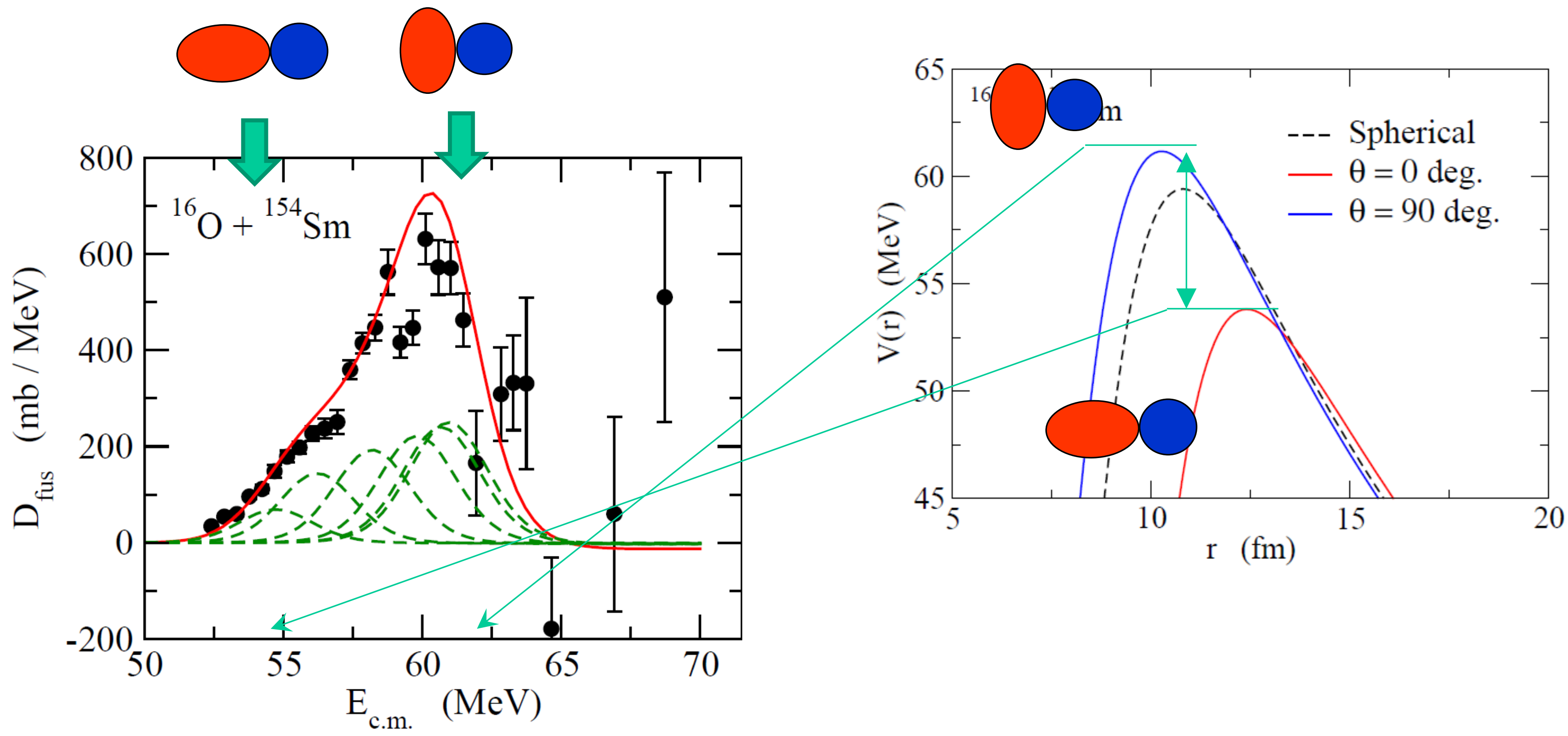




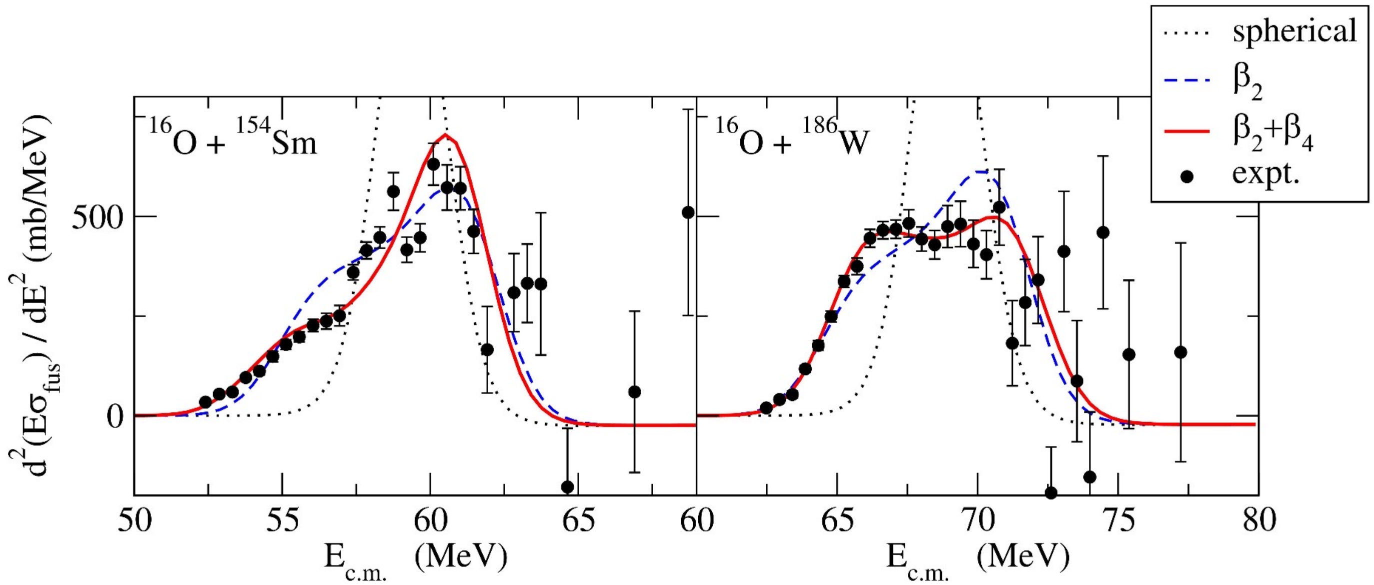


✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91) 25)

$$D_{\mathsf{fus}}(E) = \frac{d^2(E\sigma_{\mathsf{fus}})}{dE^2} \sim \pi R_b^2 \frac{dP_{l=0}}{dE}$$



Data: J.R. Leigh et al., PRC52 (1995) 3151



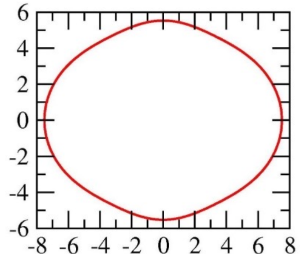
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

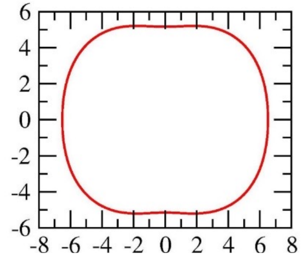
$\beta_2 = 0.33$

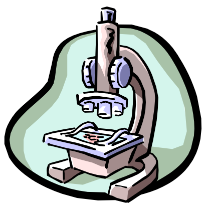
$\beta_4 = +0.05$

$\beta_2 = 0.29$

$\beta_4 = -0.03$

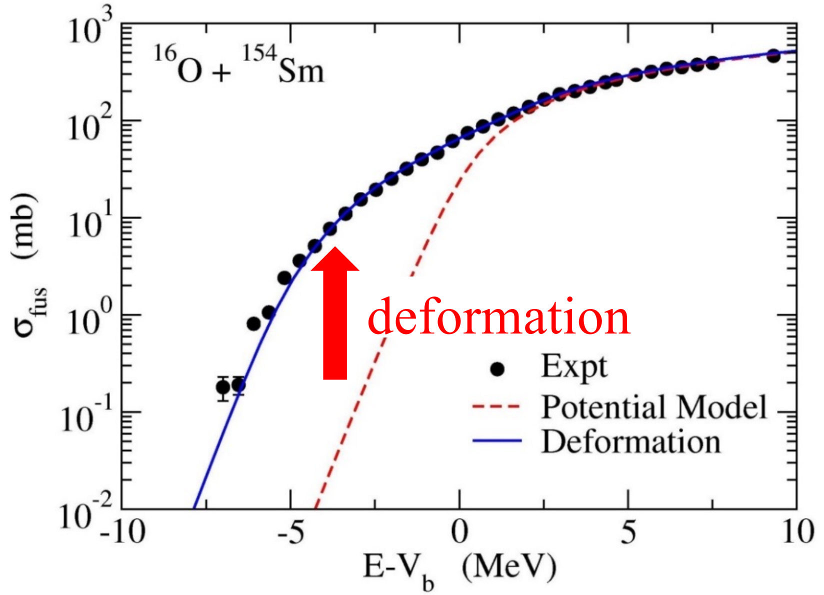




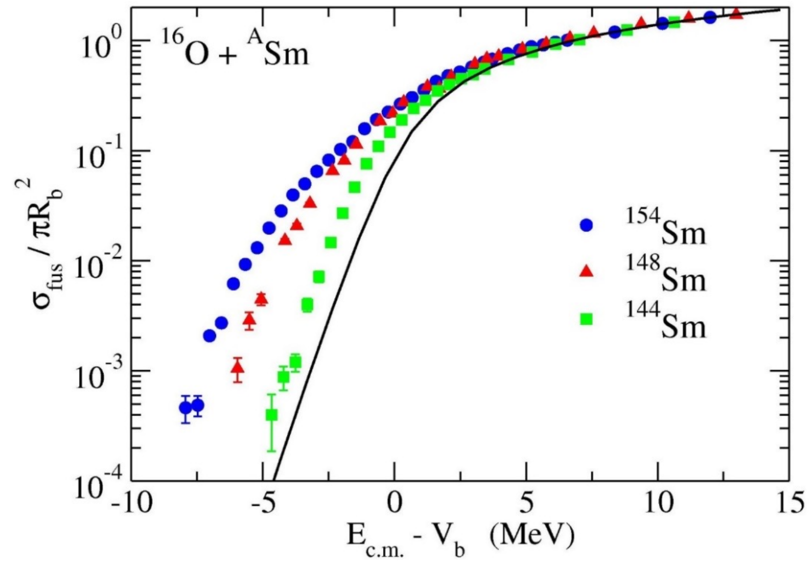


sensitive to the sign of β_4!

→ Fusion as a quantum tunneling microscope for nuclei

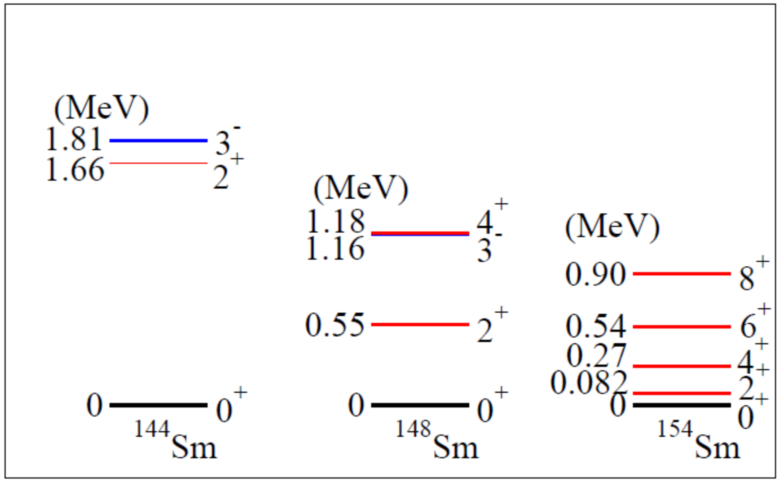


similar enhancement for non-deformed nuclei

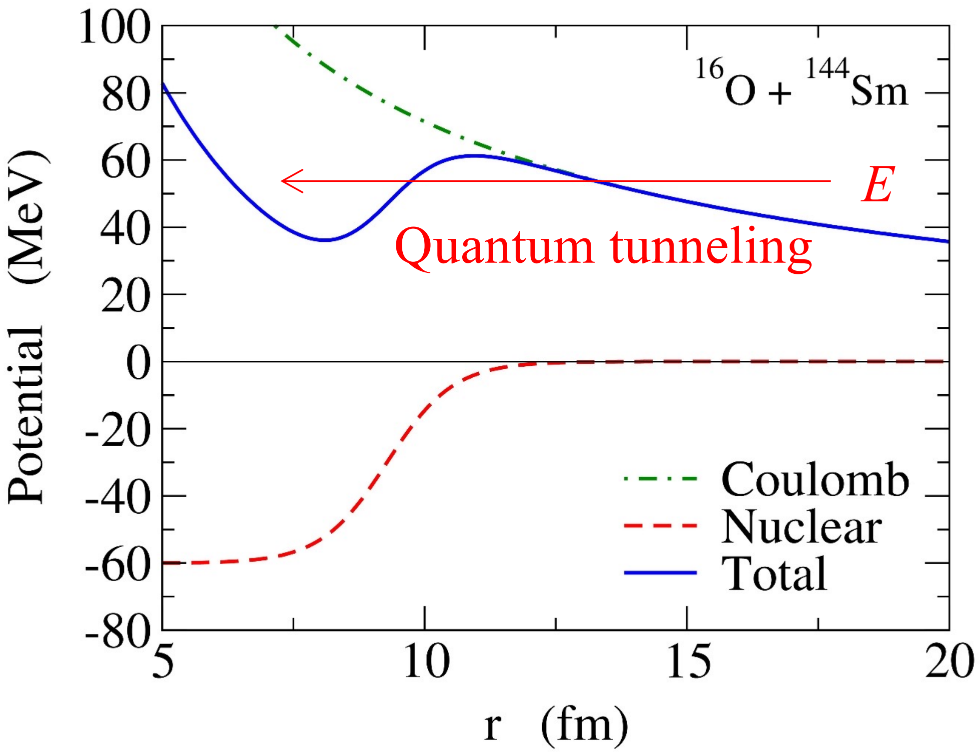


strong correlation with nuclear spectrum

→ coupling assisted tunneling phenomena



Coulomb barrier



1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

Potential barrier (Coulomb barrier)

Fusion: takes place by overcoming the barrier

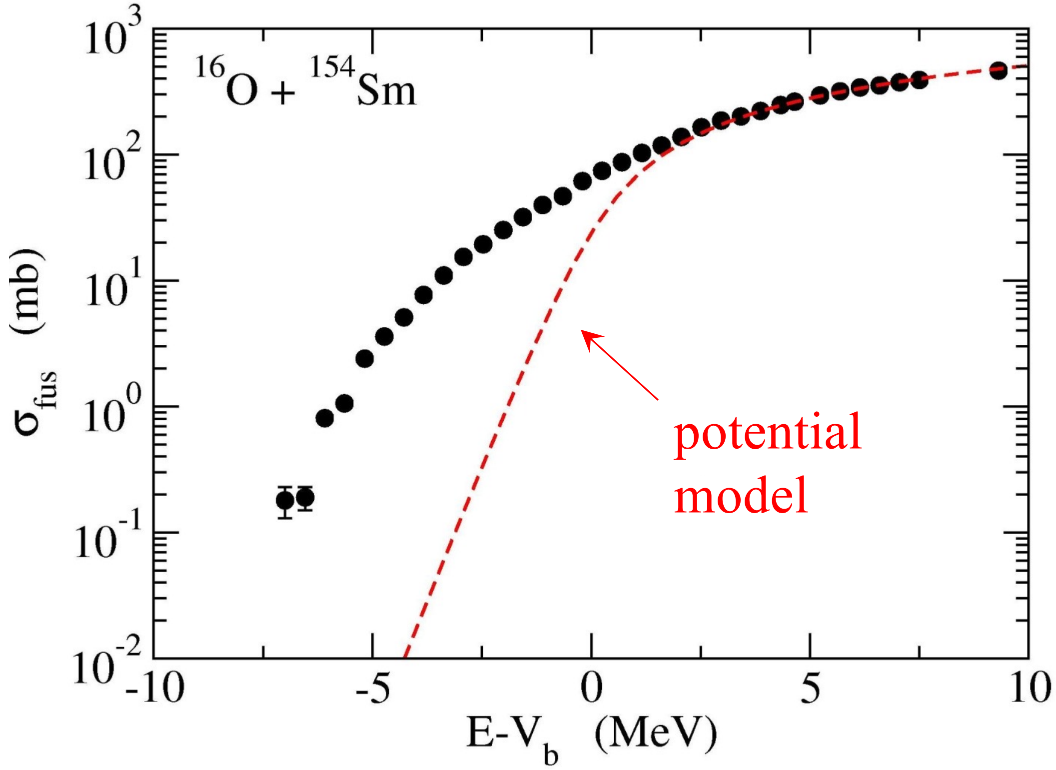
the barrier height → defines the energy scale of a system

Fusion reactions at energies around the Coulomb barrier

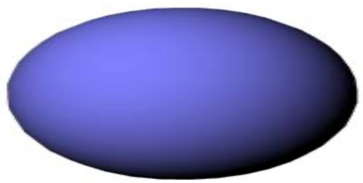
Discovery of large sub-barrier enhancement of σ_{fus} (~80's)

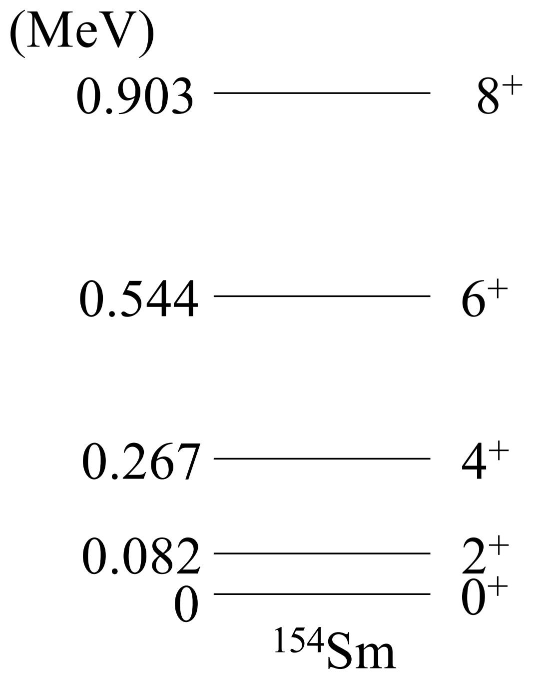
the potential model: inert nuclei (no structure)

$$\sigma_{\text{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

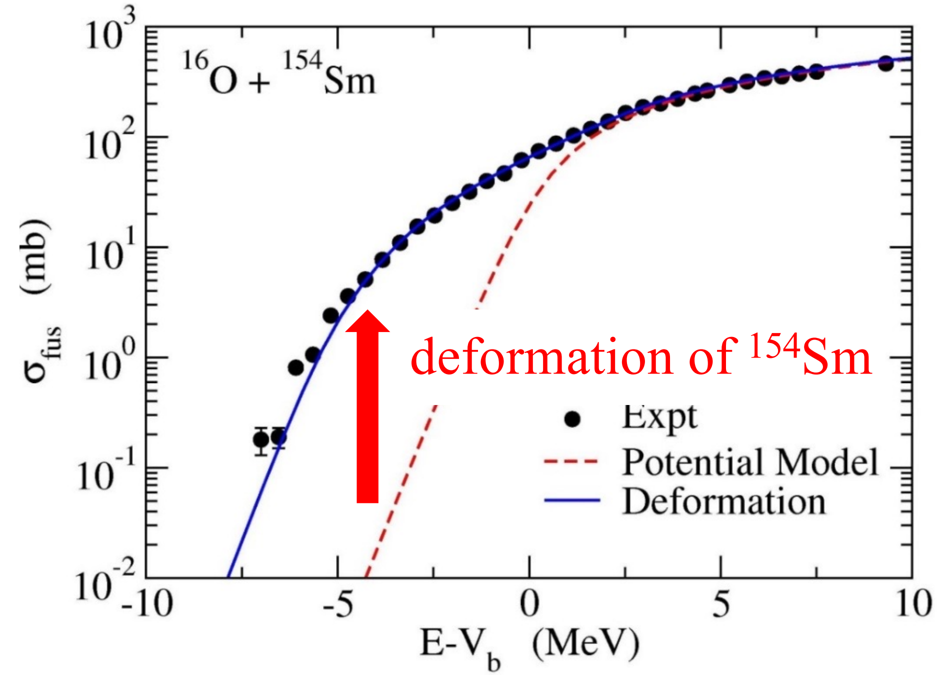


^{154}Sm : a typical deformed nucleus



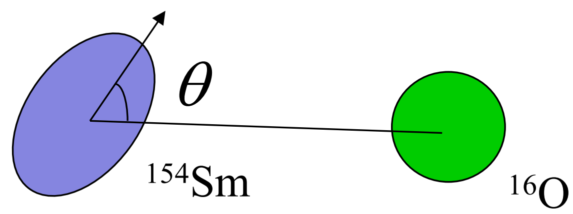


rotational spectrum



K. H. and N. Takigawa, Prog. Theo. Phys.128 ('12)1061.

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$



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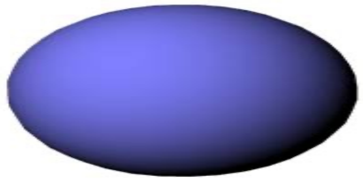
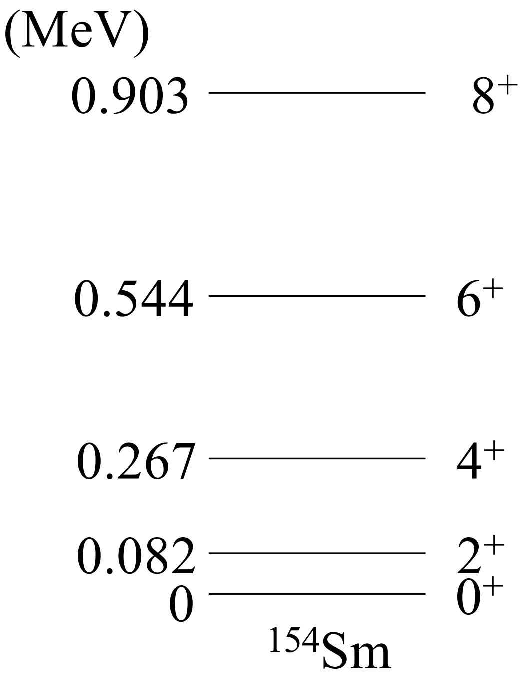
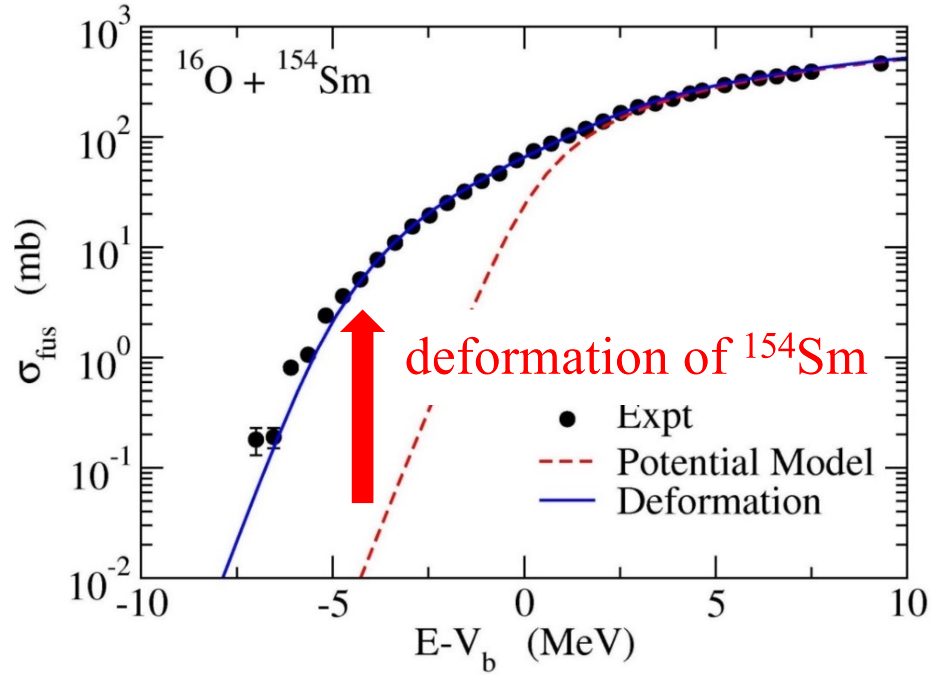


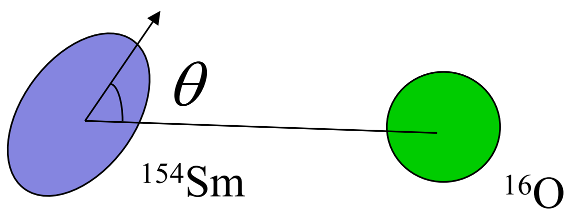


rotational spectrum

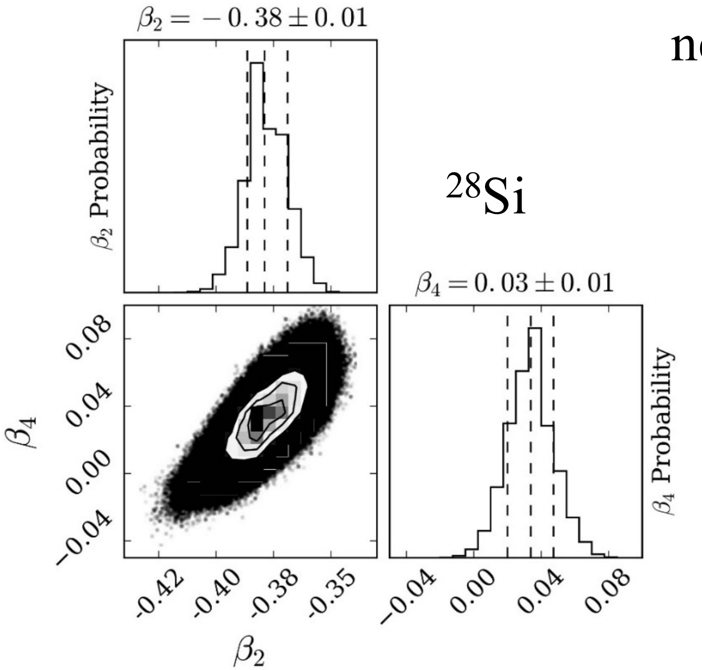


K. H. and N. Takigawa, Prog. Theo. Phys.128 ('12)1061.

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$



Emulator for multi-channel scattering



^{28}Si

Y.K. Gupta, V.B. Katariya, G.K. Prajapati, K.Hagino et al., PLB845, 138120 (2023).

needs to repeat many calculations with different (β_2,β_4)

→ an emulator to speed-up the calculations

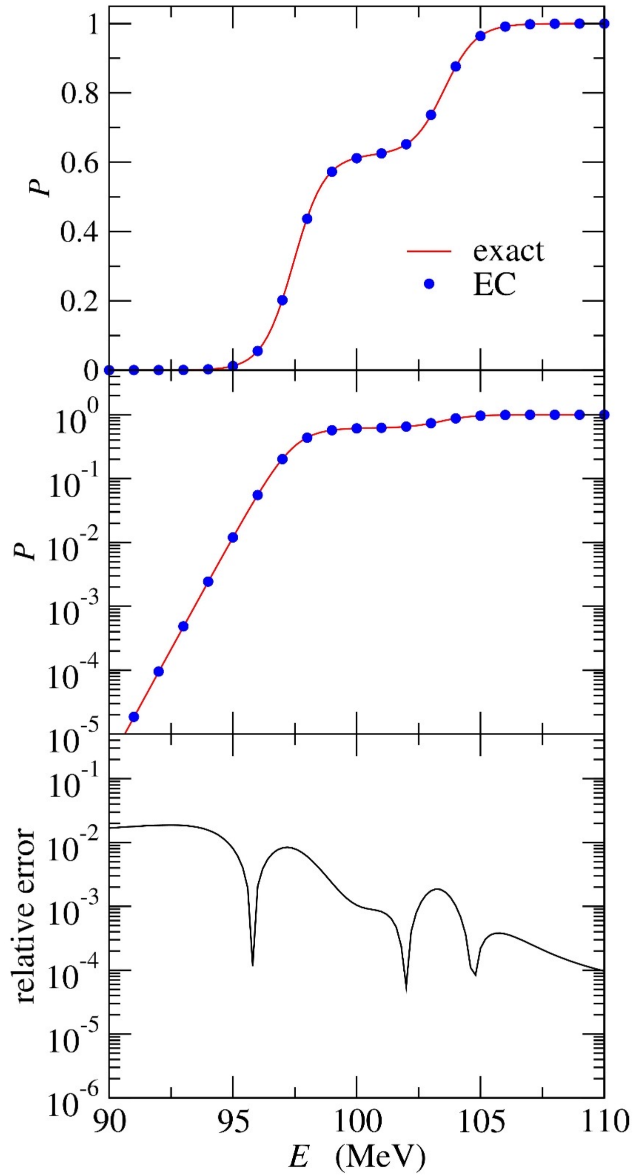
Eigenvector continuation

- bound state problems:

$$H(\theta)|\Psi(\theta)\rangle = E(\theta)|\Psi(\theta)\rangle \qquad \Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i)$$

T. Duguet et al.,
Rev. Mod. Phys. 96, 031002 (2024)

- Extension to scattering problems:
  - R. Furnstahl et al., PLB809, 135719 (2020)
  - C. Drisshler et al., PLB823, 136777 (2021)
  - J. Liu, J. Lei, and Z. Ren, PLB858, 139070 (2024)
  - K. Hagino, Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, arXiv: 2504.14922



1D two-channel problem:

$$H = \begin{pmatrix} V(x) & F(x) \\ F(x) & V(x) + \epsilon \end{pmatrix}$$

$$V(x) = V_0 \, e^{-x^2/2s^2}$$
$$F(x) = F_0 \, e^{-x^2/2s_f^2}$$

V_0 = 100 MeV, s=s_f=3 fm,
F_0 = 3 MeV

$$\Psi_E(x, F_0) = \sum_{i=1-5} c_i \Psi_E(x, F_{0i})$$

F_{0i} = 1.5, 2.0, 2.5, 3.5, 4.5 MeV

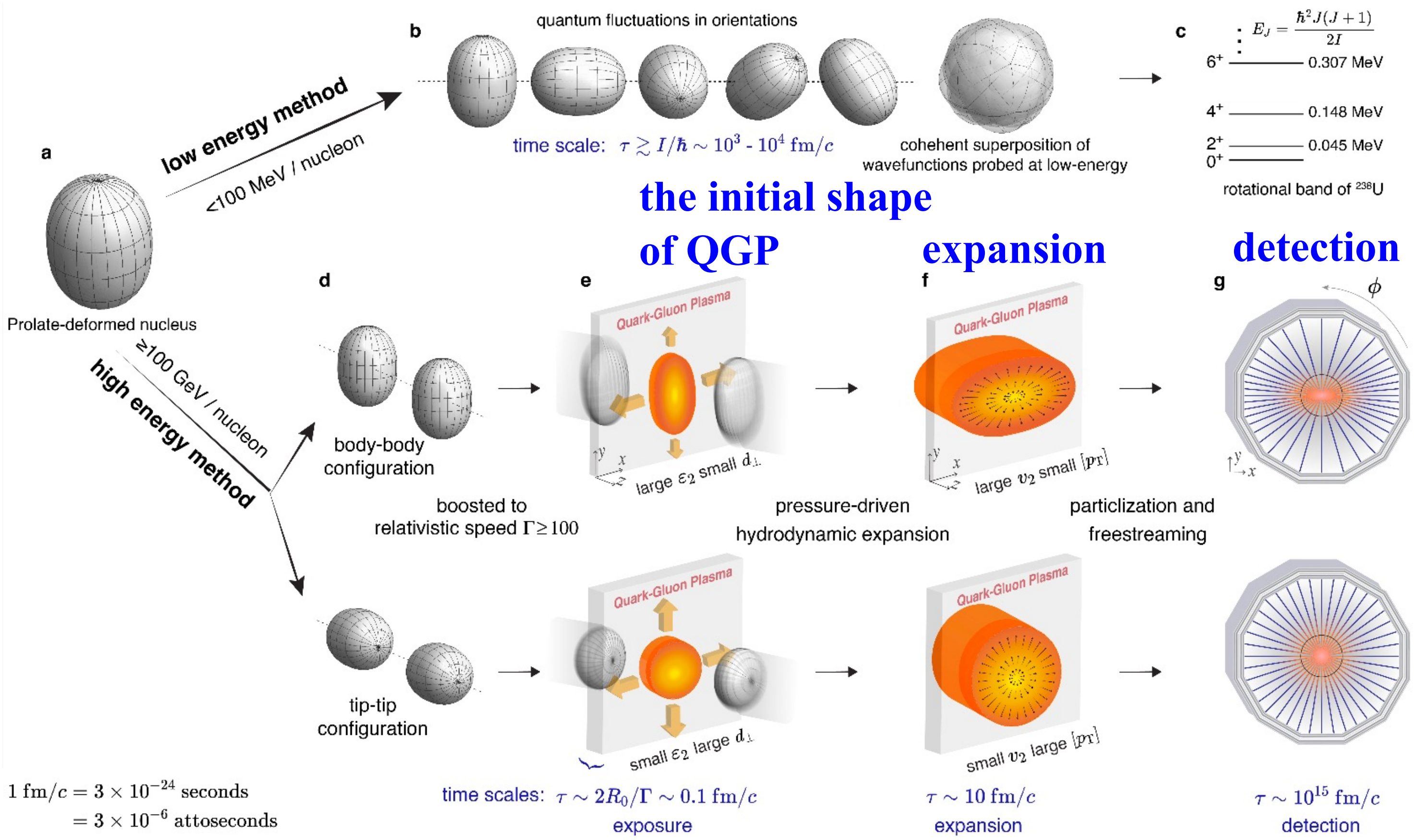
to simulate F_0 = 3 MeV

EC: the discrete basis method + Kohn variation principle

cf. K.H. and G.F. Bertsch, PRC110, 054610 (2024)

K. H., Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, arXiv: 2504.14922

Probing nuclear shapes in Rel. H.I. collisions



M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)