Perspectives on nuclear reaction theory and superheavy elements

Kouichi Hagino
(Sendai →) Kyoto University, Kyoto, Japan

1. Nuclear Reactions: overview
2. Coupled-channels approach with a beyond-mean-field method
3. Time-dependent GCM for many-body tunneling
4. Fusion for superheavy elements and TDHF
5. Summary

Workshop on new generation nuclear density functionals, 2019.11.18-22, Peking University
Introduction: low-energy nuclear physics

- behaviors of atomic nuclei as a quantum many-body systems
  - understanding based on strong interaction

- static properties: nuclear structure
  - ground state properties
    - (mass, size, shape, …)
  - excitations
  - nuclear matter
  - decays

- dynamics: nuclear reactions

an interplay between nuclear structure and nuclear reaction
Quantum Many-body Dynamics (nuclear reactions)

elastic scattering  inel. scattering  fusion
Quantum Many-body Dynamics (nuclear reactions)

elastic scattering  inel. scattering  fusion
Coulomb barrier

1. Coulomb interaction
   long range
   repulsion

2. Nuclear interaction
   short range
   attraction

the barrier height → defines the energy scale of a system

Fusion reactions at energies around the Coulomb barrier

Potential barrier
(Coulomb barrier)
Fusion reactions: compound nucleus formation

energy production in stars (Bethe ‘39)

nucleosynthesis

superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: an ultimate goal of nuclear physics
Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~80’s)

potential model: inert nuclei (no structure)

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2)$$

potential model
Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~80’s)

$^{154}\text{Sm}$: a typical deformed nucleus

Effects of nuclear deformation

$^{154}\text{Sm}$: a typical deformed nucleus
Effects of nuclear deformation

$^{154}\text{Sm}$ : a typical deformed nucleus

Fusion: strong interplay between nuclear structure and reaction

$$\sigma_{\text{fus}}(E) = \int_{0}^{1} d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$
Enhancement of fusion cross sections: A general phenomenon

Strong correlation with nuclear spectrum → coupling assisted tunneling
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

still very challenging
TDHF simulation

TDHF = Time Dependent Hartree-Fock
(a single Slater determinant)

S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 (‘15) 020056

ab-initio, but no tunneling

C. Simenel, EPJA48 (’12) 152
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

two-body problem, but with excitations (coupled-channels approach)

coupling

still very challenging
Coupled-channels method: a quantal scattering theory with excitations

Coupling between rel. and intrinsic motions

\[ \psi(r, \xi) = \sum_k \psi_k(r) \phi_k(\xi) \]

coupled Schroedinger equations for \( \psi_k(r) \)
Inputs for C.C. calculations

i) Inter-nuclear potential
   ✓ a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom
   ✓ types of collective motions (rotation / vibration) a/o transfer
   ✓ coupling strengths and excitation energies
   ✓ how many states


![Graph of nuclear fusion cross-section vs. energy]

- Simple harmonic oscillator
- States: $0^+, 2^+, 4^+$
- Energies: $\varepsilon, \sqrt{2\beta}, 2\varepsilon$
- Levels: $0^+, 2^+$
- Hamiltonian: $H = \hbar \omega (\hat{N} - \frac{1}{2}) - \frac{1}{2} \hbar \omega \hat{N}$
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91(‘15) 064606

multi-phonon excitations

\[ \begin{align*}
0^+ & \quad 2.94 \\
2^+ & \quad 2.78 \\
4^+ & \quad 2.45 \\
126(8) \ e^2\text{fm}^4 & \quad 1.45 \\
58\text{Ni} & \quad 0 \\
\end{align*} \]

Simple harmonic oscillator → justifiable?

\[ Q(2^+_1) = -10 +/- 6 \ e\text{fm}^2 \]
Anharmonic vibrations

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

\[ |J M\rangle = \int d\beta \, f_J(\beta) \hat{P}_M^J |\Phi(\beta)\rangle \]

- MF + ang. mom. projection
- + particle number projection
- + generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121
J.M. Yao et al., PRC89 (‘14) 054306

K.H. and J.M. Yao, PRC91 (‘15) 064606

cf. Harmonic limit:
\[ B(E2: I_{2ph}^+ \rightarrow 2^+_1) = 2 \times B(E2: 2^+_1 \rightarrow 0^+_1) \]
Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 (‘15) 064606

\[ V_{\text{coup}} \sim -R_T \frac{dV_N}{dr} \alpha_\lambda \cdot Y_\lambda(\hat{r}) \rightarrow -R_T \frac{dV_N}{dr} Q_\lambda \cdot Y_\lambda(\hat{r}) \]

✓ \( M(E2) \) from MR-DFT calculation
✓ scale to the empirical \( B(E2; 2_1^+ \rightarrow 0_1^+) \)
✓ still use a phenomenological potential
✓ use the experimental values for \( E_x \)

* axial symmetry (no 3\(^+\) state)
$^{58}\text{Ni} + ^{58}\text{Ni}$

anharmonicity of $2^+$ phonon → only a minor improvement

Next, more non-trivial case with $2^+ - 3^-$ coupling:

anharmonicity of oct. vib. in $^{208}\text{Pb}$
Application to $^{16}\text{O} + ^{208}\text{Pb}$ fusion reaction
double-octupole phonon states in $^{208}\text{Pb}$

\[ \begin{array}{c}
\text{6+} \\
\text{6+} \\
\text{6+} \\
\text{2+} \\
\text{0+}, 2^+, 4^+, 6^+ \\
\text{3-} \\
\text{0+} \\
\end{array} \]

$^{208}\text{Pb}$

M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631
V. Yu. Pnomarev and P. von Neumann-Cosel, PRL82 (‘99) 501
B.A. Brown, PRL85 (‘00) 5300

large fragmentations, especially 6$^+$ state
Application to $^{16}\text{O} + ^{208}\text{Pb}$ fusion reaction

$\sqrt{2}\beta_3$  

$\beta_2$  

$\beta_3$  

$2^+ \times 3^-$  

$\beta_3$  

$3^-$  

$2^+$  

$0^+$  

$\sigma_{\text{fus}}$ (mb)  

$^ {16}\text{O} + ^{208}\text{Pb}$  

No Coupling  

2 phonon (HO)  

too high!

cf. C.R. Morton et al., PRC60(‘99) 044608
fluctuation both in $\beta_3$ and $\beta_2$

2$_1^+$ state: strong coupling both to g.s. and 3$_1^-$

$$|2_1^+\rangle = \alpha |2^+\rangle_{HO} + \beta |[3^- \otimes 3^-]^{(I=2)}\rangle_{HO} + \cdots$$
J.M. Yao and K.H.,
PRC94 ('16) 11303(R)
From phenomenological approach to microscopic approach

Macroscopic (phenomenological)

- C.C. with collective model
- C.C. with inputs from microscopic nuclear structure calculations
- C.C. with inputs based on TDHF
- TDHF simulations

Microscopic

TDHF = Time Dependent Hartree-Fock

S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 (‘15)

ab initio, but no tunneling
Time-dependent GCM for many-body tunneling

N. Hasegawa, K.H., and Y. Tanimura, in preparation

TDHF simulations
ab initio, but no tunneling

need to go Beyond the mean-field app.

✓ Time-dependent GCM

a single Slater determinant (SD) to multi-SD

\[ |\psi(t)\rangle = \int dq f(q, t)|\Phi_q(t)\rangle \]

dynamics with a superposition of many “TDHF trajectories (Slater determinants)”

cf. Stochastic mean-field method
B. Yilmaz et al.,
PRC90 ('14) 054617
TDHF

\[ \psi(t) = \Phi_{SD}(t) \]

\[ e^{ikx} \varphi_i(x + x_0) \quad e^{-ikx} \varphi_i(x - x_0) \]

\( \alpha + \alpha \) in 1D

\[ E < E_{th} \]

\[ E \geq E_{th} \]
TDHF

\[ \alpha + \alpha \text{ in 1D} \]

\[ E < E_{\text{th}} \]

\[ E \geq E_{\text{th}} \]
$$\Psi(t) = \sum_k f_k(t) \Phi_{SD,k}(t)$$

time-dep. variational principle

$$\delta \int dt \frac{\langle \Psi(t)|i\hbar \partial_t - H|\Psi(t)\rangle}{\langle \Psi(t)|\Psi(t)\rangle} = 0$$
\( \alpha + \alpha \) in 1D

A simplification:

\[
\varphi_i^{(k)}(x, t) \sim \exp \left[ -\nu \left( x - \frac{Z_i^{(k)}(t)}{\sqrt{\nu}} \right)^2 \right]
\]

\[
\Phi_{SD, k}(t) = \mathcal{A} \left[ \{ \varphi_i^{(k)}(x, t) \} \right]
\]
TDGCM (a superposition of two SD)

N. Hasegawa, K.H., and Y. Tanimura, in preparation
N. Hasegawa, K.H., and Y. Tanimura, in preparation

superposition (TDGCM)
TDGCM (a superposition of two SD)

N. Hasegawa, K.H., and Y. Tanimura, in preparation
TDGCM: a promising microscopic theory for many-body tunneling

\[ \psi = \sum_i f_i \Phi_{SD,i} \]

\[ \psi = \Phi_{SD} \]

TDHF

\[ 16O + ^{208}Pb \]

C. Simenel, EPJA48 ('12) 152
Yuri Oganessian

a prediction of island of stability
(Swiatecki et al., 1966)
Future directions of SHE

Superheavy elements synthesized so far

Towards Z=119 and 120 isotopes

➢ Towards Z=119 and 120 isotopes
  Hot fusion reactions with $^{48}$Ca, $^{50}_{22}$Ti, $^{51}_{23}$V, $^{54}_{24}$Cr etc.

➢ Towards the island of stability
  neutron-rich beams: indispensable → reaction dynamics?
Fusion for SHE: fusion hindrance

→

strong Coulomb repulsion

→ re-separation

compound nucleus

fusion hindrance
Fusion for SHE: more fusion hindrance

- P + T → P+T
- Fusion
- Fusion hindrance
- Strong Coulomb repulsion
- Quasi-fission
- Fission

- Evaporation process (cooling)
- n, p, α emission
- γ decay
- A very rare event
Theoretical challenges

formation of SHE: very rare
→ a large theoretical uncertainty

✓ no exp. data for $P_{CN}$
✓ exp. data: $P_{ER}$ only

CN = 複合核, ER = 蒸発残留核

theoretical challenges: to reduce the uncertainties and make reliable predictions

Physics of open quantum systems
SHE formation reactions

\[
(b)^{86}\text{Kr} + ^{208}\text{Pb}
\]

\(V(r)\) (MeV)

2-body potential
1-body potential
compound nucleus
thermal fluctuation
heat up
re-separation

\(r\) (fm)
Langevin approach

multi-dimensional extension

\( q \): • internuclear separation,
• deformation,
• asymmetry of the two fragments

thermal fluctuation

\[ m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t) \]

\( \gamma \): friction coefficient

\( R(t) \): random force

V(\( \varepsilon \)) (MeV)

\( \varepsilon \) (deformation)
Extension of fusion-by diffusion model

cf. barrier distribution measurements by Tanaka et al.
Synthesis of Z=119 and 120

Towards Z=119 and 120 isotopes

hot fusion reactions with $^{48}\text{Ca}$:

$^{48}_{20}\text{Ca} + ^{99}\text{Es} \rightarrow 119$

$^{48}_{20}\text{Ca} + ^{100}\text{Fm} \rightarrow 120$

short lived $\rightarrow$ not available with sufficient amounts

$^{48}\text{Ca} \rightarrow ^{50}_{22}\text{Ti}, ^{51}_{23}\text{V}, ^{54}_{24}\text{Cr}$ projectiles

how much will cross sections be affected?

closed shell $\rightarrow$ open shells
TDHF + Langevin approach

K. Sekizawa and K. H., PRC99 (2019) 051602(R)

$^{54}\text{Cr} + ^{248}\text{Cm} \rightarrow ^{302}120$

$^{48}\text{Ca} + ^{254}\text{Fm} \rightarrow ^{302}120$

$E / V_b = 1.2$

$l = 0$

$\Delta V$

$R_{\text{min}}$

$\rightarrow$ Langevin calculation
Mapping TDHF onto a classical equation of motion

K. Washiyama and D. Lacroix, PRC78 (‘08) 024610

TDHF simulations

\[ \dot{P} = -\frac{dV}{dR} - \frac{d}{dR} \left( \frac{P^2}{2\mu} \right) - \gamma \dot{R} \]

\[ \Delta E(t) = -E_{\text{diss}}(t) = \frac{P(t)^2}{2\mu(R(t))} + V(R(t)) - E_{\text{ini}} \]
New hybrid model: TDHF + Langevin approach

K. Sekizawa and K.H., PRC99 (2019) 051602(R)
New model for fusion for SHE: TDHF + Langevin approach
K. Sekizawa and K.H., PRC99 (2019) 051602(R)

how special is $^{48}$Ca?

<table>
<thead>
<tr>
<th>System</th>
<th>CN</th>
<th>$E^*$ (MeV)</th>
<th>$R_{\text{min}}$ (fm)</th>
<th>$P_{\text{CN}}$ ($\times 10^4$)</th>
<th>$W_{\text{sur}}$ ($\times 10^9$)</th>
<th>$P_{\text{CN}} W_{\text{sur}}$ ($\times 10^{13}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}$Ca + $^{254}$Fm</td>
<td>302</td>
<td>29.0</td>
<td>12.93</td>
<td>1.72</td>
<td>176</td>
<td>302</td>
</tr>
<tr>
<td>$^{54}$Cr + $^{248}$Cm</td>
<td>302</td>
<td>33.2</td>
<td>13.09</td>
<td>1.89</td>
<td>1.31</td>
<td>2.47</td>
</tr>
<tr>
<td>$^{51}$V + $^{249}$Bk</td>
<td>300</td>
<td>37.0</td>
<td>12.94</td>
<td>3.95</td>
<td>0.117</td>
<td>0.461</td>
</tr>
<tr>
<td>$^{48}$Ca + $^{257}$Fm</td>
<td>305</td>
<td>30.5</td>
<td>12.94</td>
<td>2.49</td>
<td>0.729</td>
<td>1.82</td>
</tr>
</tbody>
</table>

similar $P_{\text{CN}}$

no special role of $^{48}$Ca in the entrance channel
From phenomenological to microscopic nuclear reaction theories

Macroscopic (phenomenological)

- C.C. with collective model
- C.C. with beyond MF
- C.C. with inputs based on TDHF

Microscopic

- TDHF simulations
- TDHF+Langevin for SHE
- TDGCM for many-body tunneling
FUSION20

November 15-20, 2020
Shizuoka, Japan

Kouichi Hagino (co-chair)  Kyoto University
Katsuhisa Nishio (co-chair) JAEA

Japan-China joint organization
Program Committee
Japanese members
+ L. Guo, X. Tang, S.G. Zhou
FUSION20

November 15-20, 2020
Shizuoka, Japan

Kouichi Hagino (co-chair)  Kyoto University
Katsuhisa Nishio (co-chair) JAEA

Japan-China joint organization
Program Committee
Japanese members
+ L. Guo, X. Tang, S.G. Zhou

Shizuoka Convention Beureau: Taichiro Ishida
FUSION20

November 15-20, 2020
Shizuoka, Japan

Kouichi Hagino (co-chair) Kyoto University
Katsuhisa Nishio (co-chair) JAEA

Japan-China joint organization
Program Committee
Japanese members
+ L. Guo, X. Tang, S.G. Zhou

Shizuoka Convention Beaureau: Taichiro Ishida（石田）