Open issues in di-neutron correlations in neutron-rich nuclei

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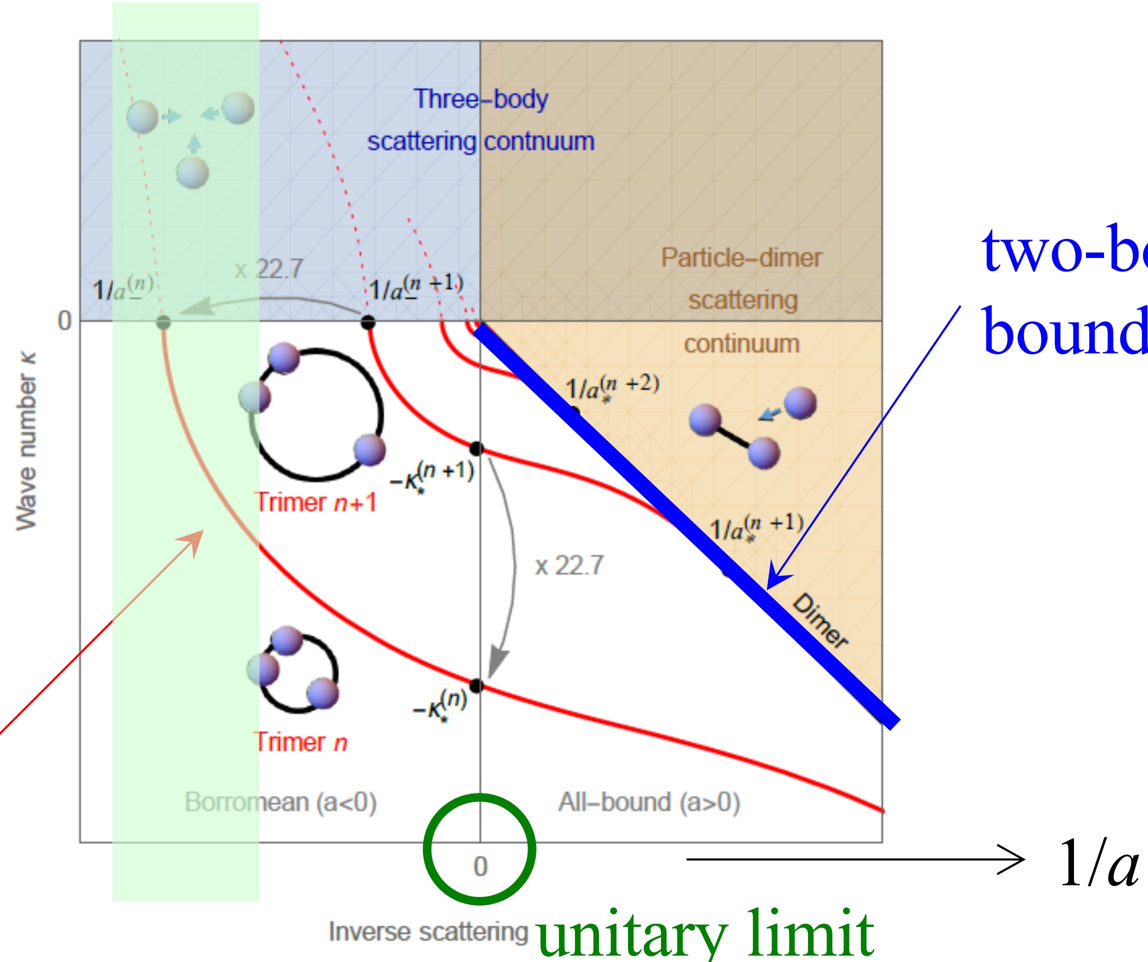
1. Introduction: di-neutron correlations
2. Correlations with a repulsive interaction
3. A measure of dineutron correlations
4. Two-neutron transfer reactions
5. Summary

A seminar at Fudan University, April 26, 2024

Borromean systems in atomic nuclei

a spectrum of 2&3 identical bosons with an attractive interaction



two-body
bound state

three-body
bound state

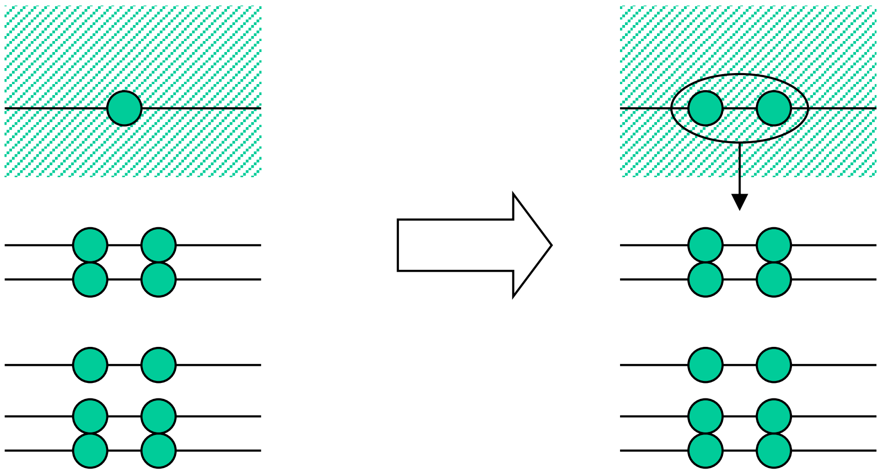
unitary limit

weakly bound
Borromean systems

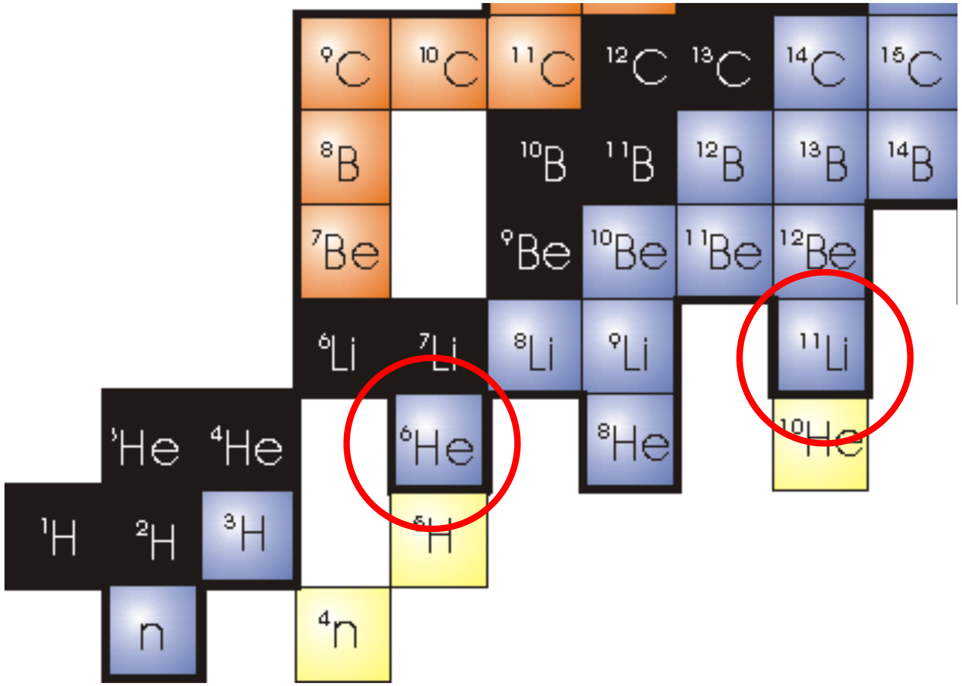
P. Naidon and S. Endo, Rep. Prog. Phys. 80 (‘17)056001

Borromean nuclei

residual interaction → attractive



particle unstable particle stable



^{11}Li = ^{9}Li + n +n : bound

^{9}Li + n : unbound

n + n : unbound

^{6}He = ^{4}He + n +n : bound

^{4}He + n : unbound

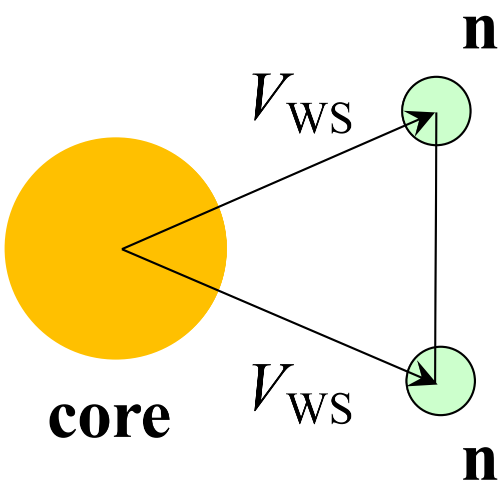
n + n : unbound

Questions to ask: the role of nn-correlation?

- Spatial structure?
- Excitation modes?
- Decay dynamics of unbound nuclei?
- Influence for nuclear reactions?

Three-body model and di-neutron correlation

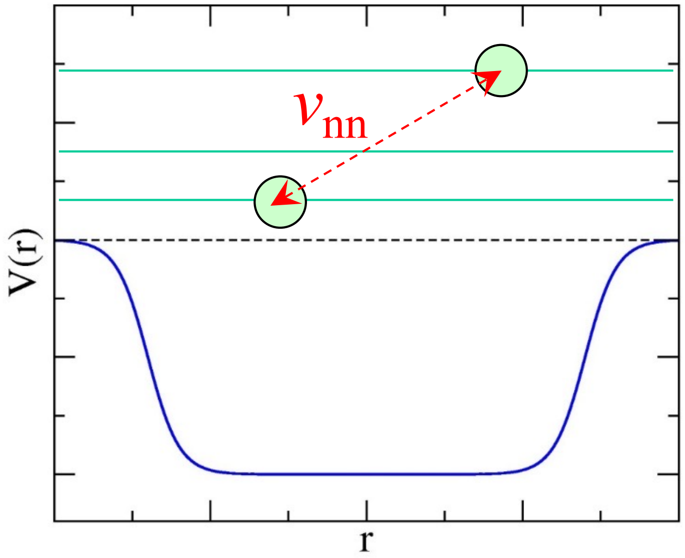
^{11}Li, ^{6}He



Density-dependent delta-force

$$v(\boldsymbol{r}_1,\boldsymbol{r}_2) = v_0(1+\alpha\rho(r)) \times\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$

v_0 ← scatt. length



continuum states:
discretized in a large box

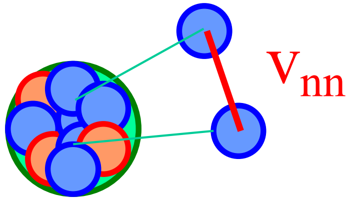
$$\Psi_{gs}(\boldsymbol{r},\boldsymbol{r}') = \mathcal{A}\sum_{nn'lj}\alpha_{nn'lj}\,\Psi^{(2)}_{nn'lj}(\boldsymbol{r},\boldsymbol{r}')$$

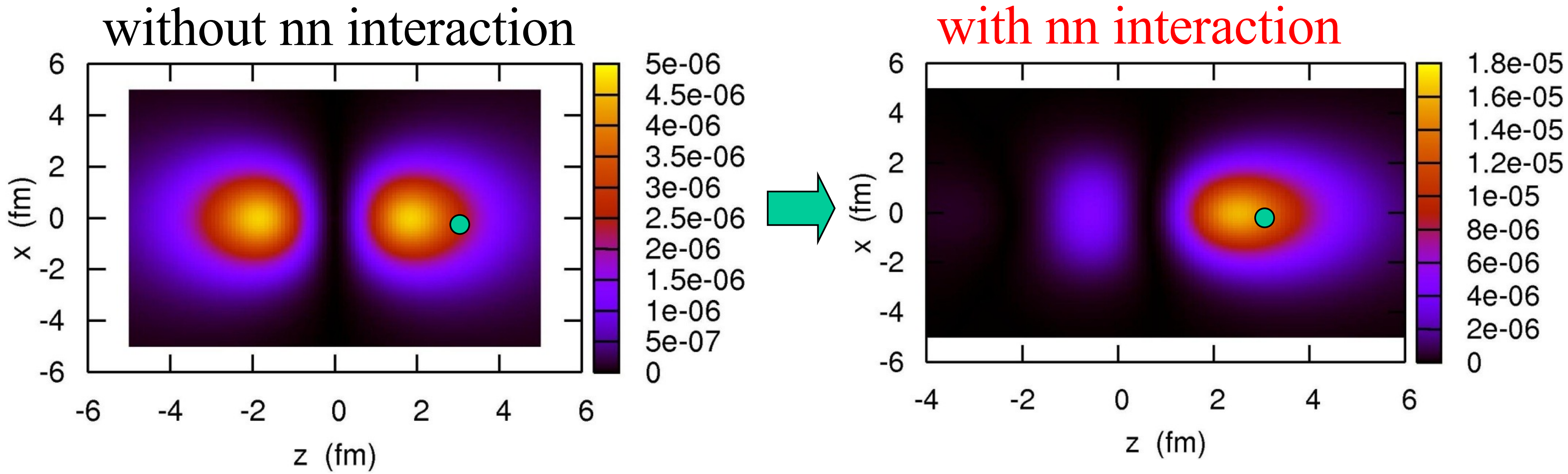
$\longrightarrow$ diagonalize the H_{3bd}

G.F. Bertsch and H. Esbensen, Ann. of Phys. 209 ('91) 327
K.H. and H. Sagawa, PRC72 ('05) 044321

The ground state density: ^{11}Li = ^{9}Li + n + n

K.H. and H. Sagawa, PRC72 ('05) 044321

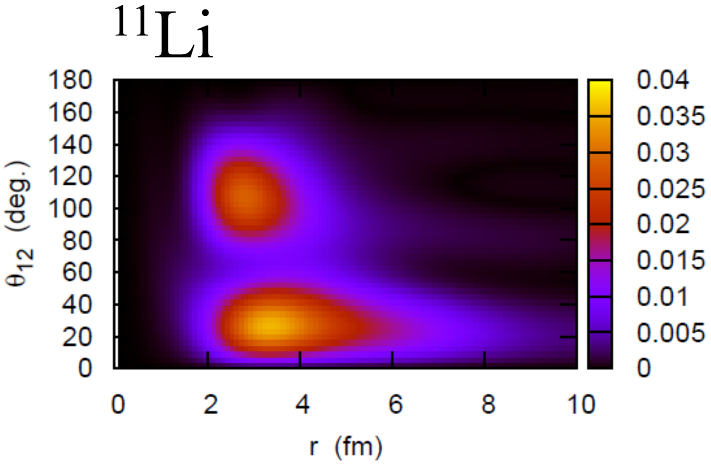



without nn interaction

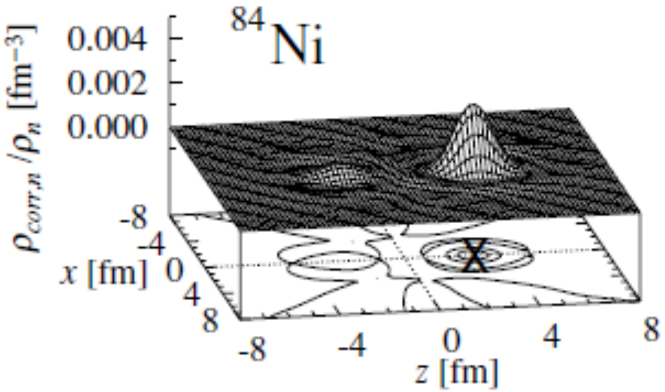
with nn interaction

large asymmetry in density distribution = **di-neutron correlation**

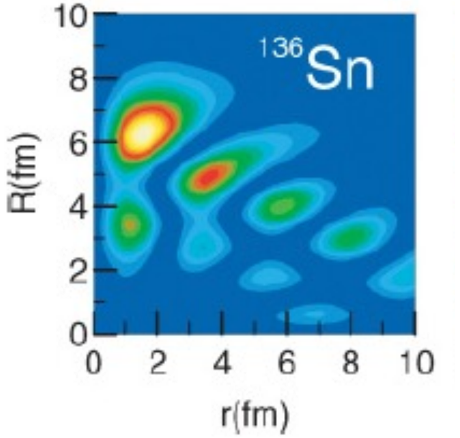
Di-neutron correlation



Bertsch-Esbensen, Ann. Phys. ('91)
Zhukov et al., Phys. Rep. ('93)
Hagino-Sagawa, PRC72 ('05)



Matsuo et al.,
PRC71 ('05)



Pillet et al.,
PRC76 ('07)

cf. coherence length in the
BCS approximation:

$$\xi = \frac{\hbar^2 k_F}{m\Delta}$$

→ much larger than nuclei

Experiments:

- Coul.-ex. (^{11}Li, ^{19}B, etc.)

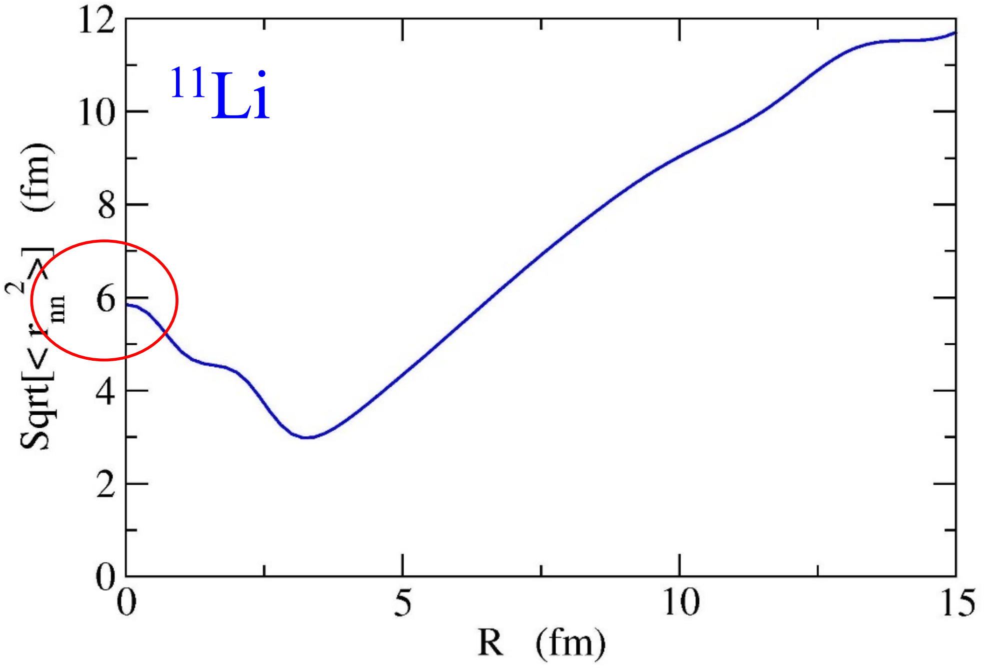
  K.J. Cook et al.,
  PRL124 ('20) 212503

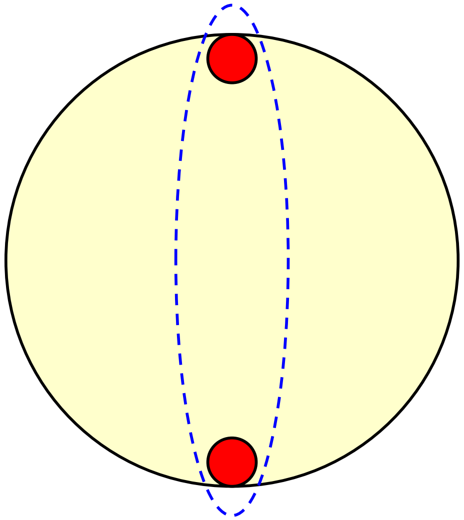
- knockout (^{11}Li)

  ^{11}Li(p,pn) ^{10}Li

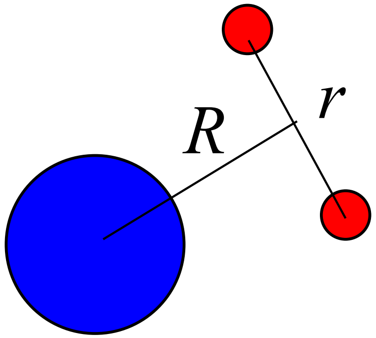
  Y. Kubota et al.,
  PRL 125 ('20) 252501

Surface dineutron correlations

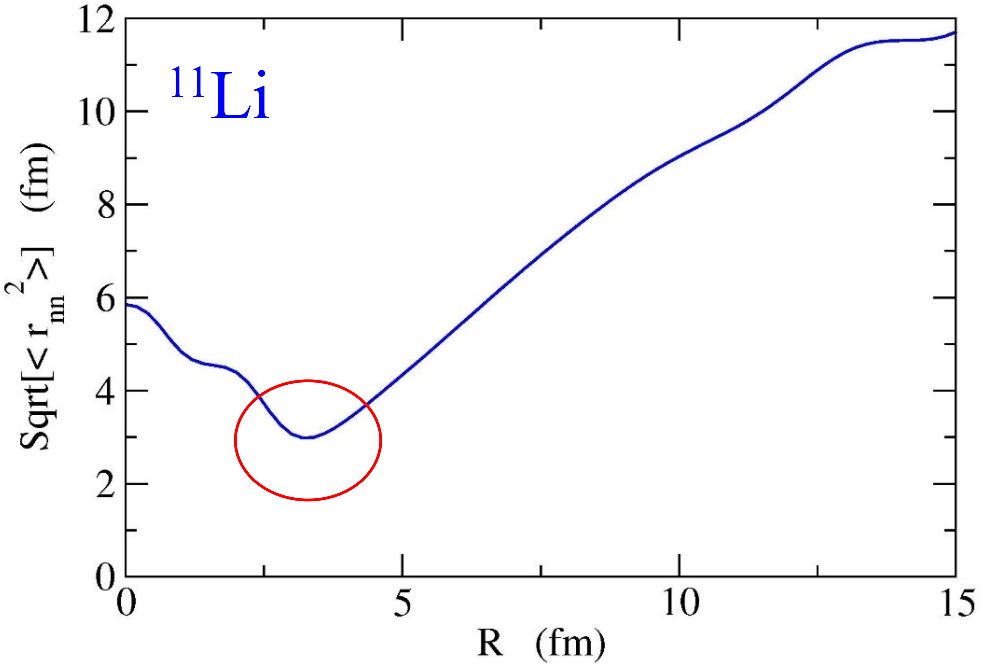




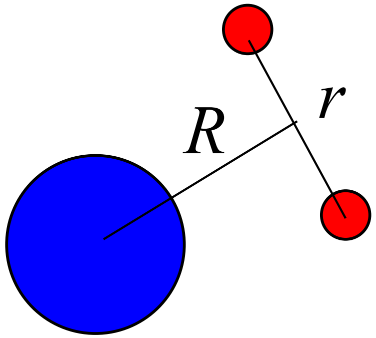
K.H., H. Sagawa, J. Carbonell, and P. Schuck,
PRL99 ('07) 022506

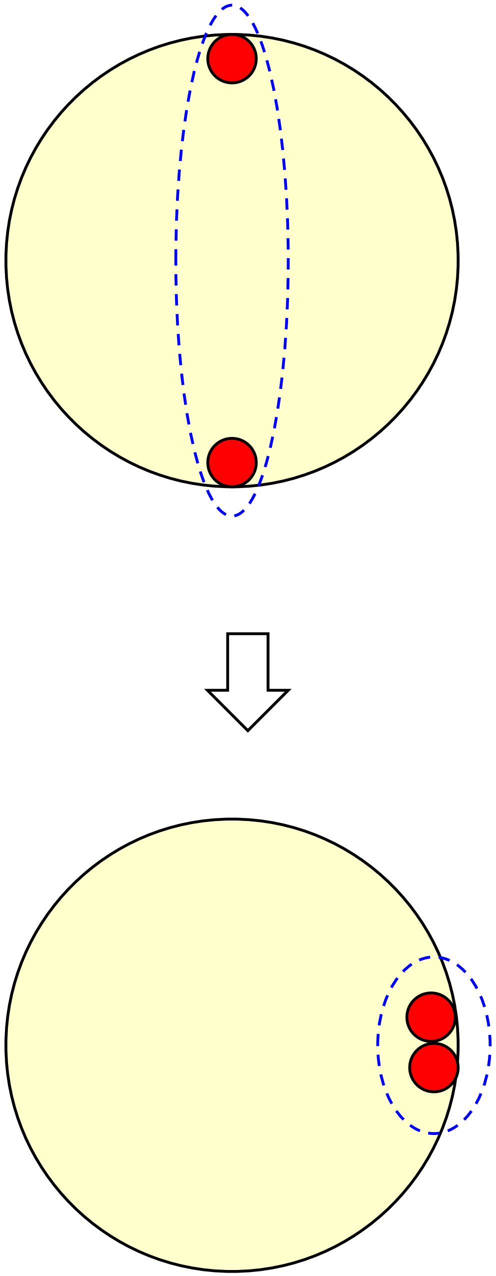


Surface dineutron correlations



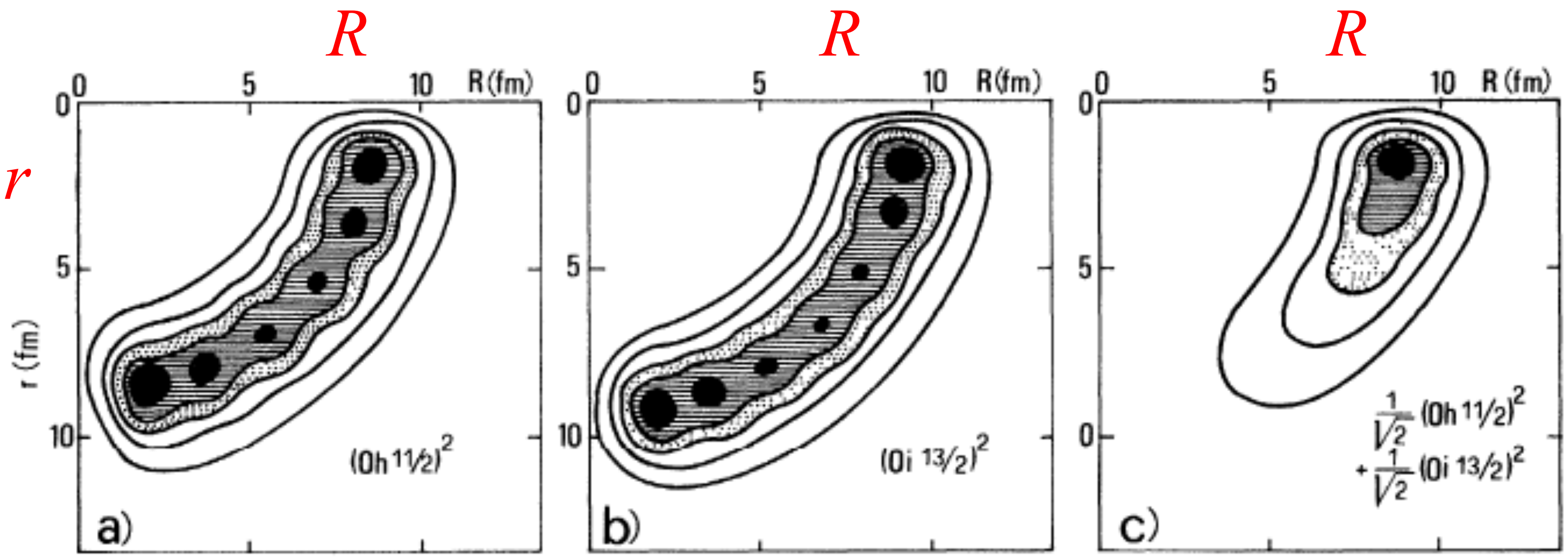
K.H., H. Sagawa, J. Carbonell, and P. Schuck,
PRL99 ('07) 022506

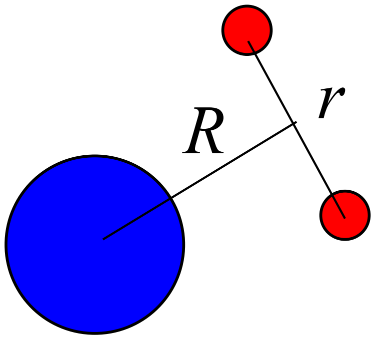




the origin of dineutron correlation: a mixing of $[jl]^2$ with different parities

$$\boxed{|\Psi\rangle = \sum_{j,l} C_{jl}|[jl]^2\rangle}$$





$(0h_{11/2})^2$

$l = 5$

$(0i_{13/2})^2$

$l = 6$

$$\frac{1}{\sqrt{2}}(0h_{11/2})^2 + \frac{1}{\sqrt{2}}(0i_{13/2})^2$$

F. Catara, A. Insolia, E. Maglione,
and A. Vitturi, PRC29('84)1091

cf. the phase of C_{jl}

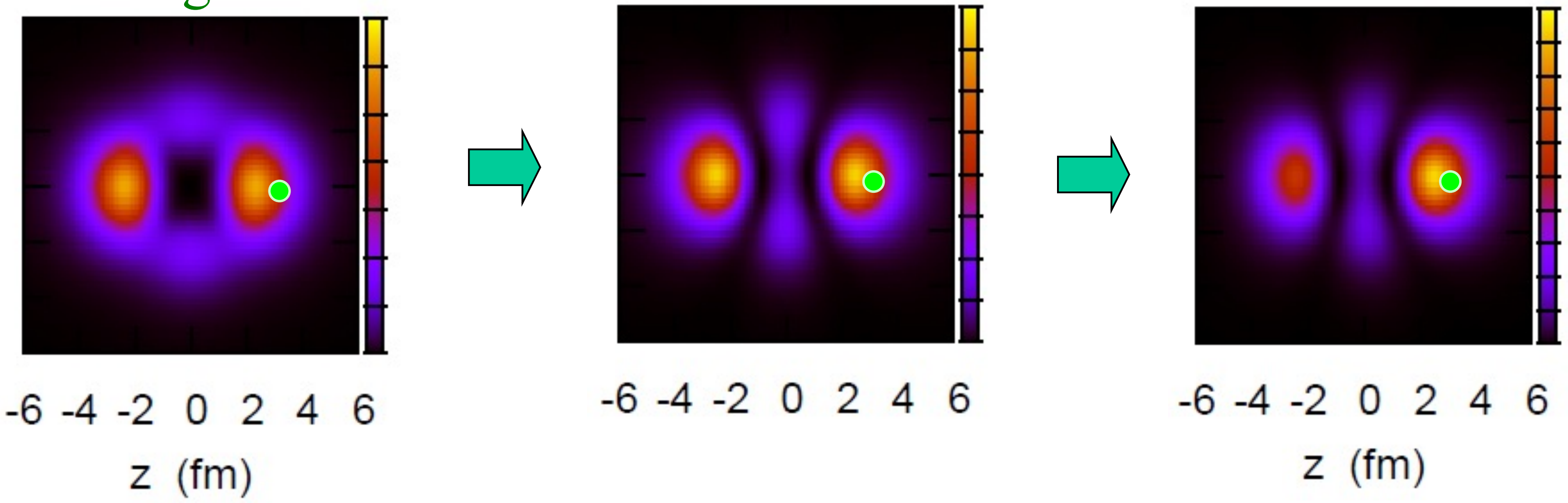
role of parity mixing

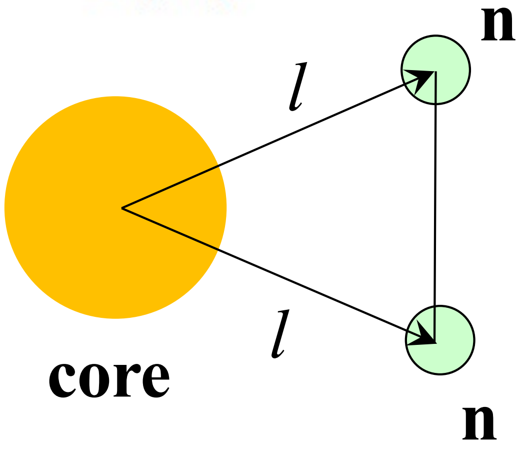
$$^{18}\mathrm{O} = {}^{16}\mathrm{O} + \mathrm{n} + \mathrm{n} \longrightarrow \quad \rho_2(\boldsymbol{r}) = |\Psi_{\mathrm{g.s.}}(\boldsymbol{r}, \boldsymbol{r}')|^2_{\boldsymbol{r}'=z_0}$$

single-*l*

multi-*l*, but even *l* only

multi-*l*, both even and odd *l*



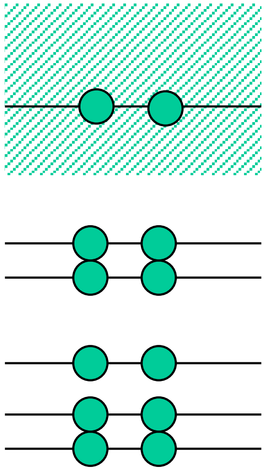


cf. F. Catara, A. Insolia, E. Maglione,
and A. Vitturi, PRC29('84)1091

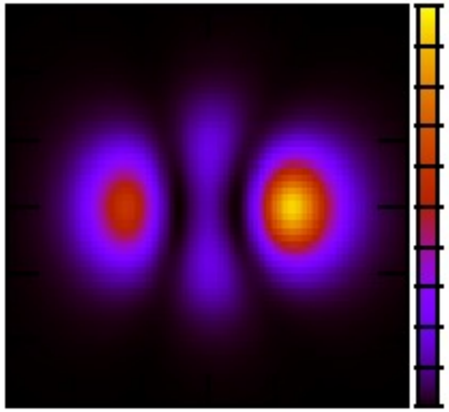
weakly bound systems

✓ continuum states

several l's

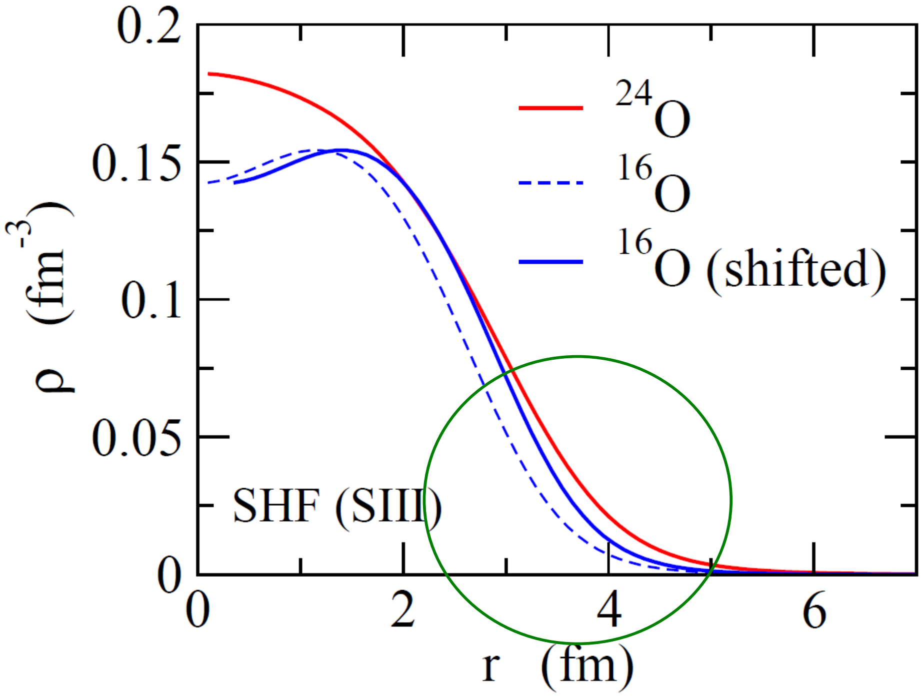


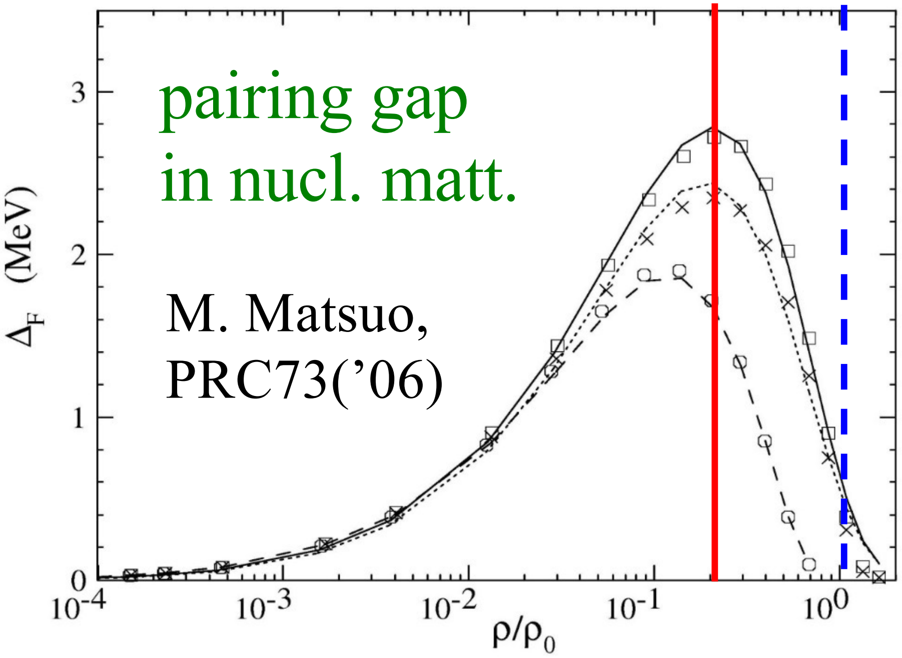
parity mixing: easy



➡ enhanced dineutron correlation

✓ extended density distribution



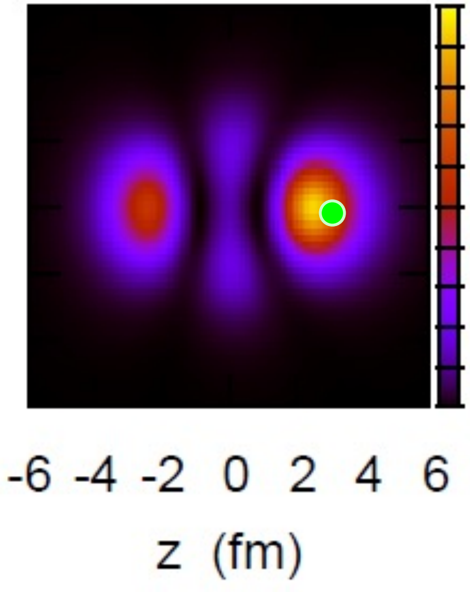


pairing gap
in nucl. matt.

M. Matsuo,
PRC73('06)

Two-nucleon correlation with a repulsive interaction

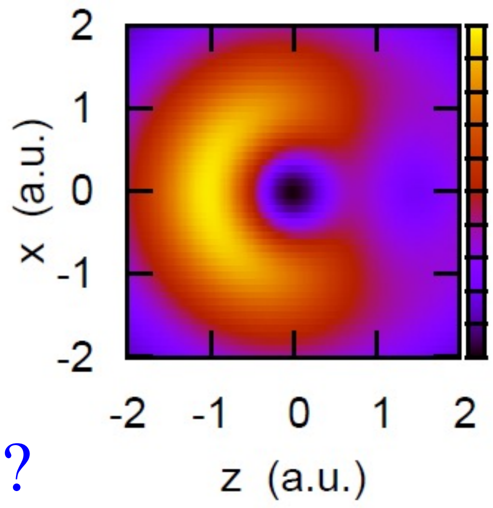
$$|\Psi\rangle = \sum_{j,l} C_{jl}|[jl]^2\rangle$$

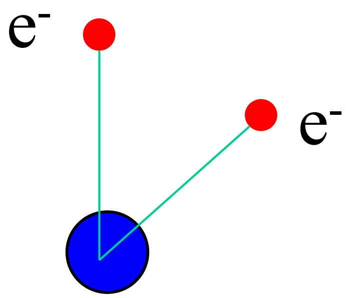


nuclear attractive interaction
→ dineutron correlation

What happens when the interaction is repulsive?

cf. A Coulomb hole in
He atoms



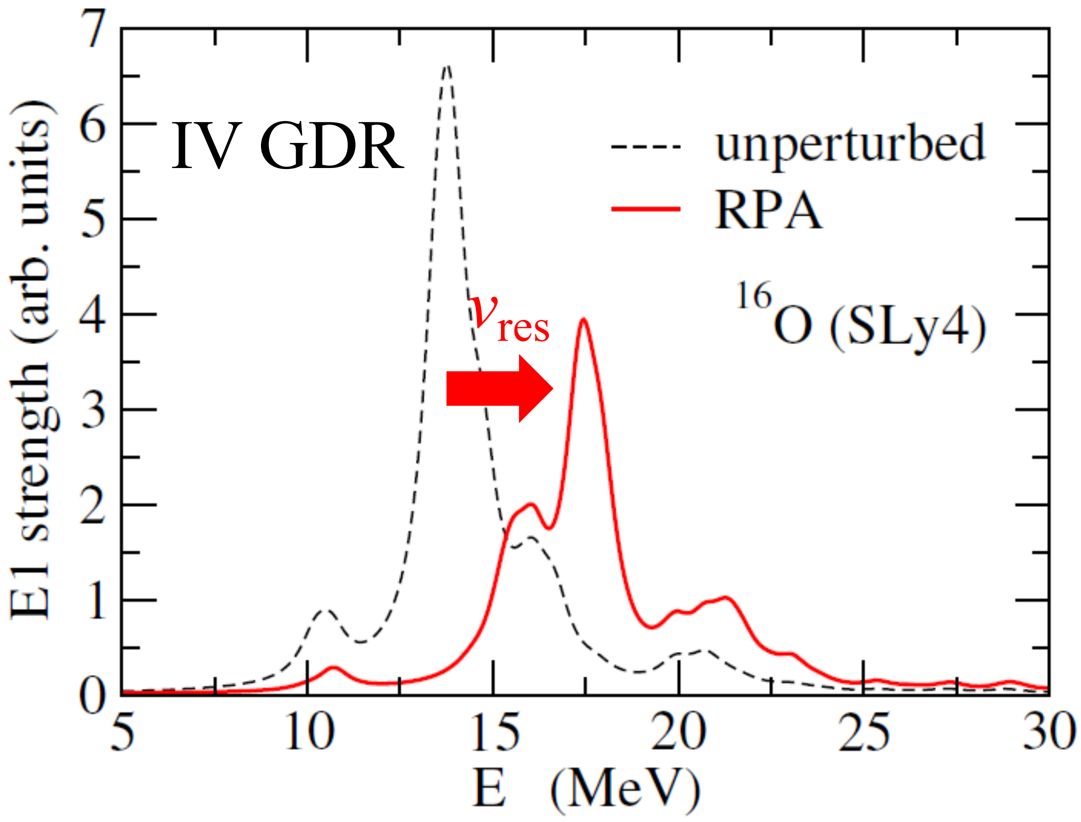


how about nuclear systems?

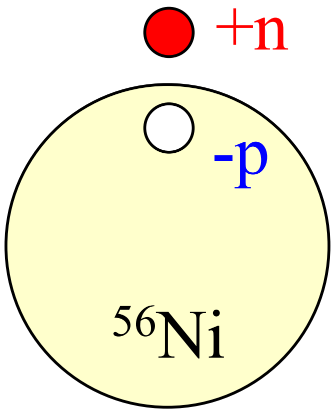
Two-nucleon correlation with a repulsive interaction

What happens when the interaction is repulsive?

IV(T=1) particle-hole interaction: repulsive



^{56}Co = ^{56}Ni + n - p



→ the particle-hole density?

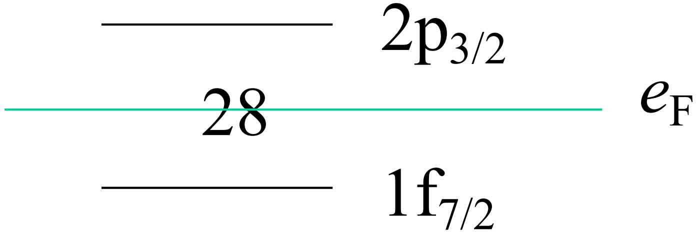
IV ph configurations

Tamm-Dancoff approximation with a Skyrme interaction

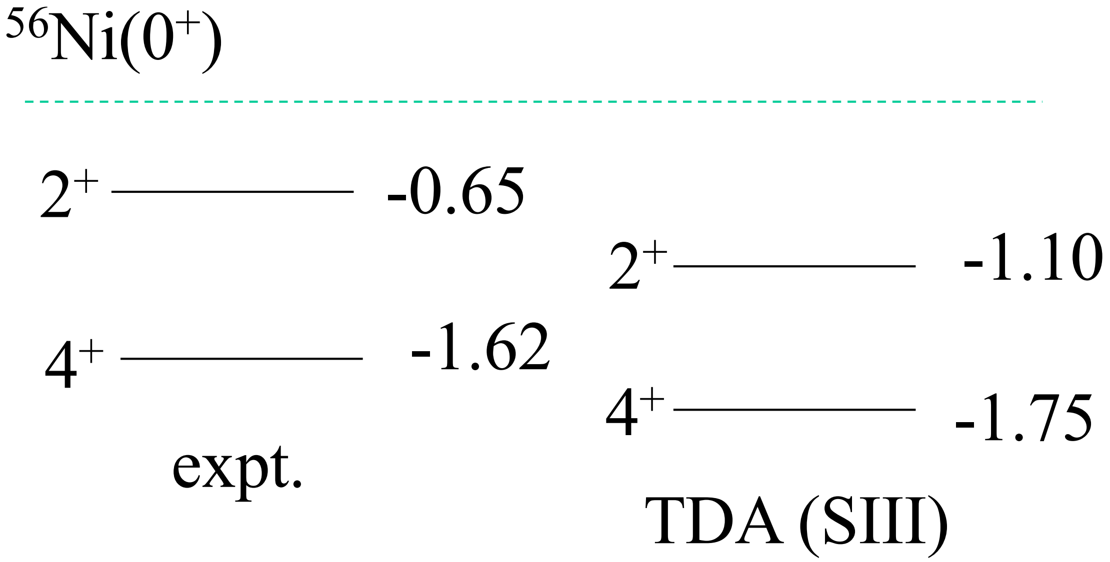
$^{56}\text{Co} = {}^{56}\text{Ni} + \text{n} - \text{p}$

$$|^{56}\mathrm{Co}\rangle = \sum_{p,h} \underset{\uparrow}{C_{ph}}\, a^{\dagger}_{\nu p} a_{\pi h}\, \underline{|^{56}\mathrm{Ni}\rangle}$$

diagonalize H_{Sk} Skyrme HF



$(2p_{3/2})_n(1f_{7/2})_p^{-1}$



IV ph configurations

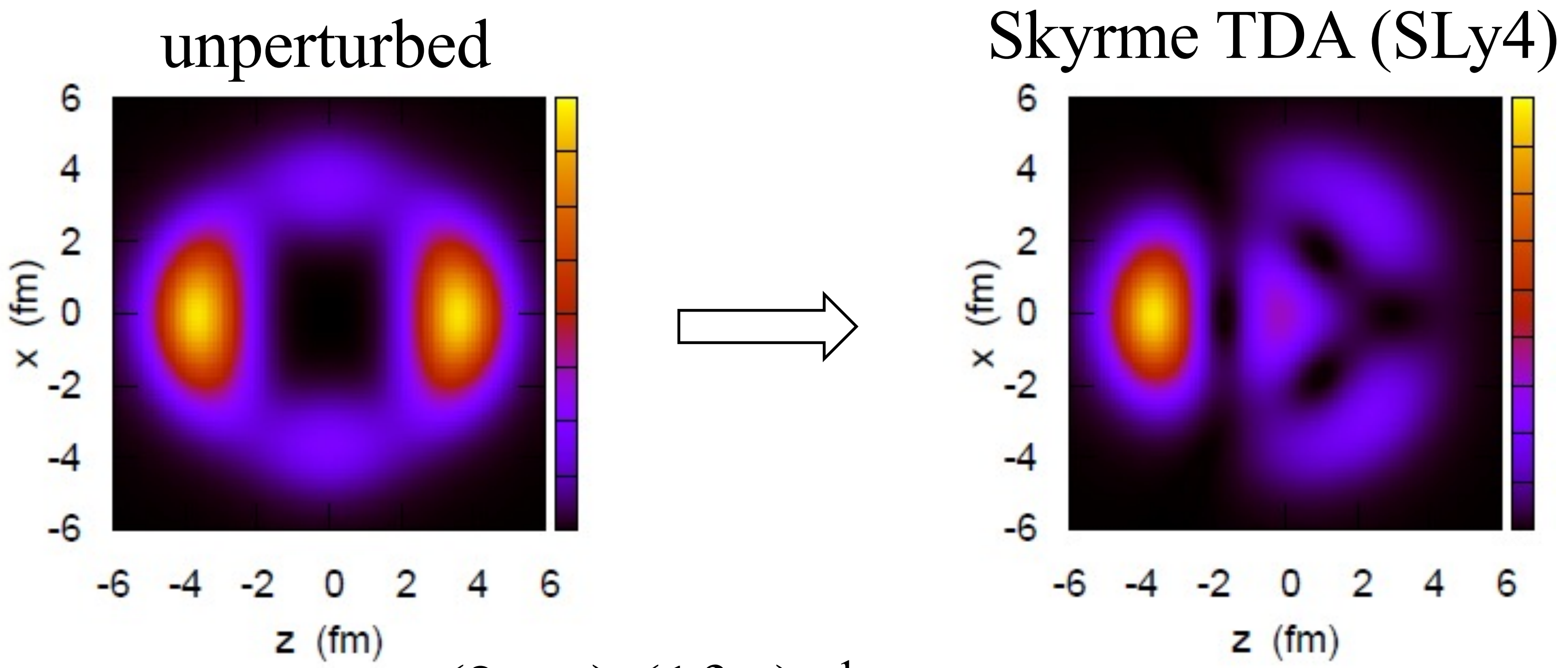
$^{56}\mathrm{Co} = {}^{56}\mathrm{Ni} + \mathrm{n} - \mathrm{p}$

$$|^{56}\mathrm{Co}\rangle = \sum_{p,h} C_{ph}\, a^{\dagger}_{\nu p} a_{\pi h}\, |^{56}\mathrm{Ni}\rangle$$

the spatial distribution of a hole configuration:

the 4^+ state of ^{56}Co (M=0)

a neutron at 3.4 fm



unperturbed

Skyrme TDA (SLy4)

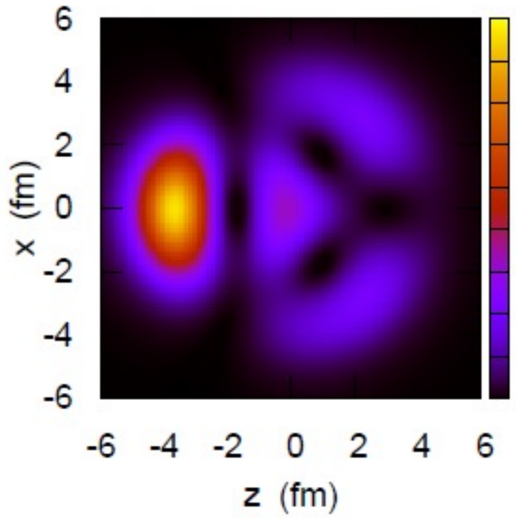
$(2p_{3/2})_n(1f_{7/2})_p^{-1}$

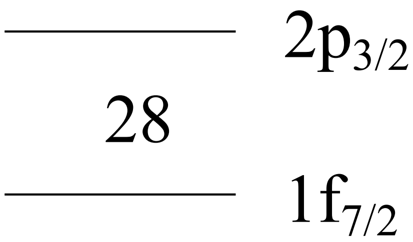
K.H. and H. Sagawa,
PRC106, 034313 (2022)

IV ph configurations

$^{56}\mathrm{Co} = {}^{56}\mathrm{Ni} + \mathrm{n} - \mathrm{p}$

$$|^{56}\mathrm{Co}\rangle = \sum_{p,h} C_{ph}\, a^{\dagger}_{\nu p} a_{\pi h}\, |^{56}\mathrm{Ni}\rangle$$





$2p_{3/2}$

28

$1f_{7/2}$

$(2p_{3/2})_n(1f_{7/2})_p^{-1}$: 97.7%

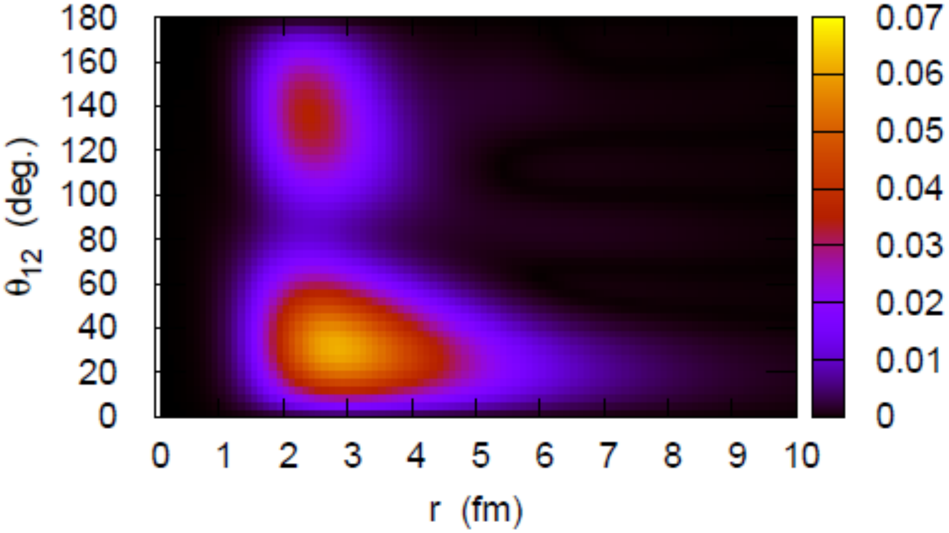
$(\mathrm{even})_n(\mathrm{even})_p^{-1}$: 0.10%
$(\mathrm{odd})_n(\mathrm{odd})_p^{-1}$: 99.9%

the origin of dineutron correlation: a mixing of $[jl]^2$ with different parities

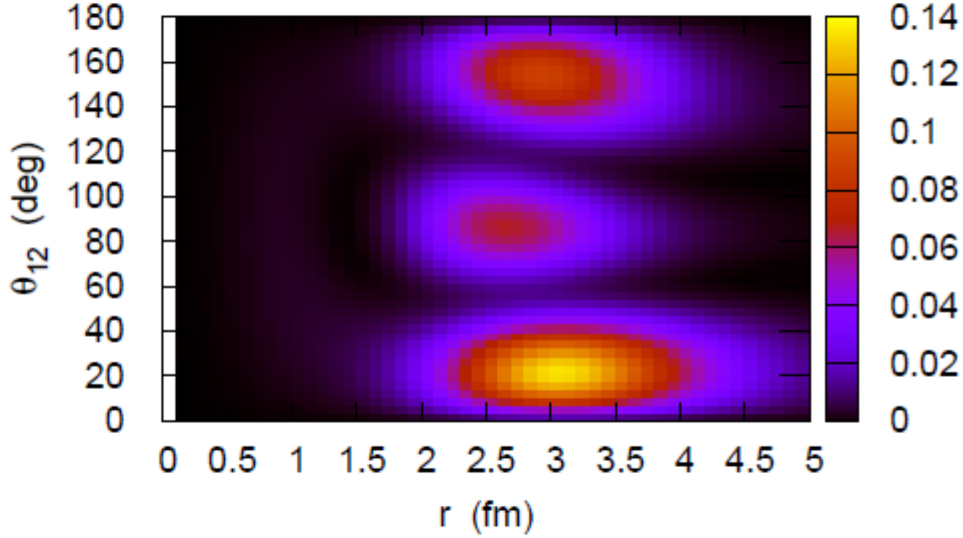
$$|\Psi\rangle = \sum_{j,l} C_{jl}|[jl]^2\rangle$$

How large should the mixing be? What is a measure of the correlation?

^{6}He (S_{2n} = 0.978 MeV)



^{18}O (S_{2n} = 12.2 MeV)



odd^2: 89.1 % [$(p_{3/2})^2$=83%]
even^2: 10.9%

odd^2: 3.37 %
even^2: 96.6% [sd shell=94.8%]

even a small mixing leads to an asymmetric distribution

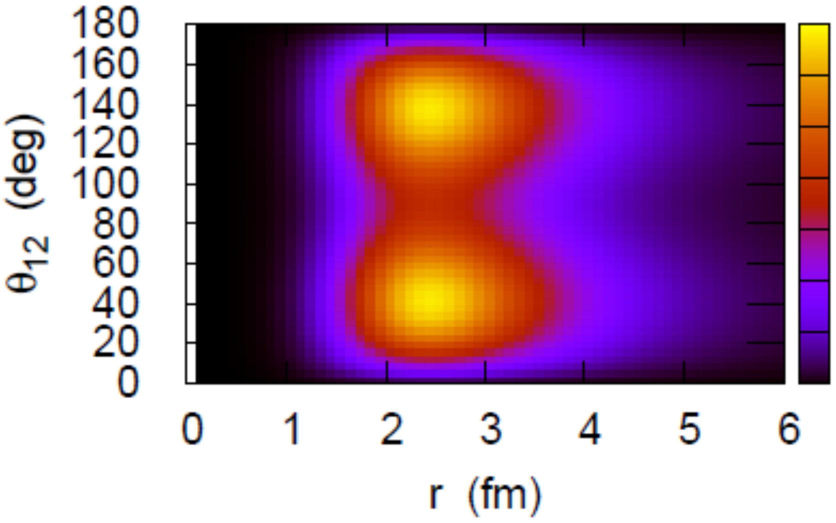
2 configuration model

$$|\Psi\rangle = \sqrt{\alpha^2}\,|(1p_{3/2})^2\rangle + \sqrt{1-\alpha^2}\,|(2s_{1/2})^2\rangle$$

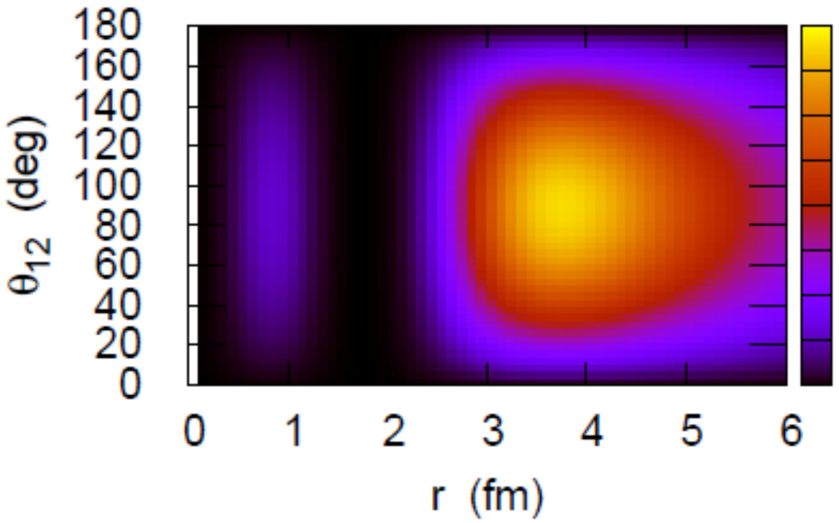
✓ wave functions of $1p_{3/2}$, $2s_{1/2}$ states ←a Woods-Saxon potential

✓ the depth of WS pot.: e_{sp} = -0.5 MeV for each state

α = 1: pure $(1p_{3/2})^2$

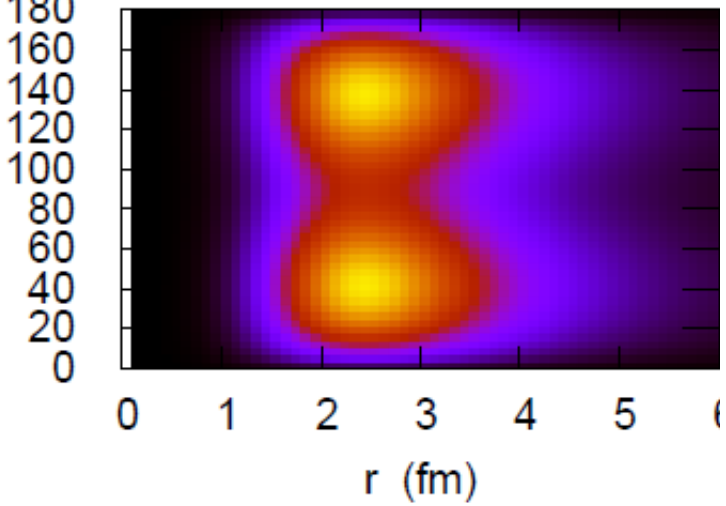


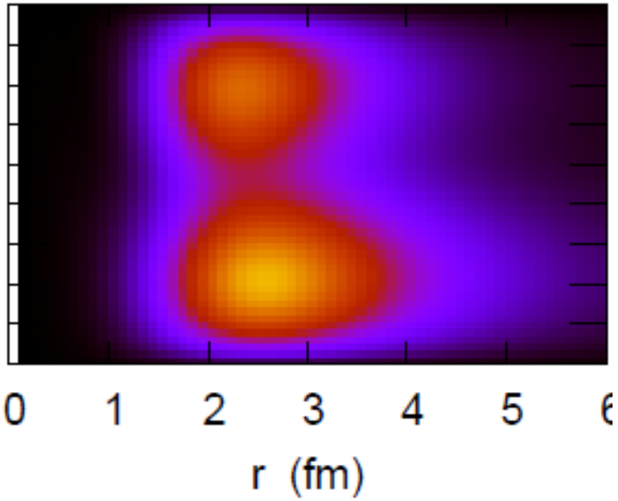
α = 0: pure $(2s_{1/2})^2$

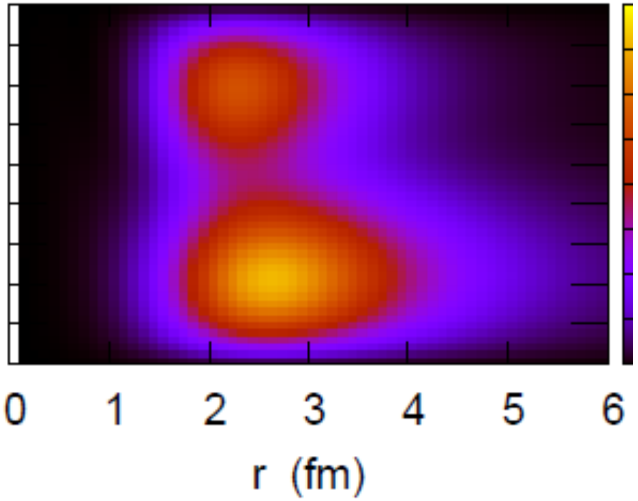


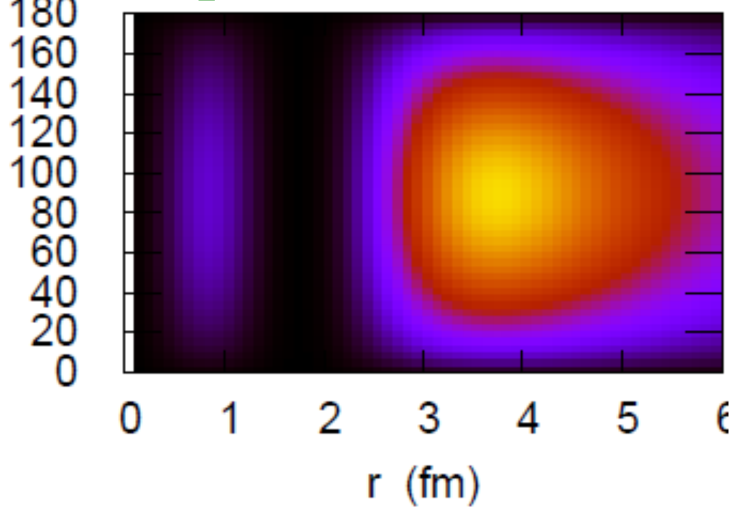
2 config. model

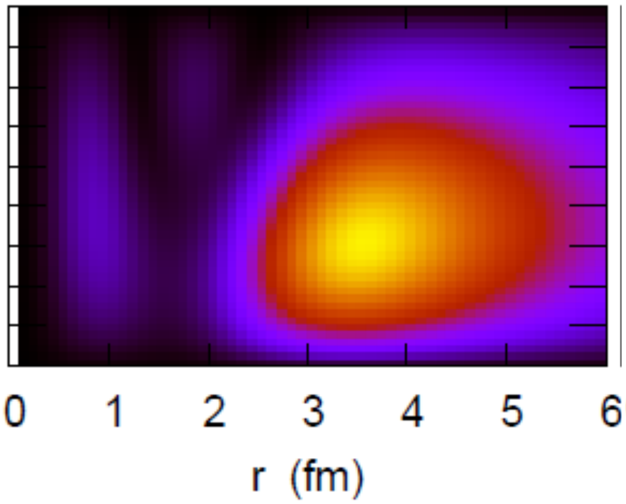
$$|\Psi\rangle = \sqrt{\alpha^2}\,|(1p_{3/2})^2\rangle + \sqrt{1-\alpha^2}\,|(2s_{1/2})^2\rangle$$

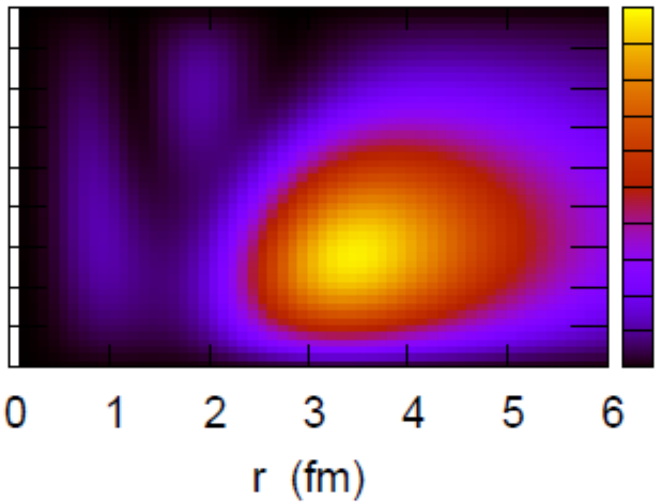

$p_{3/2}$ 100%, $s_{1/2}$ 0%


$p_{3/2}$ 95%, $s_{1/2}$ 5%


$p_{3/2}$ 90%, $s_{1/2}$ 10%

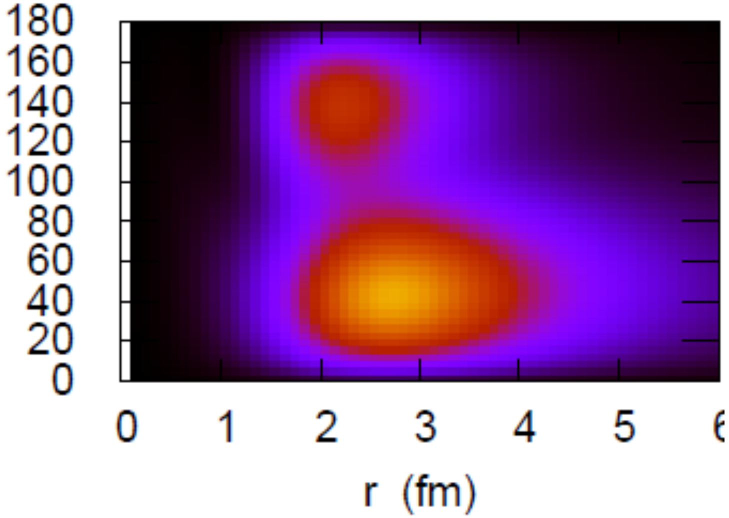

$p_{3/2}$ 0%, $s_{1/2}$ 100%


$p_{3/2}$ 5%, $s_{1/2}$ 95%

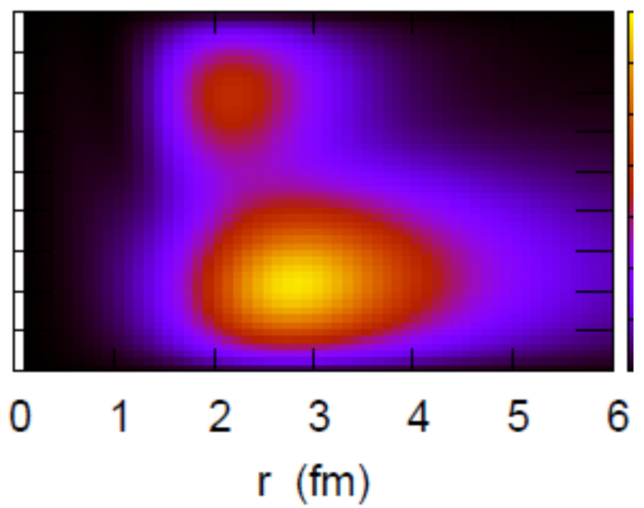

$p_{3/2}$ 10%, $s_{1/2}$ 90%

2 configuration model $|\Psi\rangle = \sqrt{\alpha^2}\,|(1p_{3/2})^2\rangle + \sqrt{1-\alpha^2}\,|(2s_{1/2})^2\rangle$

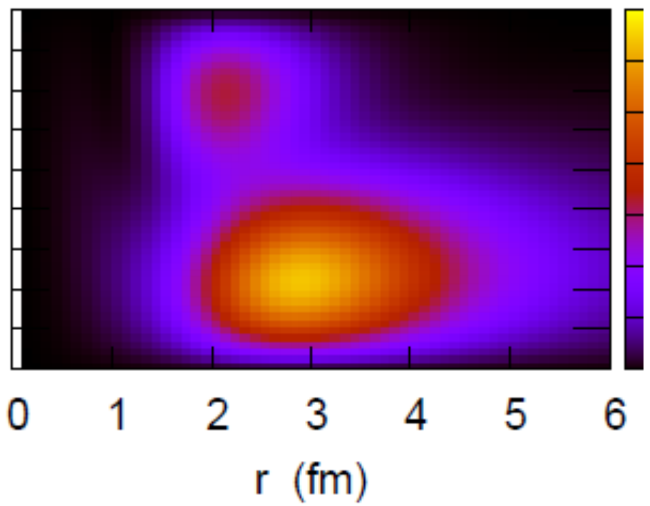
$p_{3/2}$ 80%, $s_{1/2}$ 20%



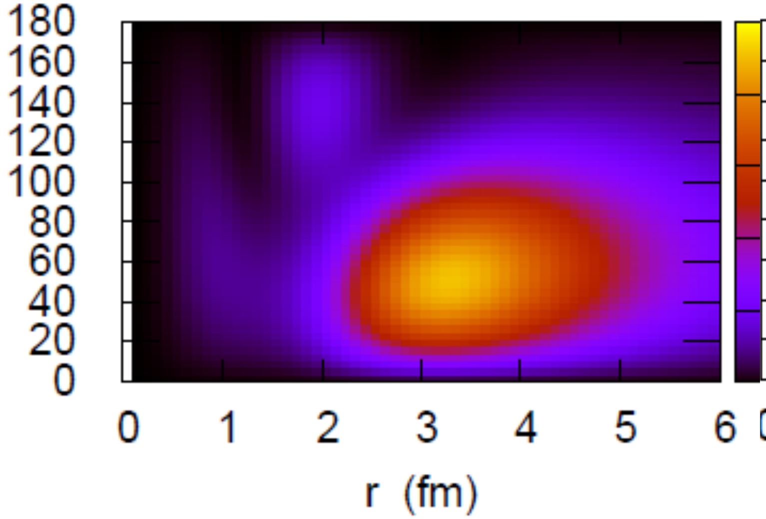
$p_{3/2}$ 70%, $s_{1/2}$ 30%



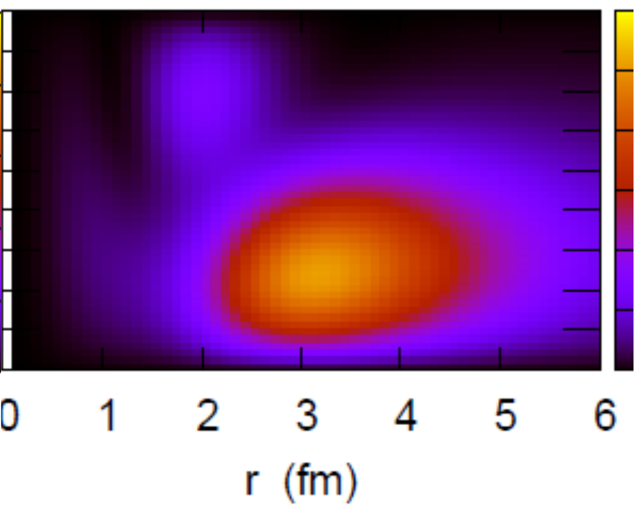
$p_{3/2}$ 60%, $s_{1/2}$ 40%



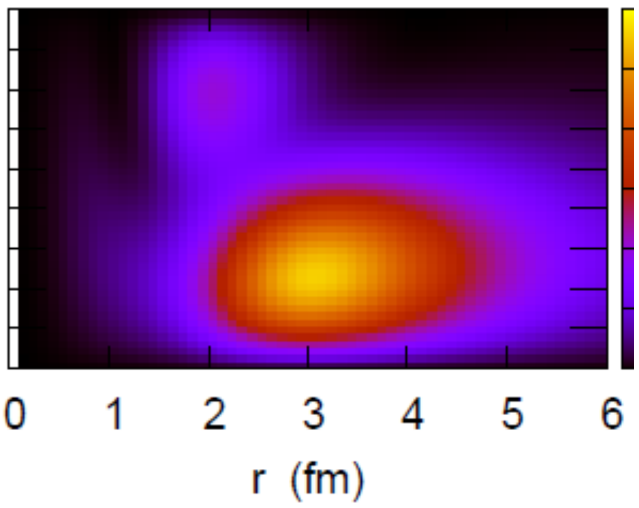
$p_{3/2}$ 20%, $s_{1/2}$ 80%



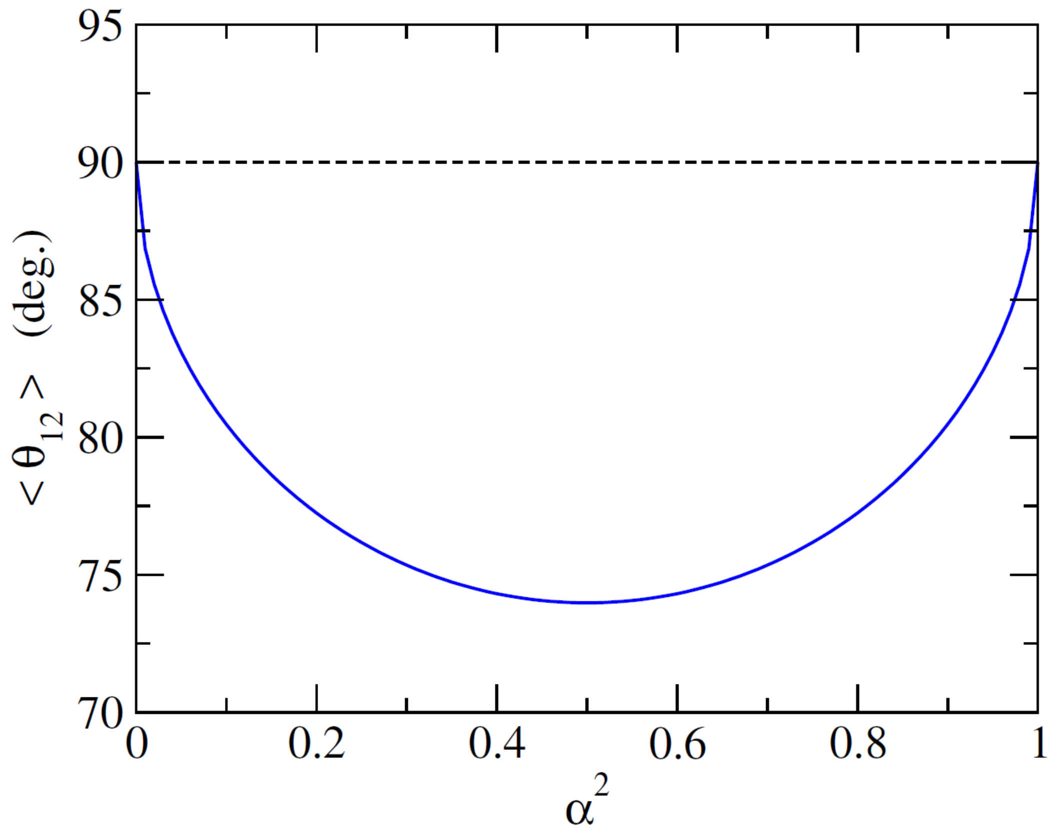
$p_{3/2}$ 30%, $s_{1/2}$ 70%



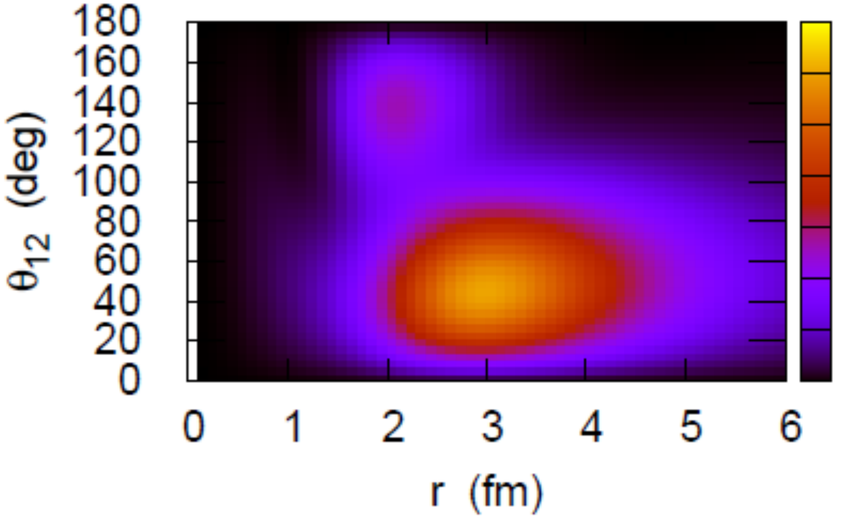
$p_{3/2}$ 40%, $s_{1/2}$ 60%



2 configuration model $|\Psi\rangle = \sqrt{\alpha^2}\,|(1p_{3/2})^2\rangle + \sqrt{1-\alpha^2}\,|(2s_{1/2})^2\rangle$



$p_{3/2}$ 50%, $s_{1/2}$ 50%



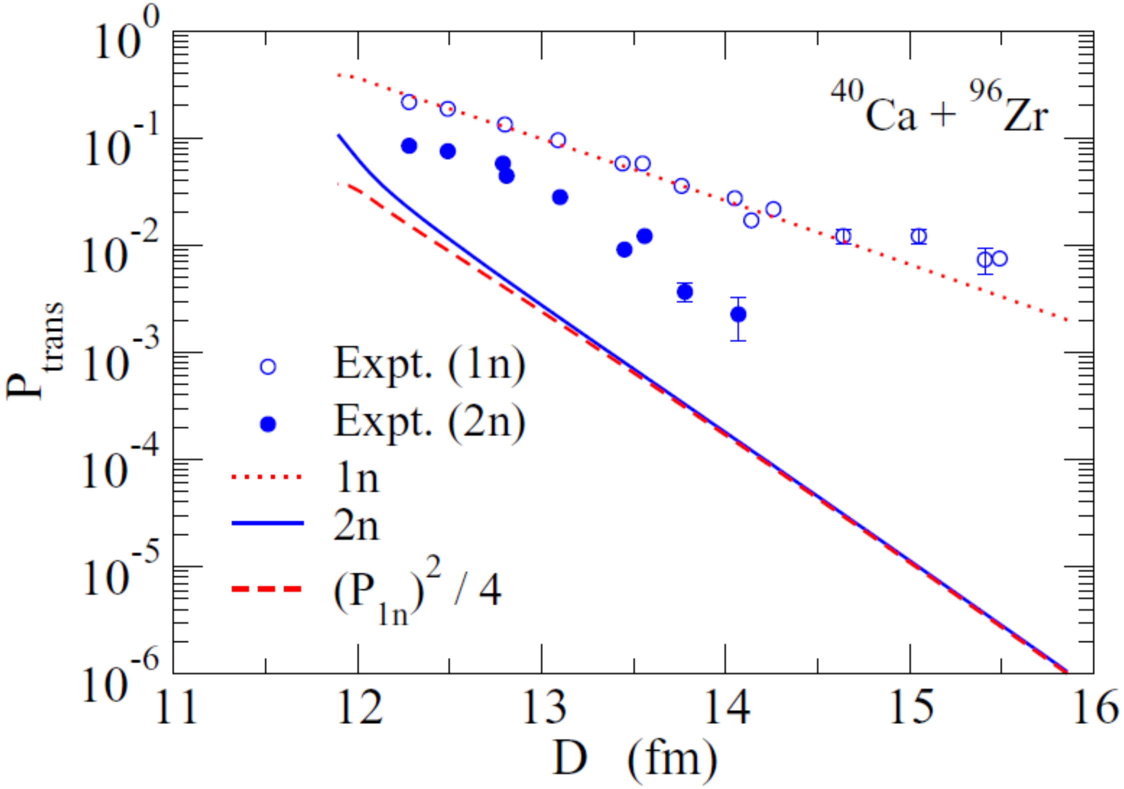
✓ symmetric at α^2=0.5→ the correlation does not matter whether the main configuration is $s_{1/2}$ or not

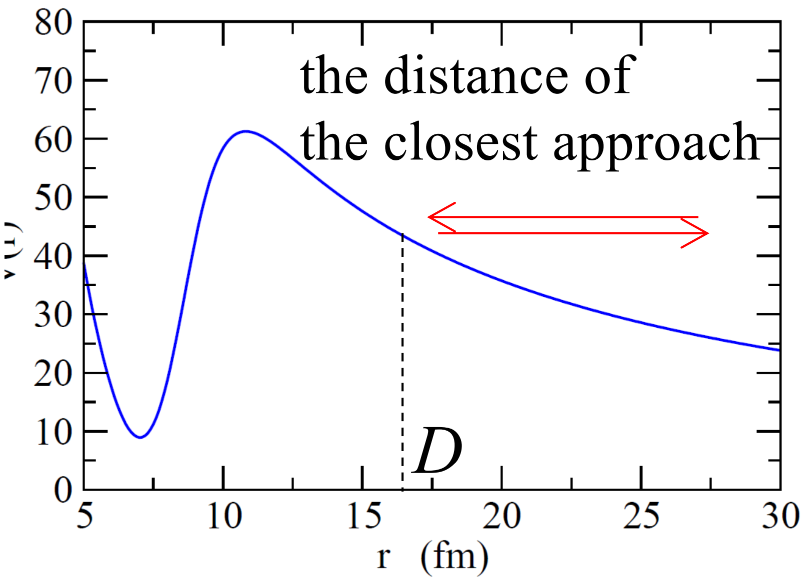
✓ even a small admixture → large asymmetry in density

What is a good measure of the degree of correlations?
(an open question)

Pair transfer and pair correlations

Sub-barrier two-neutron transfer reactions





$$D = \frac{Z_P Z_T e^2}{2E}\left[1+\sqrt{1+\cot^2\frac{\theta}{2}}\right]$$

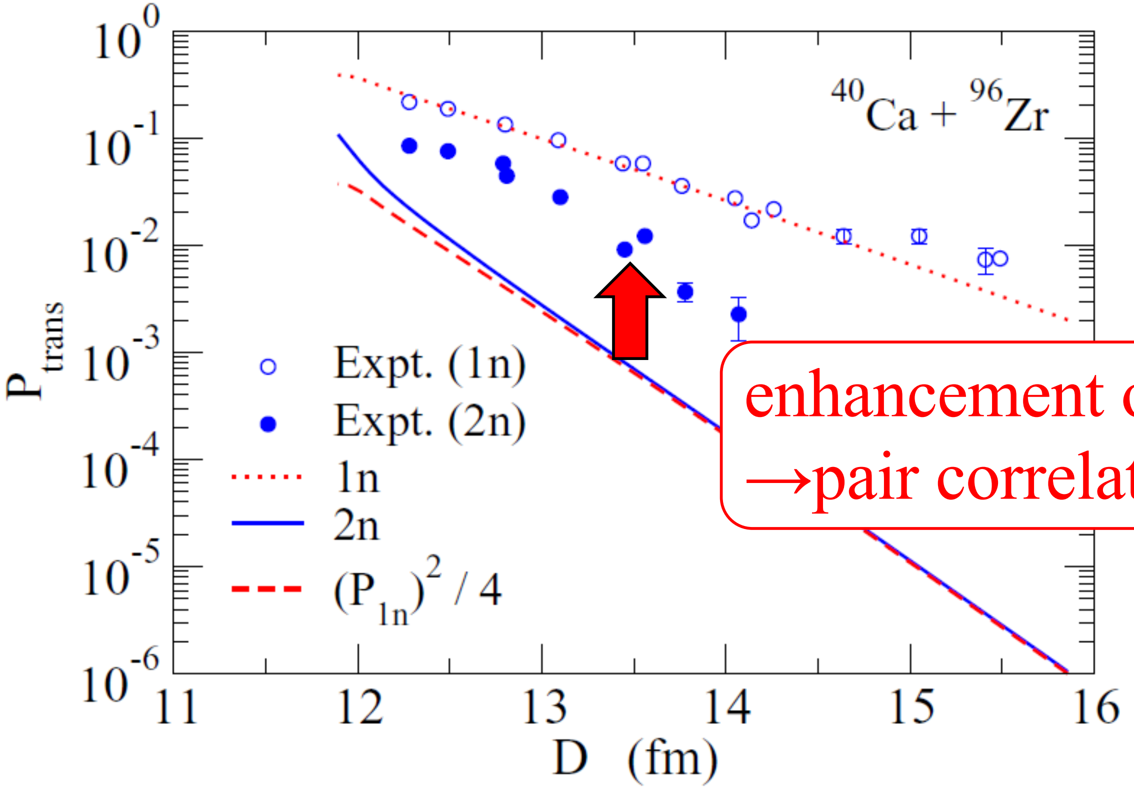
$$P_{\rm tr} \sim \frac{d\sigma_{\rm tr}}{d\sigma_R}$$

Calc. : K.H. and G. Scamps, PRC92 (‘15) 064602

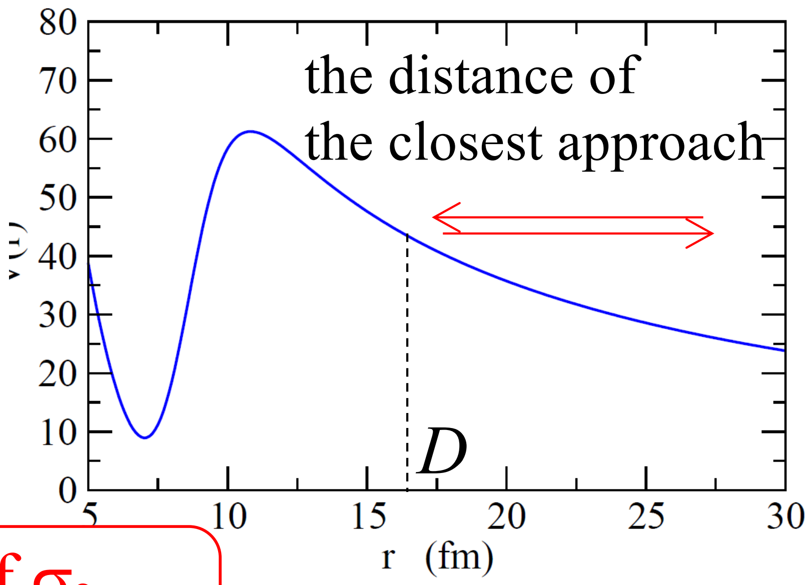
Exp. : L. Corradi et al., PRC84 (‘11) 034603

Pair transfer and pair correlations

Sub-barrier two-neutron transfer reactions



enhancement of σ_{2n}
→pair correlation

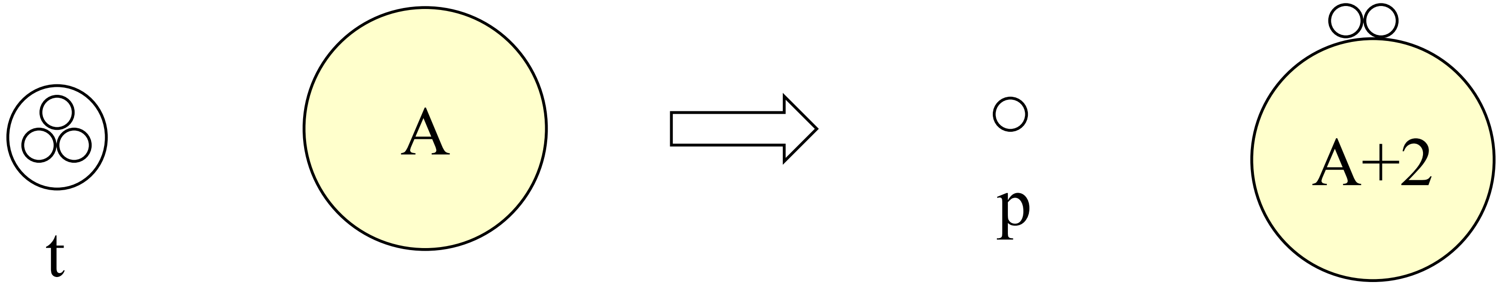


$$\frac{{}_PZ_Te^2}{2E}\left[1+\sqrt{1+\cot^2\frac{\theta}{2}}\right]$$

$$P_{\rm tr}\sim\frac{d\sigma_{\rm tr}}{d\sigma_R}$$

Calc.：K.H. and G. Scamps, PRC92 (‘15) 064602
Exp.： L. Corradi et al., PRC84 (‘11) 034603

Estimate for (t,p) and (p,t) reactions based on a one-step DWBA

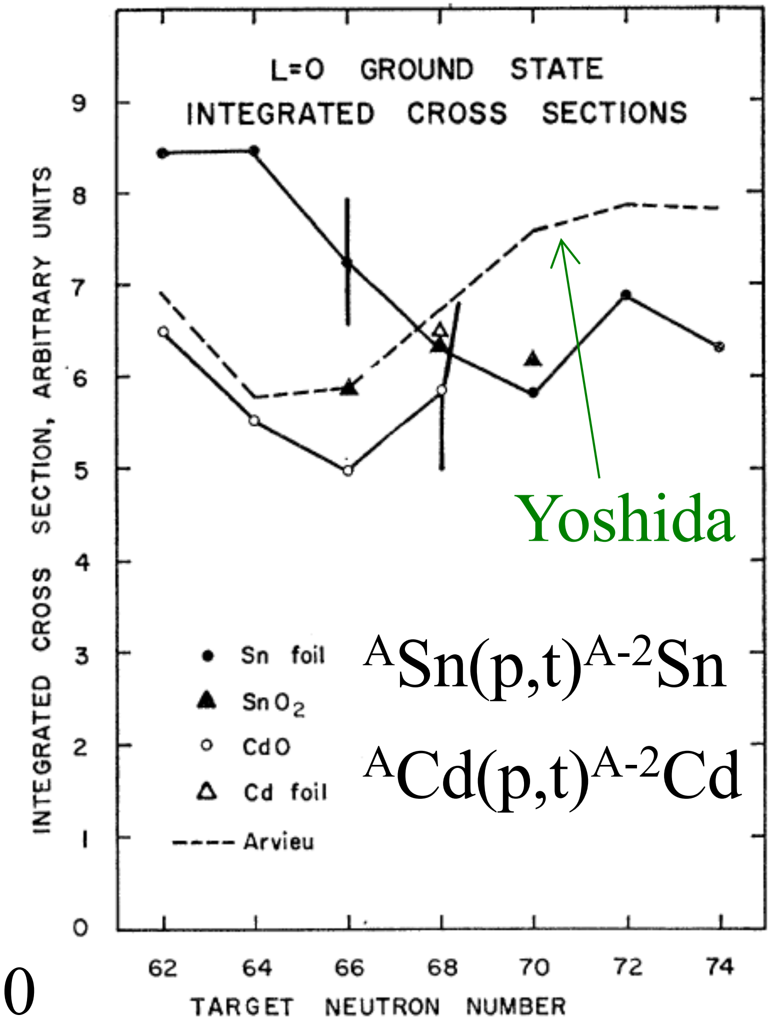


$$\boxed{\frac{\sigma(\mathrm{BCS}\to\mathrm{BCS})}{\sigma_{\mathrm{sp}}} = \frac{1}{j+1/2}\left(\frac{\Delta}{G}\right)^2}$$

S. Yoshida, Nucl. Phys. 33 (‘62) 685

for $\Delta \sim 1$ MeV, $G \sim 0.15$ MeV, j=5/2
→ enhancement: about 15 times

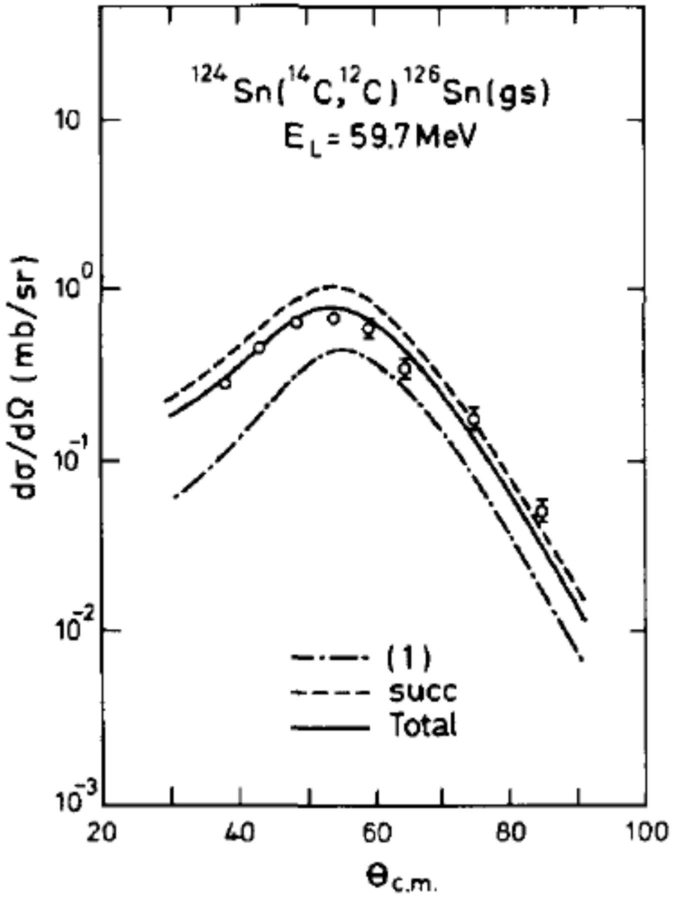
G. Bassani et al.,
Phys. Rev. 139 (‘65) B830



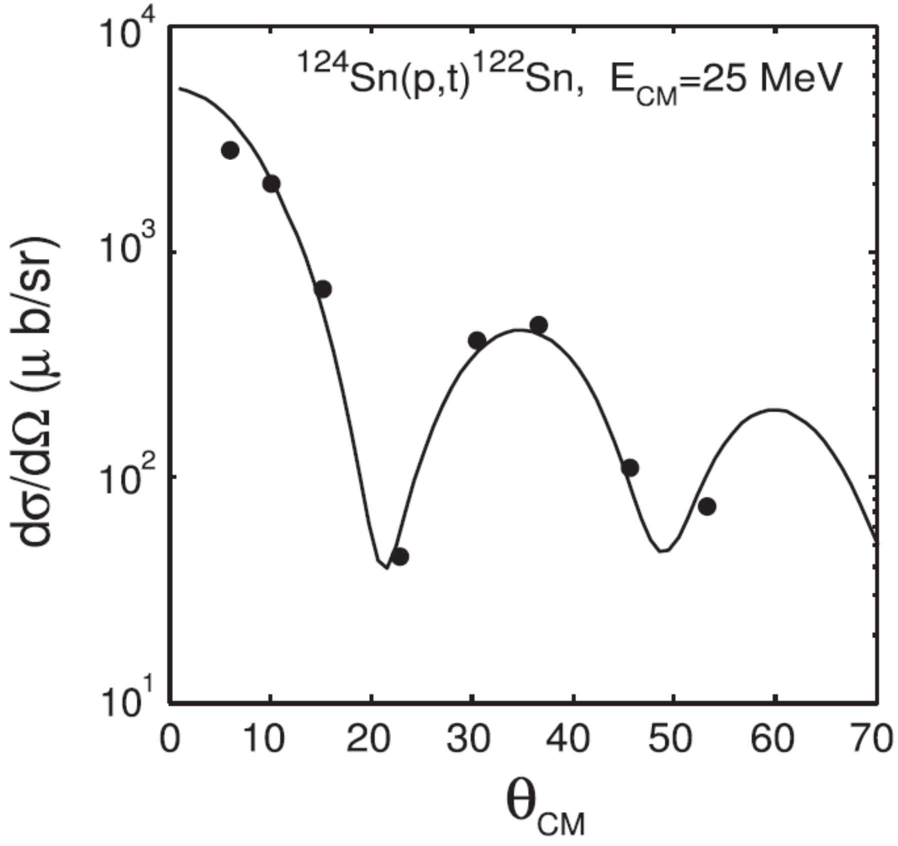
Pair transfer and pair correlations

Pair transfer reactions: complicated reaction dynamics
→ not straightforward to extract information on pairing from $\sigma_{transfer}$

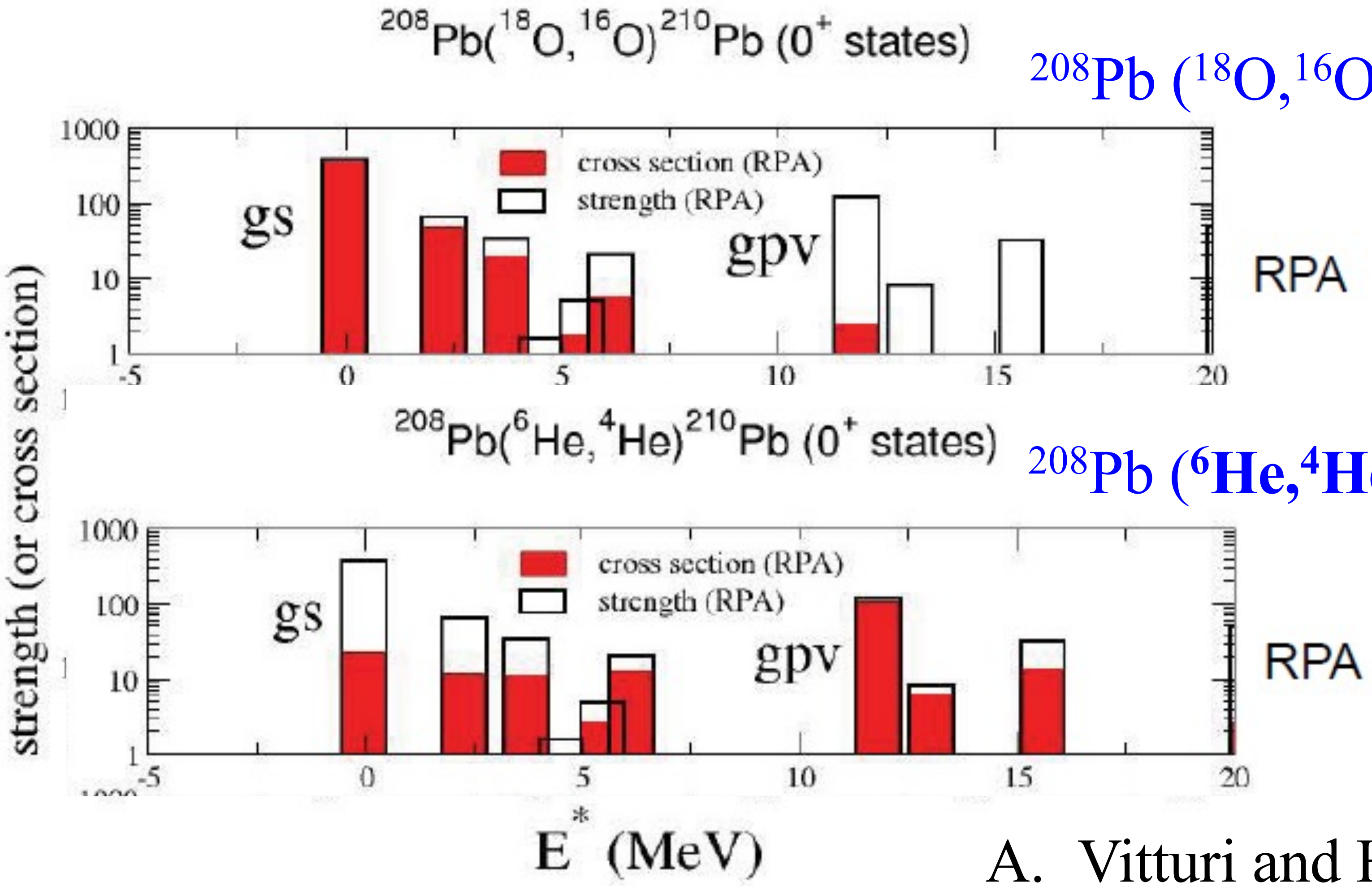
2-step DWBA



E. Maglione et al.,
Phys. Lett. 162B (‘85) 59.



G. Potel et al.,
PRL 107 (‘11) 092501



^{208}Pb (^{18}O,^{16}O)^{210}Pb (0$^+$)

^{208}Pb (**^{6}He,^{4}He**)^{210}Pb (0$^+$)

A. Vitturi and H.M. Sofia,
PTP Suppl. 196 ('12) 72

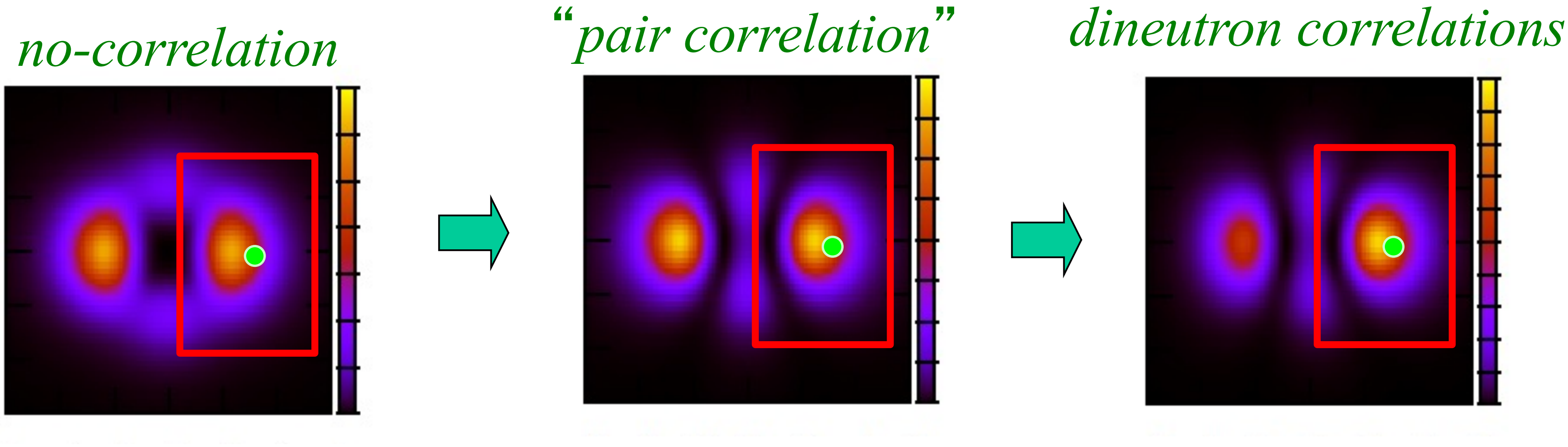
White : strength for pair addition

$$S = |\langle ^{210}\mathrm{Pb}|\psi^\dagger\psi^\dagger|^{208}\mathrm{Pb}\rangle|^2$$

Red: Pair transfer cross sections

Cross sections may not be large even when the strength is large
→ due to reaction dynamics (e.g., Q-value matching)

An additional issue : pair transfer reactions and dineutron correlations



If a pair transfer reaction probes the region of the red square

→pair transfer: distinguish between uncorrelated and correlated,
but not between the "pair correlation" and dineutron correlation ?

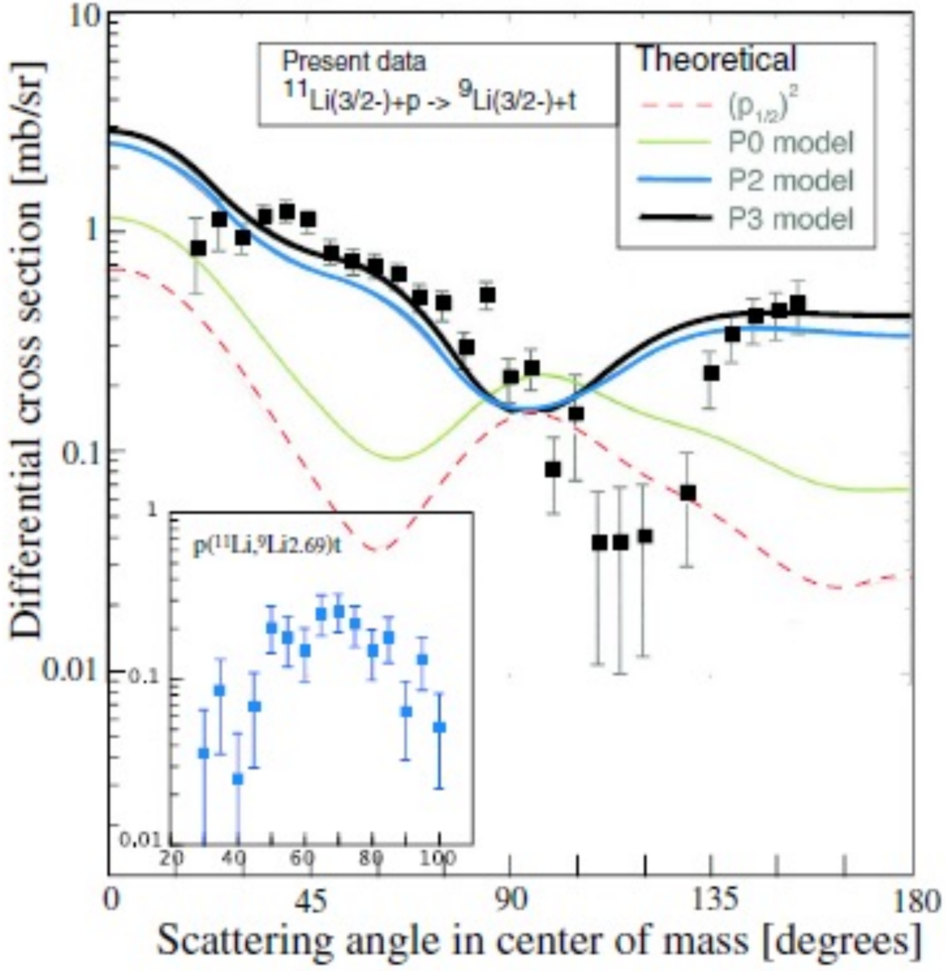
cf. A. Insolia, R.J. Liotta, and E. Maglione,
J. of Phhys. G15 ('89) 1249

→an open problem: need a new perspective

cf. (^{4}He,^{6}He) reaction@OEDO

Pair transfer of Borromean nuclei (Expt.)

^{1}H(^{11}Li,^{9}Li)^{3}H (TRIUMF)



- Uncorrelated: not reproduce the data
- P2 (31% $(s_{1/2})^2$) and P3 (45%) reproduce the data at forward angles
- But not for backward angles (Opt. pot.? intermediate states?)

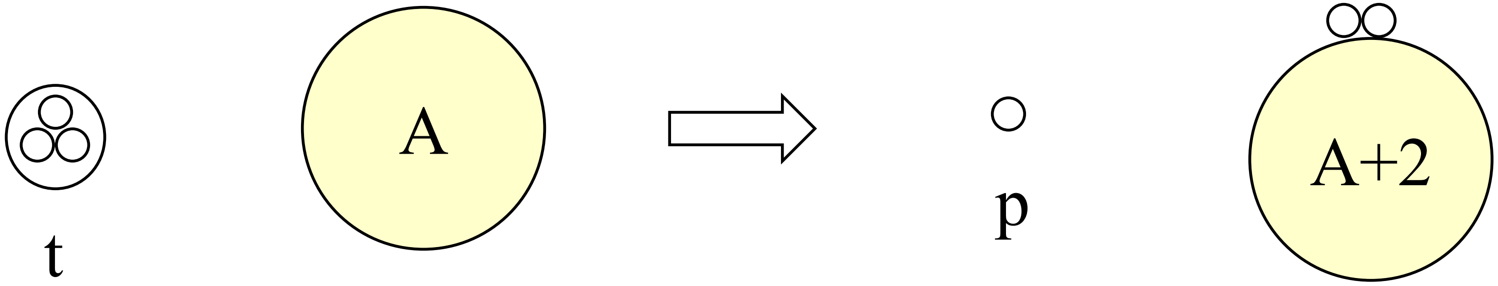
a treatment of ^{10}Li
as intermediate states

E_{lab} = 3 MeV/A

I. Tanihata et al., PRL100('08)192502

A further additional issue

After all, a one-step pair transfer process is not dominant



Remarks

- ＊ 1-step and 2-step are terminologies based on perturbation theory
- ＊ a relative importance of each process depends also on the post form or the prior form formulations (a choice of H_0)

$$h = \underline{\underline{t} + V_T(r)} + \underline{V_P(r)}$$

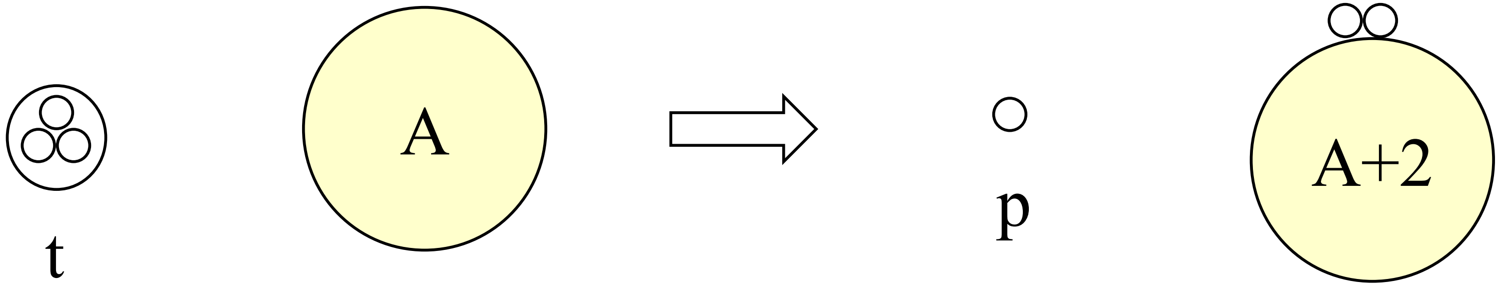
Broglia et al.,

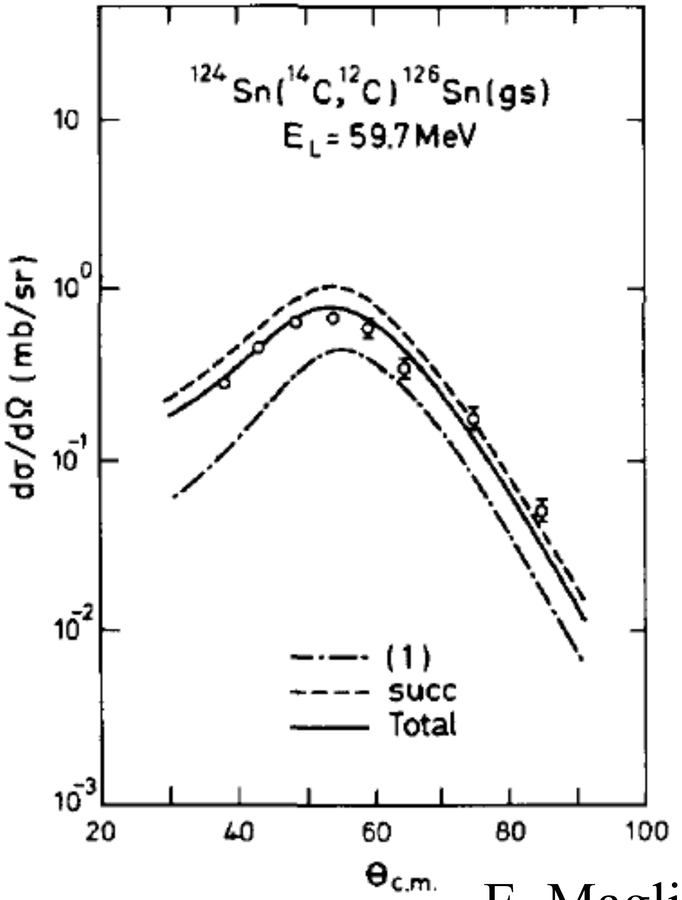
$$a_{\mathsf{tr}} = a_{\mathsf{sim}} + a_{\mathsf{succ}} + a_{\mathsf{non-orthog}} \sim a_{\mathsf{succ}}$$

$$= \tilde{a}_{\mathsf{sim}} + \tilde{a}_{\mathsf{succ}} + \tilde{a}_{\mathsf{non-orthog}}$$

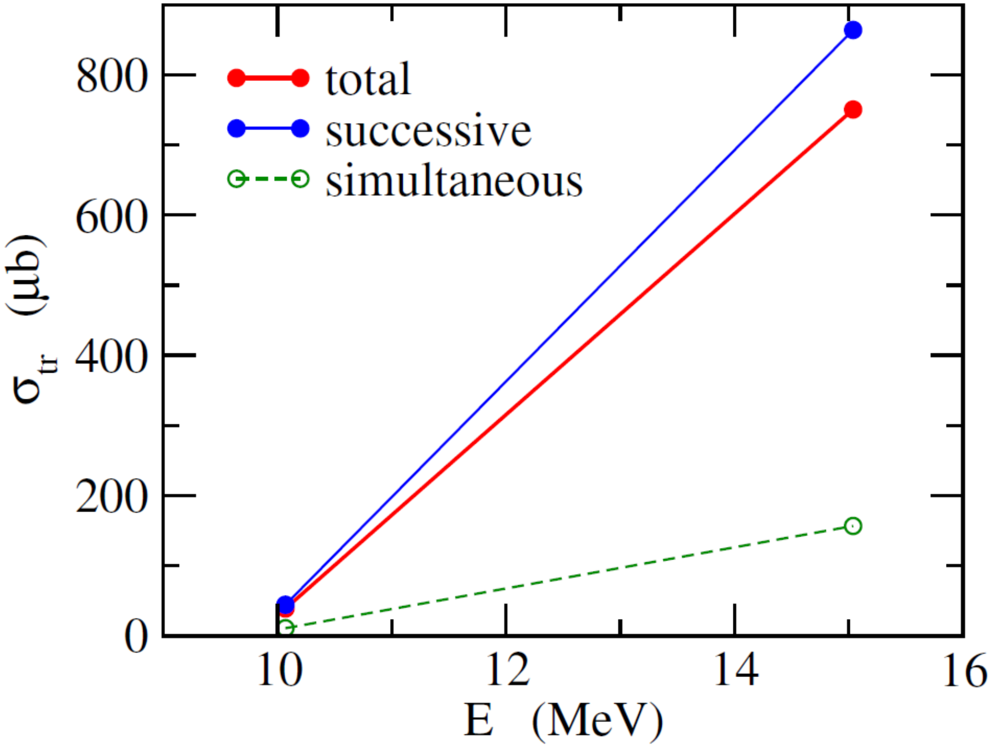
A further additional issue

After all, a one-step pair transfer process is not dominant





^{132}Sn(p,t)^{130}Sn



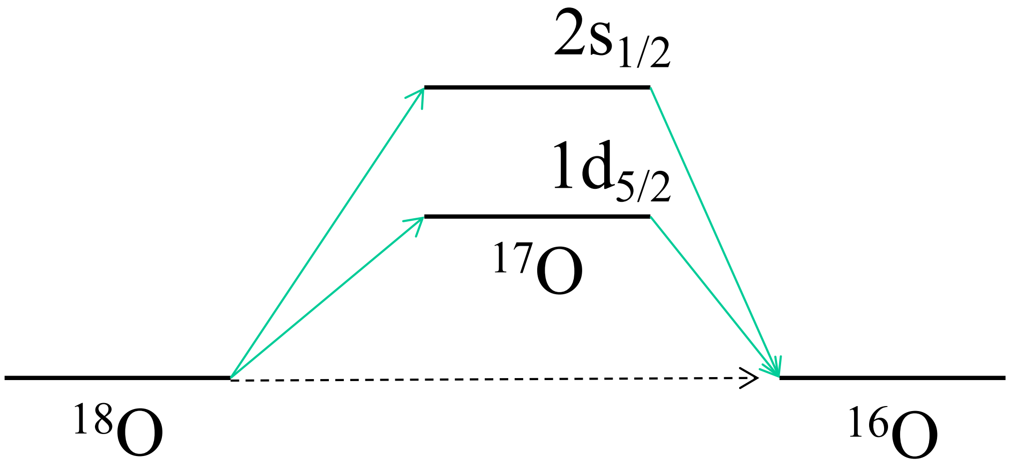
E. Maglione et al. PLB ('85)

G. Potel et al., PRL ('11)

A further additional issue

After all, a one-step pair transfer process is not dominant

→the main process is a sequential 1n transfer



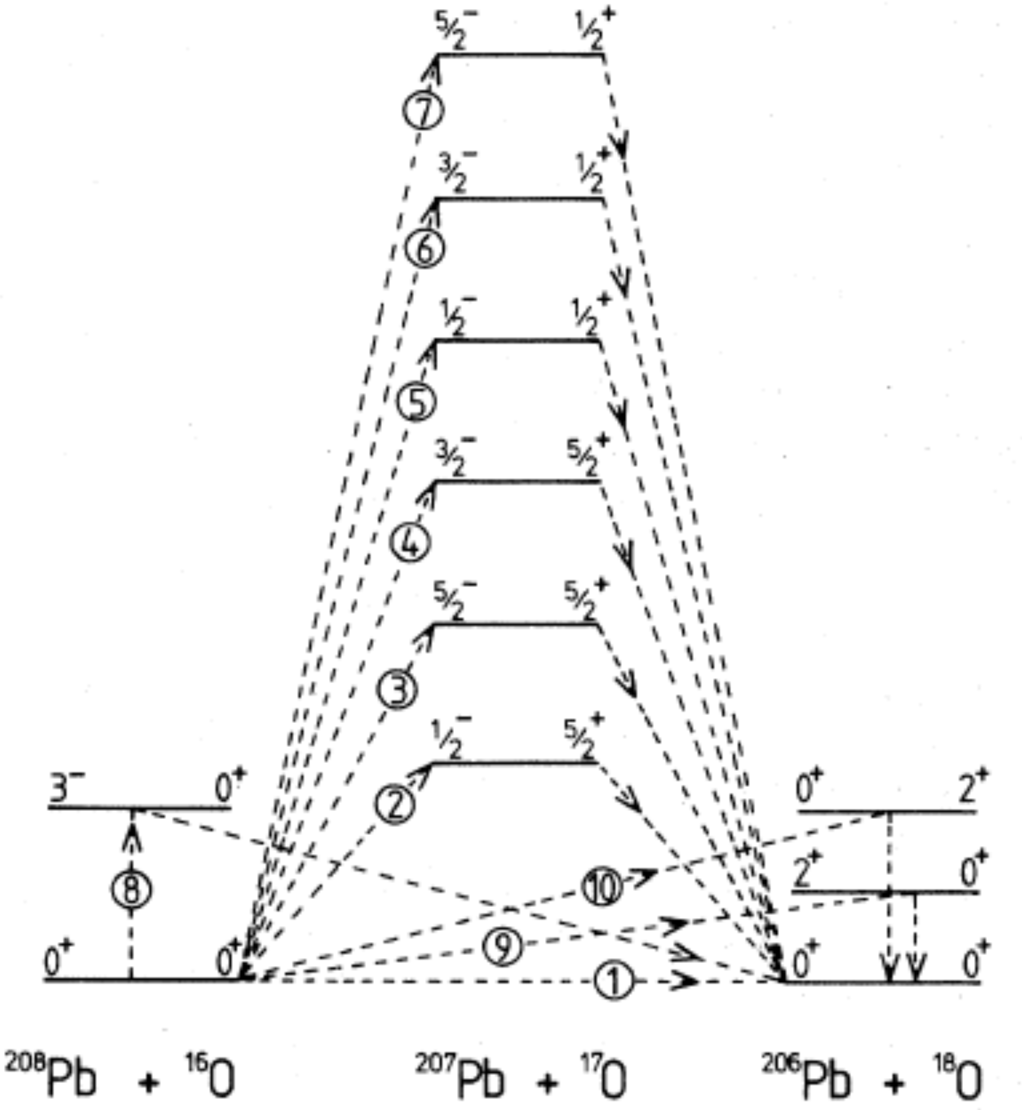
$0.8\ (1d_{5/2})^2$
$+0.6\ (2s_{1/2})^2$

pair correlation→a coherent superposition of many 1n transfer processes

* In reality, superfuidity in a target nucleus has also to be taken into account

dependence of incident energy? → still an open problem

A related problem: Pair transfer reactions of neutron-rich nuclei



For neutron-rich nuclei, many intermediate states will be unbound

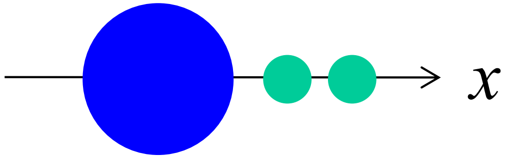
⇩

How much will the reaction dynamics be altered?

↑

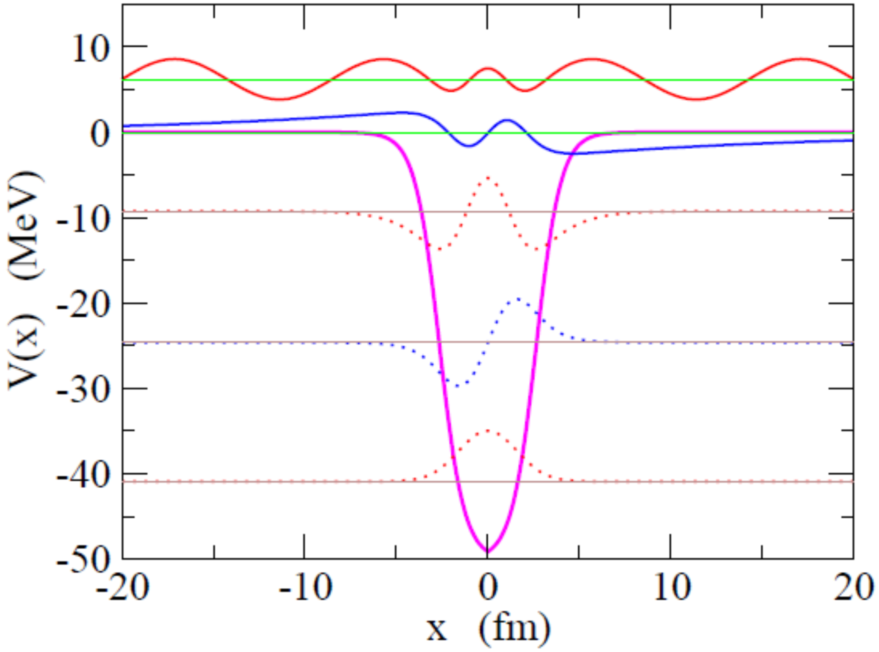
Another open problem

Pair transfer reaction with a one-dimensional 3-body model



based on
K.H., A. Vitturi, F. Perez-Bernal,
and H. Sagawa, J. of Phys. G38 ('11) 015105

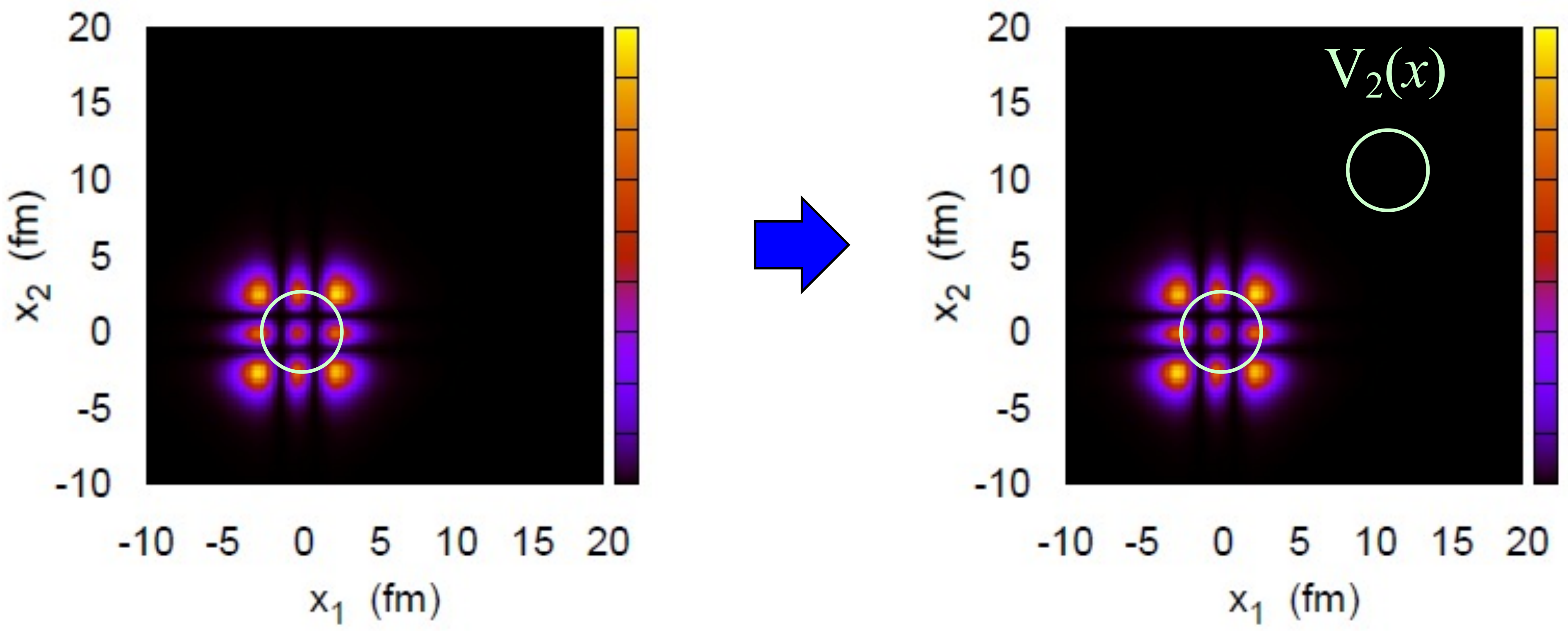
$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} + V(x_1) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V(x_2) + v_{nn}(x_1,x_2)$$



$$v_{nn}(x_1,x_2) = -g\left(\frac{V(\bar{x})}{V_0}\right)\delta(x_1 - x_2)$$

pairing correlation only
inside a nucleus

$$\rho(x_1, x_2) = |\Psi_{\rm gs}(x_1, x_2)|^2$$

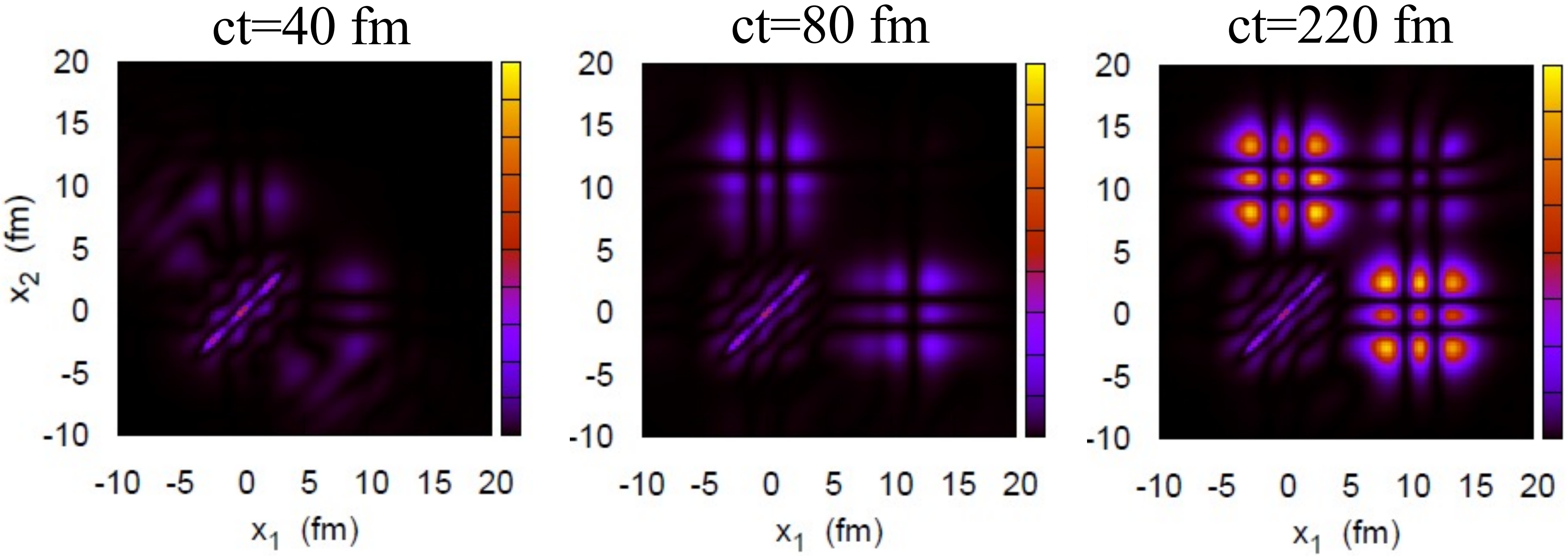


time-evolution

$$i\hbar\frac{\partial}{\partial t}\Psi(x_1, x_2, t) = H\Psi(x_1, x_2, t)$$

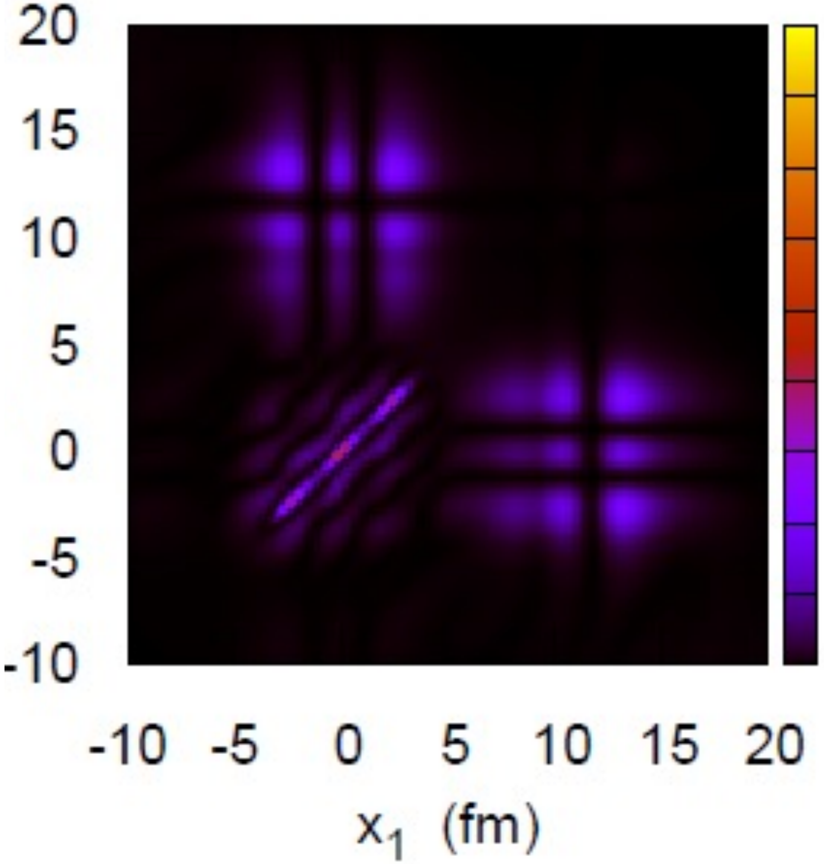
$$\Psi(x_1, x_2, t) = \alpha \Psi_{\mathrm{gs}}(x_1, x_2) + \tilde{\Psi}(x_1, x_2, t)$$

$$\rightarrow \tilde{\rho}(x_1, x_2, t) = |\tilde{\Psi}(x_1, x_2, t)|^2$$

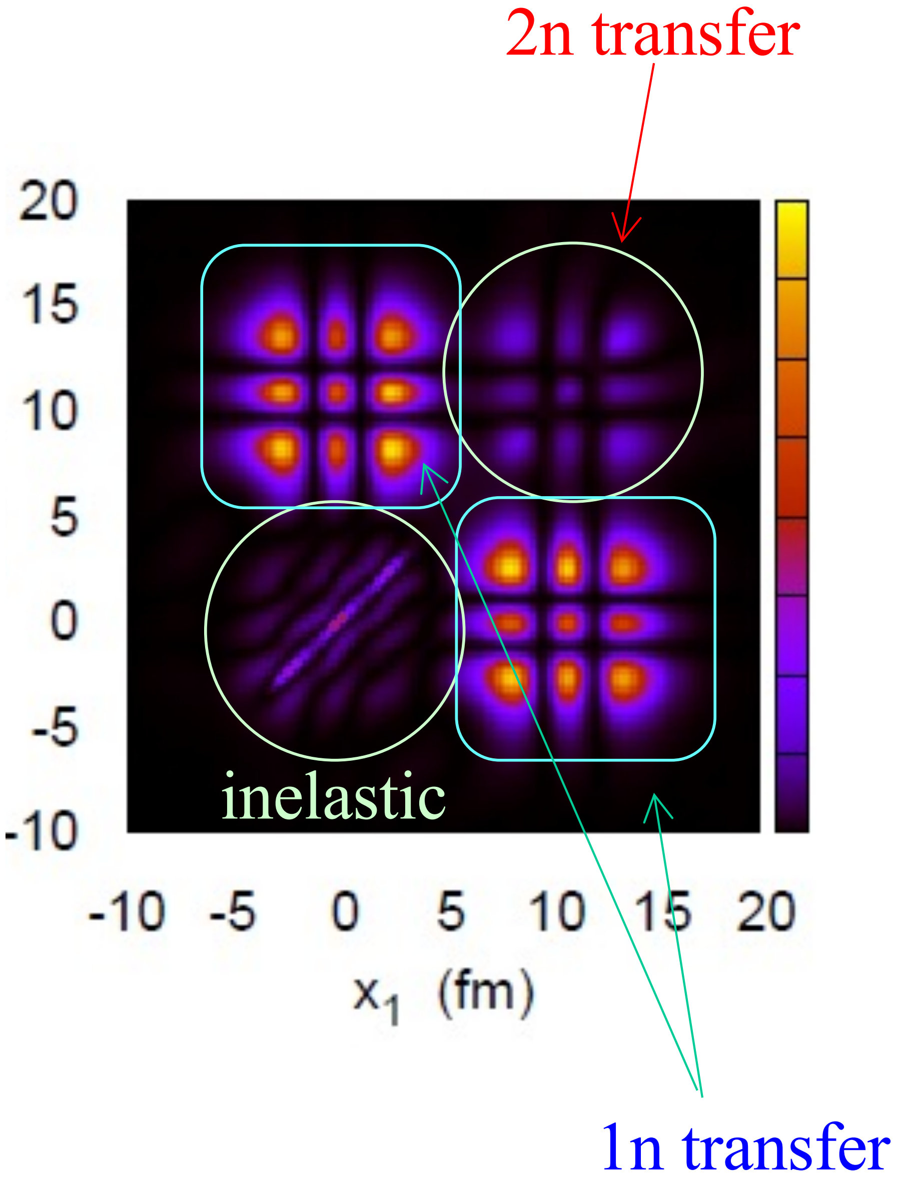


$$\Psi(x_1, x_2, t) = \alpha\Psi_{\rm gs}(x_1, x_2) + \tilde{\Psi}(x_1, x_2, t)$$

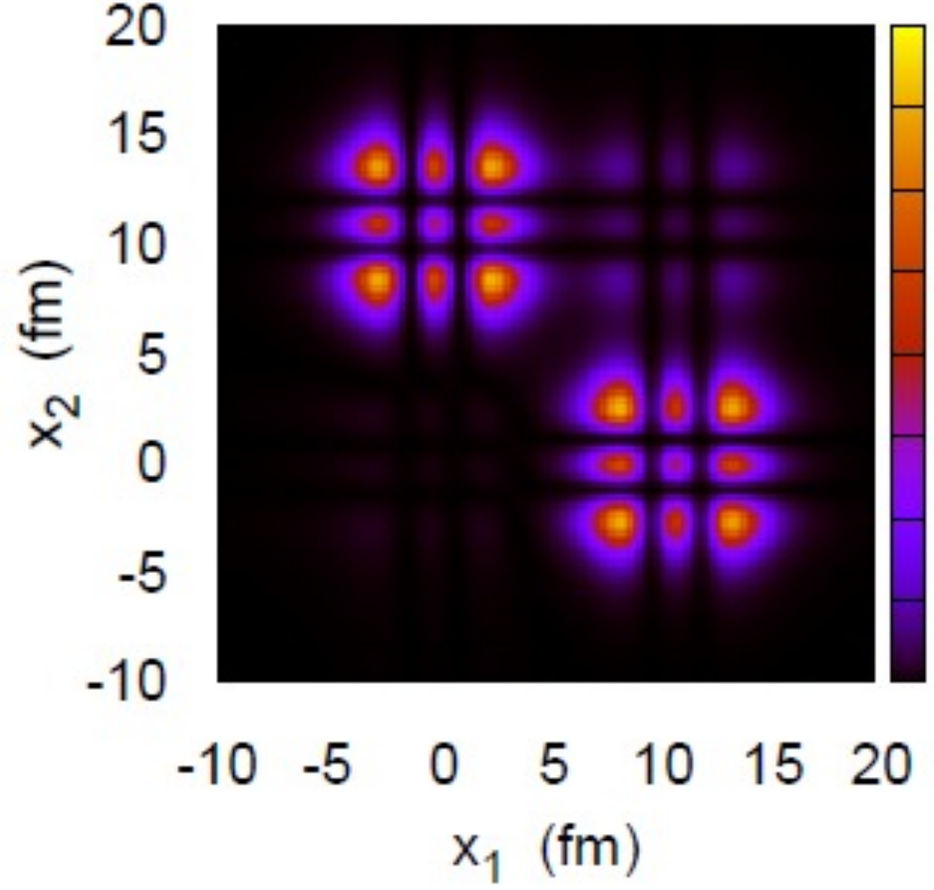
$$\to \tilde{\rho}(x_1, x_2, t) = |\tilde{\Psi}(x_1, x_2, t)|^2$$

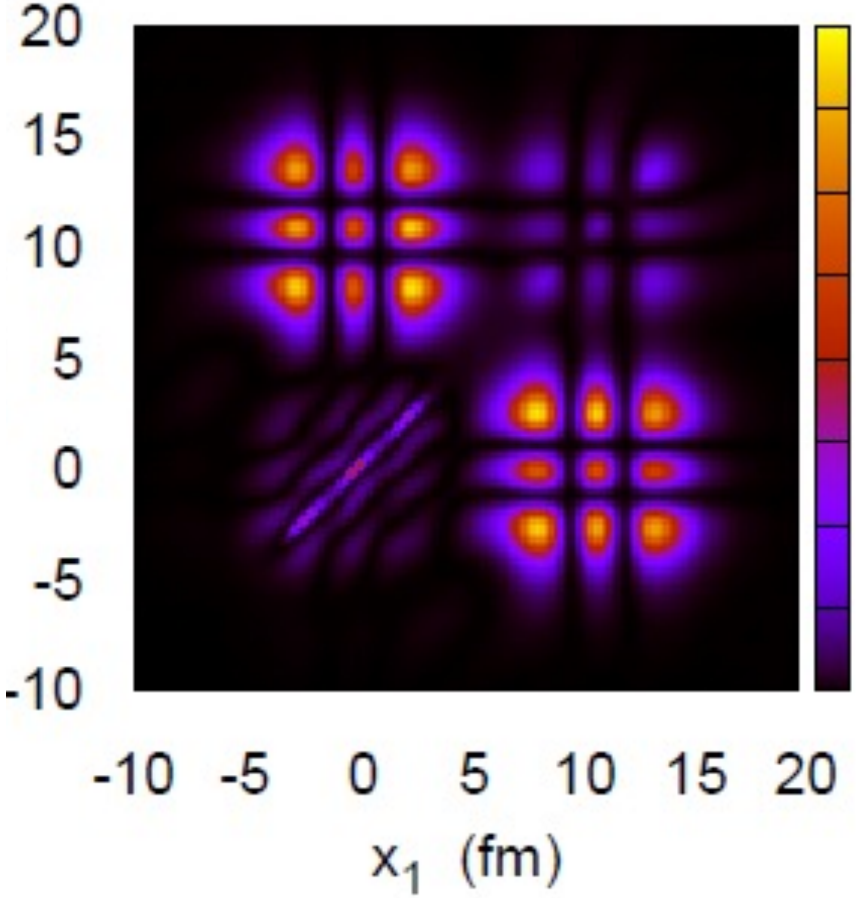


sequential: the main process



ct=220 fm


without correlation

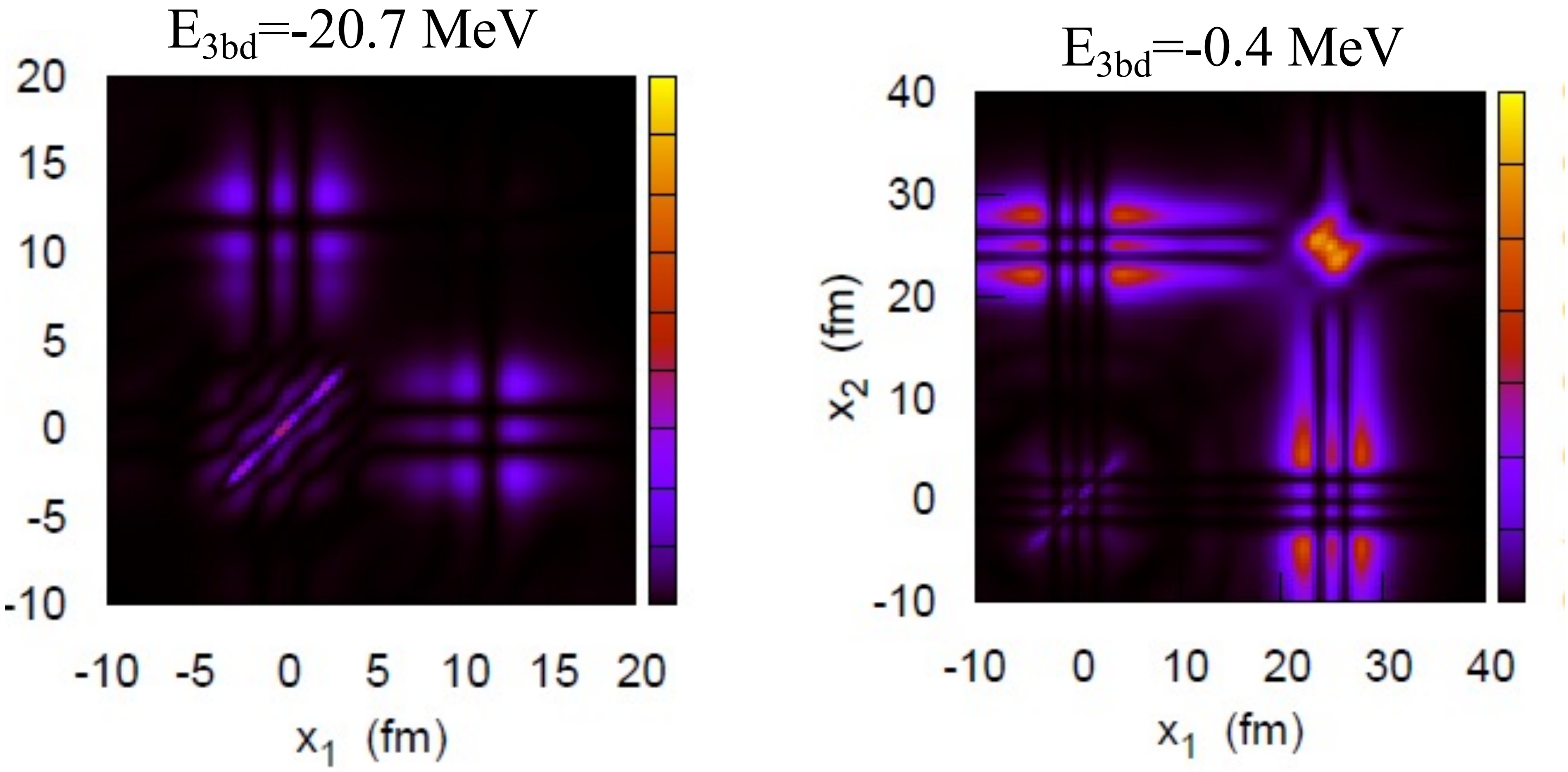

with correlation

Due to correlations

- inelastic scattering
- 2n transfer reaction

are enhanced

ct=80 fm



E_{3bd}=-20.7 MeV

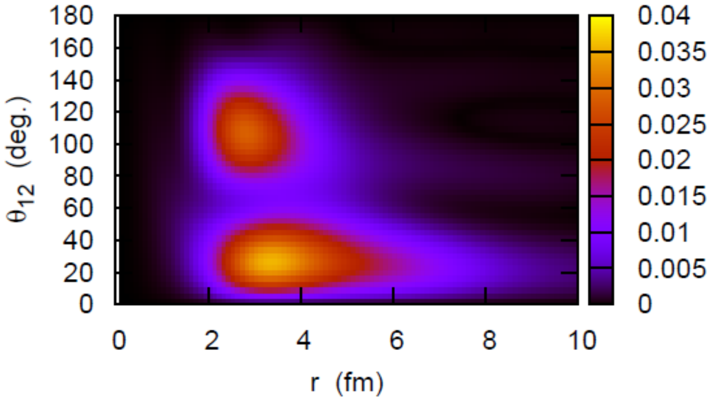
E_{3bd}=-0.4 MeV

For weakly bound situation: $P_{2n} > P_{1n}$ (consistent with expt.)

Time-dep. approach: a good method to understand complicated pair transfer processes

Future problesms: 3D calculations, dynamical calculations

Summary



➢ Dineutron correlations ← mixing of config. consisted of opposite parity states

- an attractive pairing interaction → dineutron

  even a small mixing → a large asymmetry in density

- anti-correlation if the interaction is repulsive

  ✓ T=1 particle-hole interaction

➢ Future theoretical perspectives

- An extension of 3-body model with core deformation
- An extention to a 5-body mode: double dineutrons? ← ^{28}O
- two-nucleon transfer reactions: time-dependent approach?