Beyond-mean-field approach to low-lying spectra of Λ hypernuclei

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1. Introduction
2. Mean-field approximation and beyond
3. Microscopic particle-rotor model for hypernuclei
4. Results for $^{13}\Lambda C$, $^{21}\Lambda Ne$, and $^{155}\Lambda Sm$
5. Summary

H. Mei, K.H., J.M. Yao, and T. Motoba, PRC90 (‘14) 064302
PRC91 (‘15) 064305
Impurity effects: one of the main interests of hypernuclear physics

how does $\Lambda$ affect several properties of atomic nuclei?

- size, shape, density distribution, single-particle energy,
  shell structure, fission barrier……

Theoretical approaches:

- cluster model
- shell model
- AMD (Isaka’s talk)
- self-consistent mean-field models
  (Zhou and Vesely’s talks)

A figure from a recent review:
Mean-field approximation and beyond

**Self-consistent mean-field (Hartree-Fock) method:**

- independent particles in a mean-field potential
- global theory for **the whole nuclear chart**
- intuitive picture for nuclear deformation
- optimized shape can be automatically determined

Suitable for a discussion on shape of hypernuclei

Myaing Thi Win and K.H., PRC78('08)054311
Mean-field approximation and beyond

Drawbacks of the mean-field approximation: nuclear spectrum

✓ body-fixed frame formalism $\rightarrow$ intuitive picture of nuclear def.
✓ spectrum: lab-frame $\leftrightarrow$ transformation from intrinsic to lab. frames

\[ |\psi_{I_cM_c}(\beta)\rangle = \hat{P}_{M_cK_c}^{I_c} \hat{P}^N \hat{P}^Z |\psi_{\text{MF}}(\beta)\rangle \]

angular momentum + particle number projections

nuclear spectrum: requires to go beyond the mean-field approximation

✓ quantum fluctuation

\[ |\Phi_{I_cM_c}\rangle = \int d\beta f(\beta) |\psi_{I_cM_c}(\beta)\rangle \]

generator coordinate method (GCM)

beyond the mean-field approximation
beyond mean-field approximation

J.M. Yao, K.H. et al., PRC89 (‘14) 054306
Beyond mean-field approximation

- **Projection+GCM for the whole \((A_c+1)\) system**
  
  H. Mei, K.H. and J.M. Yao, in preparation

- **Microscopic particle-rotor model for single-\(\Lambda\) hypernuclei**
  
  H. Mei, K.H., J.M. Yao, and T. Motoba, PRC90('14)064302, PRC91('15) 064305

  i) Beyond mean-field calculations for e-e core: \(|\Phi_{0+}\rangle, |\Phi_{2+}\rangle, |\Phi_{4+}\rangle, \cdots\)

  ii) Coupling of \(\Lambda\) to the core states

\[
|\Phi_{IM}\rangle = \sum_{j,l,I_c} \begin{bmatrix} \Lambda \\ j,l \end{bmatrix} (IM) \quad I_c
\]

\(\Lambda+\)core model with core excitations
Microscopic Particle-Rotor Model for $\Lambda$ hypernuclei

Example: $^{13}\Lambda\text{C}$

i) beyond mean-field calculations for e-e core ($^{12}\text{C}$) : GCM + projections

$$|\Phi_{IcMc}\rangle = \int d\beta f(\beta) |\Psi_{IcMc}(\beta)\rangle = \int d\beta f(\beta) \hat{P}^{Ic}_{McKc} \hat{P}^N \hat{P}^Z |\Psi_{\text{MF}}(\beta)\rangle$$

- axial symmetry
- relativistic PC-F1
Microscopic Particle-Rotor Model for $\Lambda$ hypernuclei

Example: $^{13}_\Lambda$C

(i) beyond mean-field calculations for e-e core ($^{12}$C)
(ii) coupling of $\Lambda$ to the core states

\[
|\Phi_{IM}\rangle = \sum_{j,l,I_c} \left[ \psi_{jl}(r_\Lambda) \otimes |\Phi_{I_c}\rangle \right]^{(IM)}
\]

\[
|1/2^+\rangle = \alpha |s_{1/2} \otimes 0^+\rangle + \beta |d_{5/2} \otimes 2^+\rangle + \cdots
\]

particle-core model with core excitations

cf. conventional particle-rotor model:

core states $\rightarrow$ macroscopic rotor (Wigner’s D-functions) with a fixed deformation

our approach: a microscopic version of particle-rotor model
Results for $^{13}_\Lambda C$

$\mathcal{L}_{\Lambda N} = -\alpha_{V}^{N \Lambda} \delta(r_{\Lambda} - r_{N}) - \alpha_{S}^{N \Lambda} \gamma_{\Lambda}^{0} \delta(r_{\Lambda} - r_{N}) \gamma_{N}^{0}$

✓ coupling to $0^+, 2^+$, and $4^+$ of $^{12}C$

Parameters $\leftarrow B_{\Lambda}$ of $^{13}_\Lambda C$

$ls$ splitting:

- $199$ keV
- cf. expt.
- $152 +/- 90$ keV

* qualitatively similar to the previous cluster model calc.
Results for $^{13}_Λ$C

B(E2) transition rates ($e^2$ fm$^4$)

- $B(E2)$ : $\sim 14\%$ reduction

H. Mei, K.H., J.M. Yao, T. Motoba, PRC91(’15) 064305

<table>
<thead>
<tr>
<th></th>
<th>$^{12}$C</th>
<th>$^{13}_Λ$C</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>-0.29</td>
<td>-0.23</td>
</tr>
<tr>
<td>$r_p$ (fm)</td>
<td>2.44</td>
<td>2.39</td>
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</tbody>
</table>
Results for $^{21}_{\Lambda}$Ne

$^{20}$Ne $^\rightarrow$ $^{21}_{\Lambda}$Ne

$E^*$ [MeV]  $B$(E2) [$e^2fm^4$]

9.52 $^{1/2-}$ $^{1/2-}$  $\sim 42\% |p \otimes 0^+\rangle$

9.48 $^{3/2-}$

$\beta_{\text{min}} = 0.376, 0.67$

3.11 $^{4+}$  3.08 $^{7/2^+, 9/2^+}$  $99\% |s_{1/2} \otimes 4^+\rangle + \cdots$

$76.2$

$73.2$

$56.1$

$54.3$

$1.12$ $^{2+}$  1.18 $^{3/2^+, 5/2^+}$  $98\% |s_{1/2} \otimes 2^+\rangle + \cdots$

$2^{+}$  $3/2^+$  $1/2^+$

$2^{+}$  $3/2^+$

$2^{+}$  $0^+$

$^{21}_{\Lambda}$Ne

$^{20}$Ne

$\beta_{\text{min}} = 0.376, 0.63$
Results for $^{155}_{\Lambda}$Sm

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$^{155}_{\Lambda}$Sm

\[ \begin{align*}
E^* [\text{MeV}] & \quad B(\text{E2}) [e^2b^2] \\
6.28 & \quad 1/2^- & \sim 32\% |p \otimes 0^+\rangle \\
6.27 & \quad 3/2^- & \quad + 66\% |p \otimes 2^+\rangle + \cdots
\end{align*} \]

\[ \begin{align*}
0.346 & \quad 4^+ \\
1.35 & \quad (\beta_{\text{min}} = 0.3604)
\end{align*} \]

\[ \begin{align*}
0.345 & \quad 7/2^+, 9/2^+ \\
1.34 & \quad 98\% |s_{1/2} \otimes 4^+\rangle + \cdots
\end{align*} \]

\[ \begin{align*}
0.105 & \quad 2^+ \\
0.936 & \quad (\beta_{\text{min}} = 0.3604)
\end{align*} \]

\[ \begin{align*}
0.106 & \quad 3/2^+, 5/2^+ \\
0.928 & \quad 98\% |s_{1/2} \otimes 2^+\rangle + \cdots
\end{align*} \]

\[ \begin{align*}
\text{\^{154}Sm} & \quad 0^+ \\
\text{\^{155}_{\Lambda}Sm} & \quad 1/2^+
\end{align*} \]
Summary

Microscopic particle-rotor model for spectrum of $\Lambda$-hypernuclei

- $\Lambda + \text{GCM}$ states for core
- microscopic version of particle-rotor model
- first calculation for low-lying spectrum based on mean-field type calculations
- application to $^{13}\Lambda C$: good agreement with the experimental data
- reduction of B(E2) due to a change in def. param.
- application to heavier hypernuclei: $^{21}\Lambda\text{Ne}$ and $^{155}\Lambda\text{Sm}$

Future perspectives

- more consistent interaction (the derivative and tensor terms): in progress
- extension to include triaxiality (cf. $^{25}\Lambda\text{Mg}$)

Challenging problem

- application to formation reactions of hypernuclei
- description of ordinary odd-mass nuclei: Pauli principle?