

# Heavy-ion fusion reactions: quantum tunneling with many degrees of freedom and superheavy elements

Kouichi Hagino  
Tohoku University

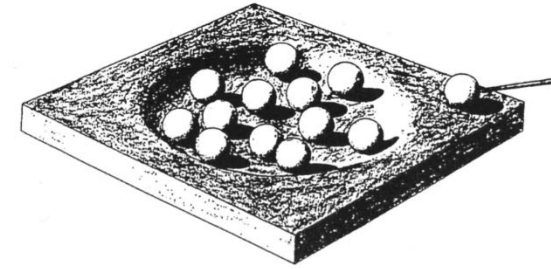
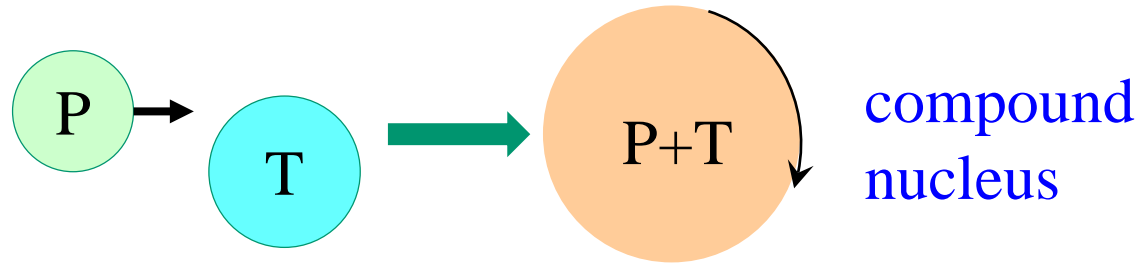


1. H.I. fusion reactions: why are they interesting?
2. Coupled-channels approach
3. Remaining challenges
4. Future perspectives: superheavy elements

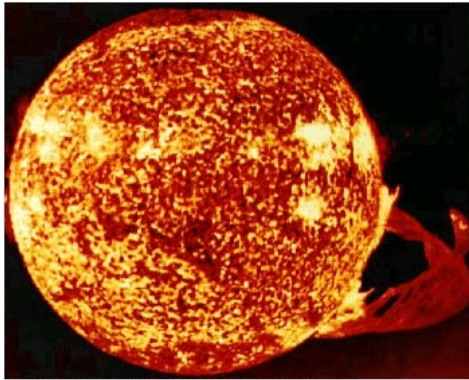
Recent review article:

K. Hagino and N. Takigawa, Prog. Theo. Phys.128 ('12)1061.

# Fusion reactions: compound nucleus formation

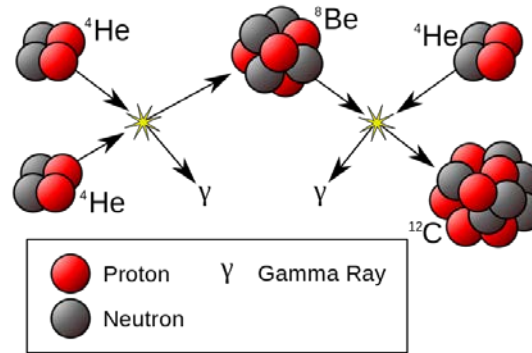


cf. Bohr '36

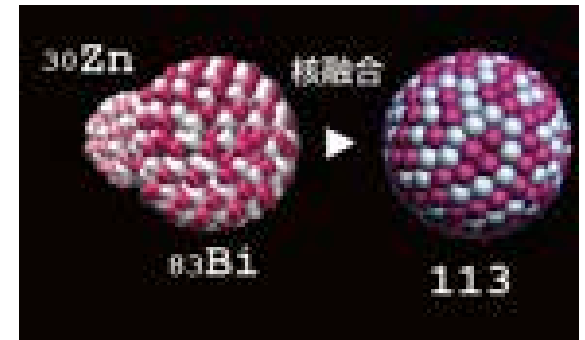


NASA, Skylab space station on December 19, 1973, solar flare reaching 588 000 km off solar surface

energy production  
in stars (Bethe '39)



nucleosynthesis

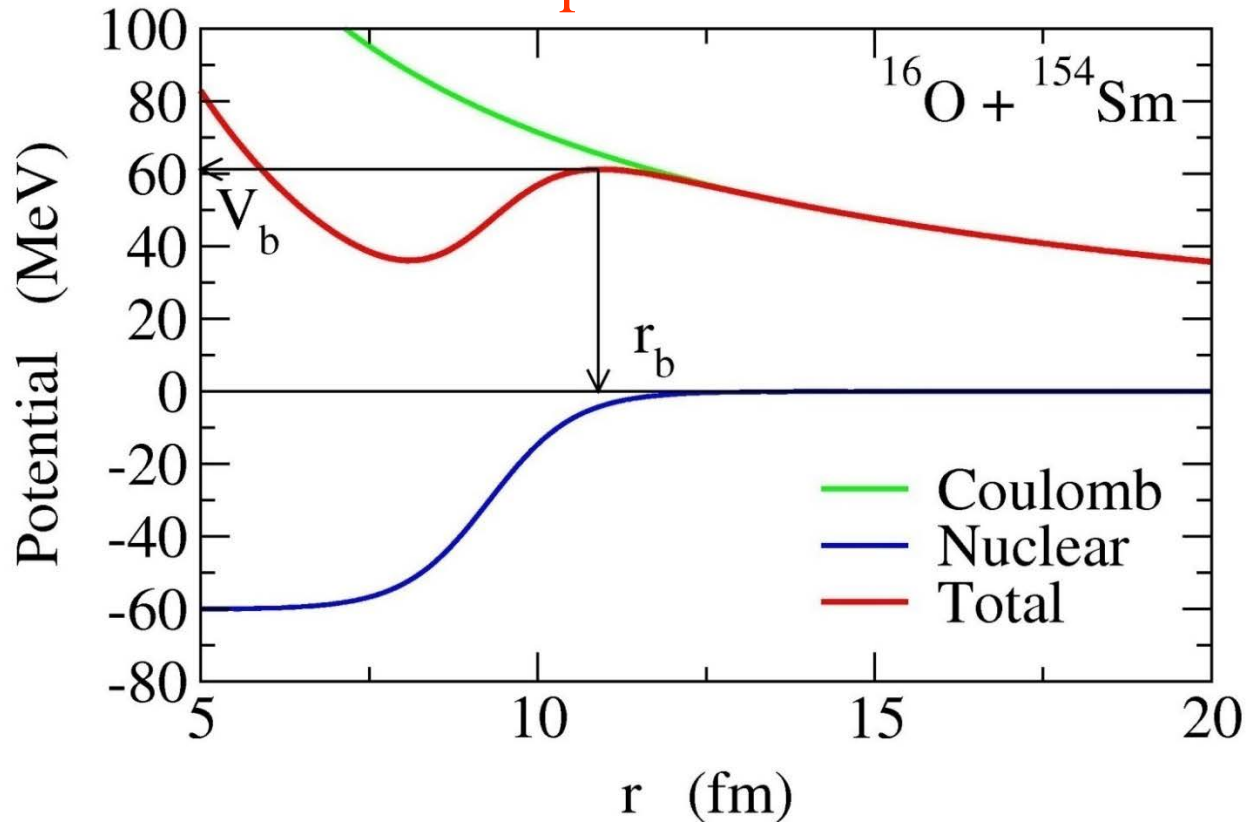


superheavy elements

**Fusion and fission:** large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: an ultimate goal of nuclear physics

## Inter-nucleus potential



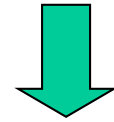
Two interactions:

1. Coulomb force

long range repulsion

2. Nuclear force

short range attraction



potential barrier  
due to a cancellation  
between the two  
(Coulomb barrier)

• Above-barrier energies



• Sub-barrier energies

• Deep sub-barrier energies

# Why sub-barrier fusion?

two obvious reasons:

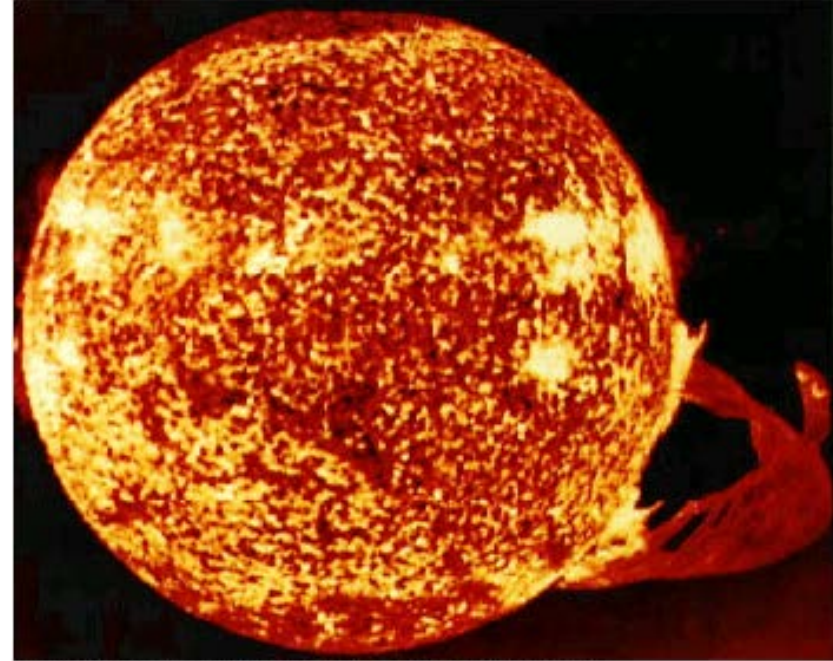


superheavy elements

cf.  $^{209}\text{Bi} (^{70}\text{Zn},n) ^{278}\text{Nh}$

$V_B \sim 260 \text{ MeV}$

$E_{\text{cm}}^{(\text{exp})} \sim 262 \text{ MeV}$



NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

nuclear astrophysics  
(nuclear fusion in stars)

cf. extrapolation of data

# Why sub-barrier fusion?

two obvious reasons:

- ✓ superheavy elements
- ✓ nuclear astrophysics

other reasons:

- ✓ reaction dynamics

**strong interplay between reaction and structure**

cf. high  $E$  reactions: much simpler reaction mechanisms

- ✓ many-particle tunneling

cf. ▪ tunneling in atomic collisions / chemical reactions

: less variety of intrinsic motions

(解離吸着)

cf. dissociative adsorption of  $H_2$  on metal surface

K. Hagino and N. Takigawa, PTP128 ('12) 1061

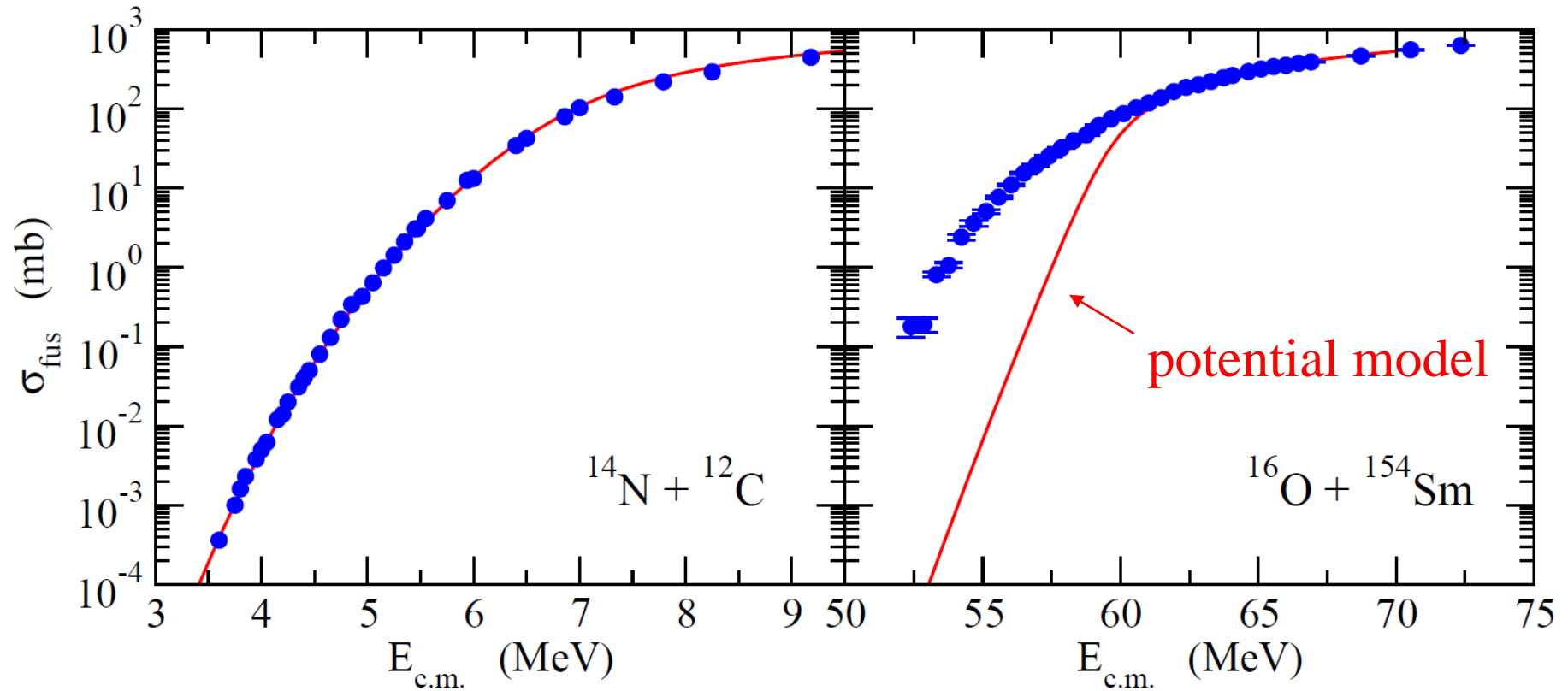
▪  $\alpha$  decays: fixed energy

H.I. fusion reaction = an ideal playground to study quantum tunneling with many degrees of freedom

# Large enhancement of fusion cross sections

Potential model:  $V(r) + \text{absorption}$

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2)$$

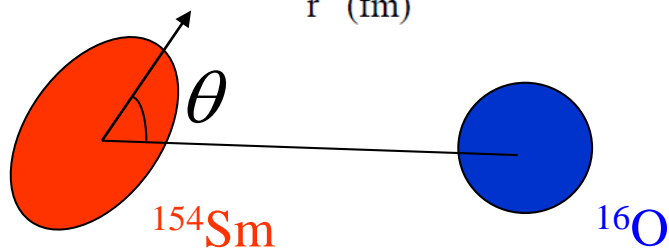
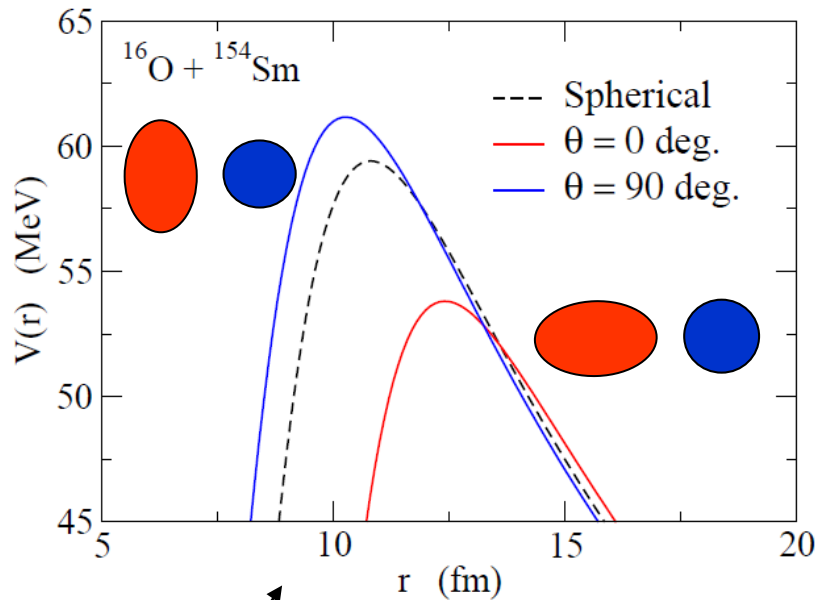


cf. seminal work:

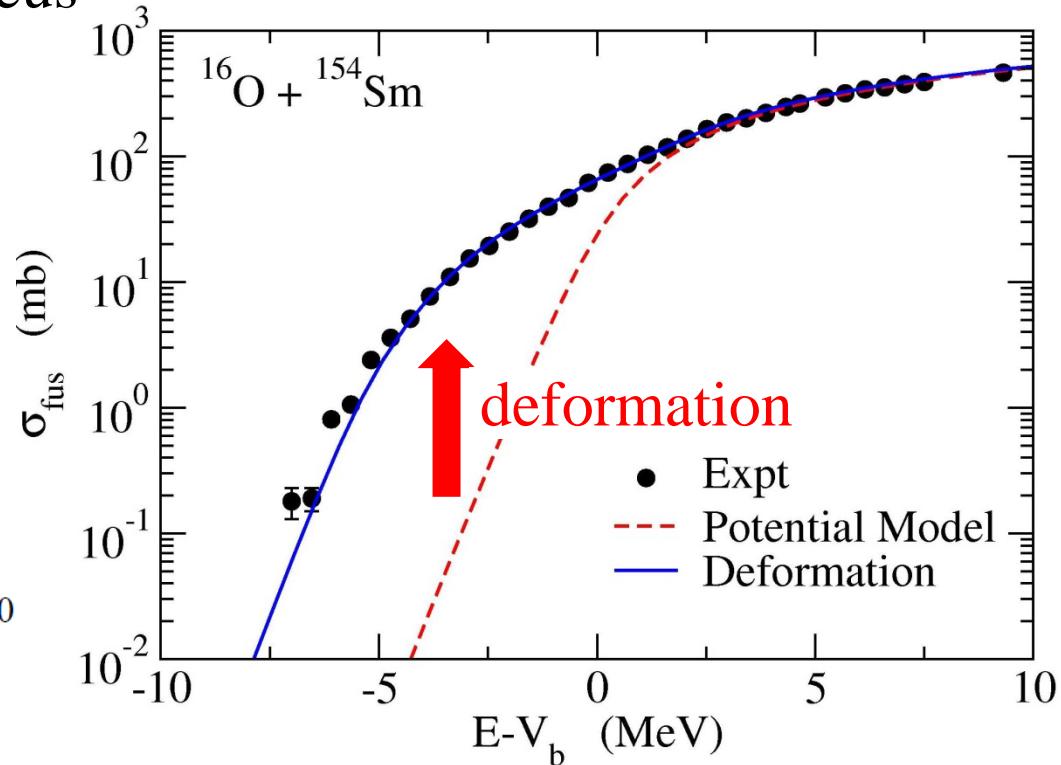
R.G. Stokstad et al., PRL41('78) 465

# Effects of nuclear deformation

$^{154}\text{Sm}$  : a typical deformed nucleus



$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

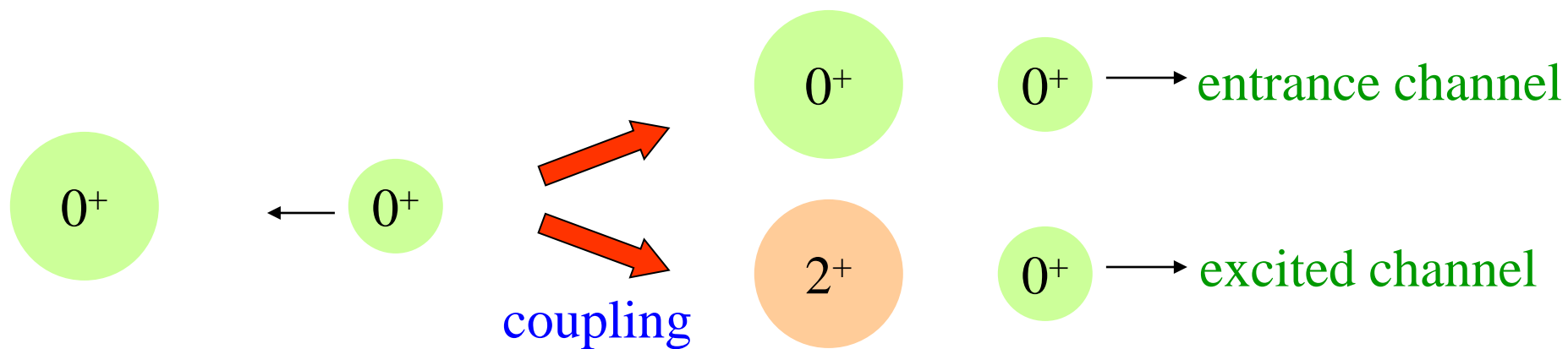


**Fusion: strong interplay between nuclear structure and reaction**

\* Sub-barrier enhancement also for non-deformed targets:

couplings to low-lying collective excitations  $\rightarrow$  coupling assisted tunneling

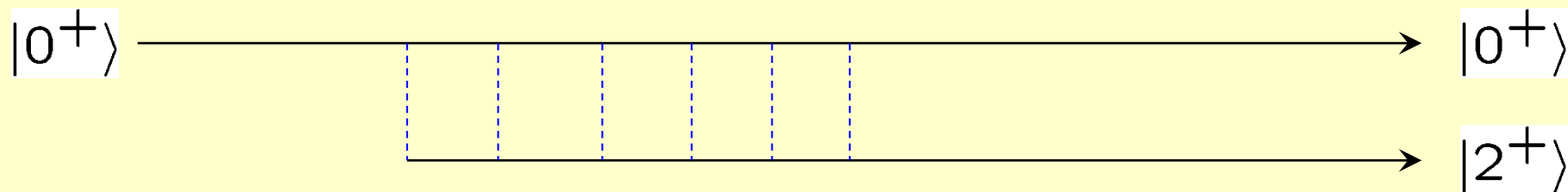
# Coupled-channels method: a quantal scattering theory with excitations



$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

excitation energy

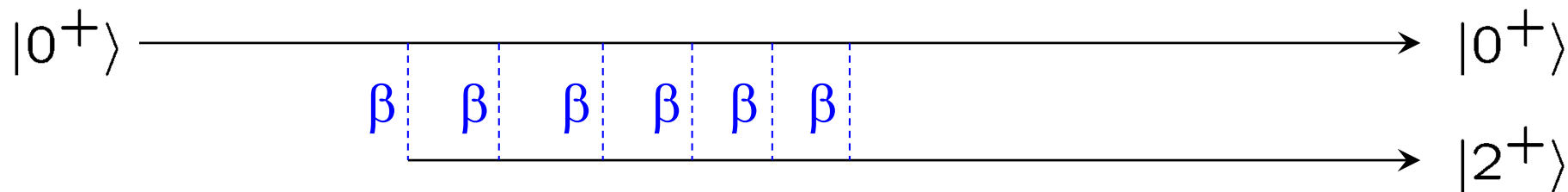
excitation operator



full order treatment of excitation/de-excitation dynamics during reaction



# Inconsistency in linear coupling approximation



full order treatment of excitation/de-excitation dynamics during reaction

but only 1st order in Hamiltonian → inconsistency

$$V_{\text{coup}}(r) \sim -R\beta \frac{dV_N(r)}{dr}$$

→ improve to include all orders also in Hamiltonian

$$\langle \phi_n | V(r-x) | \phi_m \rangle \sim V(r) \delta_{n,m} - \frac{dV(r)}{dr} \langle \phi_n | x | \phi_m \rangle$$



$$\langle \phi_n | V(r-x) | \phi_m \rangle = \int dx \phi_n^*(x) \phi_m(x) V(r-x)$$

# Inconsistency in linear coupling approximation

$$\langle \phi_n | V(r - x) | \phi_m \rangle = \int dx \phi_n^*(x) \phi_m(x) V(r - x)$$

improve to include all orders also in Hamiltonian

K. Hagino, N. Takigawa, M. Dasgupta, D.J. Hinde,  
and J.R. Leigh, PRC55 ('97) 276

later improvement: a more elegant method

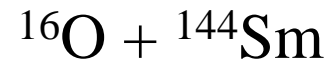
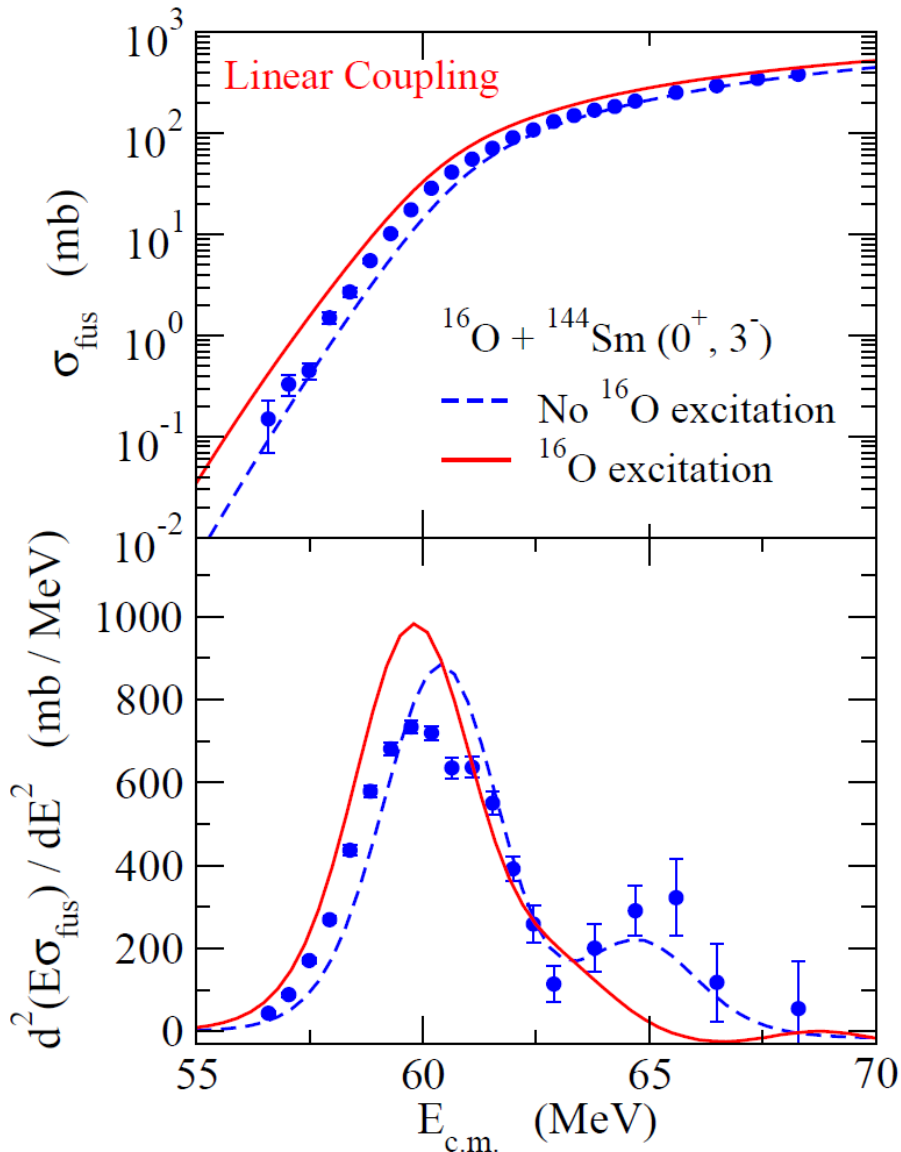
$$V(r - x) \rightarrow V(r - \{x_{nm}\}); \quad x_{nm} = \langle \phi_n | x | \phi_m \rangle$$

$$\longrightarrow \langle \phi_n | V(r - x) | \phi_m \rangle = \sum_k \langle \phi_n | u_k \rangle V(r - \lambda_k) \langle u_k | \phi_m \rangle$$

$$\hat{x} | u_k \rangle = \lambda_k | u_k \rangle$$

K. Hagino, N. Rowley, and A.T. Kruppa, CPC 123 ('99) 143

# the situation before 1997



$3^-$  ——— 6.13 MeV

$3^-$  ——— 1.81 MeV

$0^+$  ———  $0^+$

$^{16}\text{O}$

$^{144}\text{Sm}$

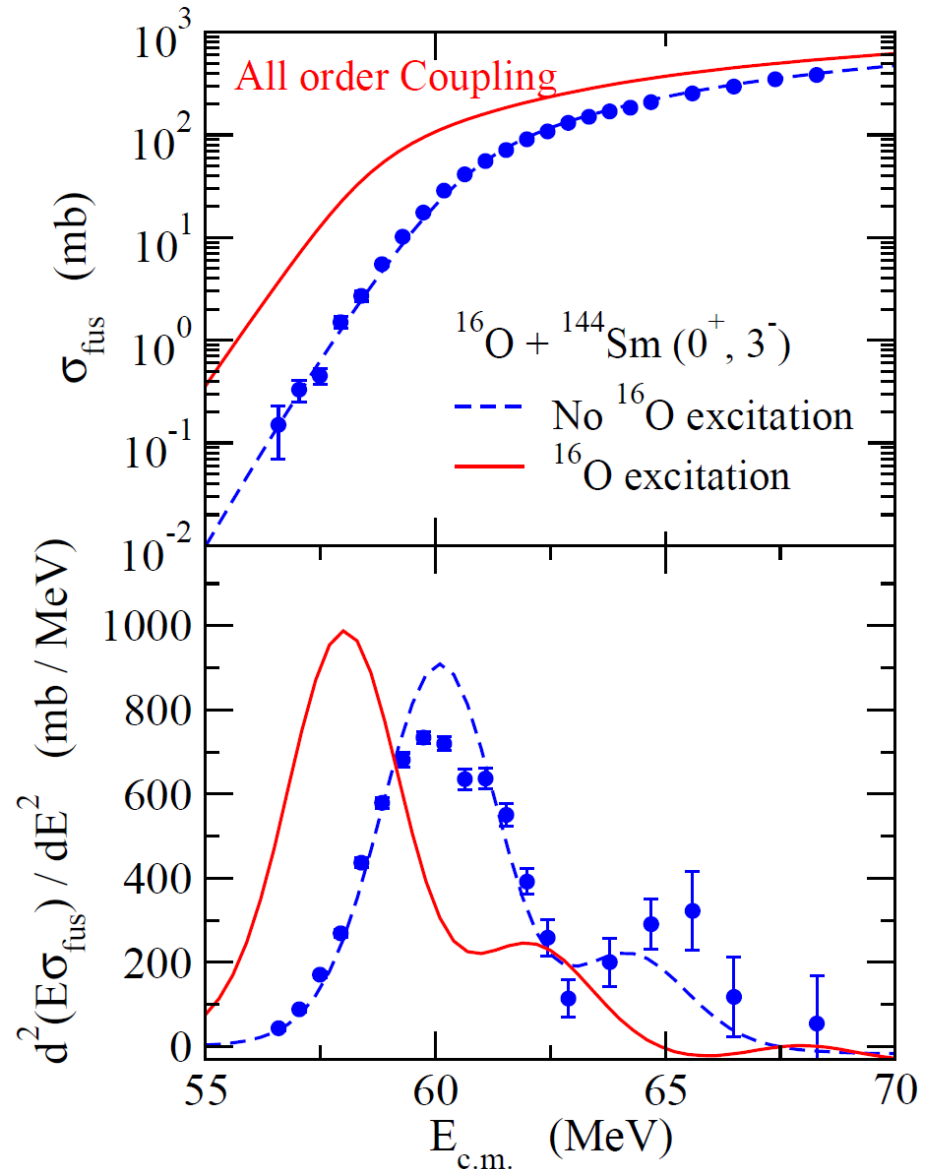
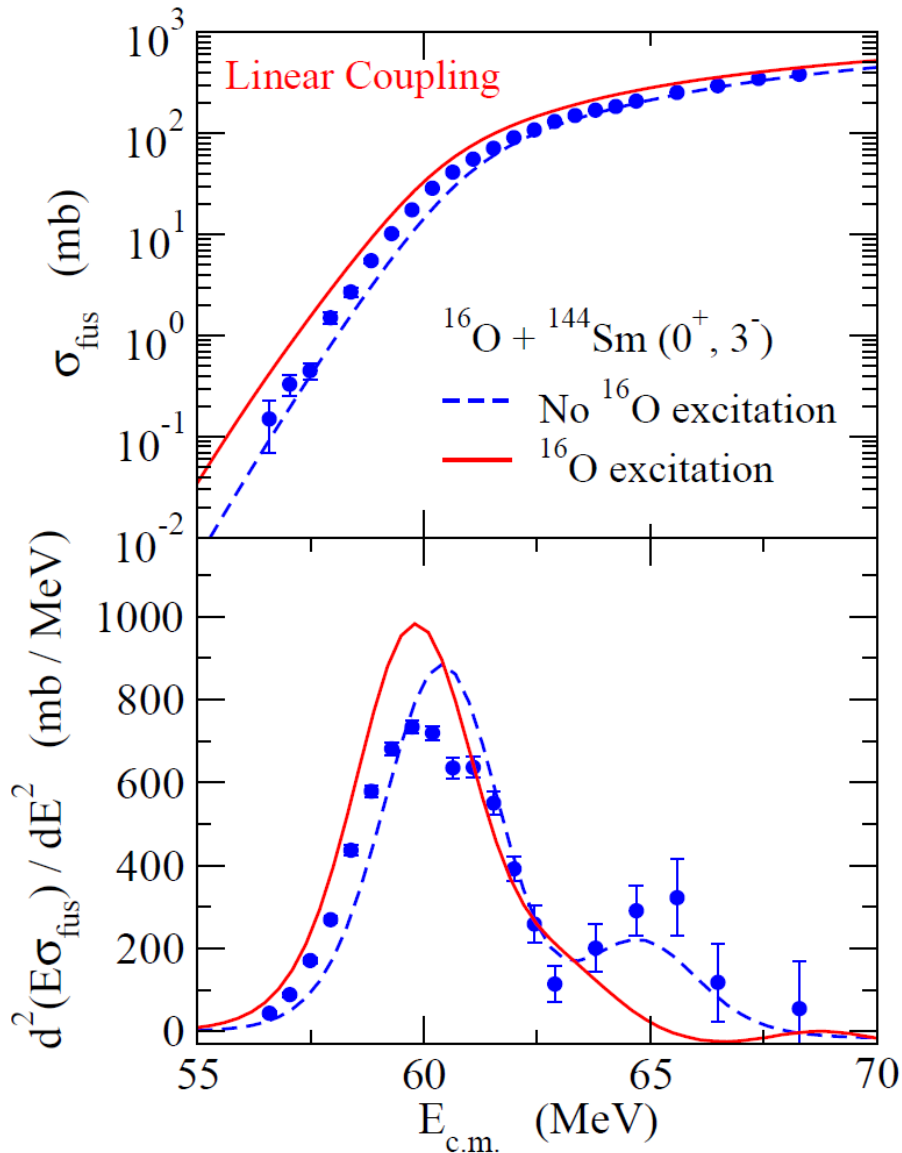
- ✓ good reproduction if  $^{16}\text{O}$  excitations are ignored
- ✓ worsen if  $^{16}\text{O}$  excitations are taken into account

similar situation also in other reactions with  $^{16}\text{O}$

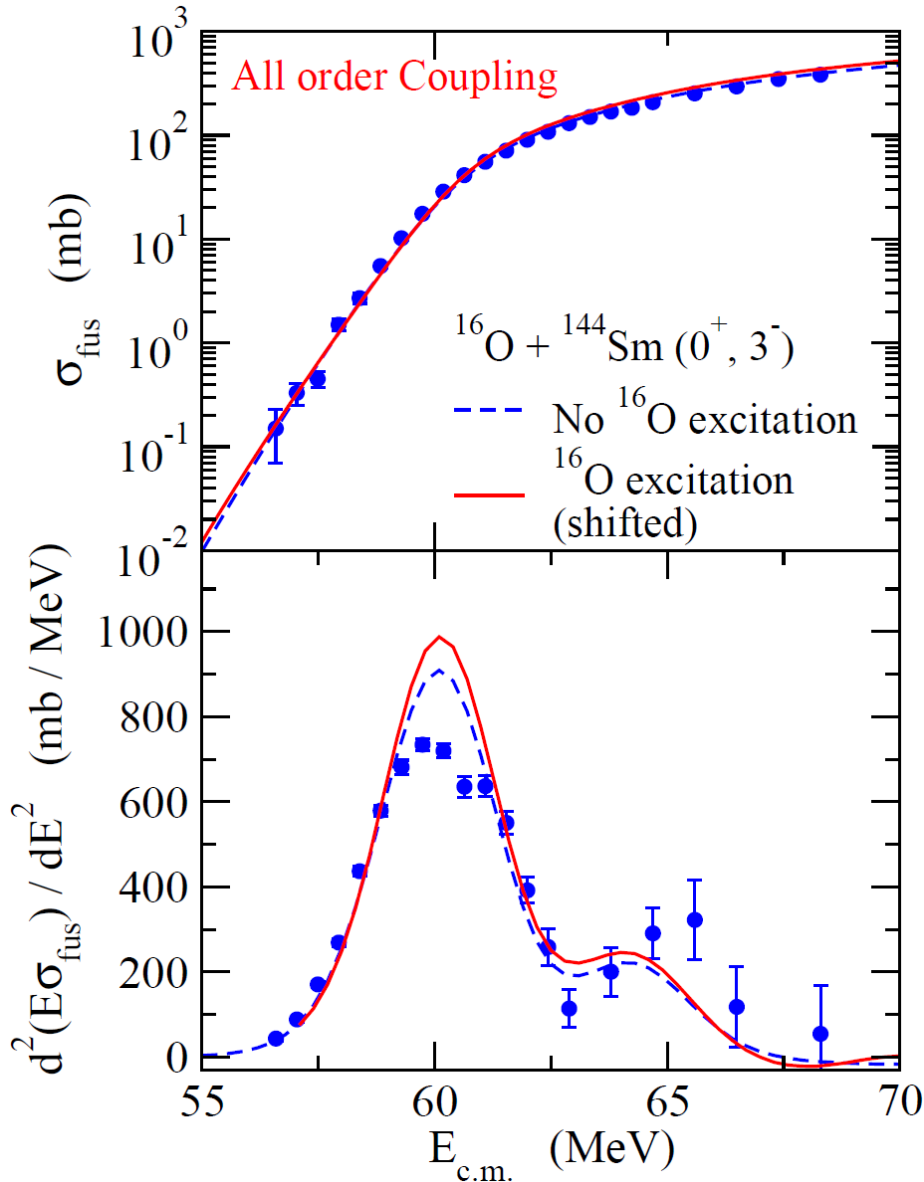
→ a big puzzle at that time

# importance of consistent treatment

# solution: all order couplings



# Adiabatic potential renormalization



- ✓ couplings to high-lying states : no change in  $E$ -dependence of fusion cross sections
- ✓ simply changes the static potential (adiabatic potential renormalization)



**coupled-channels calculations:  
sufficient only with low-lying  
excitations**

**CCFULL: a standard code in heavy-ion sub-barrier fusion reactions**

K. Hagino, N. Rowley, A.T. Kruppa, CPC123 ('99) 143

# CCFULL: a standard code in heavy-ion sub-barrier fusion reactions

K. Hagino, N. Rowley, A.T. Kruppa, CPC123 ('99) 143



my international collaborations with CCFULL (experimental groups)



# My theory collaborations on heavy-ion fusion reactions



(black: collaborations on other topics)



# Further developments : C.C. with several nuclear structure models

## Inputs for C.C. calculations

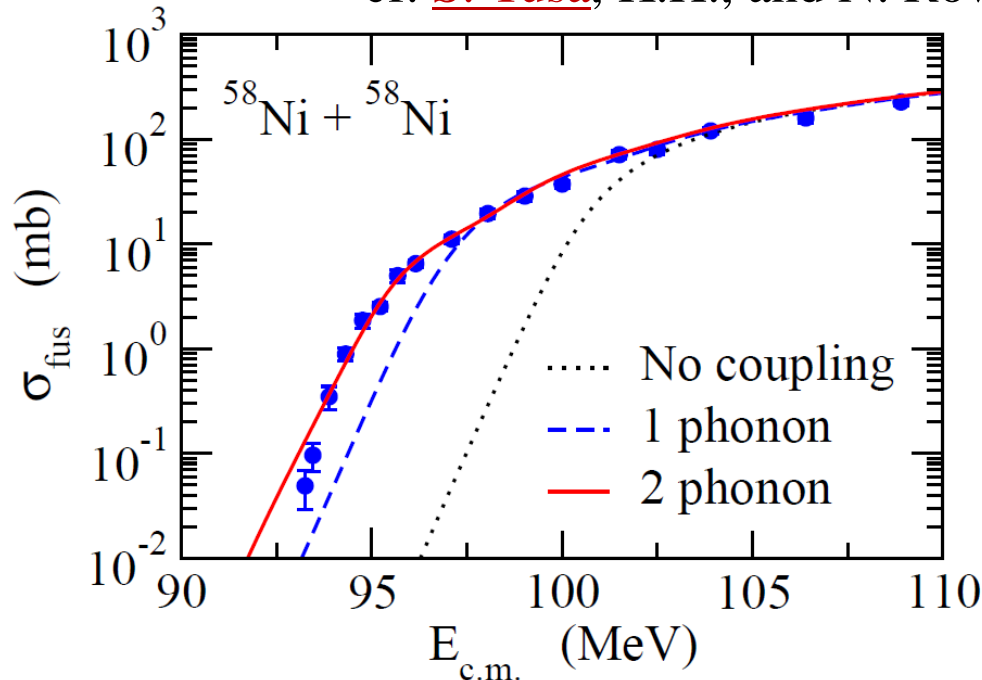
### i) Inter-nuclear potential

a fit to experimental data at above barrier energies

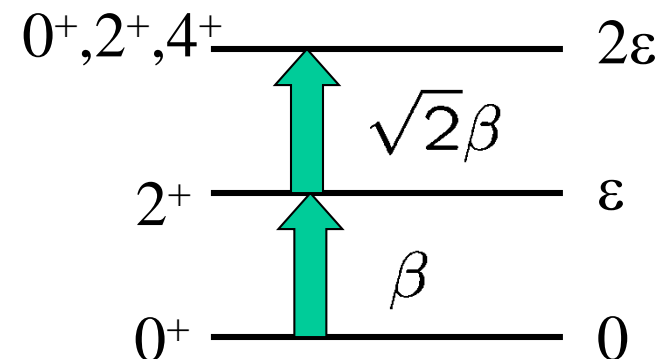
### ii) Intrinsic degrees of freedom

in most of cases, (macroscopic) collective model  
(rigid rotor / harmonic oscillator)

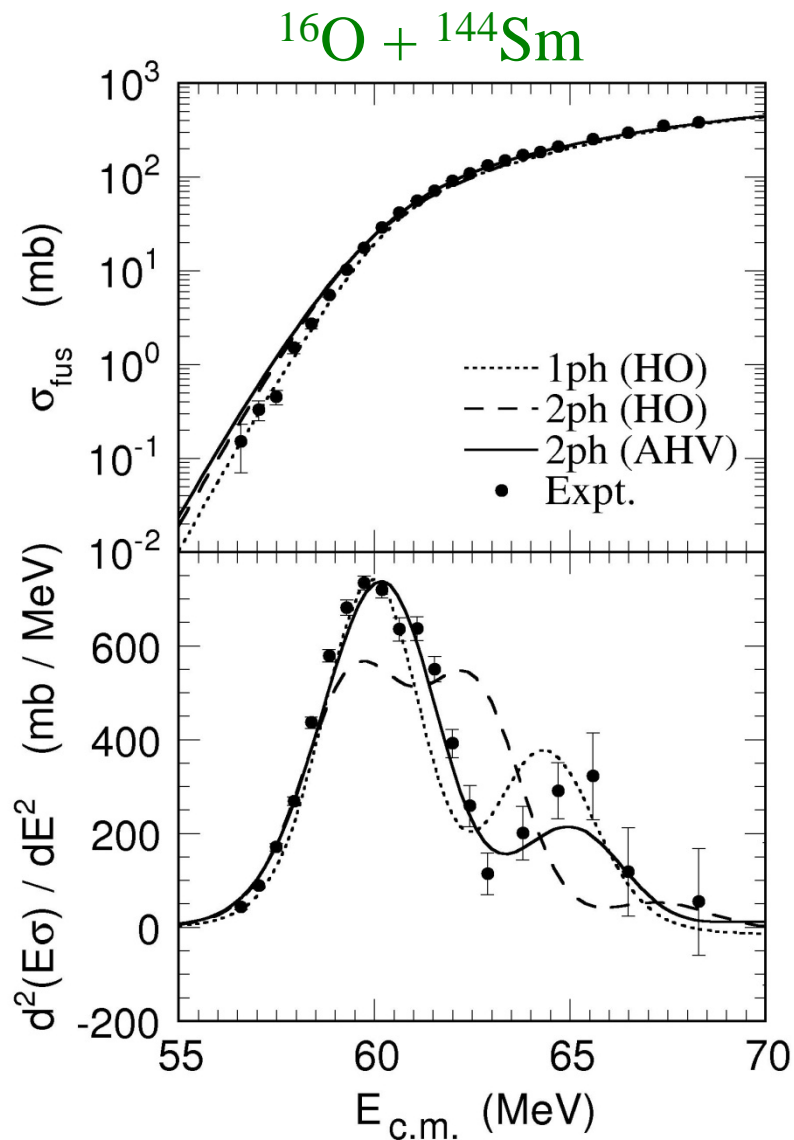
cf. [S. Yusa](#), K.H., and N. Rowley, Phys. Rev. C88 ('13) 054621



### simple harmonic oscillator

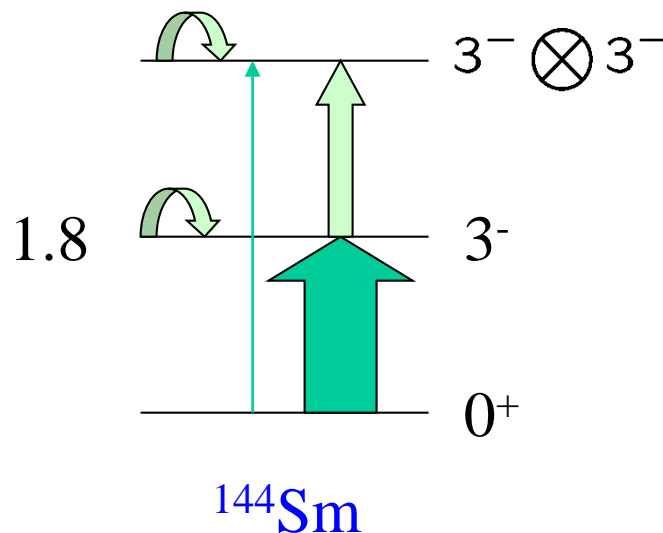


# analysis with Interacting Boson Model (IBM)



K.Hagino, N. Takigawa, and S. Kuyucak,  
PRL79('97)2943

anharmonic vibrations

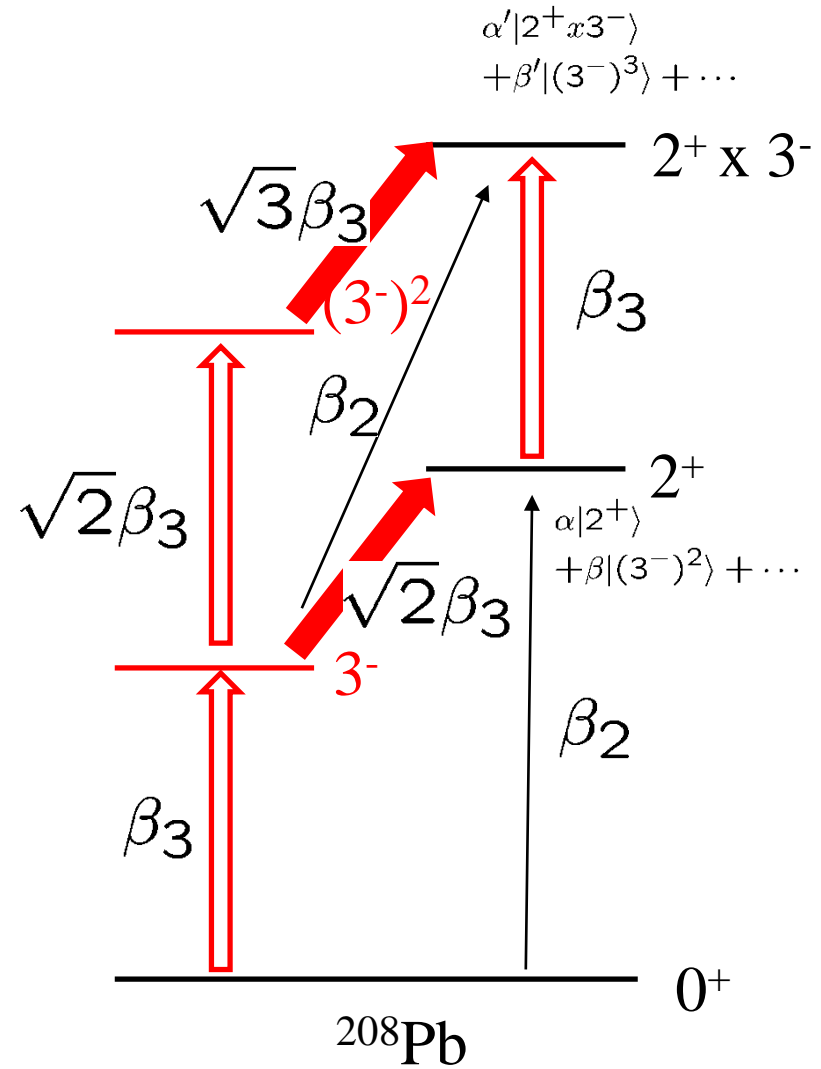
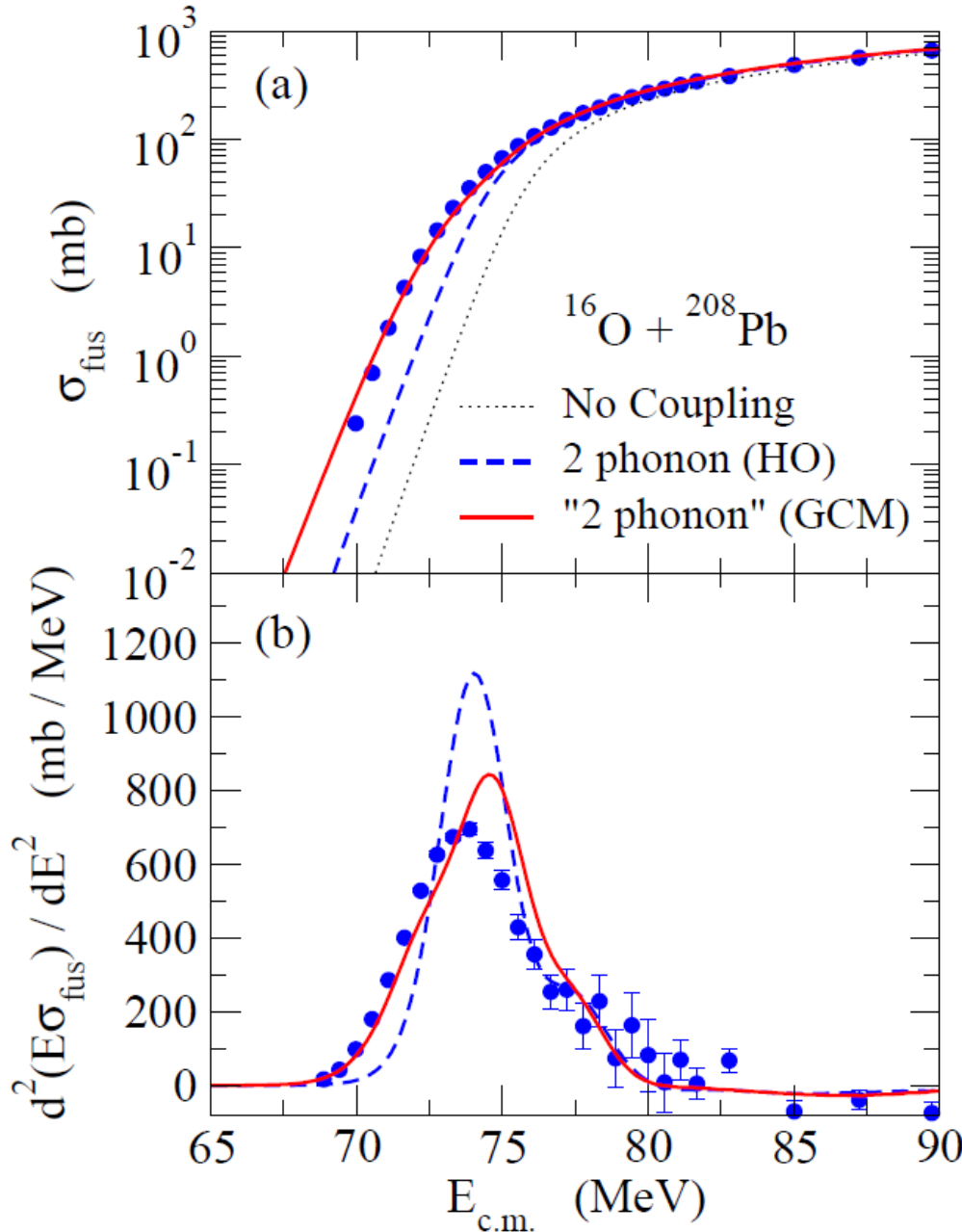


quadrupole moment:

$$Q(3^-) = -0.70 \pm 0.02b$$

\* on-going:  
a model development  
with E. Williams at ANU  
(CCFULL + IBM)

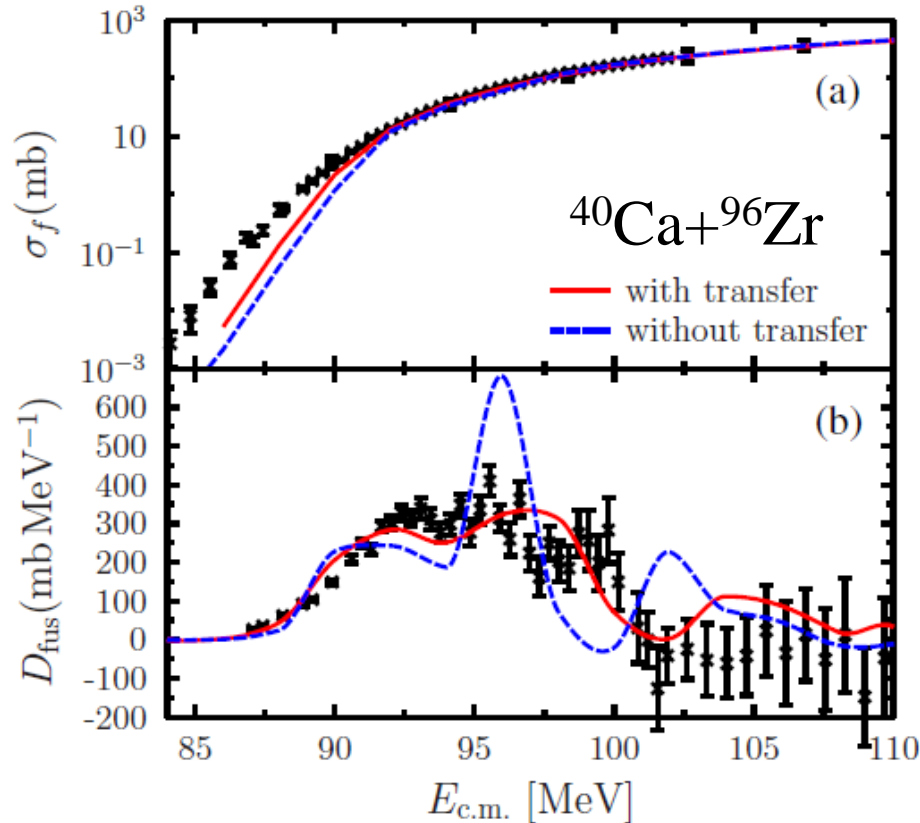
# Relativistic Mean-Field + fluctuation of MF (GCM) + C.C.



J.M. Yao and K. Hagino,  
 PRC94 ('16) 11303(R)

# Remaining challenges

## 1. Interplay between fusion and (multi-) nucleon transfer processes



fusion with multi-nucleon transfer  
: needs a further development

- ◆ explain simultaneously fusion and transfer cross sections?
- ◆ reaction dynamics of pair transfer?

cf. pairing in neutron-rich nuclei

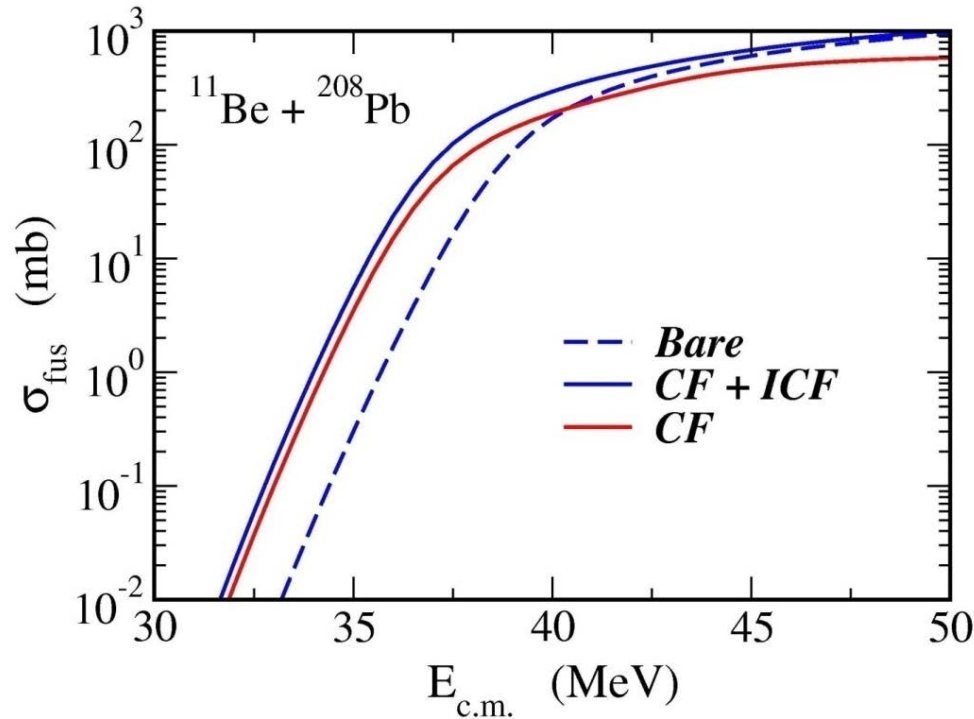
K. Hagino and H. Sagawa,  
PRC72 ('05) 044321

G. Scamps and K. Hagino,  
PRC92 ('16) 054616

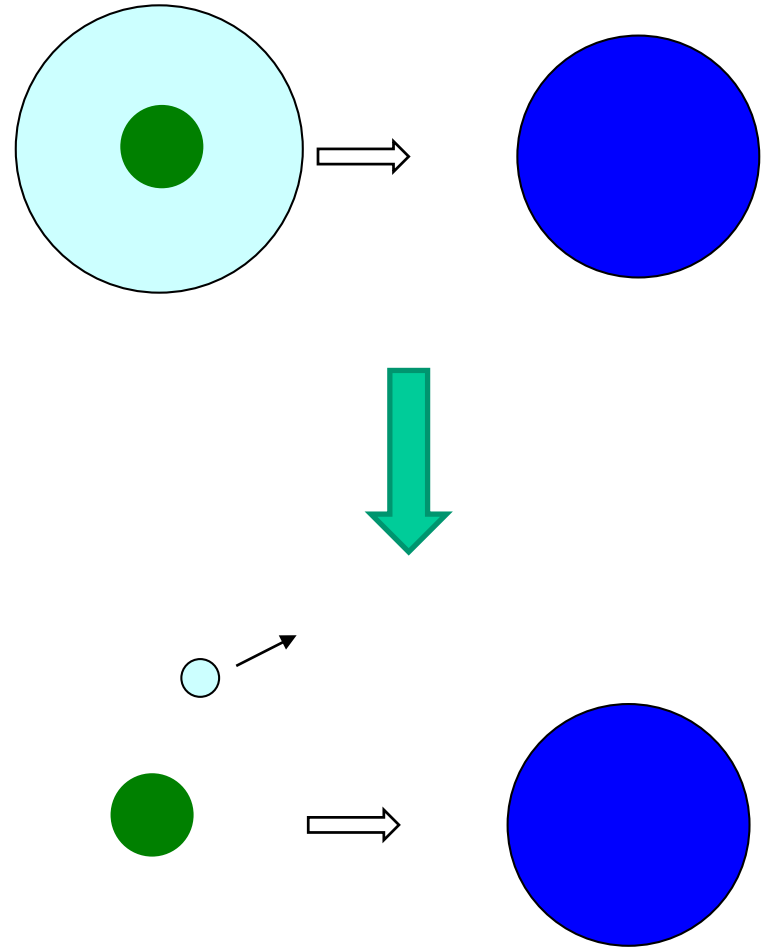
# Remaining challenges

## 2. Fusion of halo nuclei

calculations with breakup channels

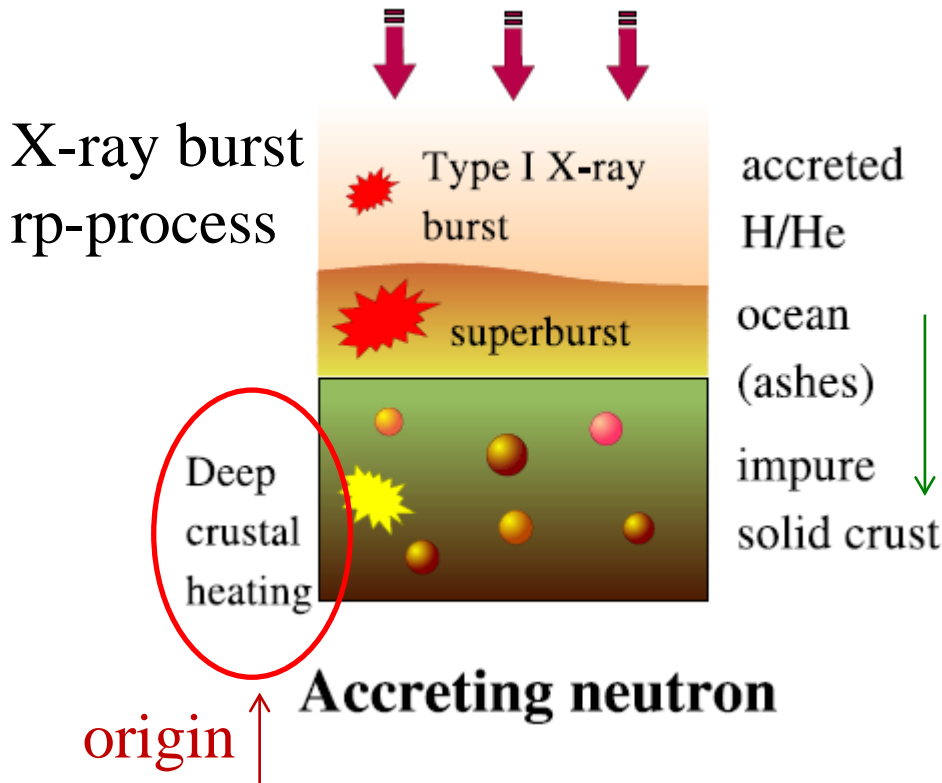


K. Hagino, A. Vitturi, C.H. Dasso,  
and S.M. Lenzi, Phys. Rev. C61 ('00) 037602



→ needs an extension with breakup and particle transfer at the same time

fusion of neutron-rich nuclei  
in accreting (質量降着) neutron stars



electron capture



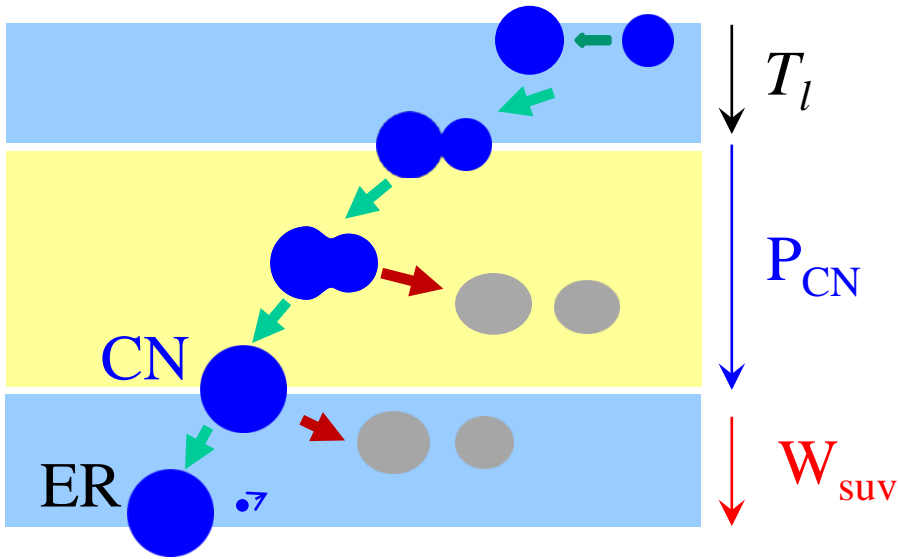
towards neutron-rich nuclei

fusion of neutron-rich nuclei  
when  $Z$  becomes small enough



N. Chamel and P. Haensel,  
Living Rev. Relativity, 11 ('08) 10.

# Future perspectives: Superheavy elements

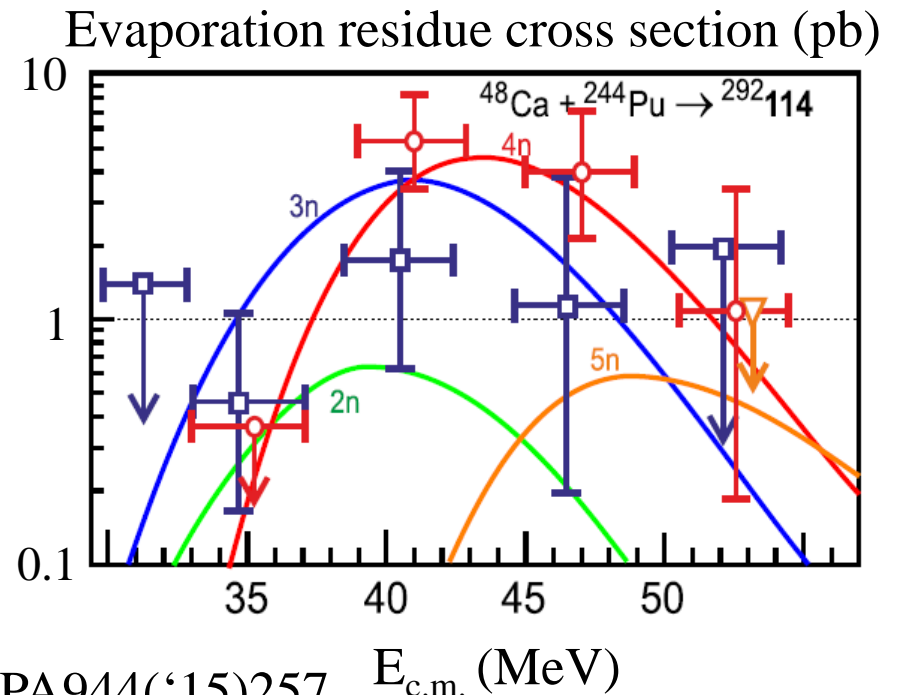


113 <b>Nh</b> nihonium	115 <b>Mc</b> moscovium
117 <b>Ts</b> tennessine	118 <b>Og</b> oganeson

CN = compound nucleus  
ER = evaporation residue

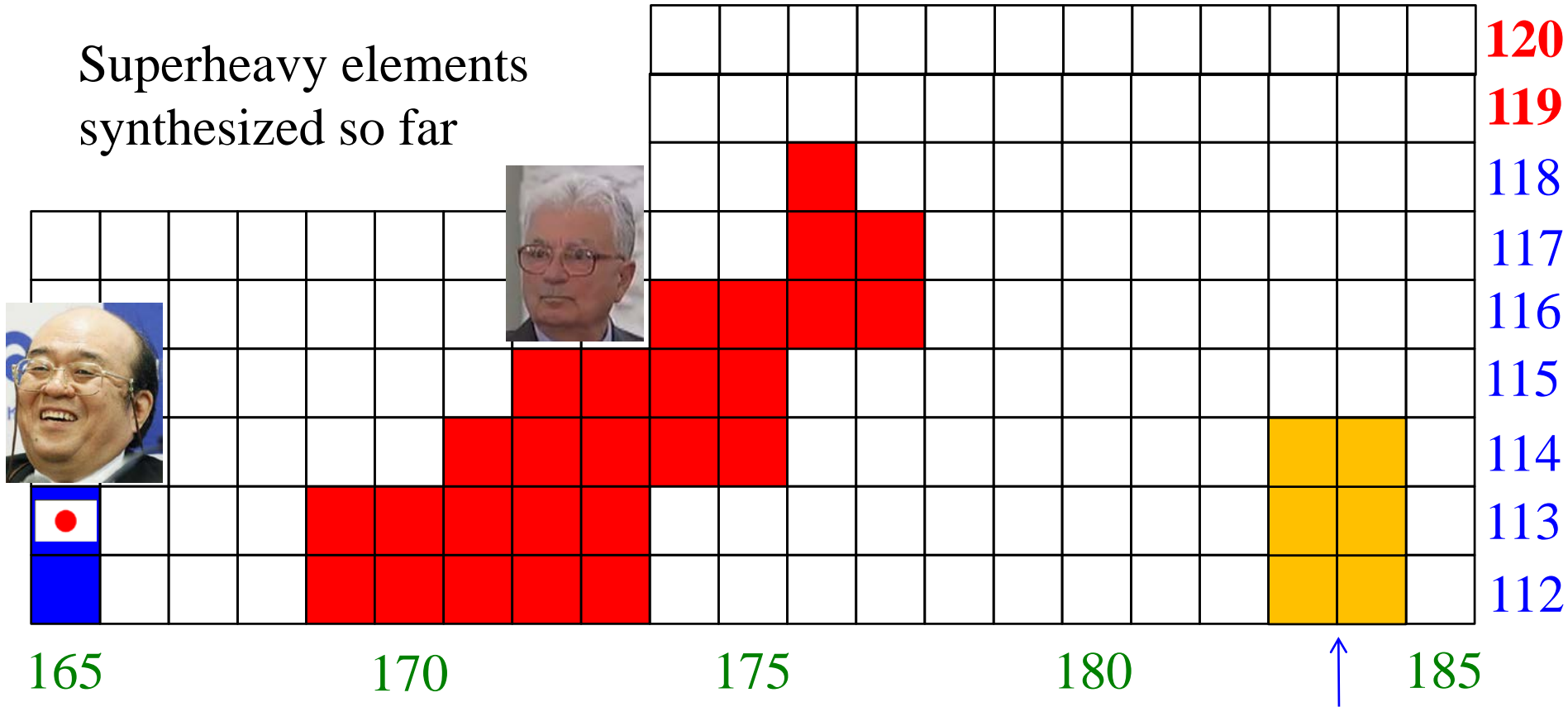
$$\sigma_{\text{ER}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E) \times P_{\text{CN}}(E, l) W_{\text{suv}}(E^*, l)$$

cf. no experimental data for  $P_{\text{CN}}$   
→ large uncertainty



# Future directions

Superheavy elements synthesized so far



➤ Towards  $Z=119$  and  $120$  nuclei

the island of stability?

reaction dynamics? reliable prediction of fusion cross sections?

➤ Towards the island of stability

neutron-rich beams: indispensable

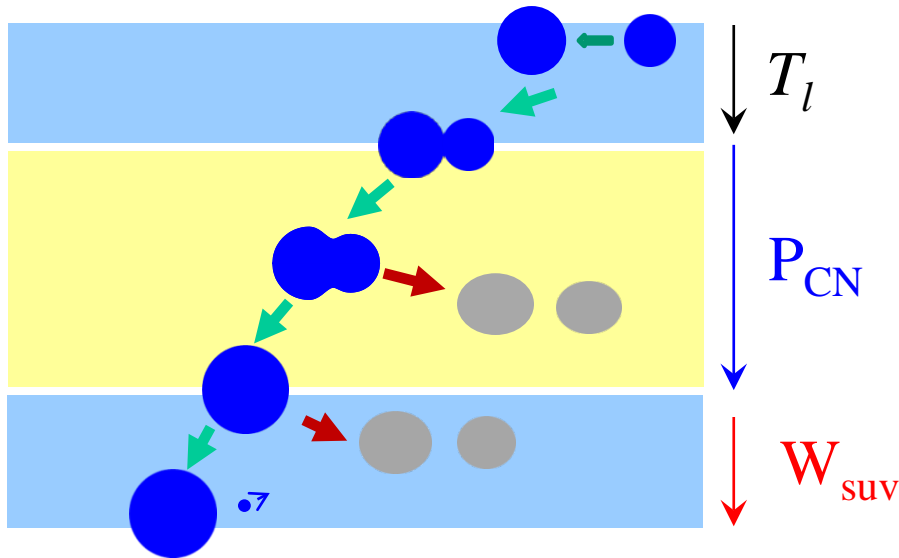


## Future directions -1

### ➤ Towards Z=119 and 120 nuclei

$^{48}\text{Ca}$  projectile (hot fusion)  $\rightarrow$   $^{50}_{22}\text{Ti}$ ,  $^{51}_{23}\text{V}$ ,  $^{54}_{24}\text{Cr}$  projectile  
+ **deformed** target nucleus

needs a proper understanding of deformation effects  
on SHE synthesis reactions

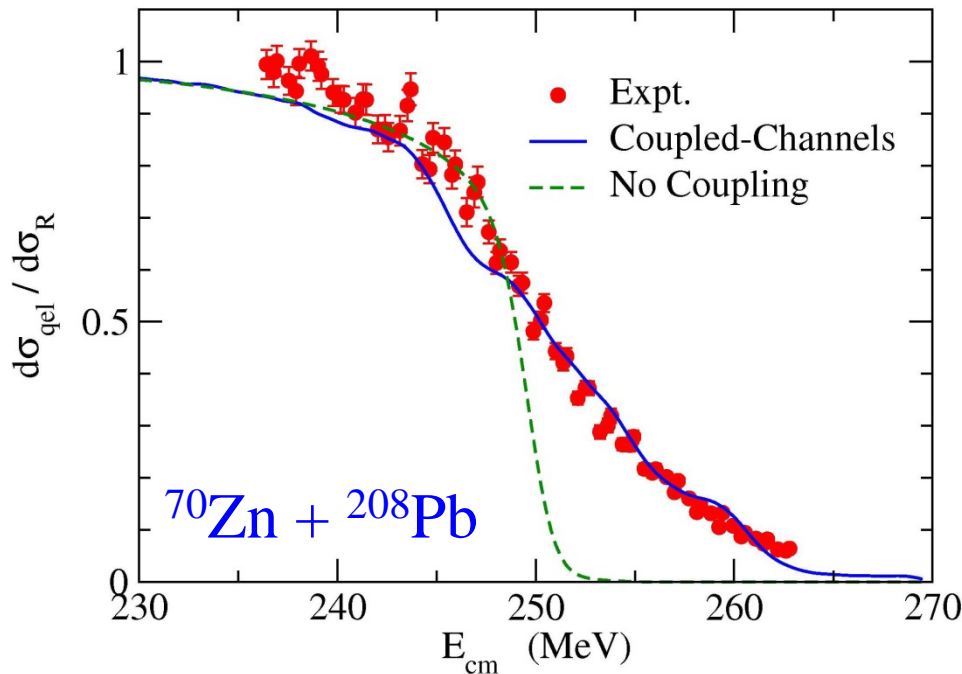


- ✓ First step:  
how to extract  $T_l$  from  $\sigma_{\text{cap}}$

$$\sigma_{\text{cap}} = \frac{\pi}{k^2} \sum_l (2l + 1) T_l$$

- ✓ Second step:  
deformation effects on  $P_{\text{CN}}$

# Extraction of $T_l$ from H.I. quasi-elastic (QEL) scattering (easier measurement than $\sigma_{\text{cap}}$ )



Exp. : Mitsuoka et al. (JAEA, 2007)

Calc.: M. Zamrun F. (Tohoku →  
Dean of Facul. Sci. at Haluoreo U.) ,  
K. Hagino, S. Mitsuoka,  
and H. Ikezoe, PRC77('08)034604.

fitting of QEL data with a few  
barriers

- ✓ Bayesian statistics for the  
number of barriers  
K. Hagino, PRC93 ('16)  
061601(R)



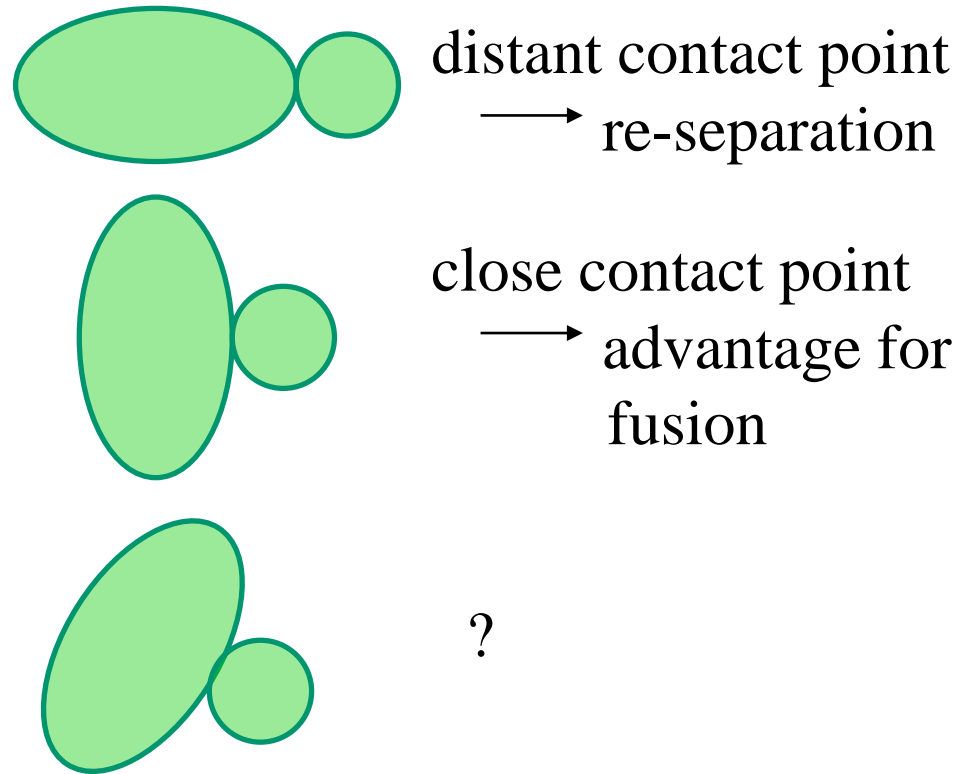
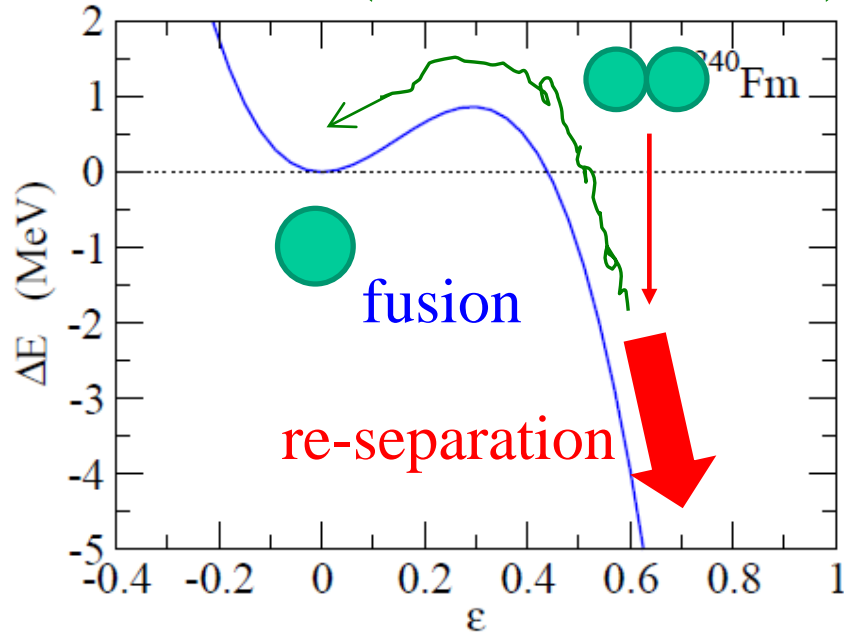
mapping from  $\sigma_{\text{qel}}$  to  
transmission probability  $T_l$   
for the  $l$ -th partial wave

cf. Expt. of  $\sigma_{\text{qel}}$  with GARIS  
(Morita et al. ,2015)

# Fusion reactions of deformed nuclei

## Thermal activation

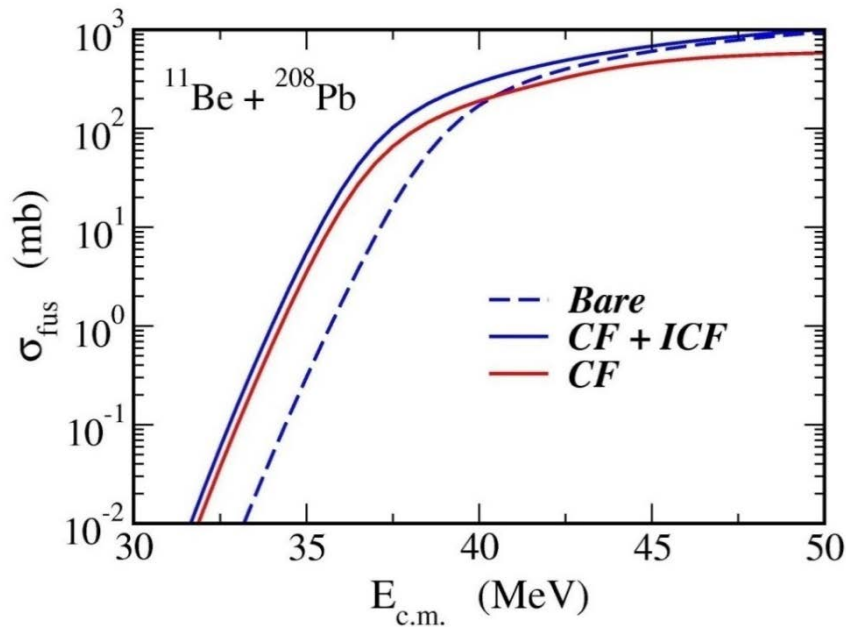
→ Langevin equation  
(Brownian motion)



- how to evolve towards a compound nucleus?  
(what are intermediate shapes?)
- deformation: quantum effects  
how deformation reduces during the heat-up process?

## Future directions - 2

- Towards the island of stability  
neutron-rich beams  
: indispensable

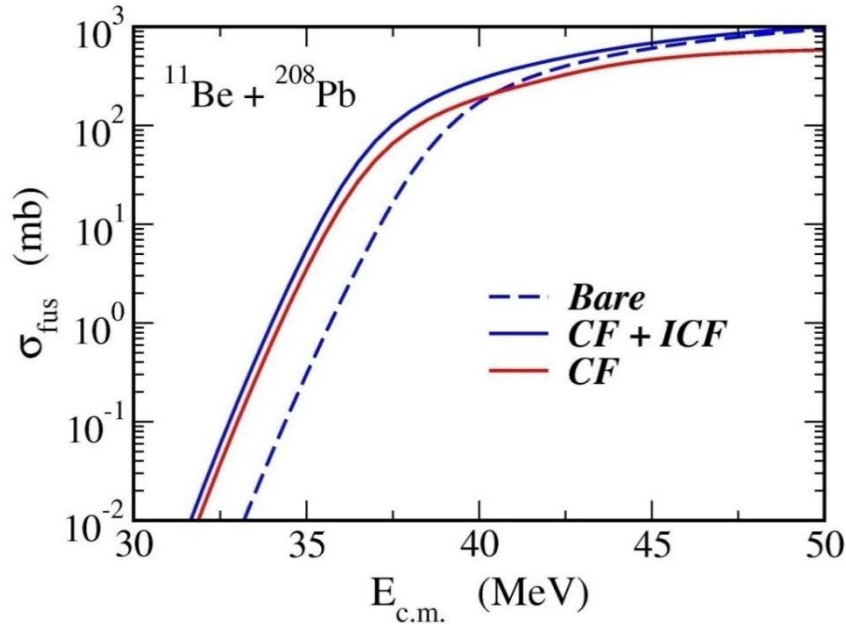


K. Hagino, A. Vitturi, C.H. Dasso,  
and S.M. Lenzi, Phys. Rev. C61 ('00) 037602

Understanding of reaction dynamics  
(breakup, particle transfer, fusion)

## Future directions - 2

➤ Towards the island of stability  
neutron-rich beams  
: indispensable

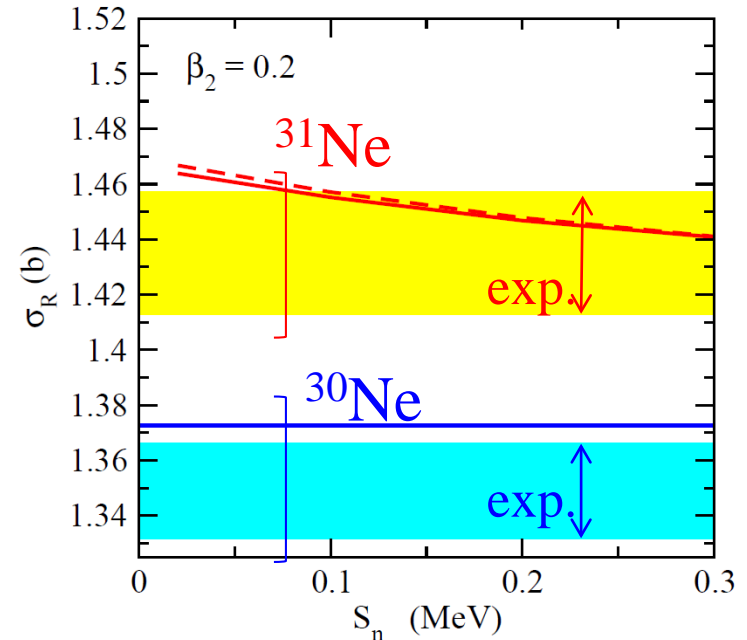


K. Hagino, A. Vitturi, C.H. Dasso,  
and S.M. Lenzi, Phys. Rev. C61 ('00) 037602

Understanding of reaction dynamics

possibilities of physics of SHE — with neutron-rich nuclei as a keyword

structure of neutron-rich nuclei  
: equally important



Y. Urata, K. Hagino, and H. Sagawa,  
PRC86('12) 044613

F. Minato (JAEA)

T. Oishi (Padova U. PD)

Y. Tanimura (GSI PD)

Y. Urata

# Summary

## Heavy-ion fusion reactions around the Coulomb barrier

- ✓ Strong interplay between nuclear structure and reaction
- ✓ Quantum tunneling with various intrinsic degrees of freedom
- ✓ theory of coupled-channels method and a world standard code
- ✓ 2 Ph.D. and 4 master students (out of 8 Ph.D. and 10 masters)

## Remaining challenges

- ✓ fusion of halo nuclei (influence of breakup and particle transfer)
- ✓ Microscopic understanding of low-E fusion reactions?
- ✓ how to understand many-particle tunneling phenomena?
- ✓ microscopic understanding of large amplitude motions

## Future perspectives: superheavy elements

- ✓ Towards heavier SHE ( $Z = 119, 120$ )
- ✓ Towards the island of stability

investigations of physics of SHE with neutron-rich nuclei as a keyword

# From phenomenological approach to microscopic approach

Macroscopic (phenomenological)

C.C. with collective model

C.C. with inputs from  
microscopic nuclear  
structure calculations

- \* Hagino-Yao
- \* Ichikawa-Matsuyanagi

C.C. with inputs based  
on TDHF

- \* Umar (DC-TDHF)
- \* Washiyama-Lacroix

TDHF simulations

- \* Simenel
- \* Sekizawa
- \* Washiyama
- \* Iwata-Otsuka etc.

ab initio, but no tunneling

Microscopic

# From phenomenological approach to microscopic approach

## TDHF simulations

- \* Simenel
- \* Sekizawa
- \* Washiyama
- \* Iwata-Otsuka etc.

ab initio, but no tunneling

## ➤ “Beyond mean-field” approximations

Nuclear structure: GCM  $|\Psi\rangle = \int dq f(q) |\Phi_q\rangle$

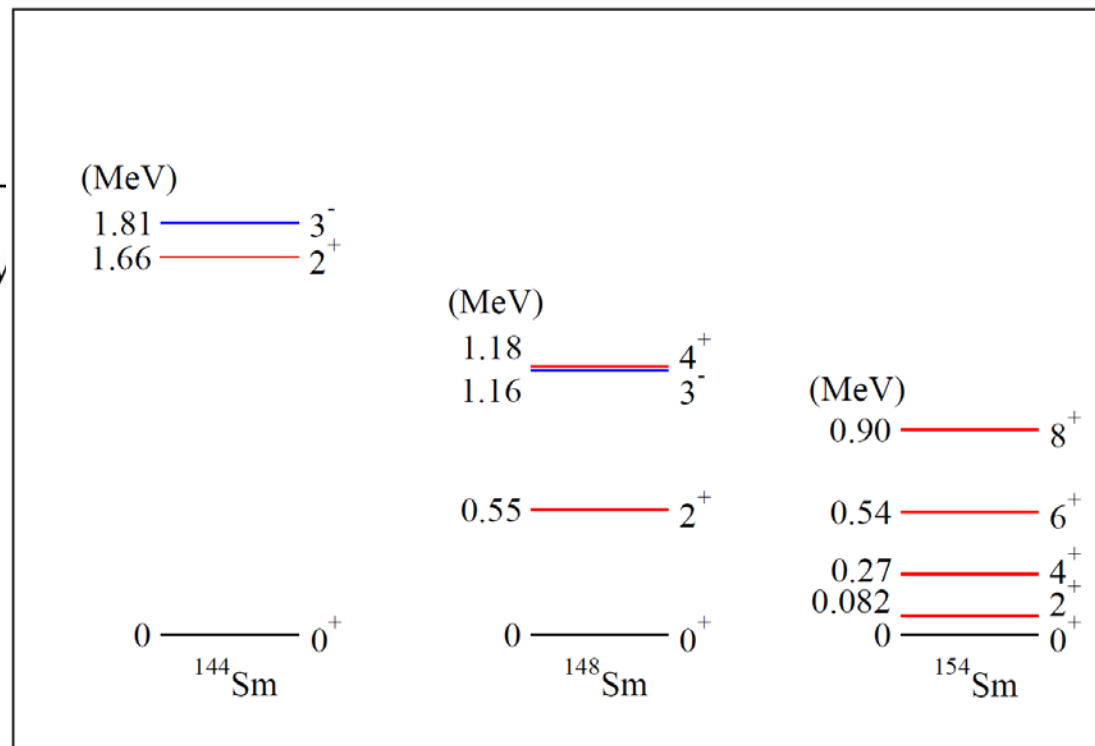
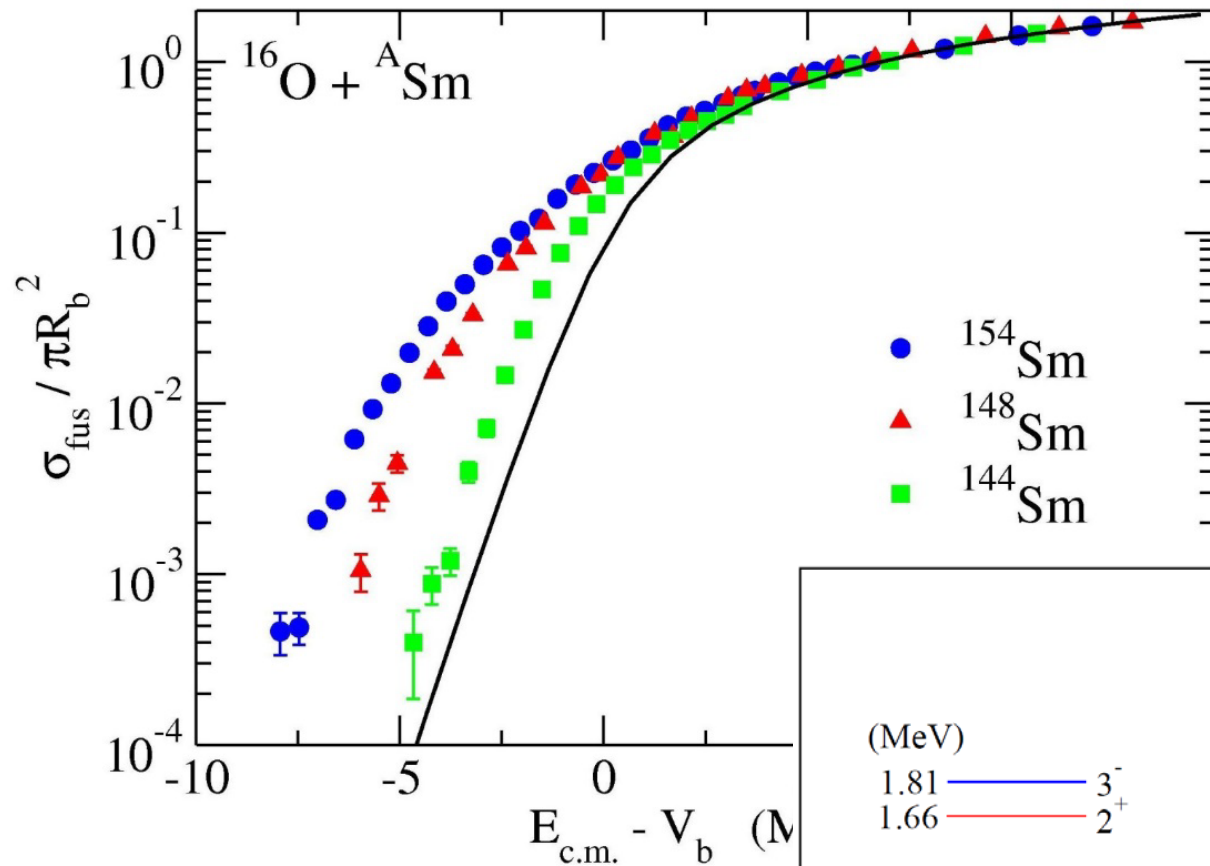
Full time-dependent GCM?  $|\Psi(t)\rangle = \int dq f(q, t) |\Phi_q(t)\rangle$

→ many-particle tunneling and large amplitude collective motions

- \* discussions with prof. Tsuyoshi Kato  
(AP at U. of Tokyo, theoretical chemistry)



Back-up slides

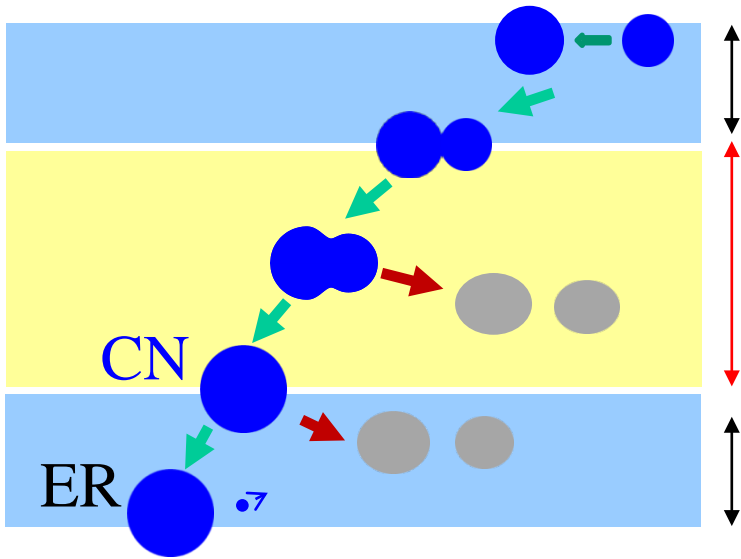


Strong target dependence  
at  $E < V_b$

→ couplings to  
low-lying collective  
excitations

# Future perspectives: Superheavy elements

113 <b>Nh</b> nihonium	115 <b>Mc</b> moscovium
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Coupled-channels method

Langevin equation

$$m \frac{d^2 q}{dt^2} = -\frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

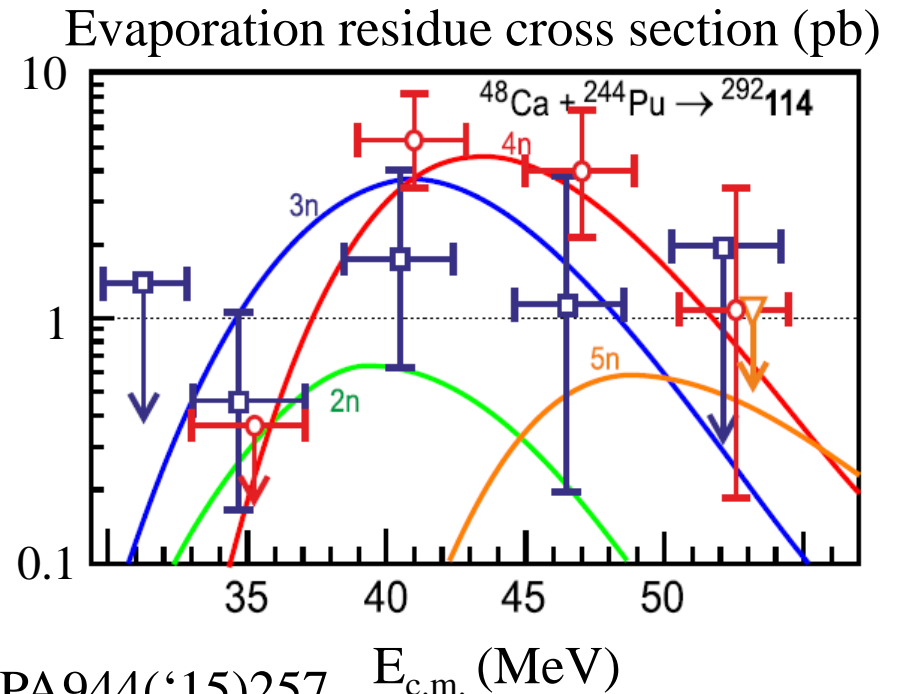
Statistical model (CN decay)

CN = compound nucleus

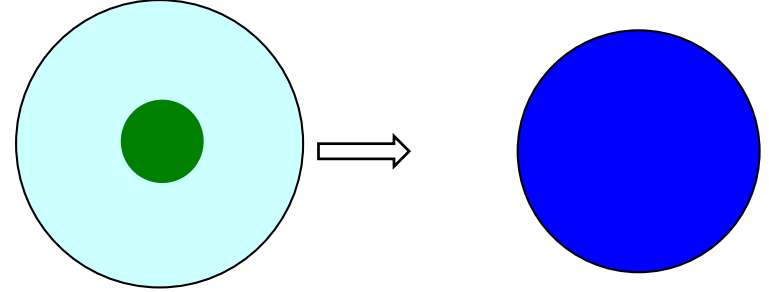
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$$\sigma_{\text{ER}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E) \times P_{\text{CN}}(E, l) W_{\text{SUV}}(E^*, l)$$

cf. no experimental data for  $P_{\text{CN}}$   
→ large uncertainty

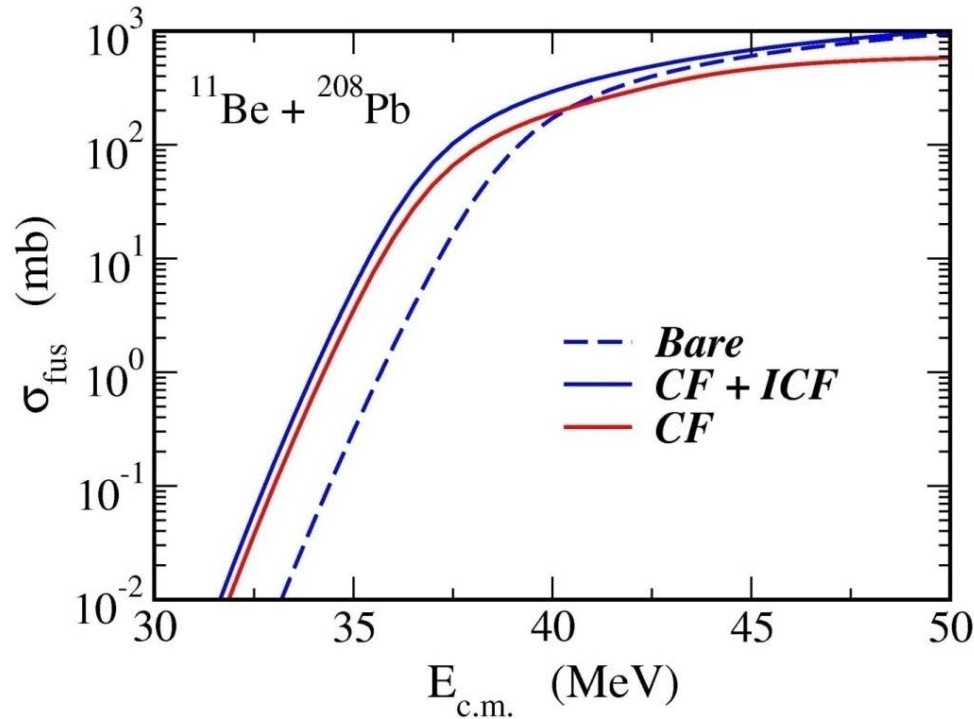


# Remaining challenges

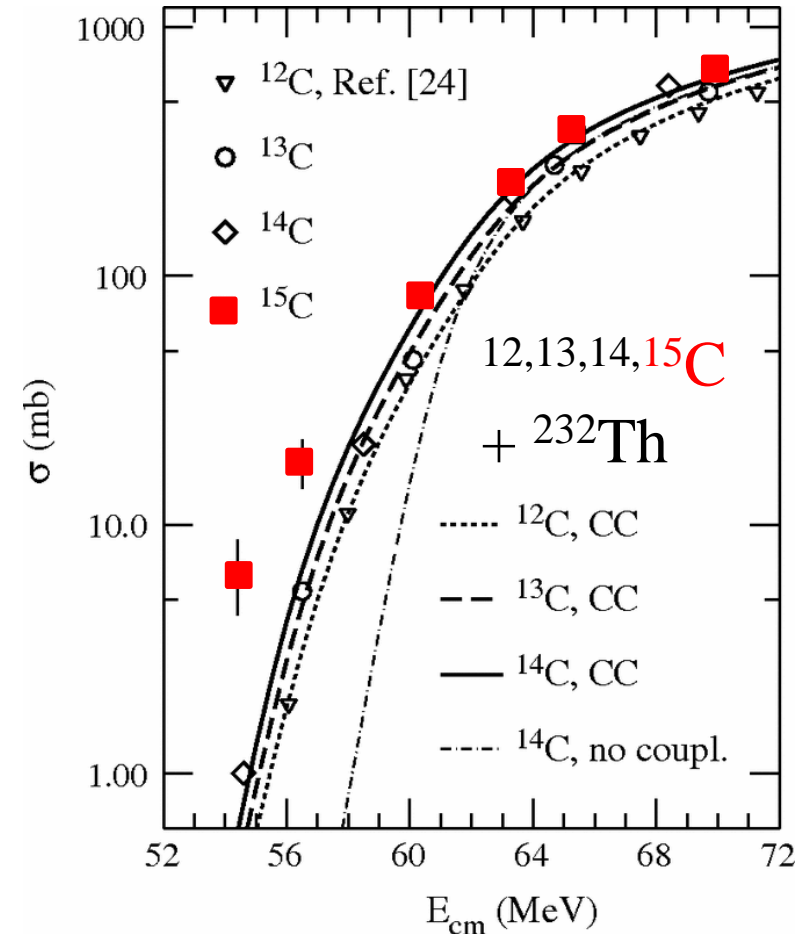


## 2. Fusion of halo nuclei

calculations with breakup channels



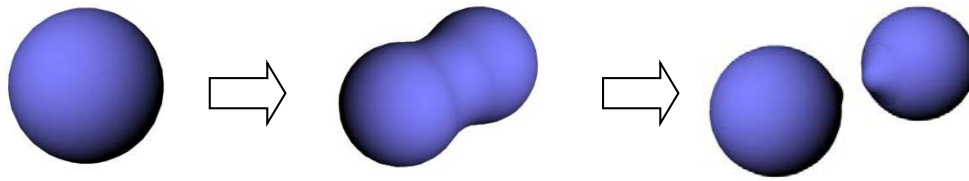
K. Hagino, A. Vitturi, C.H. Dasso,  
and S.M. Lenzi, Phys. Rev. C61 ('00) 037602



M. Alcorta et al., PRL106('11)

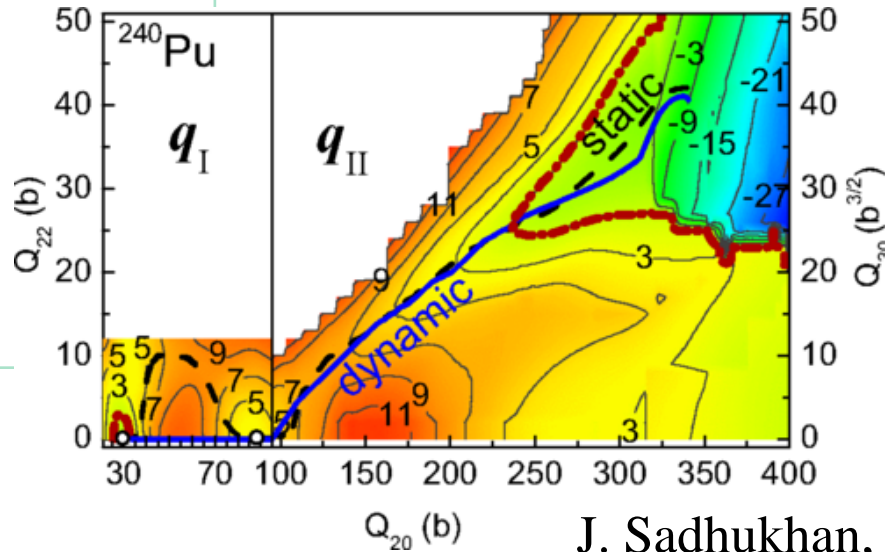
→ needs an extension with breakup and particle transfer at the same time

# Fission



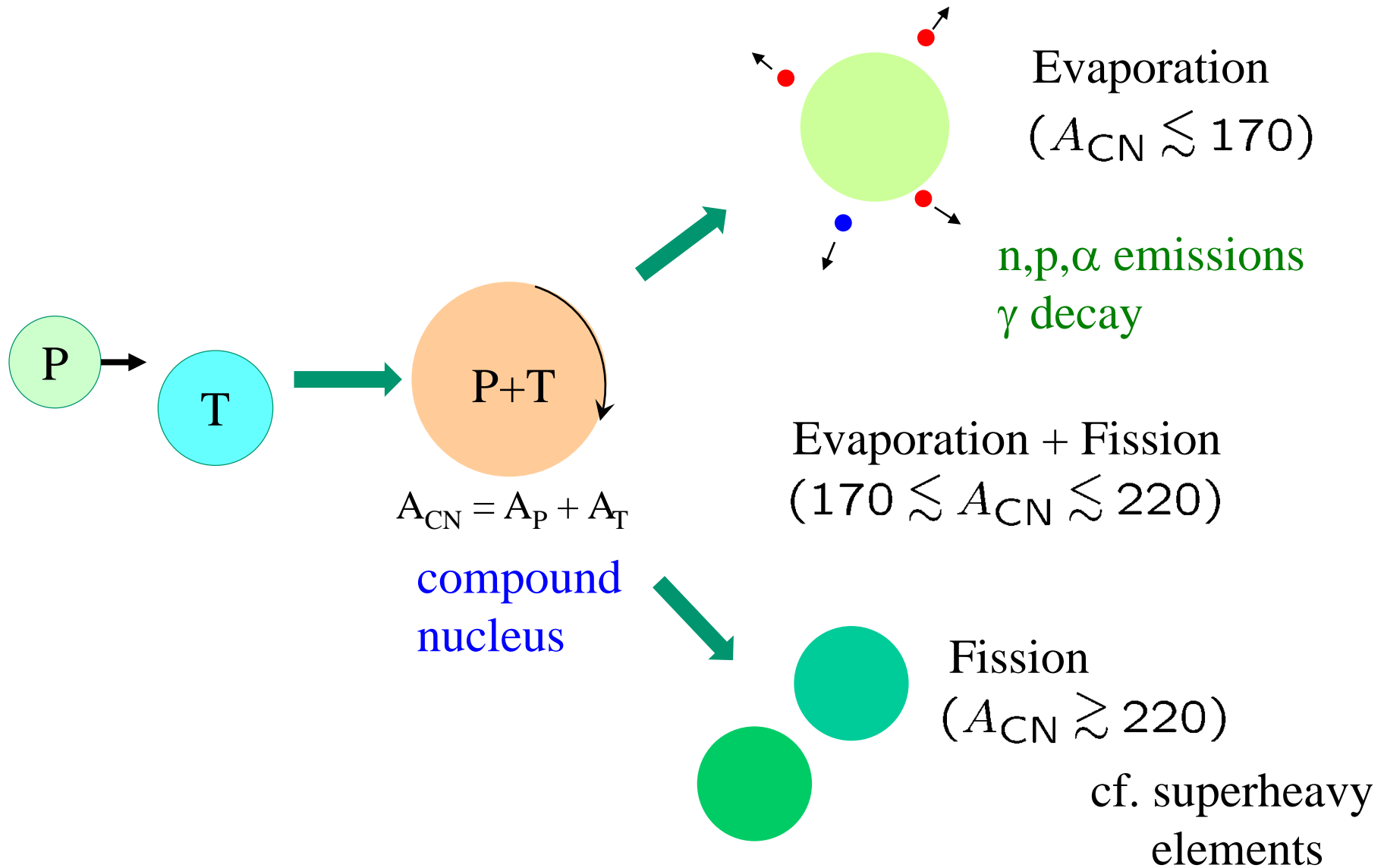
still a very challenging problem for nuclear theory

	<b>Time-indep. approach</b>	<b>Time-dep. approach</b>
Induced fission	✓ Bohr-Wheeler	✓ Langevin-type ✓ Discrete basis (Bertsch)
Spontaneous fission	✓ PES+Mass+WKB	✓ Im.-time TDHF (Negele) ✓ Time-dep. Hill-Wheeler (Goutte et al.) ✓ TDHF (after the barrier)

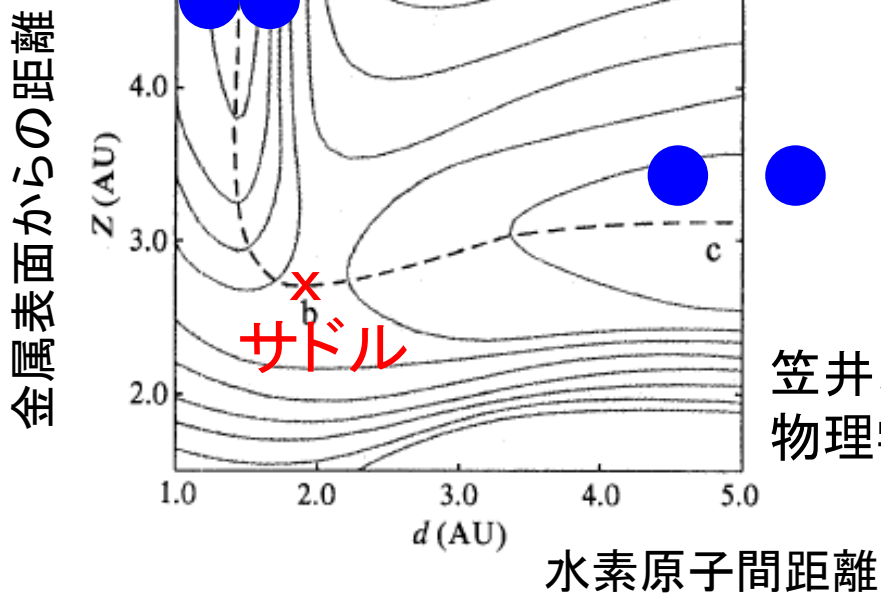
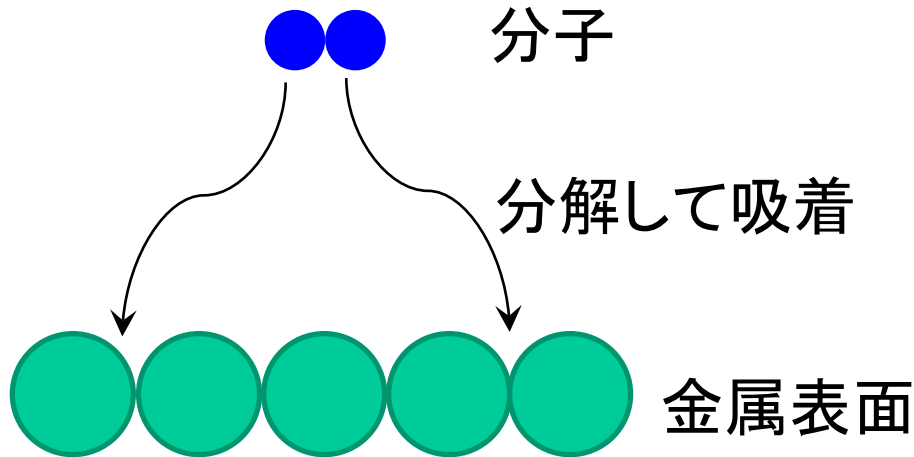


J. Sadhukhan, W. Nazarewicz, N. Schunck,  
PRC93('16)011304(R)

# Fusion reactions: compound nucleus formation

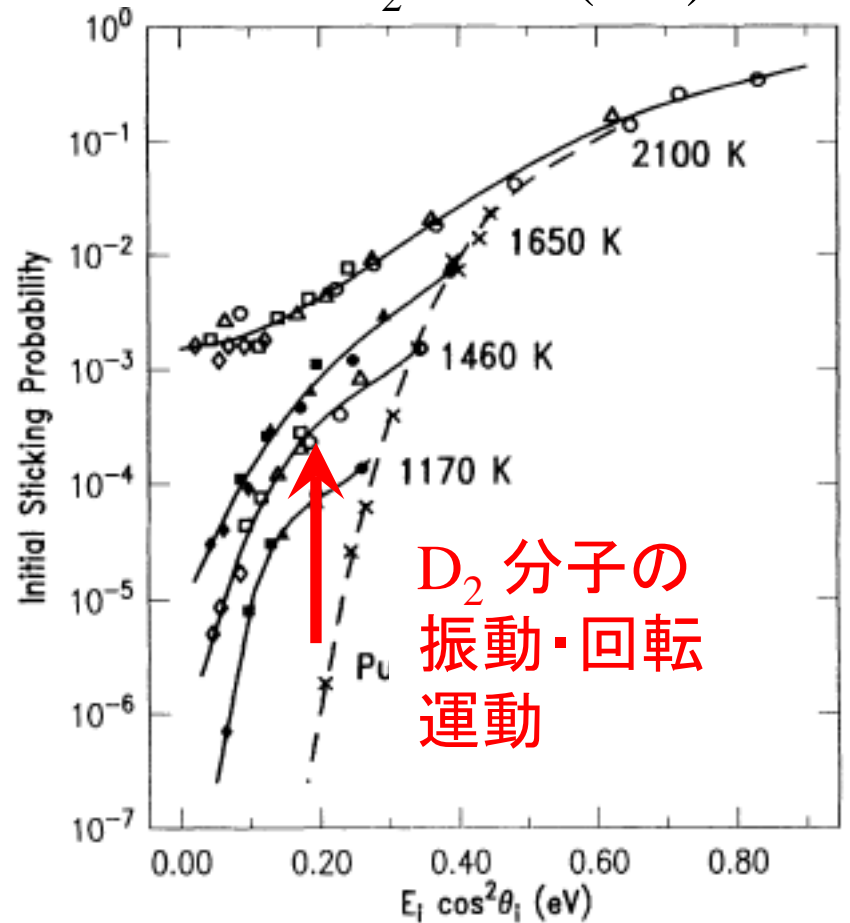


# H<sub>2</sub>/D<sub>2</sub> 分子の金属表面への解離吸着



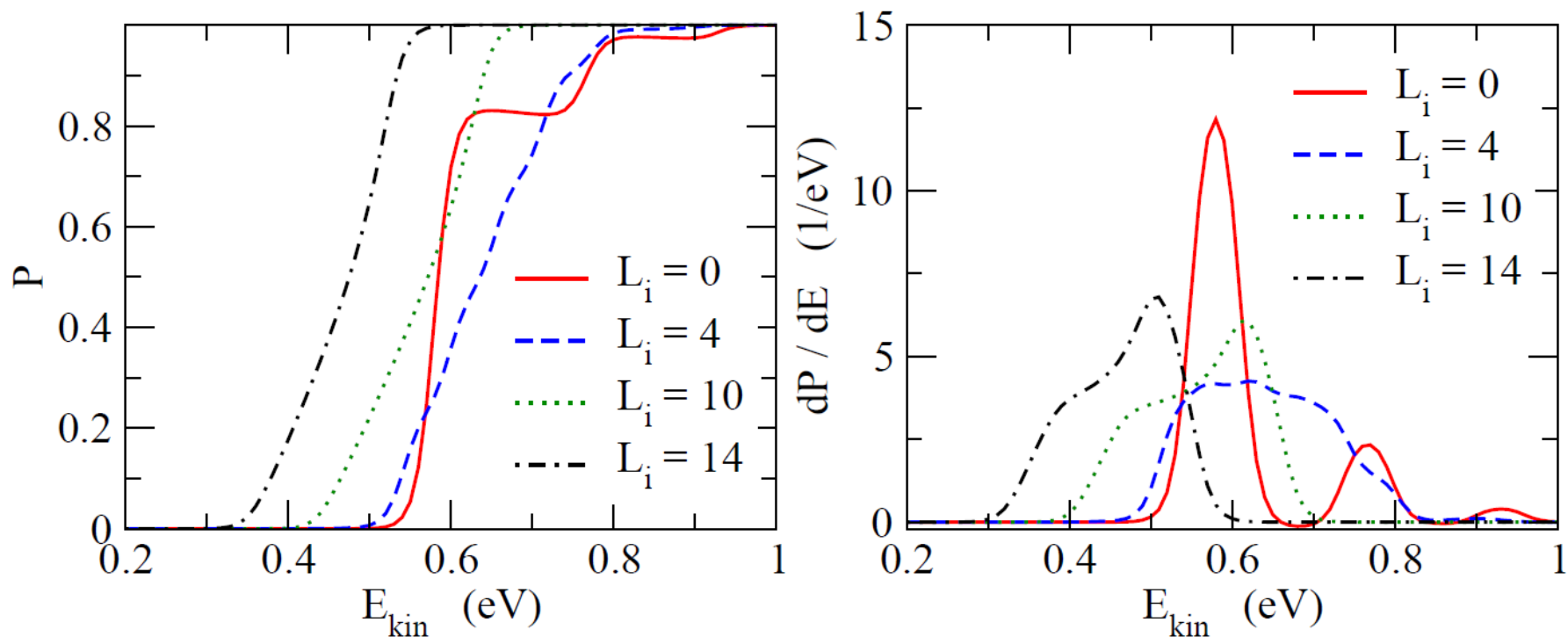
笠井、Dino、興地  
 物理学会誌 52 ('97) 824

## D<sub>2</sub> on Cu(111)



C.T. Rettner et al.,  
 PRL68('92)1164

## H<sub>2</sub> 分子の解離吸着に対する H<sub>2</sub> 分子の回転運動の効果

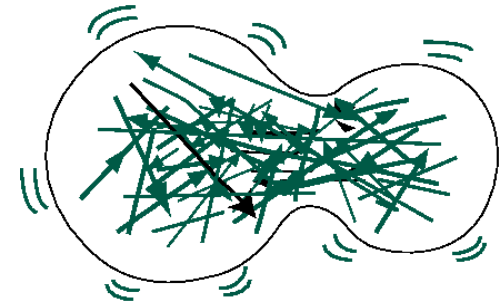
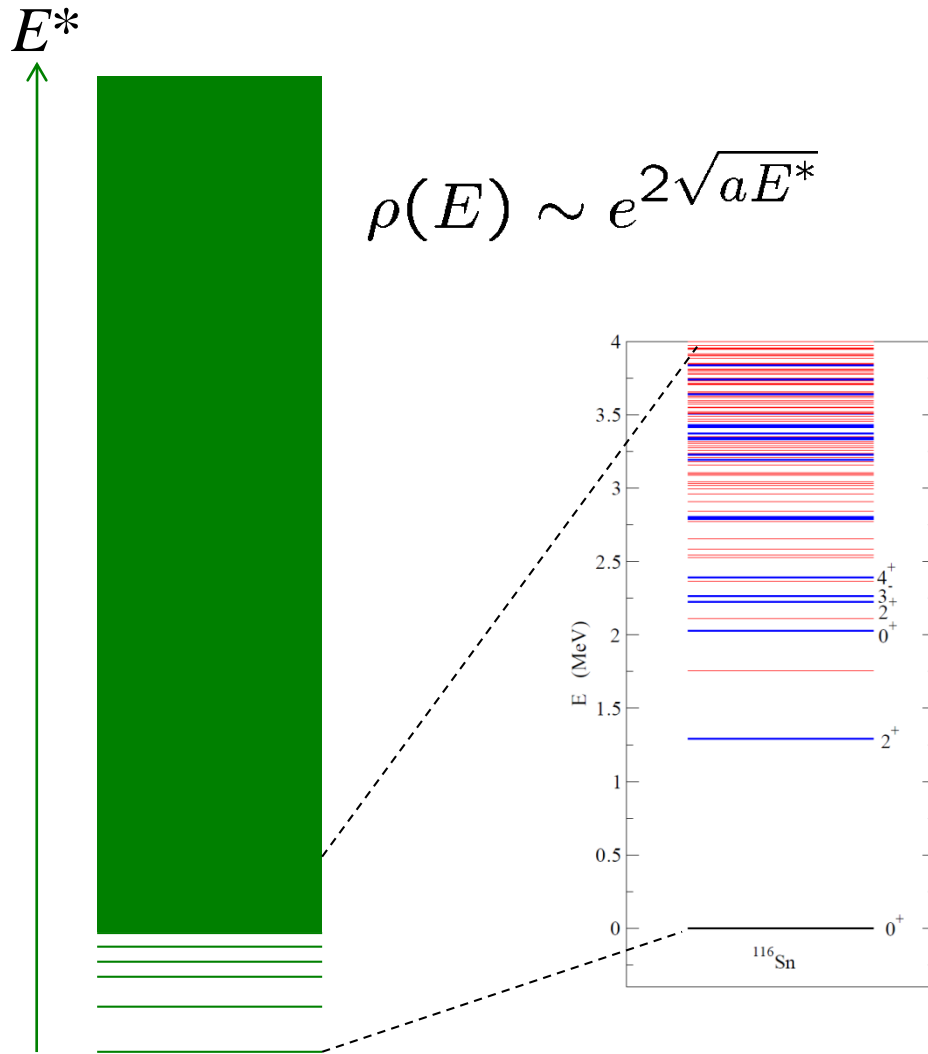


$L_i$ : 反応が始まる前の回転状態の角運動量

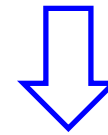
$$\frac{6\hbar^2}{2\mu r_0^2} = 0.046 \text{ eV}$$



# 原子核の摩擦と重イオン核融合反応



これらの無数の状態は核反応の途中で複雑に励起



原子核の内部自由度は核反応に対して「環境」のように振舞う  
「内的環境自由度」

原子核のスペクトル