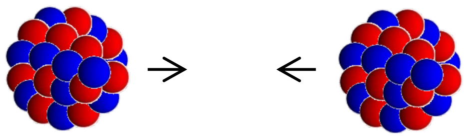
Nuclear shape dynamics in low-energy heavy-ion reactions

INPC 2025
May 25-30, 2025
DCC, Daejeon, Korea



Kouichi Hagino
Kyoto University, Kyoto, Japan

1. Introduction to low-energy nuclear reactions
2. Potential model: optical potentials
3. Multi-channel dynamics: coupled-channels approach
4. A short comment on Relativistic Heavy-ion collisions
5. Summary

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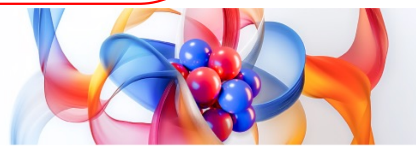
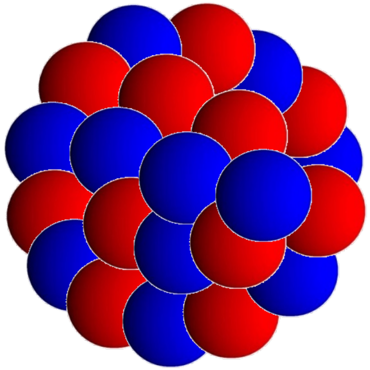


Low-energy nuclear physics

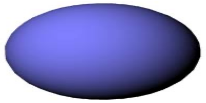


Nuclei as quantum many-body systems of nucleons

← to understand several properties in terms of nucleon d.o.f.

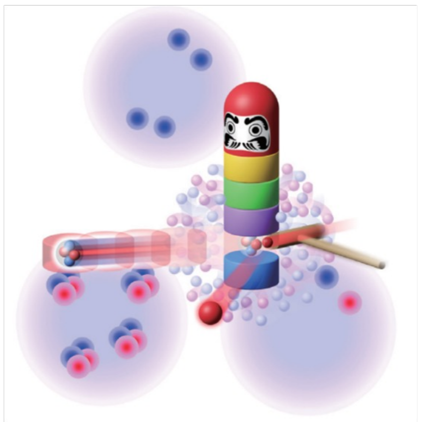
- static properties: nuclear structure

  the mass, the size, the shape....

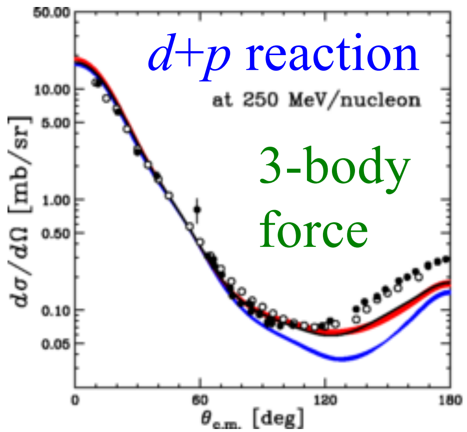


- dynamics: nuclear reactions

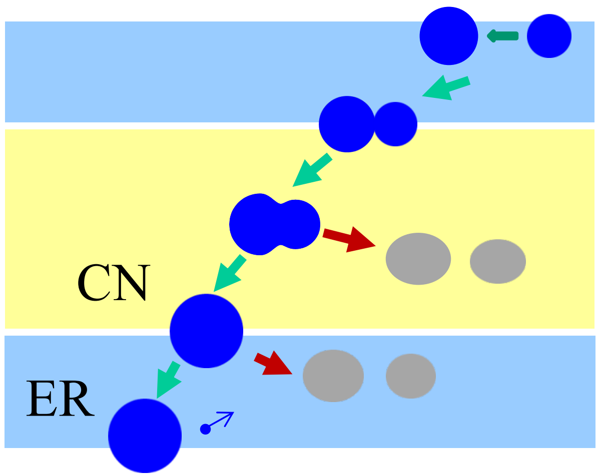
✓ Nuclear Reactions as a tool to investigate nuclear structure



knock-out reactions



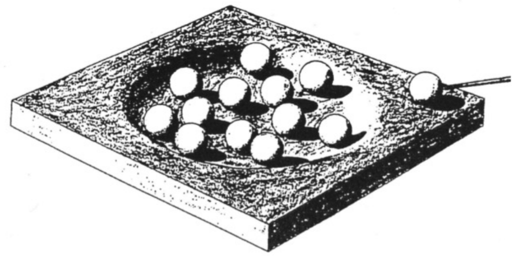
K. Sekiguchi et al., PRC89 ('14) 064007



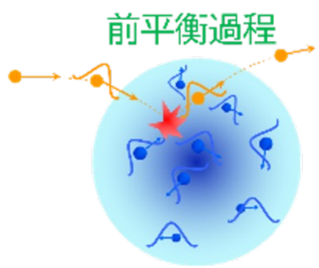
a synthesis of SHE

Two aspects of nuclear reactions

- ✓ a tool for nuclear structure
- ✓ reaction dynamics ⇐ this talk

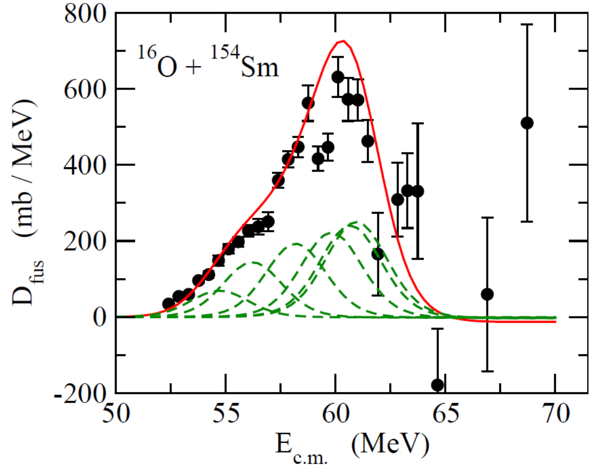


compound nucleus and quantum chaos



pre-equilibrium reactions

fusion barrier distribution



nucleus: a composite system

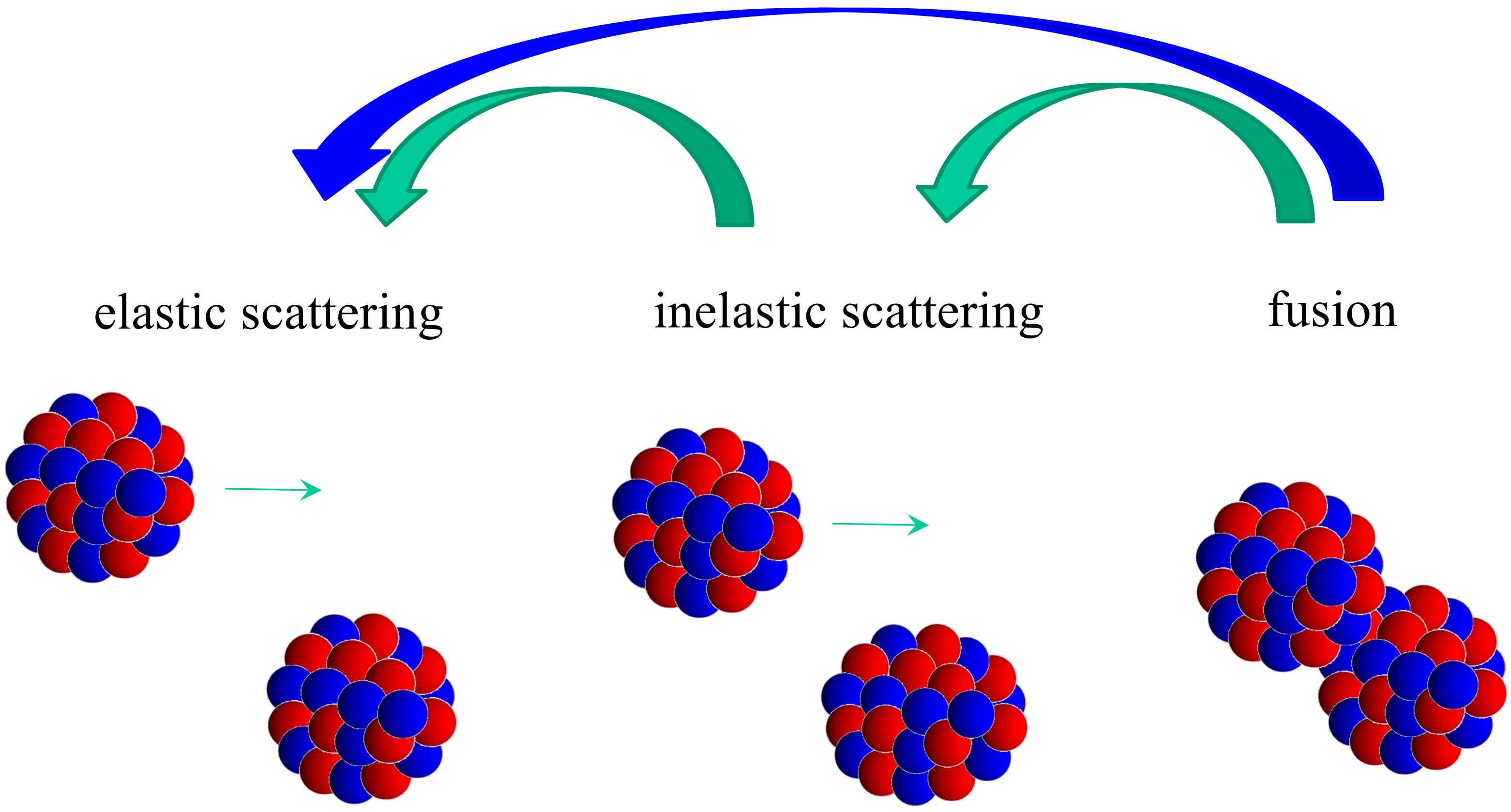
- ✓ rich reaction processes
- ✓ a rich interplay between nuclear structure and reaction

- ✓ elastic scattering
- ✓ inelastic scattering
- ✓ particle transfer reactions
- ✓ fusion reactions

etc.

- ✓ g.s. properties (mass, size, shape….)
- ✓ excitations

quantum many-body dynamics (nuclear reactions)

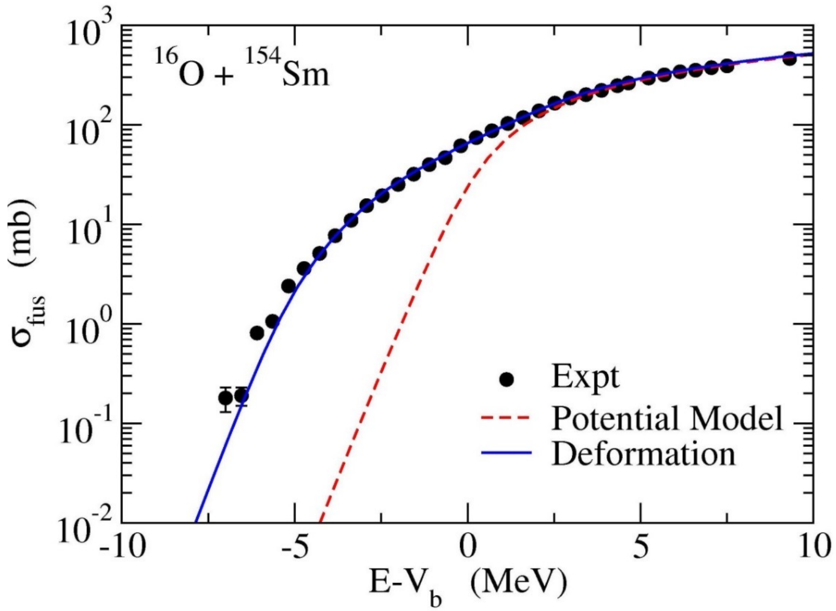


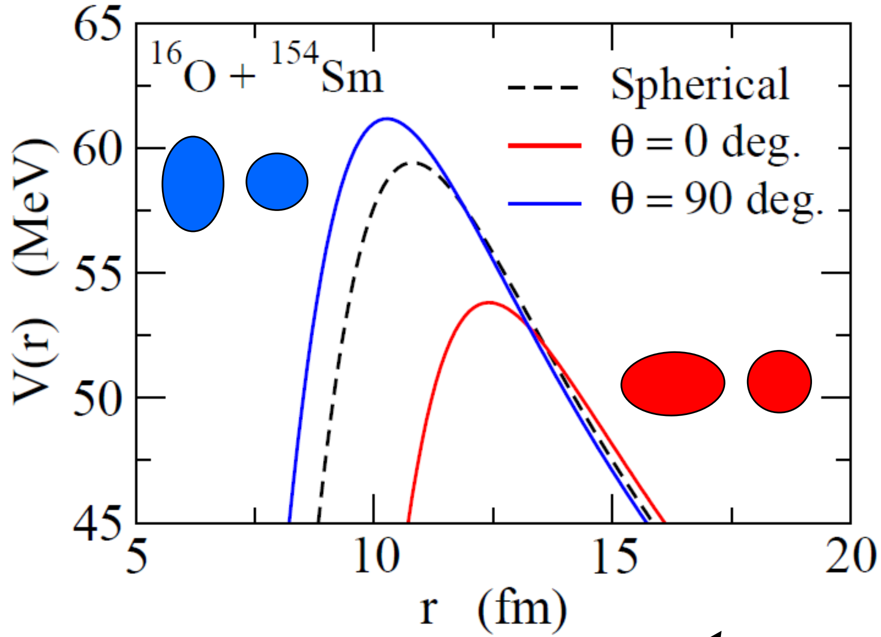
physics of nuclear reactions:
a unified description of these processes

A good example of the interplay between structure and reaction

Sub-barrier enhancement of fusion cross sections

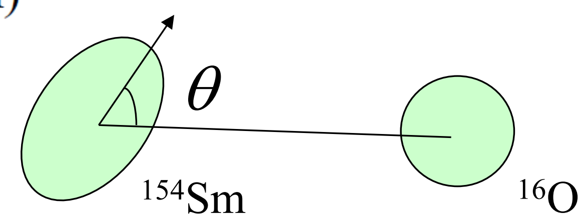
: a large impact of inelastic processes on fusion





a "snapshot" of a rotating nucleus

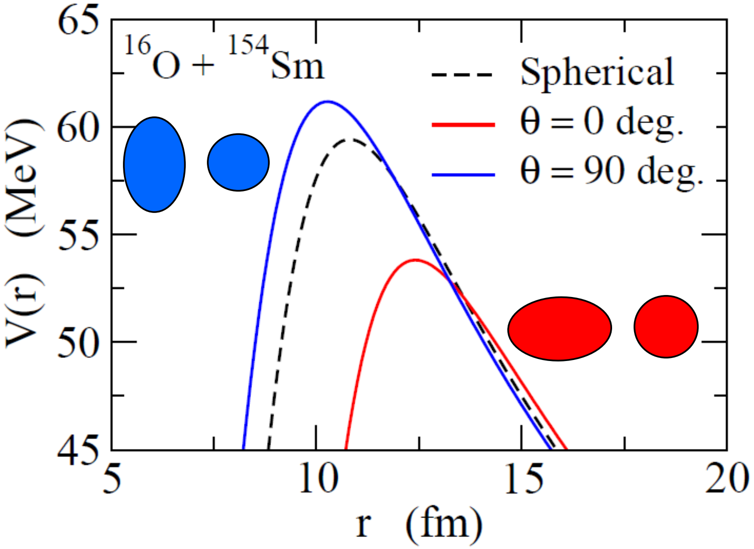
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$



K.H., N. Takigawa, PTP128 (2012) 1061

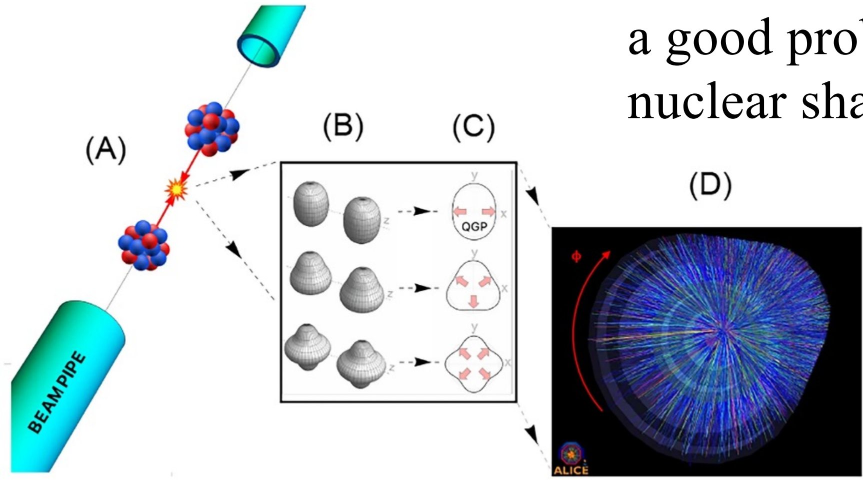
[Taking a snapshot of a nucleus with a “fast” nuclear reaction]

low-energy H.I. fusion reactions of a deformed nucleus



in recent years increasing interests in:

relativistic H.I. collisions with a deformed nucleus



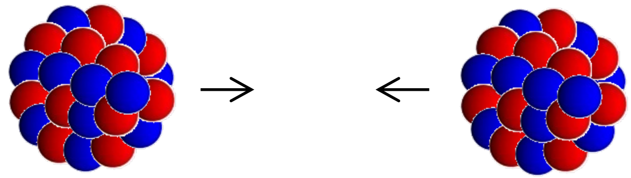
a good probe of nuclear shapes

J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

A talk by C. Zhang (Tuesday)

Large similarities → intersection of **High *E*** and **Low *E*** HI collisions

Single-channel calculations



g.s. g.s.

optical potential

$$V_{\rm opt}(r) = V(r) - \underline{iW(r)}$$

- elastic scattering
- absorption process

✓ inelastic scattering
✓ transfer reaction
✓ fusion
etc.

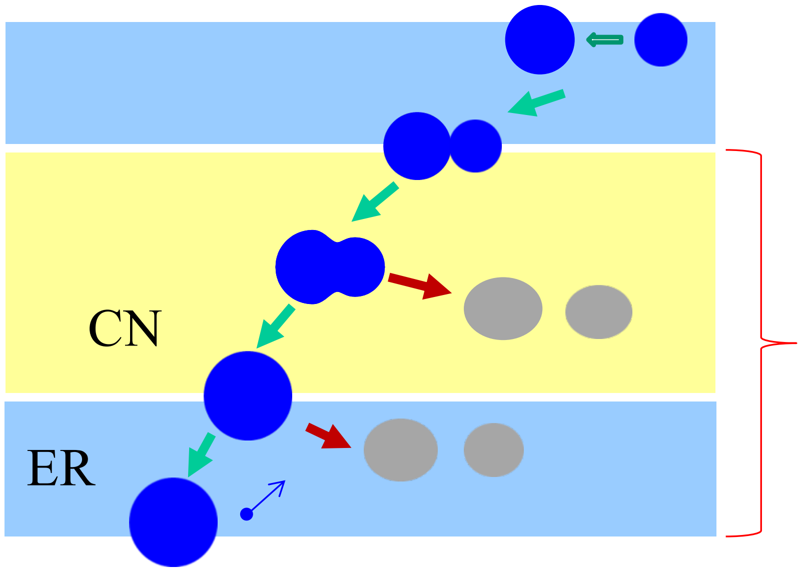
Optical model calculations

$$V_{\rm opt}(r) = V(r) - iW(r) \;\; \to \sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

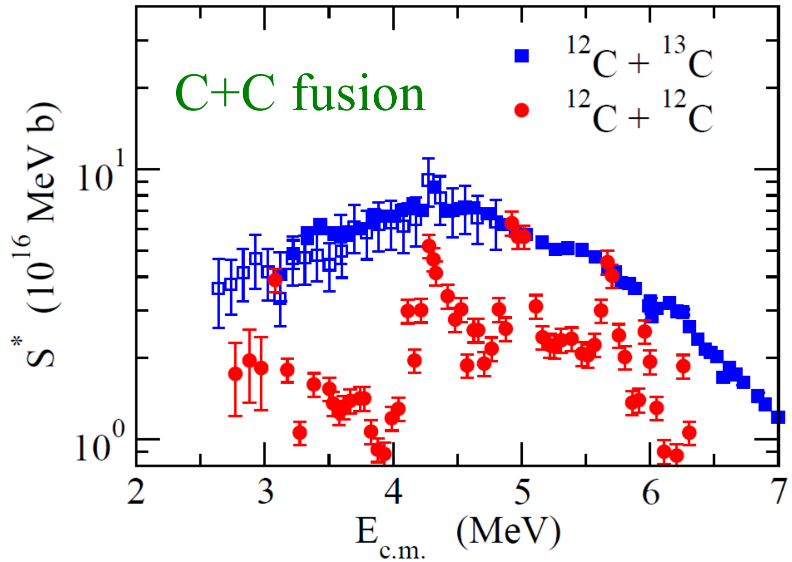
◆ no dynamics after the capture: only the absorption of elastic flux

◆ dynamics are important in several cases:

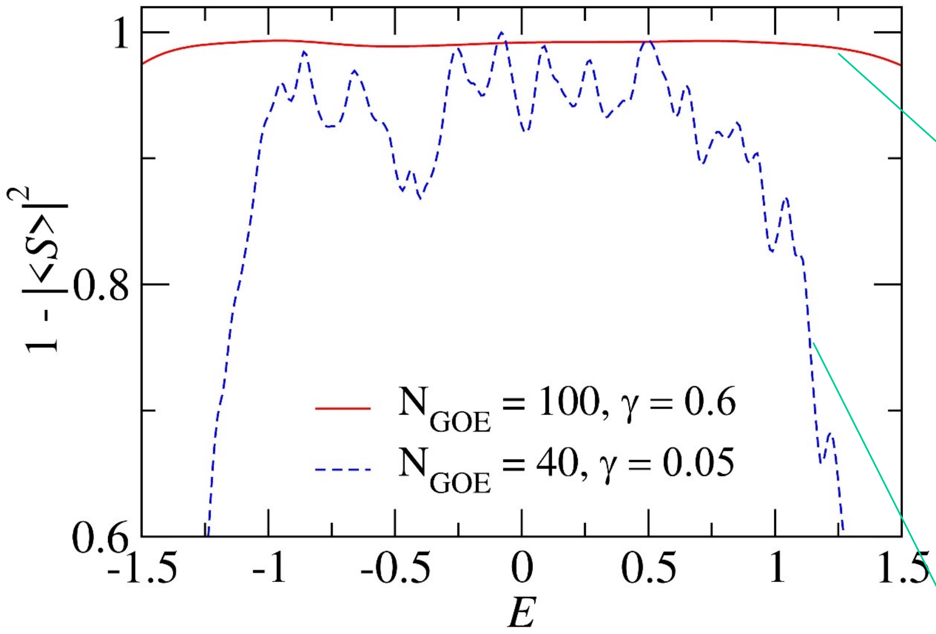
syntheses of superheavy elements

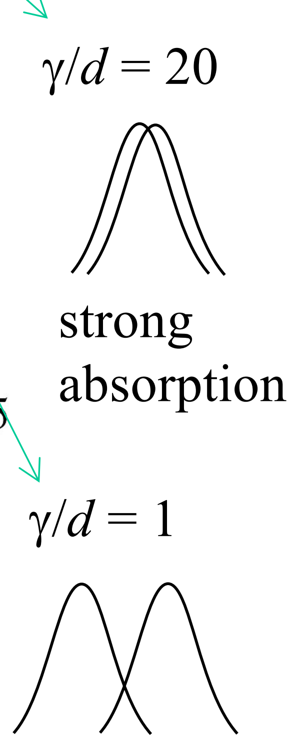


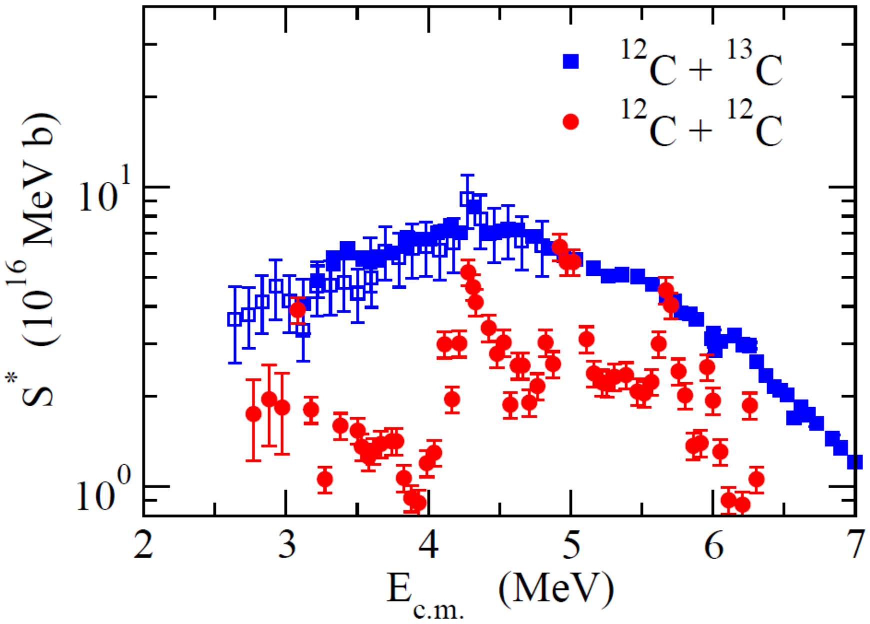
light-ion fusion reactions



a schematic reaction model with explicit inclusion of CN states





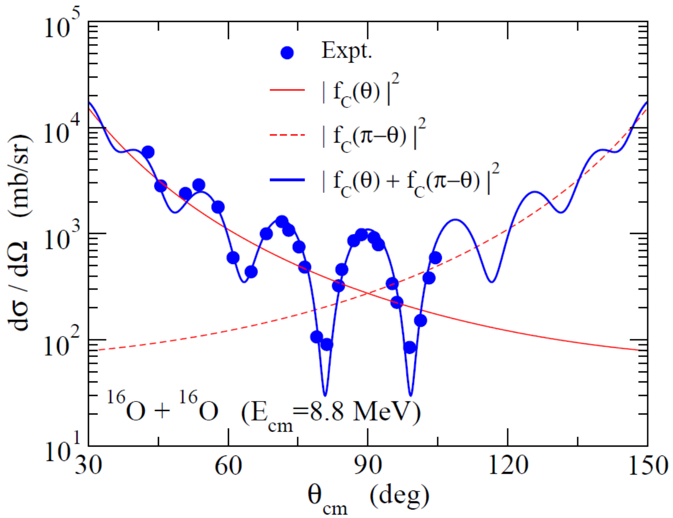


K.Hagino, arXiv: 2504.01371

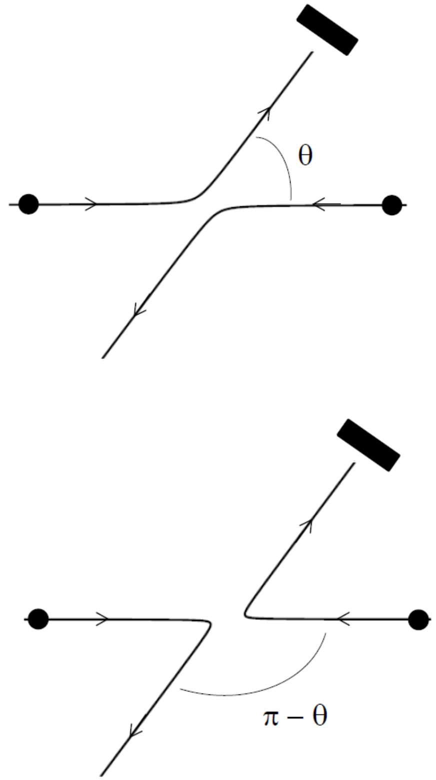
imaging quantum interference phenomena

quantum interference phenomena in heavy-ion reactions

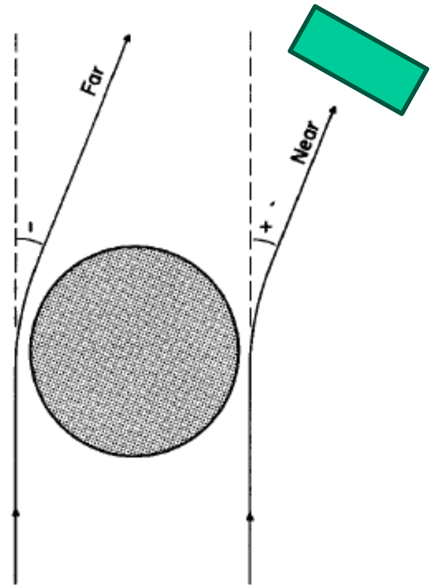
✓ Mott Scattering



expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878



✓ near side-far side interference

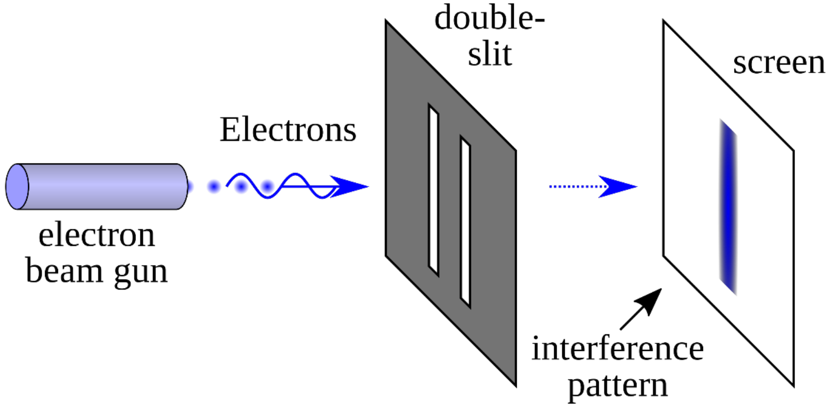


✓ nuclear-Coulomb interference

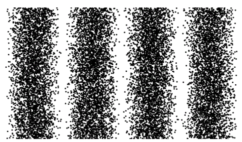
✓ barrier wave-internal wave interference

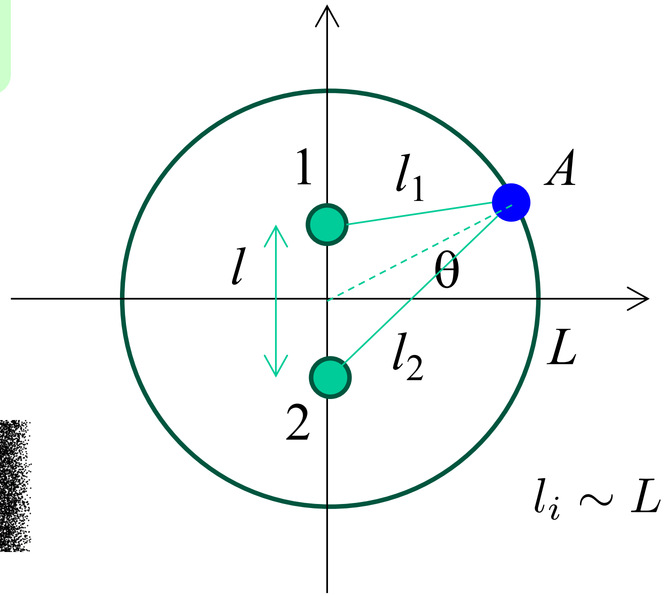
imaging quantum interference phenomena

a double slit problem



Wikipedia





$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$

the amplitude at A

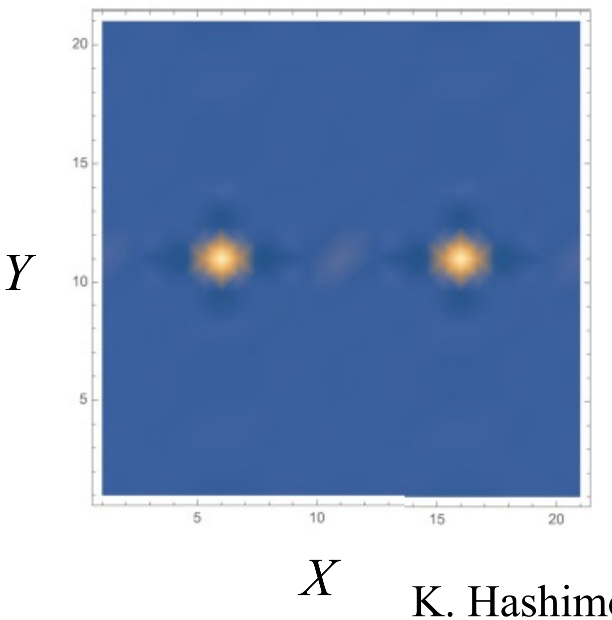
$$\begin{aligned} f(\theta) &= f_1(\theta) + f_2(\theta) \to P = |f_1(\theta) + f_2(\theta)|^2 \\ f_i(\theta) &= A\sin\left(\frac{2\pi}{\lambda}l_i - \omega t\right) \end{aligned}$$

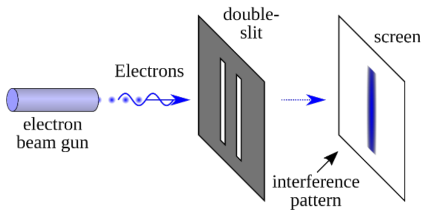
<u>the Fourier transform of $f(\theta)$:</u>

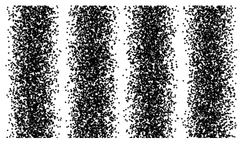
$$\Phi(X,Y) \propto \int_{\theta_2}^{\theta_2} d\theta \int_{\varphi_1}^{\varphi_2} d\varphi\, e^{ik(\theta X + \varphi Y)} f(\theta,\varphi)$$

imaging quantum interference phenomena

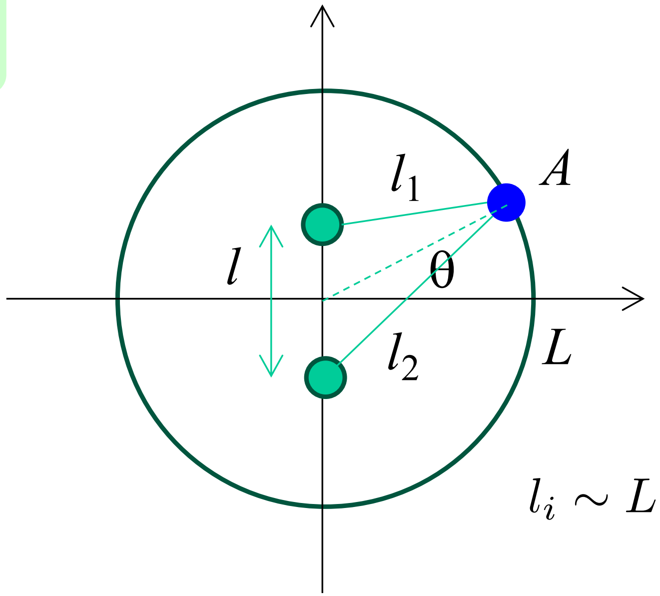
$|\Phi(X,Y)|^2$







K. Hashimoto et al.,
PTEP2023, 043B04 (2023)



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imaging quantum interference phenomena

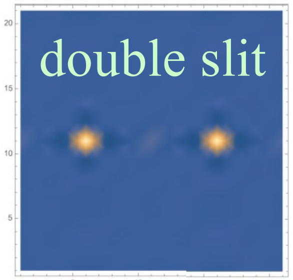
K. Hagino and T. Yoda, PLB848, 138326 (2024).
K. Heo and K.Hagino, PRC111, 034612 (2025).

cf. a talk by K. Heo (Tue.)

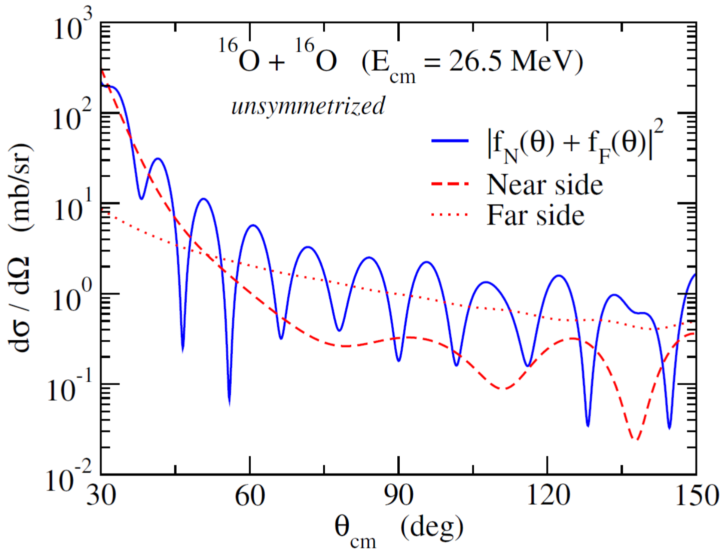
Application of nuclear reactions:

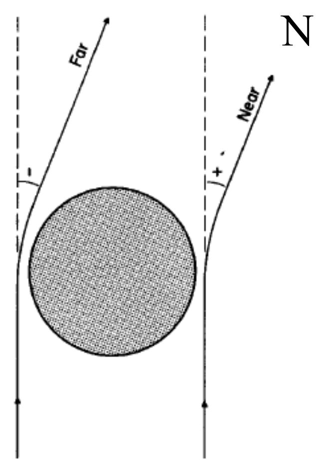
$$V_{\rm opt}(r) = V(r) - iW(r) \longrightarrow f(\theta) \longrightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$\xrightarrow{\text{F.T.}} \Phi(X,Y) \rightarrow |\Phi(X,Y)|^2$$

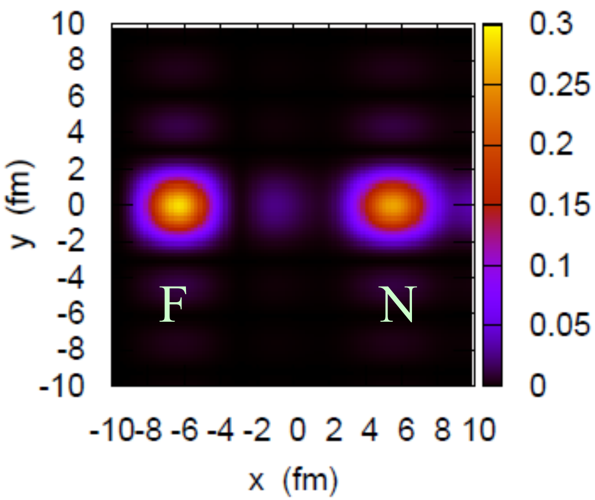


^{16}O+^{16}O at $E = 26.5$ MeV

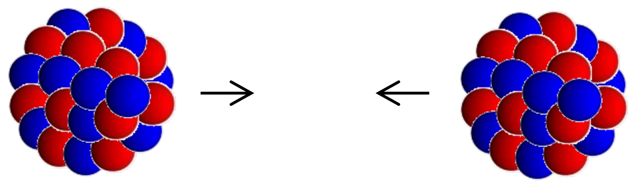


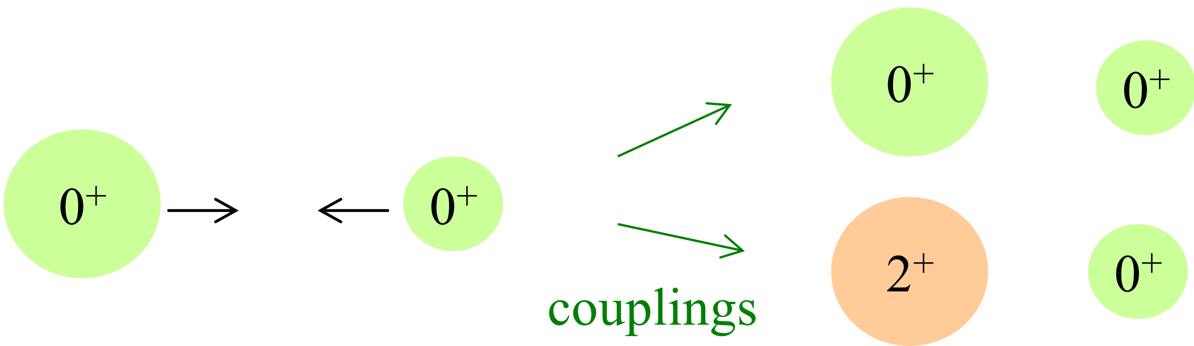


θ_0=55 deg., Δθ=15 deg.



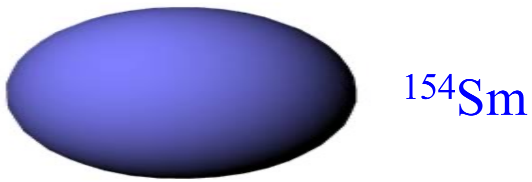
Multi-channel dynamics

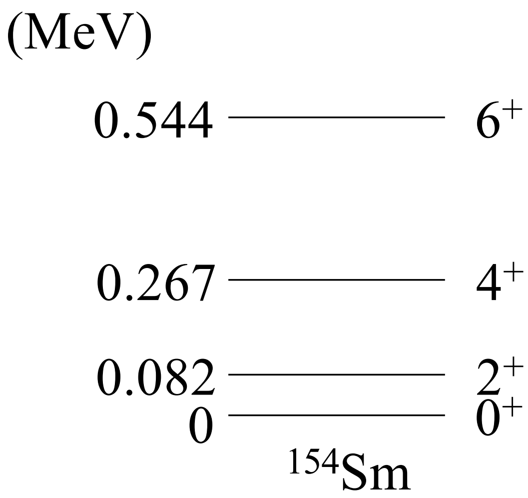




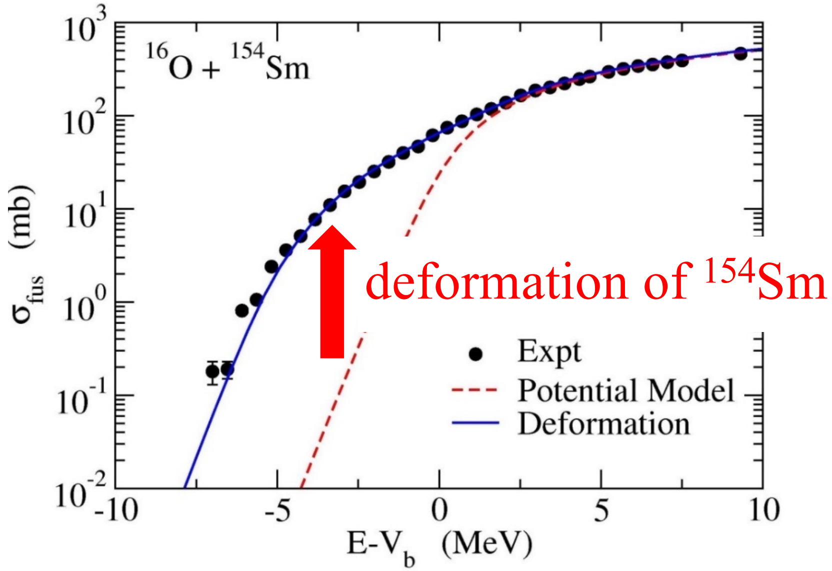
Sub-barrier fusion enhancement

^{154}Sm : a typical deformed nucleus

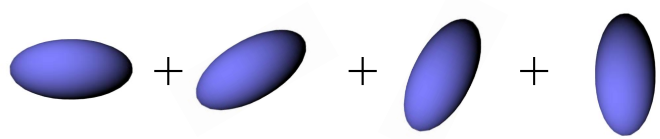




rotational spectrum



K. H. and N. Takigawa,
Prog. Theo. Phys.128 ('12)1061.

$\Psi_{0+} =$ 

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

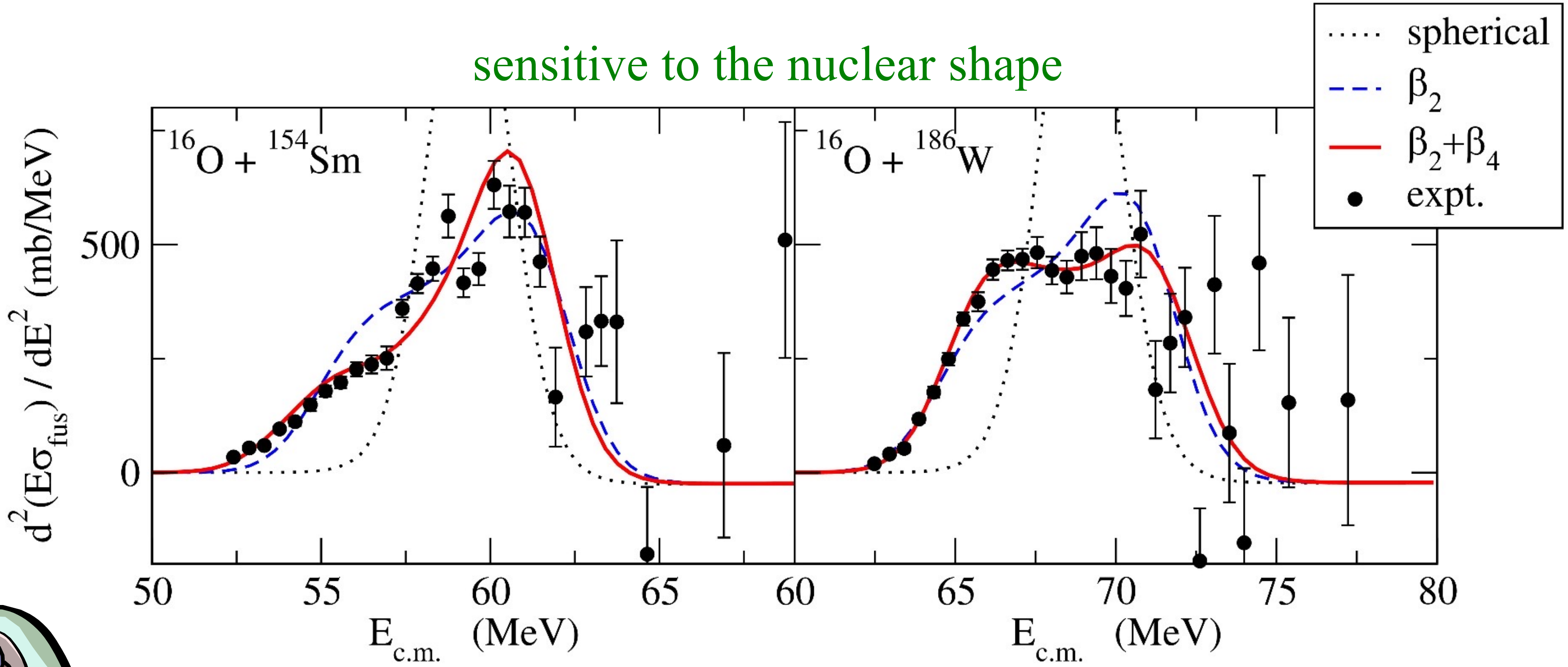
"a snapshot of a rotating nucleus"

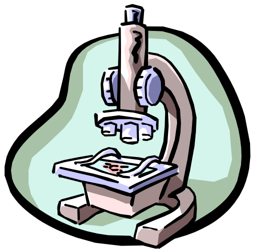
Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 (‘91) 25

sensitive to the nuclear shape





$\beta_2 = 0.33$
$\beta_4 = +0.05$

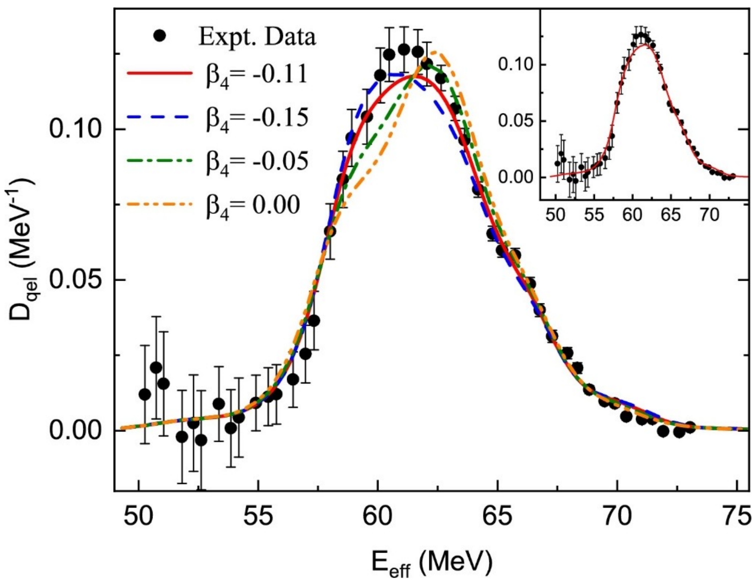
$\beta_2 = 0.29$
$\beta_4 = -0.03$

J.R. Leigh et al., PRC52 (‘95) 3151

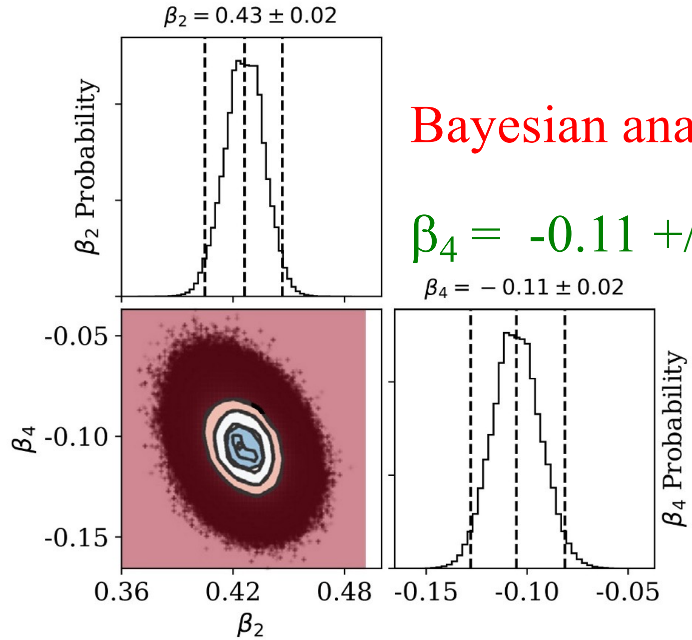
Determination of β_4 of ^{24}Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

^{24}Mg + ^{90}Zr



$\beta_2 = 0.43$ +/- 0.02



Bayesian analysis

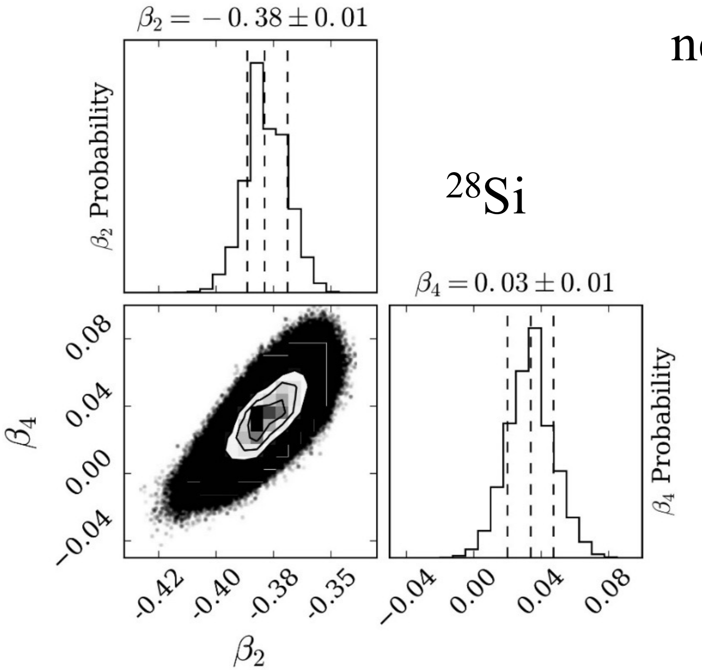
$\beta_4 = -0.11$ +/- 0.02

high precision determination of β_4
→ for the first time

cf. (p,p'): $\beta_4 = -0.05$ +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

Emulator for multi-channel scattering



^{28}Si

Y.K. Gupta et al.,
PLB845, 138120 (2023).

needs to repeat many calculations with different (β_2,β_4)

→ an emulator to speed-up the calculations

Eigenvector continuation

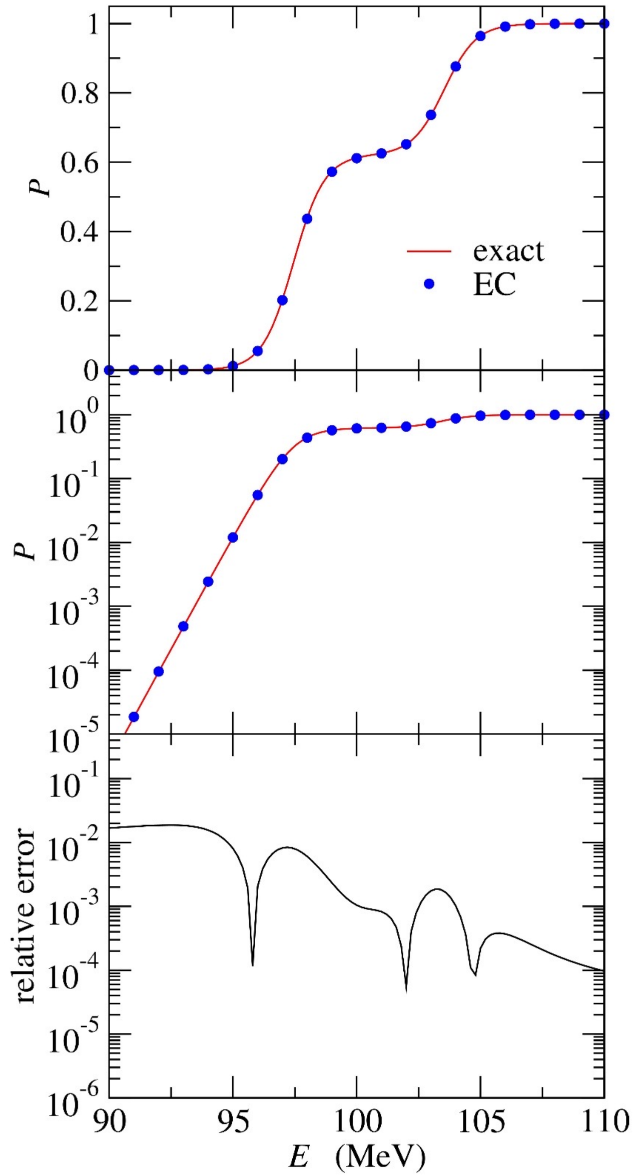
➢ bound state problems:

$$H(\theta)|\Psi(\theta)\rangle = E(\theta)|\Psi(\theta)\rangle \qquad \Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i)$$

T. Duguet et al.,
Rev. Mod. Phys. 96, 031002 (2024)

➢ Extension to scattering problems:

- R. Furnstahl et al., PLB809, 135719 (2020)
- C. Drisshler et al., PLB823, 136777 (2021)
- J. Liu, J. Lei, and Z. Ren, PLB858, 139070 (2024)
- K. Hagino, Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, arXiv: 2504.14922



1D two-channel problem:

$$H = \begin{pmatrix} V(x) & F(x) \\ F(x) & V(x)+\epsilon \end{pmatrix} \qquad \begin{aligned} V(x) &= V_0\, e^{-x^2/2s^2} \\ F(x) &= F_0\, e^{-x^2/2s_f^2} \end{aligned}$$

V_0 = 100 MeV, s=s_f=3 fm,
F_0 = 3 MeV

$$\Psi_E(x, F_0) = \sum_{i=1-5} c_i \Psi_E(x, F_{0i})$$

F_{0i} = 1.5, 2.0, 2.5, 3.5, 4.5 MeV

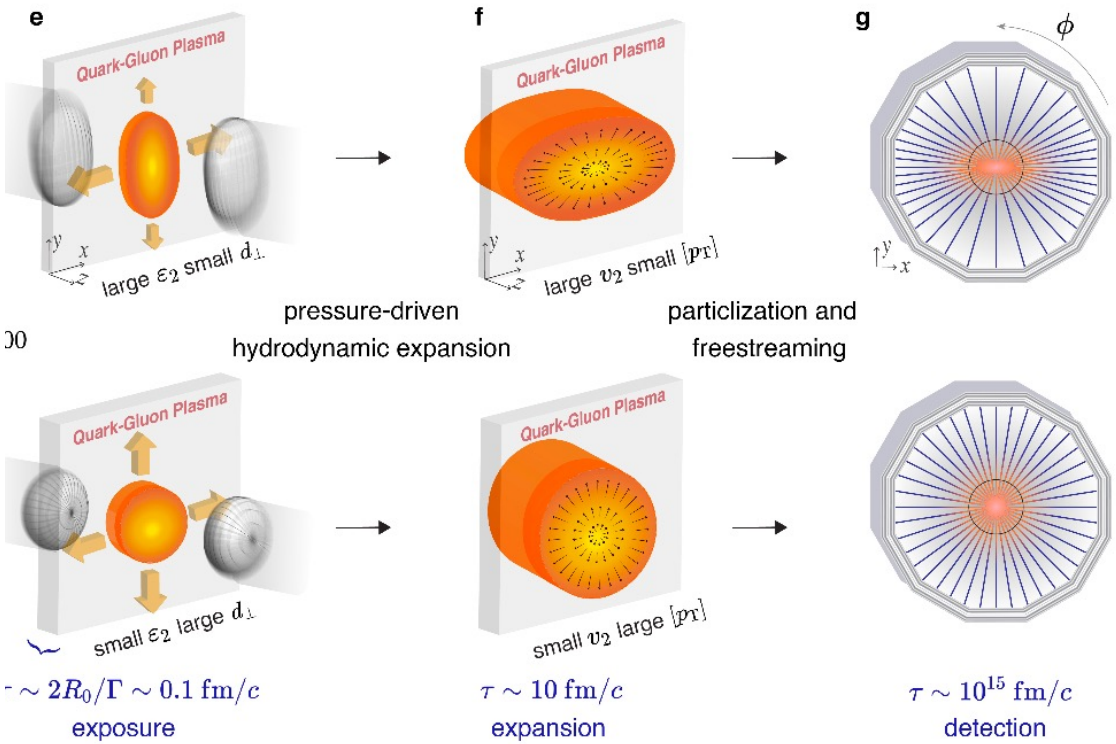
to simulate F_0 = 3 MeV

EC: the discrete basis method + Kohn variation principle

cf. K.H. and G.F. Bertsch, PRC110, 054610 (2024)

K. H., Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, arXiv: 2504.14922

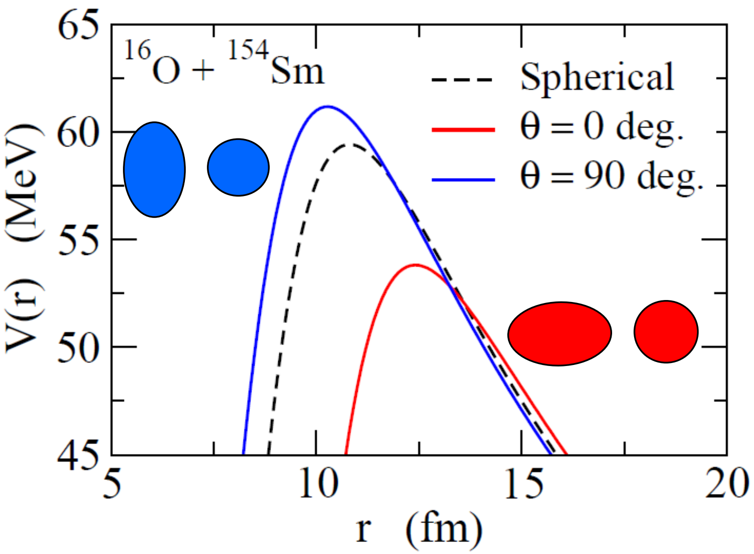
Relativistic Heavy-Ion collisions



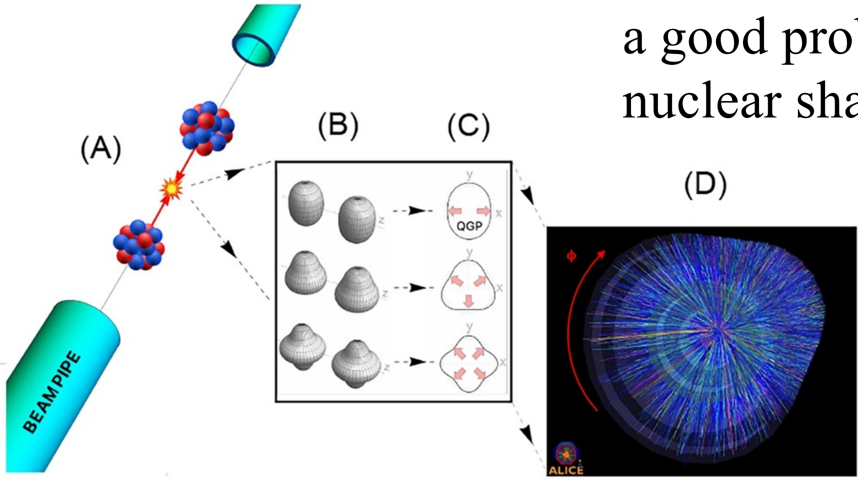
M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

Taking a snapshot of a nucleus with a "fast" nuclear reaction

low-energy H.I. fusion reactions of a deformed nucleus



relativistic H.I. collisions with a deformed nucleus



a good probe of nuclear shapes

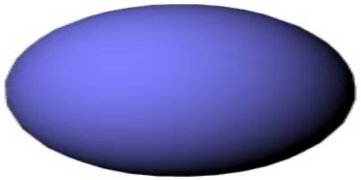
J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

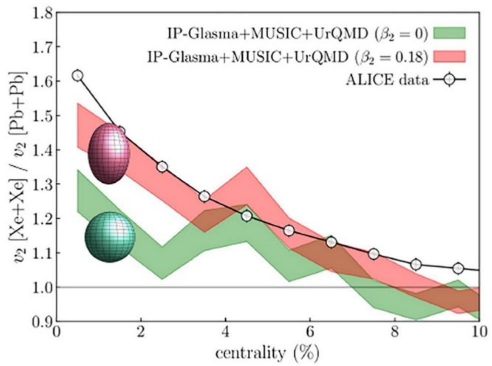
increasing interests in recent years

Large similarities → intersection of **High *E*** and **Low *E*** HI collisions

Probing nuclear shapes in Relativistic H.I. collisions

So far, the main focus of ref. HIC has been on a static deformation of a nucleus





H. Esbensen, Nucl. Phys. A352, 147 (1981)

FUSION AND ZERO-POINT MOTIONS

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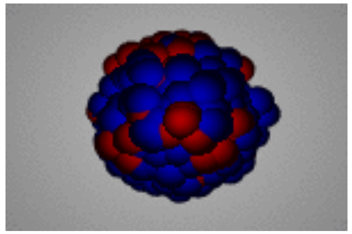
Received 14 July 1980

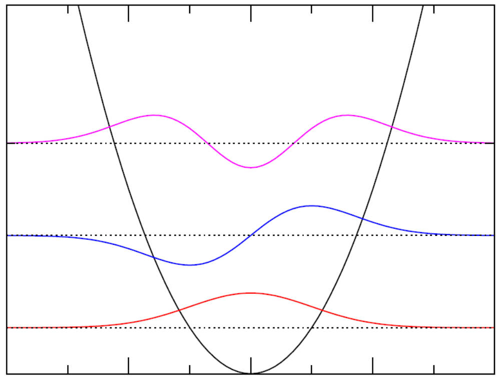
$$\sigma_{\rm fus}(E) \sim \int d\beta\, w(\beta)\sigma_0(E;\beta); \quad w(\beta) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\beta^2/2\sigma^2}$$

There are also several dynamical deformations of a spherical nucleus



$$\langle\beta\rangle = 0$$

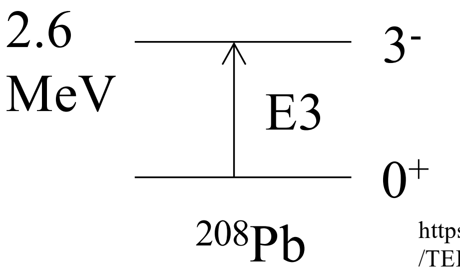
but fluctuates around β=0 $\rightarrow \langle\beta^2\rangle \neq 0$

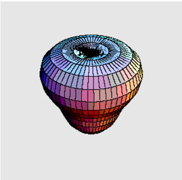


Probing nuclear shapes in Relativistic H.I. collisions

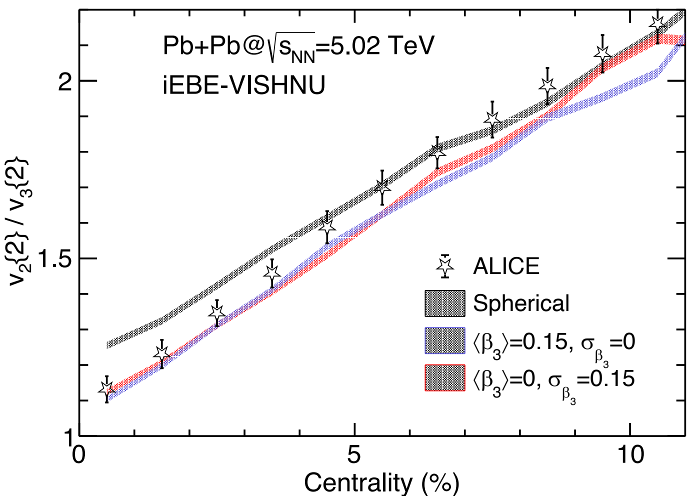
Duoduo Xu et al., arXiv: 2504.19644

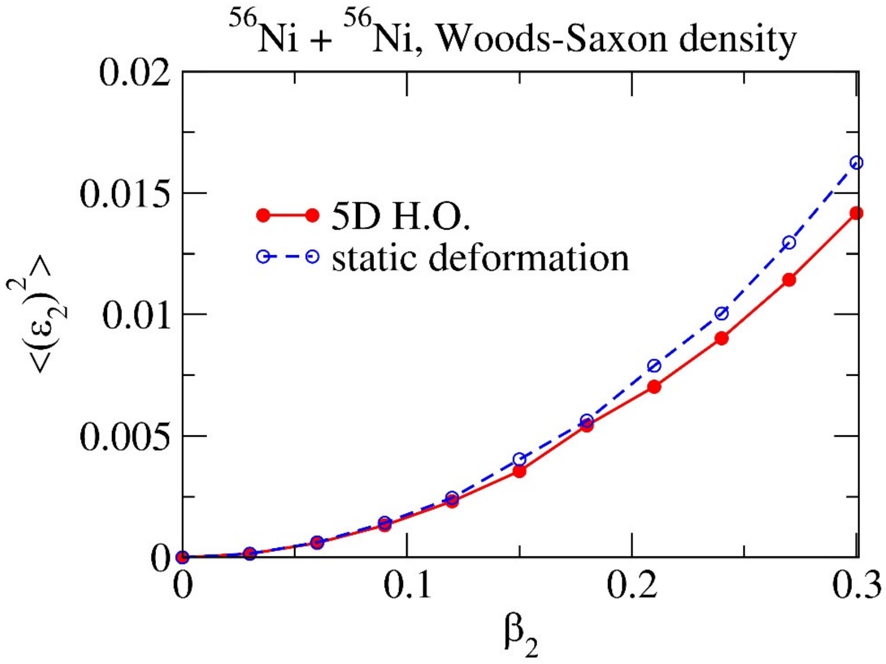
octupole vibration of ^{208}Pb





https://web-docs.gsi.de/~wolle
/TELEKOLLEG /KERN/index-s.html



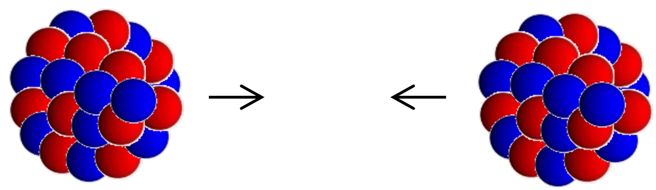


eccentricity parameter:

$$\epsilon_2 e^{2i\Phi} = -\frac{\int d\boldsymbol{r}\, r^2 \sin^2\theta\, e^{2i\phi} \sum_i \rho_i(\boldsymbol{r},\boldsymbol{\alpha}_i)}{\int d\boldsymbol{r}\, r^2 \sin^2\theta\, \sum_i \rho_i(\boldsymbol{r},\boldsymbol{\alpha}_i)}$$

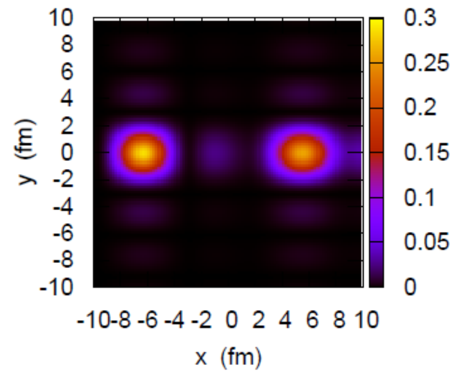
K. Hagino and M. Kitazawa, in preparation

Summary



✓ Nuclear Reaction Dynamics

- a microscopic approach to absorption due to CN formation
- imaging elastic scattering ← Fourier transform of $f(\theta)$
- an emulator for multi-channel reactions



✓ Similarities between low-*E* H.I. fusion and Relativistic H.I. Collisions

→ a snapshot of a nucleus

A tool to probe nuclear deformations

→ surface vibrations of a spherical nucleus