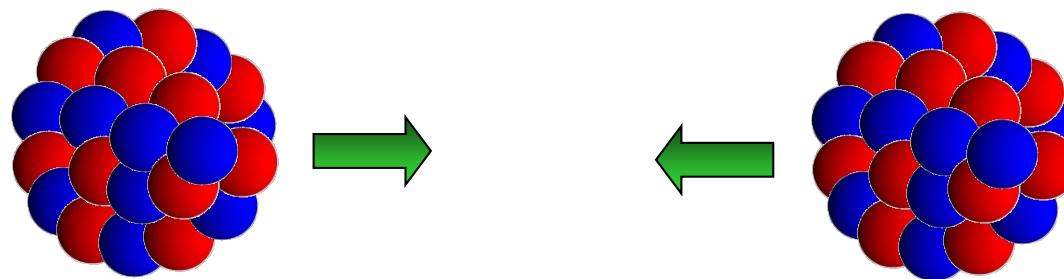


# Perspectives on nuclear reaction theory and superheavy elements

Kouichi Hagino

Tohoku University, Sendai, Japan



1. Nuclear Reactions: overview
2. Heavy-ion fusion reactions
3. Fusion for superheavy elements
4. Summary

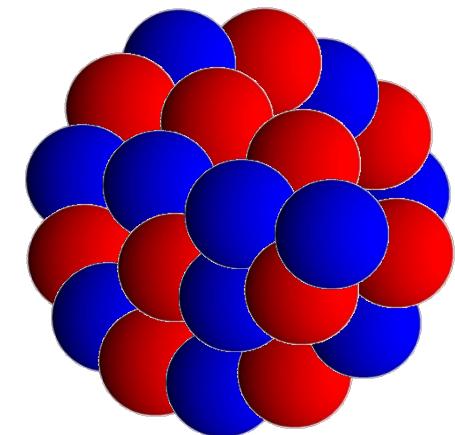
# Introduction: low-energy nuclear physics

- ❑ behaviors of atomic nuclei as a quantum many-body systems
  - ← understanding based on strong interaction

➤ static properties: nuclear structure

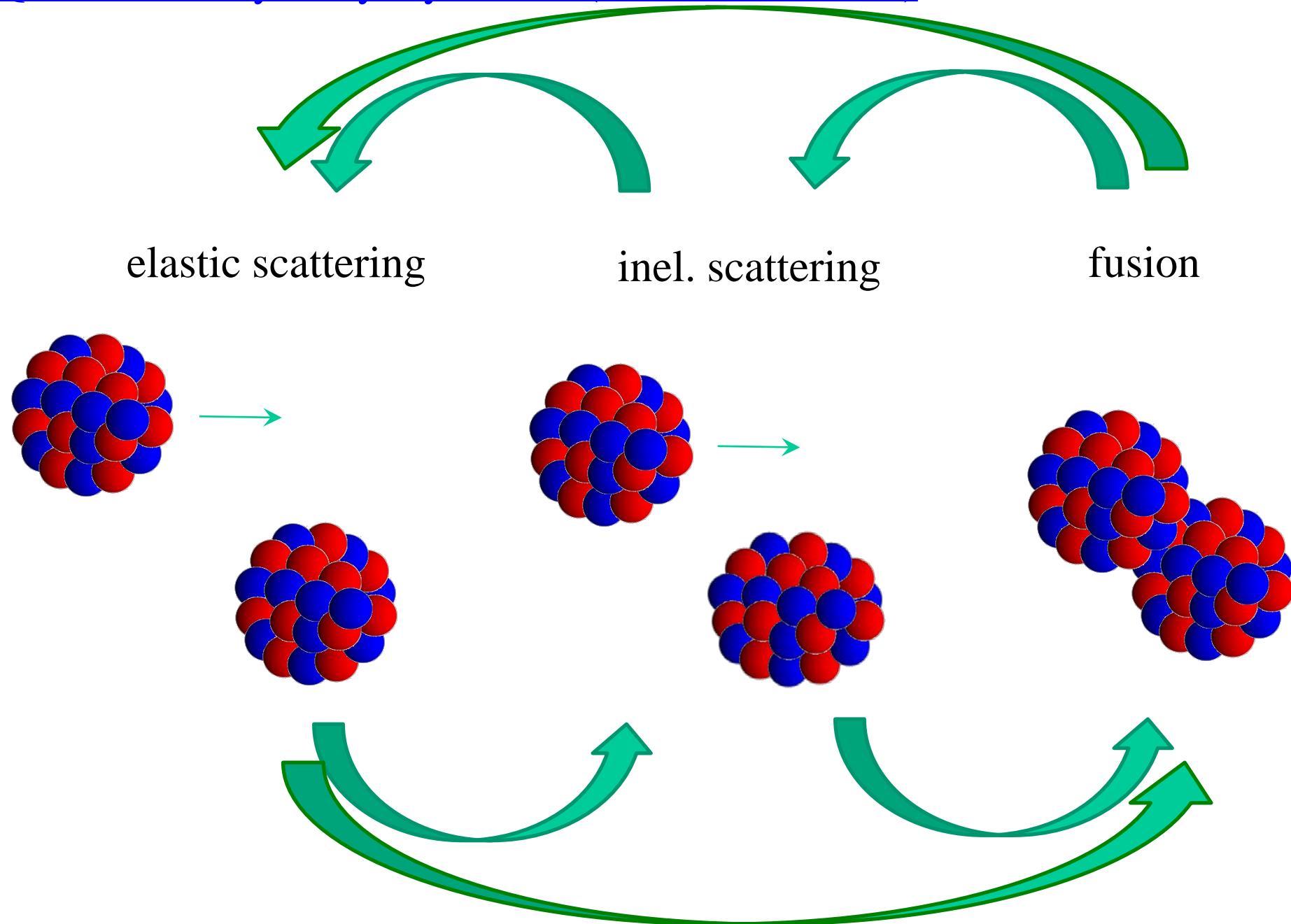
- ✓ ground state properties  
(mass, size, shape,...)
- ✓ excitations
- ✓ nuclear matter

➤ dynamics: nuclear reactions

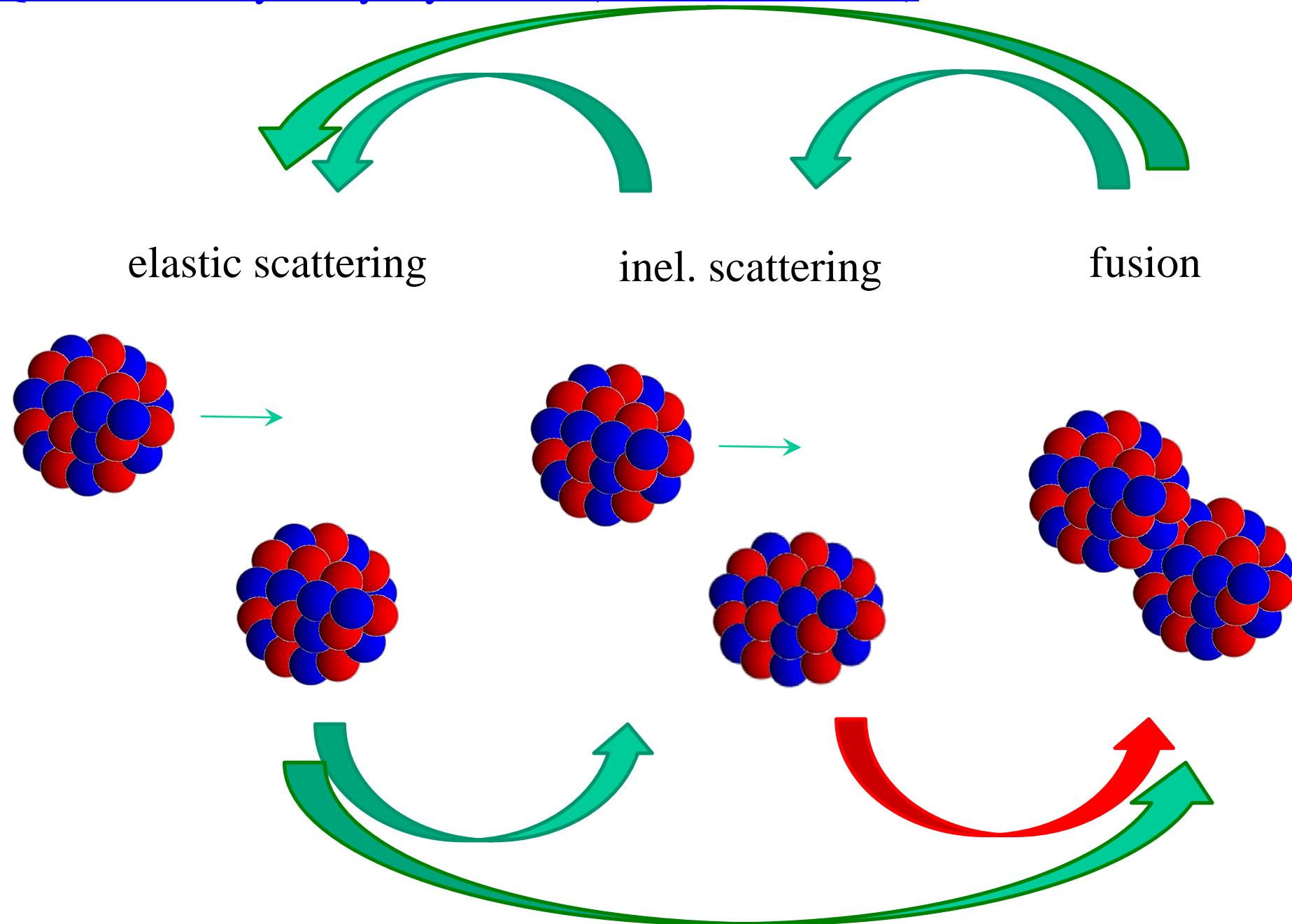


an interplay between these

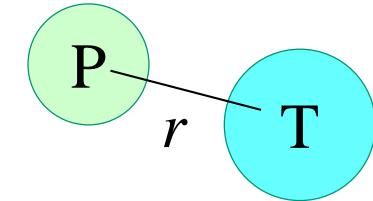
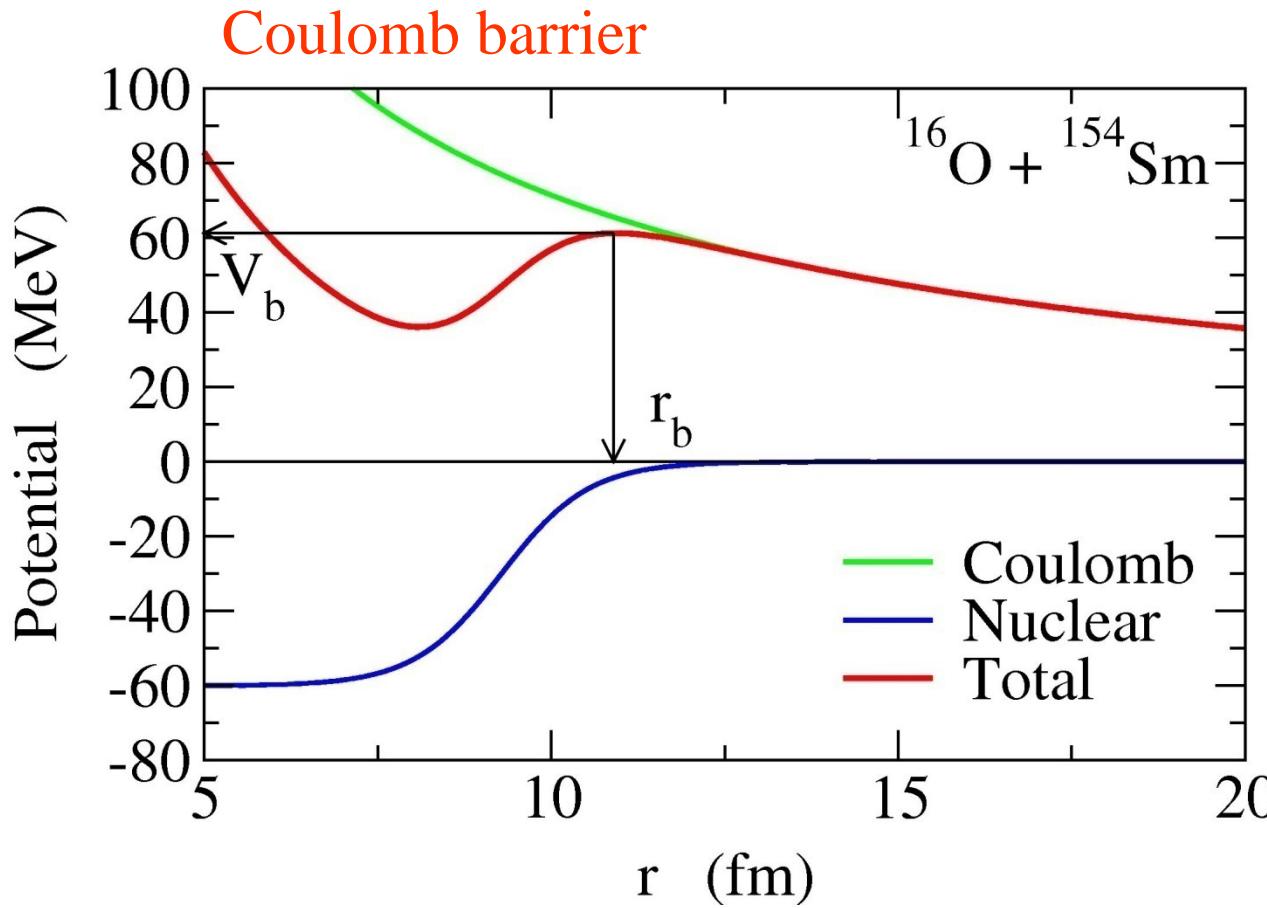
# Quantum Many-body Dynamics (nuclear reactions)



# Quantum Many-body Dynamics (nuclear reactions)



## Coulomb barrier



1. Coulomb interaction  
long range  
repulsion
2. Nuclear interaction  
short range  
attraction



Potential barrier  
(Coulomb barrier)

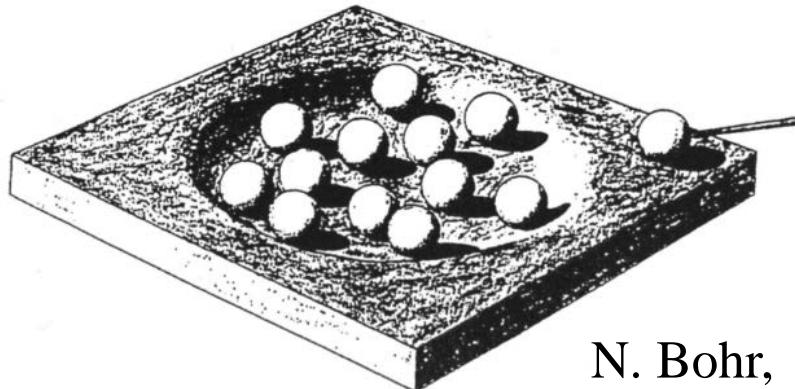
the barrier height → defines the energy scale of a system

Fusion reactions at energies around the Coulomb barrier

# Fusion reactions: compound nucleus formation

Niels Bohr (1936)

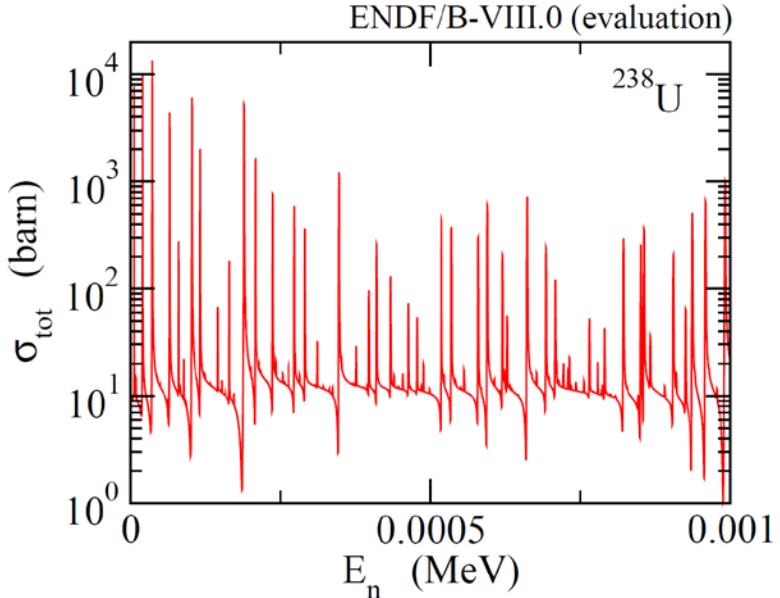
Neutron capture of nuclei → compound nucleus



N. Bohr,  
Nature 137 ('36) 351



Wikipedia

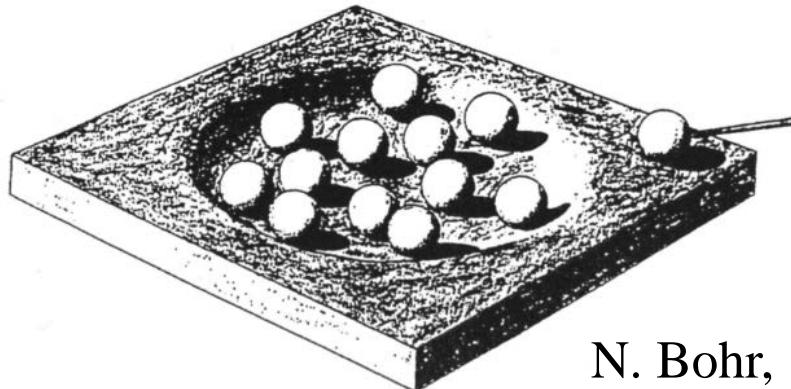


cf. Experiment of Enrico Fermi (1935)  
many very narrow (=long life-time)  
resonances (width ~ eV)

# Fusion reactions: compound nucleus formation

Niels Bohr (1936)

Neutron capture of nuclei → compound nucleus

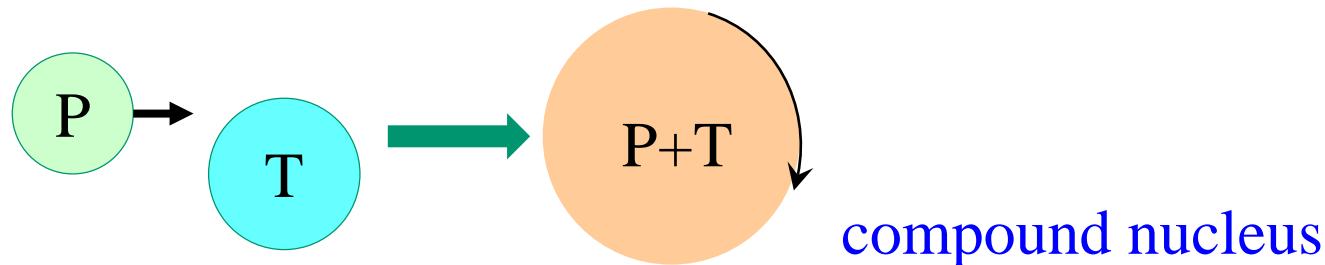


N. Bohr,  
Nature 137 ('36) 351

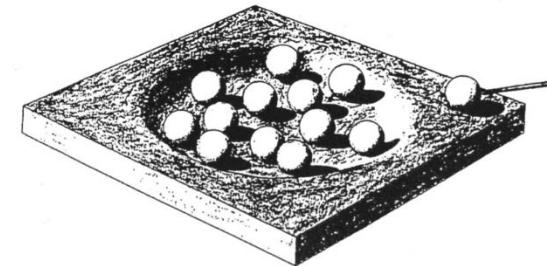
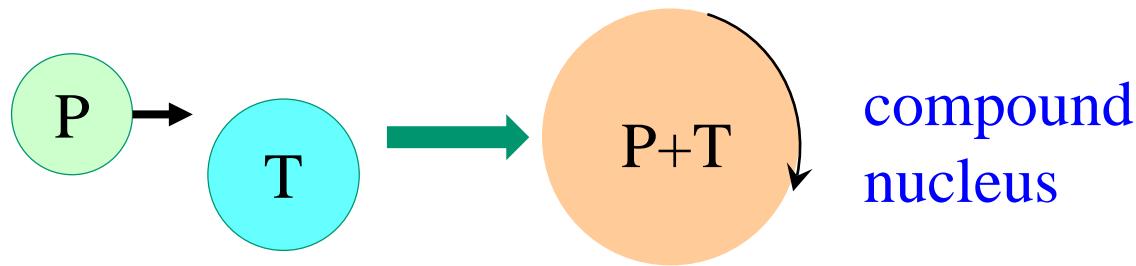


Wikipedia

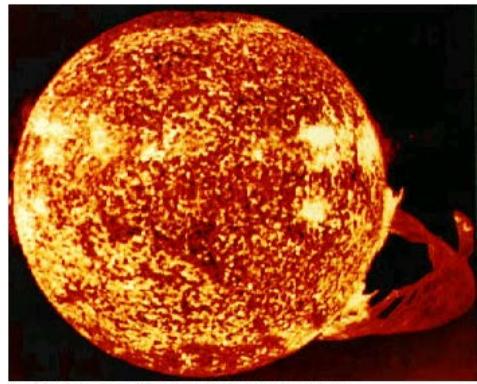
forming a compound nucleus with heavy-ion reactions = H.I. fusion



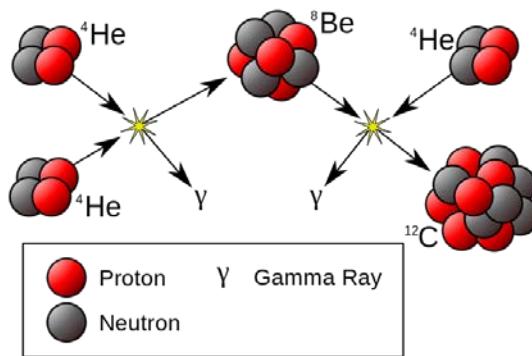
# Fusion reactions: compound nucleus formation



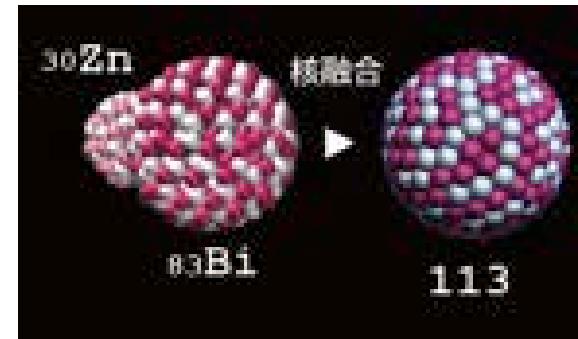
cf. Bohr '36



energy production  
in stars (Bethe '39)



nucleosynthesis



superheavy elements

**Fusion and fission:** large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: an ultimate goal of nuclear physics

# Low-energy heavy-ion fusion reactions and quantum tunneling

## ✓ Reaction dynamics

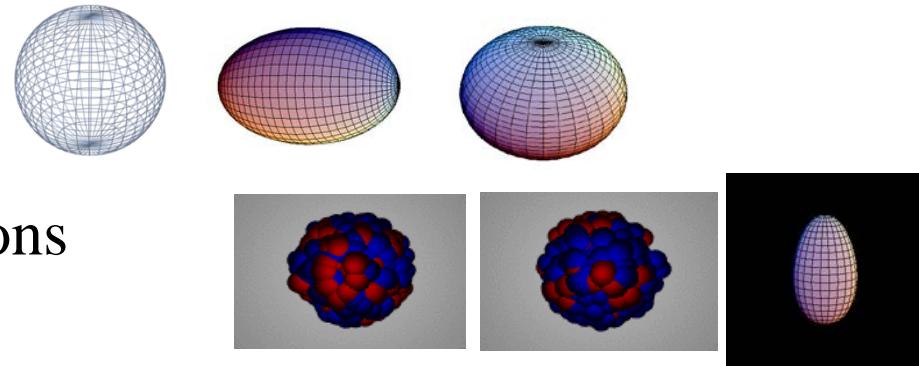
strong interplay between reaction and structure

cf. high  $E$  reactions: much simpler reaction mechanisms

## ✓ Many-particle tunneling

cf. - rich intrinsic motions

- several nuclear shapes
- several surface vibrations



several modes  
and adiabaticities

# Low-energy heavy-ion fusion reactions and quantum tunneling

## ✓ Reaction dynamics

strong interplay between reaction and structure

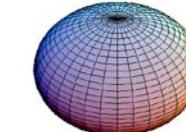
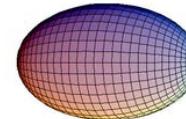
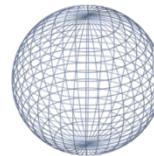
cf. high  $E$  reactions: much simpler reaction mechanisms

## ✓ Many-particle tunneling

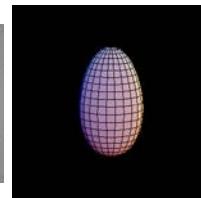
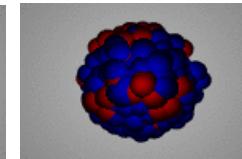
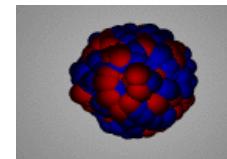
cf.

- rich intrinsic motions

- several nuclear shapes



- several surface vibrations



- several types of nucleon transfers

“environment” can be changed relatively freely

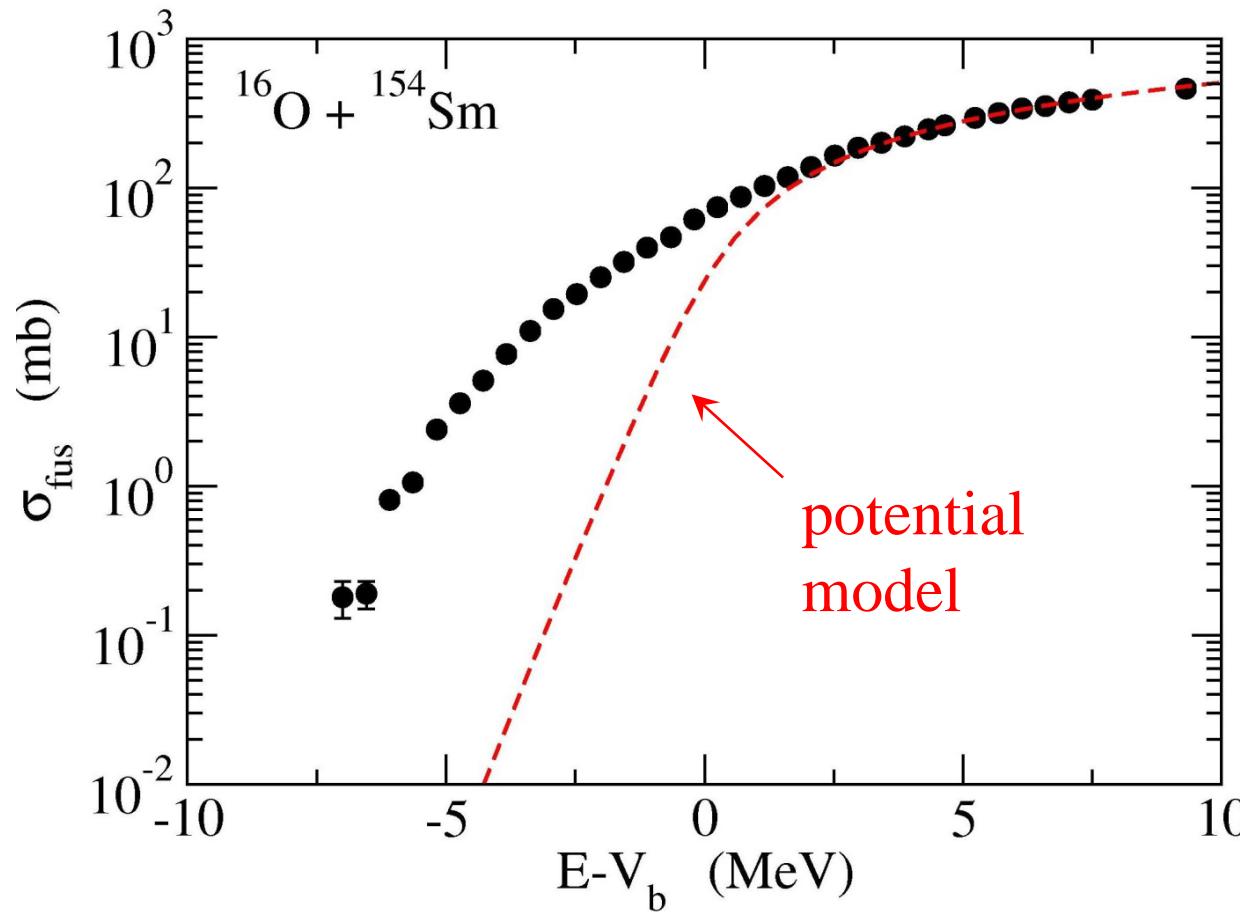
- $E$ : variable cf.  $\alpha$  decays: fixed energy

H.I. fusion reaction = an ideal playground to study quantum tunneling with many degrees of freedom

## Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~80's)

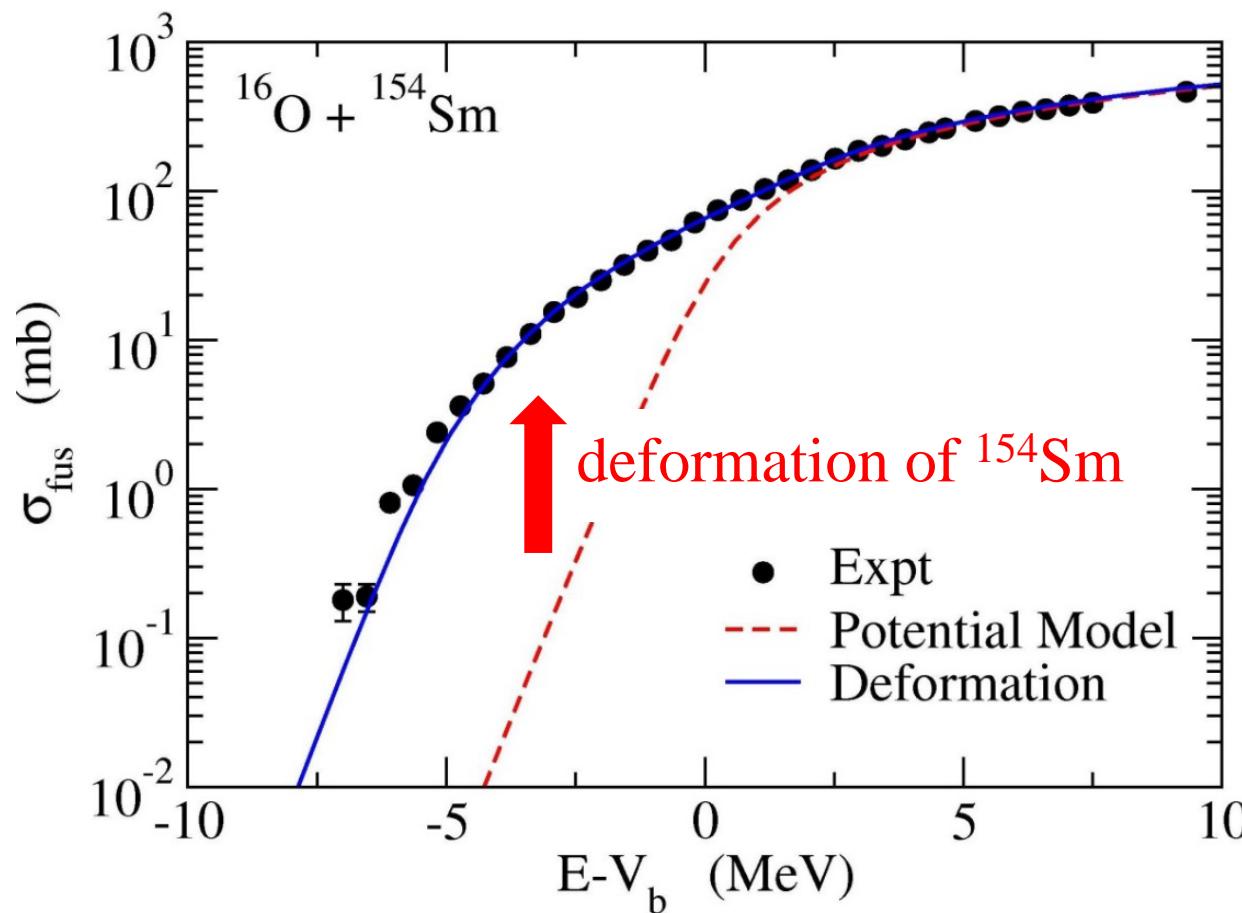
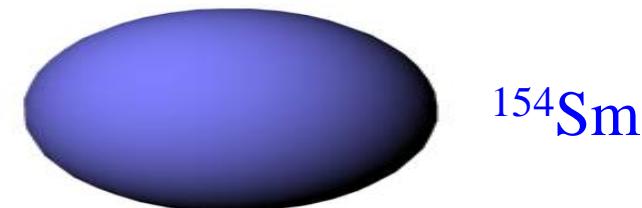
potential model: inert nuclei (no structure)

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2)$$



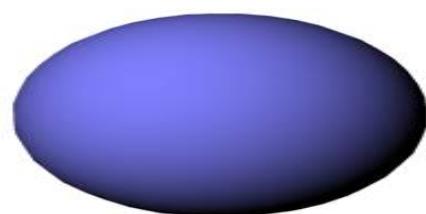
## Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~80's)

$^{154}\text{Sm}$  : a typical deformed nucleus

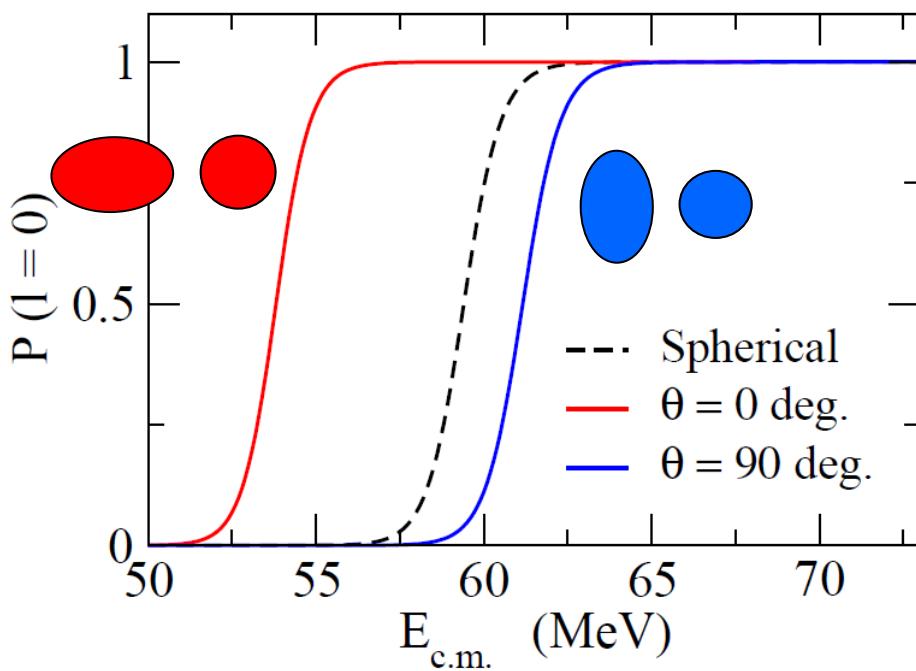
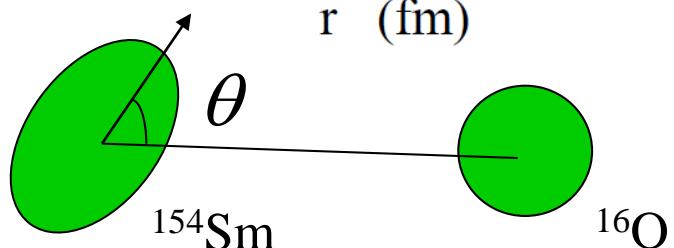
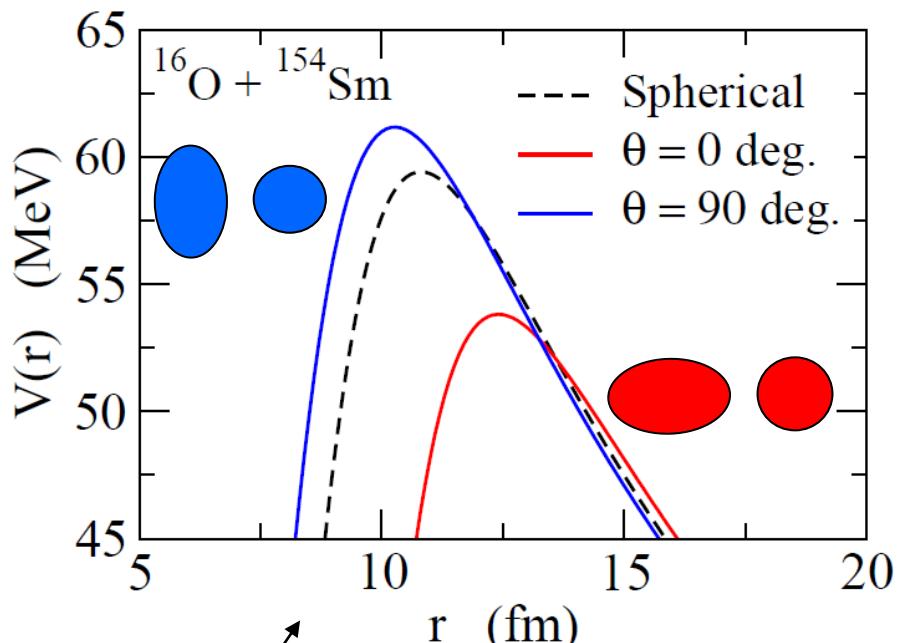


## Effects of nuclear deformation

$^{154}\text{Sm}$  : a typical deformed nucleus

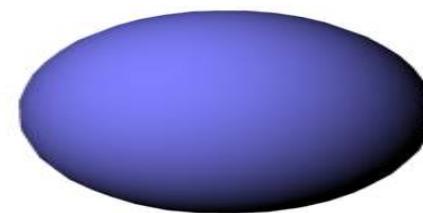


$^{154}\text{Sm}$

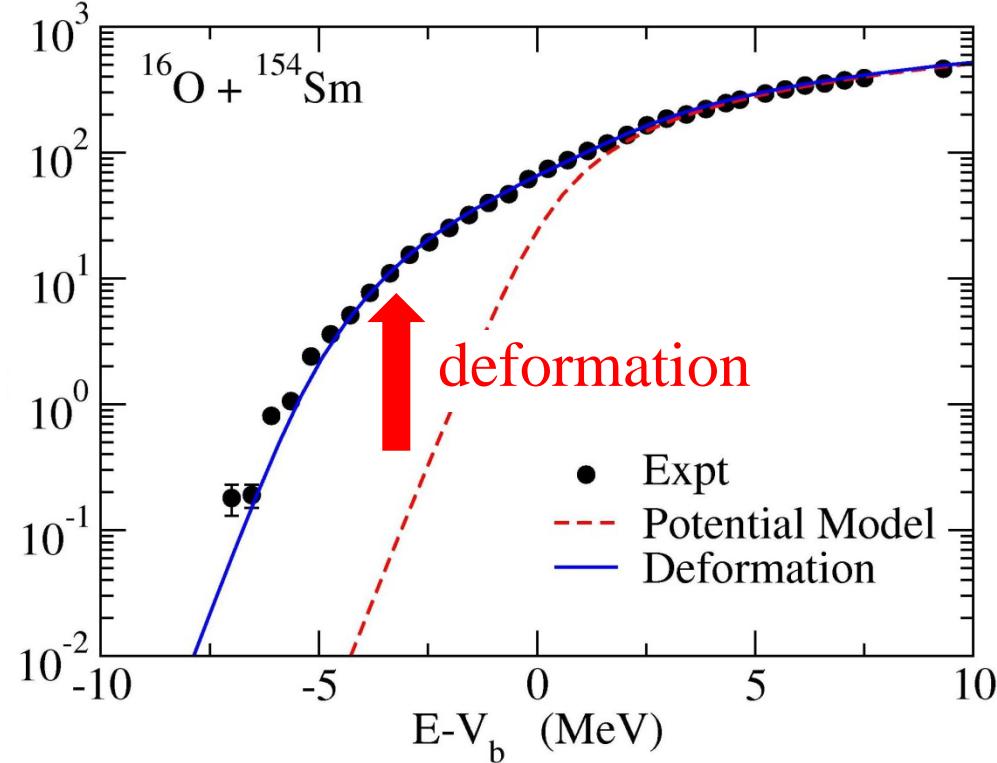
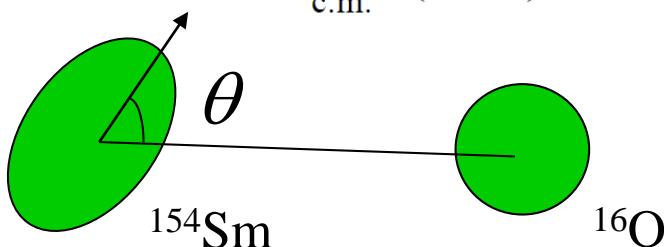
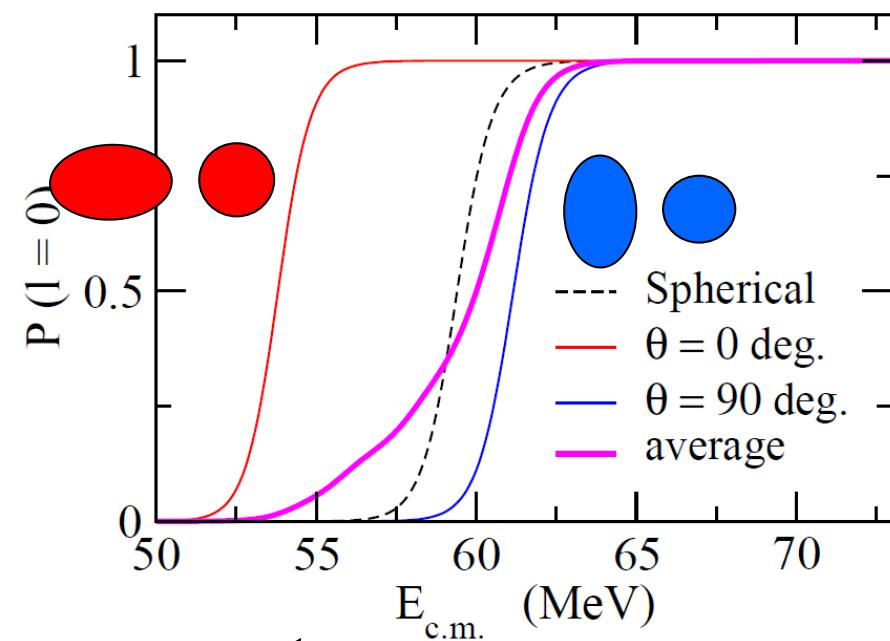


## Effects of nuclear deformation

$^{154}\text{Sm}$  : a typical deformed nucleus

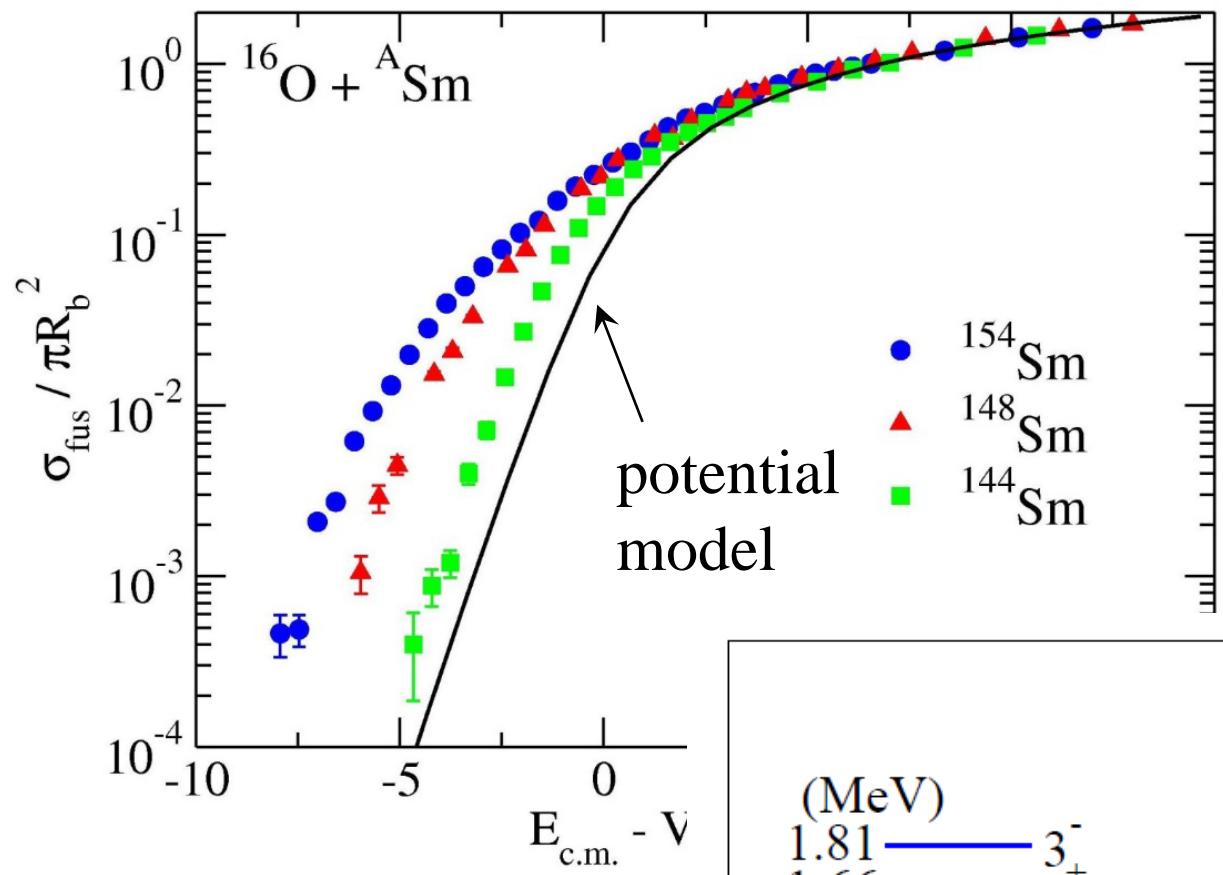


$^{154}\text{Sm}$



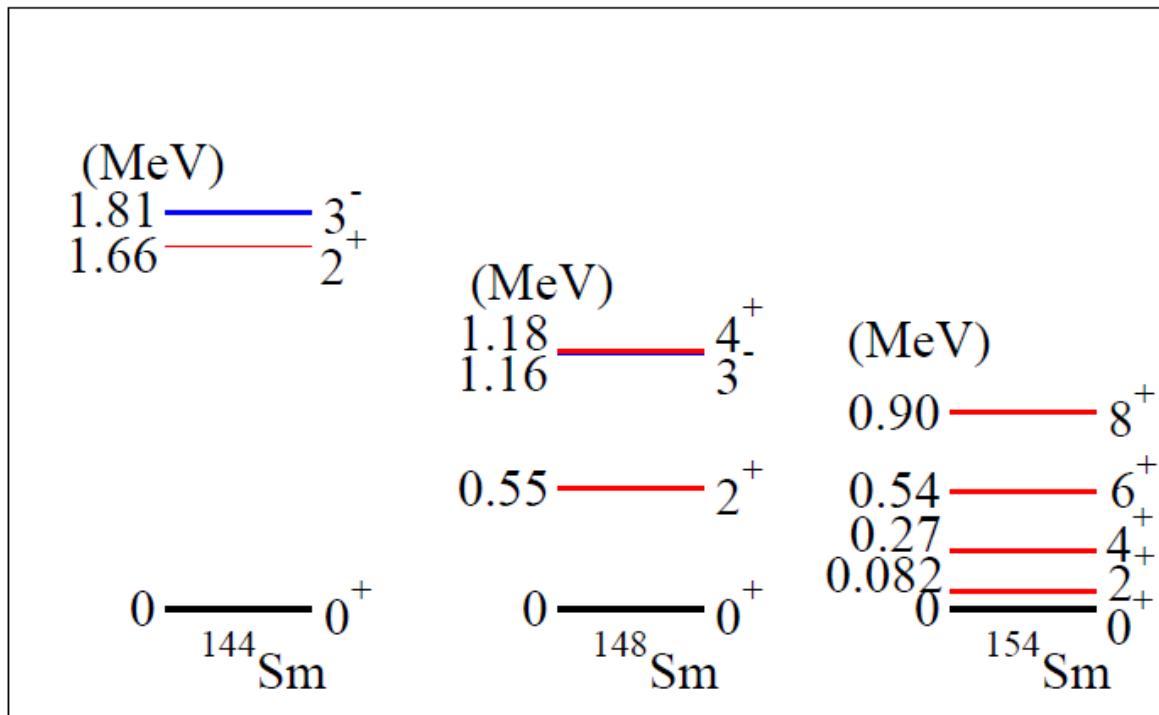
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

Fusion: strong interplay between  
nuclear structure and reaction



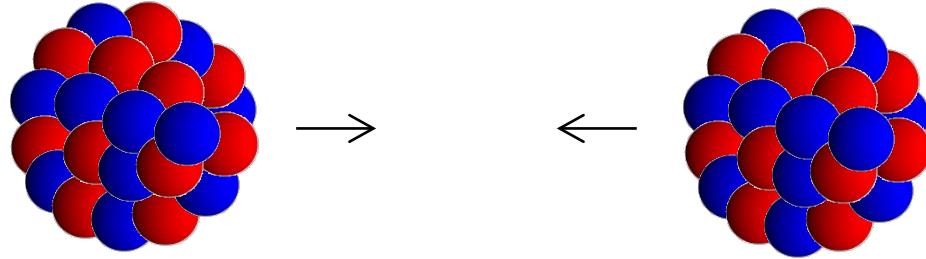
enhancement of fusion cross sections  
: a general phenomenon

strong correlation with nuclear spectrum  
→ coupling assisted tunneling



# Coupled-channels method: a quantal scattering theory with excitations

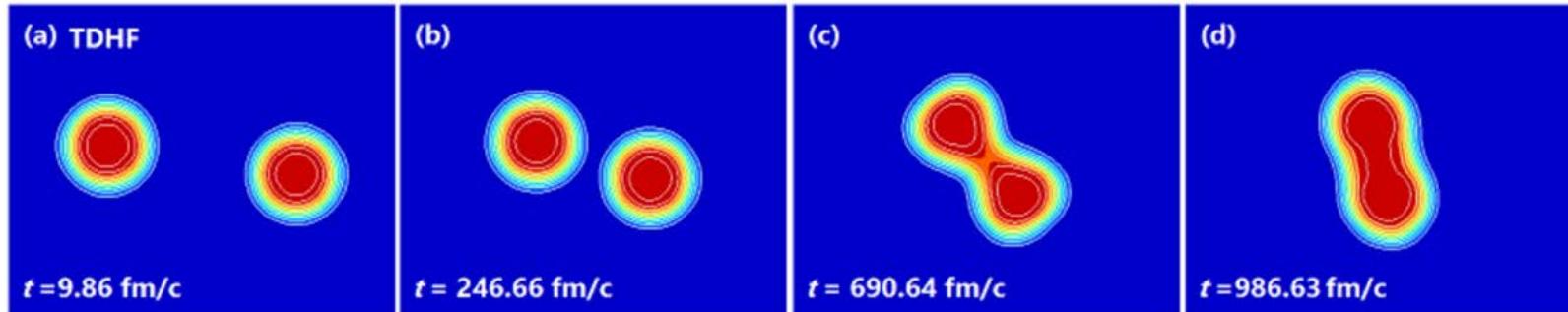
## many-body problem



still very challenging

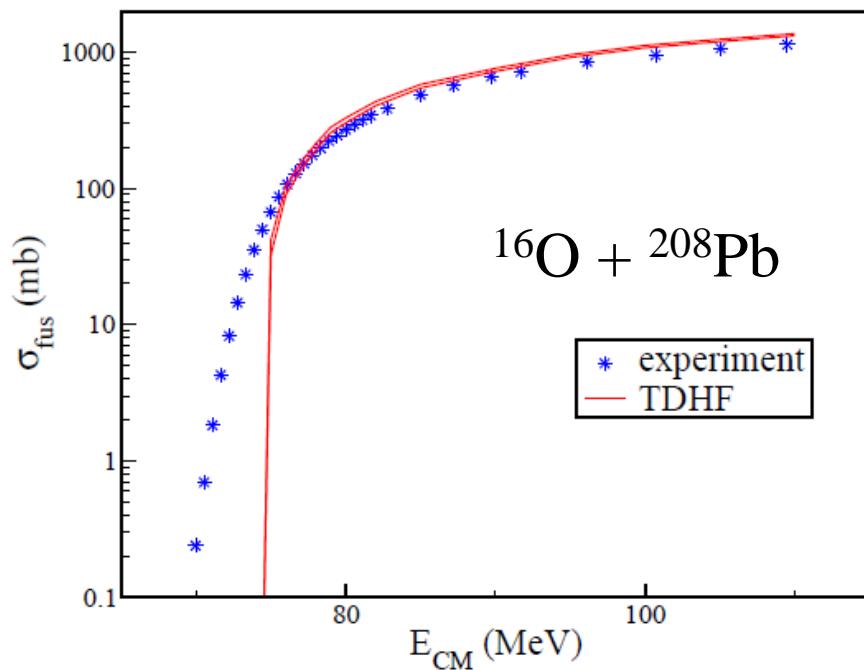
## TDHF simulation

TDHF = Time Dependent Hartree-Fock  
(a single Slater determinant)



S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 ('15) 020056

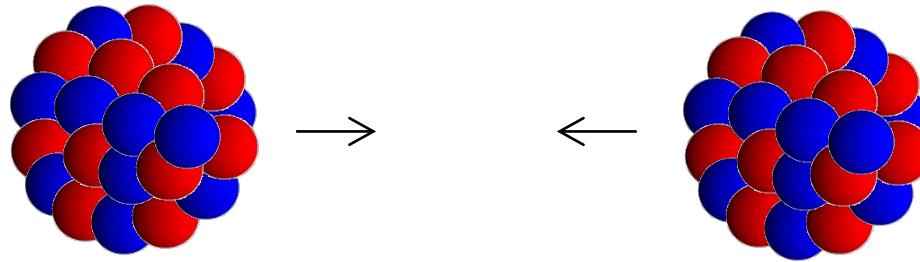
ab-initio, but no tunneling



C. Simenel,  
EPJA48 ('12) 152

# Coupled-channels method: a quantal scattering theory with excitations

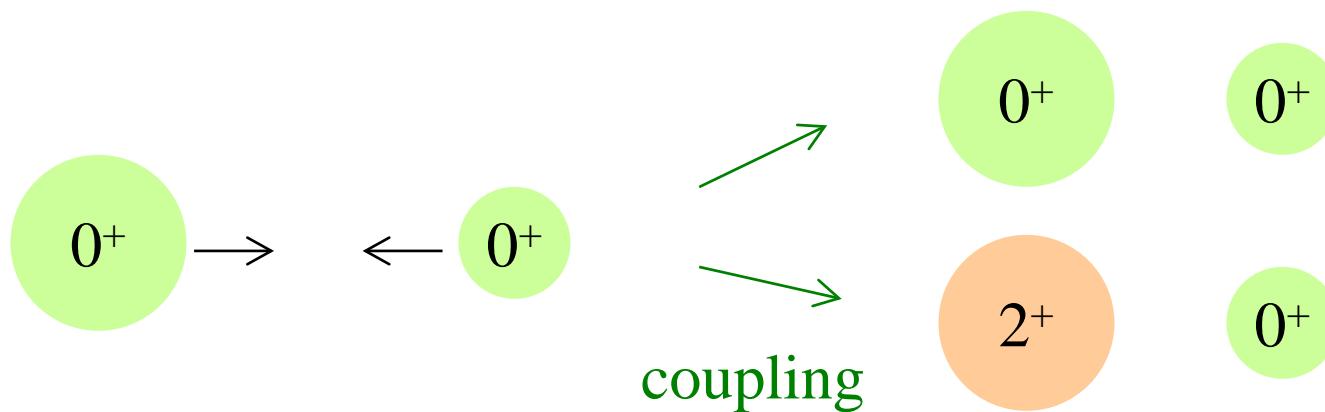
## many-body problem



still very challenging

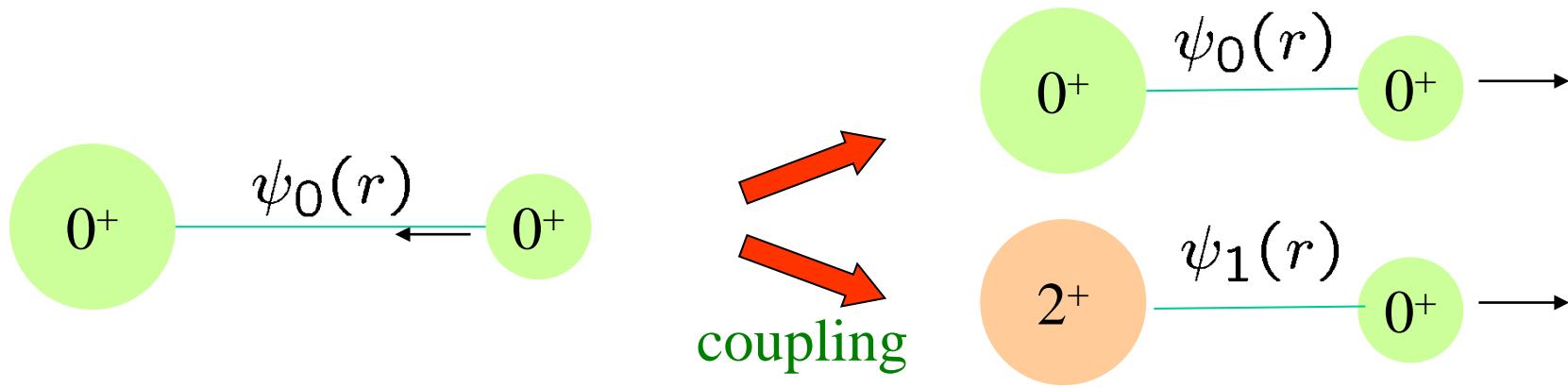


two-body problem, but with excitations  
(coupled-channels approach)

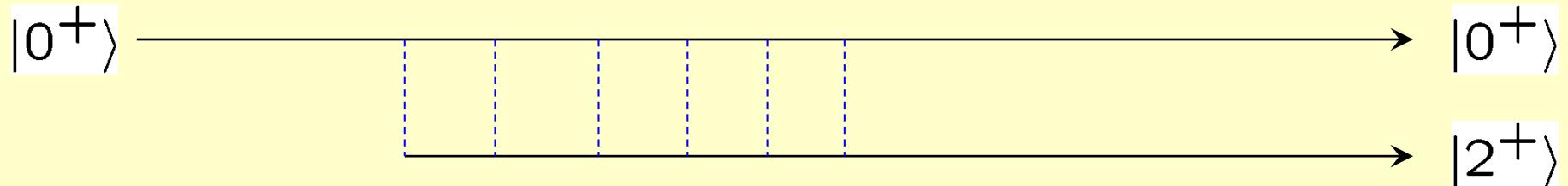


scattering theory with excitations

## Coupled-channels method: a quantal scattering theory with excitations



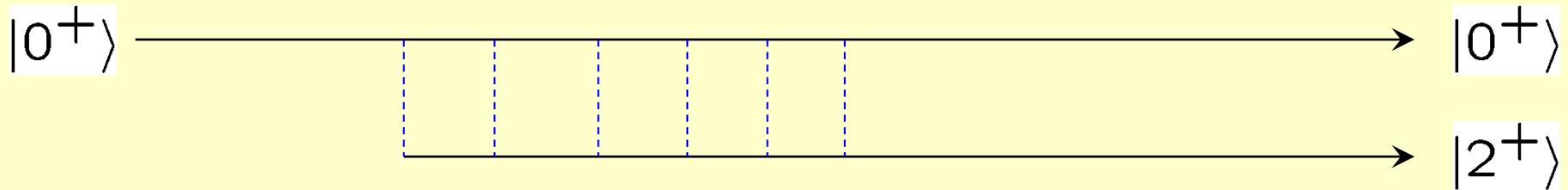
$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + \stackrel{\longleftrightarrow}{V}(r) - \stackrel{\longleftrightarrow}{E} \right] \vec{\psi}(r) = 0$$



dynamics of excitations/de-excitations during reaction

- ✓ Non-perturbative (full order)
- ✓ Non-adiabatic (excitation energy)

## Coupled-channels method: a quantal scattering theory with excitations



dynamics of excitations/de-excitations during reaction

- ✓ Non-perturbative (full order)
- ✓ Non-adiabatic (excitation energy)

in the past, the linear coupling approximation in a Hamiltonian :

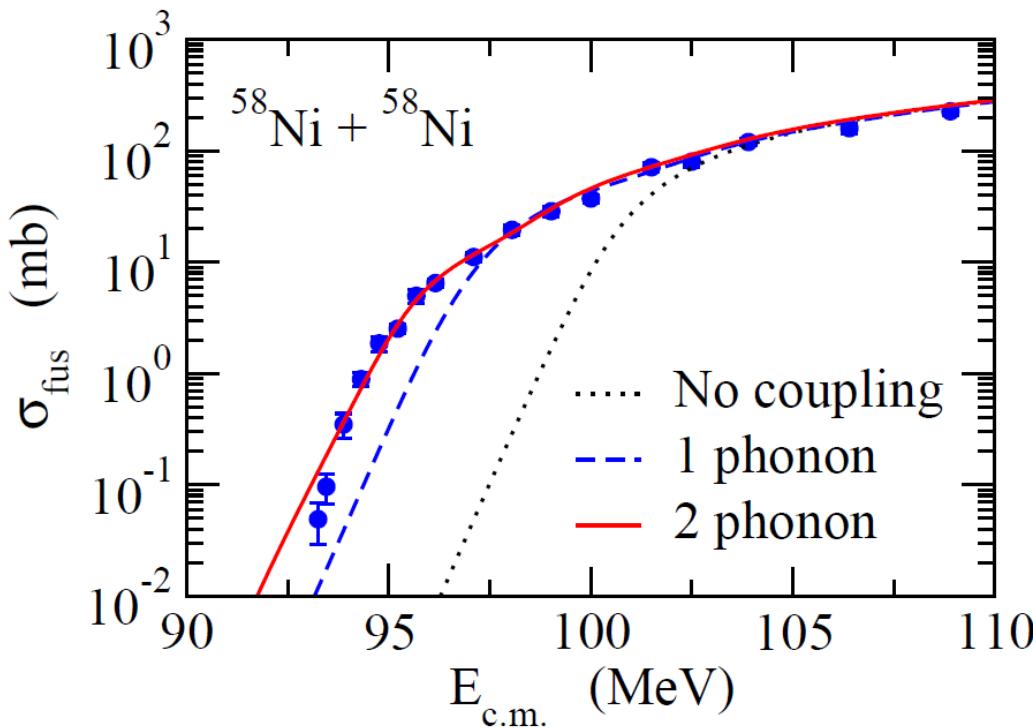
$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \beta f(r) \hat{O}$$

full order treatment

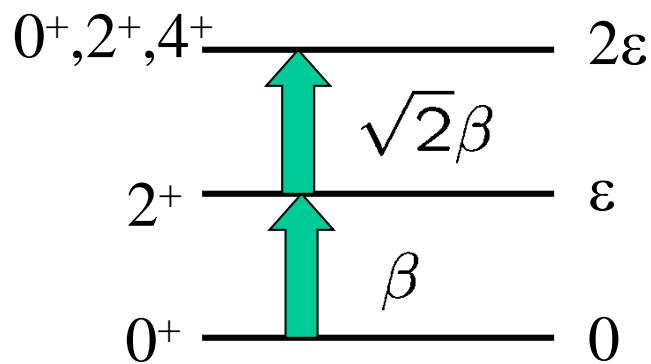
$$V(r, \beta \hat{O})$$

## modellings of coupled-channels calculations

- K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143
- M. Zamrun, K.H., S. Mitsuoka, H. Ikezoe, PRC77 ('08) 034604
- T. Ichikawa, K.H., A. Iwamoto, PRL103 ('09) 202701
- S. Yusa, K.H., and N. Rowley, PRC88 ('13) 044620
- J.M. Yao and K.H., PRC94 ('16) 11303(R) etc.



harmonic oscillations

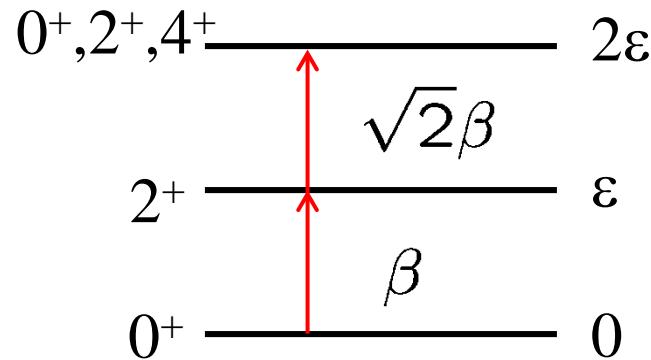


## Further development: semi-microscopic modelling

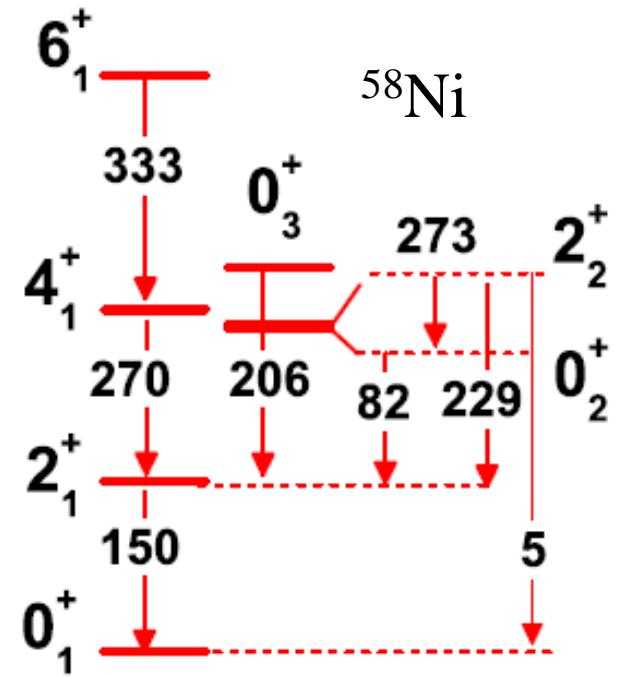
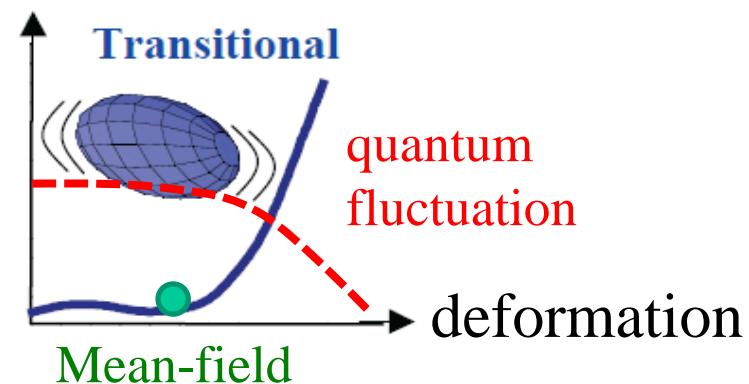
K.H. and J.M. Yao, PRC91('15) 064606

Coupled-channels  
+ microscopic nuclear structure  
calculations  
(GCM, Shell Model, IBM.....)

simple harmonic  
oscillator

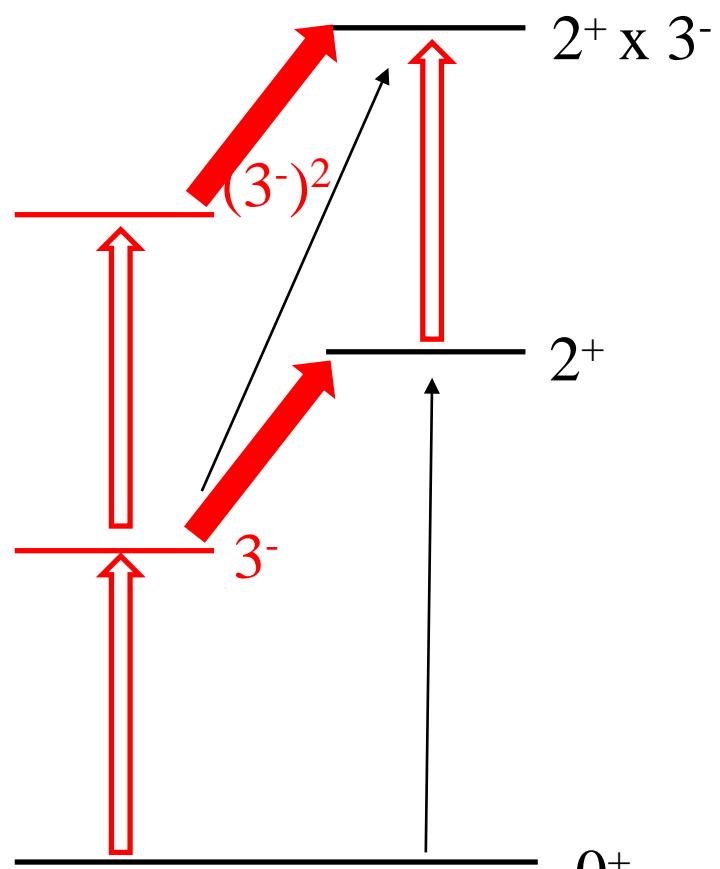
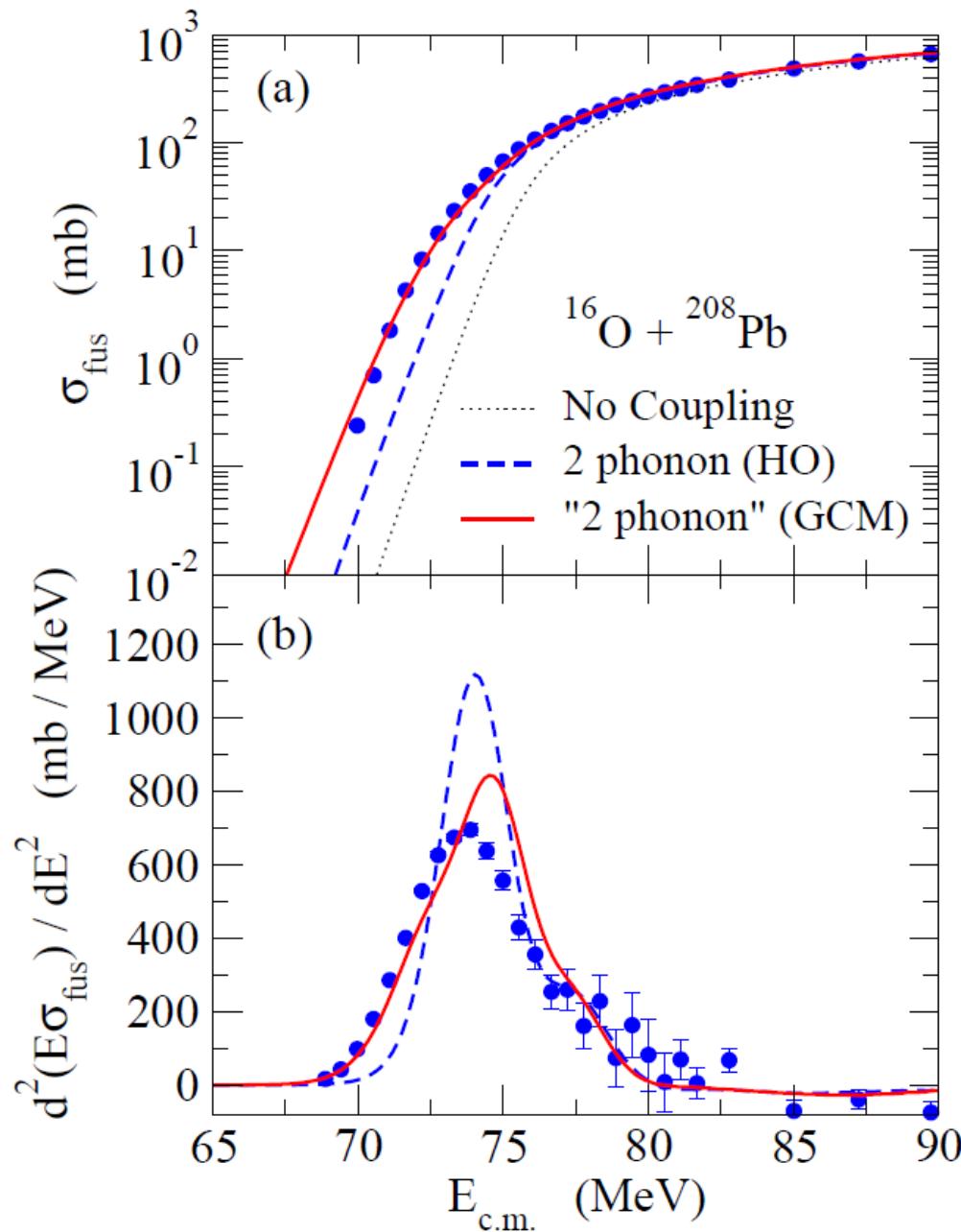


anharmonicity  
in phonon spectra



relativistic MF + GCM

# Relativistic Mean-Field + Quantum fluctuation + coupled-channels



J.M. Yao and K.H.,  
PRC94 ('16) 11303(R)

# From phenomenological approach to microscopic approach

Macroscopic (phenomenological)



C.C. with collective model

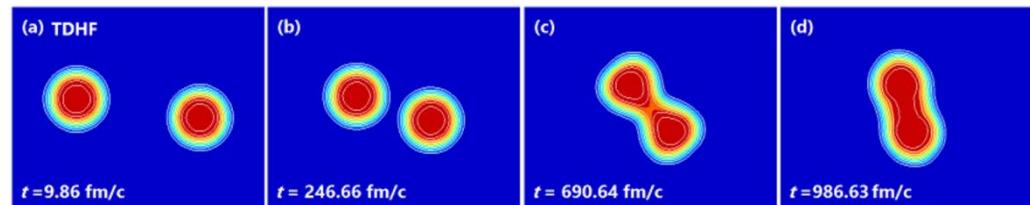
C.C. with inputs from  
microscopic nuclear  
structure calculations

C.C. with inputs based  
on TDHF

TDHF simulations

Microscopic

TDHF = Time Dependent Hartree-Fock



S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 ('15)

ab initio, but no tunneling

# From phenomenological approach to microscopic approach

## TDHF simulations

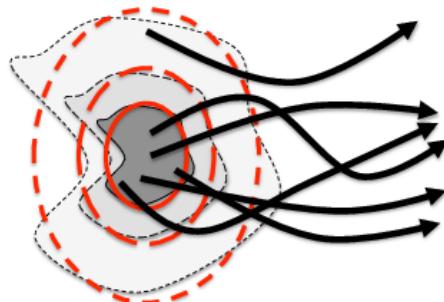
ab initio, but no tunneling

➤ “Beyond mean-field” approximations

✓ Time-dependent GCM?

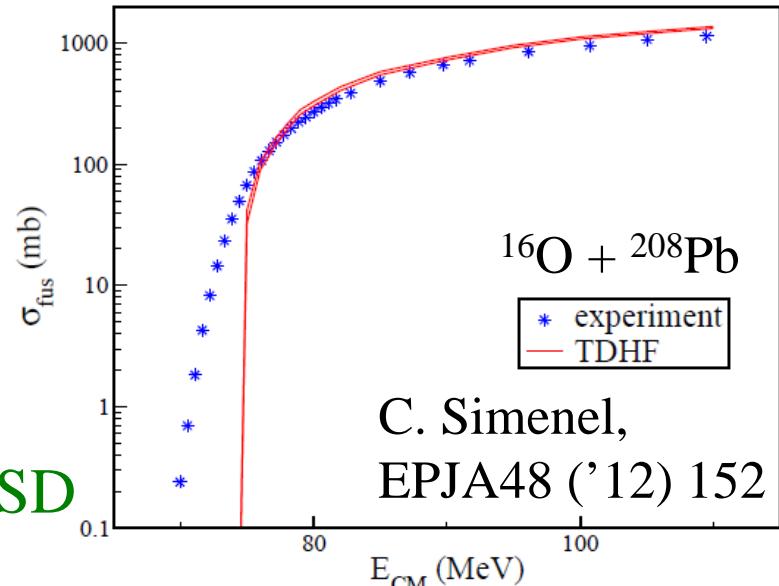
a single Slater determinant (SD) to multi-SD

$$|\Psi(t)\rangle = \int dq f(q, t) |\Phi_q(t)\rangle$$



an open problem

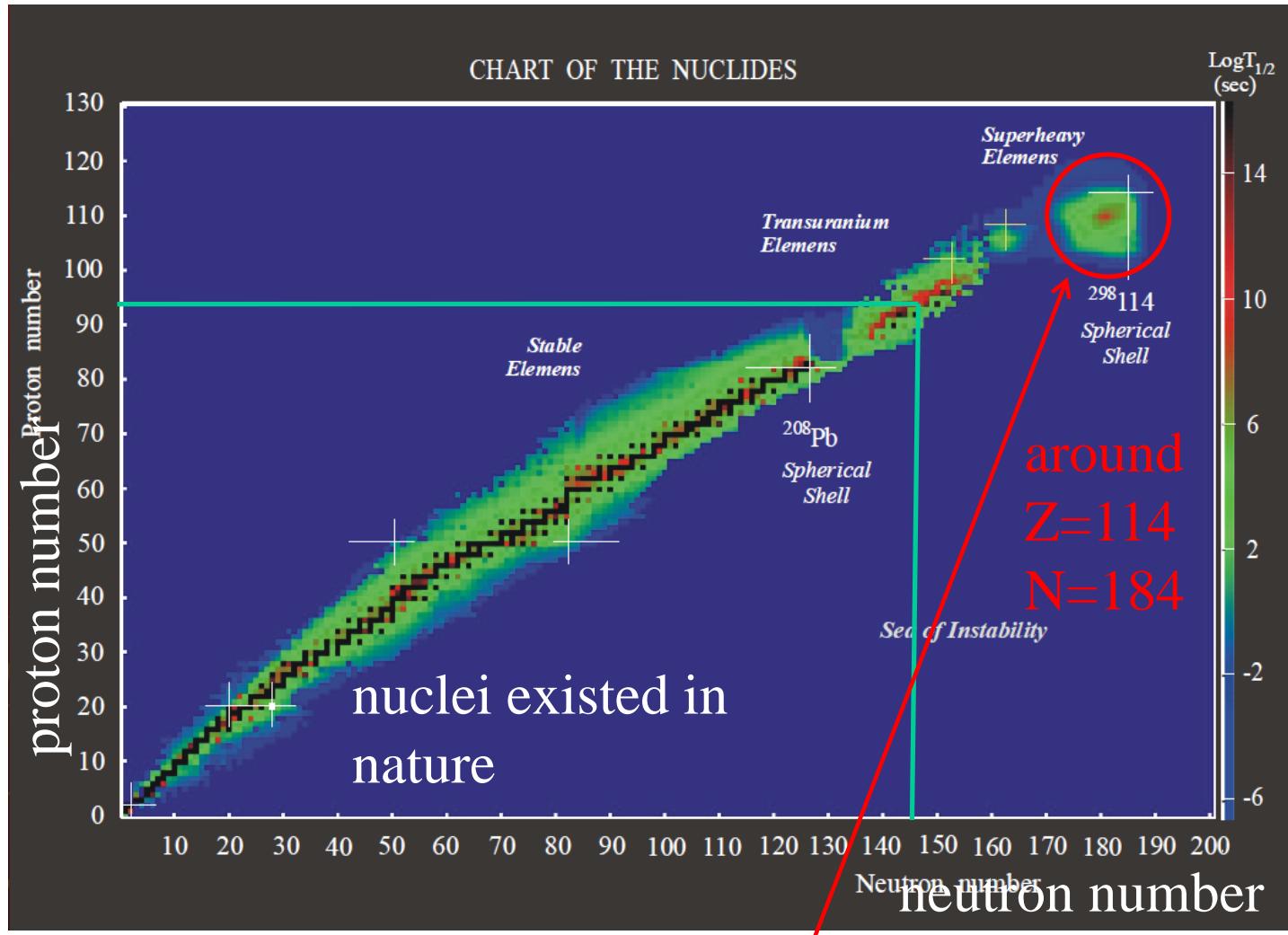
K.H., N. Hasegawa, and Y. Tanimura,  
a work in progress



dynamics with a superposition of many  
“TDHF trajectories (Slater determinants)”

cf. Stochastic mean-field method  
B. Yilmaz et al.,  
PRC90 ('14) 054617

# Future perspectives: fusion for superheavy elements



a prediction of island of stability  
(Swiatecki et al., 1966)

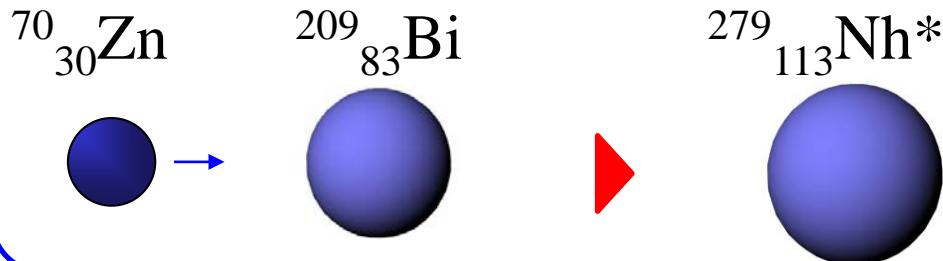
Yuri Oganessian

# Fusion reactions for SHE

# the element 113: Nh



# November, 2016



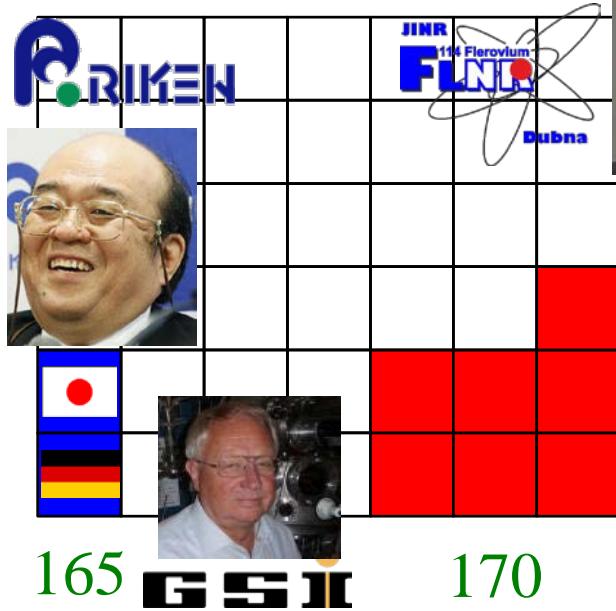
Group →	1	2	3	
Period ↓	1	2	3	
1	H			He
2	Li	Be		Ne
3	Na	Mg		Ar
4	K	Ca	Sc	
5	Rb	Sr	Y	
6	Cs	Ba	La	
7	Fr	Ra	Ac	
	*	104	Rf	
	*	90	Th	
	*	91	Pa	
	*	92	U	
	*	93	Np	
	*	94	Pu	
	*	95	Am	
	*	96	Cm	
	*	97	Bk	
	*	98	Cf	
	*	99	Es	
	*	100	Fm	
	*	101	Md	
	*	102	No	
	*	103	Lr	
	*	58	Ce	
	*	59	Pr	
	*	60	Nd	
	*	61	Pm	
	*	62	Sm	
	*	63	Eu	
	*	64	Gd	
	*	65	Tb	
	*	66	Dy	
	*	67	Ho	
	*	68	Er	
	*	69	Tm	
	*	70	Yb	
	*	71	Lu	

Wikipedia

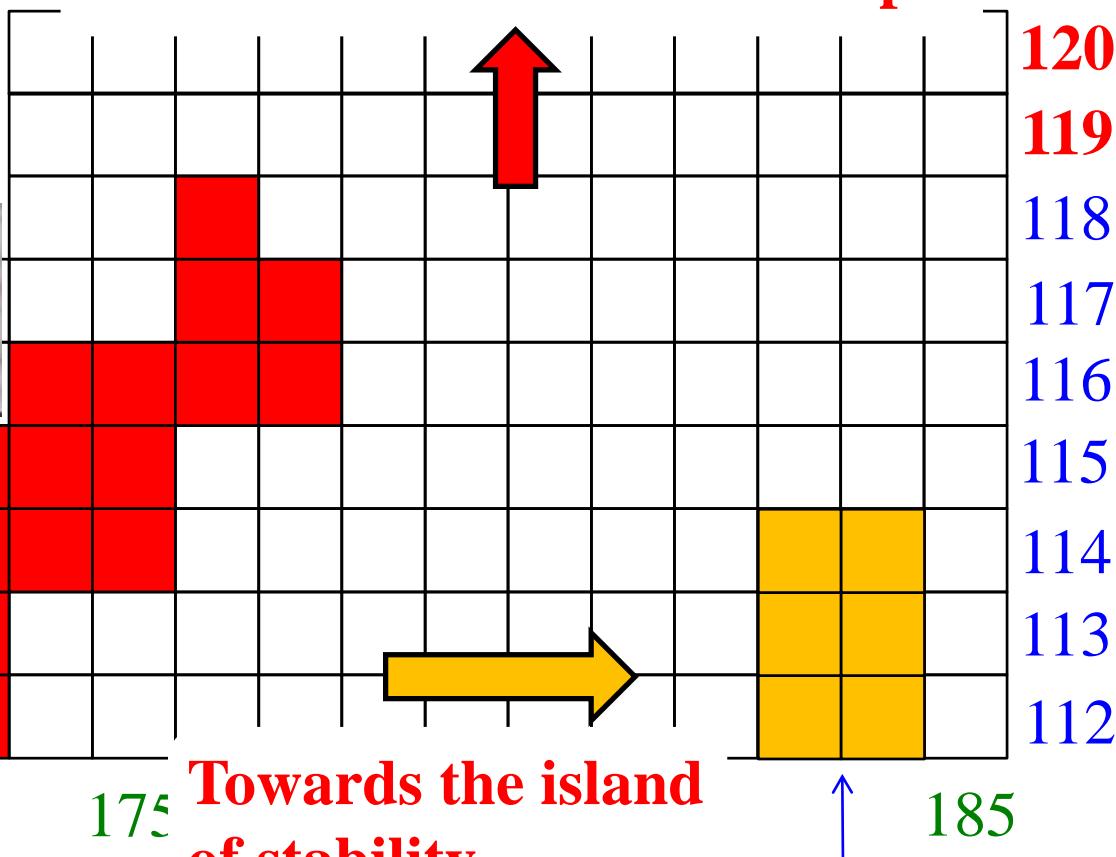
## Heavy-ion fusion reaction

## Future directions of SHE

Superheavy elements synthesized so far



## Towards Z=119 and 120 isotopes



➤ Towards Z=119 and 120 isotopes

Hot fusion reactions with  $^{48}\text{Ca}$ ,  $^{50}_{22}\text{Ti}$ ,  $^{51}_{23}\text{V}$ ,  $^{54}_{24}\text{Cr}$  etc.

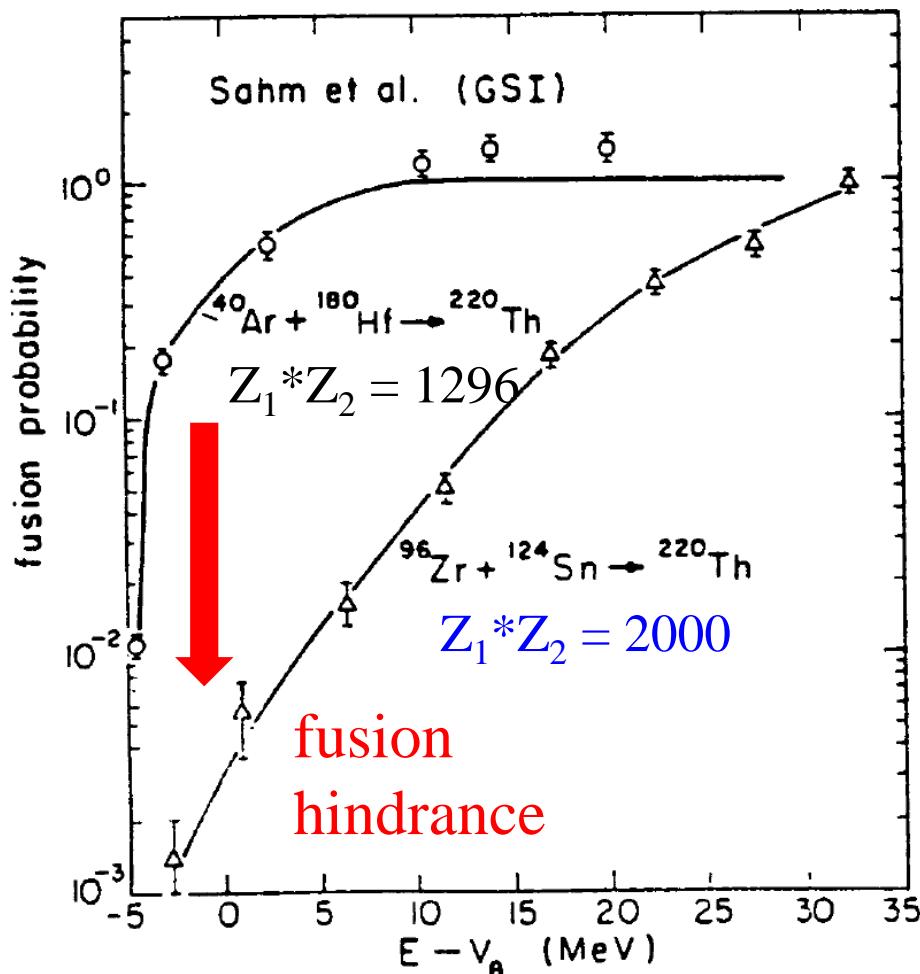
➤ Towards the island of stability

neutron-rich beams: indispensable → reaction dynamics?

the island  
of stability?

# Fusion reactions in the SHE region ( $Z_p^*Z_T > 1600 \sim 1800$ )

fusion hindrance



C.-C. Sahm et al., Z. Phys. A319('84)113

superheavy nuclei

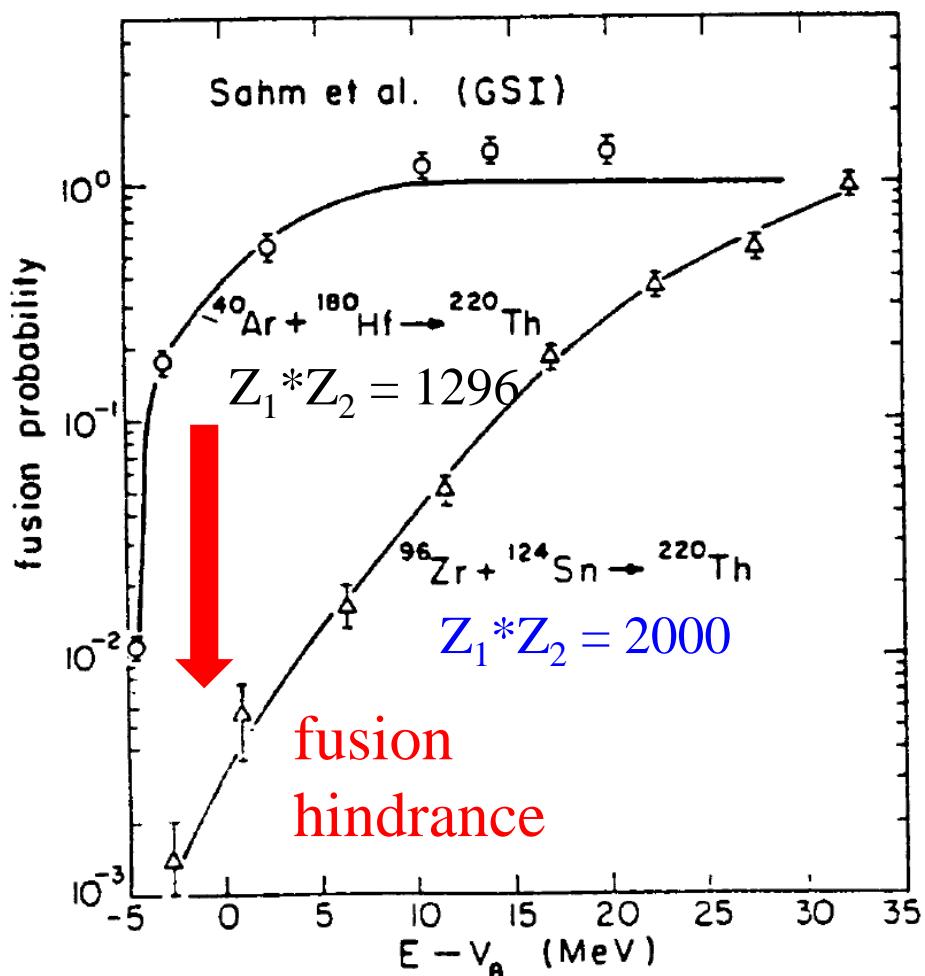
113 <b>Nh</b> nihonium	115 <b>Mc</b> moscovium
117 <b>Ts</b> tennessine	118 <b>Og</b> oganesson

syntheses with heavy-ion fusion reactions

theoretical issues:  
understanding the reaction dynamics

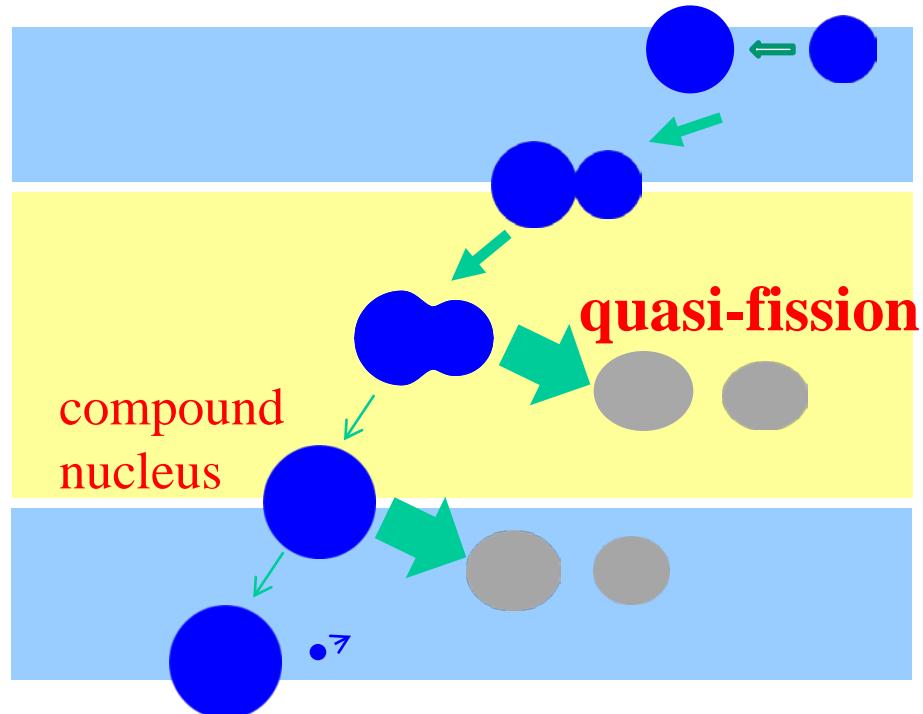
# Fusion reactions in the SHE region ( $Z_p^*Z_T > 1600 \sim 1800$ )

fusion hindrance



C.-C. Sahm et al., Z. Phys. A319('84)113

modern interpretation of hindrance



strong Coulomb repulsion  
→ re-separation before the  
compound nucleus

# Fusion reactions in the SHE region ( $Z_p^*Z_T > 1600\sim1800$ )

SHE formation: a very rare event

→ large theoretical uncertainties

✓ No data for  $P_{CN}$

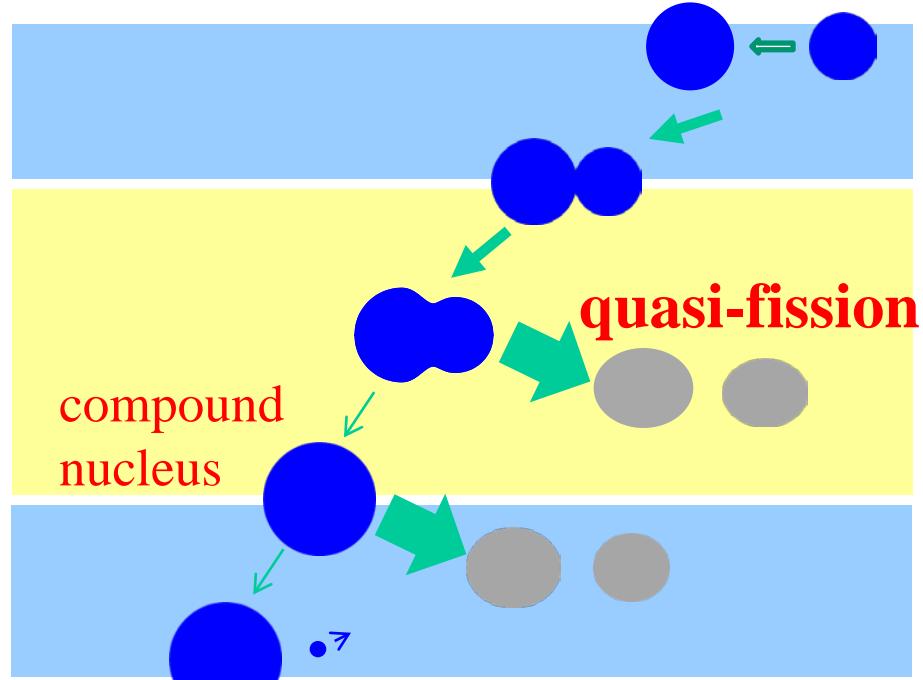
✓ Data: only for  $P_{ER}$

CN = compound nucleus

ER = evaporation residues

theoretical challenge:  
to reduce theoretical uncertainties  
and make a reliable prediction

modern interpretation of hindrance

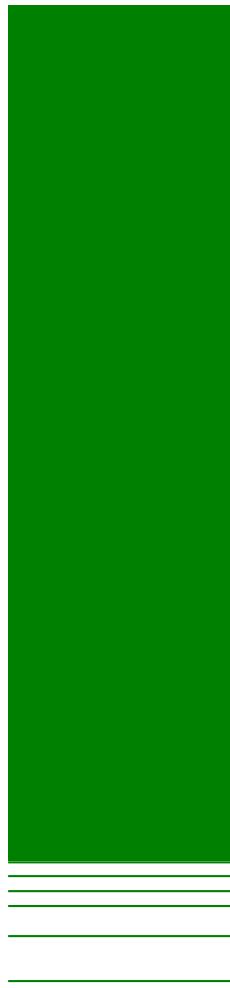


evaporation  
residues

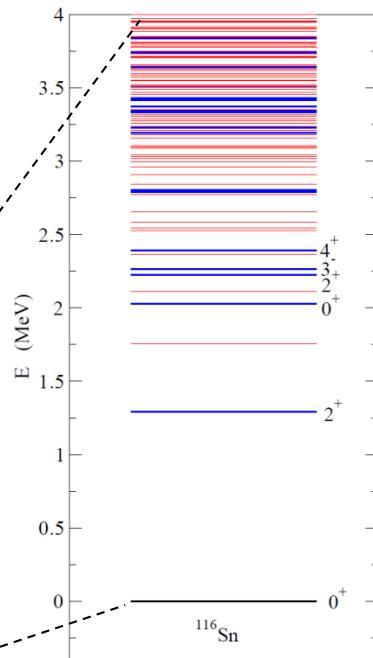
strong Coulomb repulsion  
→ re-separation before the  
compound nucleus

# Nuclear friction and heavy-ion fusion reactions

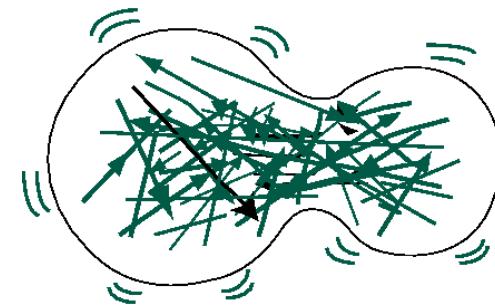
$E^*$



$$\rho(E) \sim e^{2\sqrt{aE^*}}$$



nuclear spectrum



These states:  
are excited in a complicated way.

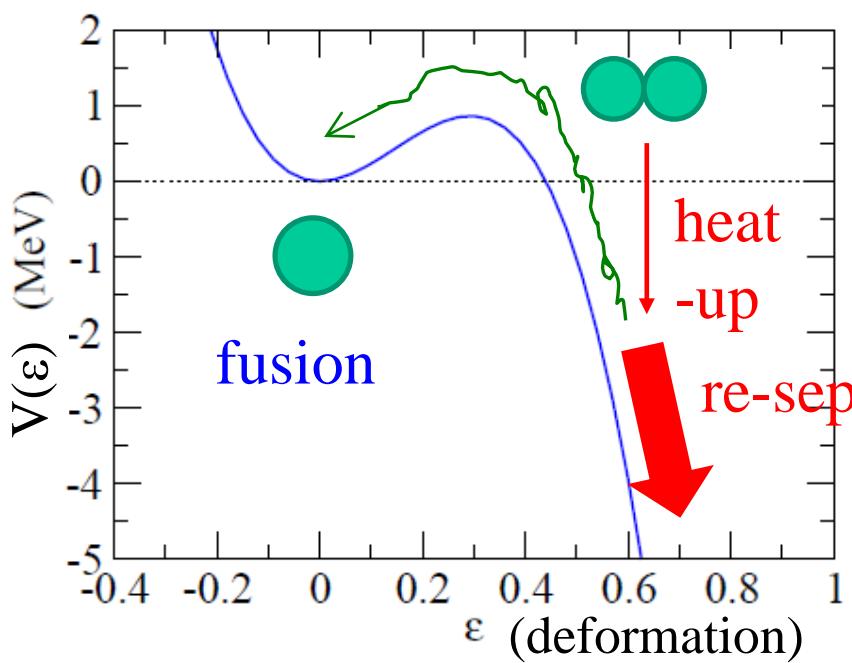


nuclear intrinsic d.o.f.  
: act as environment

“*intrinsic* environment”

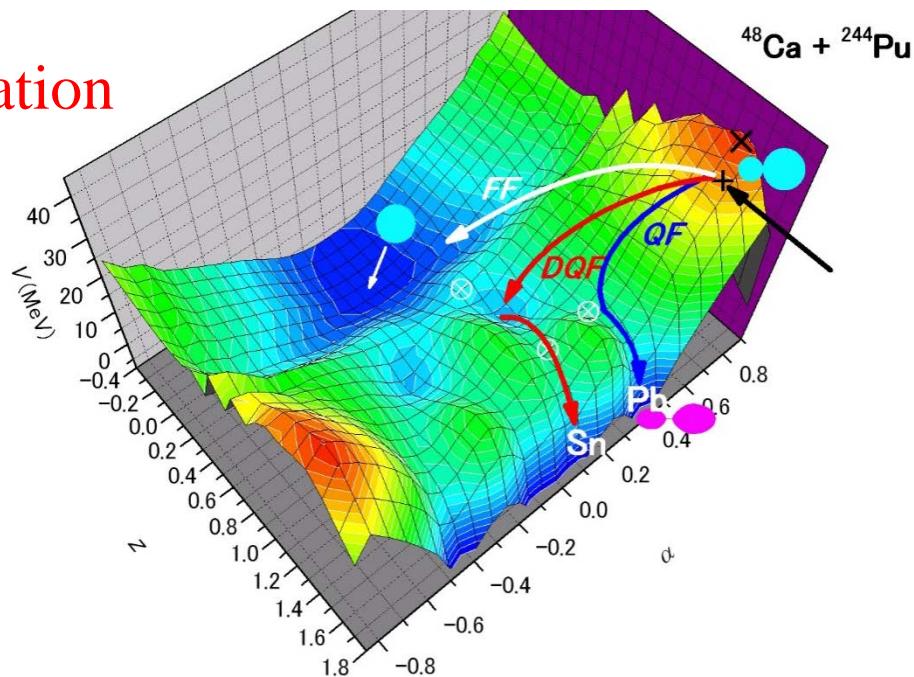
→ friction

## Langevin approach



## Multi-dimensional space

- internuclear separation
- deformation
- mass asymmetry of the two fragments



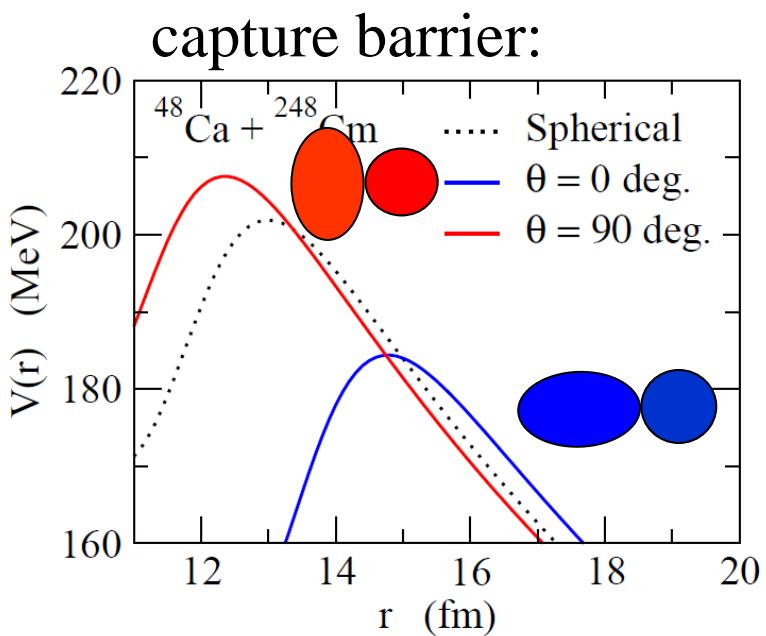
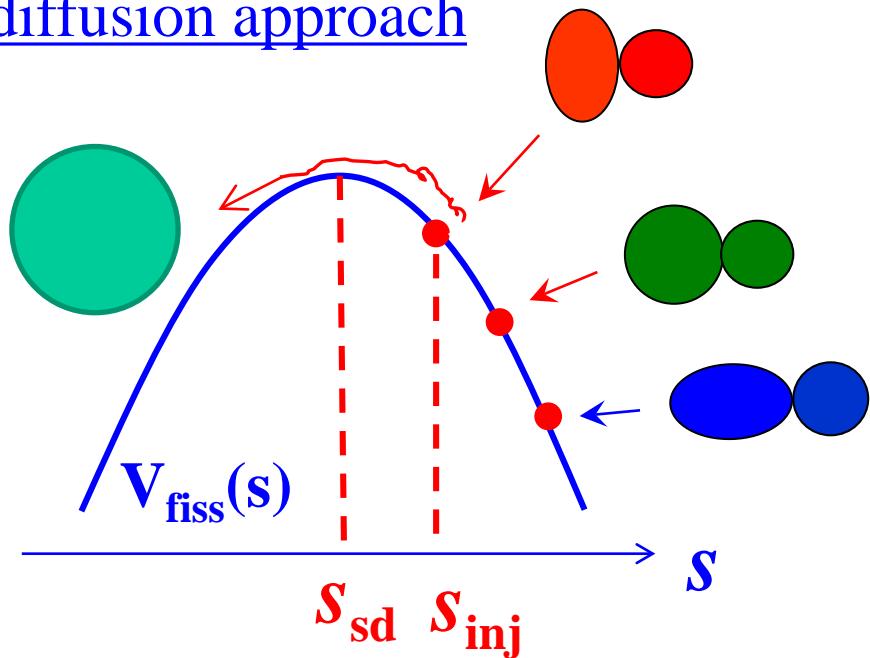
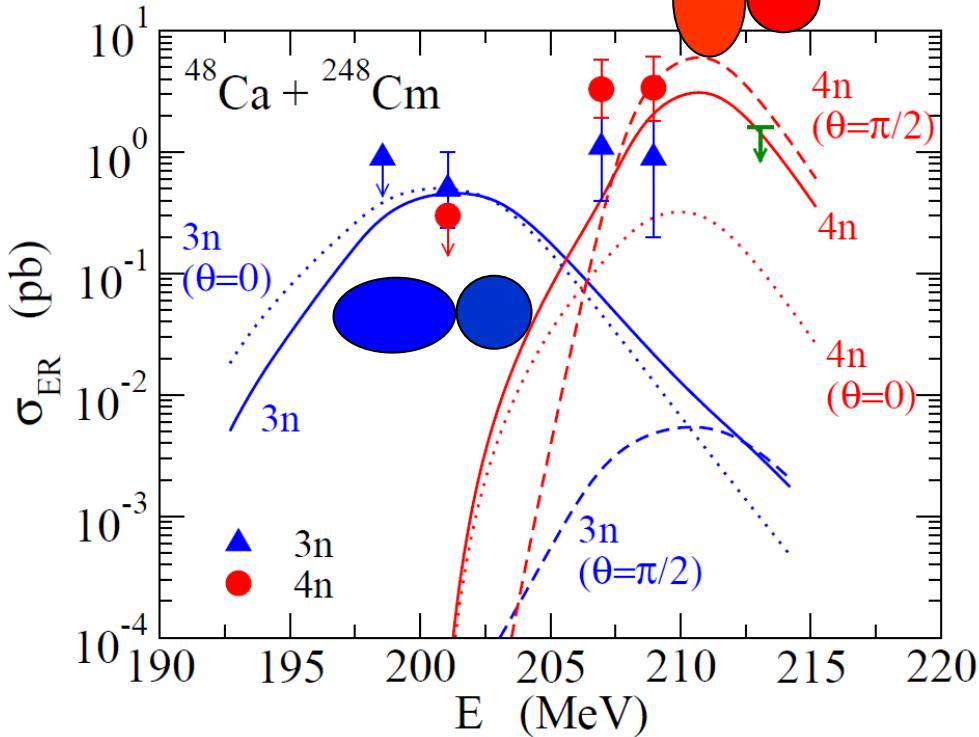
## thermal diffusion

→ Langevin approach  
(Brownian motion)

$$m \frac{d^2 q}{dt^2} = -\frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

# Analysis with an extended fusion-by-diffusion approach

K.H., PRC98 ('18) 014607



# New hybrid model: TDHF + Langevin approach

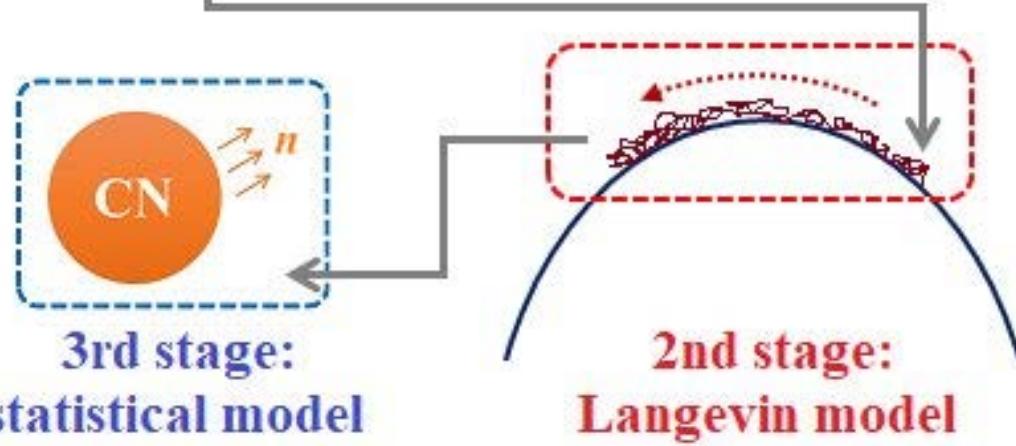
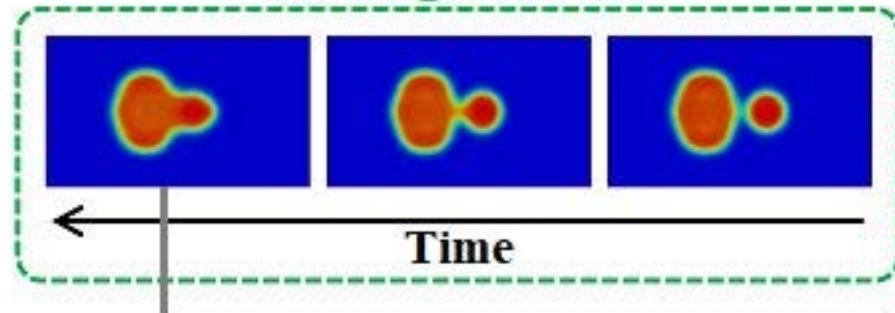
K. Sekizawa and K.H., PRC99 (2019) 051602(R)



## TDHF+Langevin:

a new hybrid model of fusion reactions for superheavy elements

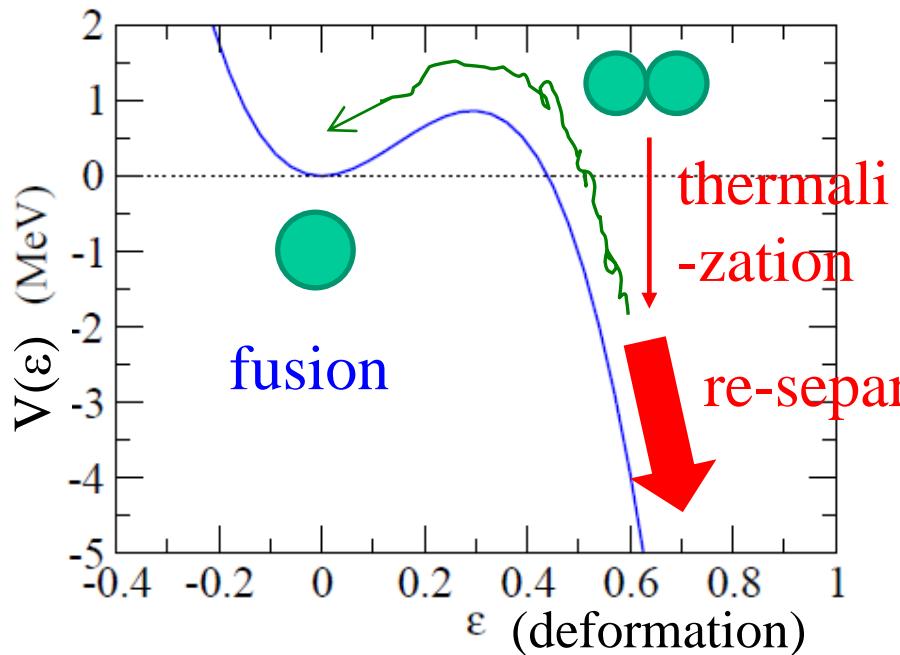
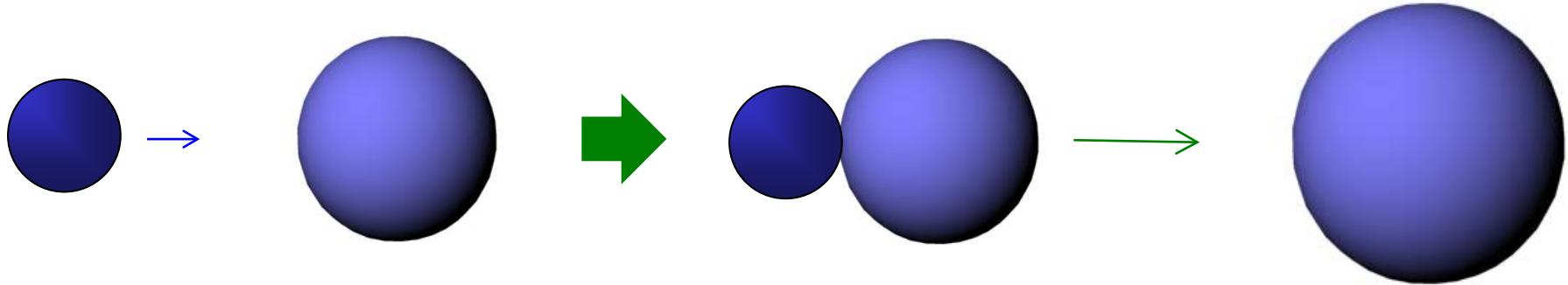
### 1st stage: TDHF



System	CN (fm)	$R_{\min}$ fm	$P_{\text{fus}}$ ( $\times 10^{13}$ )
$^{48}\text{Ca} + ^{254}\text{Fm}$	$^{302}$	120	12.93
$^{54}\text{Cr} + ^{248}\text{Cm}$	$^{302}$	120	13.09
$^{51}\text{V} + ^{249}\text{Bk}$	$^{300}$	120	12.94
$^{48}\text{Ca} + ^{257}\text{Fm}$	$^{305}$	120	0.461
			1.82

a special role of  $^{48}\text{Ca}$ ?

## issues from the theoretical physics point of view



**thermal diffusion**

→ Langevin approach

✓ how to thermalize?

Quantum friction theory

c.f. tunneling with quantum friction

M. Tokieda and K.H.,  
PRC95 ('17) 054604

✓ non-Markov effect?

✓ quantum correction for diffusion over the barrier?

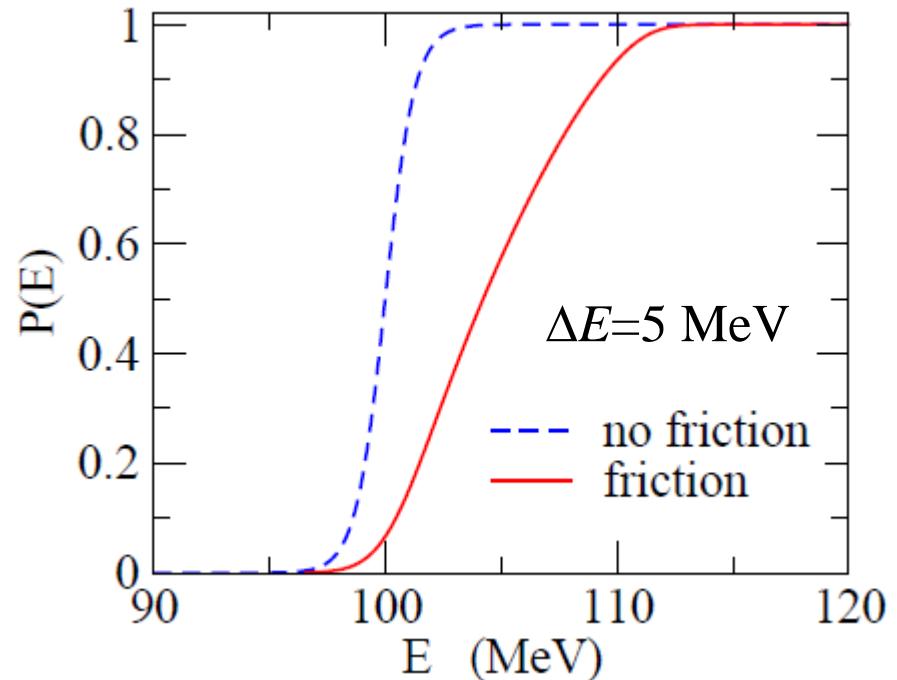
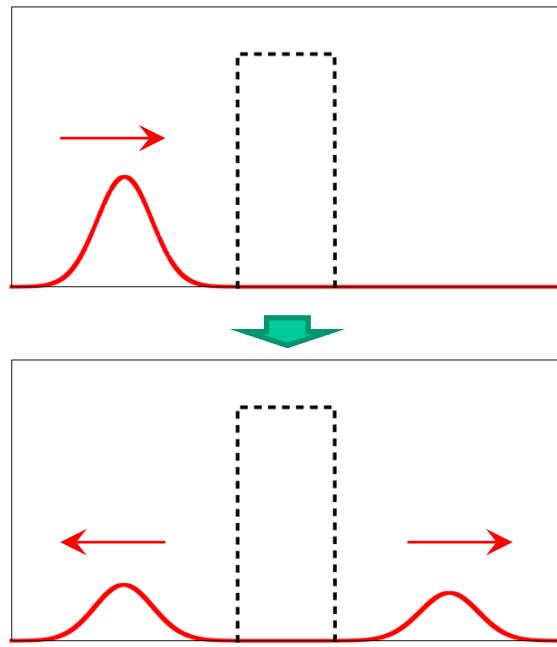
Quantum friction      classical eq. of motion       $\dot{p} = -V'(x) - \gamma p$

a quantization: Kanai model   E. Kanai, PTP 3 (1948) 440

$$H = \frac{p^2}{2m} + V(x) \rightarrow \frac{\pi^2}{2m} e^{-\gamma t} + e^{\gamma t} V(x) \quad (\pi = e^{\gamma t} p)$$

$$\longrightarrow \frac{d}{dt} \langle p \rangle = -\langle V'(x) \rangle - \gamma \langle p \rangle$$

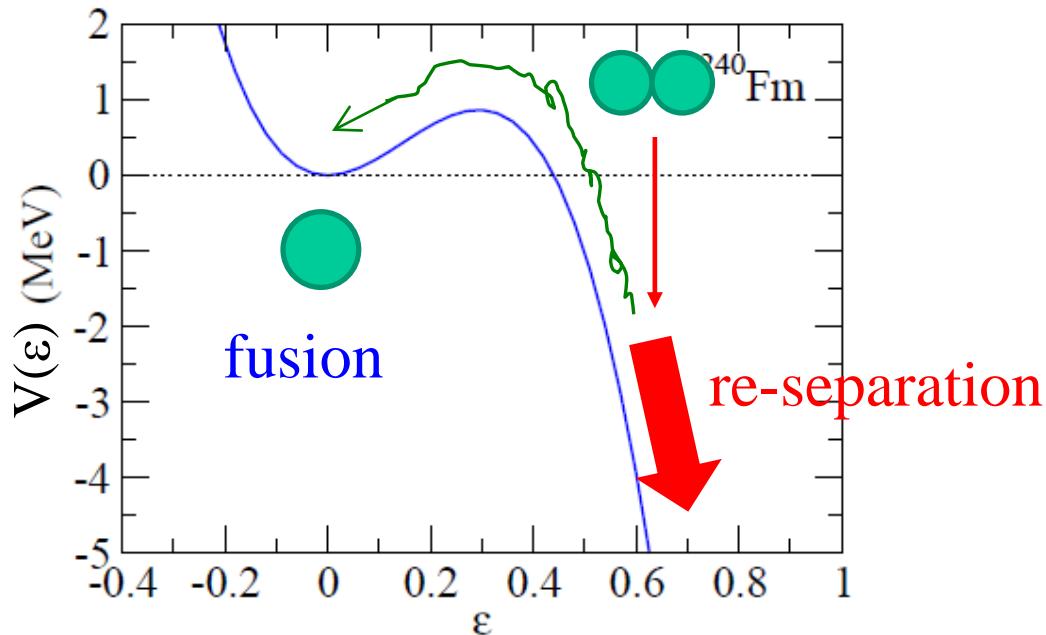
time-dep. wave packet approach



# Fusion reactions and non-equilibrium statistical mechanics

## :Langevin dynamics under a temperature gradient

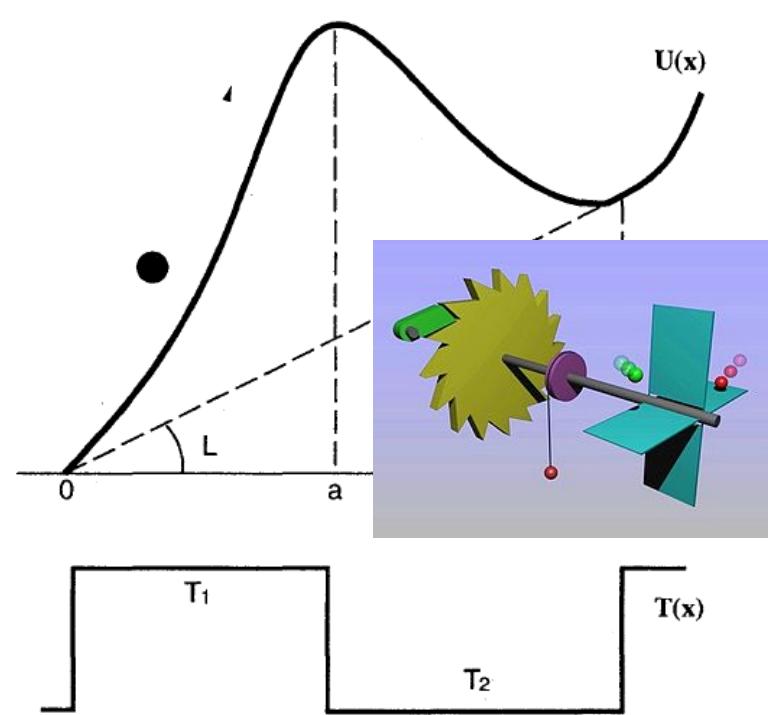
➤ Superheavy elements



$$E_{\text{int}} = E^* - E_{\text{kin}} - V(\epsilon) = aT^2$$

← coordinate dependent  
temperature

➤ a math model for molecular motors

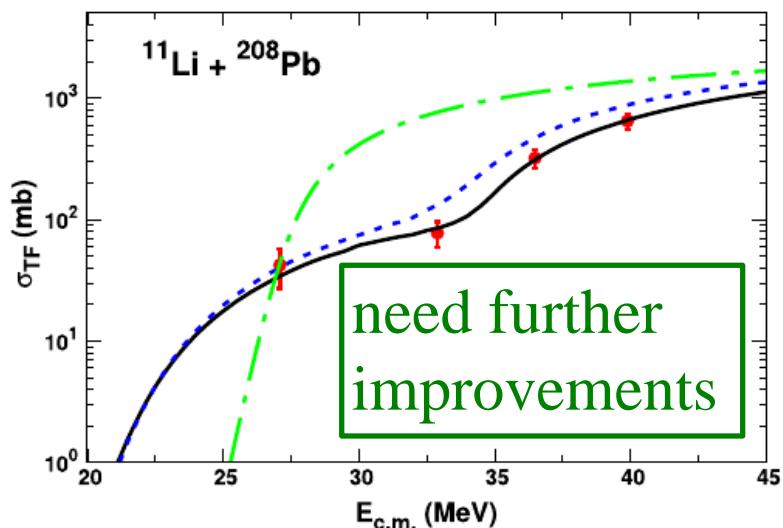
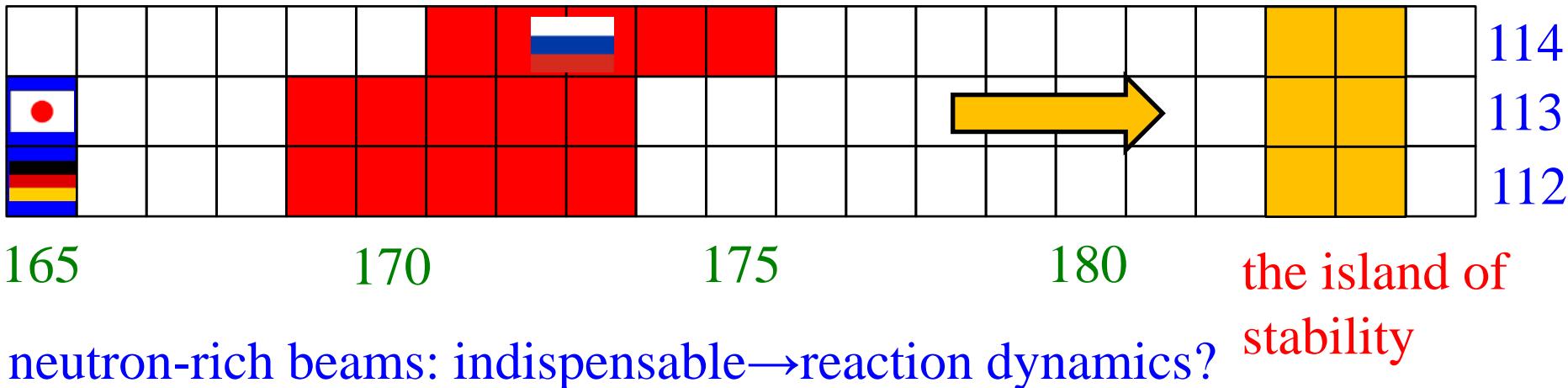


松尾美希、物性研究 73 ('99) 557

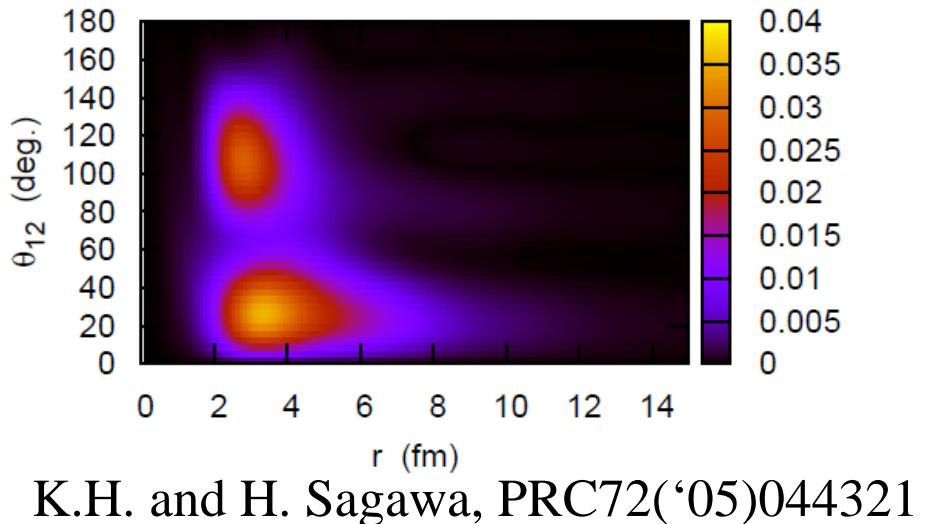
temperature gradient  
→ one-way dynamics

SHE formation reactions as a general problem of non-eq. stat. mechanics?

# Fusion of unstable nuclei

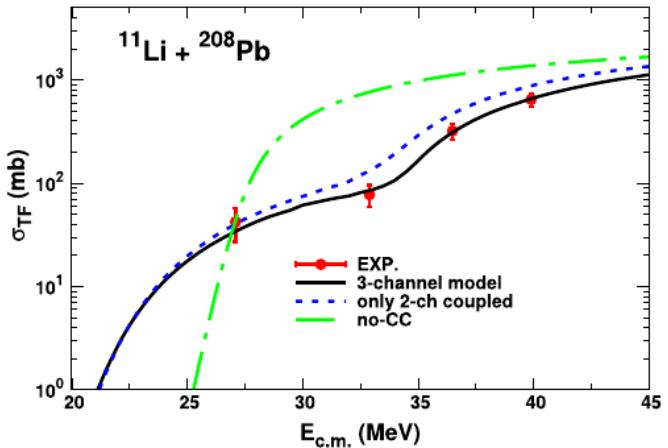


K.-S. Choi, K. Hagino et al.,  
Phys. Lett. B780 ('18) 455



good understandings of the structure  
of neutron-rich nuclei is also important

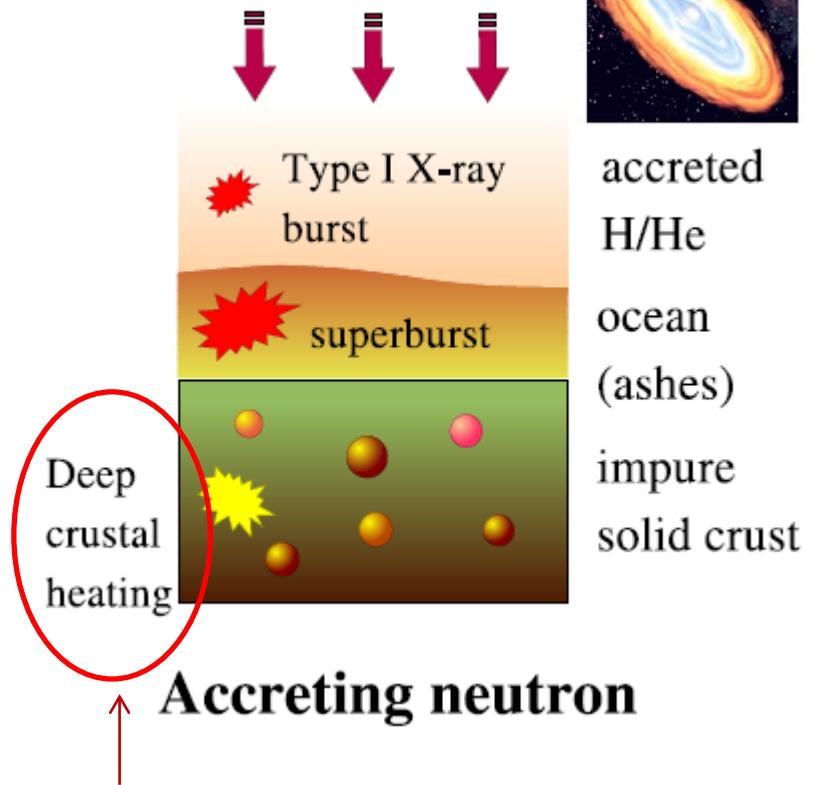
# reactions of neutron-rich nuclei



- ✓ fusion
- ✓ transfer



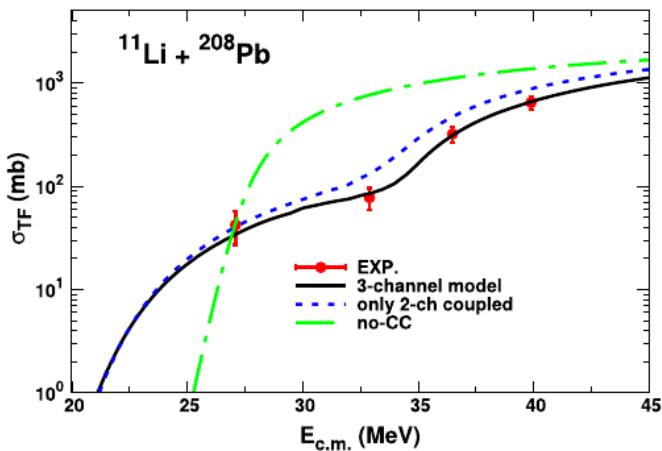
- development of microscopic nuclear reaction theory
- nuclear reactions in neutron stars



fusion of neutron-rich nuclei

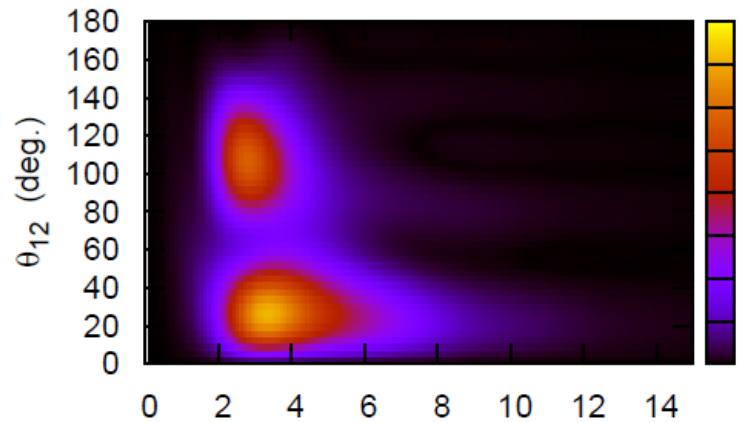


## reactions of neutron-rich nuclei



- ✓ fusion
- ✓ transfer

## structure of neutron-rich nuclei



- ✓ nucleon correlations
- ✓ collective motions
- ✓ fission

- development of microscopic nuclear reaction theory
- nuclear reactions in neutron stars

from few-body to many-body

**Physics of SHE with n-rich nuclei as important ingredient**

# Summary

## SHE: quantum many-body systems with a strong Coulomb field

physics

113 <b>Nh</b> nihonium	115 <b>Mc</b> moscovium
117 <b>Ts</b> tennessine	118 <b>Og</b> oganesson

reaction dynamics

- ✓ quantum friction
- ✓ neutron-rich nuclei

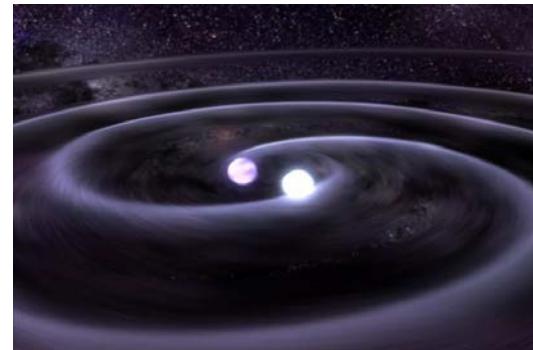
chemistry

Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
↓ Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	1 H															2 He			
2	3 Li	4 Be															10 Ne		
3	11 Na	12 Mg															18 Ar		
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	
7	87 Fr	88 Ra	89 Ac	*	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				*	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	



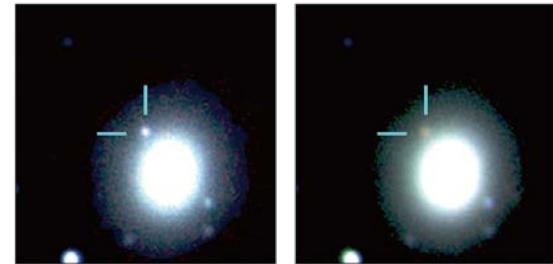
International Year  
of the Periodic Table  
of Chemical Elements

astronomy



2017.08.18-19

2017.08.24-25



- ✓ origin of elements
- ✓ r-process
- ✓ kilonova

interdisciplinary SHE science

$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \beta f(r) \hat{O}$$

full order treatment

$$V(r, \beta \hat{O})$$

$$\hat{O}|\phi_k\rangle = \lambda_k |\phi_k\rangle \quad \leftarrow \text{diagonalize } \langle n | \hat{O} | m \rangle$$

→  $\langle n | V(r, \beta \hat{O}) | m \rangle = \sum_k \langle n | \phi_k \rangle \langle \phi_k | m \rangle V(r, \beta \lambda_k)$

K.H., N. Rowley, and A.T. Kruppa,  
Comp. Phys. Comm. 123('99) 143.