Semi-microscopic modelling of heavy-ion fusion reactions

Kouichi Hagino
Tohoku University, Sendai, Japan

TOHOKU UNIVERSITY

How to do C.C. calculations if there is only limited experimental information on intrinsic degrees of freedom?

1. Introduction
   - H.I. sub-barrier fusion reactions
   - Coupled-channels (C.C.) approach
2. Phenomenological approach: Bayesian statistics
3. C.C. with nuclear structure calculations
4. Summary

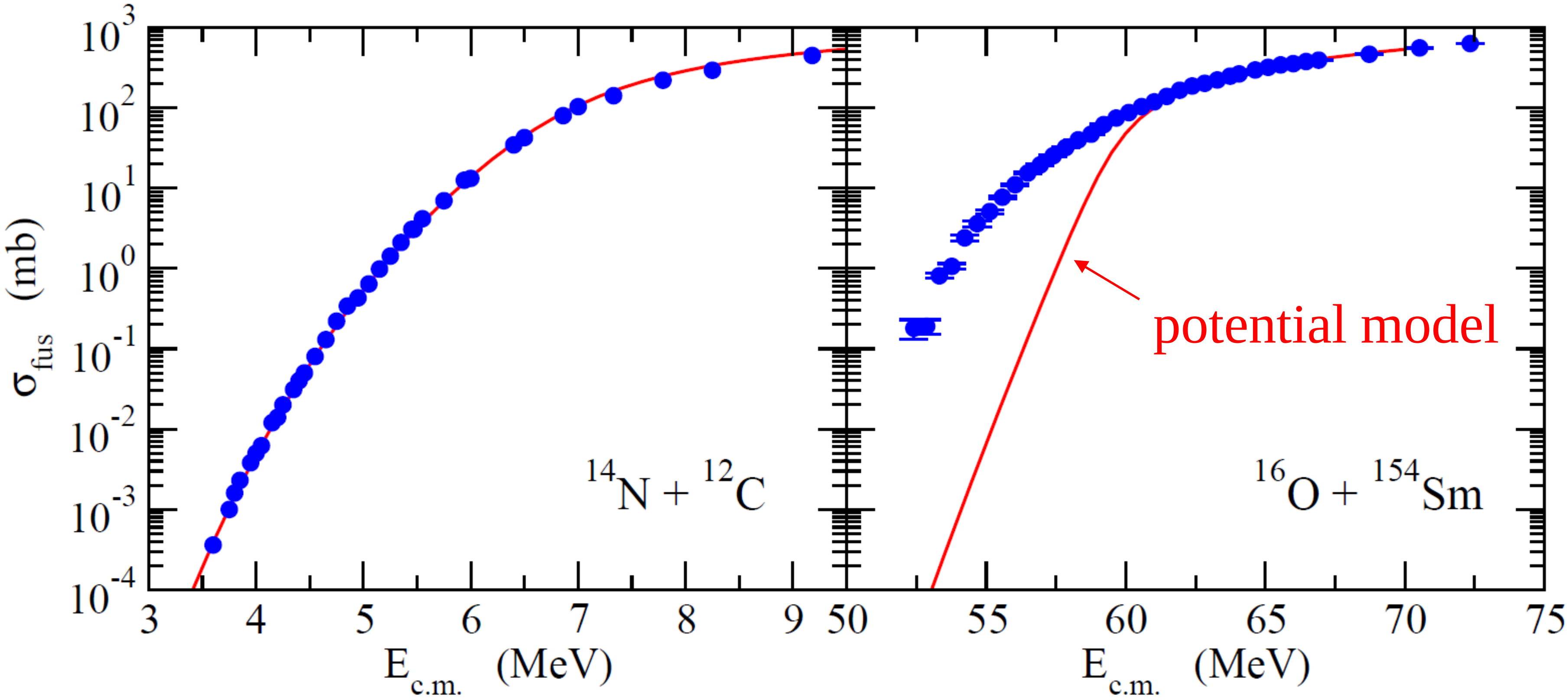
Workshop on Low-Energy Nuclear Reaction Theory (LENRT), Canberra, Feb. 15-17

Introduction: heavy-ion sub-barrier fusion reactions

Discovery of large sub-barrier enhancement of σ_{fus} (~ the late 70's)

potential model: $V(r)$ + absorption

$$\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$



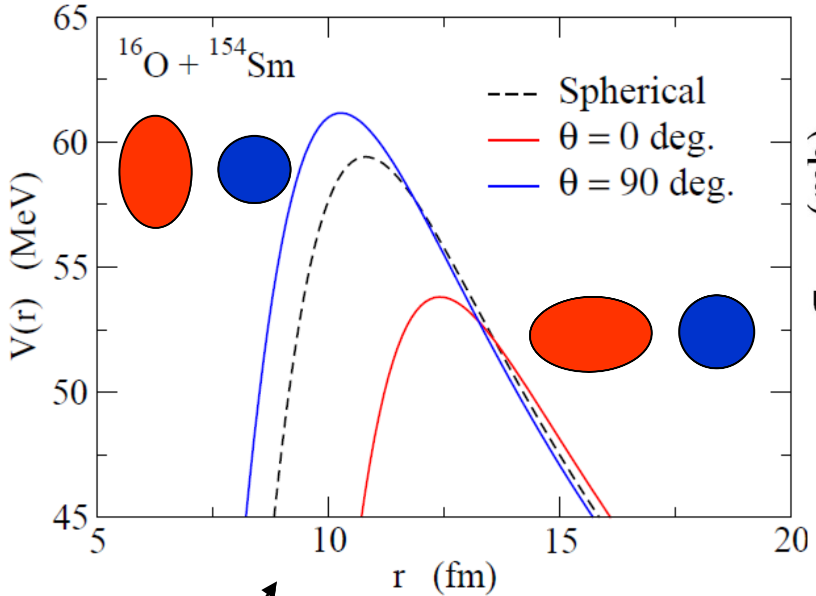
cf. seminal work:

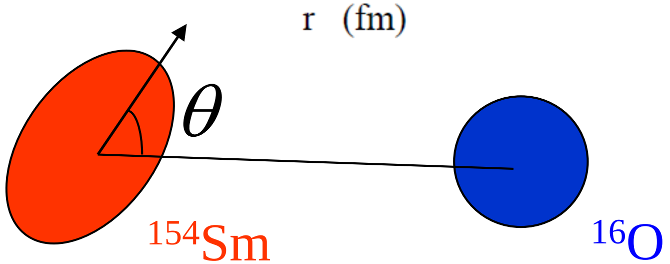
R.G. Stokstad et al., PRL41('78) 465

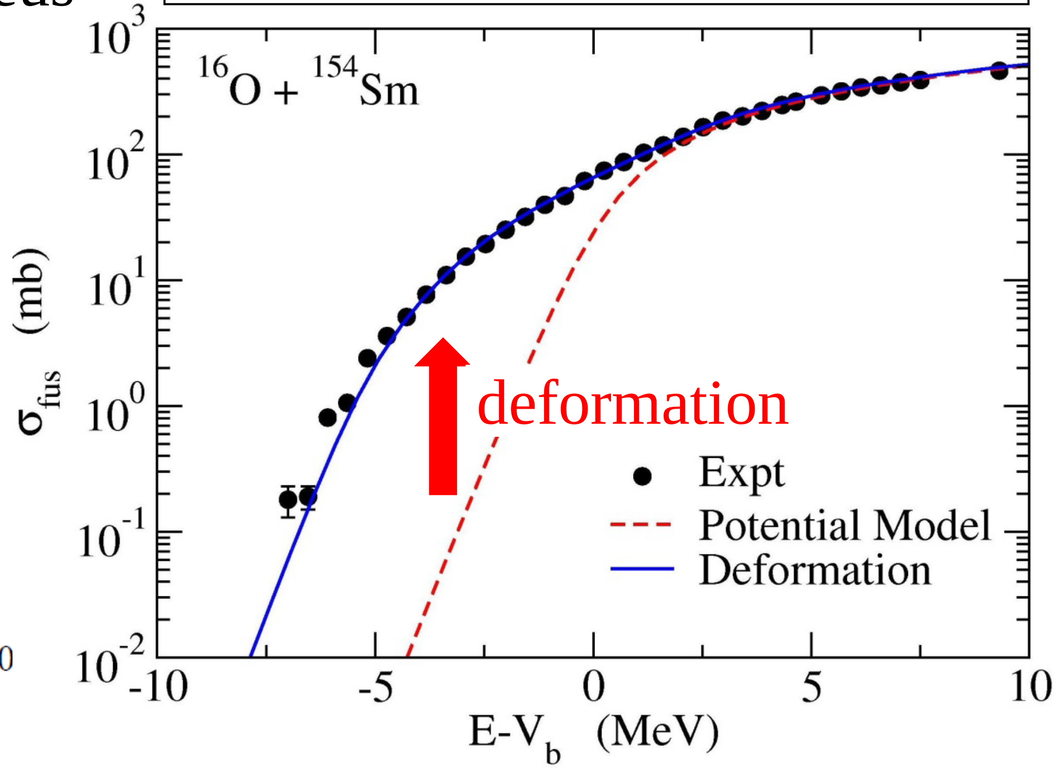
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus
with $\beta_2 \sim 0.3$

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$





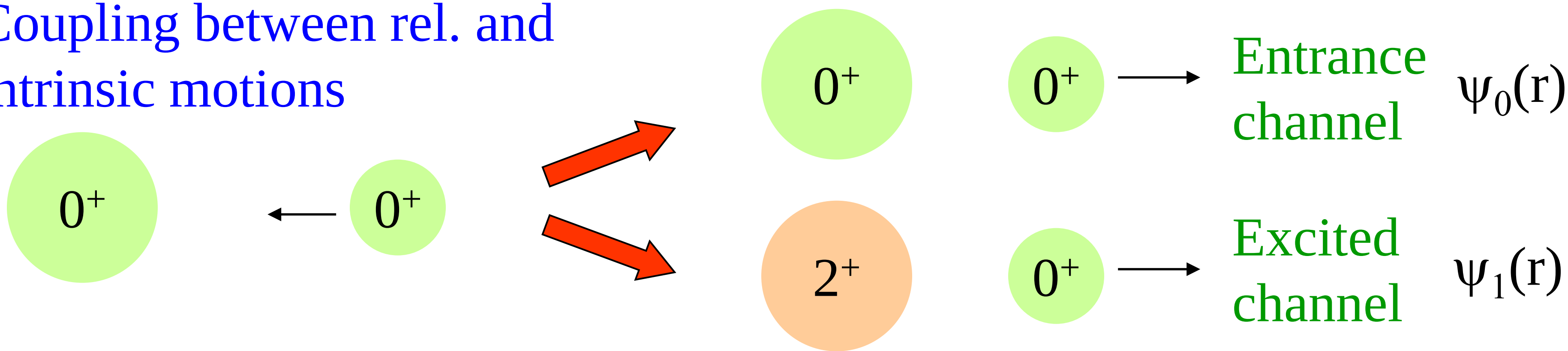


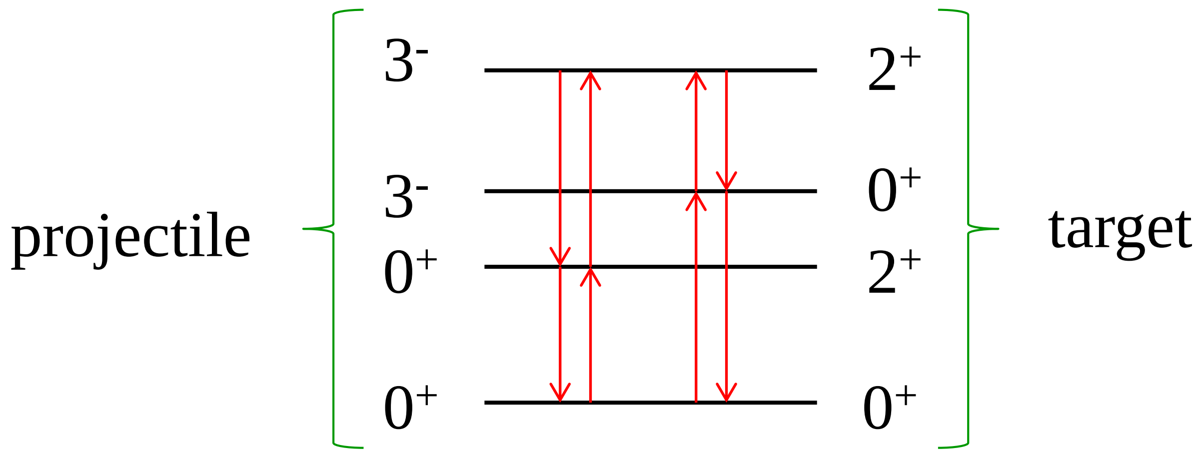
Fusion: strong interplay between nuclear structure and reaction

* Sub-barrier enhancement also for non-deformed targets:
couplings to low-lying collective excitations → coupling assisted tunneling

Coupled-Channels method

Coupling between rel. and intrinsic motions





$$\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)$$

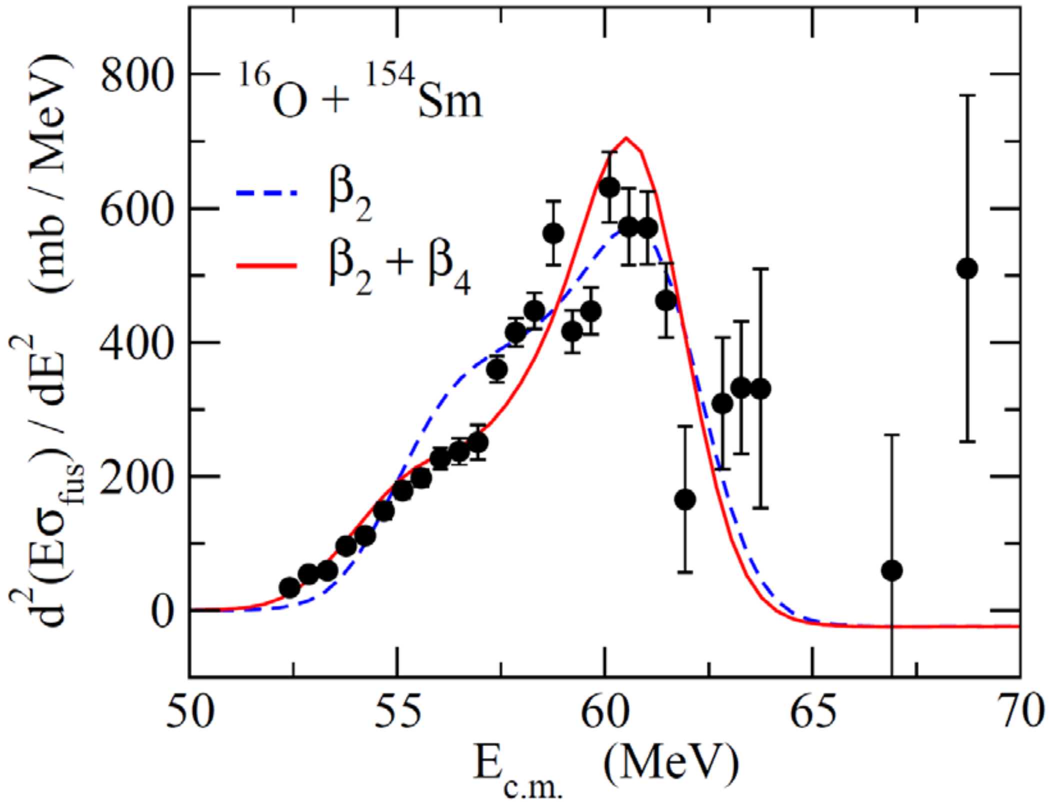
coupled Schroedinger equations for $\psi_k(r)$

C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}$$



◆ N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254(‘91) 25

◆ J.X. Wei, J.R. Leigh et al., PRL67(’91) 3368

◆ M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48(’98)401

many barriers are “distributed” due to the channel coupling effects

sensitive to nuclear structure

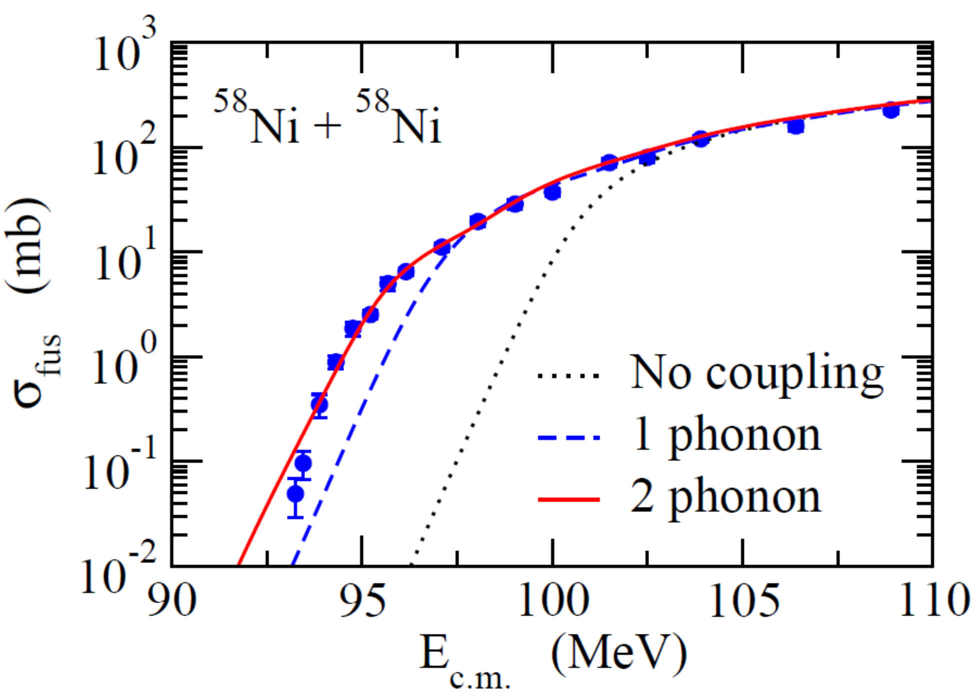
Inputs for C.C. calculations

i) Inter-nuclear potential

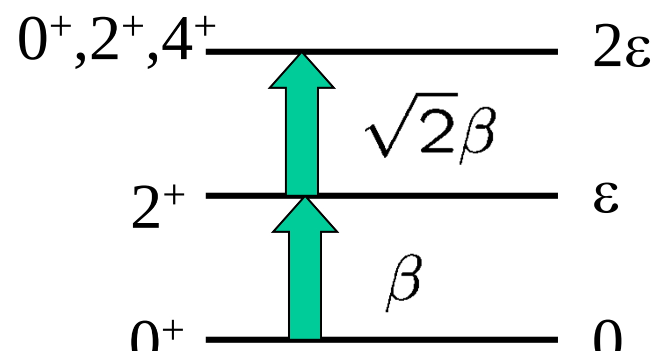
- ✓ a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom

- ✓ types of collective motions (rotation / vibration) a/o transfer
- ✓ coupling strengths and excitation energies
- ✓ how many states



simple harmonic oscillator



Inputs for C.C. calculations

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What to do if there is only limited experimental information on intrinsic degrees of freedom?

1. Phenomenological fit with a few barriers
   K.H., PRC93 (‘16) 061601(R)

2. C.C. + nuclear structure calculations
   K.H. and J.M. Yao, PRC91 (‘15) 064606
   J.M. Yao and K.H., PRC94 (‘16) 011303(R)

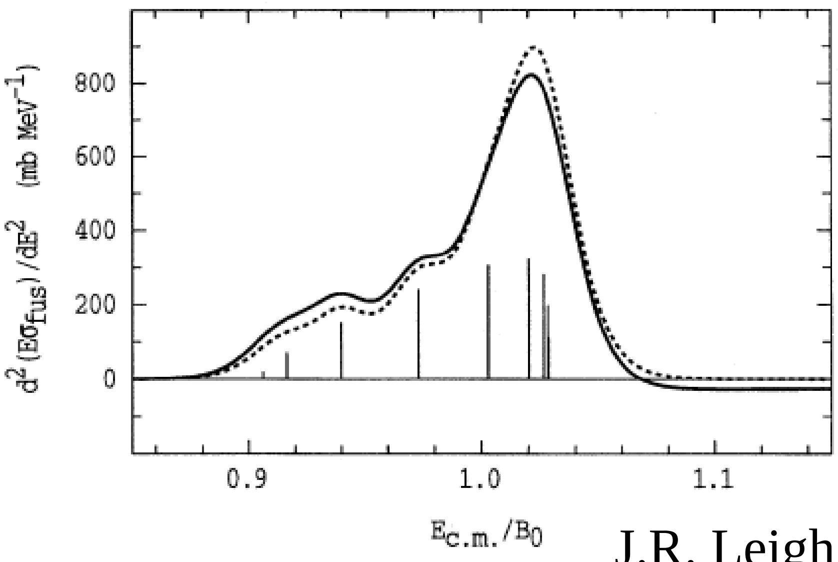
A Bayesian approach to fusion barrier distributions

Fusion barrier distributions

K.H., PRC93 ('16) 061601(R)

- Coupled-channels analyses
  - ✓ a standard approach
  - ✓ need to know the nature of collective excitations
- Direct fit to experimental data

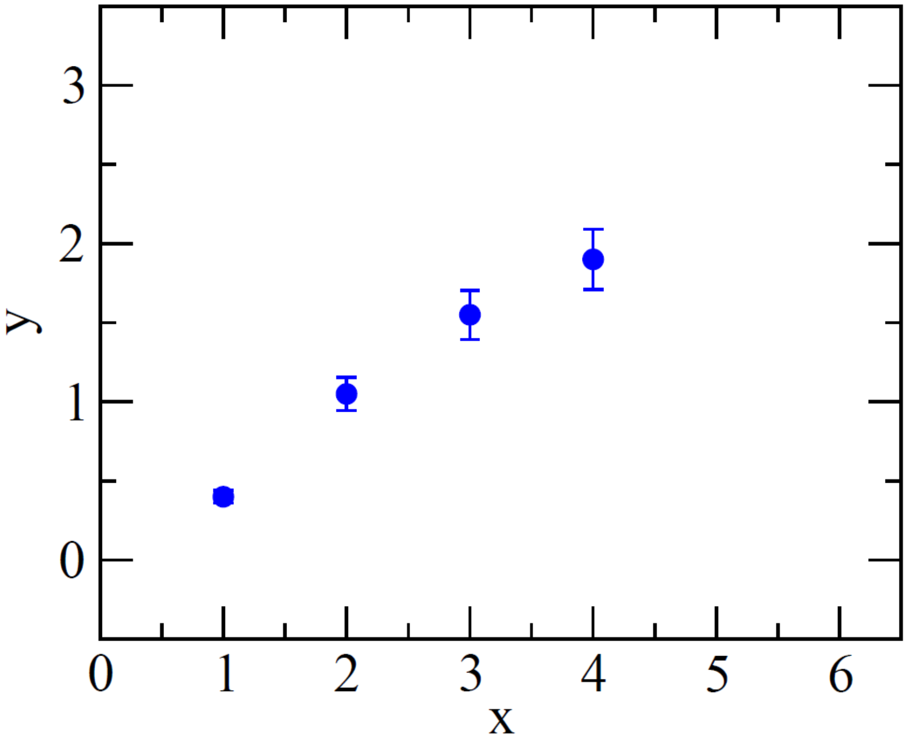
$$D_{\rm fus}(E) = \sum_k w_k D_0(E; B_k, R_k, \hbar\Omega_k)$$



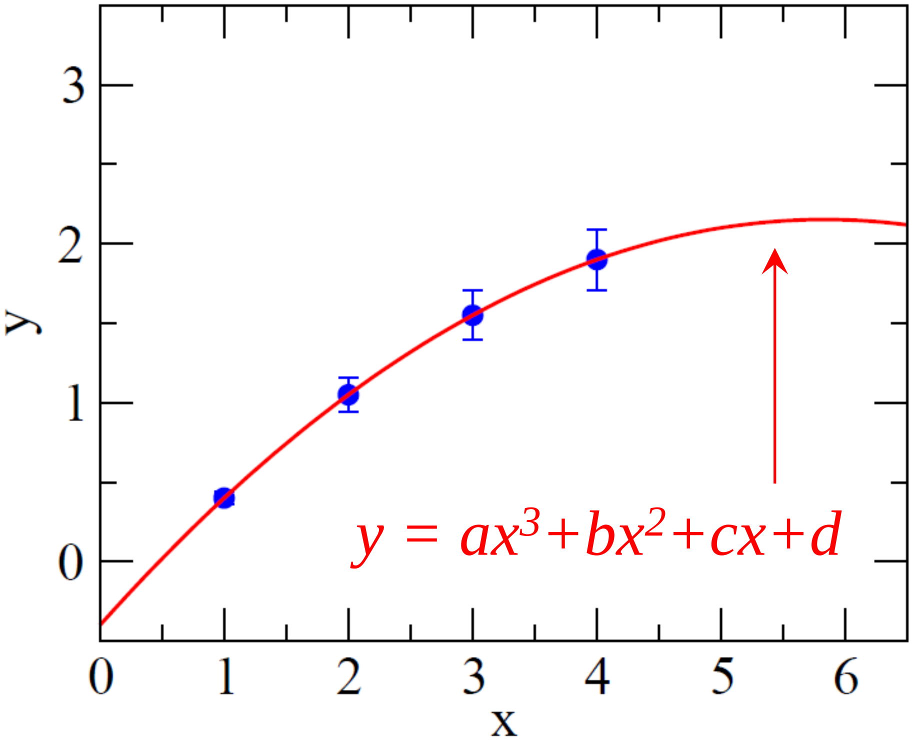
J.R. Leigh et al., PRC52 ('95) 3151

- ✓ phenomenological
- ✓ no need to know the nature of coll. excitations
- ✓ quick and convenient way
- ✓ mapping from D to T_l (cf. SHE)
- ✓ the number of barriers? (over-fitting problem)

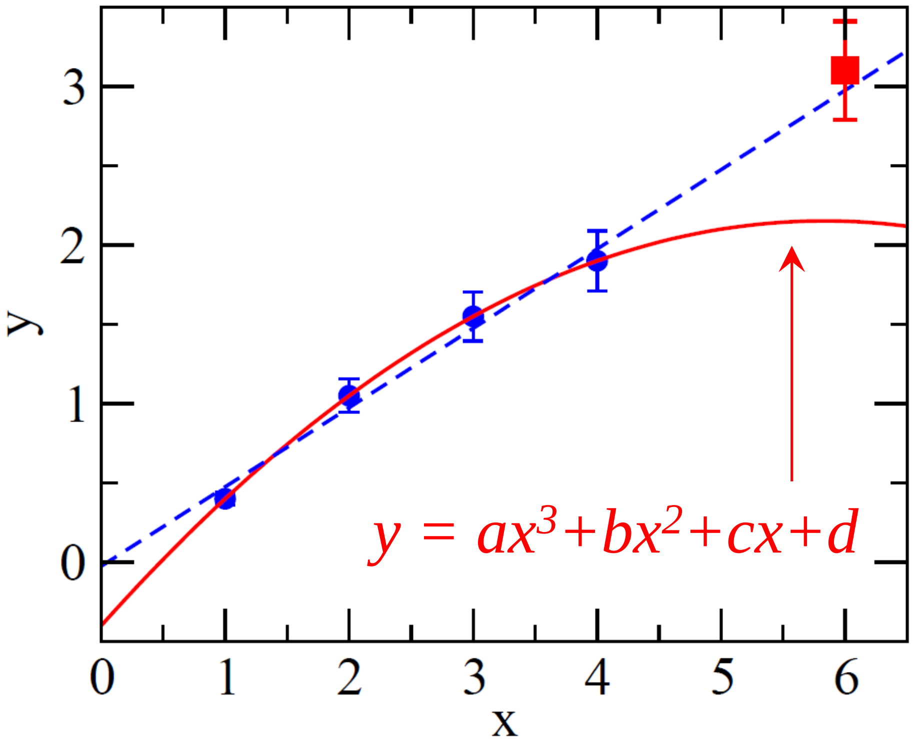
[over-fitting problem]

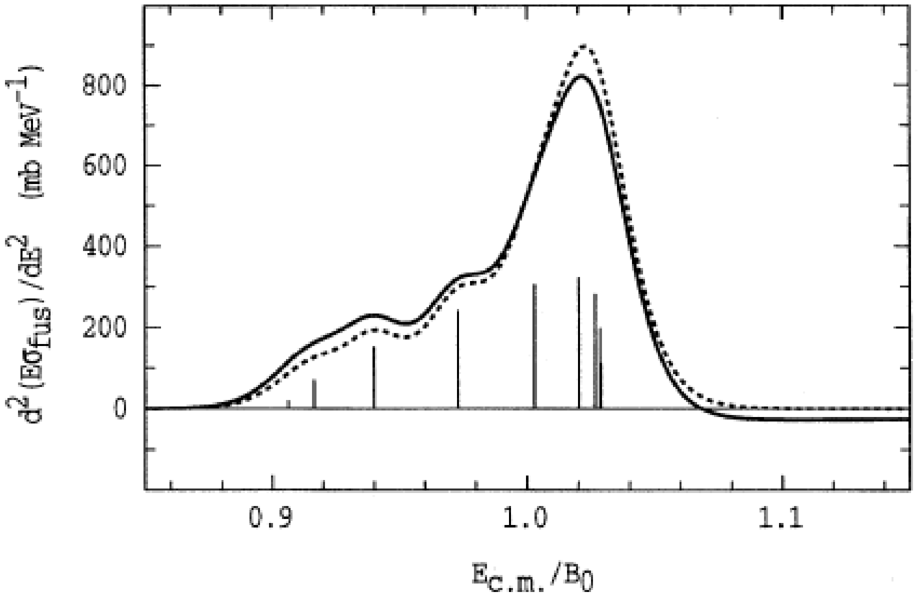


over-fitting problem



over-fitting problem





J.R. Leigh et al., PRC52 ('95) 3151

one can make χ^2 small
by increasing the number of barriers

how many barriers?

Bayesian spectrum deconvolution

K. Nagata, S. Sugita, and M. Okada, Neural Networks 28 ('12) 82

- ✓ data set: $D_{exp} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
- ✓ fitting function: $D_{\mathrm{fit}}(E; \tilde{\boldsymbol{\theta}}, K) = \sum_{k=1}^{K} w_k \, \phi_k(E; \boldsymbol{\theta}_k)$, $\quad \tilde{\boldsymbol{\theta}} \equiv \{w_k, \boldsymbol{\theta}_k\}$

K: the number of barriers

Bayes theorem

$$P(K|D_{\mathrm{exp}}) = \frac{P(D_{\mathrm{exp}}|K)P(K)}{P(D_{\mathrm{exp}})}$$

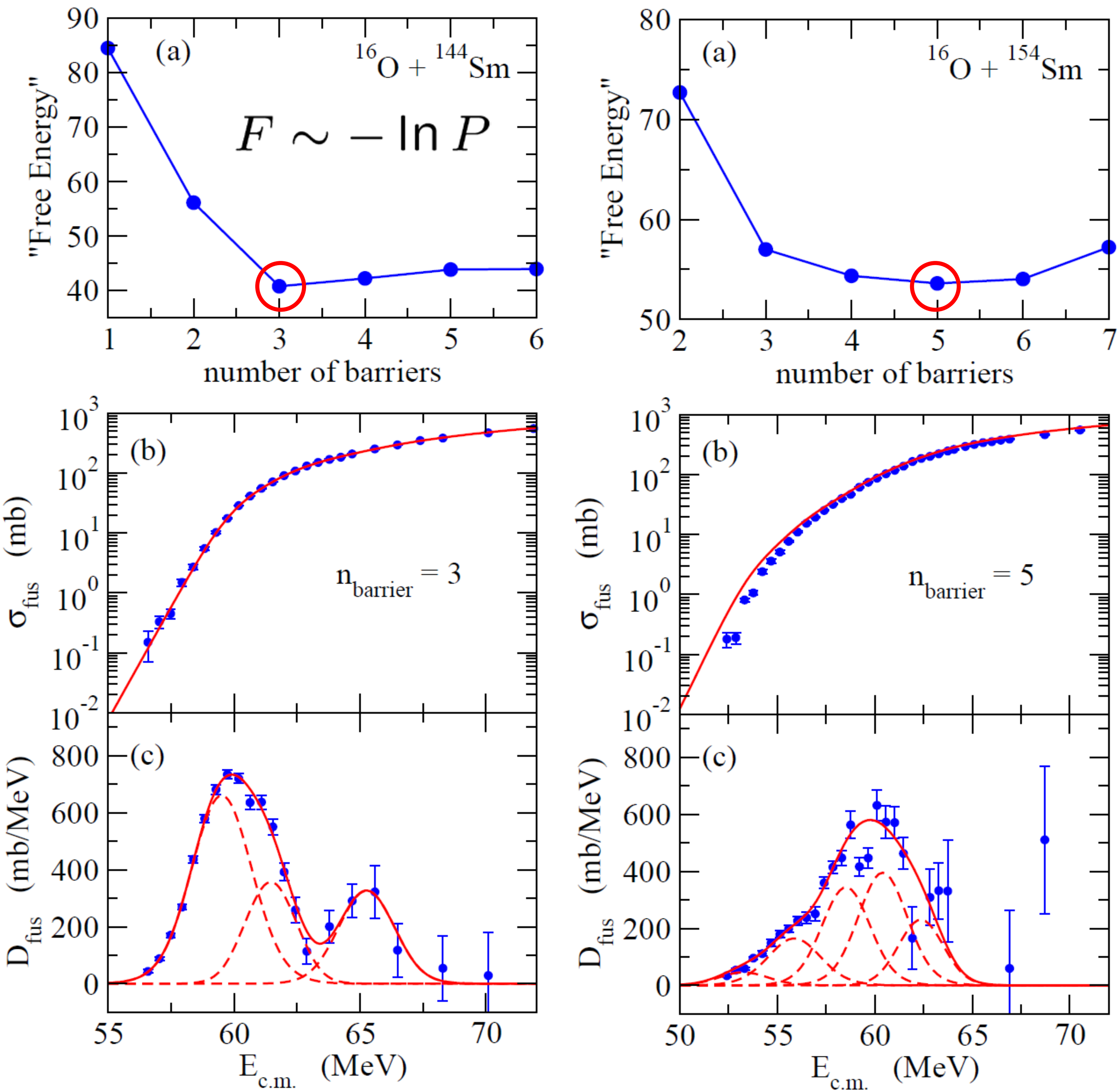
$$\propto P(D_{\mathrm{exp}}|K) = \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}},K)/2}$$

$$\chi^2(\tilde{\boldsymbol{\theta}}, K) = \sum_{i=1}^{M} \left(\frac{d_i - D_{\mathrm{fit}}(E_i; \tilde{\boldsymbol{\theta}}, K)}{\delta d_i} \right)^2$$

most probable value of K: maximize $P(K|D_{exp})$

or, equivalently, minimize $F = -\ln P(K|D_{exp})$

⟶ optimize the other parameters for a given value of *K*



K.H., PRC93 ('16) 061601(R)

Bayesian approach to σ_{ER}

$$D_{\exp}(E) = \sum_{i=1}^{K} w_k D_0(E; V_k(r))$$

either D_{fus} or D_{qel}

$$\Rightarrow \quad T_l = \sum_{k=1}^{K} w_k T_l(E; V_k(r))$$

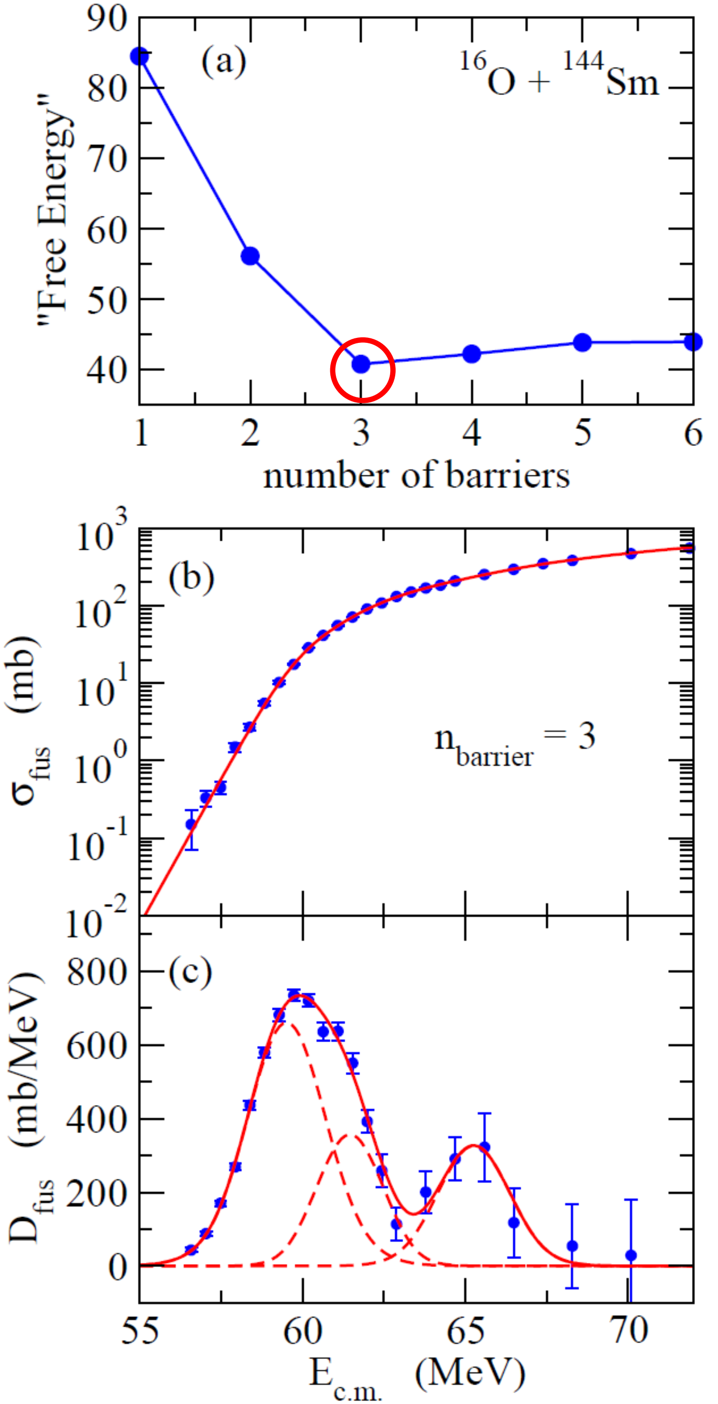
* no need to know the details of the couplings

+

Langevin + stat. model calculations

$$\sigma_{ER}(E) = \frac{\pi}{k^2} \sum_l (2l+1) T_l(E) \times P_{CN}(E,l)\, W_{suv}(E^*,l)$$

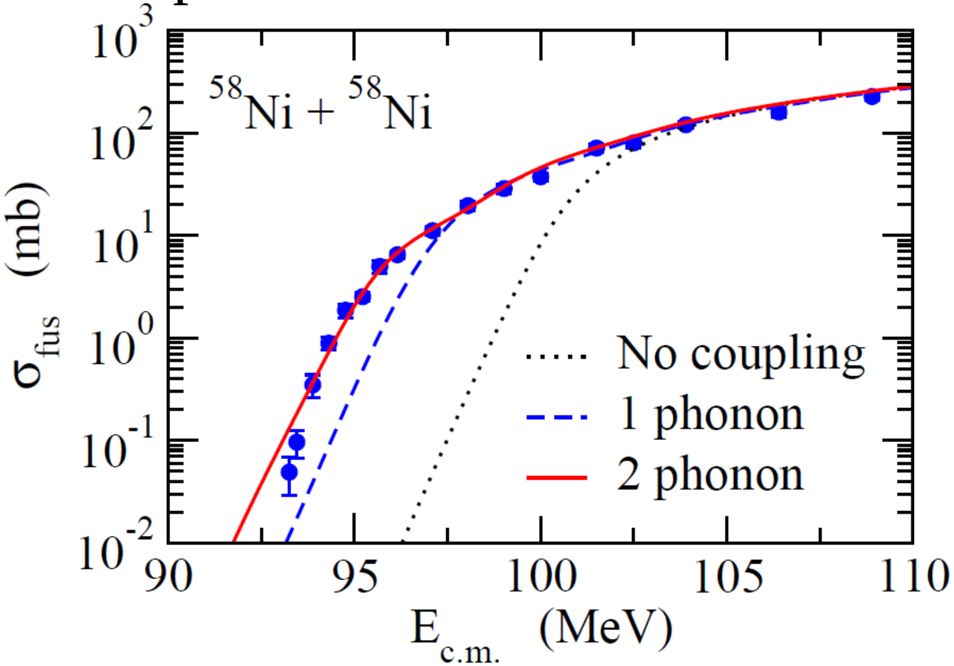
superheavy elements

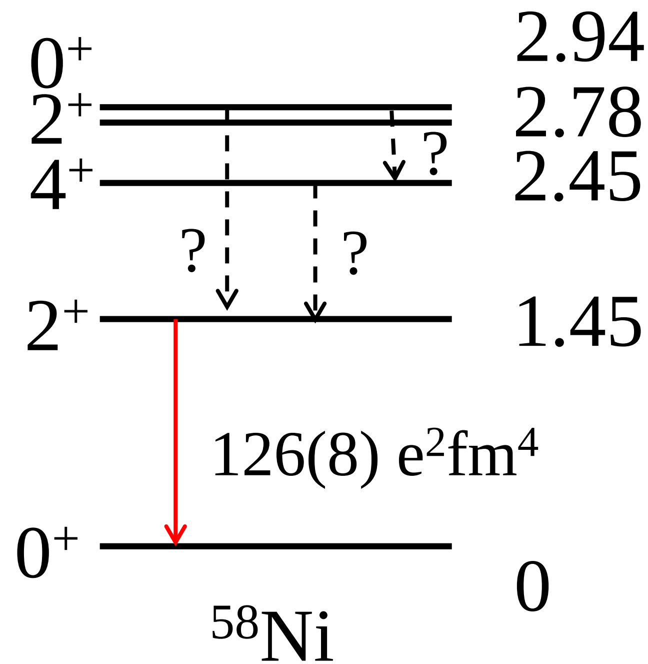


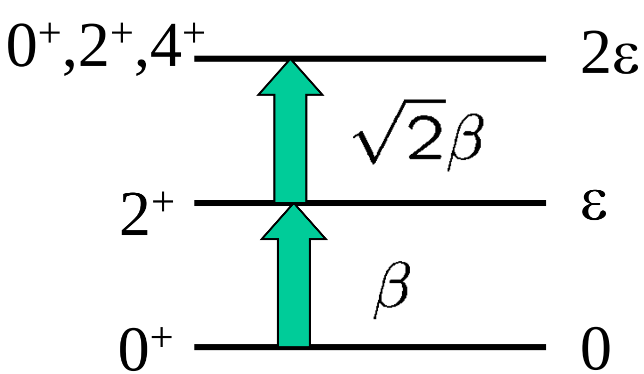
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15) 064606

multi-phonon excitations







$Q(2_1^+) = -10 +/- 6\ e\mathrm{fm}^2$

Simple harmonic oscillator
→ justifiable?

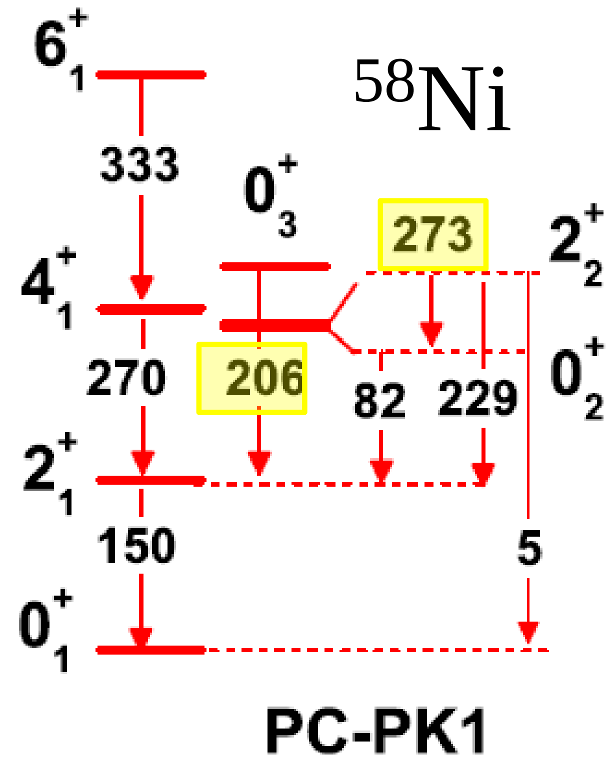
Anharmonic vibrations

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

$$|JM\rangle = \int d\beta\, f_J(\beta) \hat{P}^J_{M0} |\Phi(\beta)\rangle$$

✓ MF + ang. mom. projection
+ particle number projection
+ generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 ('03) 121
J.M. Yao et al., PRC89 ('14) 054306

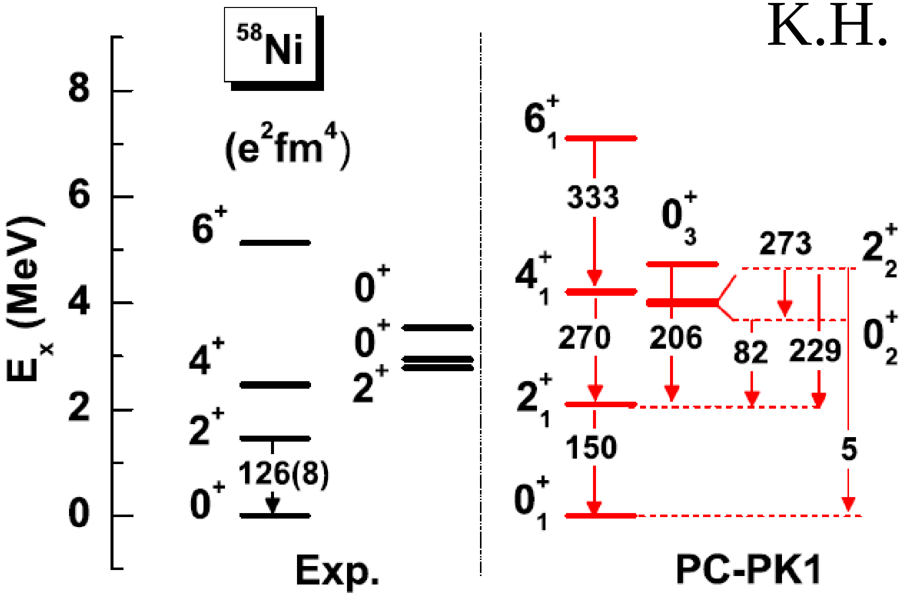


cf. Harmonic limit:
B(E2: $I_{2ph}{}^+ \to 2_1{}^+$) =2 x B(E2: $2_1{}^+ \to 0_1{}^+$)

K.H. and J.M. Yao, PRC91('15) 064606

Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 ('15) 064606



microscopic multi-pole operator

✓ $$V_{\rm coup} \sim -R_T \frac{dV_N}{dr} \alpha_\lambda \cdot Y_\lambda(\hat{\boldsymbol{r}}) \to -R_T \frac{dV_N}{dr} Q_\lambda \cdot Y_\lambda(\hat{\boldsymbol{r}})$$

✓ M(E2) from MR-DFT calculation ← among higher members of phonon states

✓ scale to the empirical B(E2; $2_1^+ \to 0_1^+$)

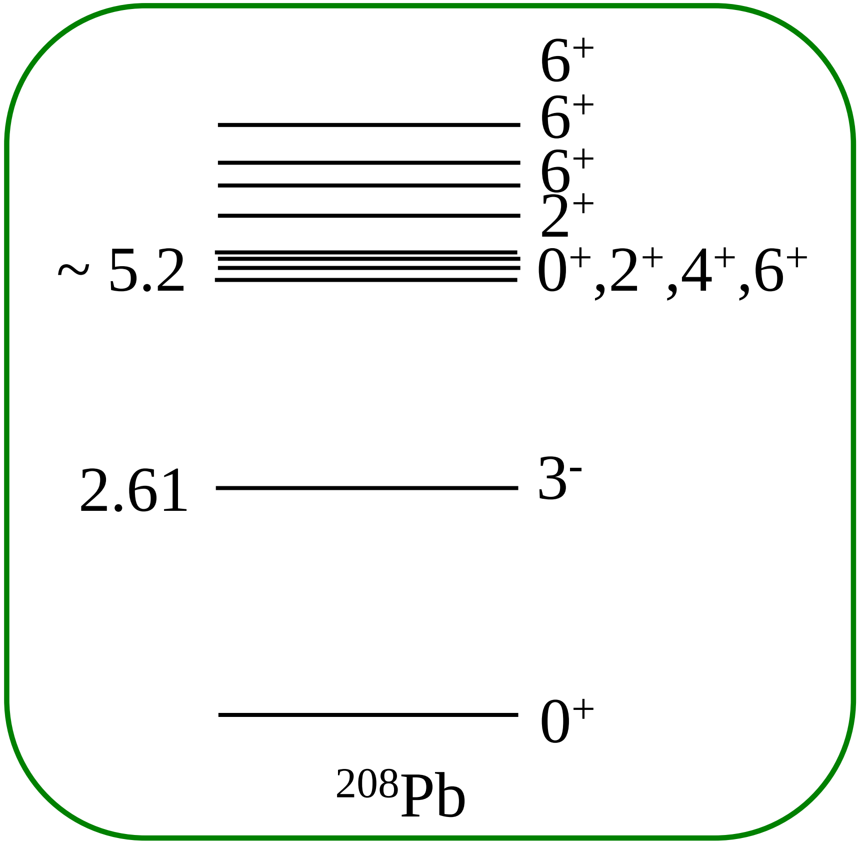
✓ still use a phenomenological potential

✓ use the experimental values for E_x

* axial symmetry (no 3^+ state)

Application to ^{16}O + ^{208}Pb fusion reaction

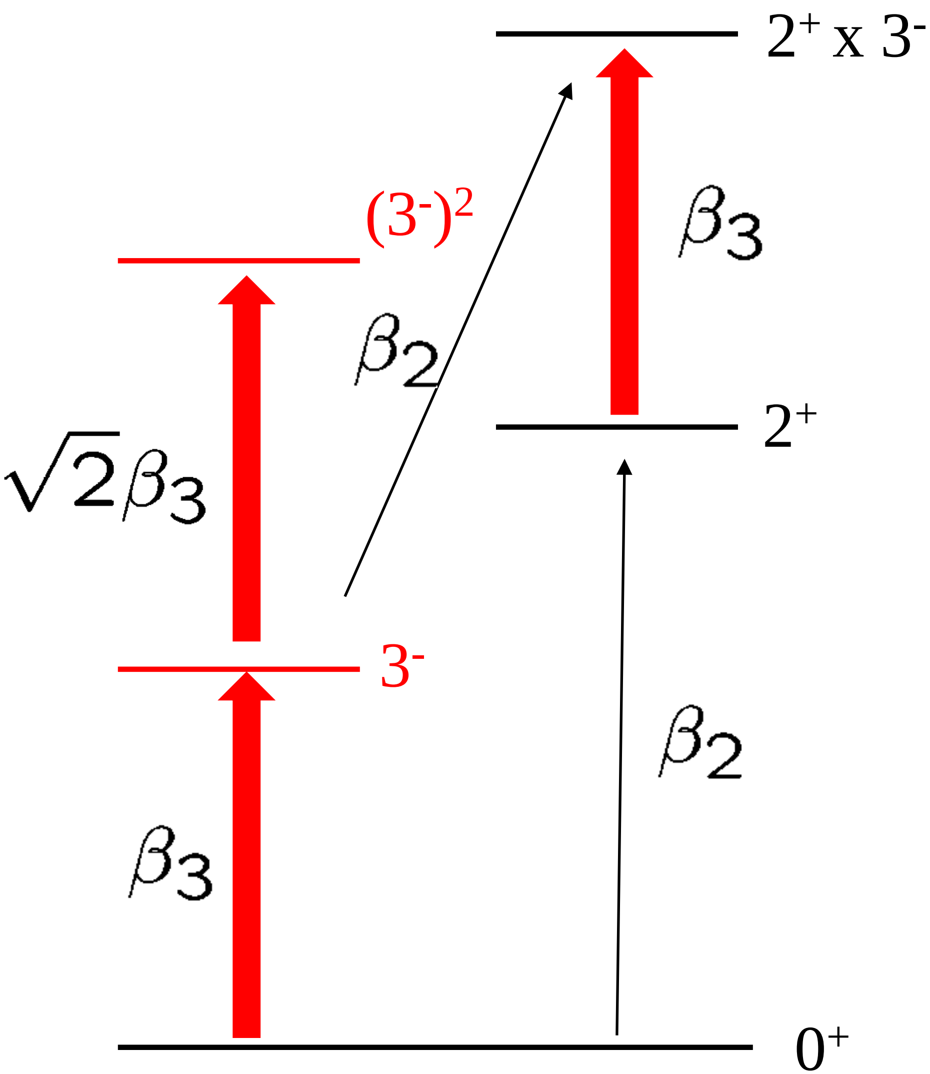
double-octupole phonon states in ^{208}Pb

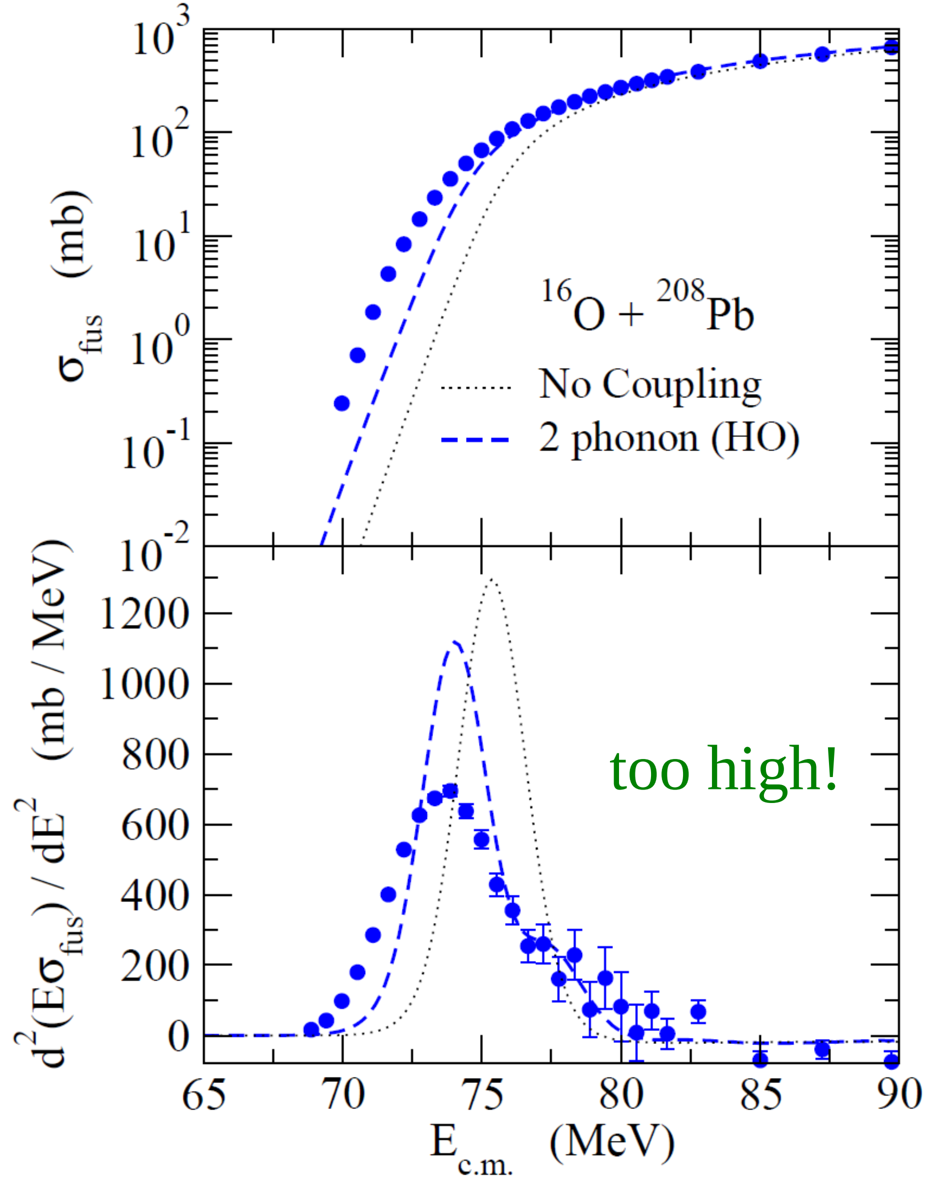


M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631
V. Yu. Pnomarev and P. von Neumann-Cosel, PRL82 (‘99) 501
B.A. Brown, PRL85 (‘00) 5300

large fragmentations, especially 6^+ state

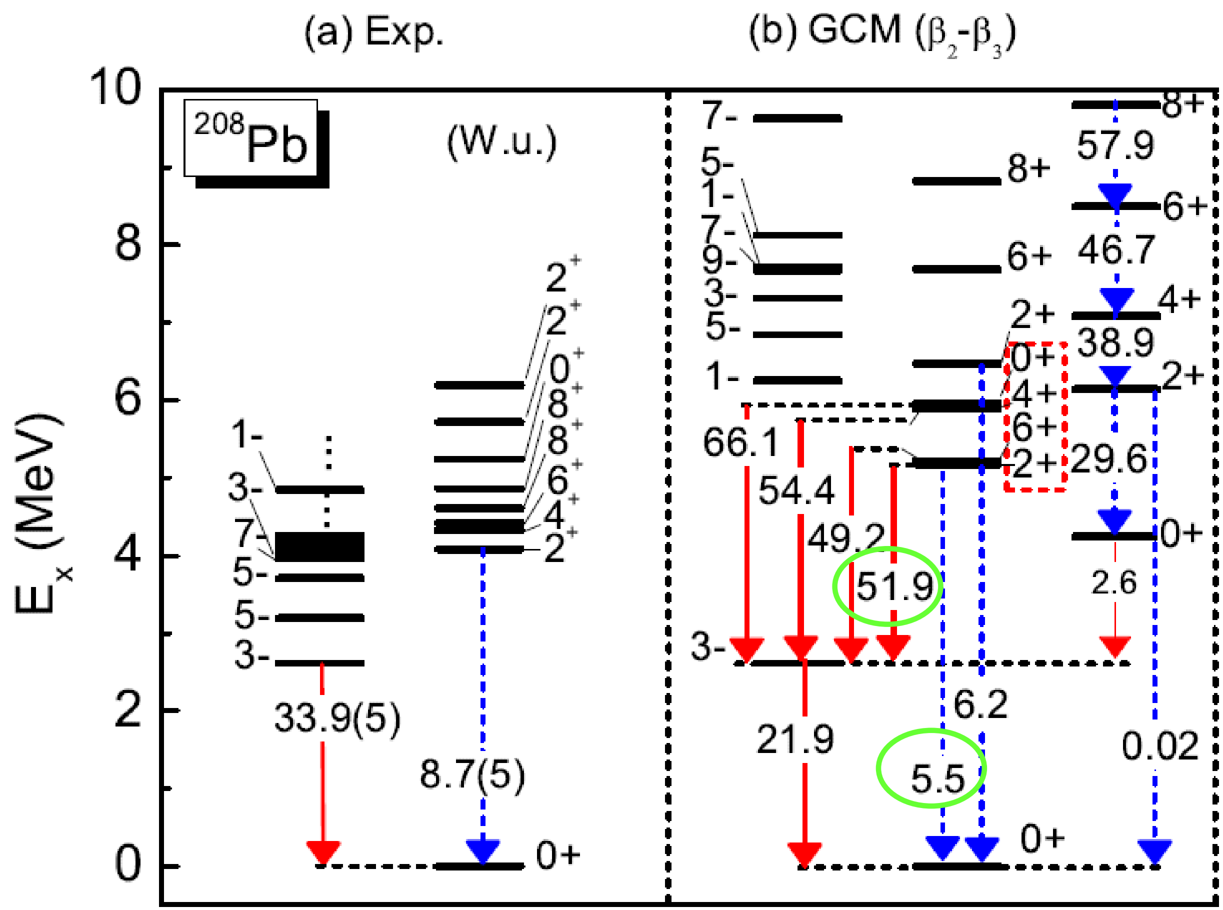
Application to ^{16}O + ^{208}Pb fusion reaction





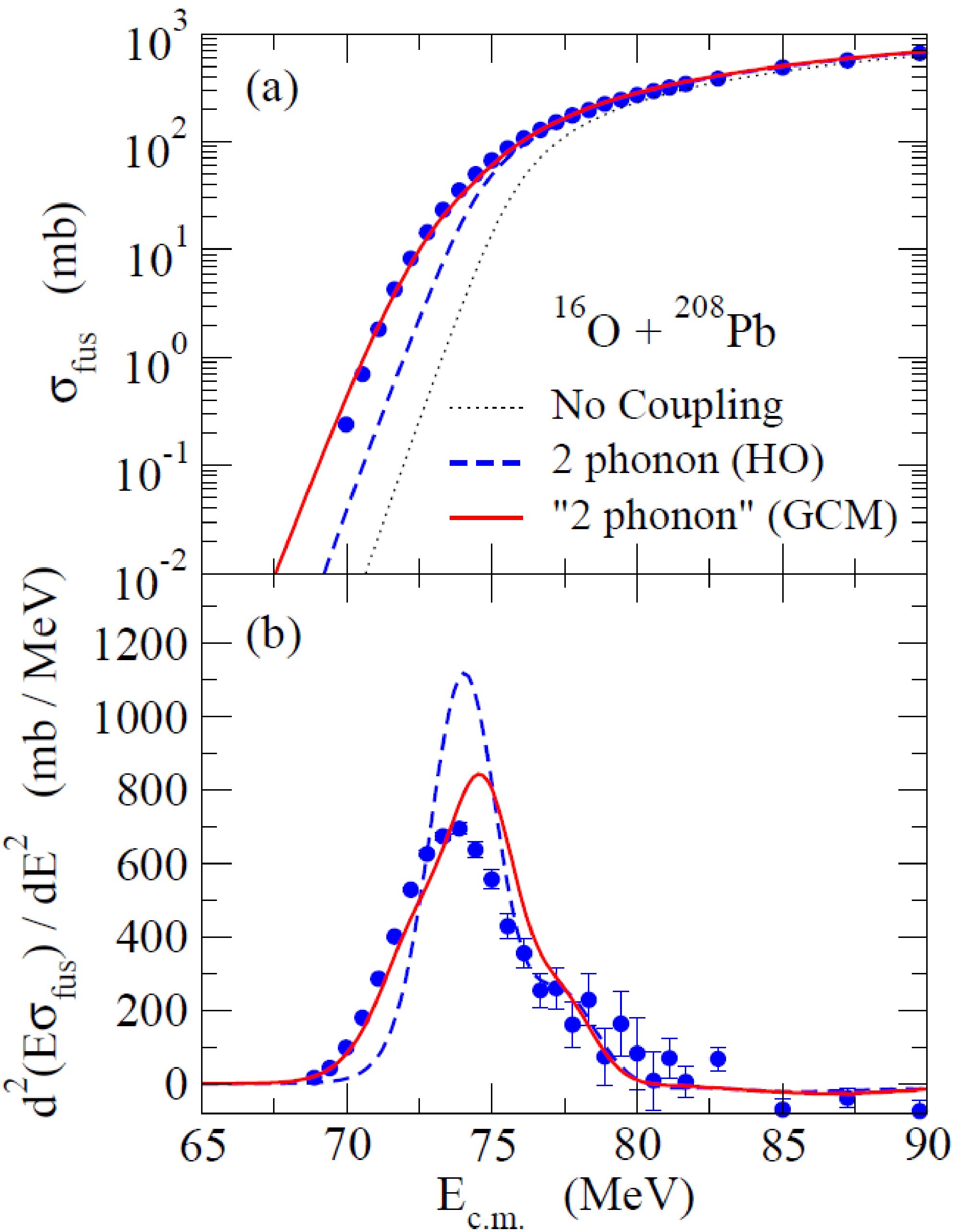
cf. C.R. Morton et al., PRC60('99) 044608 expt. data

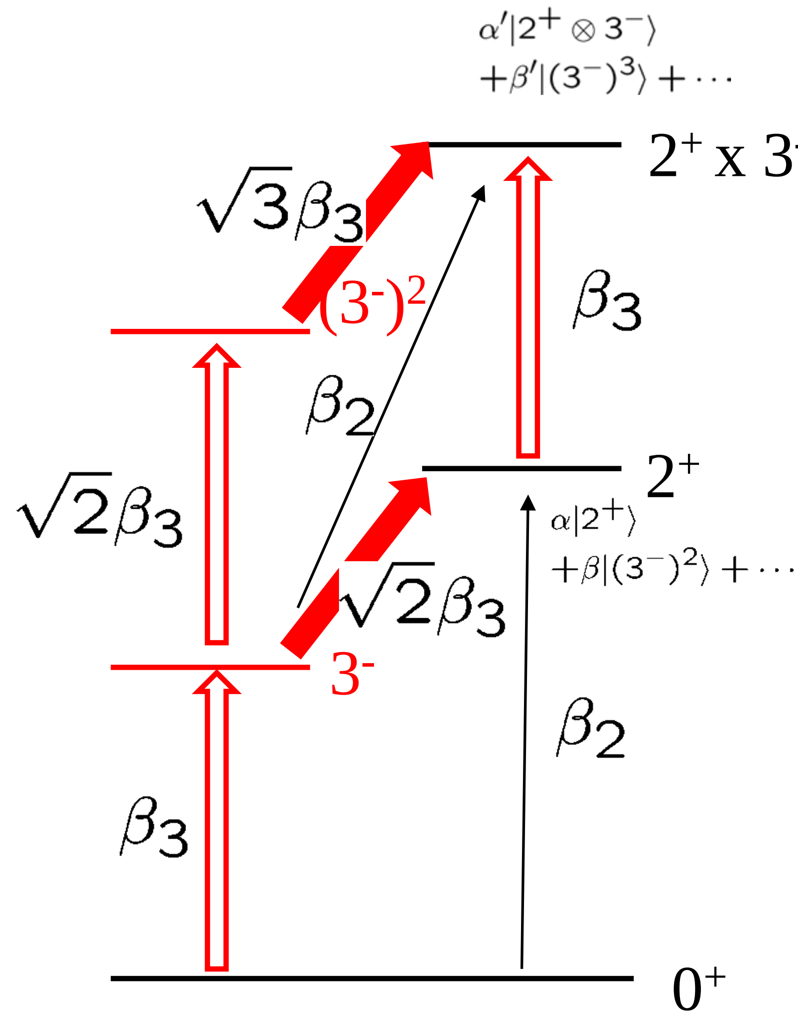
fluctuation both in β_3 and β_2



2_1^+ state: strong coupling both to g.s. and 3_1^-

$$\longrightarrow \quad |2_1^+\rangle = \alpha|2^+\rangle_{\rm HO} + \beta|[3^- \otimes 3^-]^{(I=2)}\rangle_{\rm HO} + \cdots$$





J.M. Yao and K.H.,
PRC94 ('16) 11303(R)

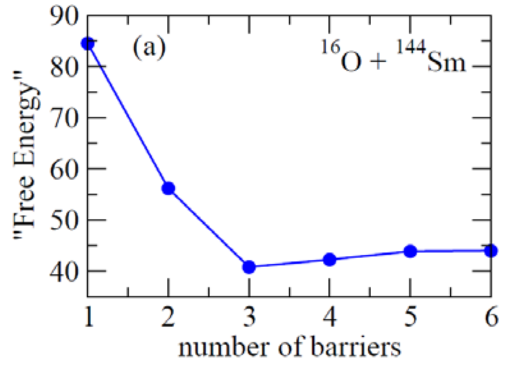
Summary

Heavy-ion subbarrier fusion reactions

✓ strong interplay between reaction and structure
cf. fusion barrier distributions

➢ A Bayesian approach to fusion barrier distributions

✓ a quick and convenient way to analyze data
✓ determination of the number of barriers



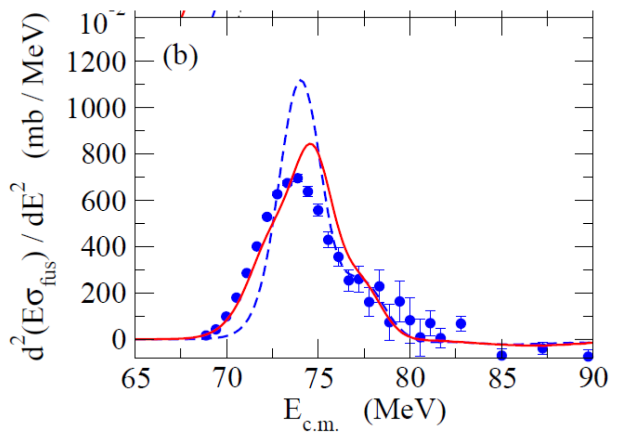
➢ C.C. calculations with MR-DFT method

✓ anharmonicity
✓ truncation of phonon states
✓ octupole vibrations: $^{16}O + ^{208}Pb$

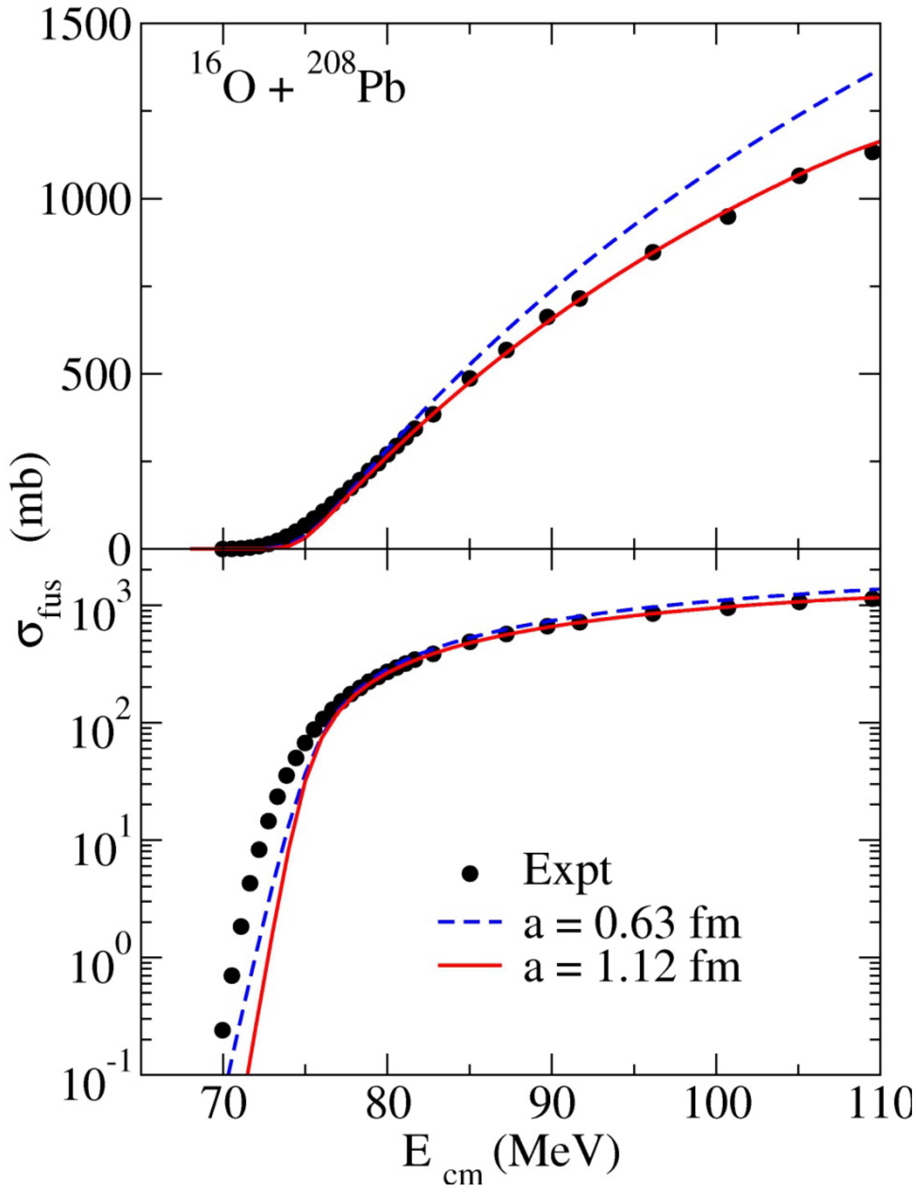
more flexibility:

- application to transitional nuclei

C.C. with shell model?



Why not full microscopic treatment?



microscopic potential
(e.g., double folding potential)

$\longrightarrow$ $a \sim 0.63$ fm

does not work for fusion