New approach to coupled-channels calculations for heavy-ion fusion reactions around the Coulomb barrier

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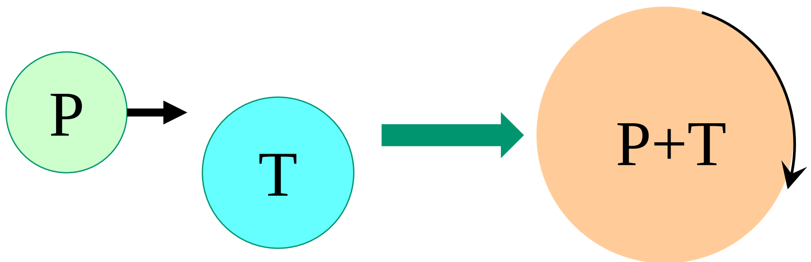
1. Introduction
   - H.I. sub-barrier fusion reactions
   - Coupled-channels (C.C.) approach
2. Phenomenological approach: Bayesian statistics
3. C.C. with nuclear structure calculations
4. Summary

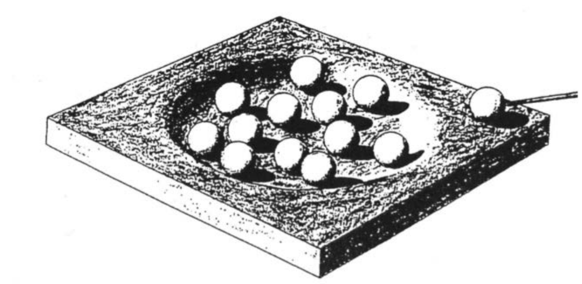
How to do C.C. calculations if there is only limited experimental information on intrinsic degrees of freedom?

Introduction: heavy-ion fusion reactions

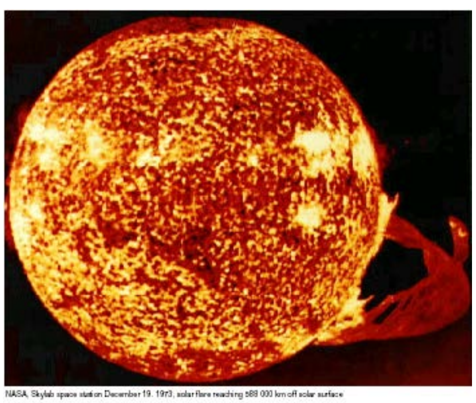
Fusion: compound nucleus formation



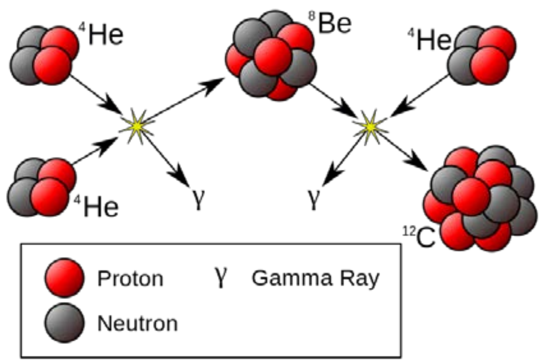
compound nucleus



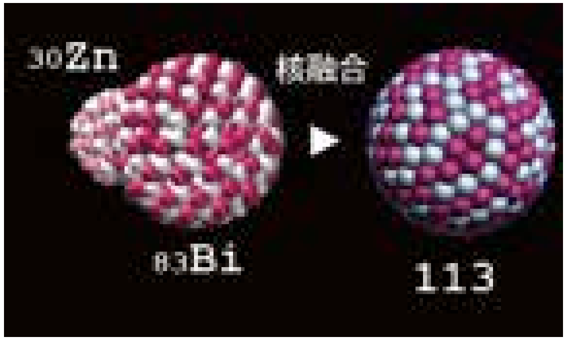
cf. Bohr '36



energy production in stars (Bethe '39)



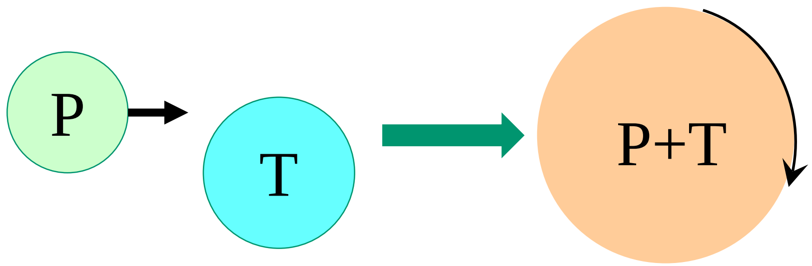
nucleosynthesis



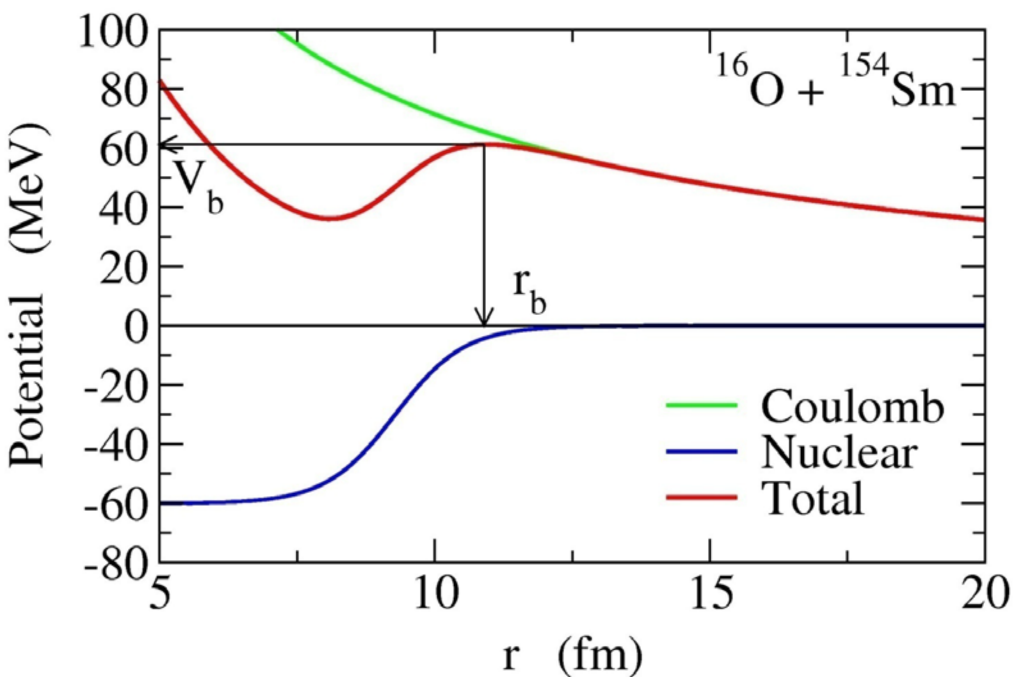
superheavy elements

Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation



compound nucleus

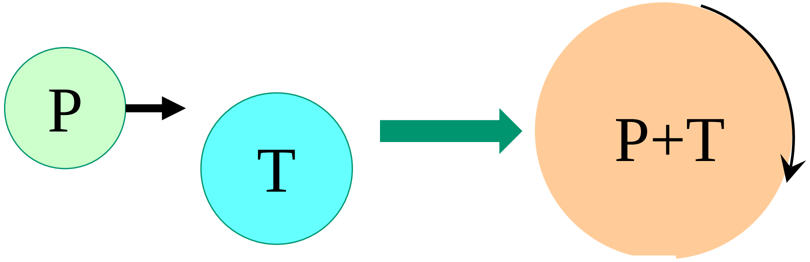


1. Coulomb force : long range, repulsive
2. Nuclear force : short range, attractive

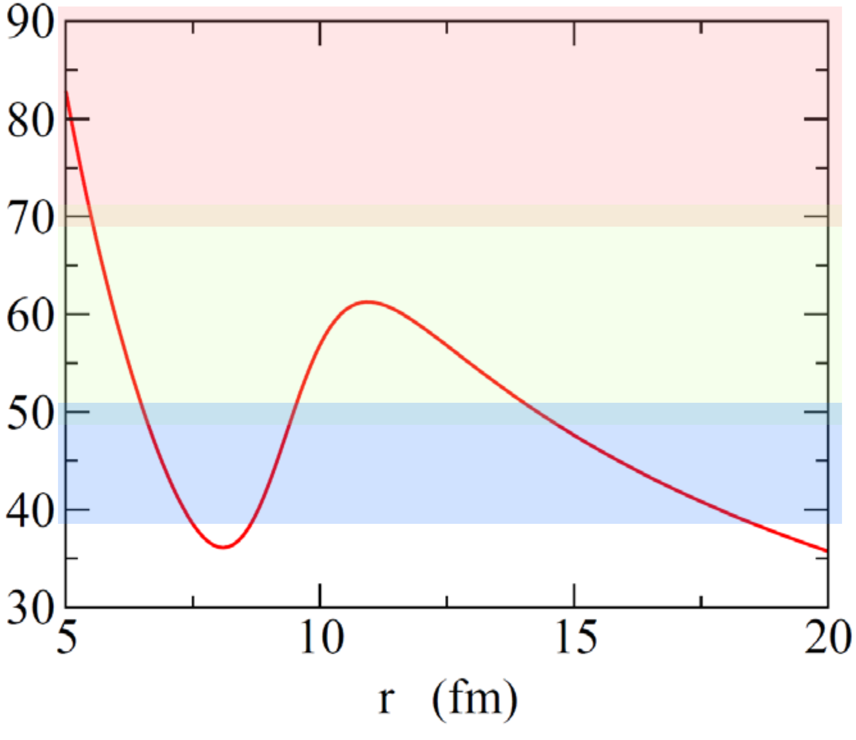
→ Coulomb barrier

Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation



compound nucleus



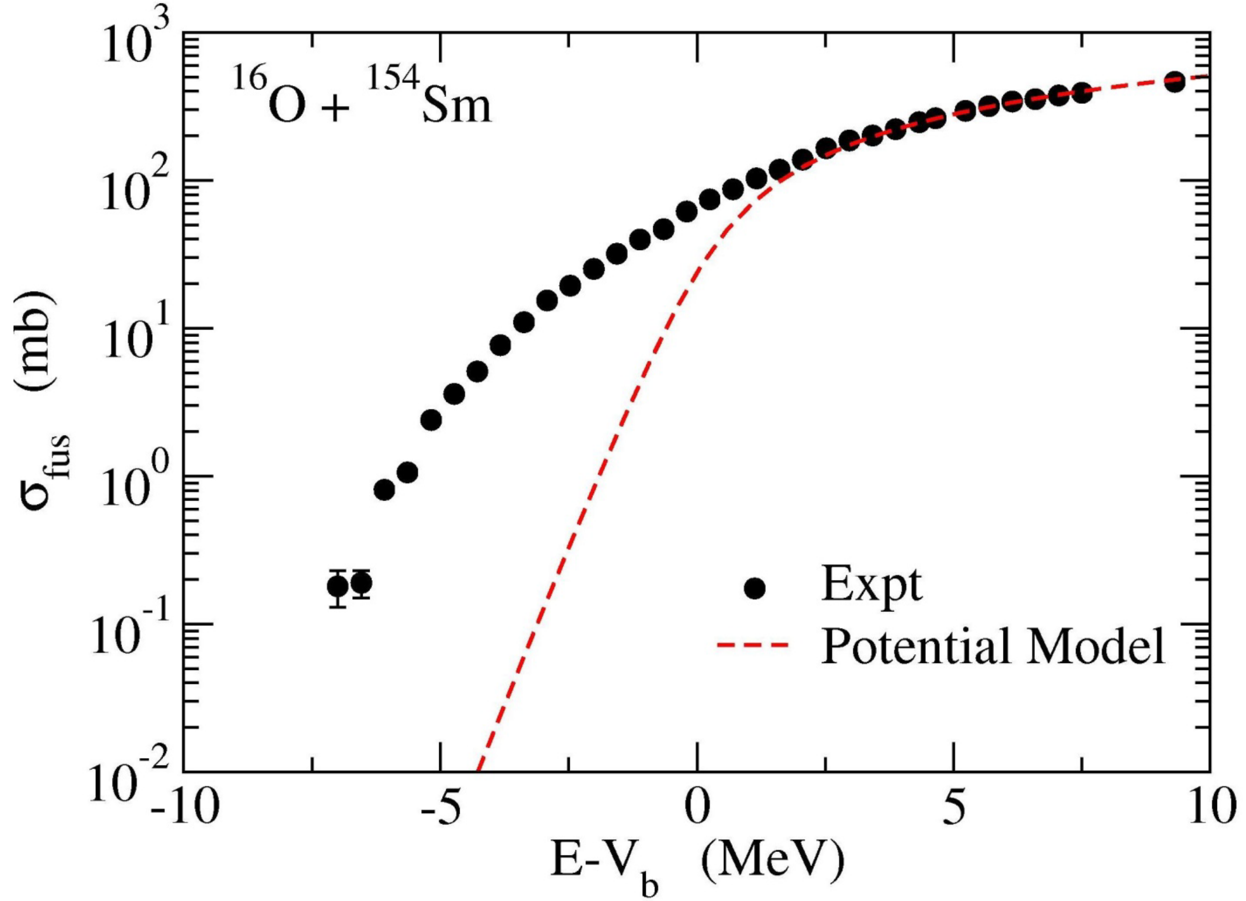
fusion reactions
in the sub-barrier energy region

$(|E - V_b| \lesssim 10\text{MeV})$

1. Coulomb force : long range, repulsive
2. Nuclear force : short range, attractive

⇨ Coulomb barrier

Discovery of large sub-barrier enhancement of σ_{fus}



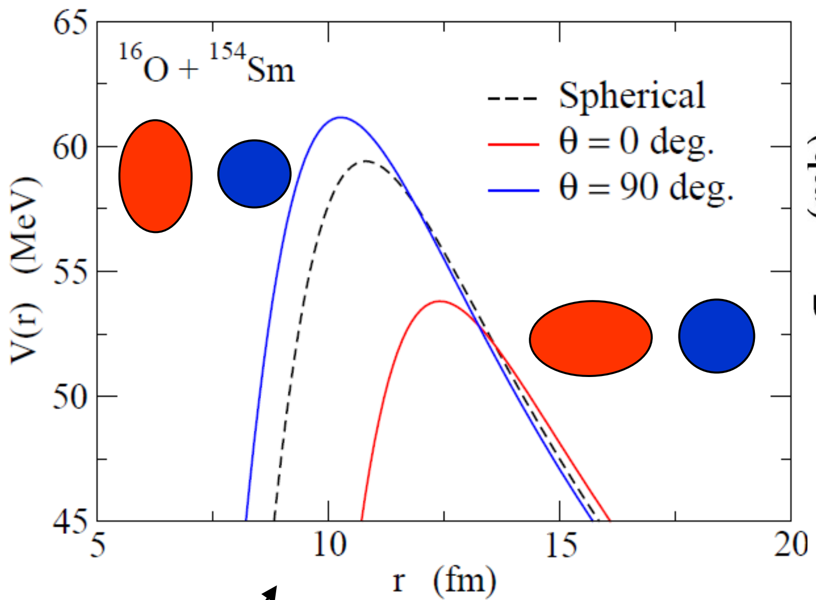
potential model: $V(r)$ + absorption

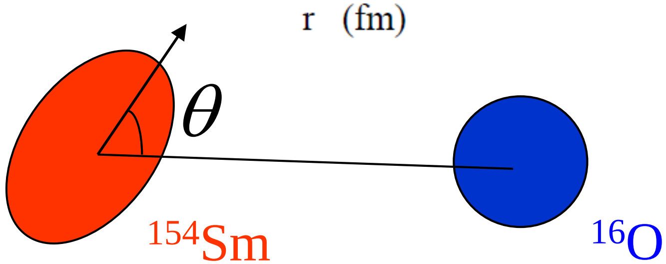
cf. seminal work:
R.G. Stokstad et al., PRL41('78) 465

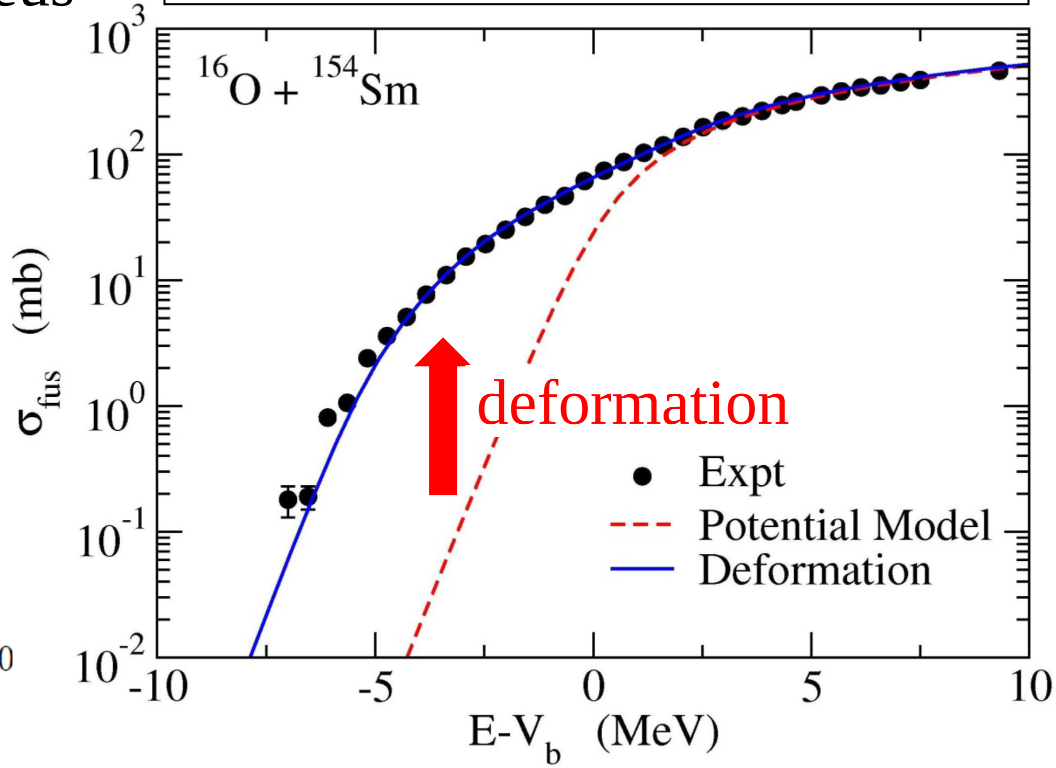
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus
with $\beta_2 \sim 0.3$

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\rm fus}(E;\theta)$$

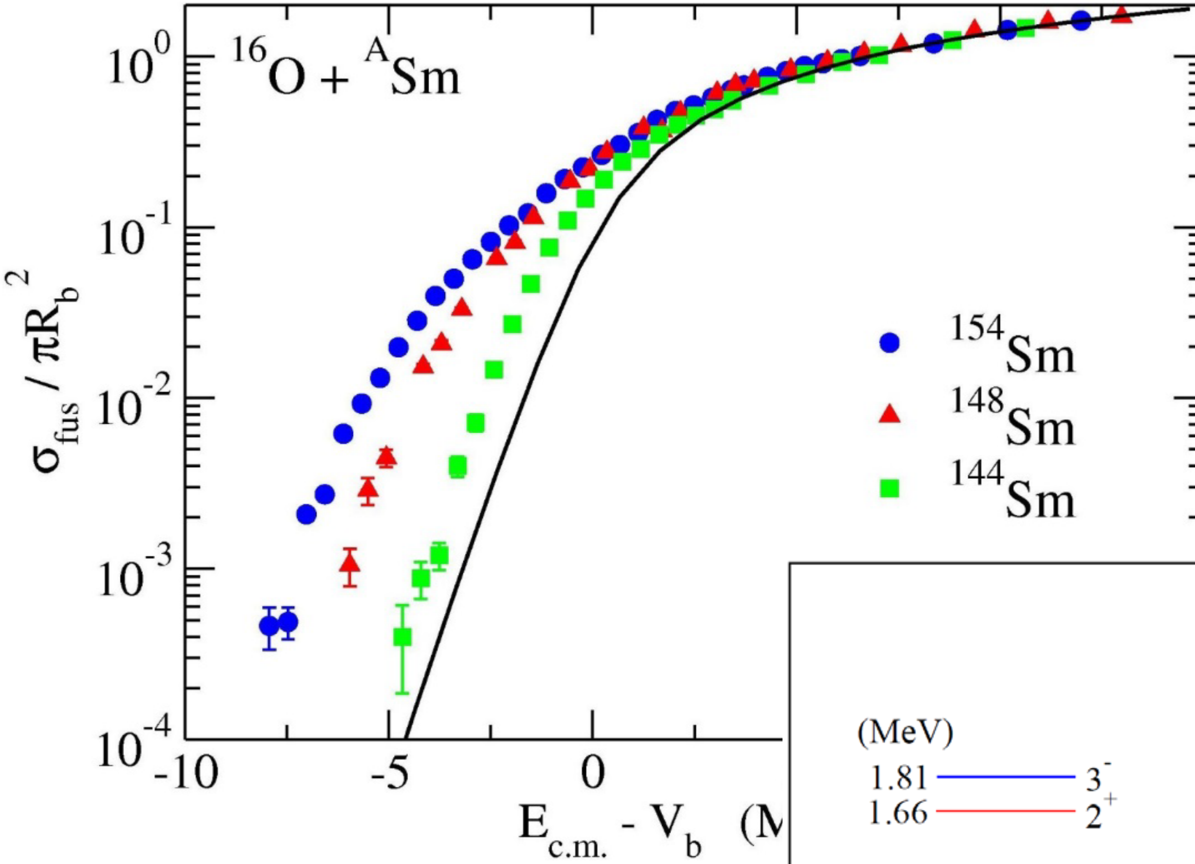


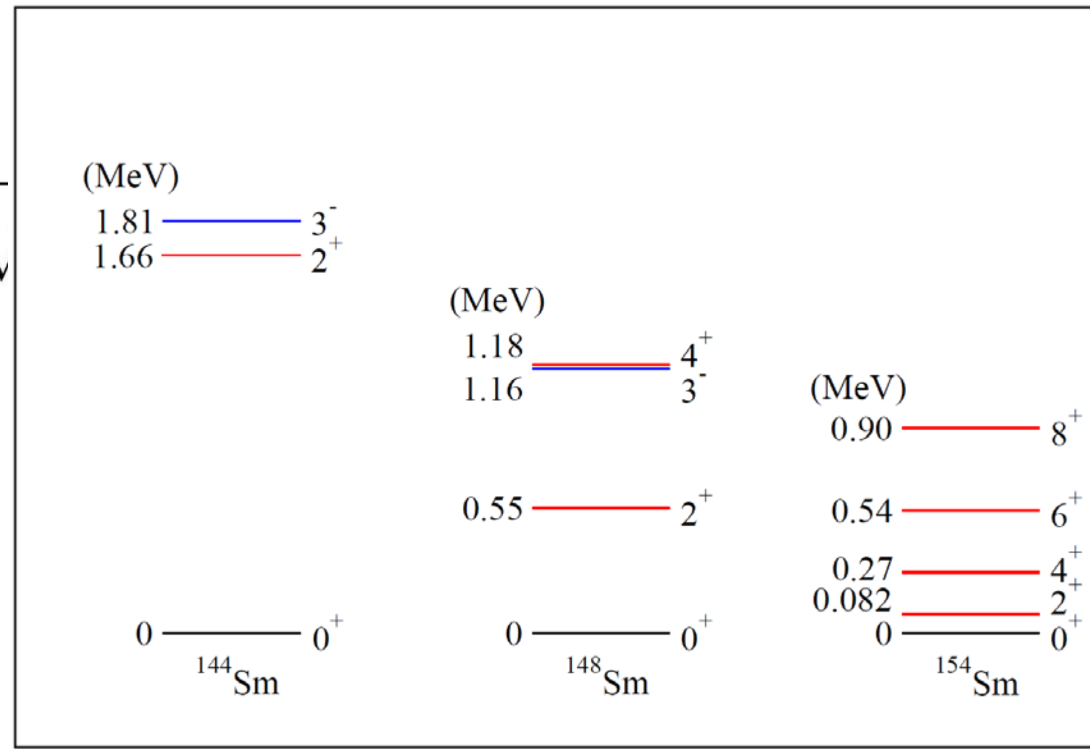




Fusion: strong interplay between nuclear structure and reaction

* Sub-barrier enhancement also in non-deformed systems:
couplings to low-lying collective excitations → coupling assisted tunneling

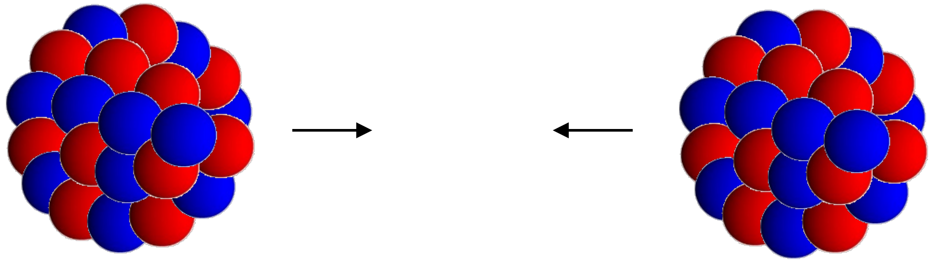




Strong target dependence
at $E < V_b$

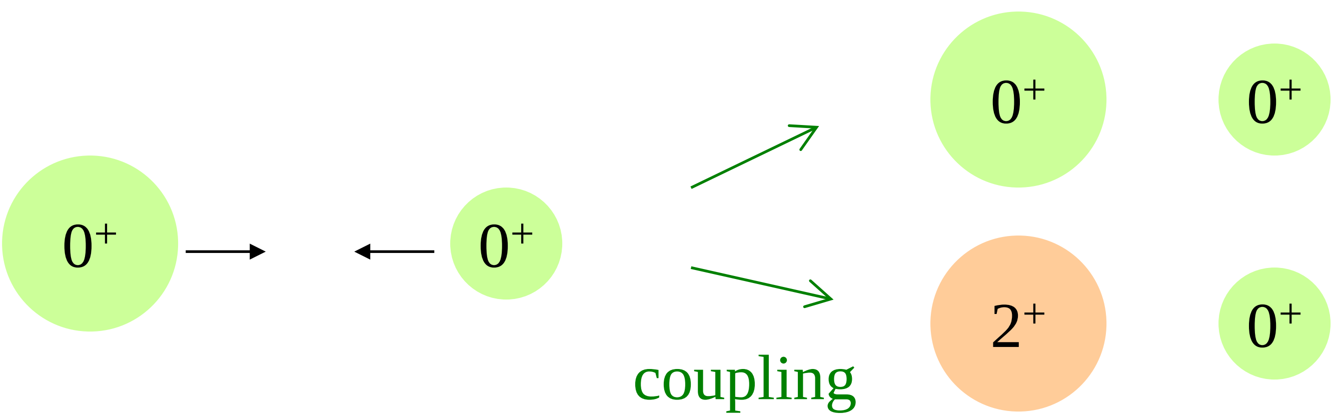
Coupled-Channels method

many-body problem

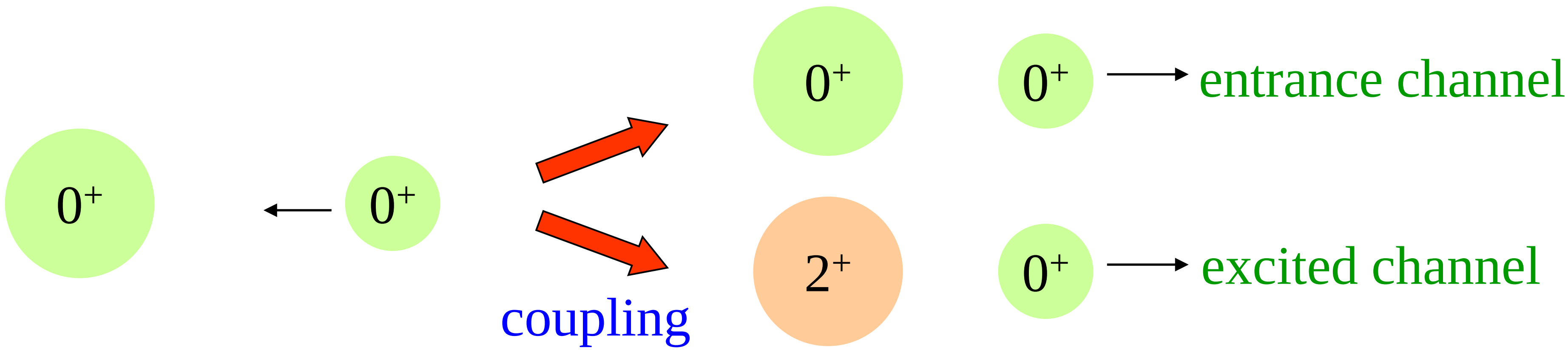


still very challenging

two-body problem, but with excitations
(coupled-channels approach)



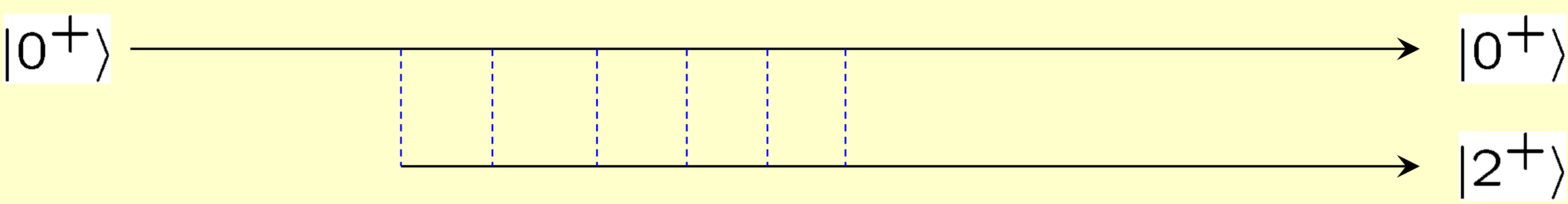
Coupled-channels method: a quantal scattering theory with excitations



$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|V_{\rm coup}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0$$

excitation energy (ϵ_k)

excitation operator ($V_{\rm coup}$)



full order treatment of excitation/de-excitation dynamics during reaction

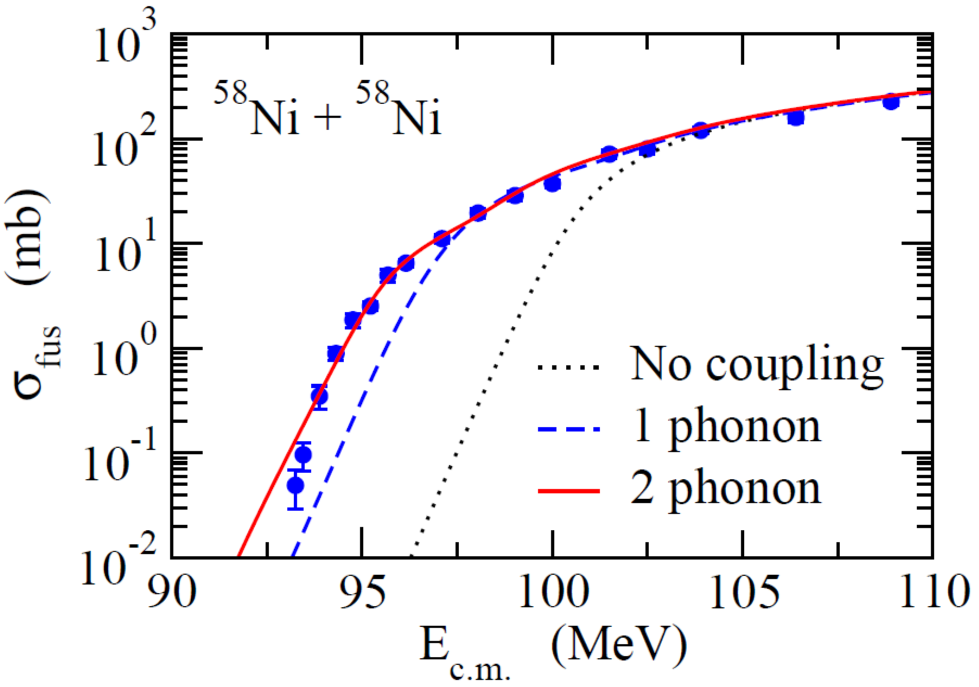
Inputs for C.C. calculations

i) Inter-nuclear potential

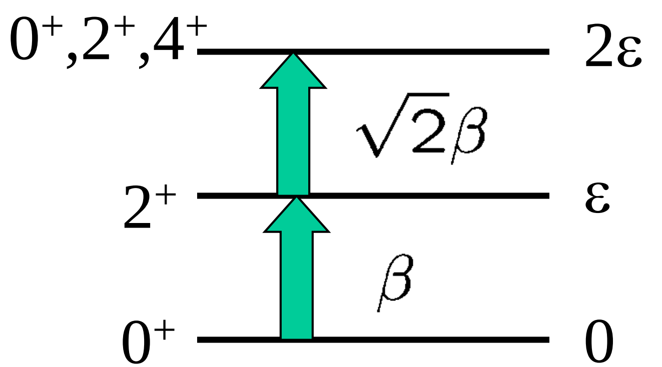
- ✓ a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom

- ✓ types of collective motions (rotation / vibration) a/o transfer
- ✓ coupling strengths and excitation energies
- ✓ how many states



simple harmonic oscillator



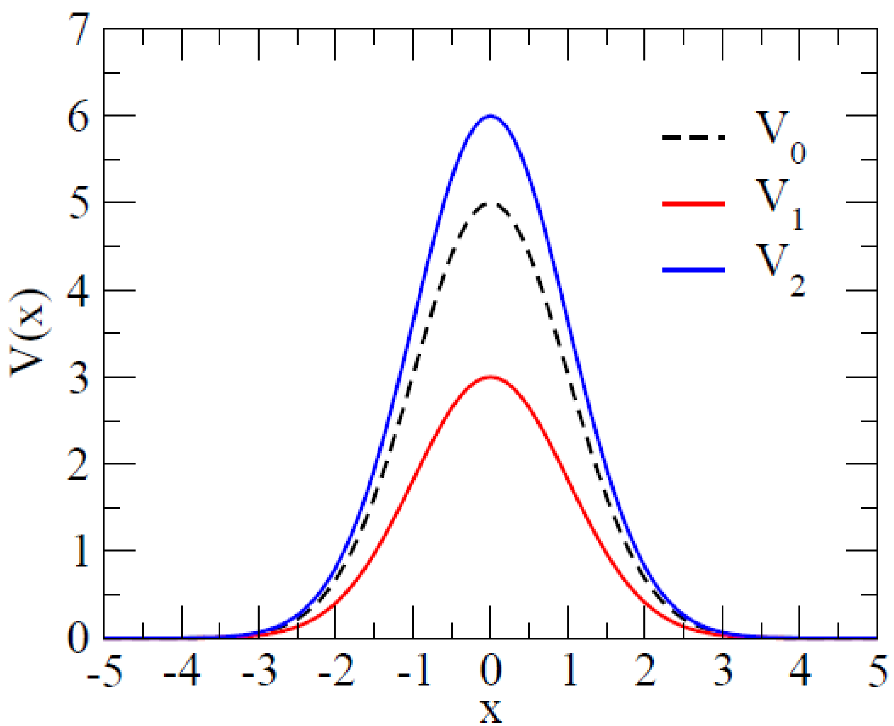
C.C. approach: a standard tool for sub-barrier fusion reactions

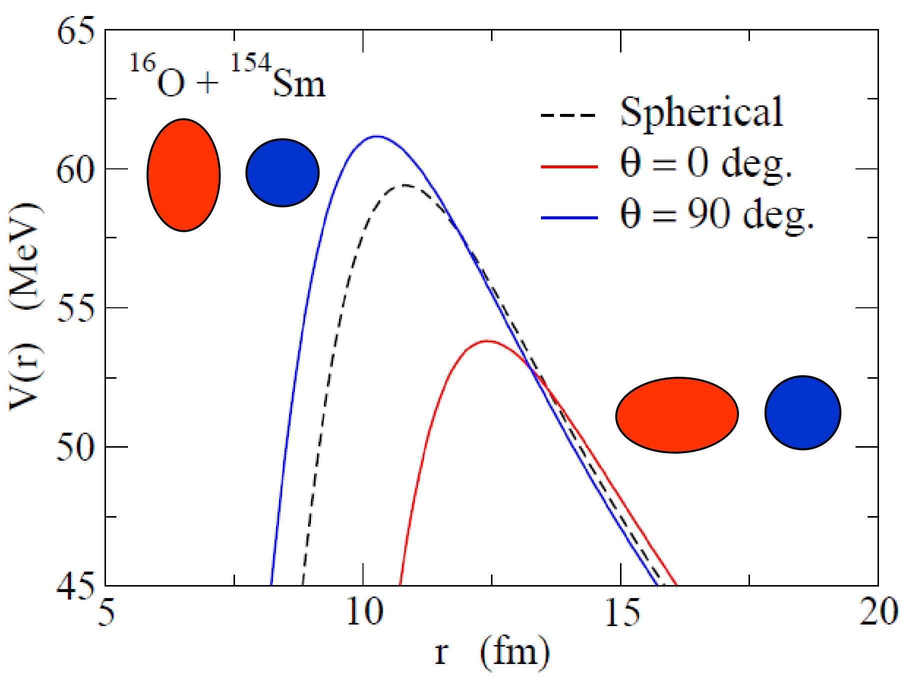
cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143)

✓ Eigen-channel representation of C.C.

$$\sigma_{\rm fus}(E) = \sum_k w_k \, \sigma_{\rm fus}(E; V_k)$$

many barriers are "distributed" due to the channel coupling effects





C.C. approach: a standard tool for sub-barrier fusion reactions

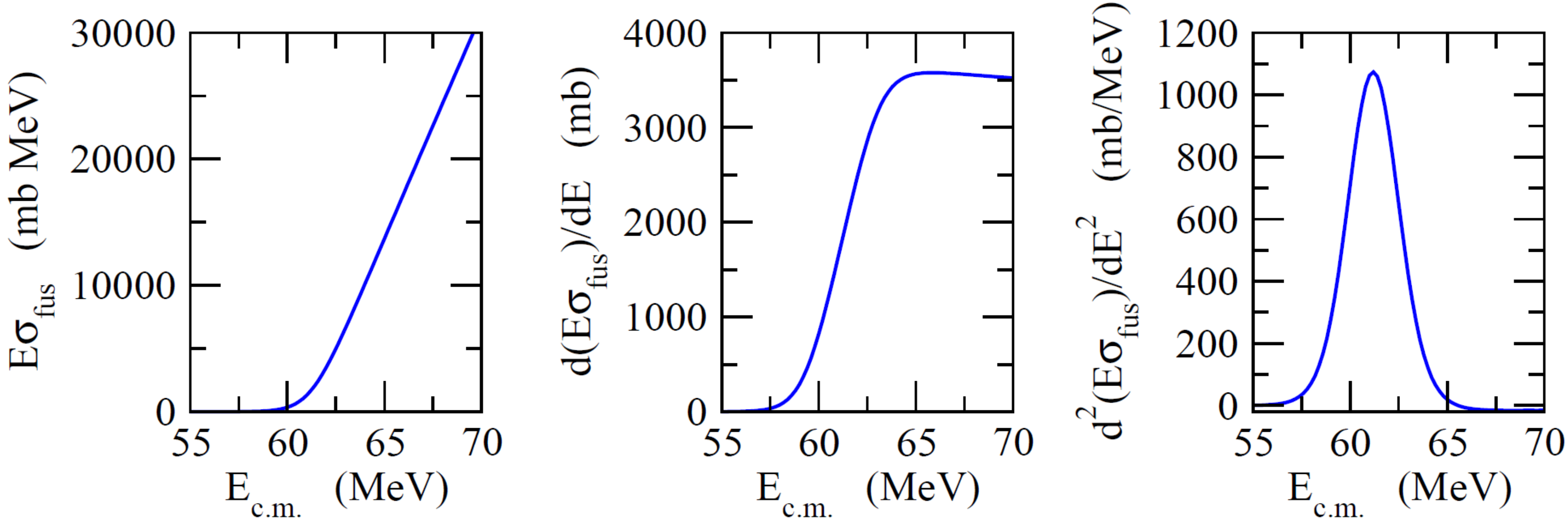
✓ Eigen-channel representation of C.C.

$$\sigma_{\rm fus}(E) = \sum_k w_k \, \sigma_{\rm fus}(E; V_k)$$

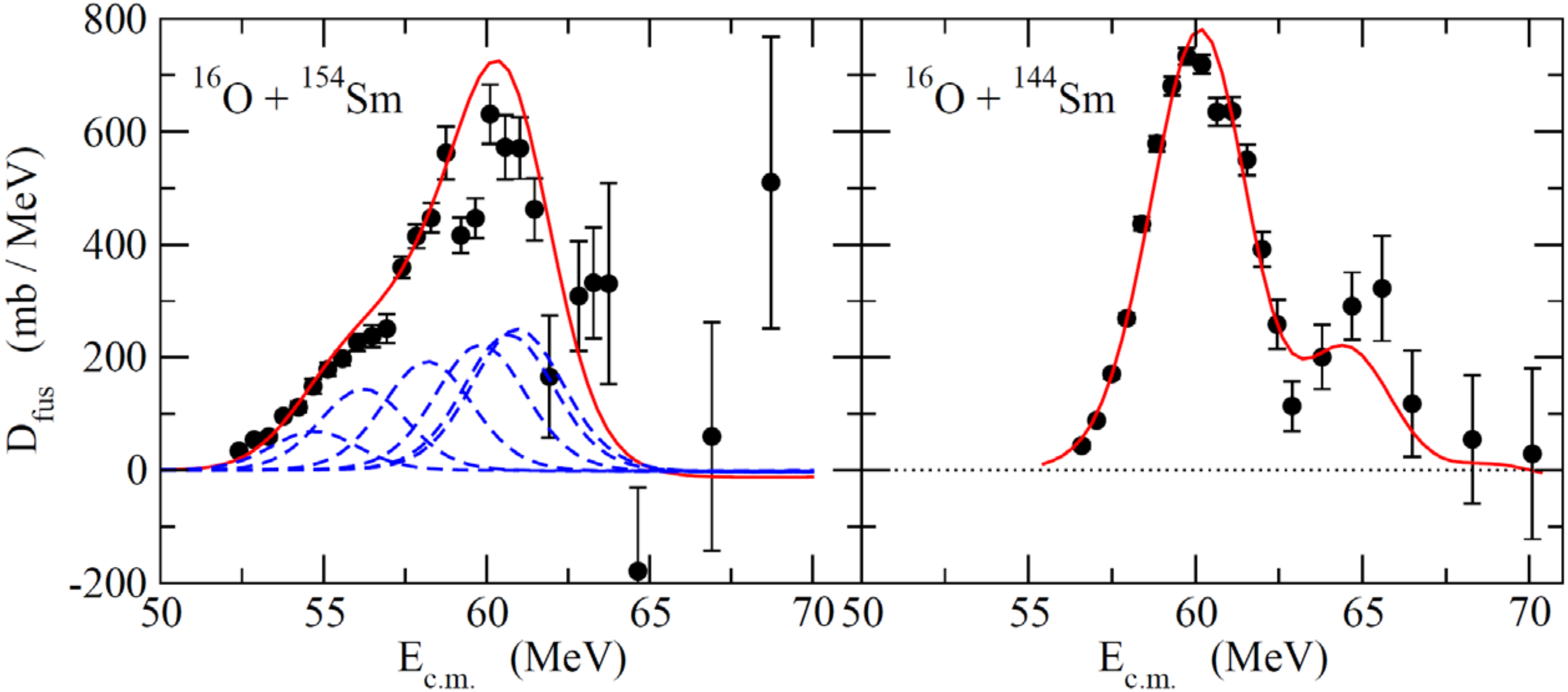
✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2} \qquad D^{(cl)}_{\rm fus}(E) = \sum_k w_k \, \delta(E - V_b^{(k)})$$

$\sim \pi R_b^2 \, P_{l=0}(E)$ $\quad$ $\sim \pi R_b^2 \cdot dP_{l=0}/dE$



$$D_{\mathsf{fus}}(E) = \sum_k w_k D_0(E; V_k)$$



sensitive to nuclear structure

- N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254('91) 25
- A.M. Stefanini et al., Phys. Rev. Lett. 74 ('95) 864
- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48('98)401

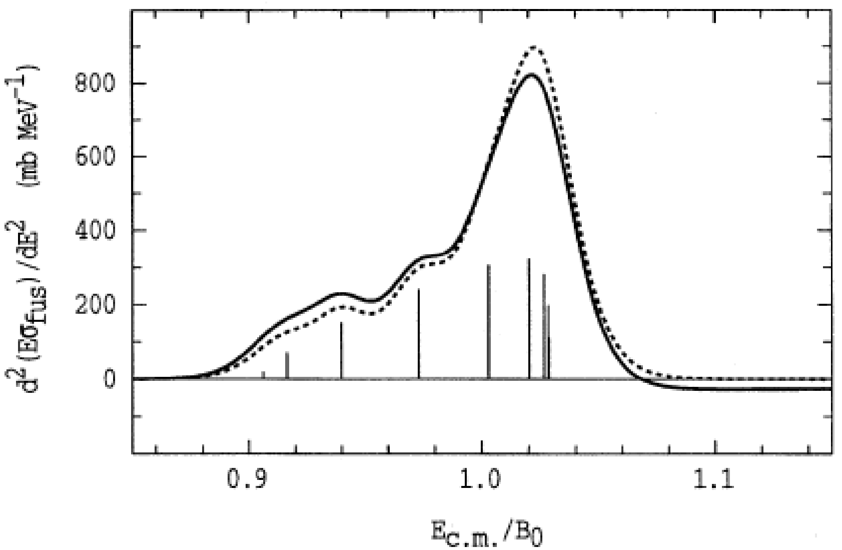
A Bayesian approach to fusion barrier distributions

<u>Fusion barrier distributions</u>

K.H., PRC93 ('16) 061601(R)

- Coupled-channels analyses
  - ✓ a standard approach
  - ✓ need to know the nature of collective excitations
- Direct fit to experimental data

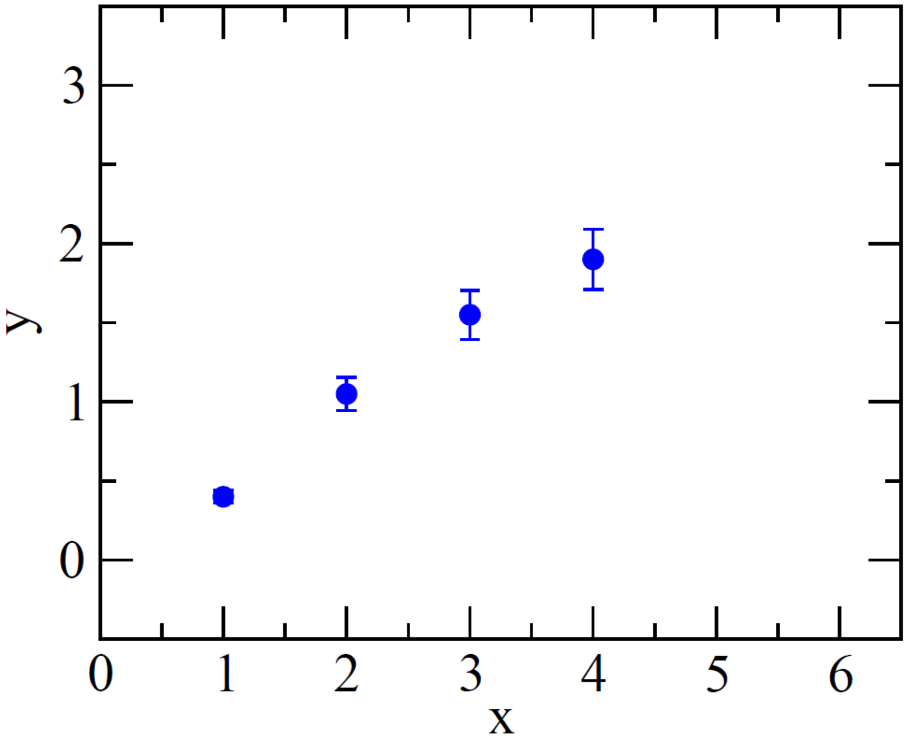
$$D_{\text{fus}}(E) = \sum_k w_k D_0(E; B_k, R_k, \hbar\Omega_k)$$



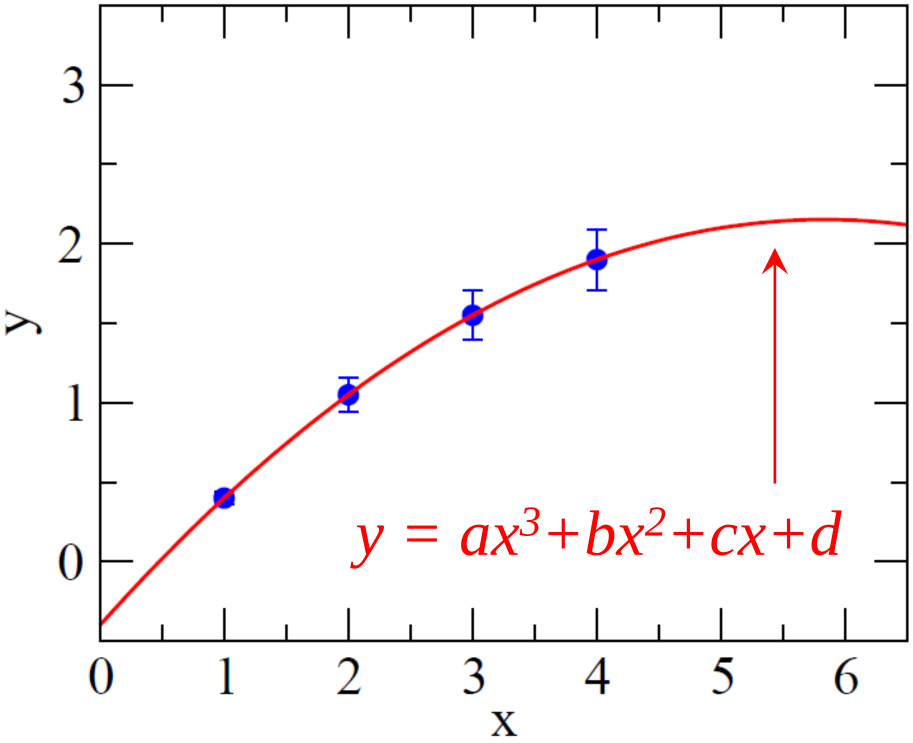
J.R. Leigh et al., PRC52 ('95) 3151

- ✓ phenomenological
- ✓ no need to know the nature of coll. excitations
- ✓ quick and convenient way
- ✓ mapping from D to T_l (cf. SHE)
- ✓ the number of barriers? (over-fitting problem)

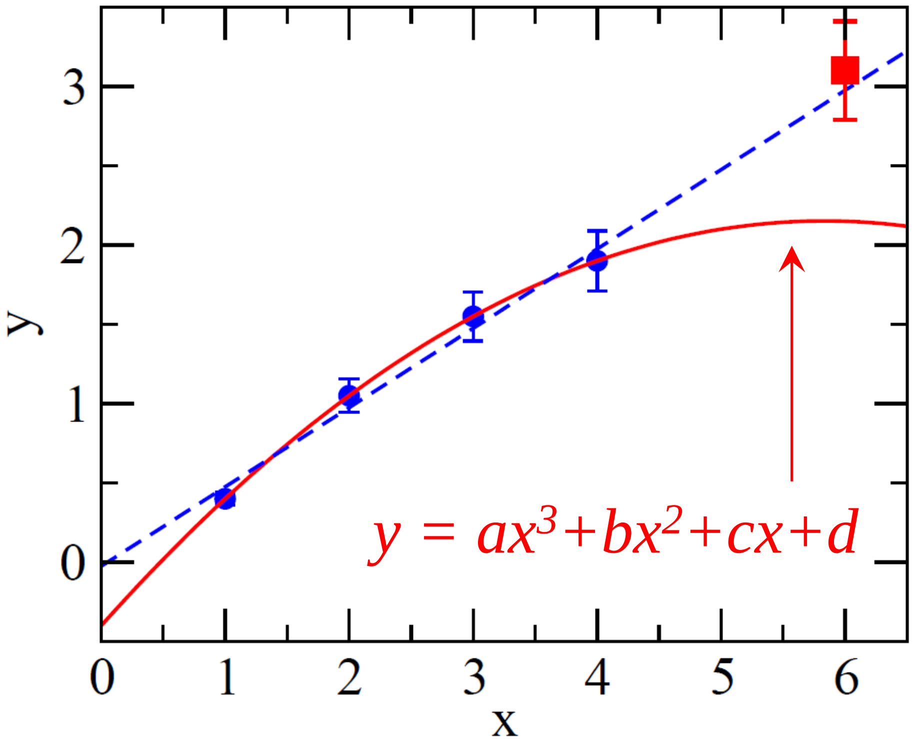
over-fitting problem

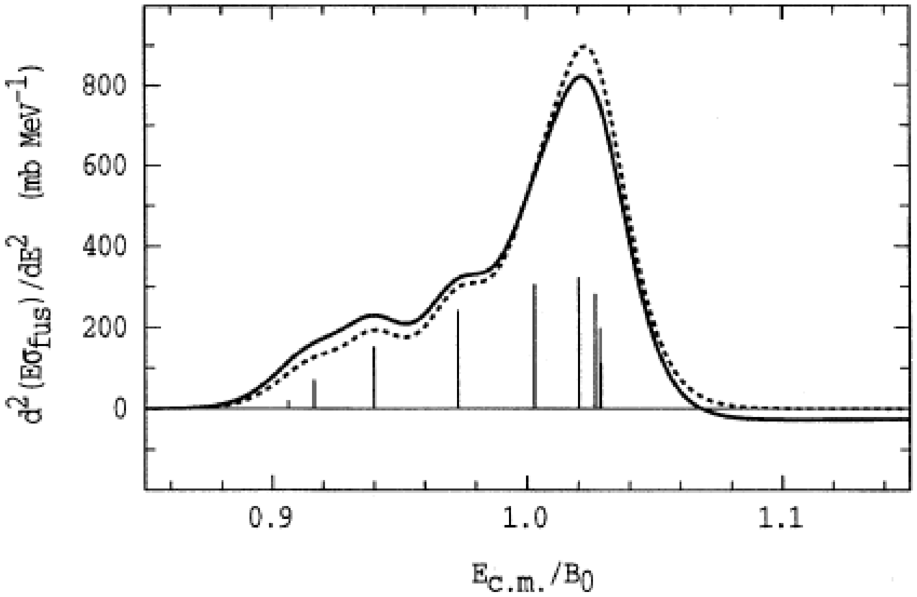


over-fitting problem



over-fitting problem





J.R. Leigh et al., PRC52 (‘95) 3151

one can make χ^2 small
by increasing the number of barriers

how many barriers?

Bayesian statistics

- ✓ data set: $D_{exp} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
- ✓ fit with $d_{model}(E; a)$ a: a model parameter

Bayes theorem

$$P(a|D_{\text{exp}}) \propto P(D_{\text{exp}}|a)P(a)$$

$P(a)$: a prior probability of a
(a guess distribution before experiment)

$P(D_{exp}|a)$: a probability to realize D_{exp} when a is given

$$P(D_{\text{exp}}|a) \propto \exp\left[-\frac{1}{2}\sum_i\left(\frac{d_i - d_{\text{model}}(E_i;a)}{\delta d_i}\right)^2\right]$$

$P(a|D_{exp})$: a posterior probability of a
(an updated distribution after knowing the data)

Bayesian statistics

Bayes theorem

$$P(a|D_{\rm exp}) \propto P(D_{\rm exp}|a)P(a)$$

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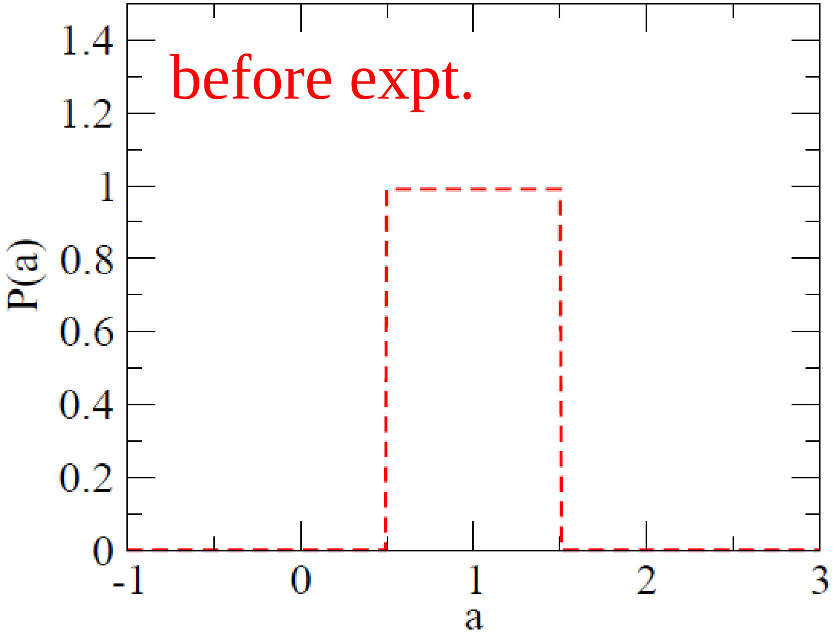
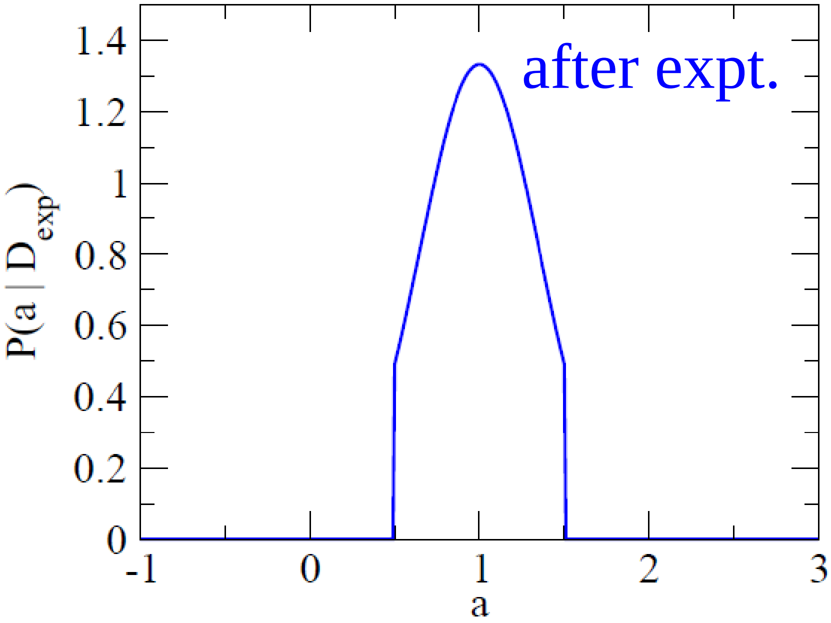


update



Bayesian spectrum deconvolution

K. Nagata, S. Sugita, and M. Okada, Neural Networks 28 ('12) 82

- ✓ data set: $D_{exp} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
- ✓ fitting function: $D_{\rm fit}(E;\tilde{\boldsymbol{\theta}},K) = \sum_{k=1}^{K} w_k\,\phi_k(E;\boldsymbol{\theta}_k), \qquad \tilde{\boldsymbol{\theta}} \equiv \{w_k, \boldsymbol{\theta}_k\}$

K: the number of barriers

Bayes theorem

$$P(K|D_{\rm exp}) \propto P(D_{\rm exp}|K)P(K)$$

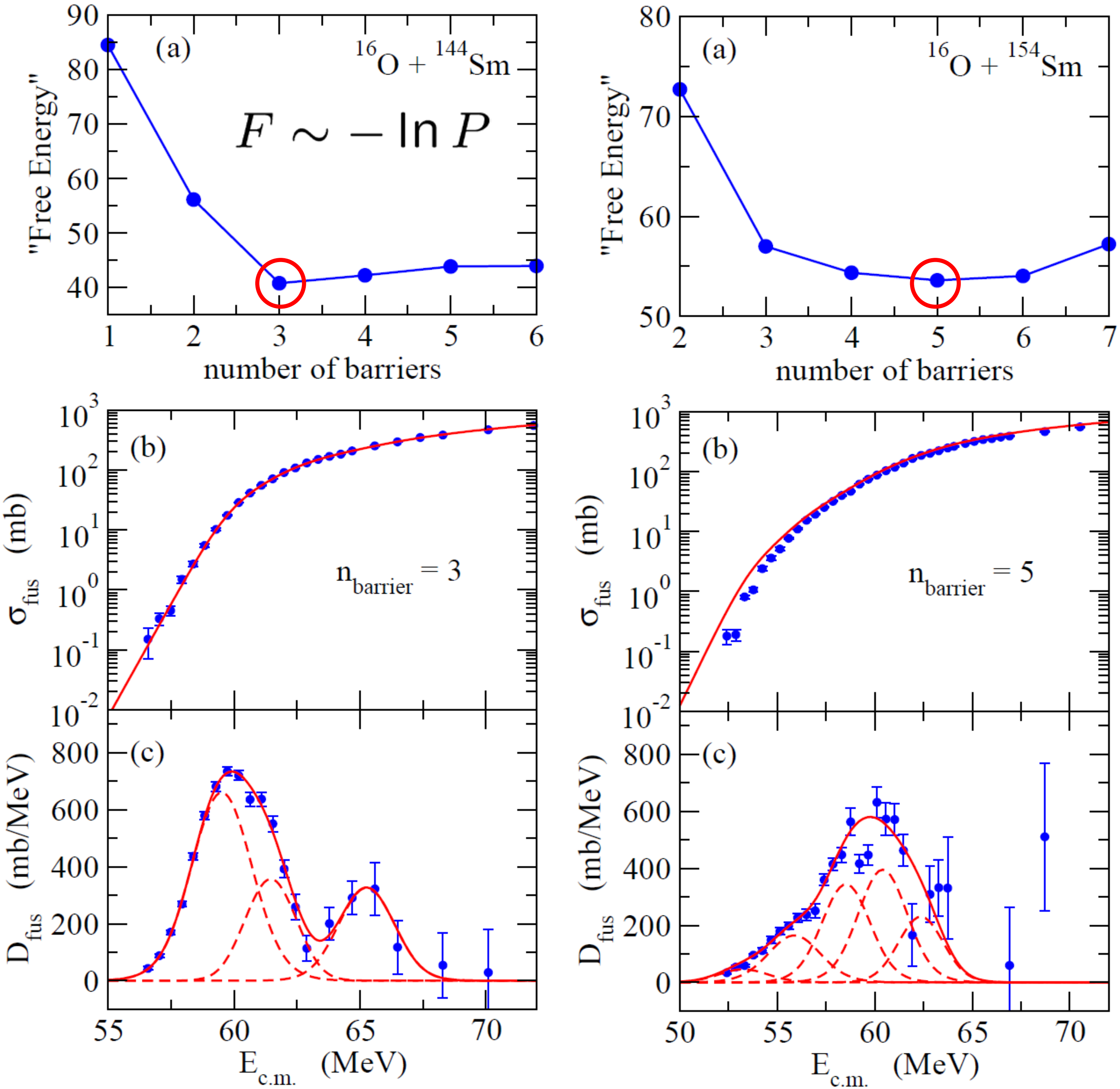
$$\propto P(D_{\rm exp}|K) = \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}},K)/2} P(\tilde{\boldsymbol{\theta}})$$

$$\chi^2(\tilde{\boldsymbol{\theta}},K) = \sum_{i=1}^{M}\left(\frac{d_i - D_{\rm fit}(E_i;\tilde{\boldsymbol{\theta}},K)}{\delta d_i}\right)^2$$

most probable value of K: maximize $P(K|D_{exp})$

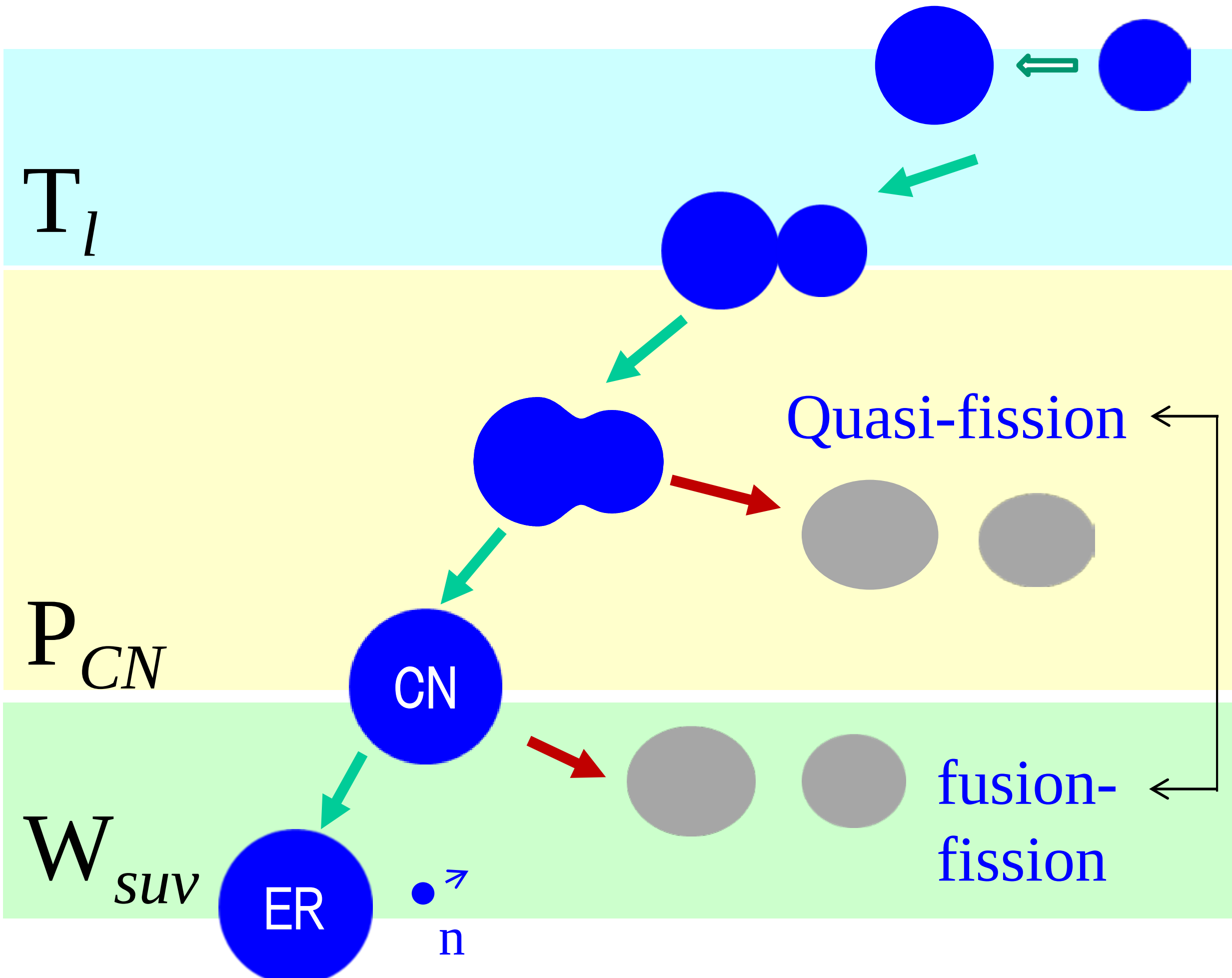
or, equivalently, minimize $F = -\ln P(K|D_{exp})$

⟶ optimize the other parameters for a given value of *K*



K.H., PRC93 ('16) 061601(R)

Future perspective: application to SHE formation reactions



Coupled-channels

Langevin approach

statistical model

CN = compound nucleus
ER = evaporation residue

$$\sigma_{\rm ER}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E)\,P_{\rm CN}(E,l)\,W_{\rm suv}(E^*,l)$$

T_l from σ_{cap}?

Bayesian approach to σ_{ER}

$$D_{\mathrm{exp}}(E) = \sum_{i=1}^{K} w_k D_0(E; V_k(r))$$

either D_{fus} or D_{qel}

$$\Rightarrow \quad T_l = \sum_{k=1}^{K} w_k T_l(E; V_k(r))$$

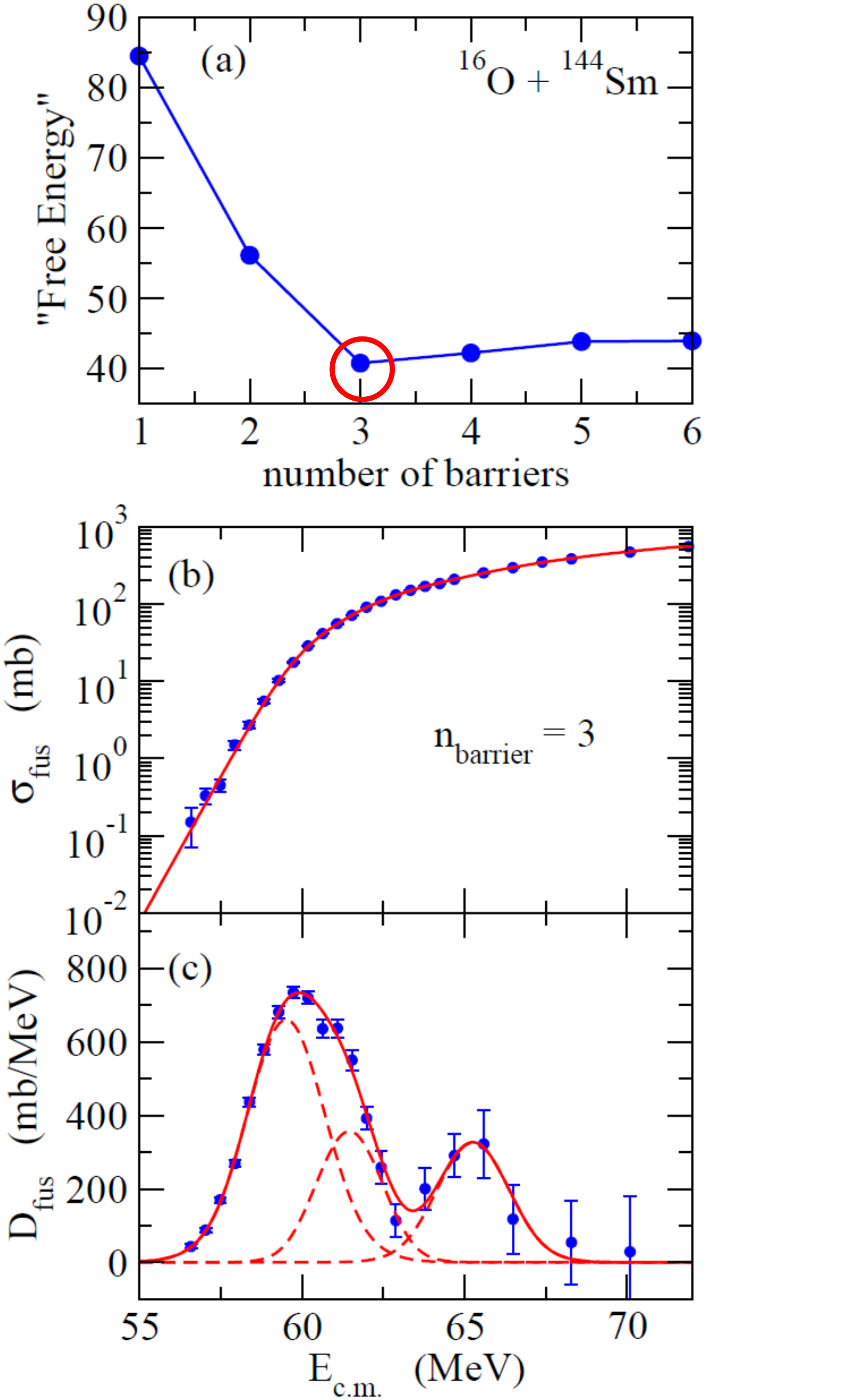
* no need to know the details of the couplings

\+

Langevin + stat. model calculations

$$\sigma_{\mathrm{ER}}(E) = \frac{\pi}{k^2}\sum_l (2l+1) T_l(E) \times P_{\mathrm{CN}}(E,l)\, W_{\mathrm{suv}}(E^*, l)$$

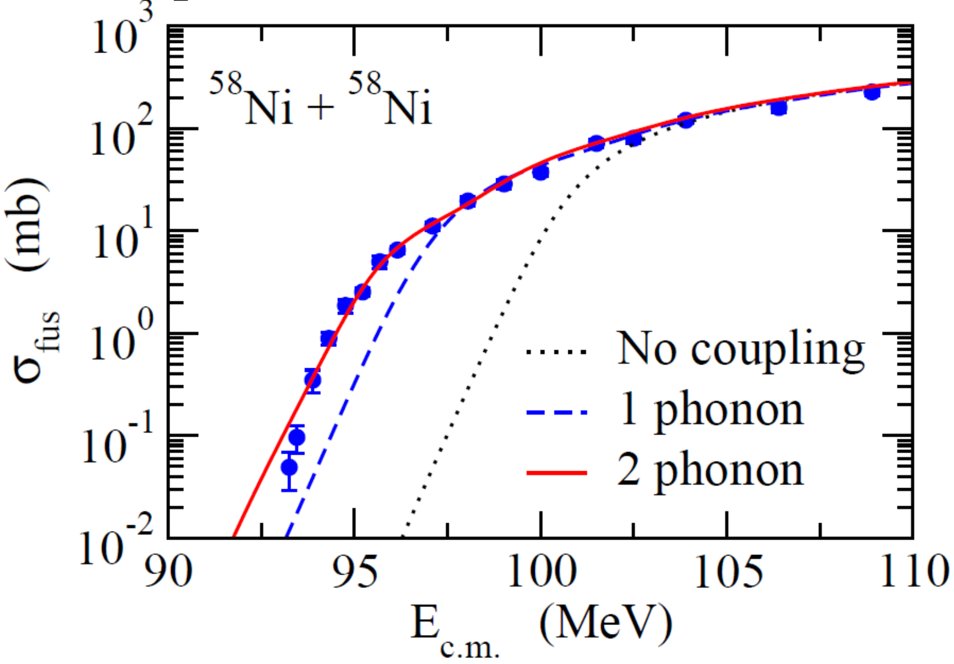
superheavy elements

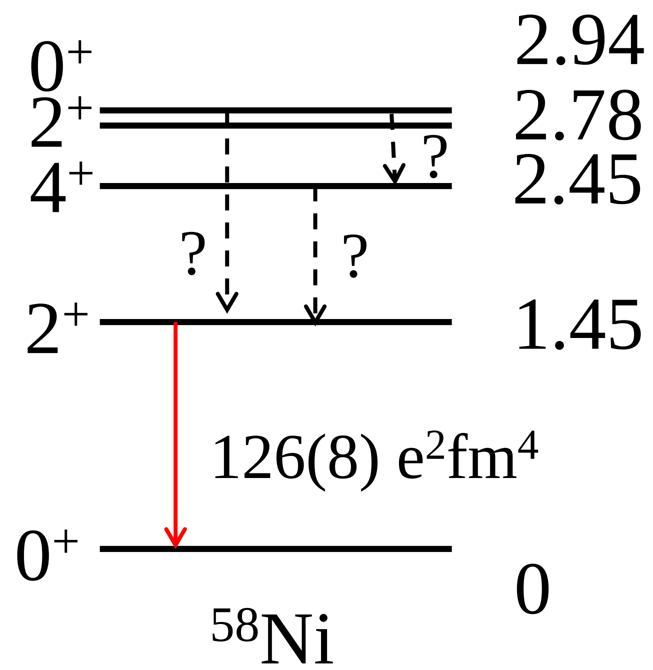


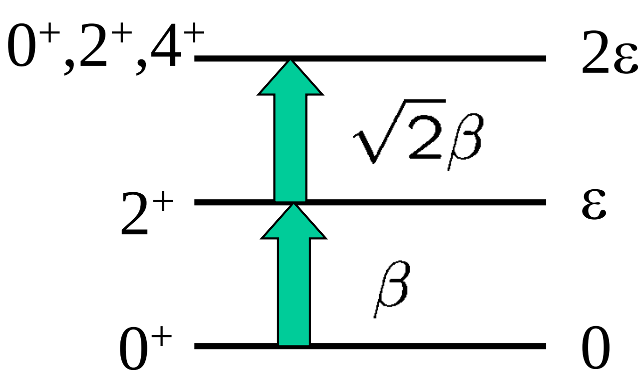
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15) 064606

multi-phonon excitations



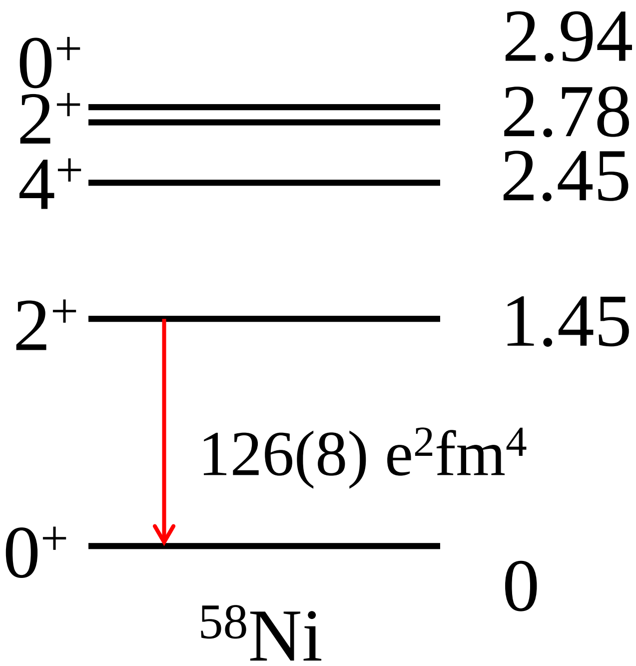




Simple harmonic oscillator
→ justifiable?

Often data available only for the 1st excited state

Anharmonic vibrations



$Q(2_1^+) = -10 \text{ +/- } 6\ e\text{fm}^2$

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

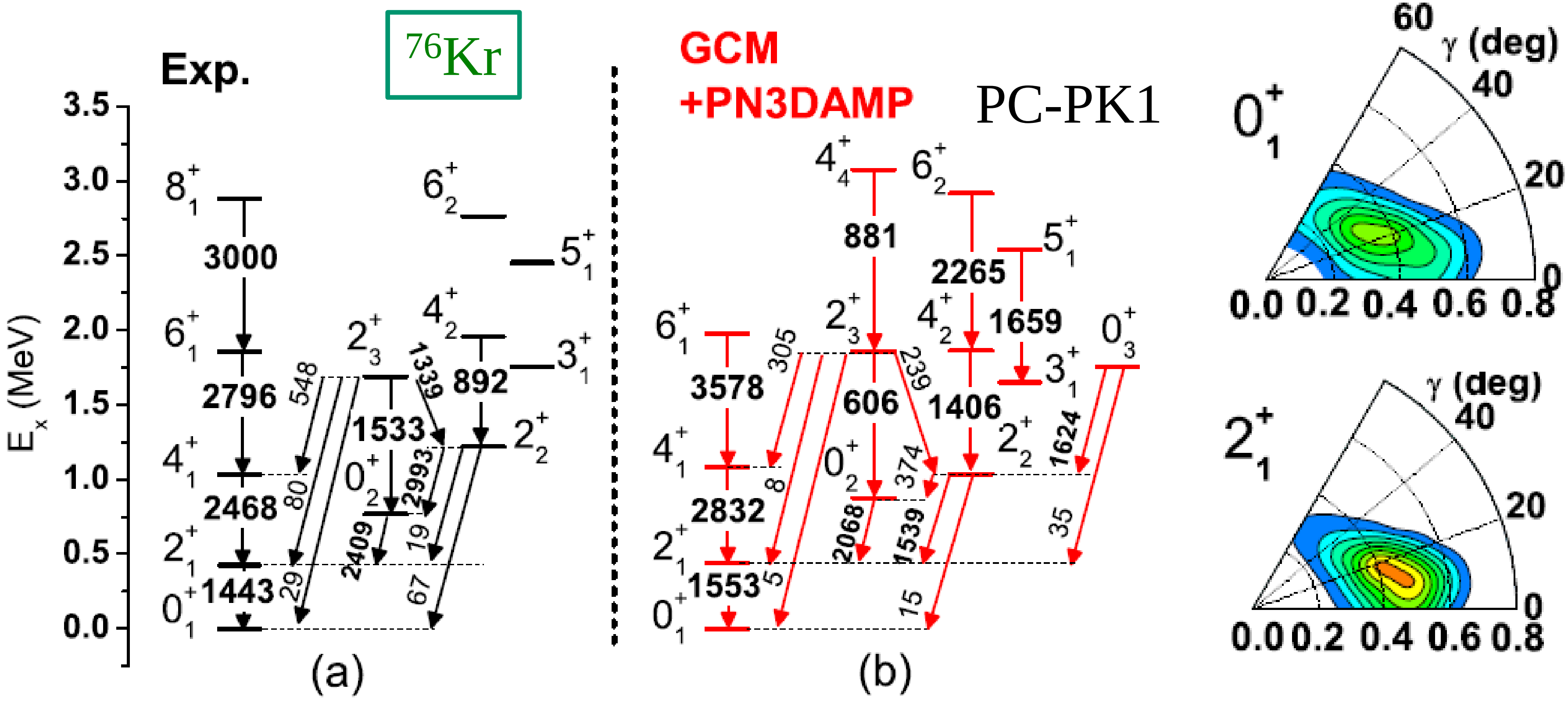
$$|JM\rangle = \int d\beta\, f_J(\beta)\hat{P}^J_{M0}|\Phi(\beta)\rangle$$

✓ MF + ang. mom. projection
+ particle number projection
+ generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121
J.M. Yao et al., PRC89 (‘14) 054306

beyond mean-field approximation

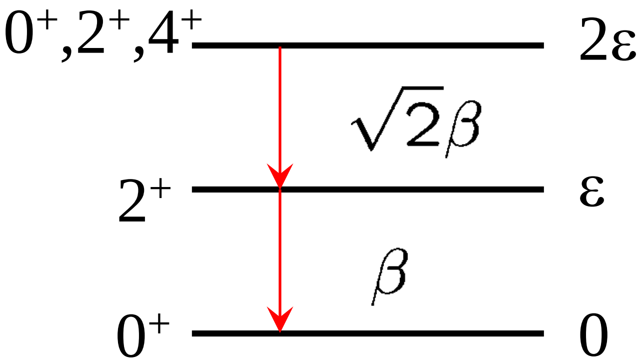
- ✓ angular momentum + particle number projections
- ✓ quantum fluctuation (GCM)

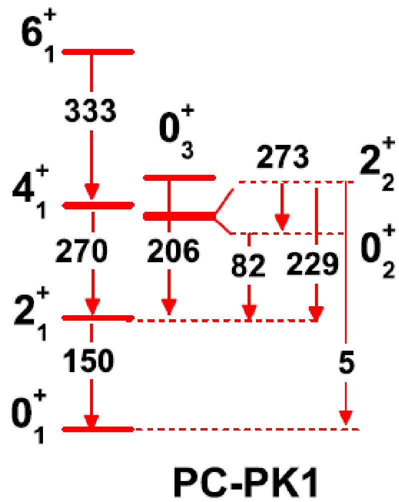


J.M. Yao, K.H., Z.P. Li, J. Meng, and P. Ring, PRC89 ('14) 054306

Semi-microscopic coupled-channels model for sub-barrier fusion

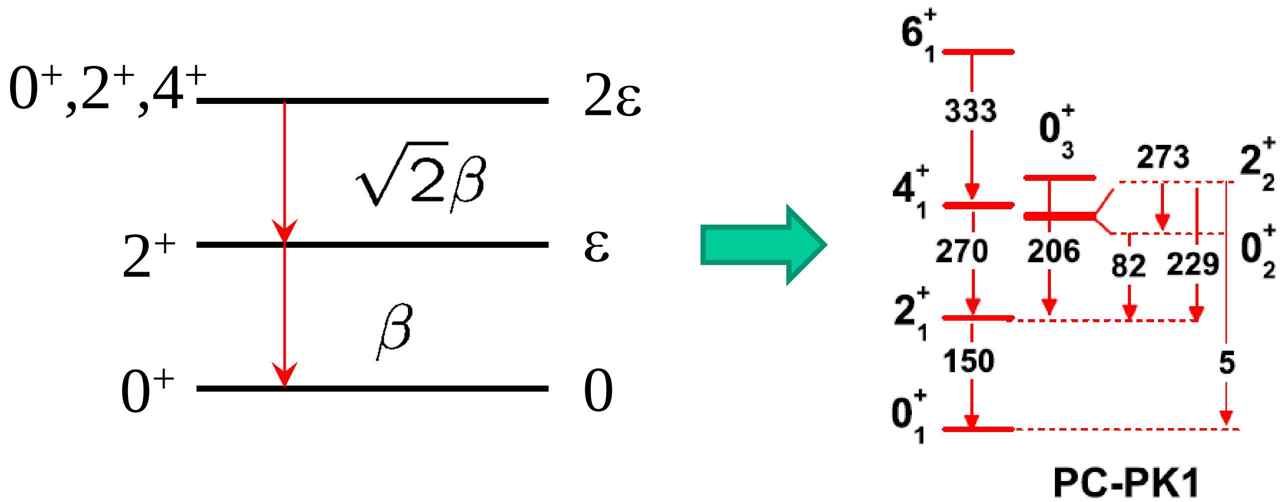
K.H. and J.M. Yao, PRC91 ('15) 064606





Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 (‘15) 064606



microscopic multi-pole operator

✓ $$V_{\rm coup} \sim -R_T\frac{dV_N}{dr}\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}) \to -R_T\frac{dV_N}{dr}Q_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}})$$

✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states

✓ scale to the empirical B(E2; $2_1^+ \to 0_1^+$)

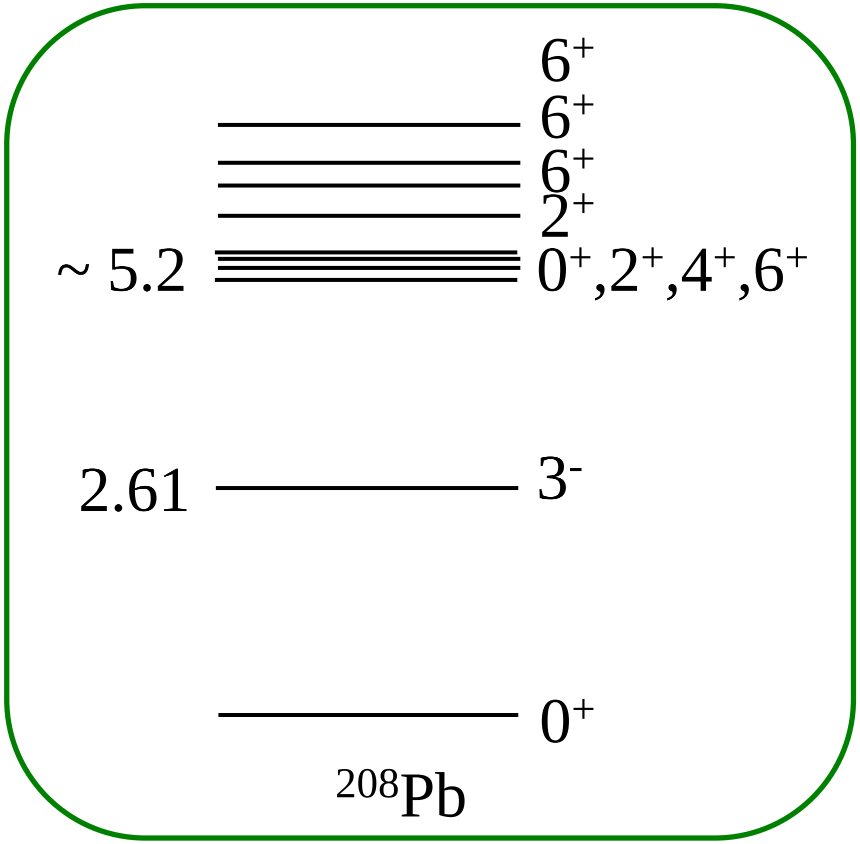
✓ still use a phenomenological potential

✓ use the experimental values for E_x

* axial symmetry (no 3^+ state)

Application to $^{16}O + ^{208}Pb$ fusion reaction

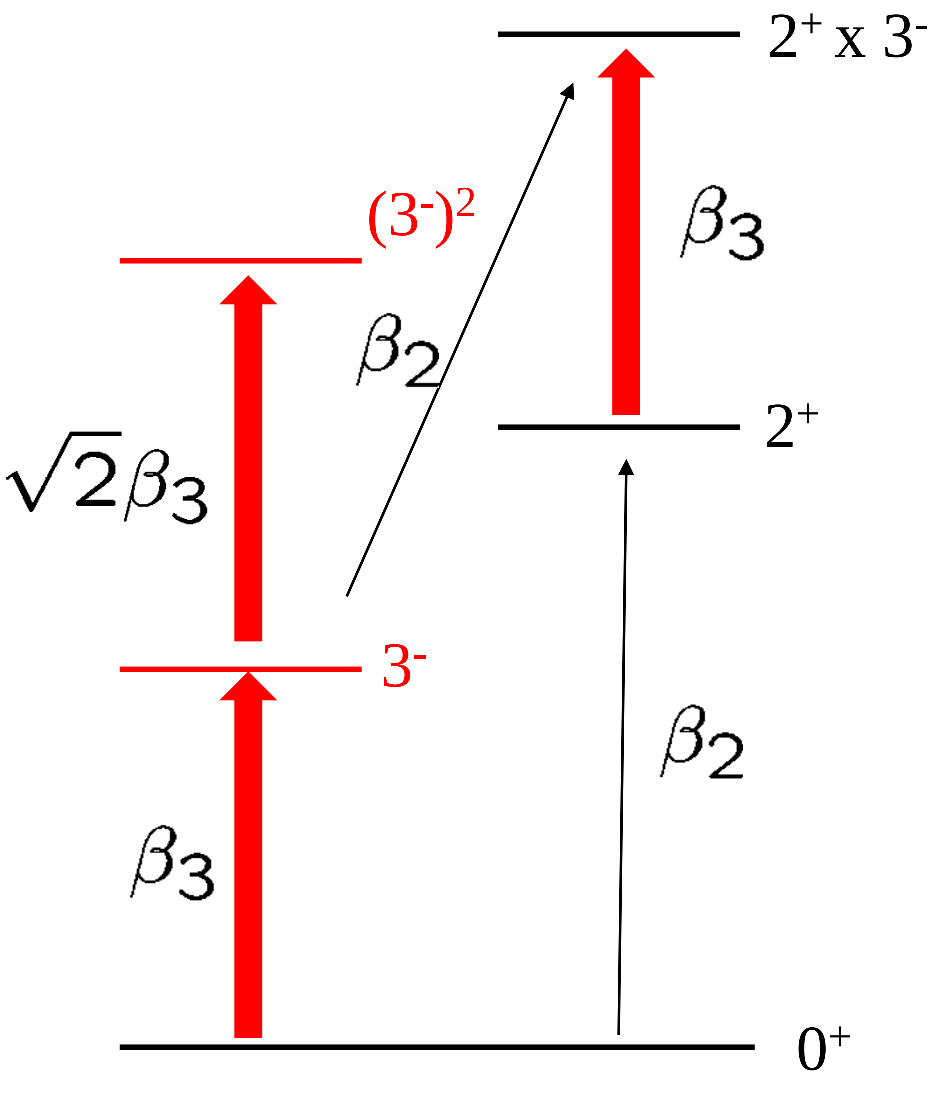
double-octupole phonon states in ^{208}Pb

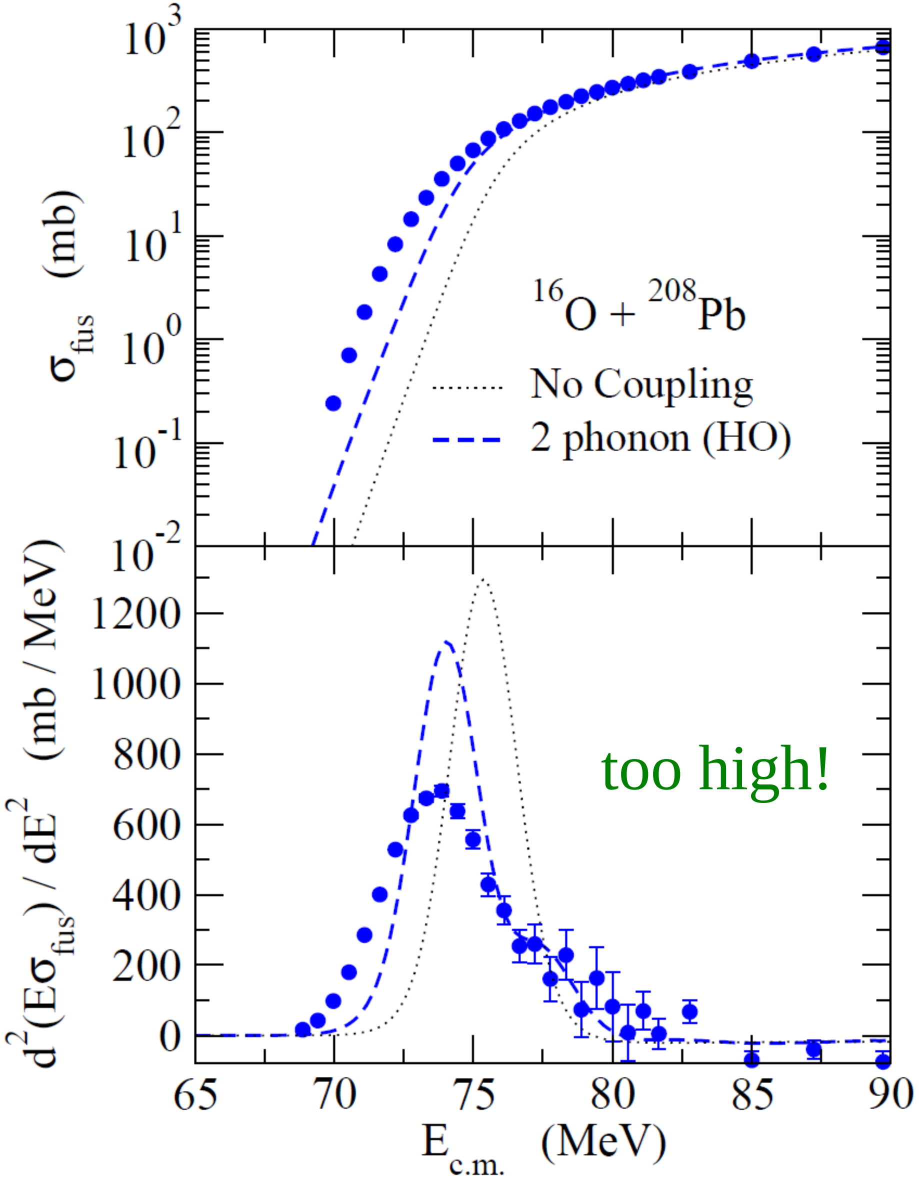


M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631
V. Yu. Pnomarev and P. von Neumann-Cosel, PRL82 (‘99) 501
B.A. Brown, PRL85 (‘00) 5300

large fragmentations, especially 6^+ state

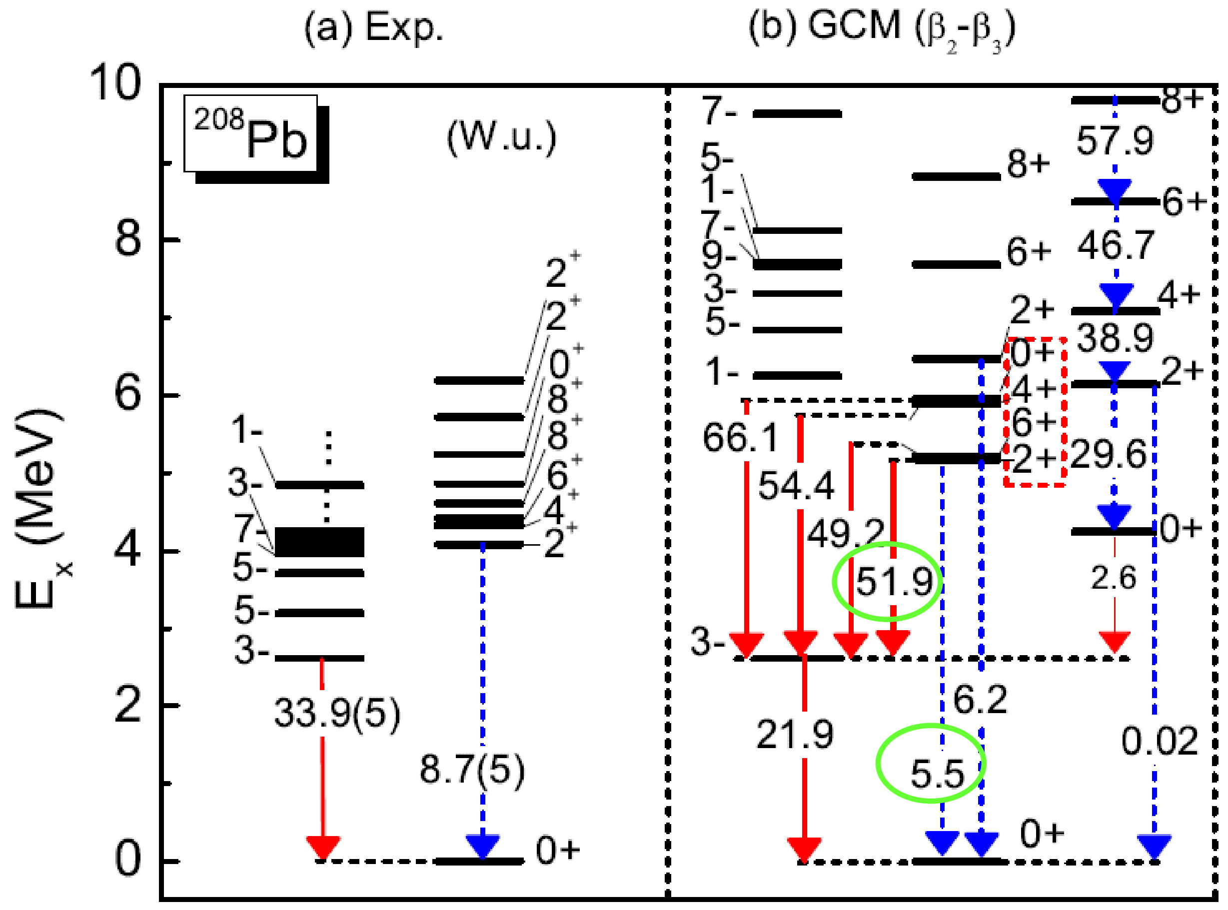
Application to $^{16}O + ^{208}Pb$ fusion reaction





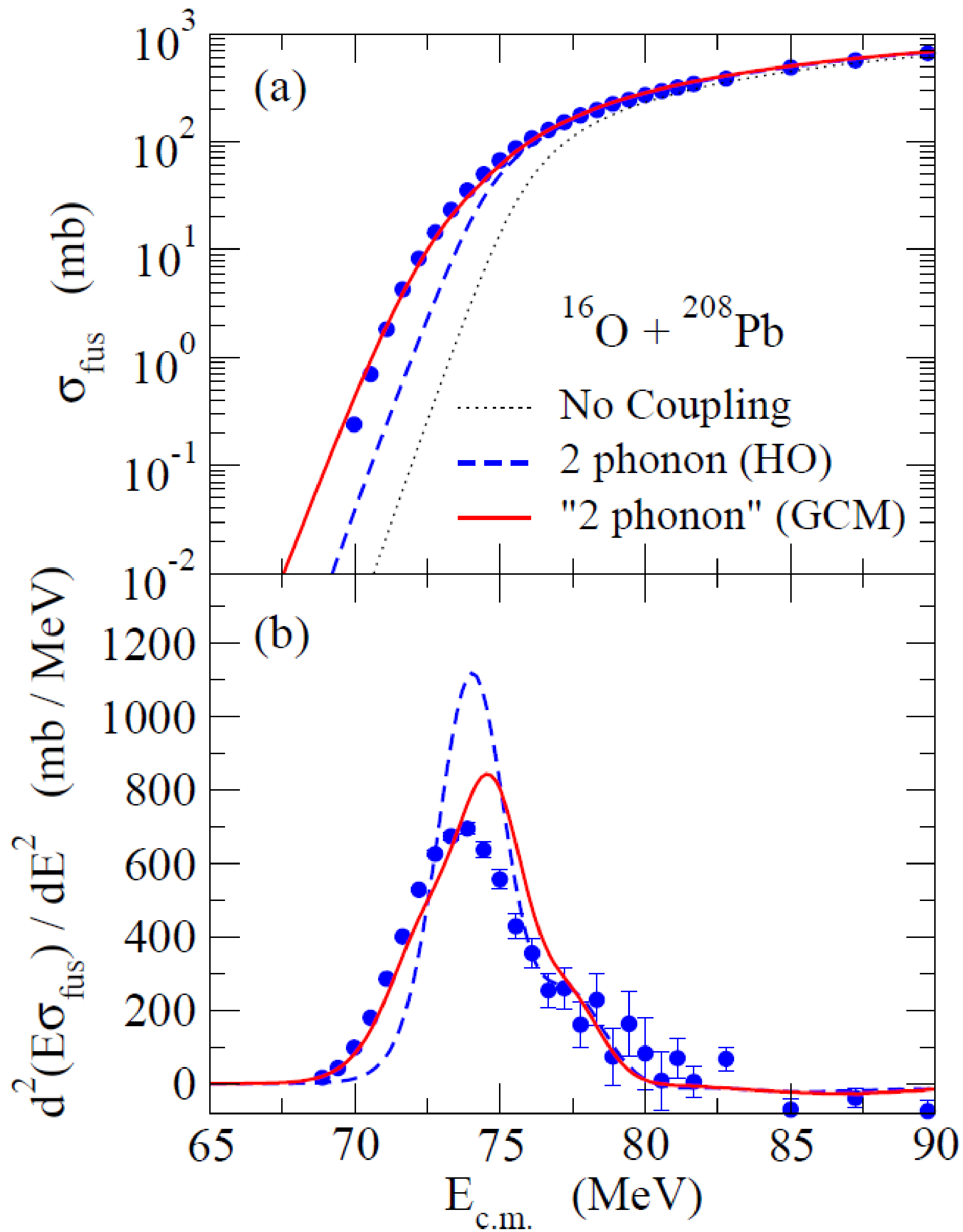
cf. C.R. Morton et al., PRC60('99) 044608 expt. data

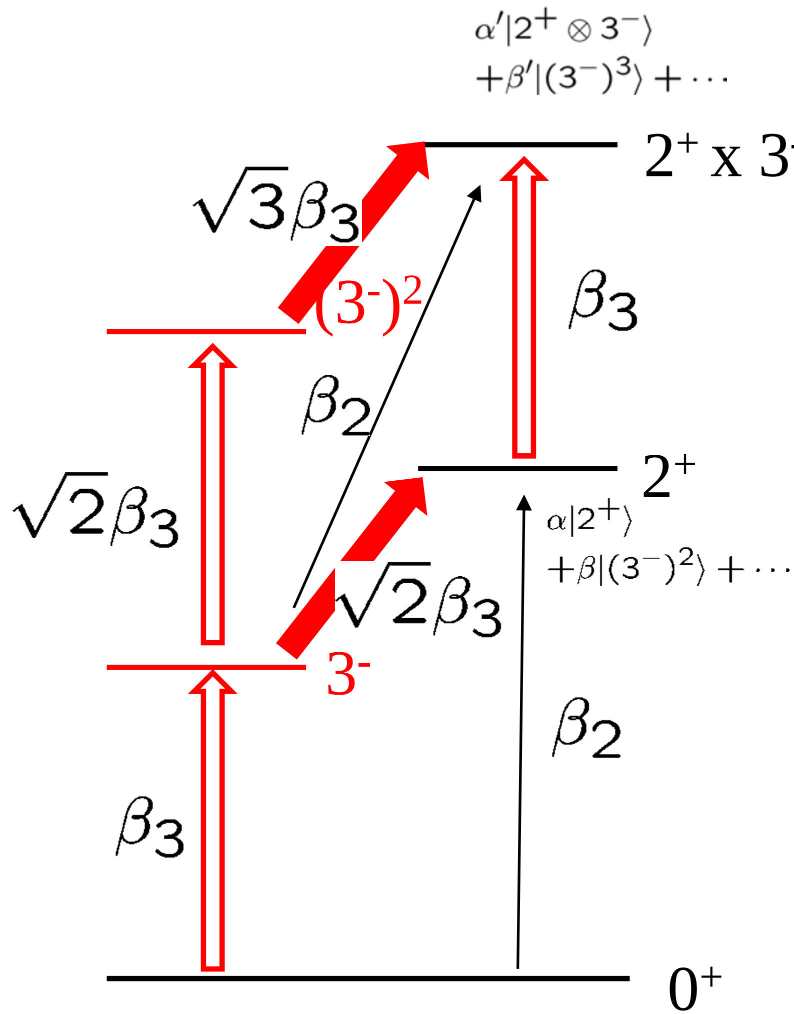
fluctuation both in β_3 and β_2



2_1^+ state: strong coupling both to g.s. and 3_1^-

$$\longrightarrow \quad |2_1^+\rangle = \alpha|2^+\rangle_{HO} + \beta|[3^- \otimes 3^-]^{(I=2)}\rangle_{HO} + \cdots$$





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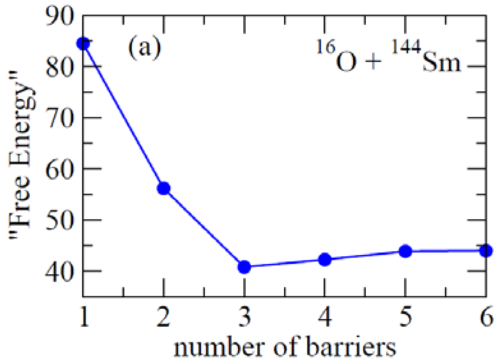
Summary

Heavy-ion subbarrier fusion reactions

✓strong interplay between reaction and structure
cf. fusion barrier distributions

➢A Bayesian approach to fusion barrier distributions

✓ a quick and convenient way to analyze data
✓ determination of the number of barriers

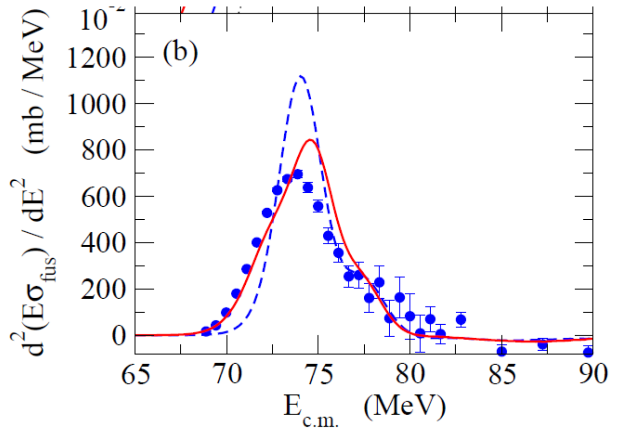


➢C.C. calculations with rel. beyond MF method

✓ anharmonicity
✓ truncation of phonon states
✓ octupole vibrations: $^{16}O + ^{208}Pb$

more flexibility:

- application to transitional nuclei



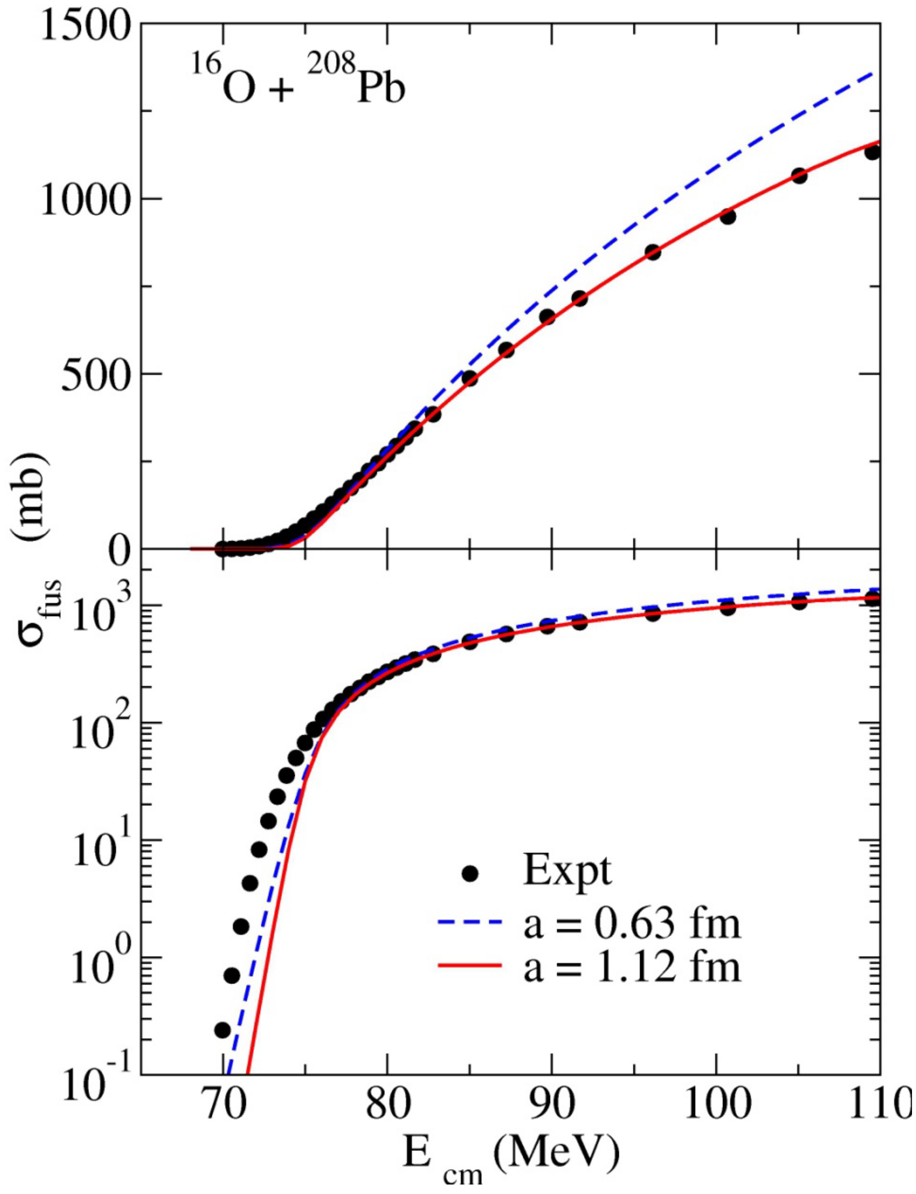
C.C. with shell model?

FUSION20

November 16-20, 2020
Shizuoka, Japan

Kouichi Hagino (co-chair) Tohoku University
Katsuhisa Nishio (co-chair) JAEA

Why not full microscopic treatment?



microscopic potential
(e.g., double folding potential)

$\longrightarrow$ $a \sim 0.63$ fm

does not work for fusion