Present status of coupled-channels calculations for heavy-ion sub-barrier fusion reactions

Kouichi Hagino
Tohoku University, Sendai, Japan

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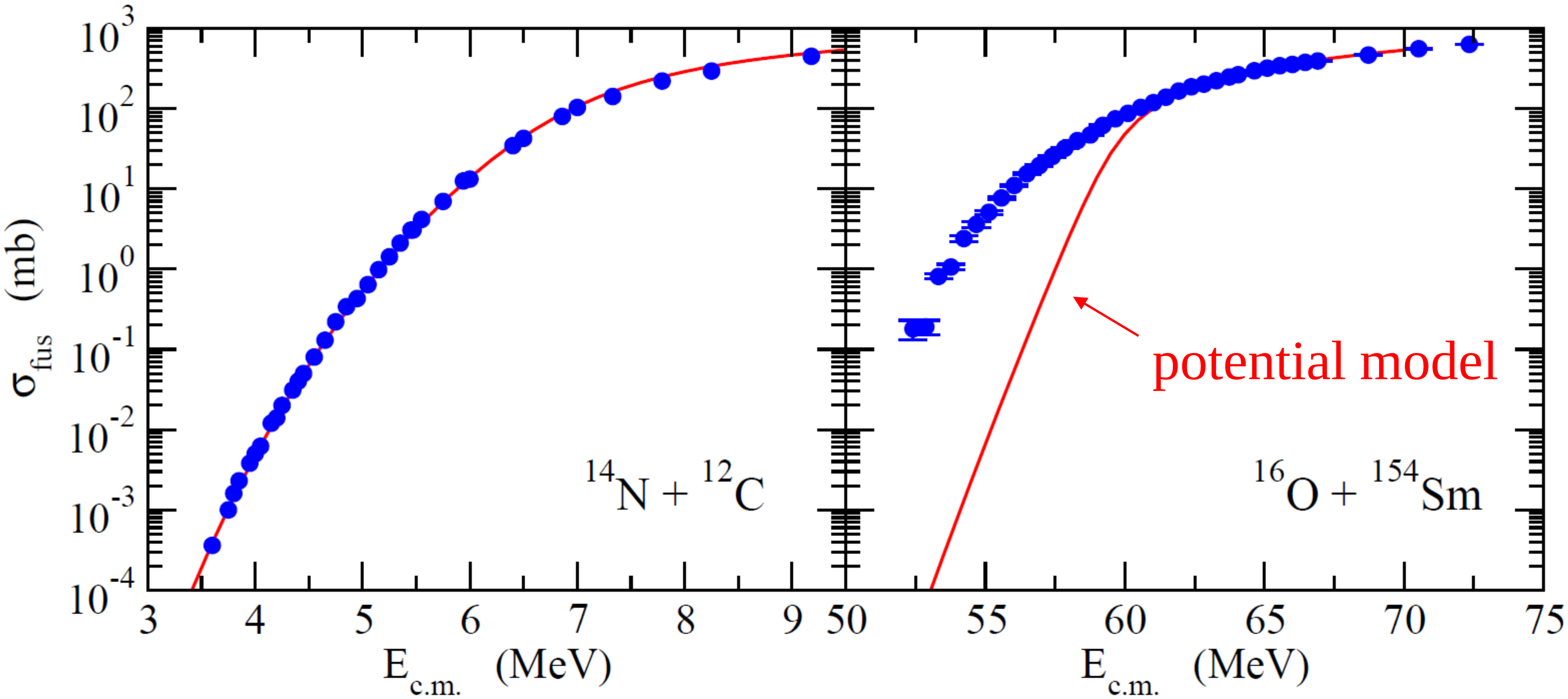
Introduction: heavy-ion sub-barrier fusion reactions

Discovery of large sub-barrier enhancement of σ_{fus} (~ the late 70's)

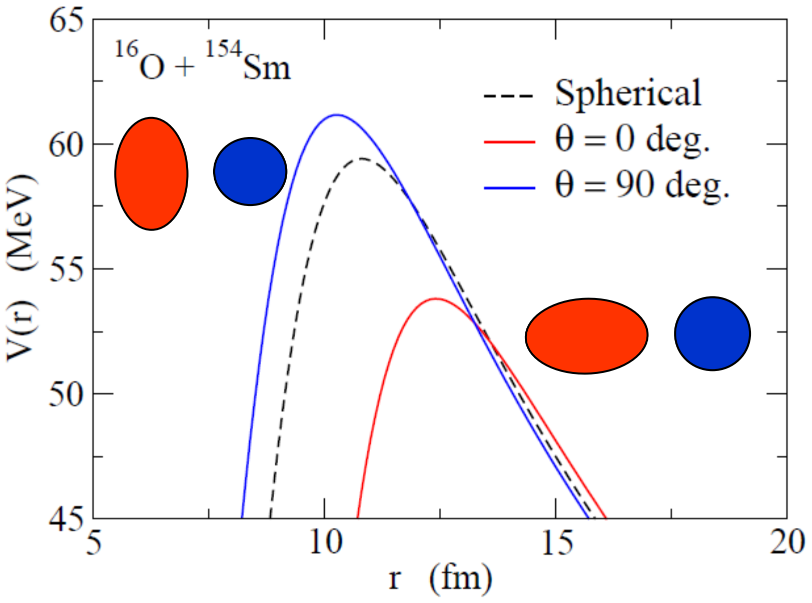
potential model: $V(r)$ + absorption

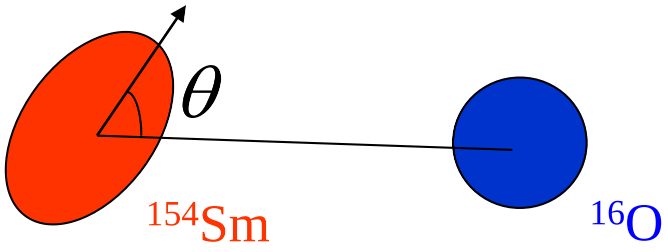


cf. seminal work:
R.G. Stokstad et al., PRL41('78) 465

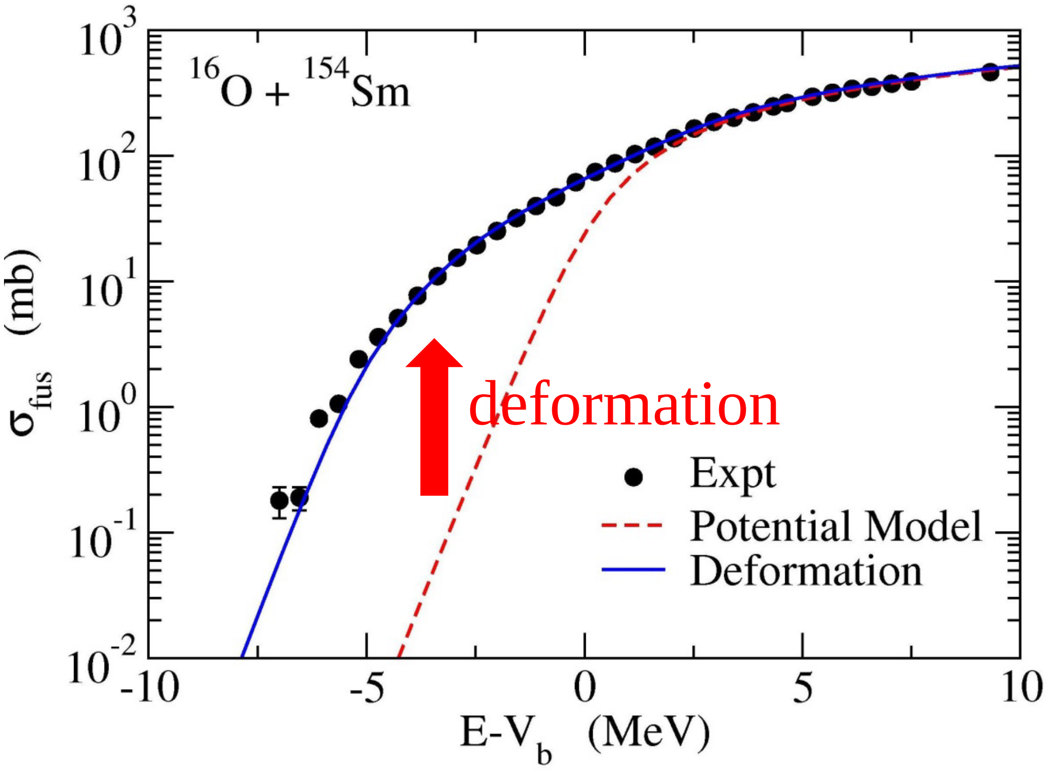
Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$





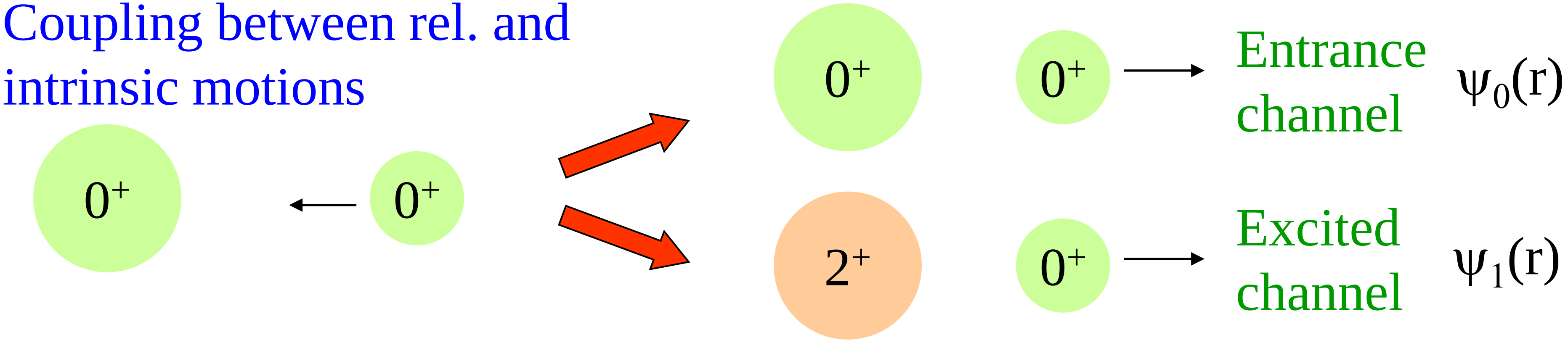
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\text{fus}}(E;\theta)$$



Fusion: strong interplay between nuclear structure and nuclear reaction

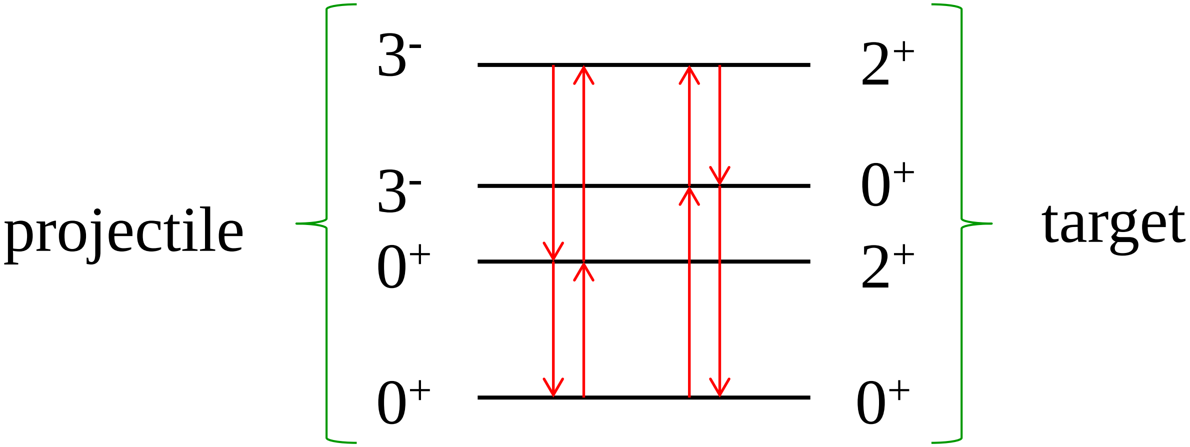
Coupled-Channels method

Coupling between rel. and intrinsic motions



Entrance channel $\psi_0(r)$

Excited channel $\psi_1(r)$



$$\boxed{\Psi(\boldsymbol{r},\xi)=\sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)}$$

coupled Schroedinger equations for $\psi_k(r)$

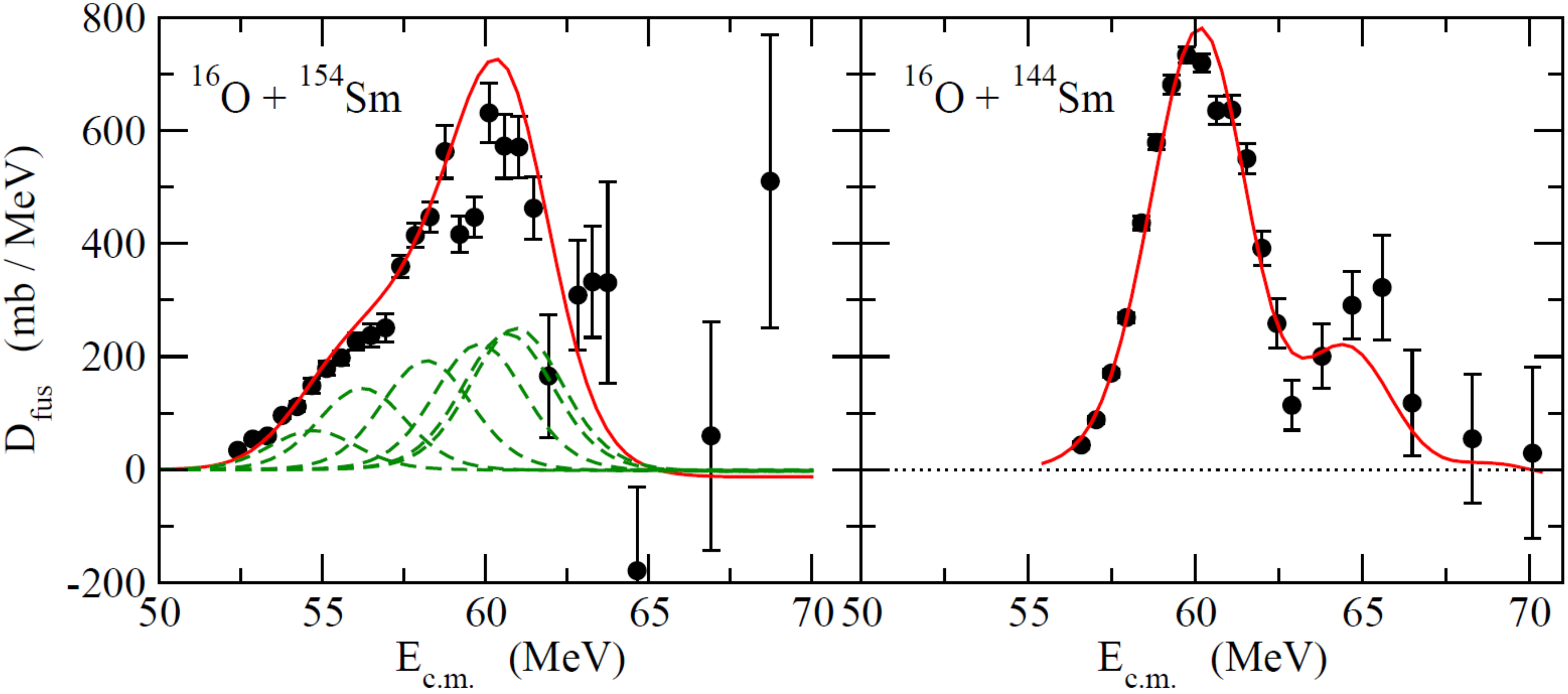
C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

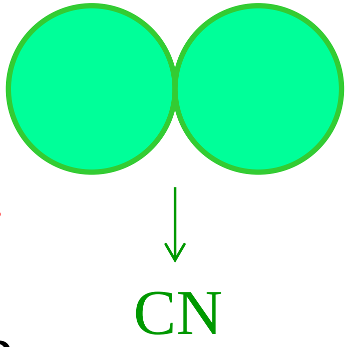
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

— c.c. calculations



K.H., N. Takigawa, PTP128 (‘12) 1061

Coupled-channels calculations for sub-barrier fusion



➢ Low-lying collective excitations only

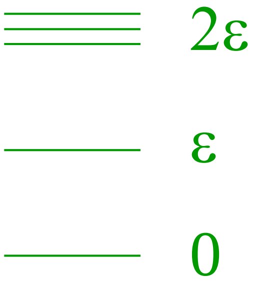
- low-lying collective excitations
  - strong coupling to the g.s., strong isotope dep.
- non-collective excitations
- giant resonances → high E_x, smooth isotope dep.

➢ Coupling strengths (B(Eλ)) and excitation energies (E_x)

identical to those in isolated nuclei

⟵ colliding nuclei: retain their identity during fusion

➢ Multiple excitations to higher collective states

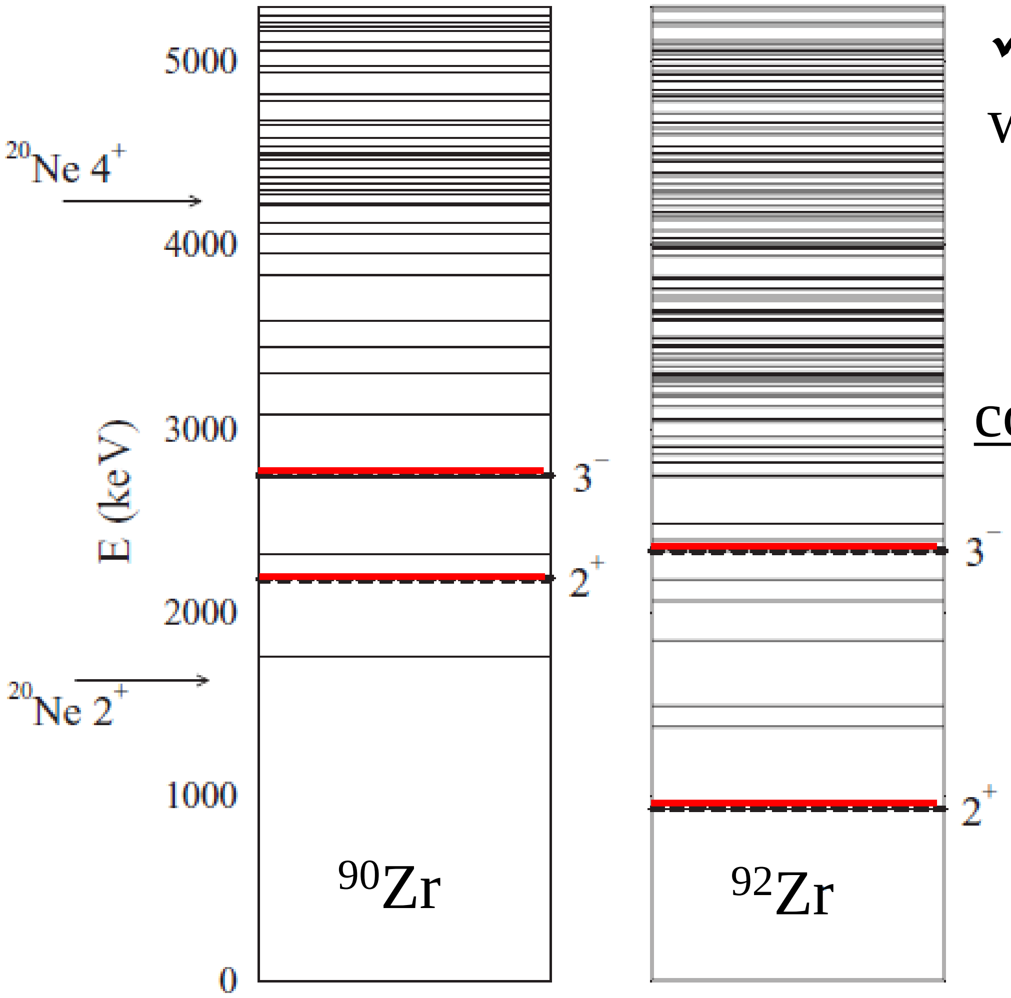


- multi-phonon excitations
- higher members in the g.s. rotational band

⟵ simple harmonic oscillator/ rigid rotor

this talk: **The validity of each of these assumptions?**

Role of non-collective excitations in fusion



✓ many non-collective states: weakly coupled, but many levels

35 levels (< 5 MeV) for ^{90}Zr
87 levels (< 5 MeV) for ^{92}Zr

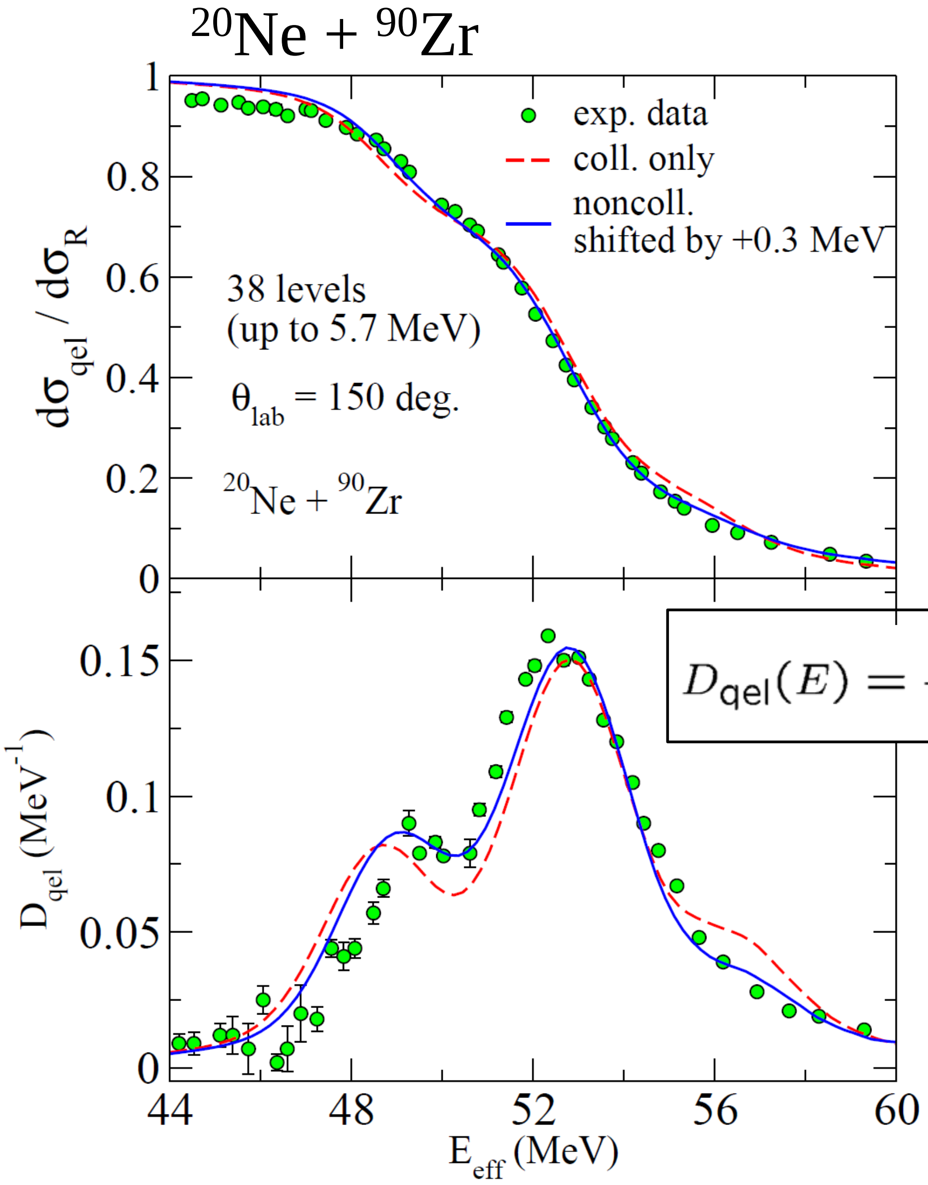
comparison between ^{90}Zr and ^{92}Zr

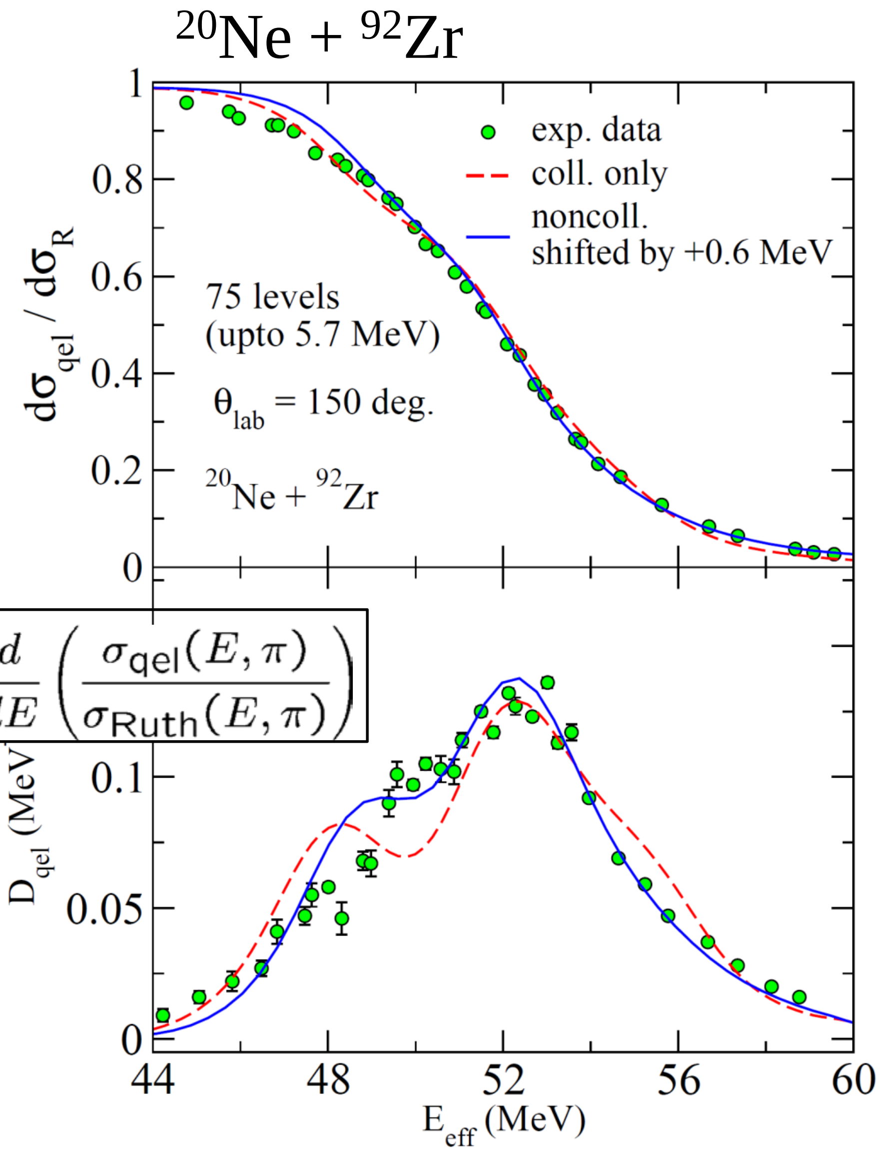
^{90}Zr (Z=40 sub-shell closure,
N=50 shell closure)
^{92}Zr = ^{90}Zr + 2n

role of these many weak channels?

the coupling strengths for non-collective excitations: poorly known

⟶ random numbers (cf. random matrix model)


^{20}Ne + ^{90}Zr


^{20}Ne + ^{92}Zr

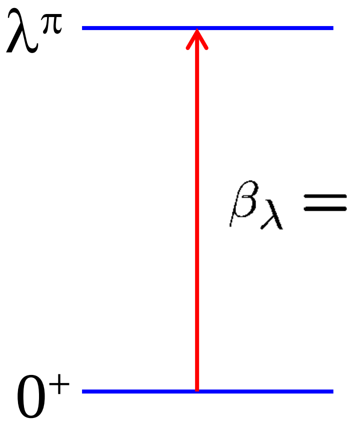
$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

$$E_{\rm eff} = 2E\frac{\sin(\theta_{\rm c.m.}/2)}{1+\sin(\theta_{\rm c.m.}/2)}$$

Expt.:
E. Piasecki et al.,
PRC80(‘09)054613

S. Yusa, K.H., and N. Rowley, PRC88(‘13)054621

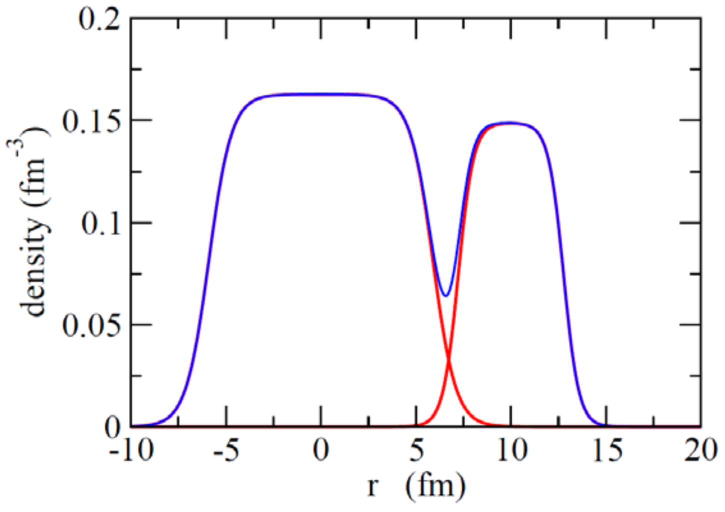
Damping of collective motions?

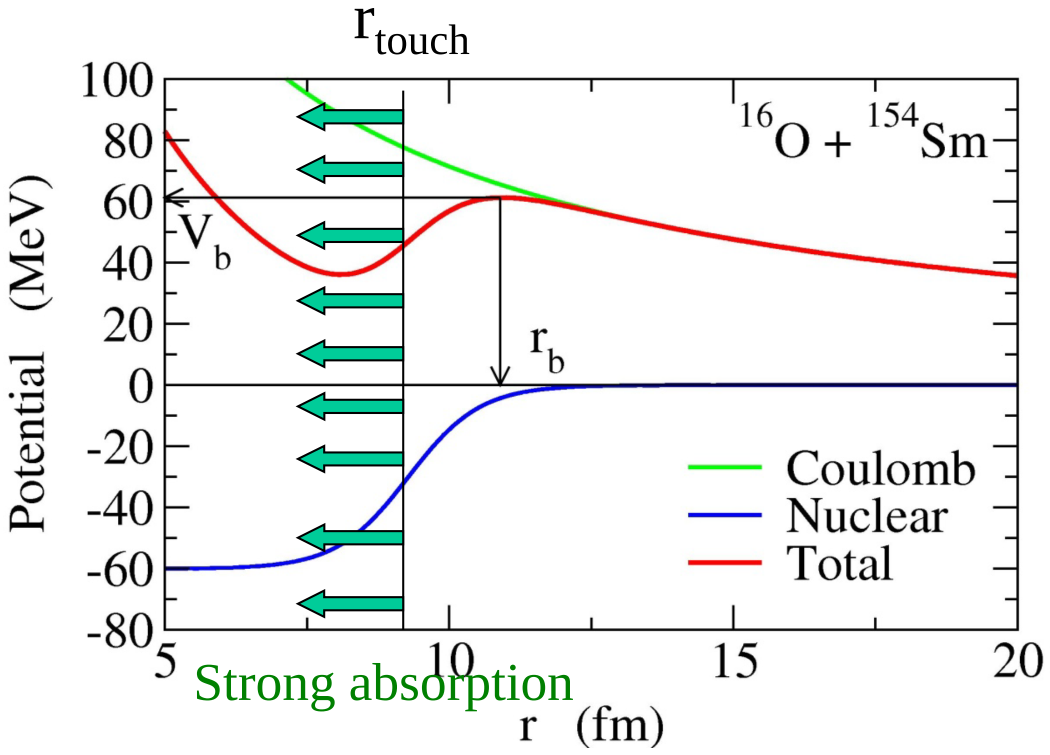


$$\beta_\lambda = \frac{4\pi}{3ZR^\lambda}\sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

$$V_{\rm coup}(r) \sim -\frac{\boxed{\beta_\lambda}}{\sqrt{4\pi}}\cdot R\frac{dV_N}{dr}$$

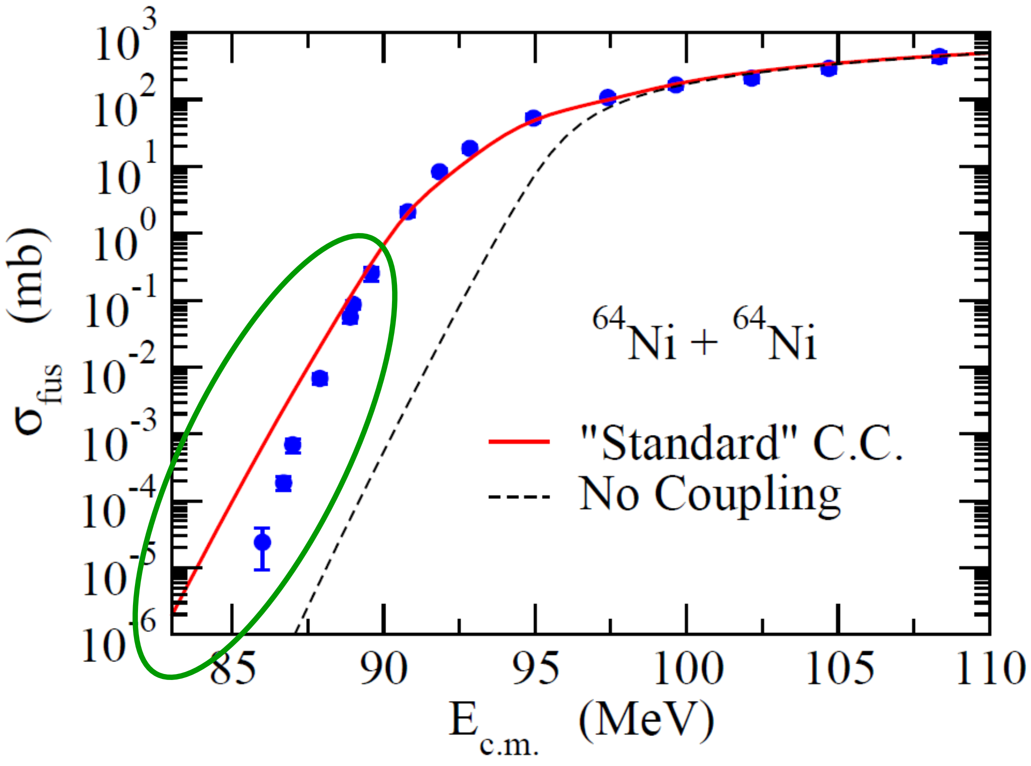
a level scheme in an isolated nucleus





Strong absorption

Fusion “hindrance” at deep sub-barrier energies



C.L. Jiang et al., PRL89(‘02)052701;
PRL93(‘04)012701

Theoretical models:

➢ Sudden model

S. Misicu and H. Esbensen,
PRL96(‘06)112701

✓ frozen density
✓ repulsive inner core

→ shallow potential

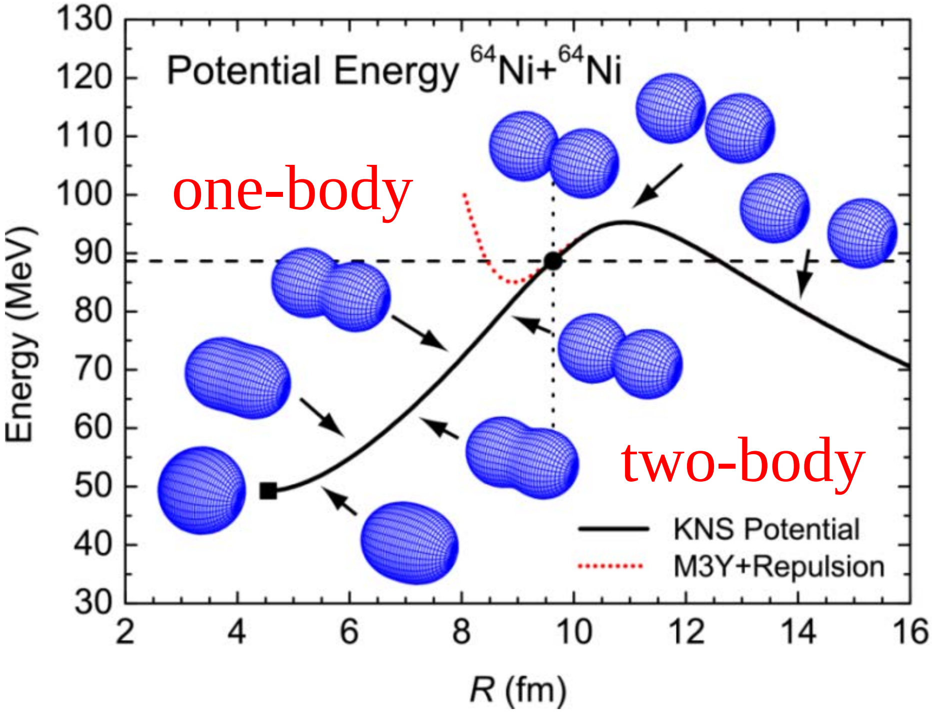
➢ Adiabatic model

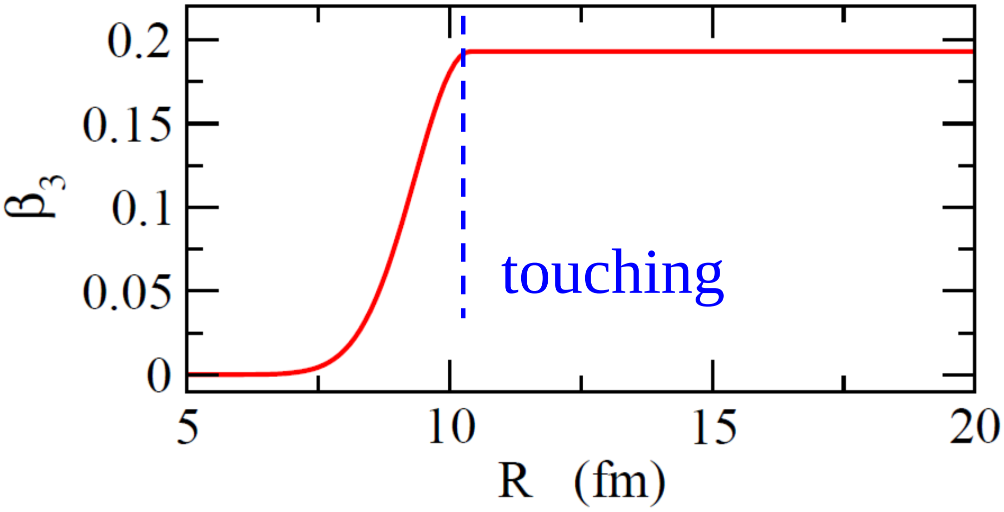
T. Ichikawa, K.H., and
A. Iwamoto,
PRL103(‘09)202701

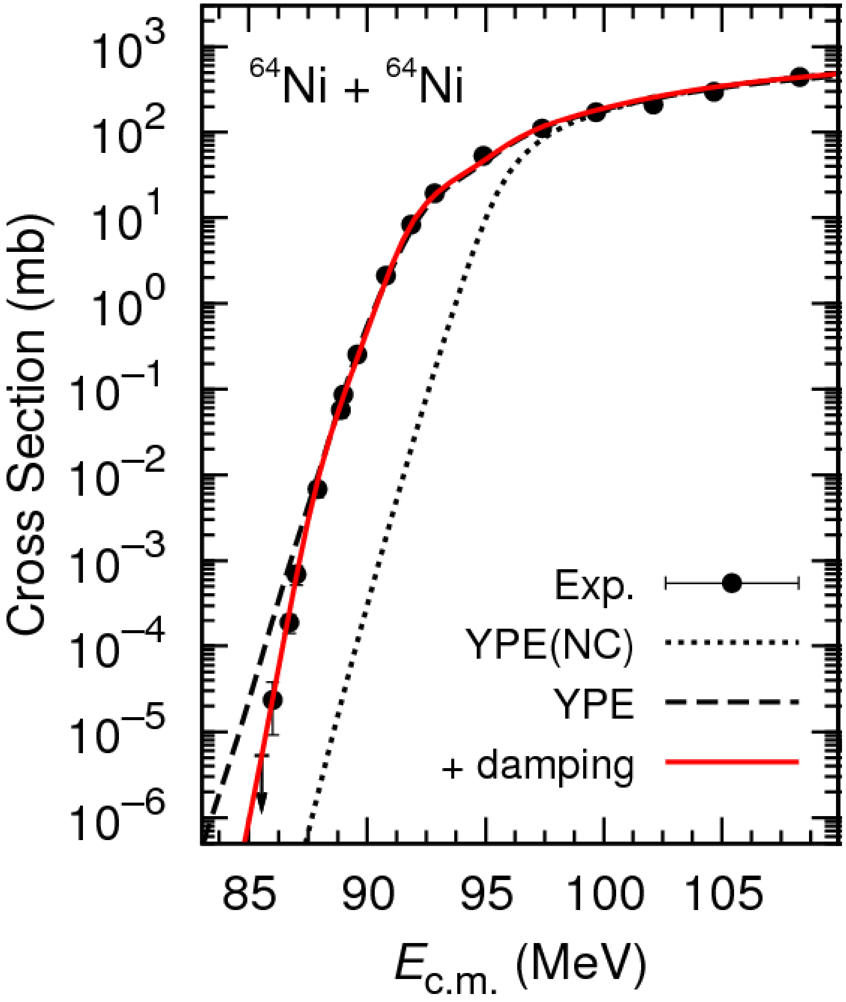
✓ density change after the touching
✓ neck formation

→ deep and thick potential

Adiabatic model for fusion hindrance (Ichikawa, Hagino, Iwamoto)



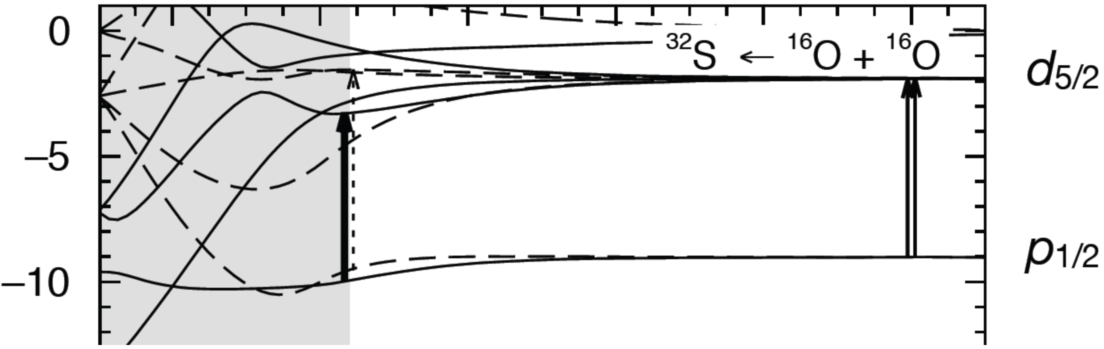


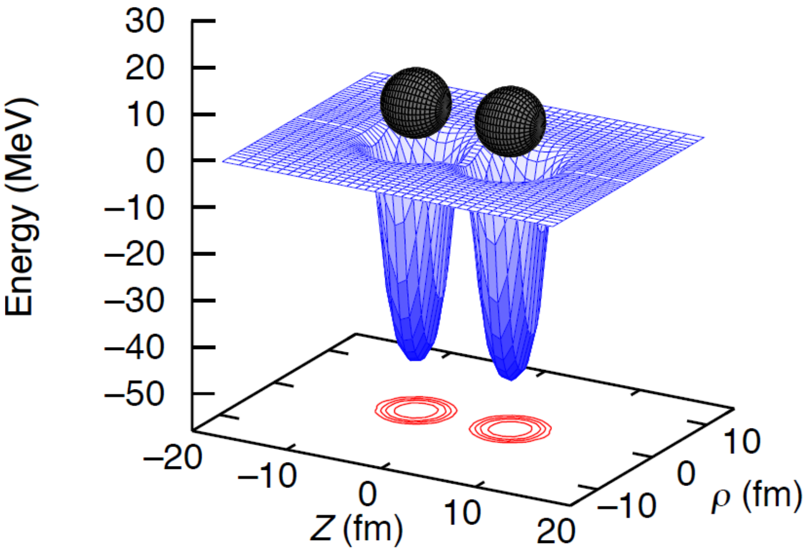


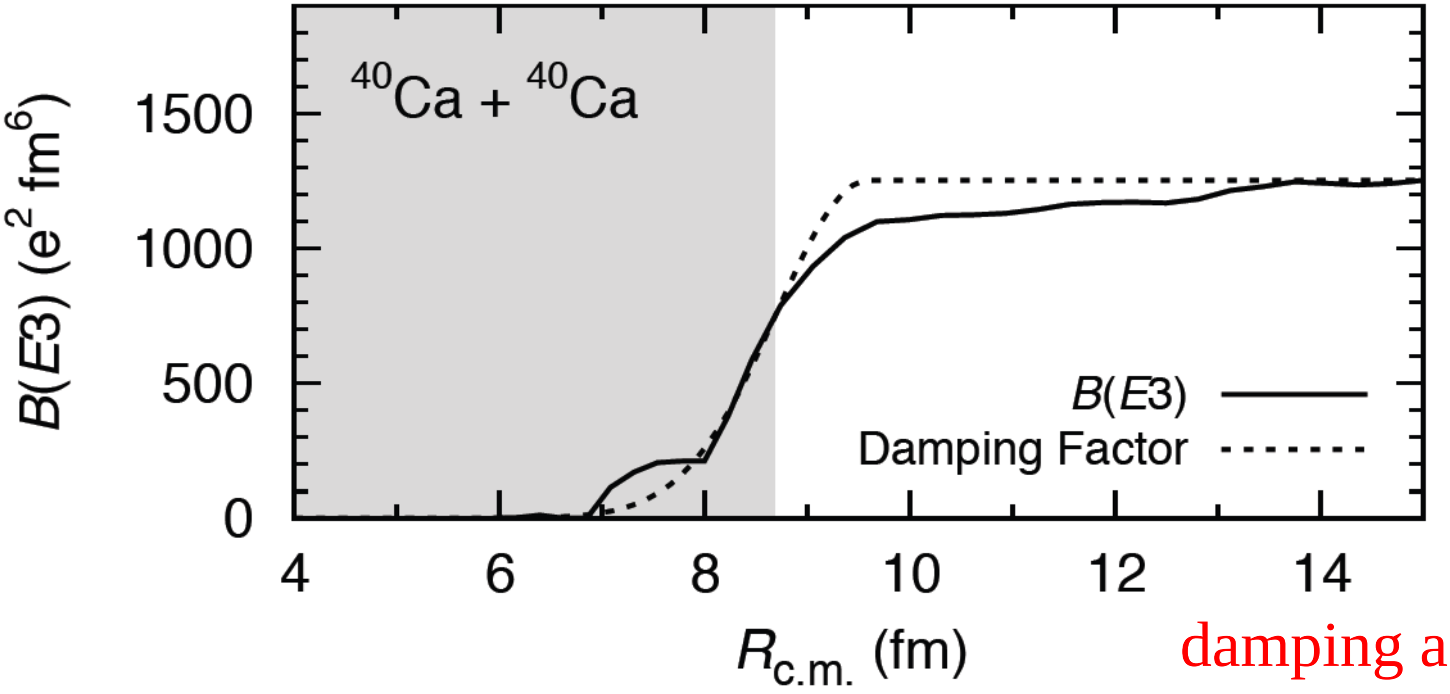
T. Ichikawa, K.H., and A. Iwamoto, PRL103(‘09)20270

RPA calculation at each separation

T. Ichikawa and K. Matsuyanagi
PRC88(‘13) 011602(R)





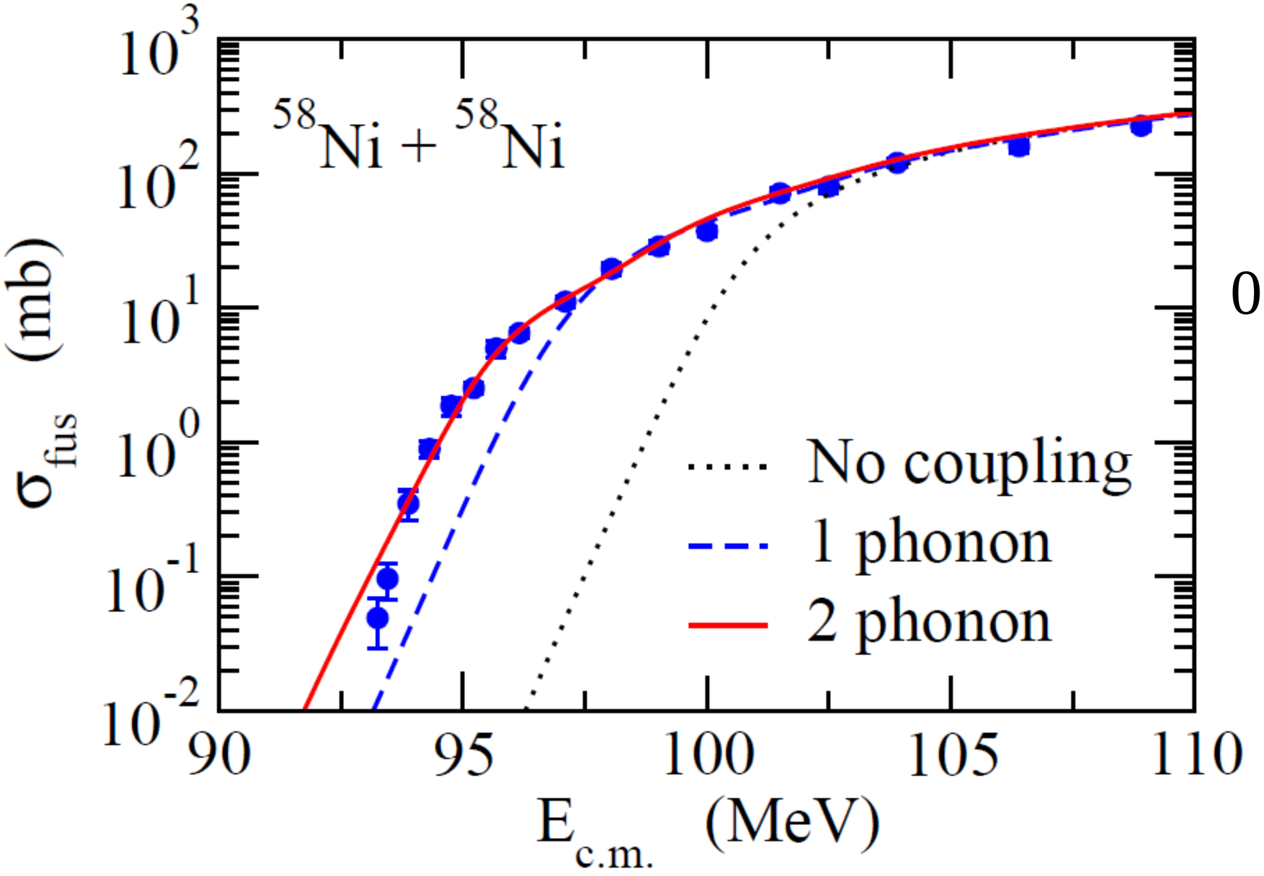


damping after the touching

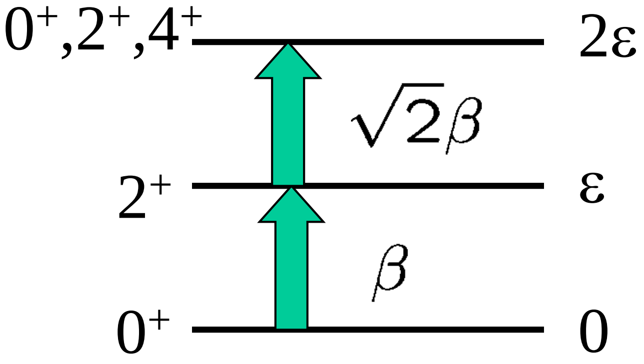
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15)064606

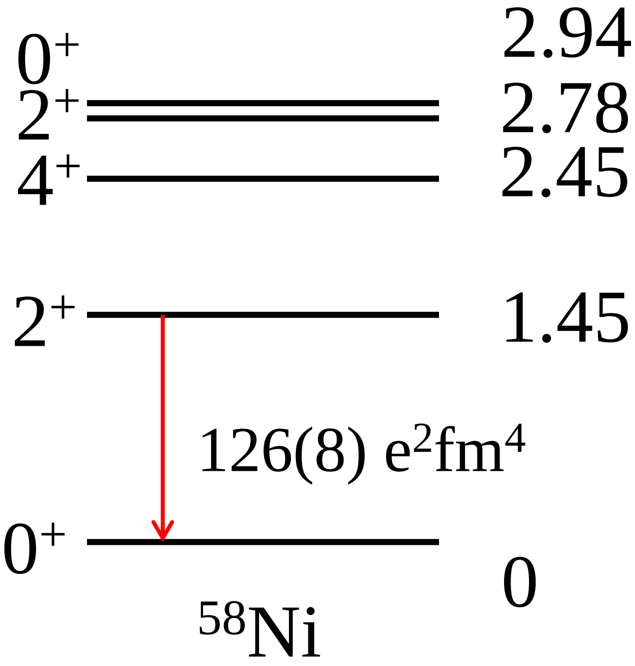
multi-phonon excitations



simple harmonic
oscillator



Anharmonic vibrations



$Q(2_1^+) = -10 \pm 6\ e\mathrm{fm}^2$

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

$$\boxed{|JM\rangle = \int d\beta\, f_J(\beta)\hat{P}^J_{M0}|\Phi(\beta)\rangle}$$

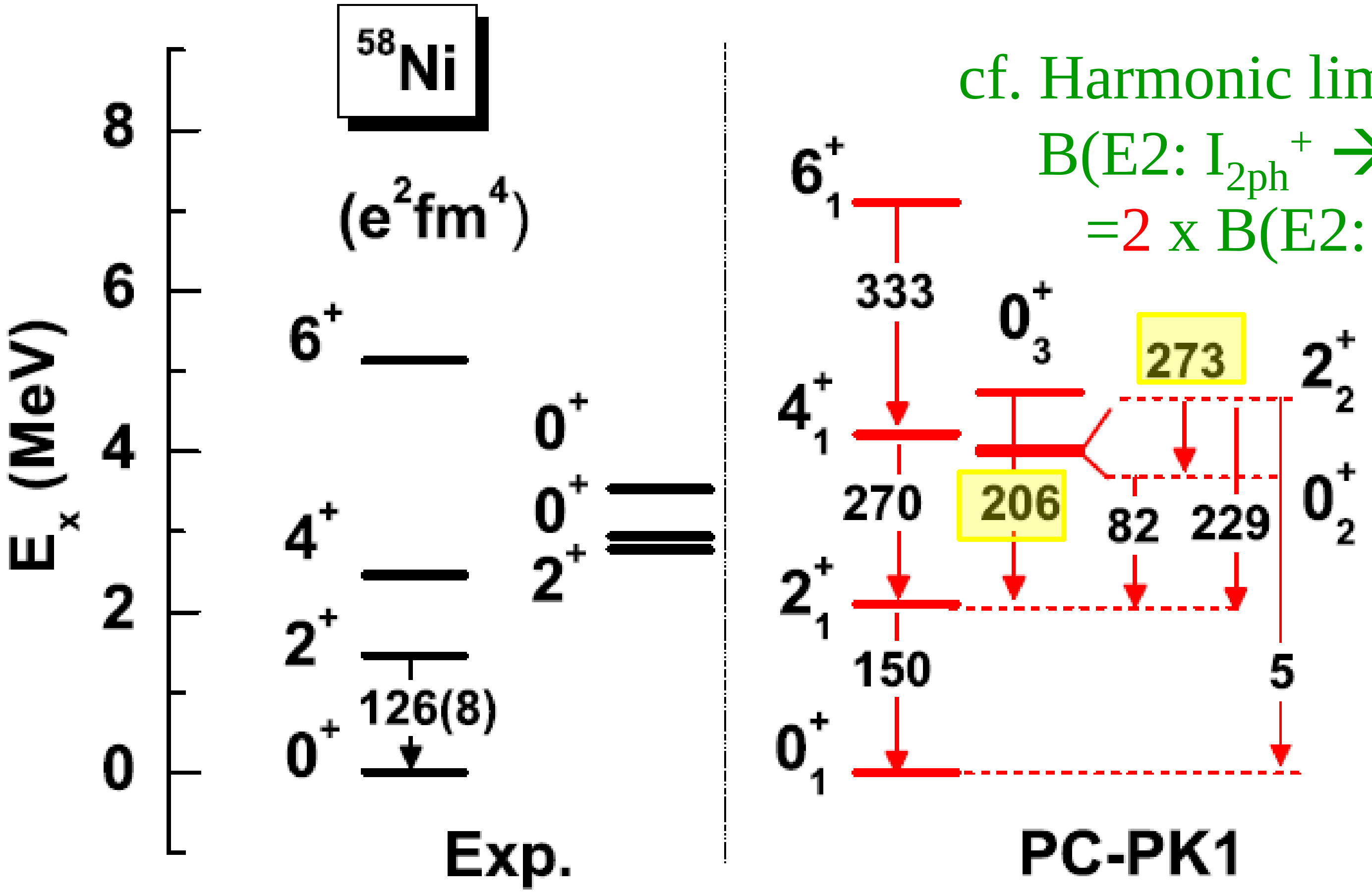
✓ MF + ang. mom. projection
+ particle number projection
+ generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121

J.M. Yao et al., PRC89 (‘14) 054306

Recent beyond-MF (MR-DFT) calculations for ^{58}Ni

K.H. and J.M. Yao, PRC91 (‘15) 064606
J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 (‘15) 024301



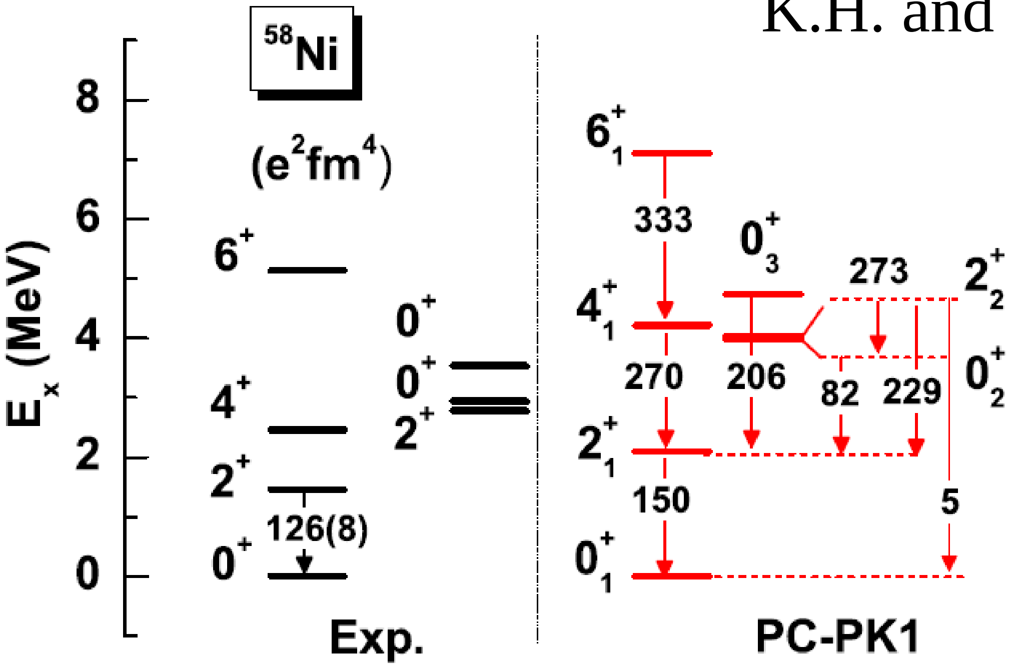
cf. Harmonic limit:
$B(E2: I_{2ph}^+ \rightarrow 2_1^+)$
$= 2 \times B(E2: 2_1^+ \rightarrow 0_1^+)$

- ✓ A large fragmentation of $(2^+ \times 2^+)_{J=0}$
- ✓ A strong transition from 2_2^+ to 0_2^+

effects on sub-barrier fusion?

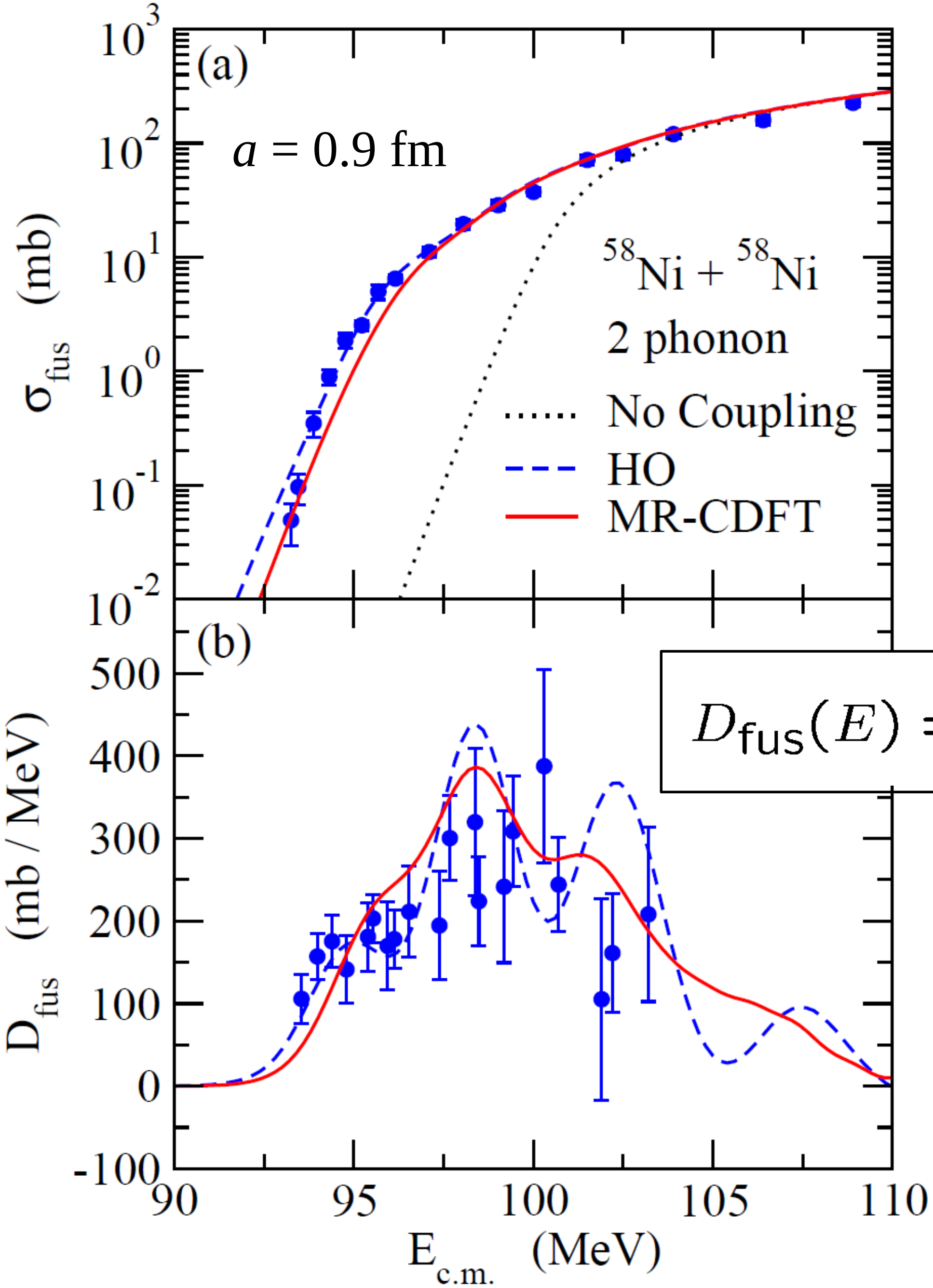
Semi-microscopic coupled-channels model for sub-barrier fusion

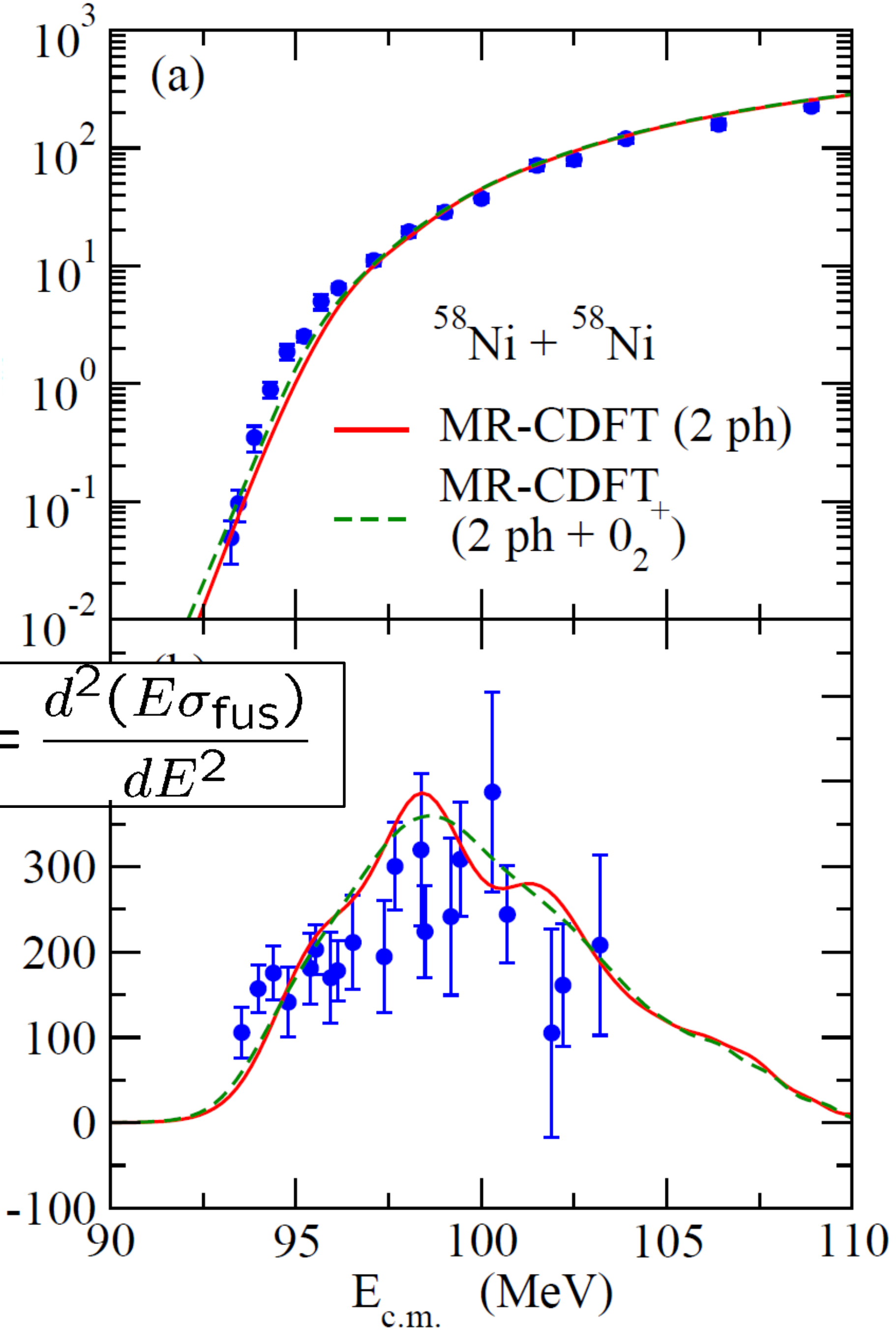
K.H. and J.M. Yao, PRC91 (‘15) 064606



microscopic
multi-pole operator

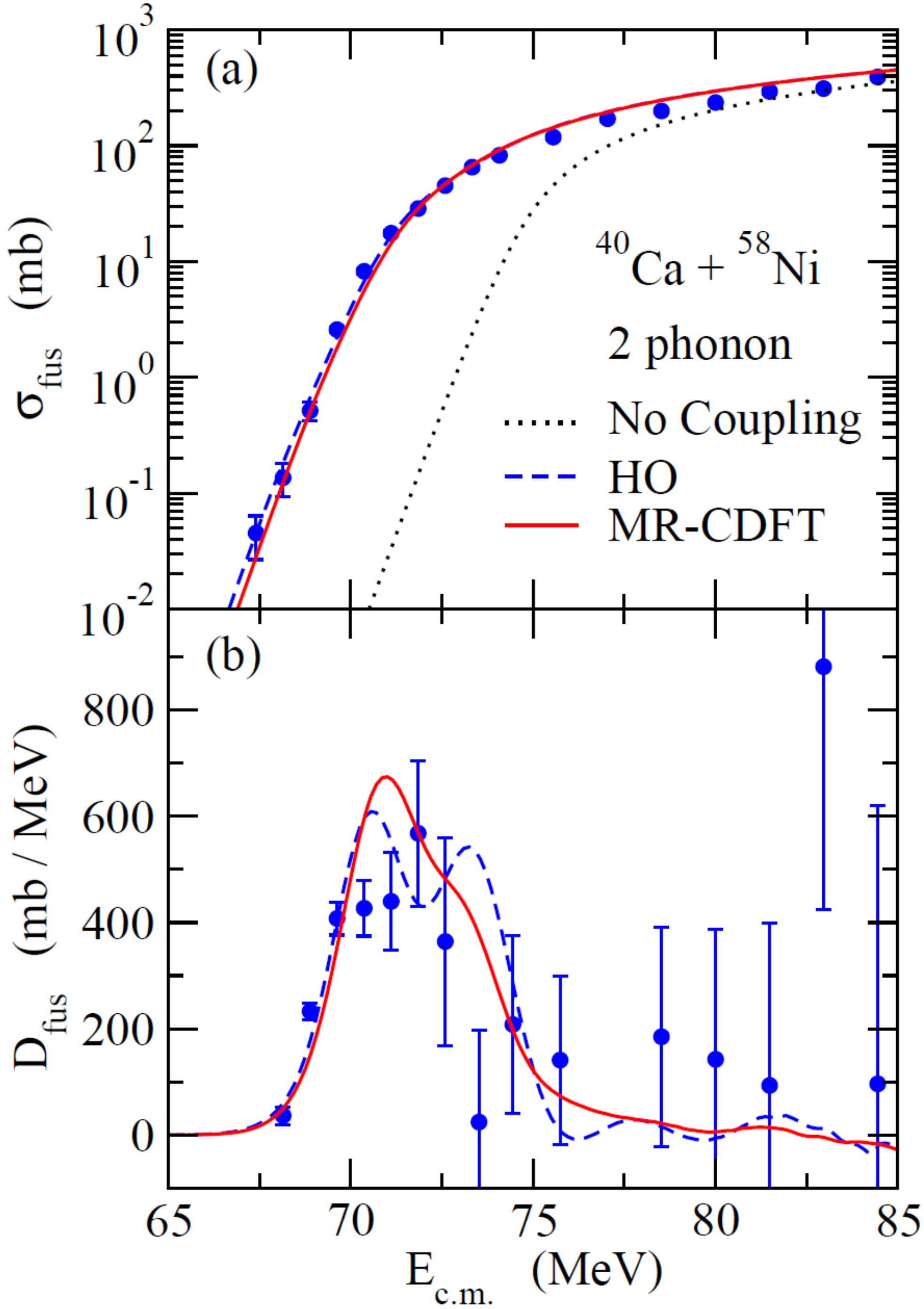
- ✓ $V_{\text{coup}}(r,\alpha_{\lambda 0}) \rightarrow V_{\text{coup}}(r,\hat{Q}_{\lambda 0})$
- ✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states
- ✓ scale to the empirical B(E2; 2_1^+ → 0_1^+)
- ✓ still use a phenomenological potential
- ✓ use the experimental values for E_x
- ✓ β_N and β_C from M_n/M_p for each transition
- ✓ axial and reflection symmetries (no 3^+ and 3^- states)





$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

K.H. and J.M. Yao, PRC91 (‘15) 064606



Experimental data:
D. Bourgin, S. Courtin et al.,
PRC90('14)044601.
c.f. S. Courtin's talk,
Tue. afternoon

Summary

Coupled-channels calculations for sub-barrier fusion

- ➢ Role of non-collective excitations
  - ✓ difference between ^{20}Ne + ^{90}Zr and ^{20}Ne + ^{92}Zr
  - ✓ heavy systems relevant to SHE

- ➢ Damping of collective excitations
  - ✓ overlapping region
  - ✓ deep sub-barrier fusion hindrance

- ➢ C.C. calculations with MR-DFT method
  - ✓ anharmonicity
  - ✓ truncation of phonon states
  - ✓ octupole vibrations and tri-axiality: in progress

more flexibility:

- application to transitional nuclei
- a good guidance to a Q-moment of excited states