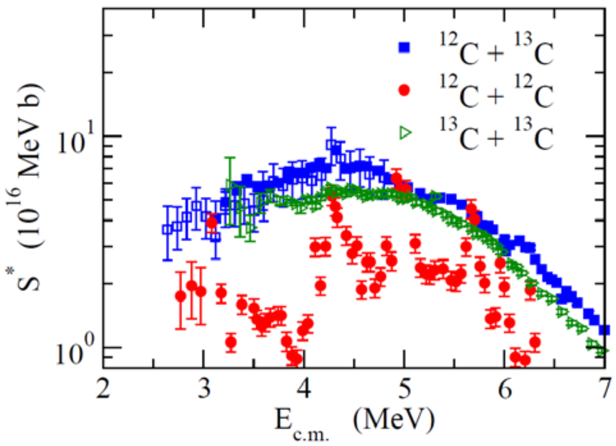
Subbarrier fusion of carbon isotopes
~ on the Wong formula and fusion oscillations ~

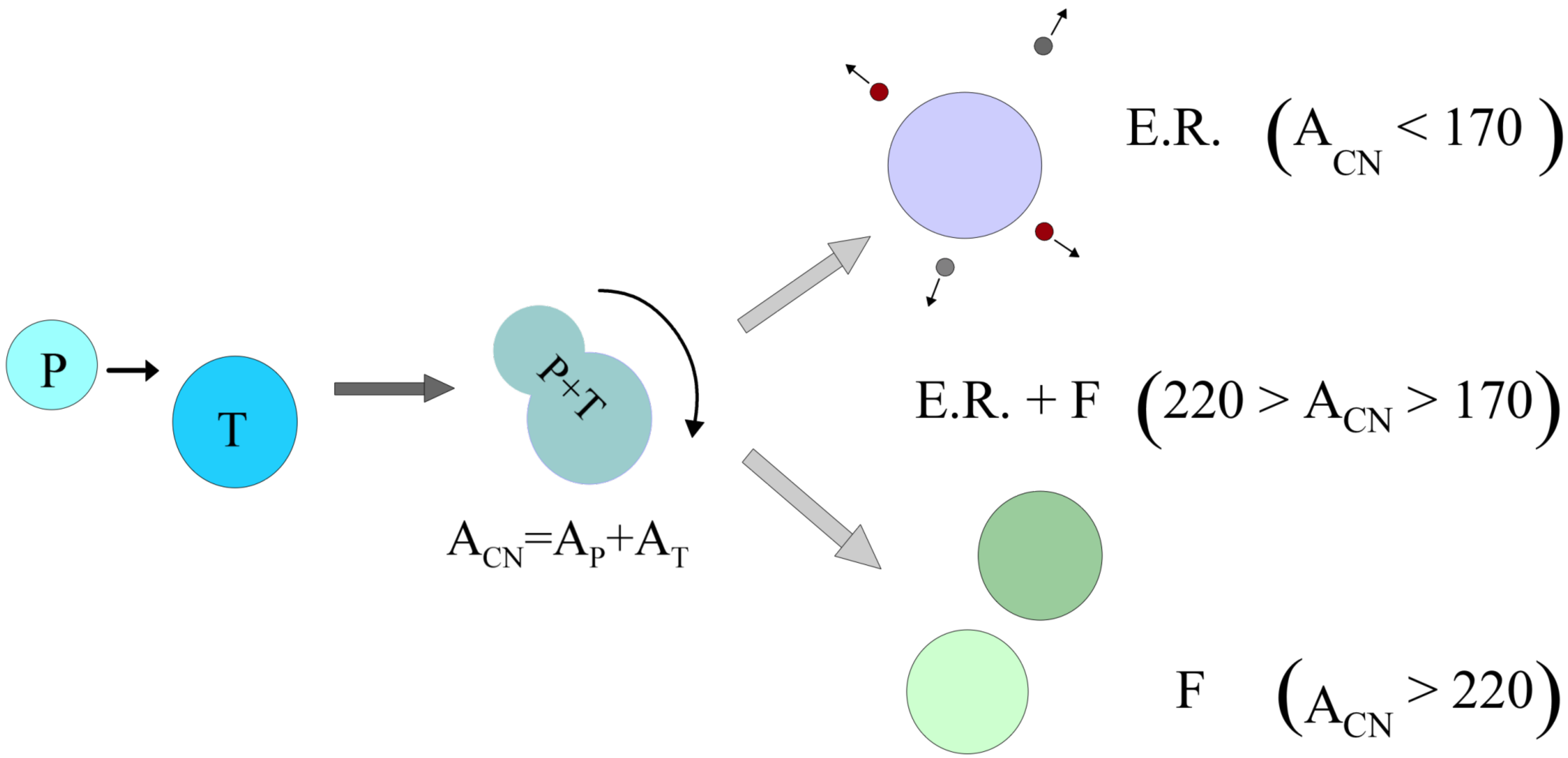


Kouichi Hagino, *Tohoku University*
Neil Rowley, *IPN Orsay*

TOHOKU UNIVERSITY

1. *Introduction: $^{12}C + ^{12}C$ fusion*
2. *Molecular resonances at subbarrier energies*
3. *Wong formula and its generalization*
4. *Fusion oscillations at above barrier energies*
5. *Summary*

Fusion: compound nucleus formation

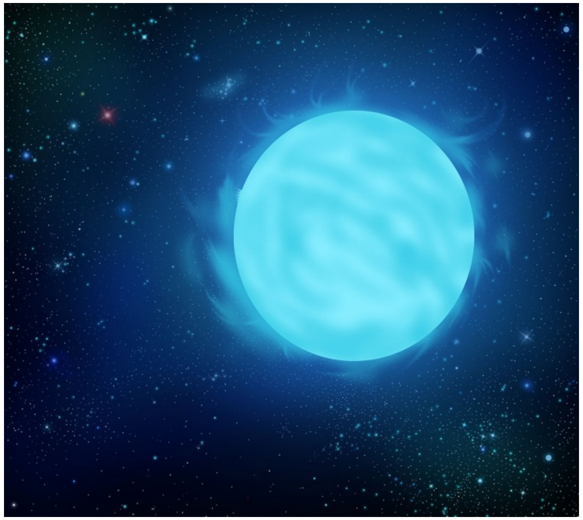


courtesy: Felipe Canto

Introduction: ^{12}C + ^{12}C fusion

$^{12}C+^{12}C$ fusion : a key reaction in nuclear astrophysics

Carbon burning
in massive stars



$^{12}C+^{12}C \rightarrow \alpha + ^{20}Ne$

$^{12}C+^{12}C \rightarrow p + ^{23}Na$

stellar evolution

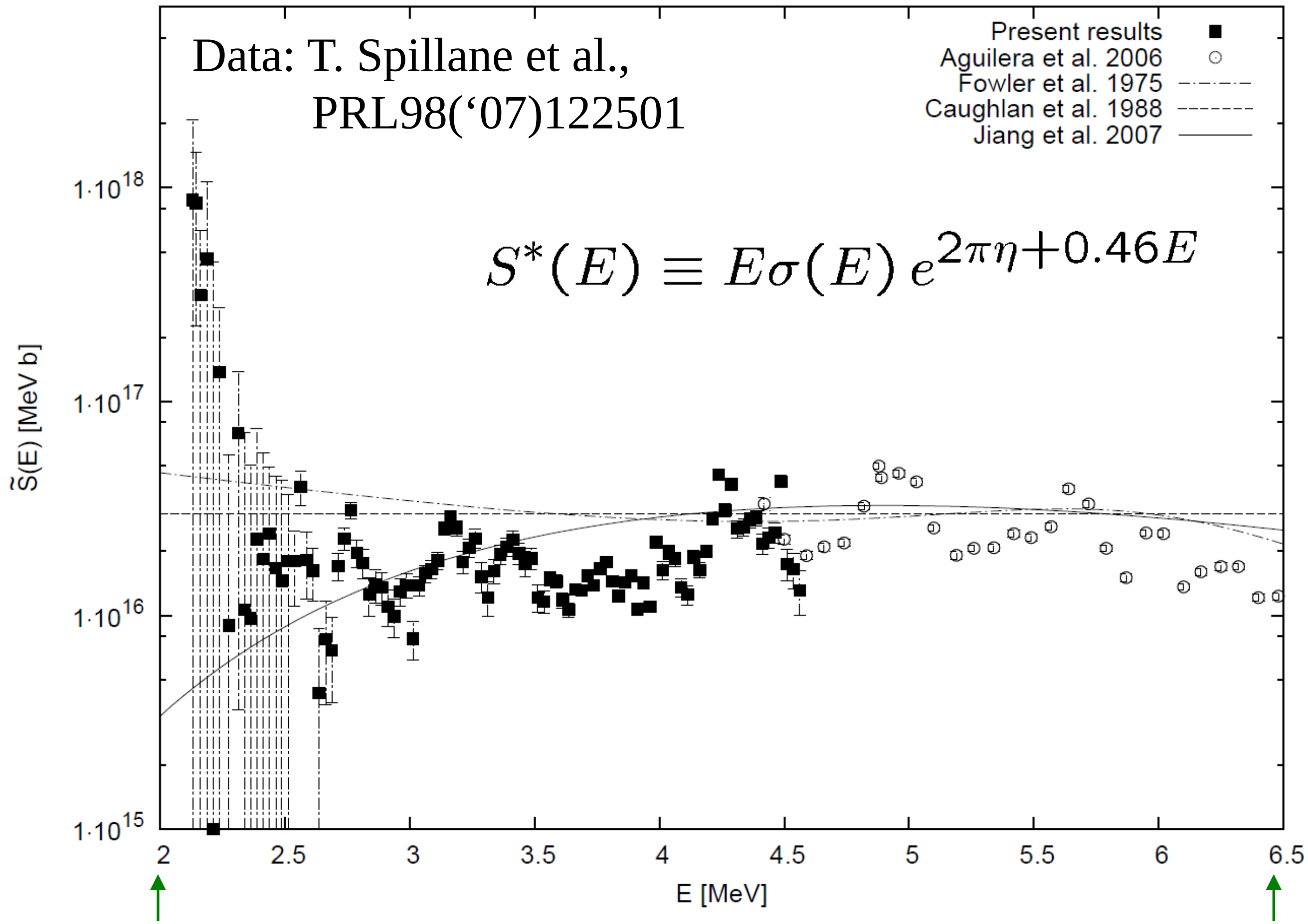
Type Ia supernovae

X-ray superburst

deep layer of the outer crust in accreting neutron stars

important to understand $^{12}C+^{12}C$ fusion at deep subbarrier energies

Experimental data at low energies



T. Spillane et al.,
PoS (NIC X) 016

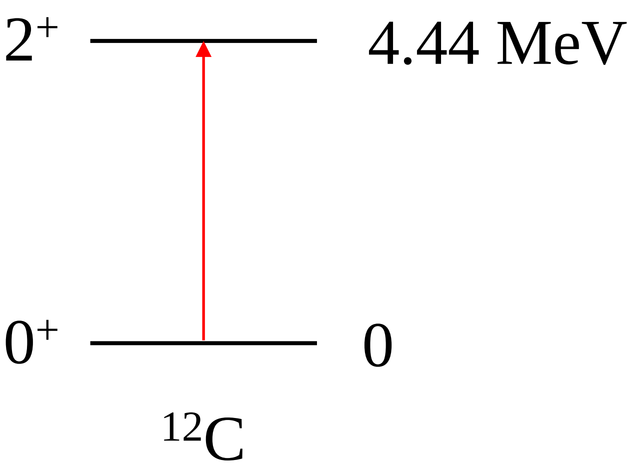
$E_G \sim 2$ MeV (T=0.85 GK) $\sim V_b$

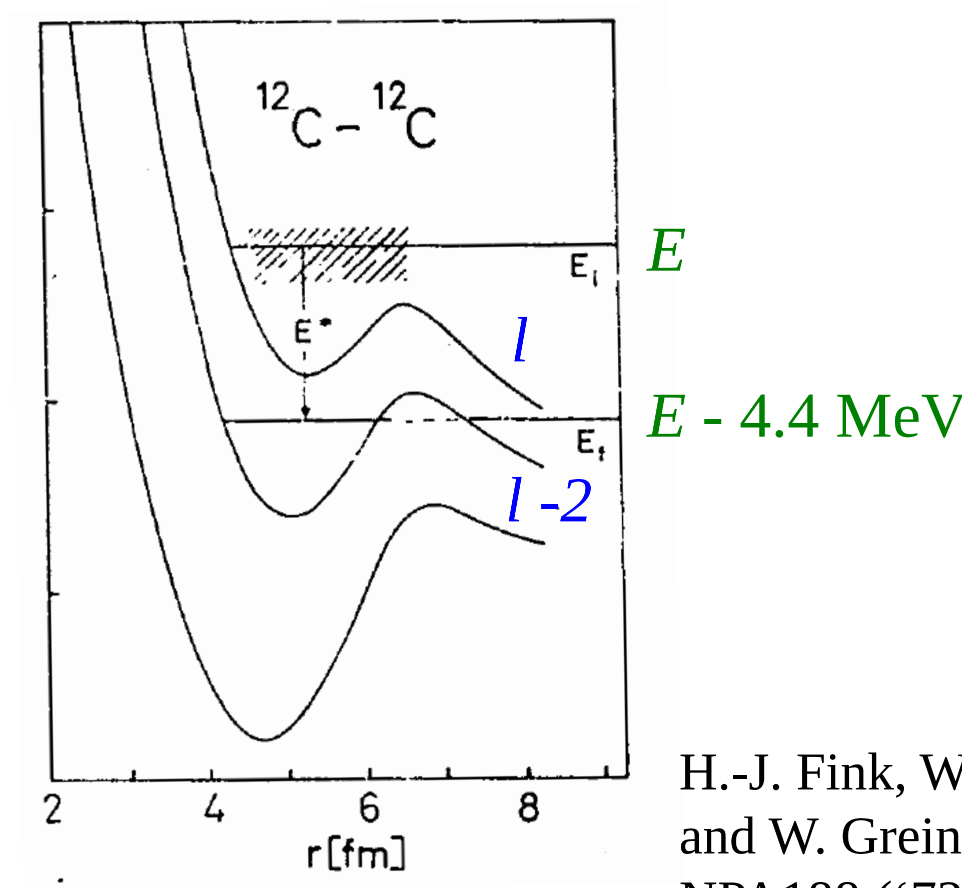
✓ pronounced resonance structures (narrow resonances)
"molecular resonances"
✓ difficult to extrapolate down to E_G

Theoretical calculations

- Nogami-Imanishi model (B. Imanishi, PL 27B (‘68) 267, NPA125 (‘69) 33)
- Band-crossing model (Y. Kondo, T. Matsuse, Y. Abe, PTP59 (‘78)465)
- Double resonance model (W. Scheid, W. Greiner, R. Lemmer, PRL25 (‘70) 176)

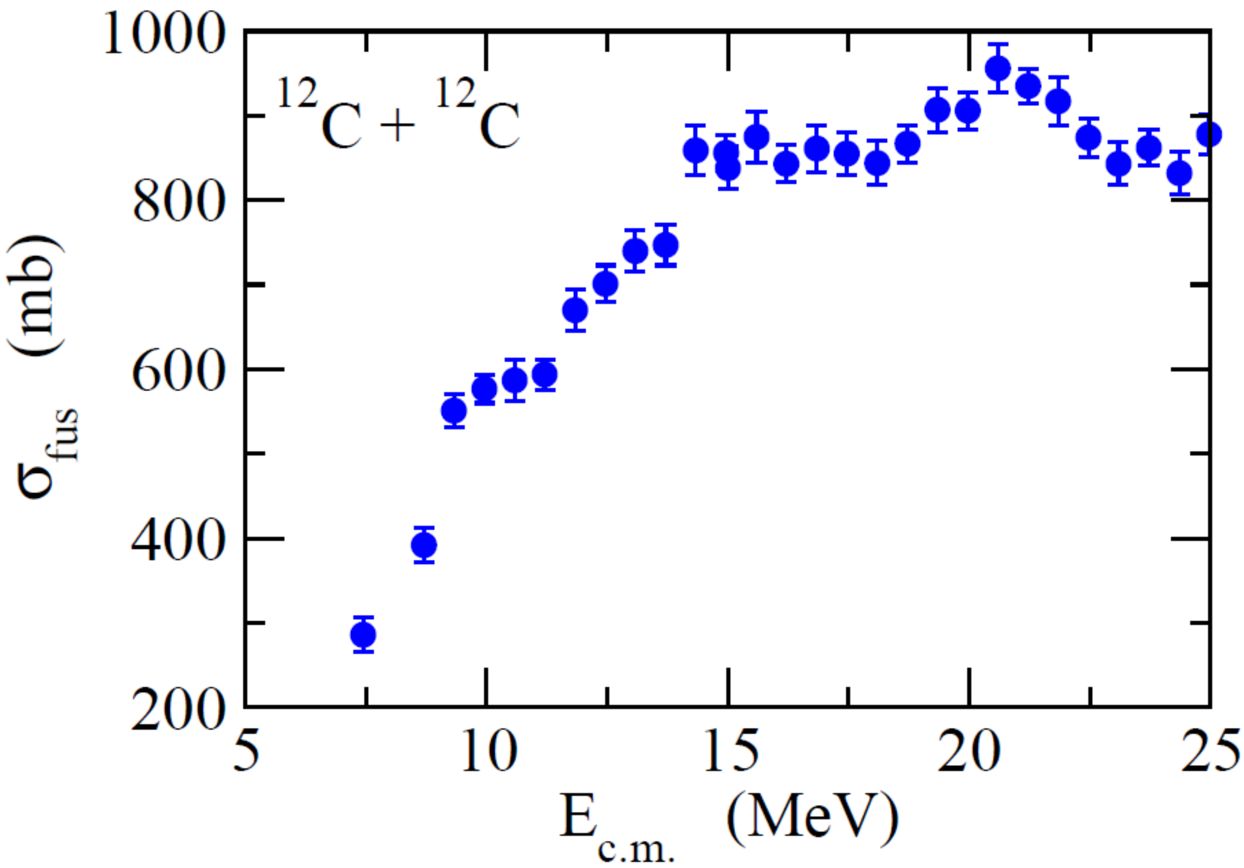
* the basic concept is all same





H.-J. Fink, W. Scheid, and W. Greiner, NPA188 (‘72) 259

Experimental data at above barrier energies



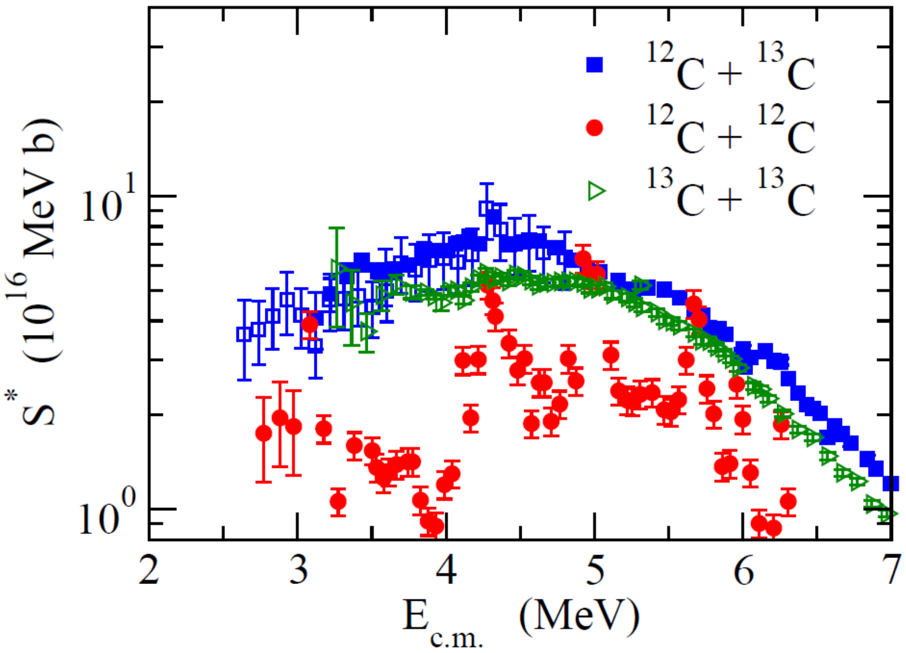
Data: D.G. Kovar et al., PRC20 ('79) 1305

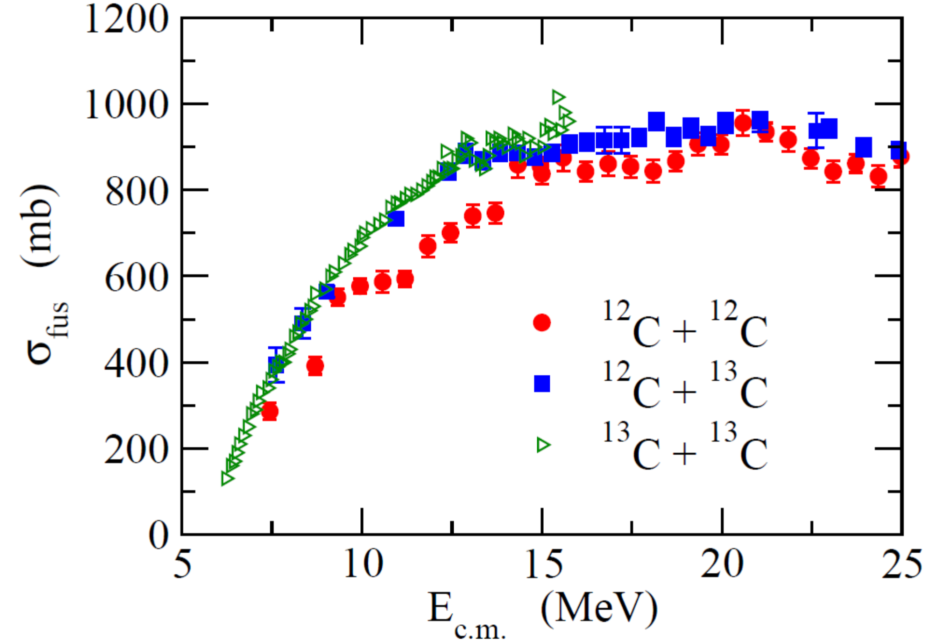
✓ fusion oscillations

⟵ successive contributions of individual partial waves

(N. Poffe, N. Rowley, and R. Lindsay, NPA410 ('83) 498)

Comparison with other C+C systems



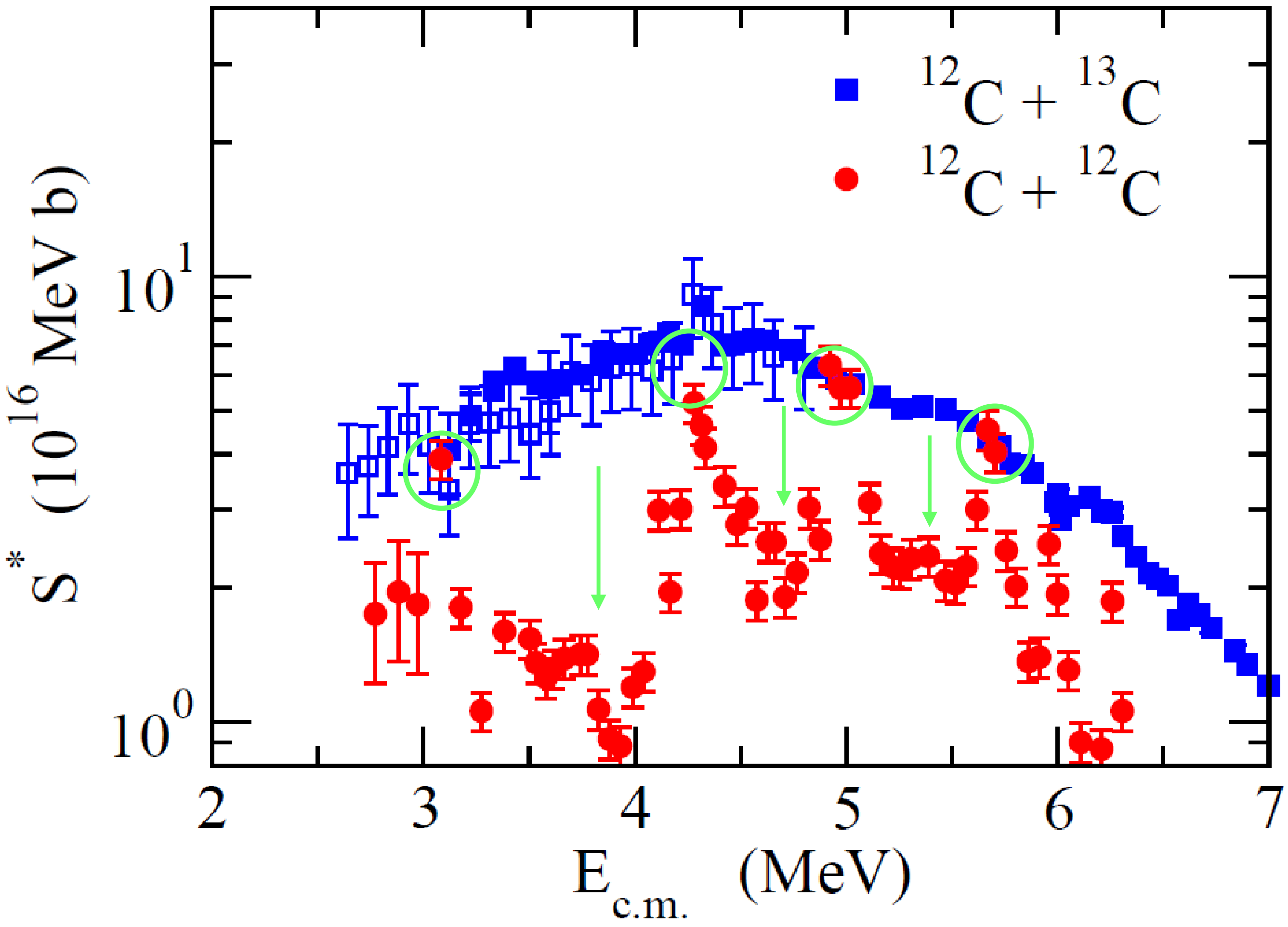


fusion cross sections for $^{12}C+^{13}C$, $^{13}C+^{13}C$: much less structured

How can one understand the systematics?
- from $^{12}C+^{12}C$ to $^{12}C+^{13}C$, $^{13}C+^{13}C$
  origins for the resonances/oscillations?
- from low to high energies

cf. most of the previous studies: $^{12}C+^{12}C$ only

Molecular resonances at subbarrier energies



M. Notani, X.D. Tang et al., PRC85(‘12)014607

off-resonance: fusion inhibition
on-resonance: match with ^{12}C+^{13}C

Jiang's conjecture: C.L. Jiang et al., PRL110('13) 072701

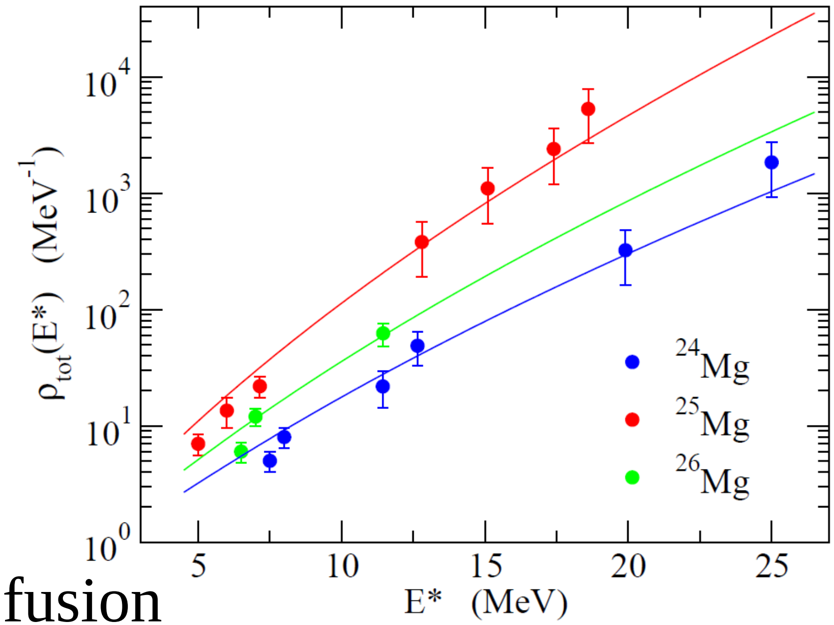
properties of compound nucleus (^{24}Mg)?

^{12}C+^{12}C reaction:

- ✓ level density of ^{24}Mg : small (e-e)
- ✓ small fusion Q-value

Q = +13.9 MeV (^{12}C+^{12}C)
+16.3 MeV (^{12}C+^{13}C)
+22.5 MeV (^{13}C+^{13}C)

⟶ small E^* for ^{24}Mg in ^{12}C+^{12}C fusion



$$\sigma \sim \sum_J \sigma^J_{\rm cap}\left[1 - e^{-2\pi\Gamma_J/D_J}\right]$$

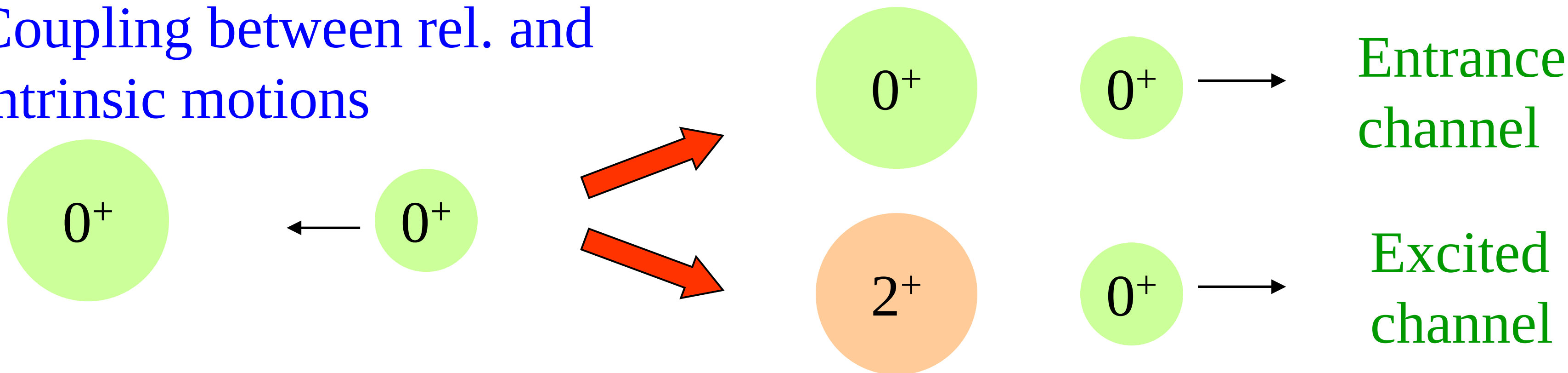
large hindrance factor

$D_J = 1/\rho_J$
Γ_J : CN width

incorporate this idea in the coupled-channels calculations?

Coupled-channels calculations: scatt. + **collective excitations**

Coupling between rel. and intrinsic motions



$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|V_{\text{coup}}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0$$

A standard tool to analyze heavy-ion subbarrier fusion reactions

e.g. CCFULL, K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143

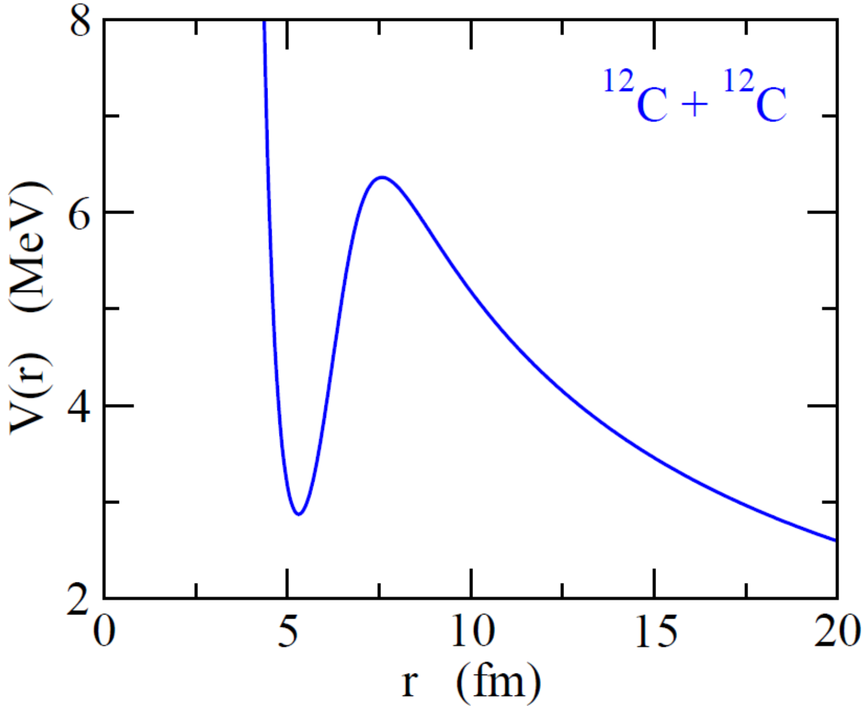
*Extension with breakup channels

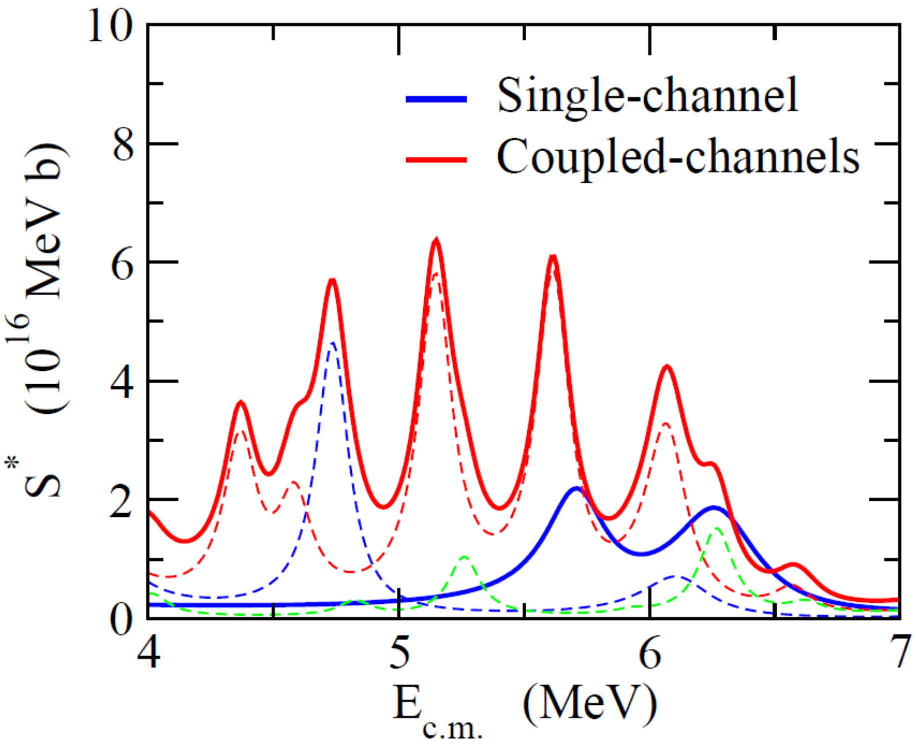
$\longrightarrow$ CDCC (Continuum Discretized Coupled-Channels) method

C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part





C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part

$$W(r) = -W_0 \cdot f_{\rm WS}(r) \to -w_0\,\rho_J(E^*) \cdot f_{\rm WS}(r)$$

G. Helling, W. Scheid, W. Greiner, PL 36B ('71) 64
H.-J. Fink, W. Scheid, W. Greiner, NPA188 ('72) 259
J.M. Quesada, M. Lozano, G. Madurga, PLB125 ('83) 14
M.V. Andres, Quesada, Lozano, Madurga, NPA443 ('85) 380

- ✓ *E* and *J* dependent imaginary potential
- ✓ system dependence through ρ(*E*)

cf. Fermi's golden rule

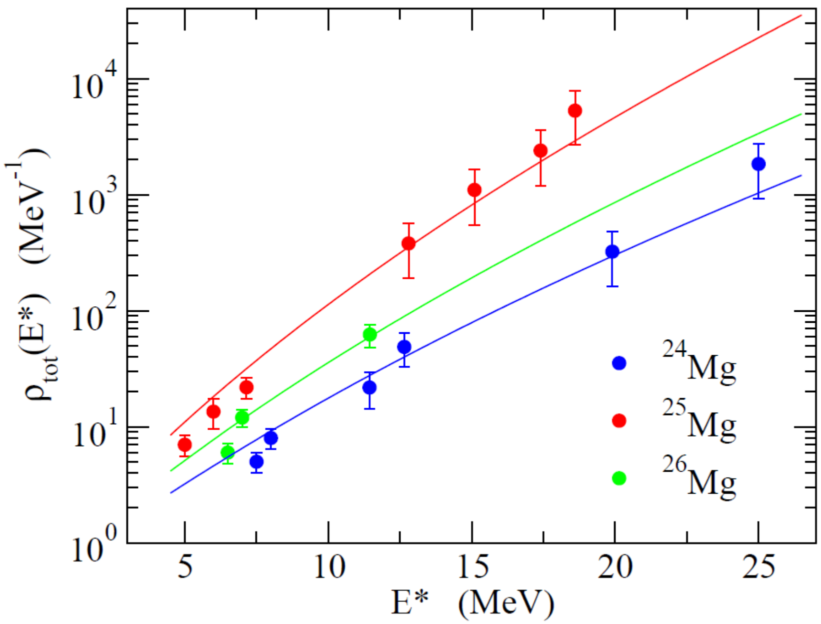
$$\frac{dw}{dt} = \frac{2\pi}{\hbar}\,|\langle\psi_{\rm CN}|V_{\rm int}|\psi_{\rm elastic}\rangle|^2\,\rho_J(E^*)$$

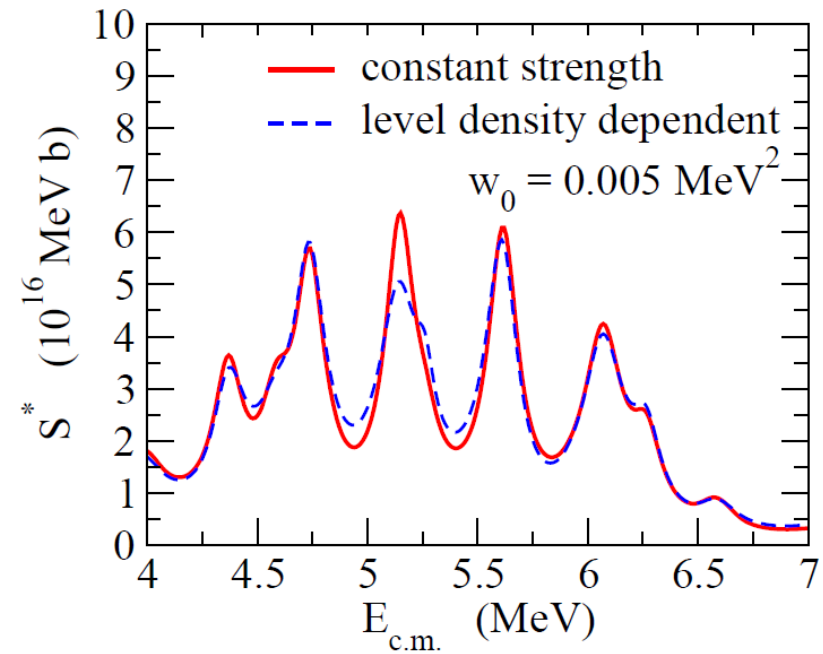
C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part

$$W(r) = -W_0 \cdot f_{WS}(r) \rightarrow -w_0\,\rho_J(E^*) \cdot f_{WS}(r)$$

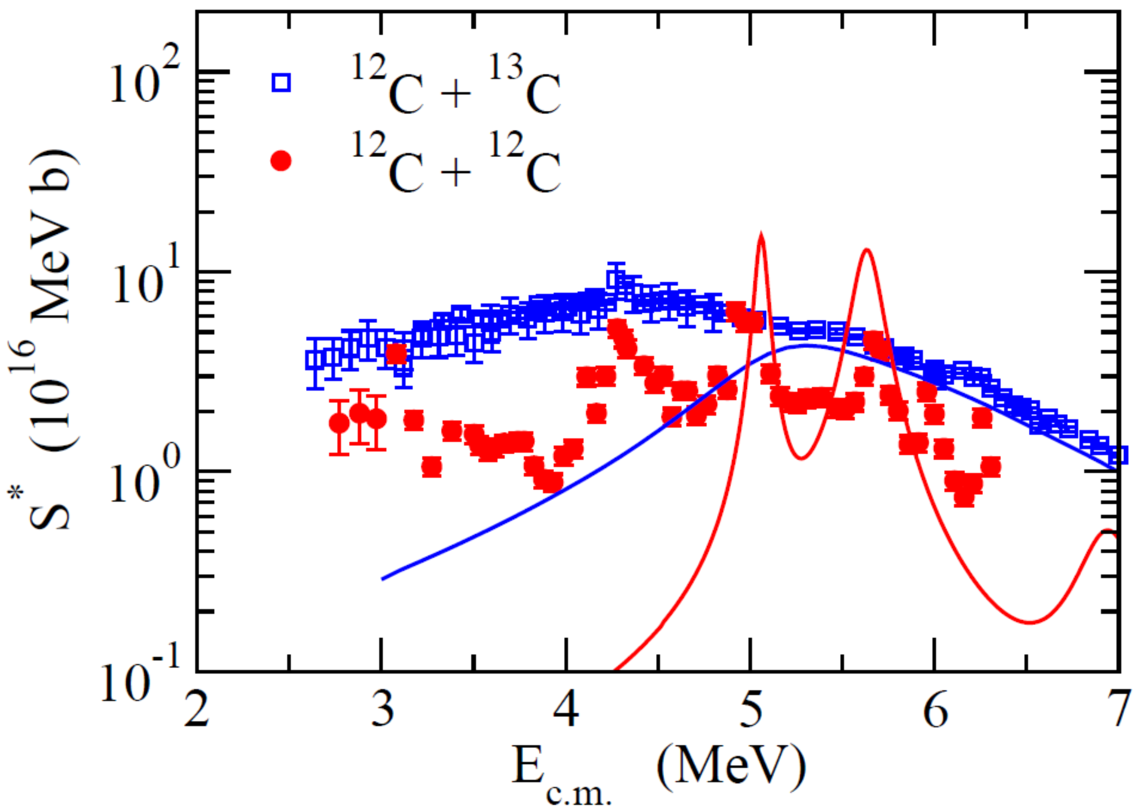




$$\rho_J(E^*) = \frac{(2J+1)e^{-(J+1/2)^2/2\sigma^2}}{4\sigma^3\sqrt{2\pi}} \frac{\sqrt{\pi}}{12} \frac{e^{2\sqrt{aE^*}}}{a^{1/4}(E^*)^{5/4}}$$

$$\left(\sigma^2 = 0.088\, a A^{2/3} \sqrt{\frac{E^*}{a}}\right)$$

Results of coupled-channels calculations



^{12}C (0^+, 2^+: 4.44)
^{13}C ($1/2^-$, $3/2^-$:3.68)
+ mutual excitations

✓ structured $^{12}C+^{12}C$
✓ smooth $^{12}C+^{13}C$

system dependence:
qualitatively reproduced

underestimate of fusion cross sections at deep subbarrier energies:
→ couplings to 3^- and 0_2^+ (Hoyle state)
a/o transfer channel $^{12}C(^{12}C,^{8}Be)^{16}O$?

cf. role of Hoyle state in $^{12}C+^{12}C$:
M. Assuncao and P. Descouvemont, PLB723 ('13) 355

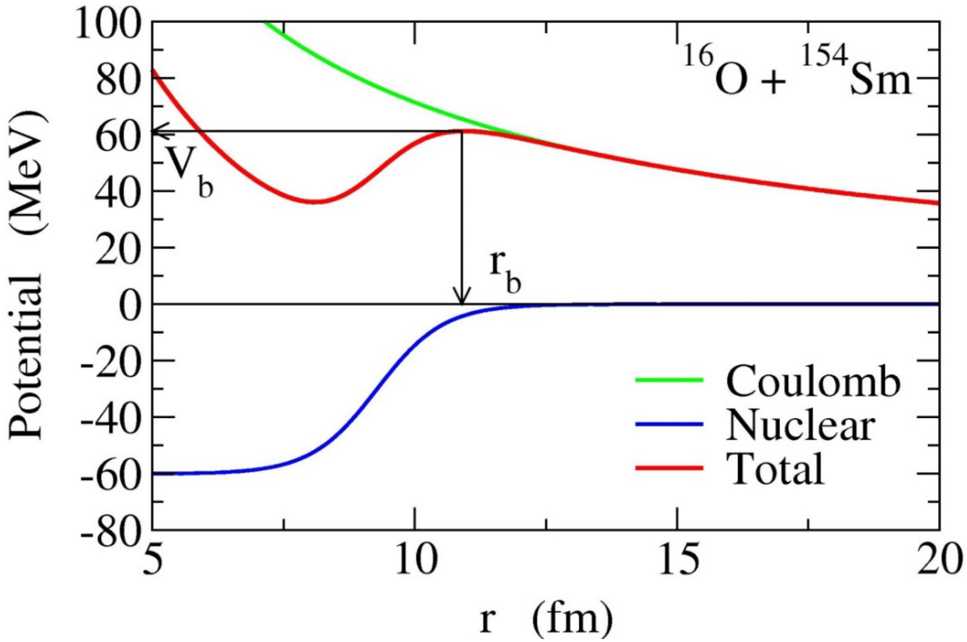
Fusion oscillations at above barrier energies

high-E : high level density of CN ⟶ overlapping resonances

⟶ strong absorption

$$\sigma_{\rm fus}(E)=\frac{\pi}{k^2}\sum_l(2l+1)P_l(E)$$

$P_l(E)$: barrier penetrability



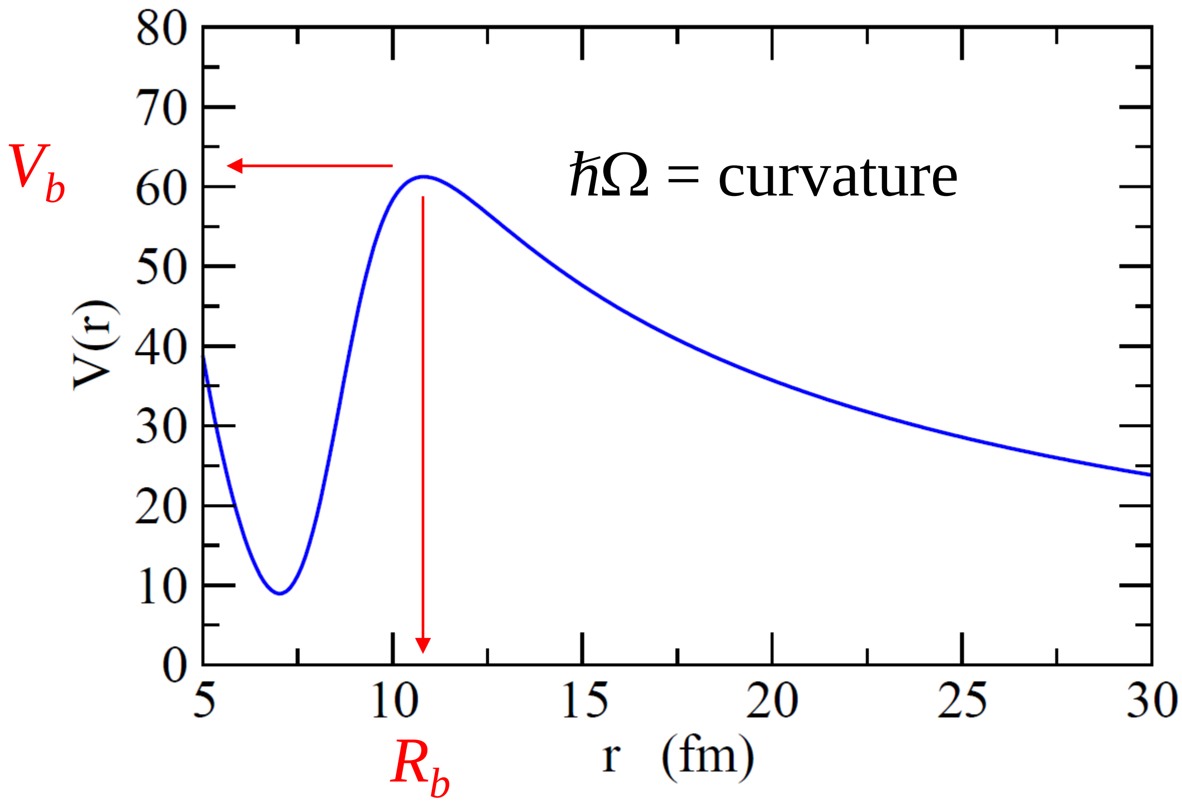
* Above barrier fusion: CC effects less important → single-ch. calcu.

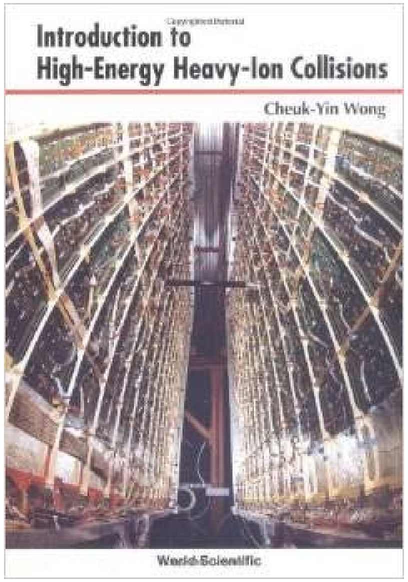
Wong's formula

C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

(single-channel)





Wong's formula

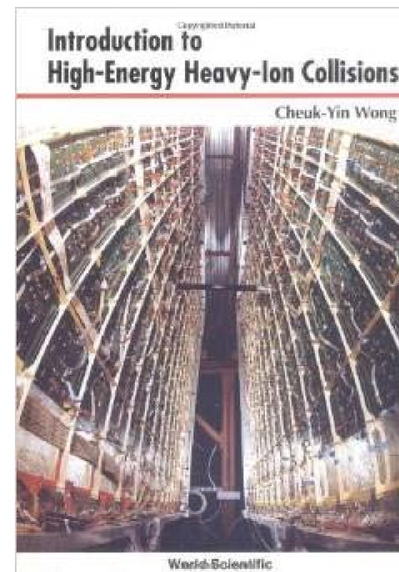
C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

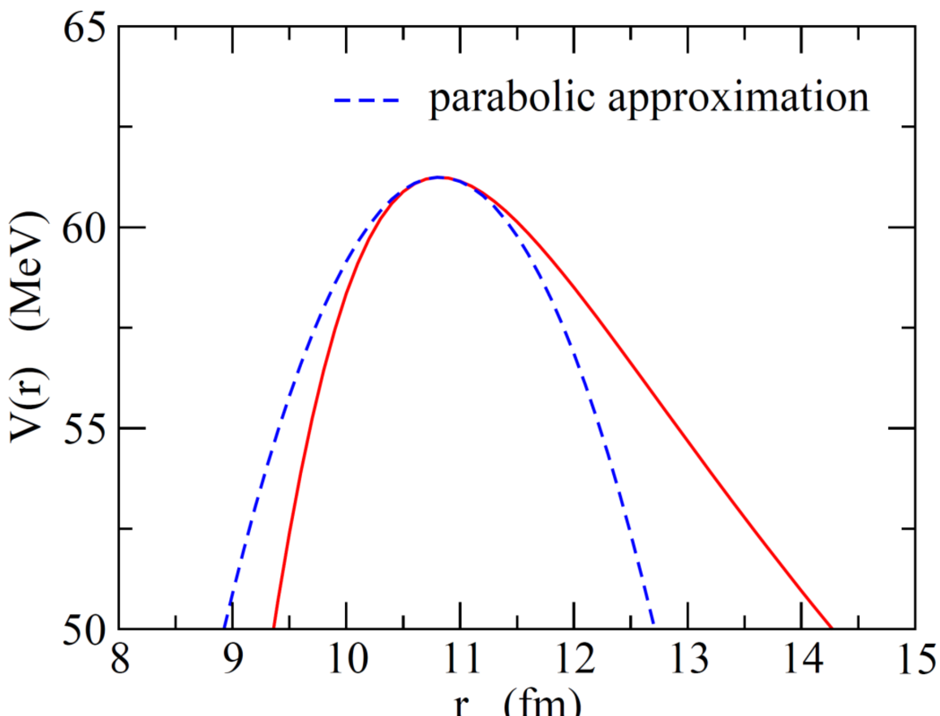
$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1+\exp\left(\frac{2\pi}{\hbar\Omega}(E-V_b)\right)\right]$$

i) Approximate the Coul. barrier by a parabola:

$$V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$$

$$\longrightarrow \quad P_0(E) = \frac{1}{1+\exp\left[\frac{2\pi}{\hbar\Omega}(V_b-E)\right]}$$





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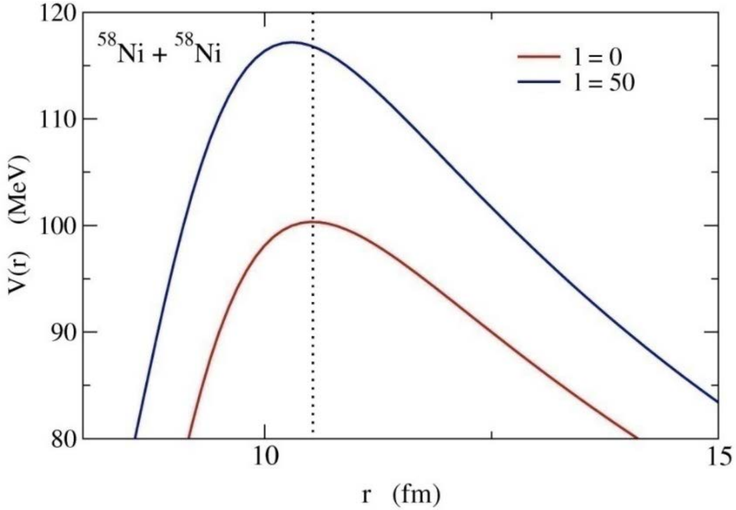
C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

i) Approximate the Coul. barrier by a parabola: $V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$

$$\longrightarrow \quad P_0(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$

ii) *l*-independent barrier position and curvature:

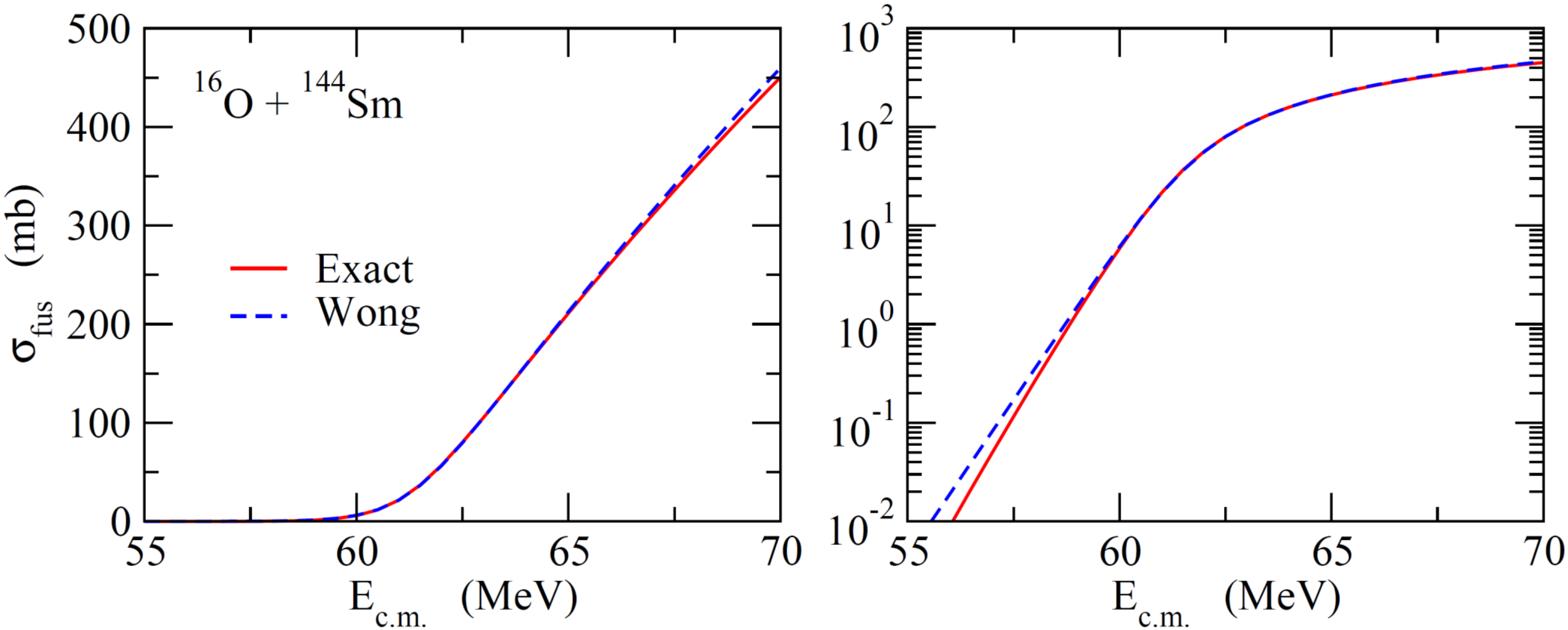
$$\longrightarrow \quad P_l(E) \sim P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$



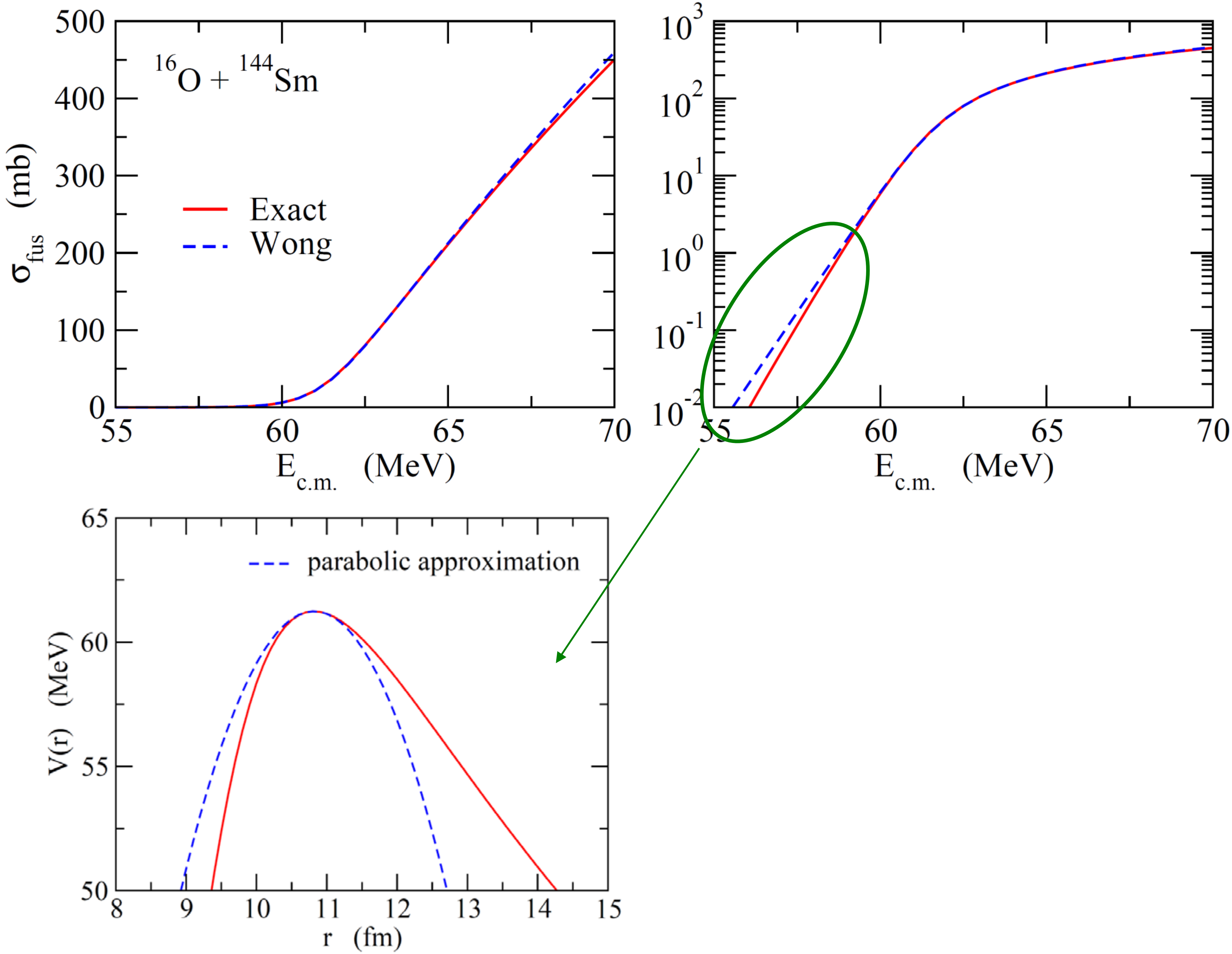
iii) Replace the sum of *l* with an integral

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \rightarrow \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$

$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E}R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

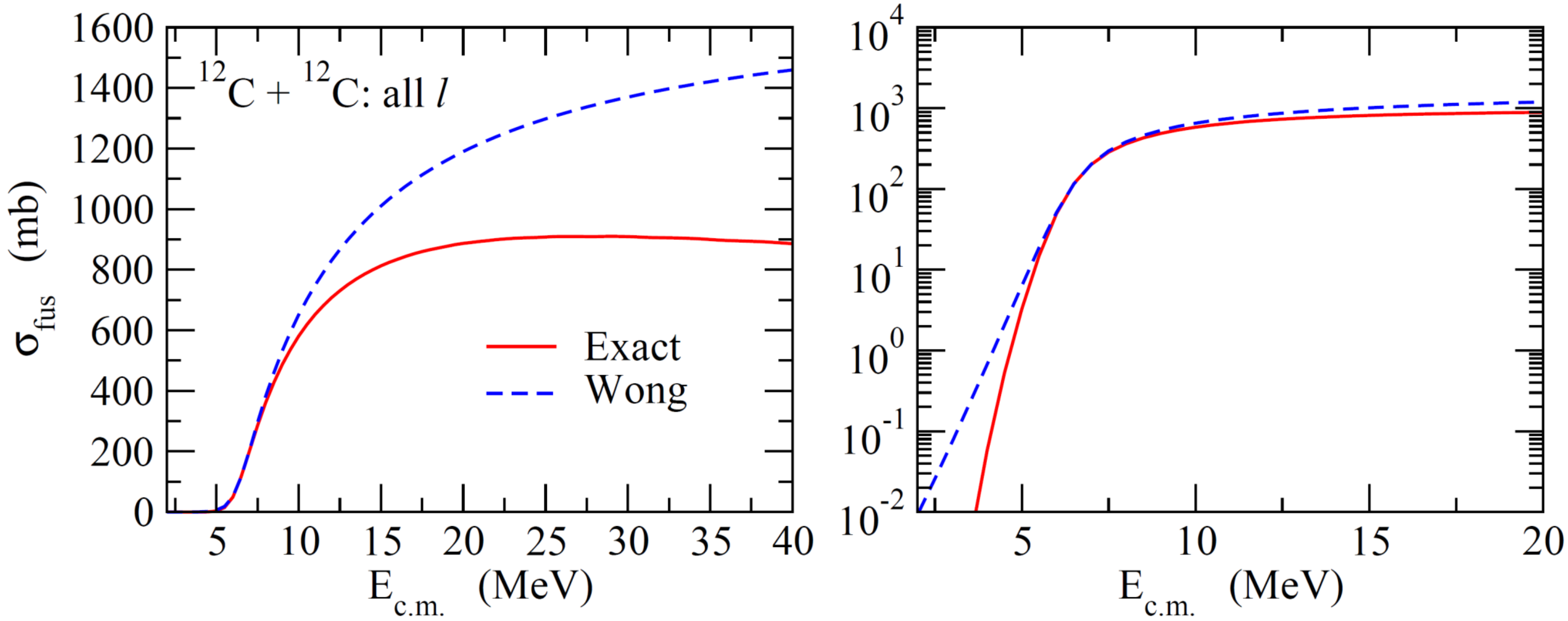


$^{16}O + ^{144}Sm$
Exact
Wong
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
$E_{c.m.}$ (MeV)



^{16}O + ^{144}Sm
Exact
Wong
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
parabolic approximation
V(r) (MeV)
r (fm)

Wong formula for light heavy-ion fusion



Wong formula:

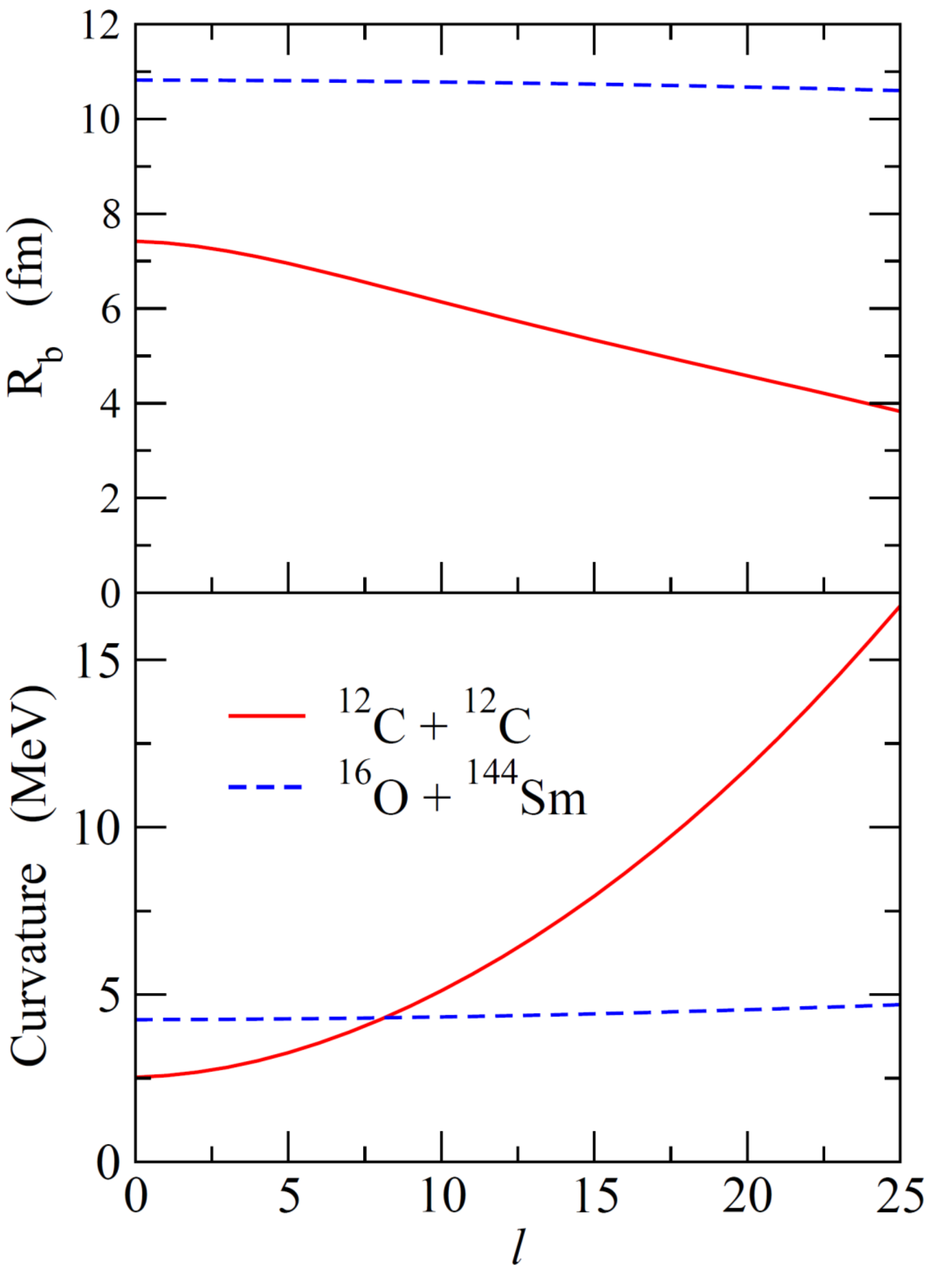
i) Approximate the Coul. barrier by a parabola

ii) l-independent barrier position and curvature ←

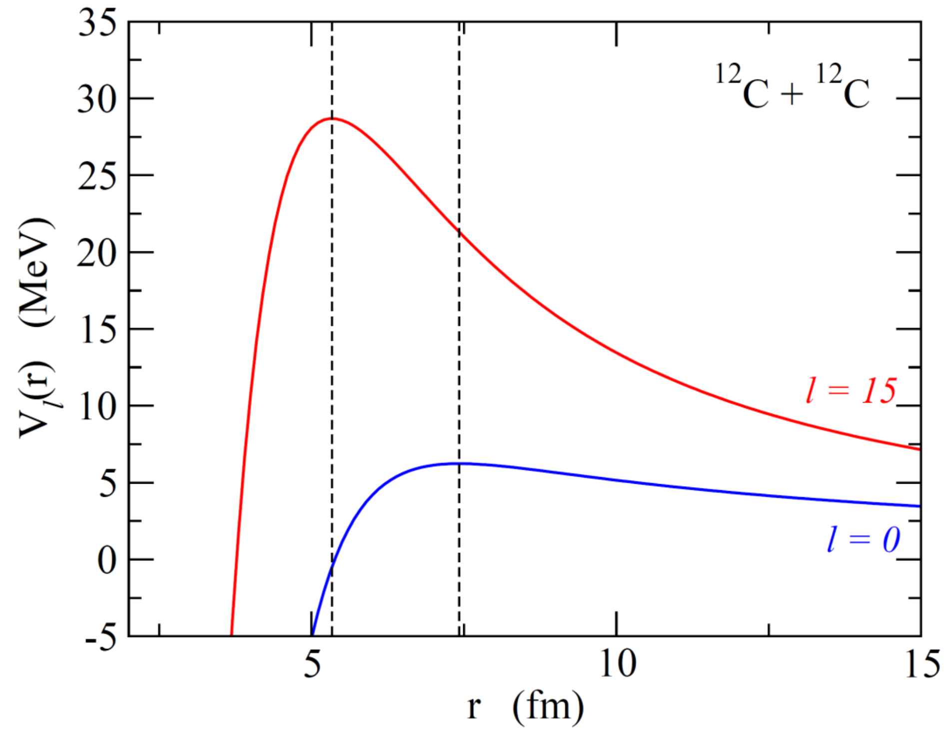
iii) Replace the sum of l with an integral

$$V_{\mathrm{cent}}(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

small



R_b (fm)
Curvature (MeV)
$^{12}C + ^{12}C$
$^{16}O + ^{144}Sm$
l

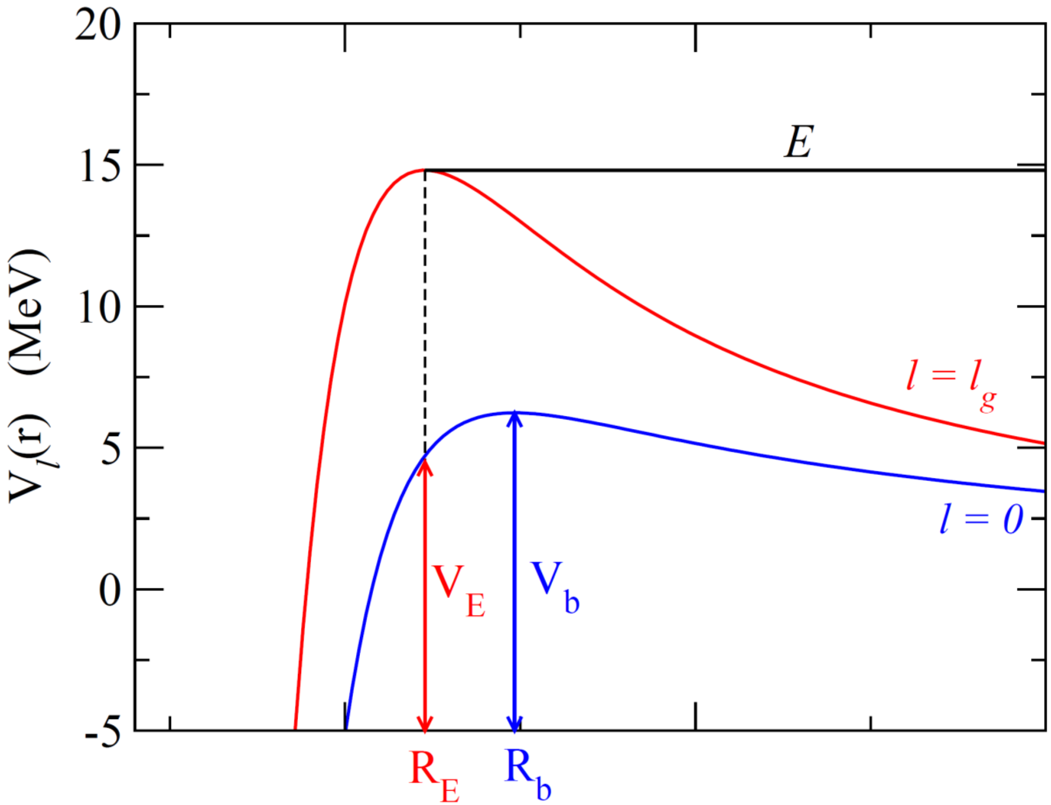


$^{12}C + ^{12}C$
$V_l(r)$ (MeV)
$l = 15$
$l = 0$
r (fm)

Generalized Wong formula

N. Rowley, A. Kabir, and R. Lindsay, J. of Phys. G15('89)L269

N. Rowley and K. Hagino, PRC91 (2015) 044617



use V_b, R_b, and Ω for the grazing angular momentum, l_g

(note)

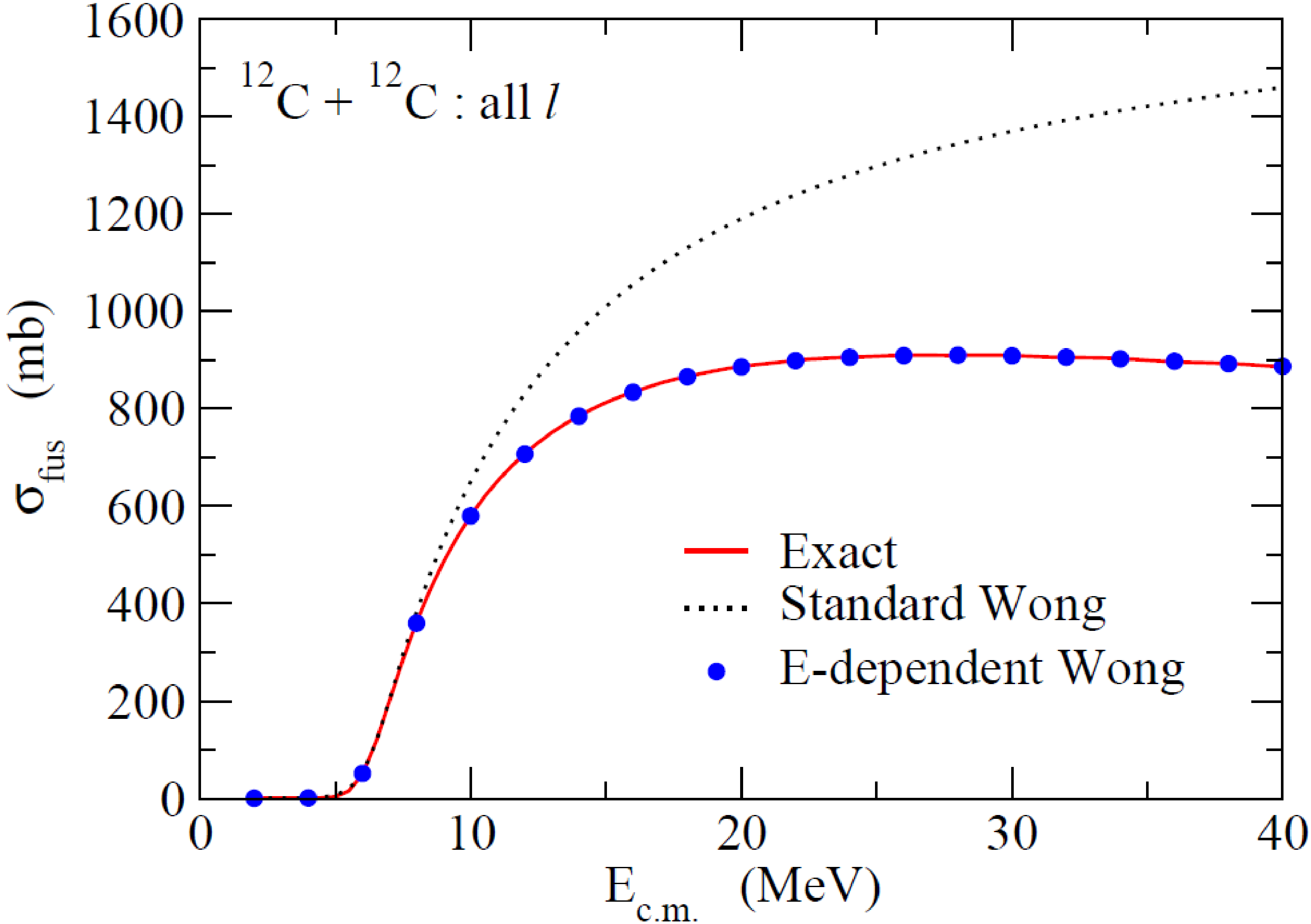
$$\begin{cases} \sigma_{\rm cl} = \pi b_g^2 \\ E = V_E + \dfrac{(kb_g)^2\hbar^2}{2\mu R_E^2} \end{cases}$$

$$\longrightarrow \sigma_{\rm cl} = \pi R_E^2(1 - V_E/E)$$

$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

$$\Longrightarrow \boxed{\sigma_{\rm fus}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]}$$

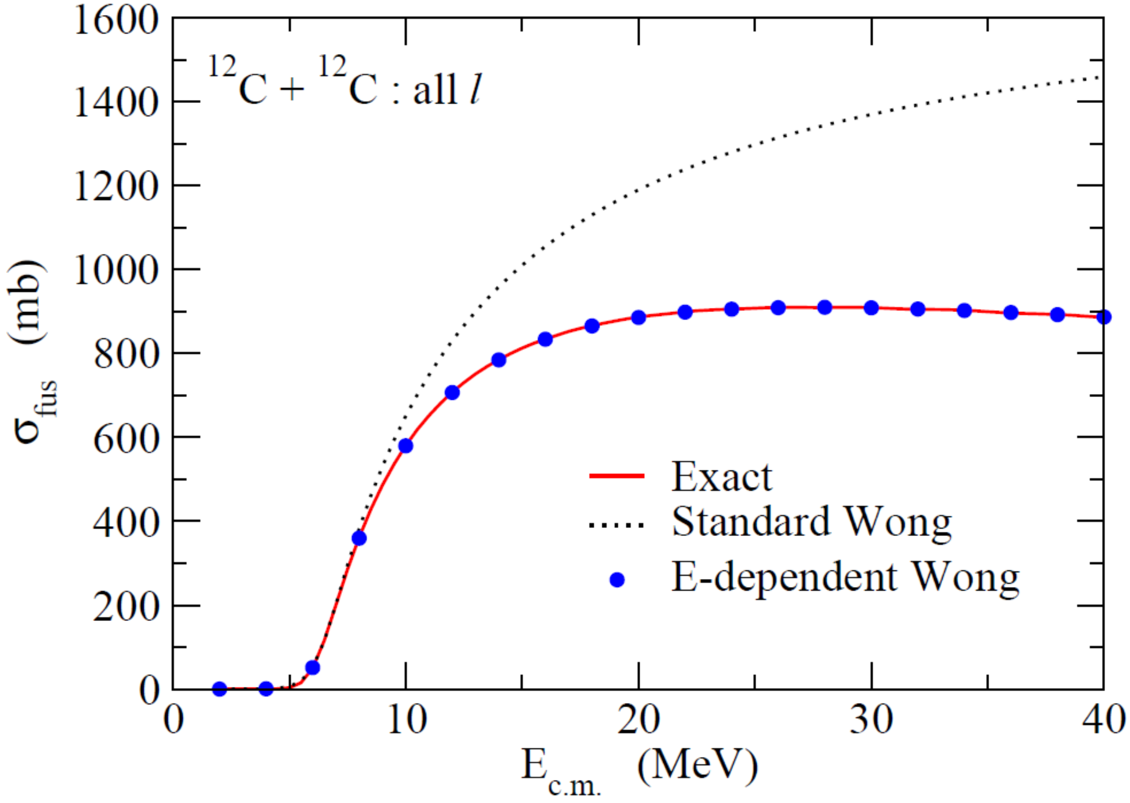
$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]$$



Continuum approximation

Wong formula:

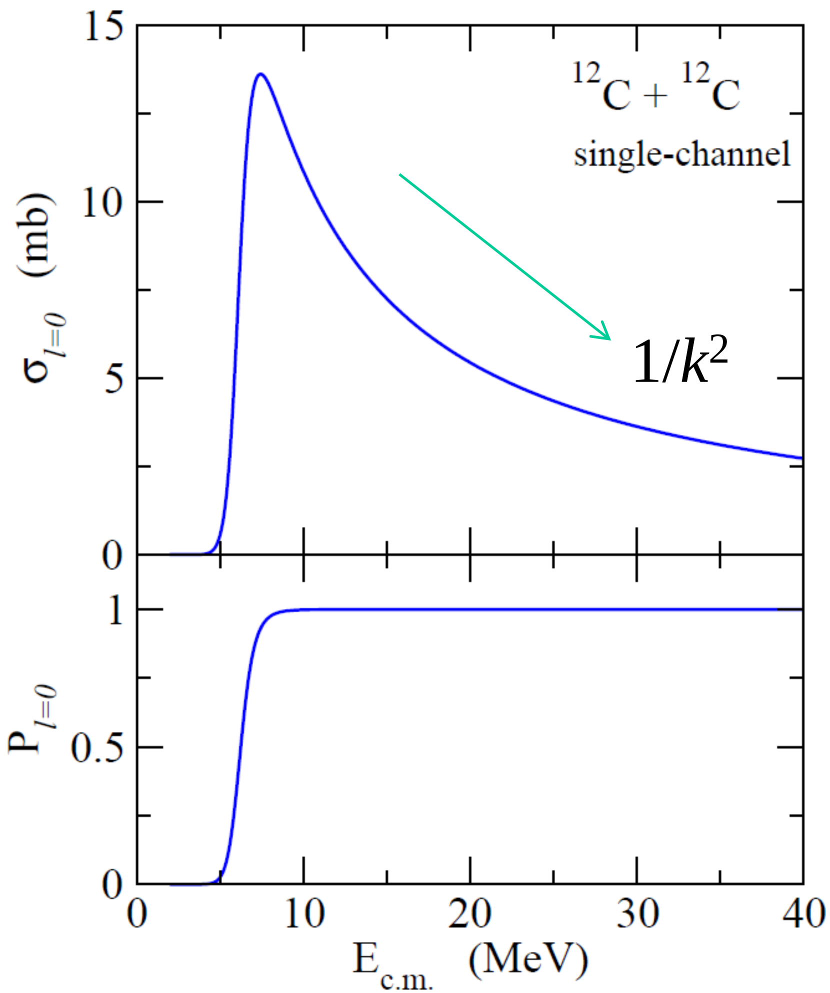
$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \rightarrow \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$

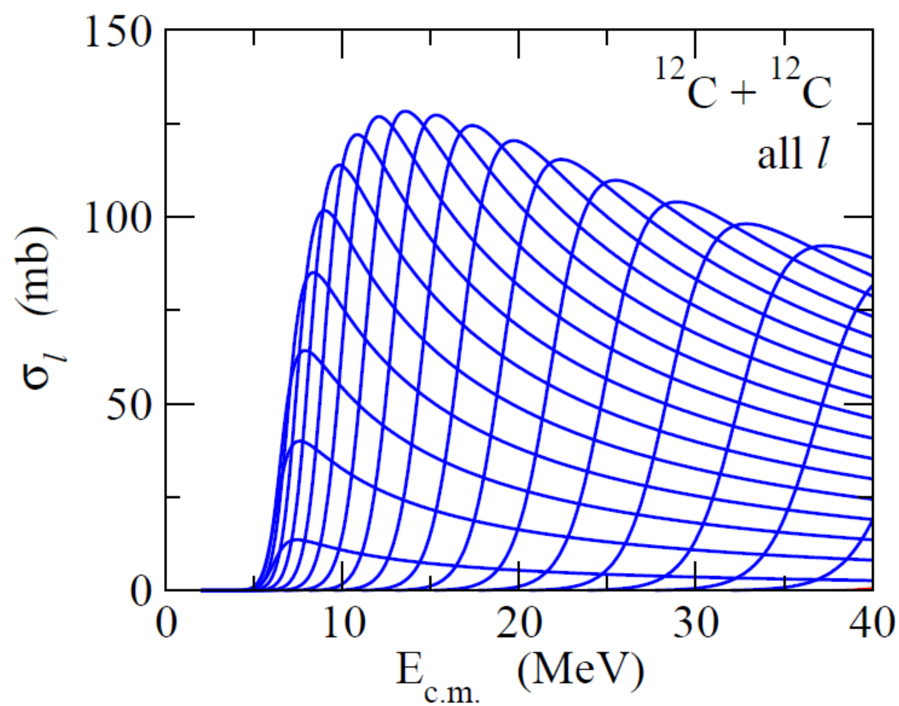


the continuum approximation: appears very good but…..

Fusion oscillations at above barrier energies

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$





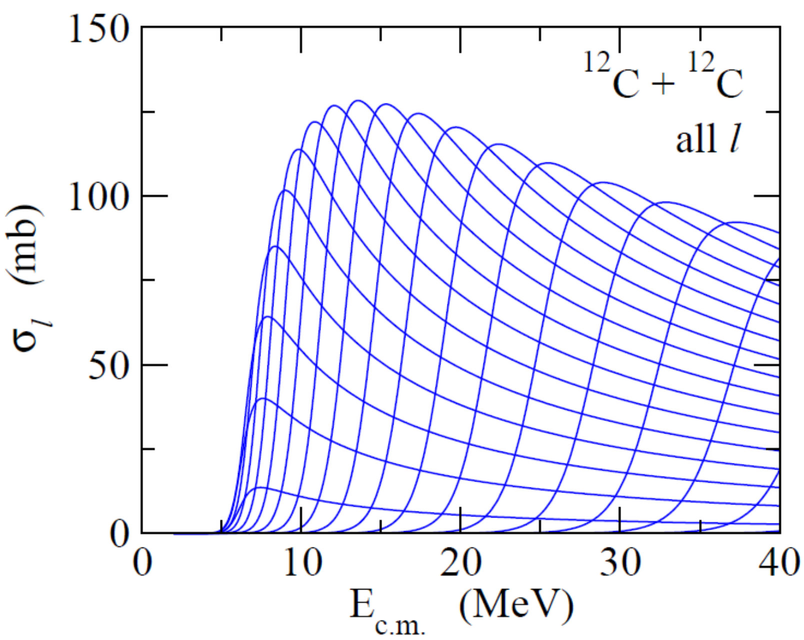
discrete l-sum

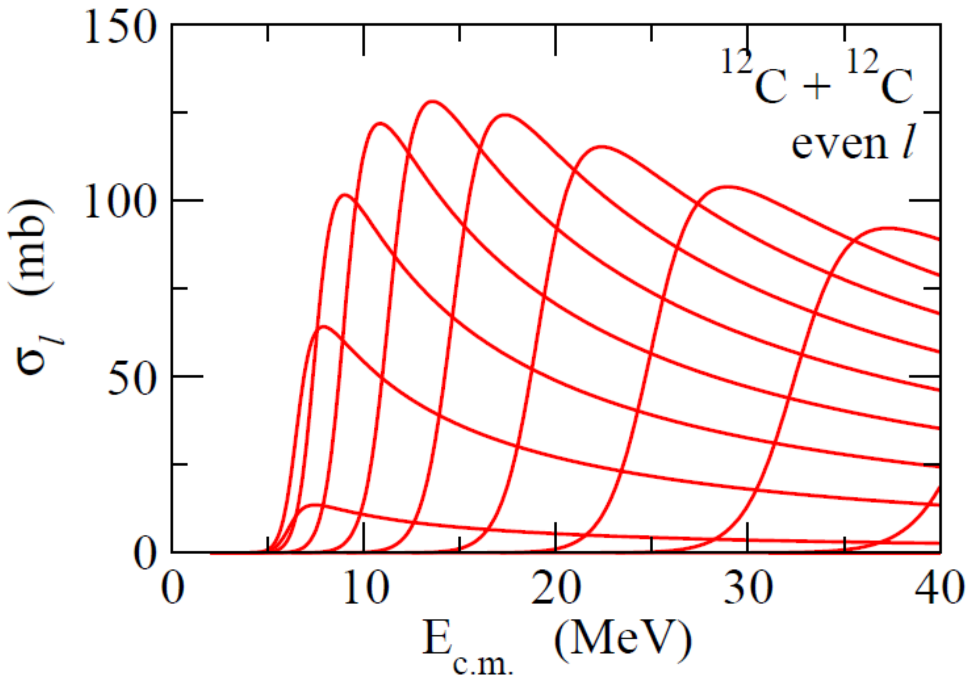
⟶ (oscillatory) structure in $\sigma_{\rm fus}$

effect of symmetrization: fusion oscillations in light symmetric systems

fusion of identical spin-zero bosons: wf has to be symmetric

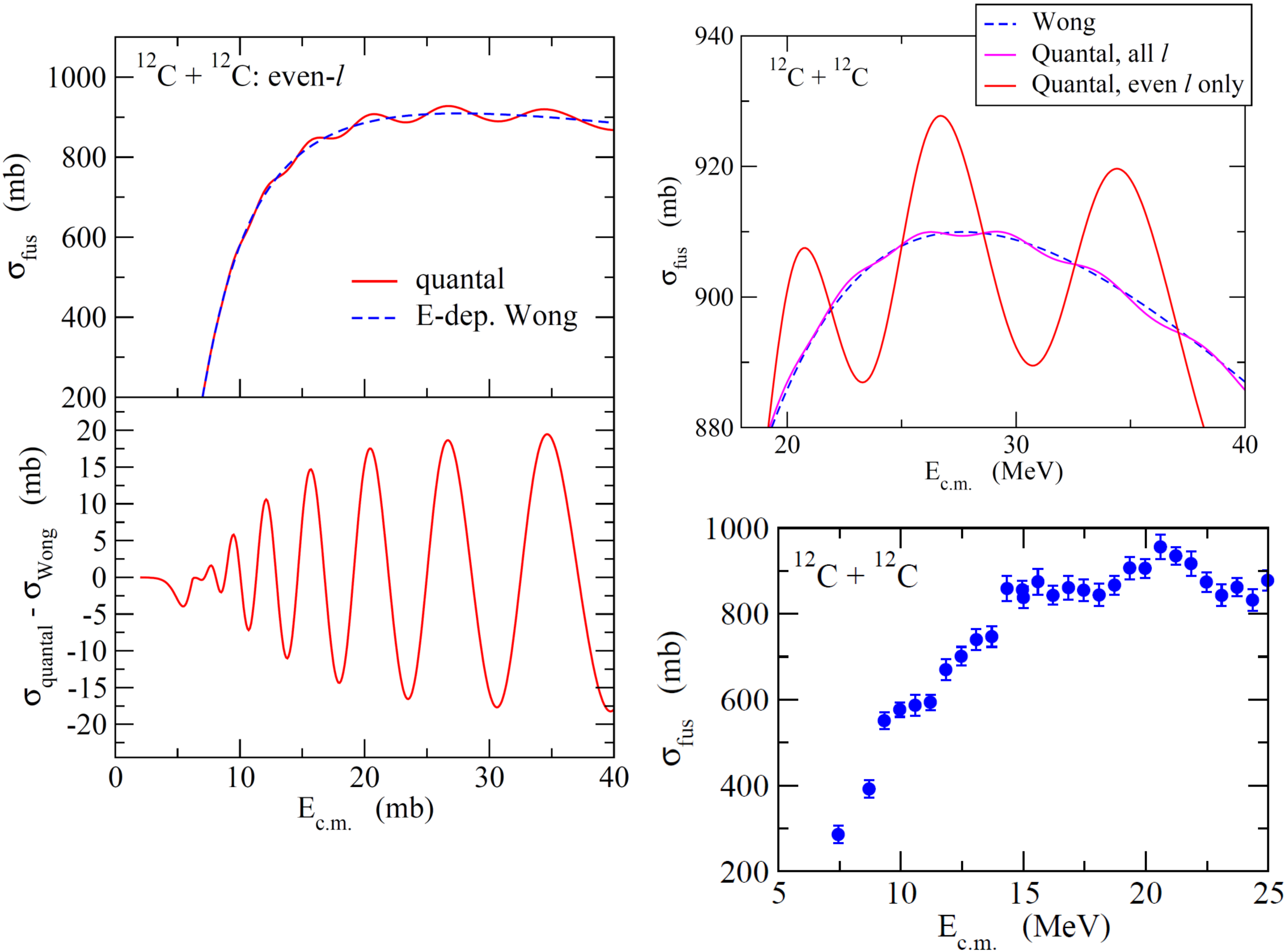
$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \rightarrow \frac{\pi}{k^2}\sum_l (1+(-)^l)(2l+1)P_l(E)$$





✓the angular mom. is quantized in units of 2-hbar

✓a larger amplitude of fusion oscillations



^{12}C + ^{12}C: even-l
quantal
E-dep. Wong
σ_{fus} (mb)
$\sigma_{quantal} - \sigma_{Wong}$ (mb)
$E_{c.m.}$ (mb)
Wong
Quantal, all l
Quantal, even l only
^{12}C + ^{12}C
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
^{12}C + ^{12}C
σ_{fus} (mb)
$E_{c.m.}$ (MeV)

Analytic formula for fusion oscillations

N. Poffe, N. Rowley, and R. Lindsay, Nucl. Phys. A410 ('83) 498
N. Rowley and K. Hagino, PRC91 (2015) 044617

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (1\pm(-)^l)(2l+1)P_l(E)$$

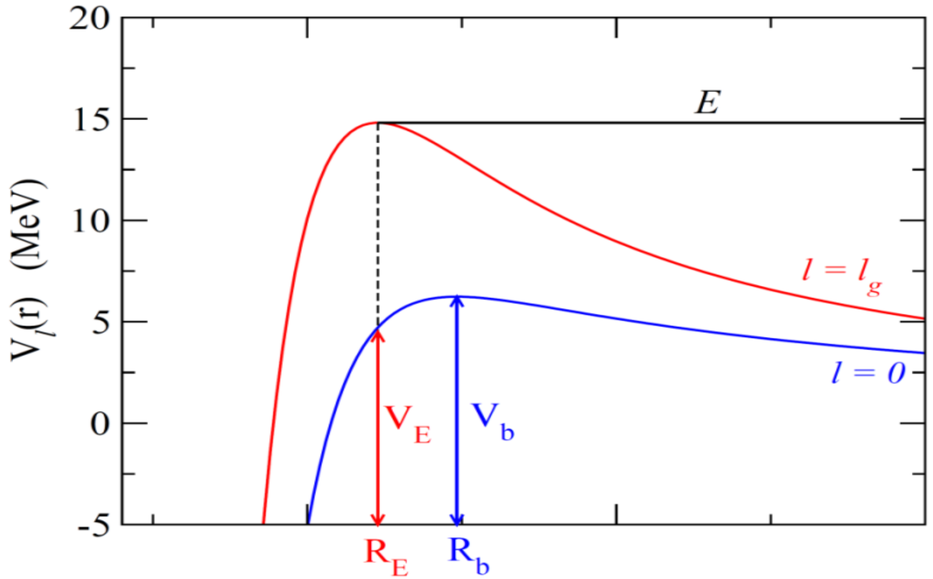
$$\sim \sigma_{\rm E-Wong} \pm 2\pi R_E^2 \frac{\hbar\Omega_E}{E} e^{-\xi}\sin(\pi l_g)$$

← Poisson sum formula

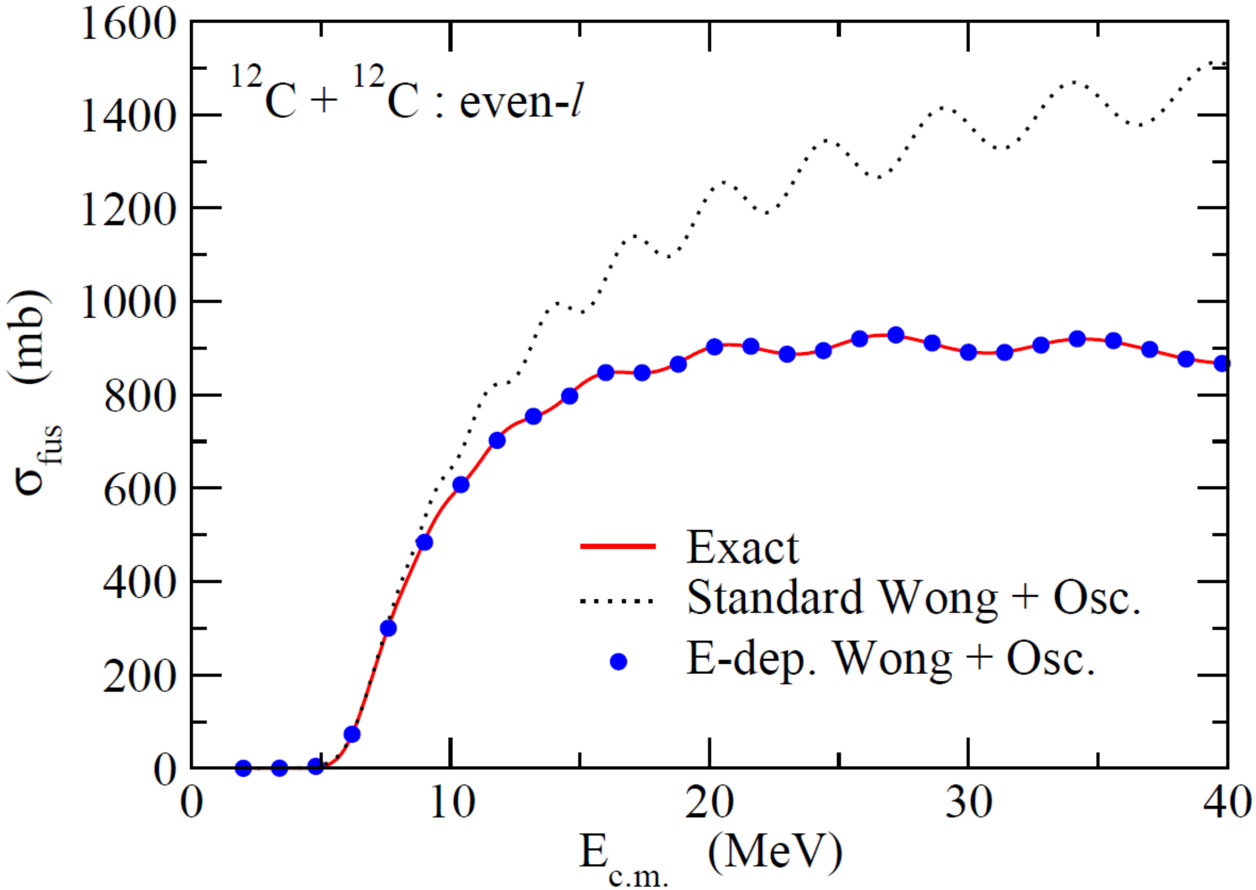
$$\xi = \pi\cdot\frac{\hbar\Omega_E}{2l_g+1}\cdot\frac{\mu R_E^2}{\hbar^2}$$

Poisson sum formula:

$$\sum_{l=0}^{\infty} f(l) = \sum_m (-1)^m \int_0^\infty f(\lambda)e^{2\pi m i\lambda}d\lambda$$



$$\sigma_{\rm osc}(E) = 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \sin(\pi l_g), \qquad \xi = \pi \cdot \frac{\hbar\Omega}{2l_g+1} \cdot \frac{\mu R_b^2}{\hbar^2}$$



(note)

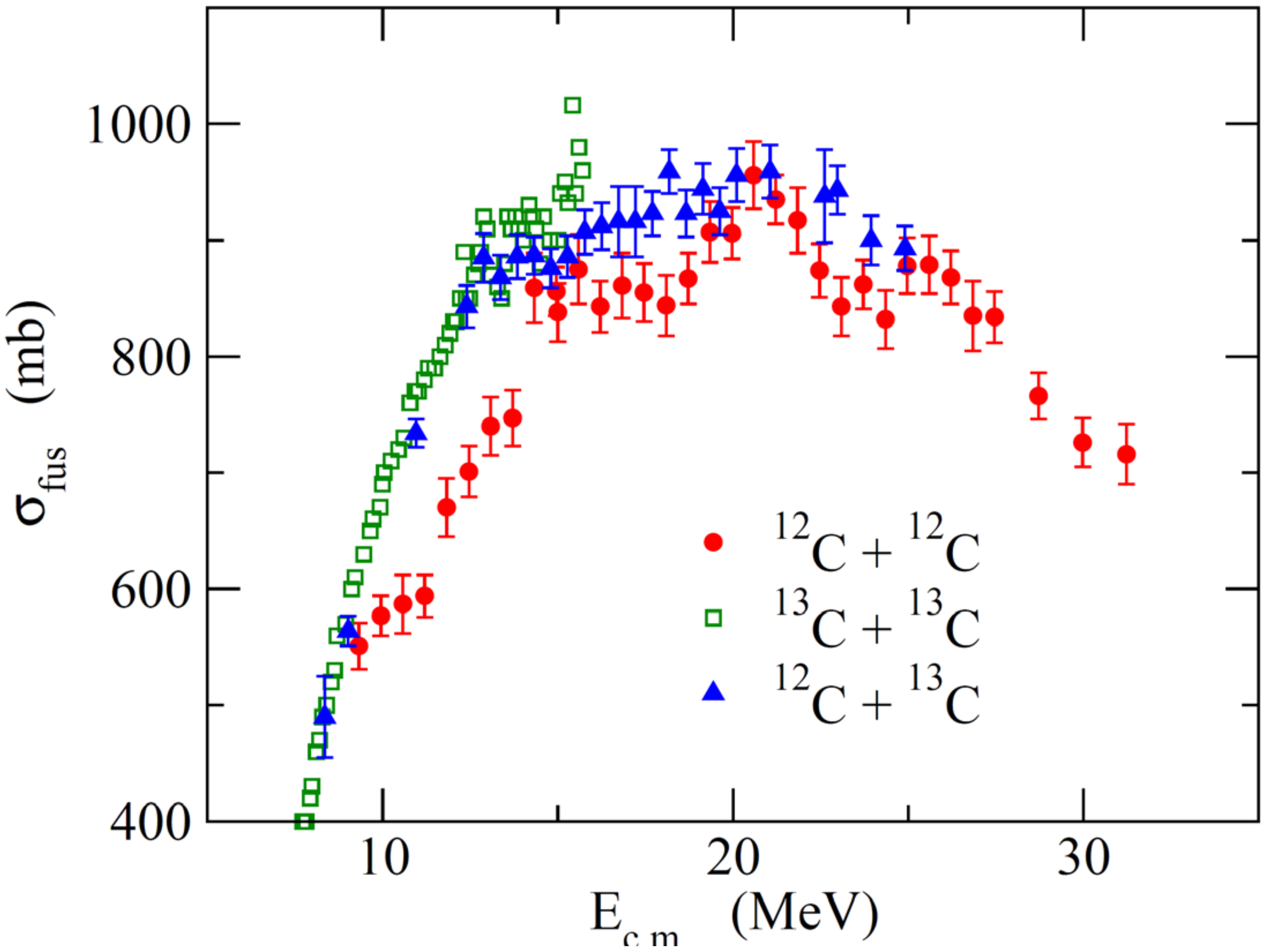
$$\frac{|\sigma_{\rm osc}|}{\sigma_{\rm Wong}} \sim \frac{2\hbar\Omega}{E - V_b} \cdot e^{-\xi}$$

$$2l_g + 1 \gg \pi\hbar\Omega \cdot \frac{\mu R_b^2}{\hbar^2}$$

in order for the osc. to be visible

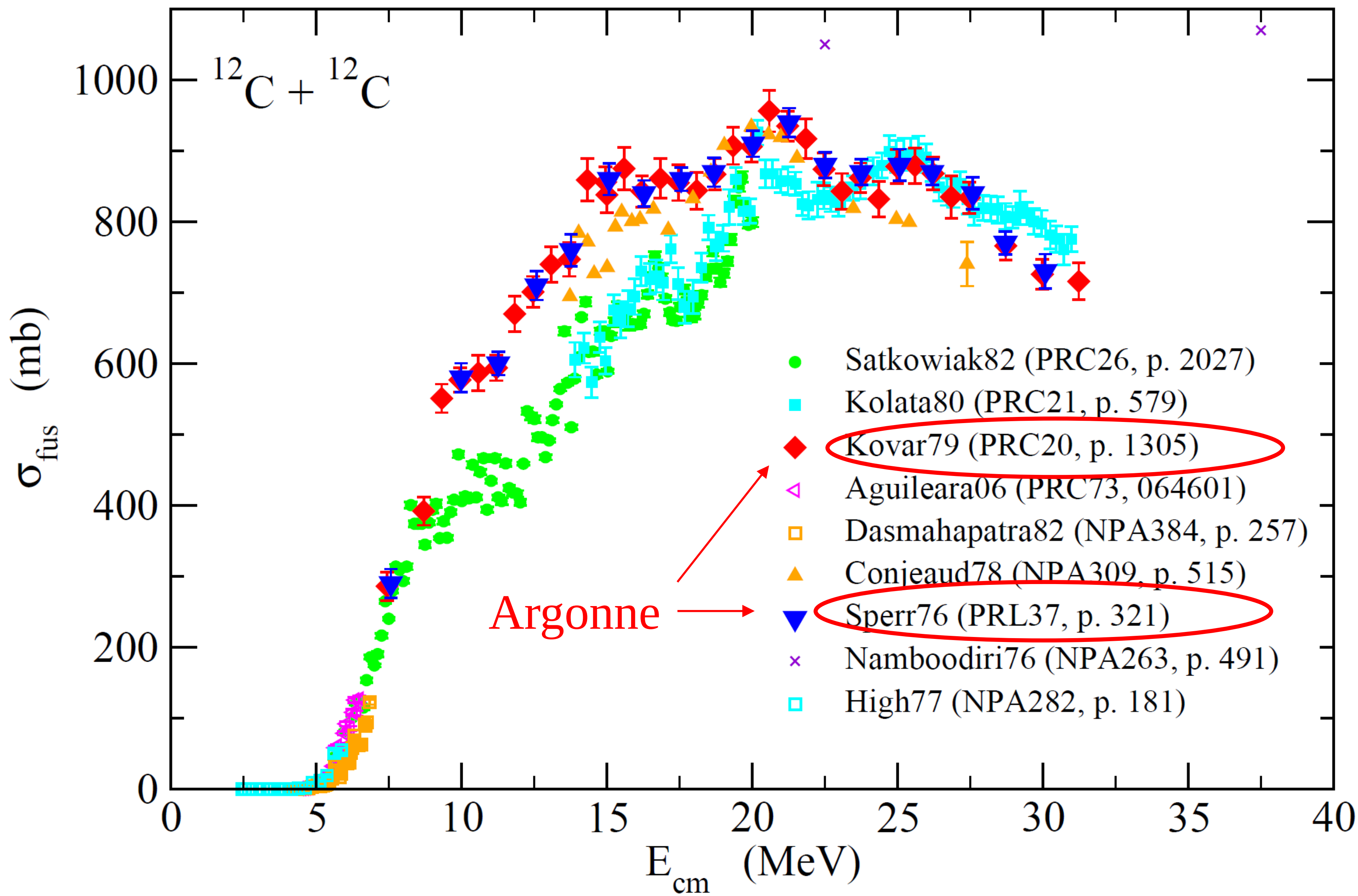
$\longrightarrow$ light symmetric systems

Comparison with experimental data



analyses with single-channel calculations

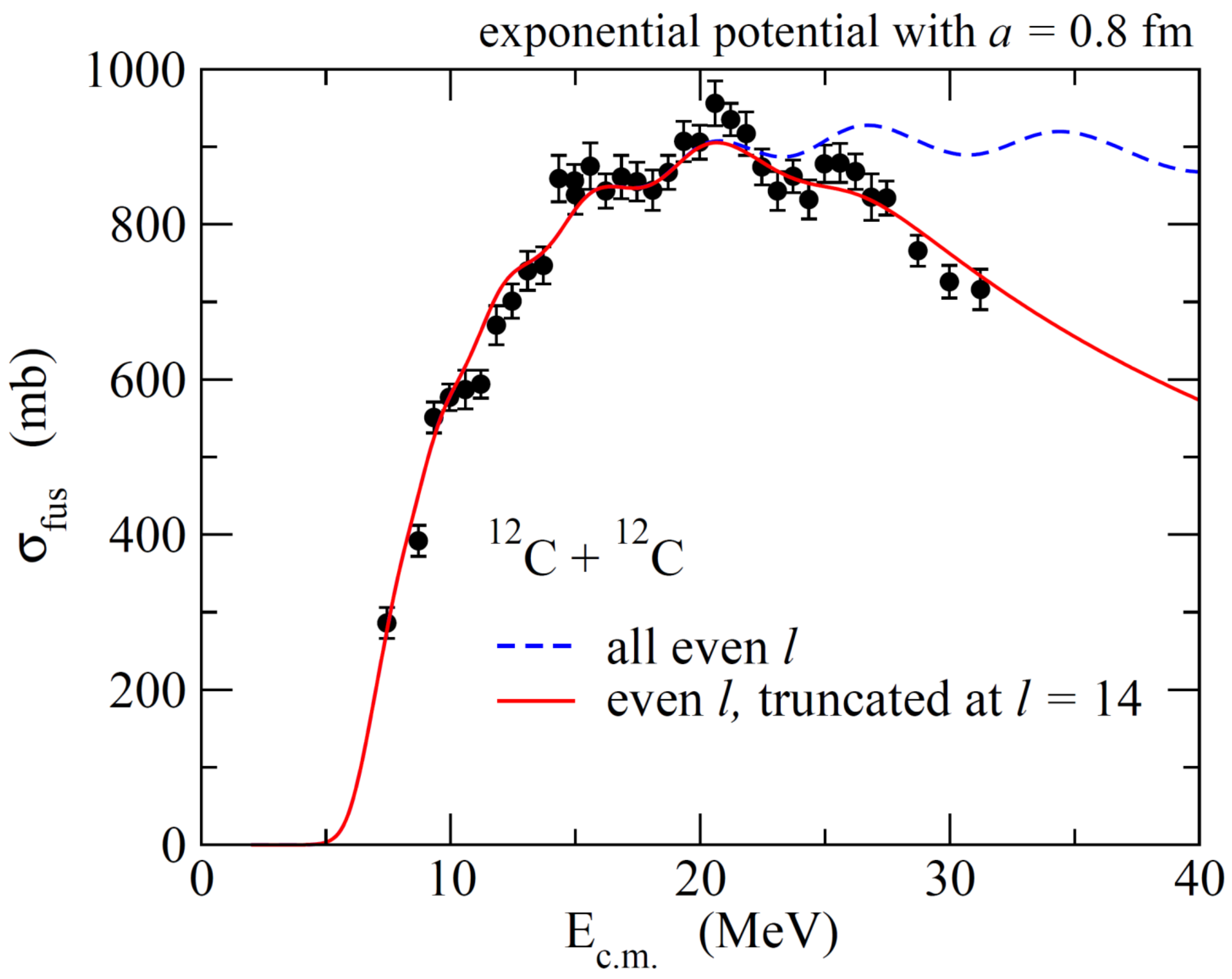
i) $^{12}C + ^{12}C$



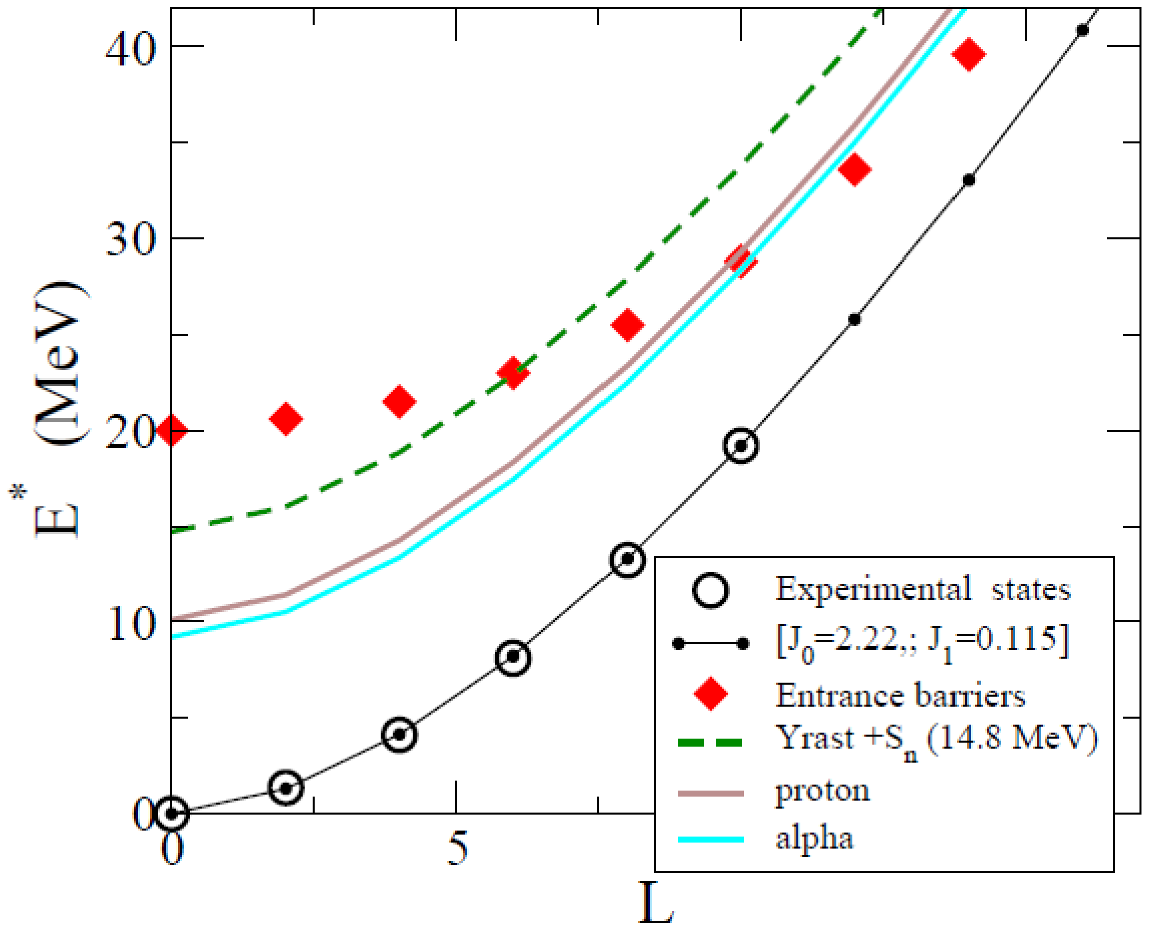
12C + 12C
1000
800
600
400
200
0
0
5
10
15
20
25
30
35
40
σfus (mb)
Ecm (MeV)
Satkowiak82 (PRC26, p. 2027)
Kolata80 (PRC21, p. 579)
Kovar79 (PRC20, p. 1305)
Aguileara06 (PRC73, 064601)
Dasmahapatra82 (NPA384, p. 257)
Conjeaud78 (NPA309, p. 515)
Sperr76 (PRL37, p. 321)
Namboodiri76 (NPA263, p. 491)
High77 (NPA282, p. 181)
Argonne

i) Comparison with the experimental data: $^{12}C + ^{12}C$

$^{12}C_{g.s.}: 0^+ \longrightarrow$ the relative w.f. has to be spatially symmetric



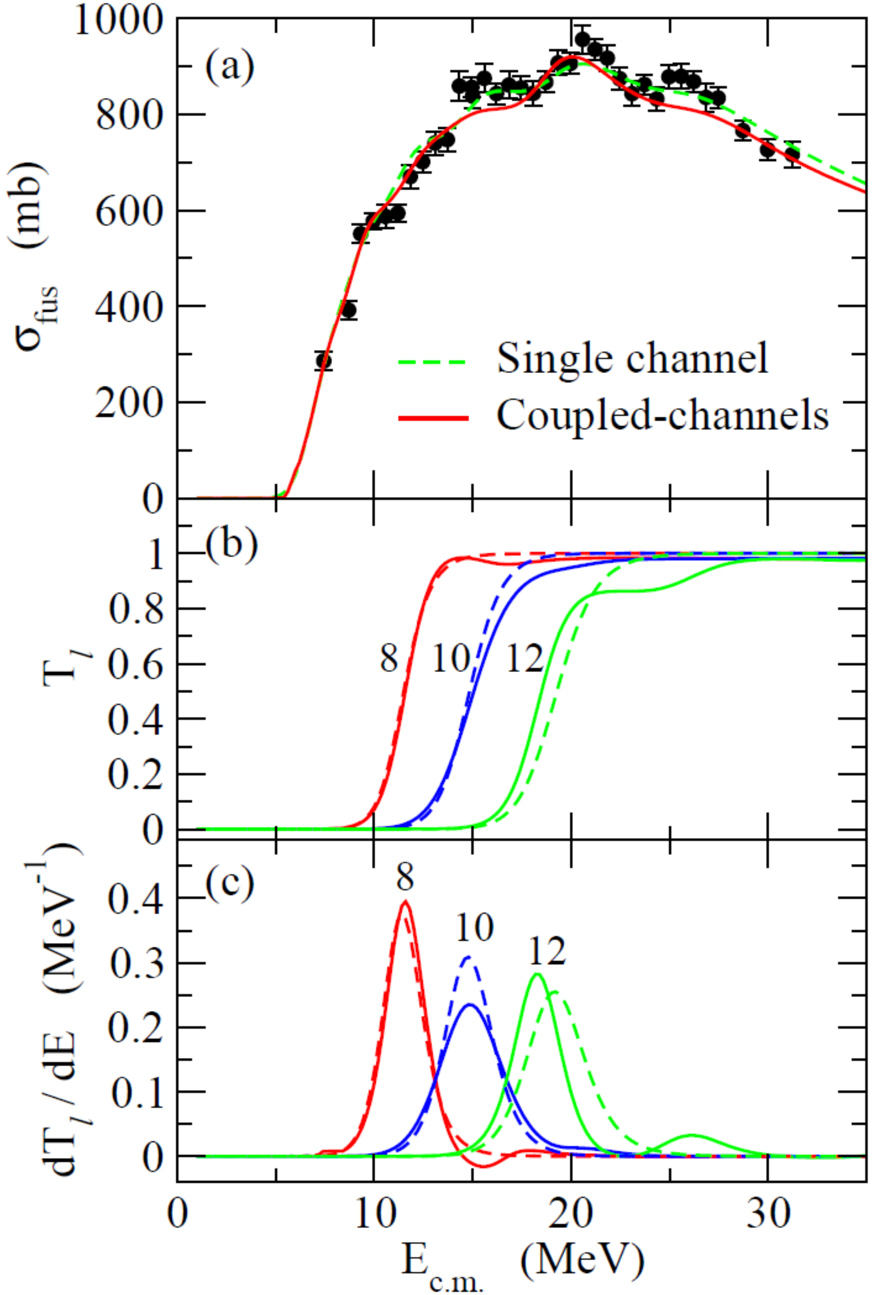
Barriers and Yrast line for ^{24}Mg



S_n = 16.5 MeV, S_p = 11.69 MeV

→ high *l*: particle evaporation inhibited
fission a/o γ-ray

Role of channel couplings



The main features of oscillations (the peak energies and the phase) : not affected much

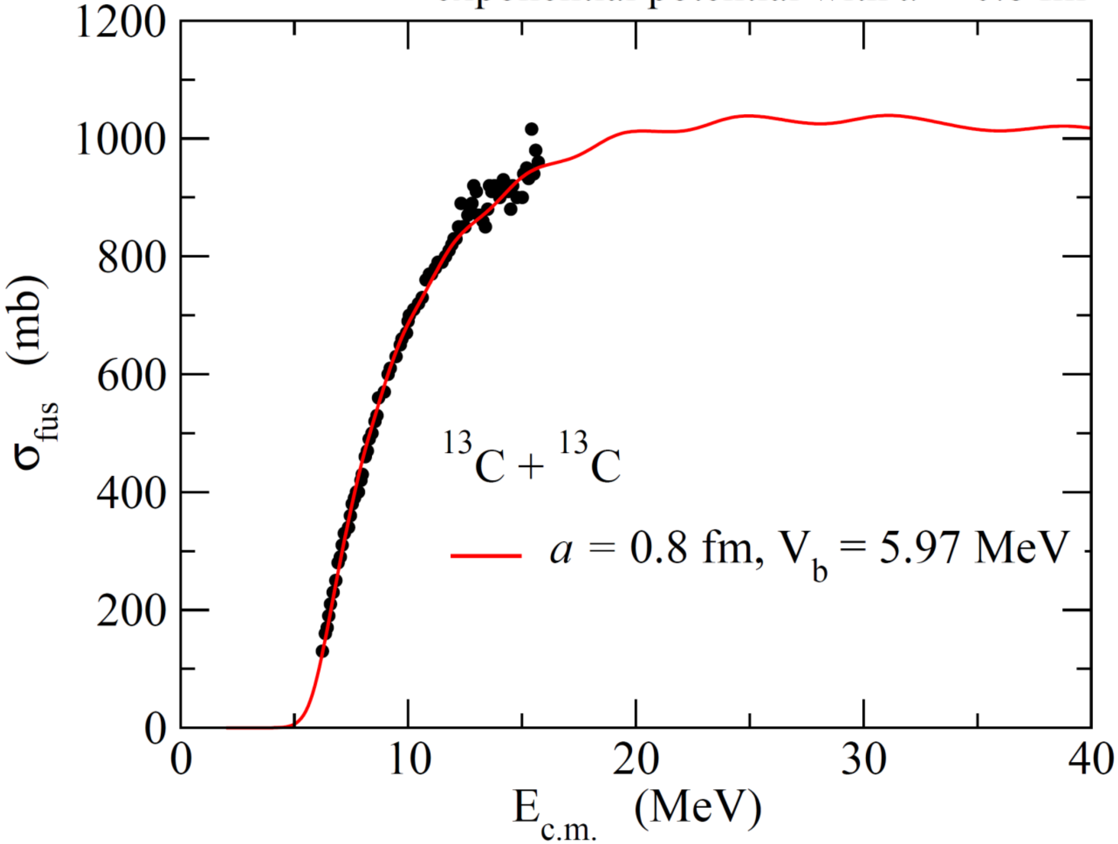
ii) $^{13}C + {}^{13}C$

$^{13}C_{g.s.}$: $1/2^-$ $\longrightarrow$ the relative w.f. has to be spatially symmetric for $S = 0$
spatially anti-symmetric for $S = 1$

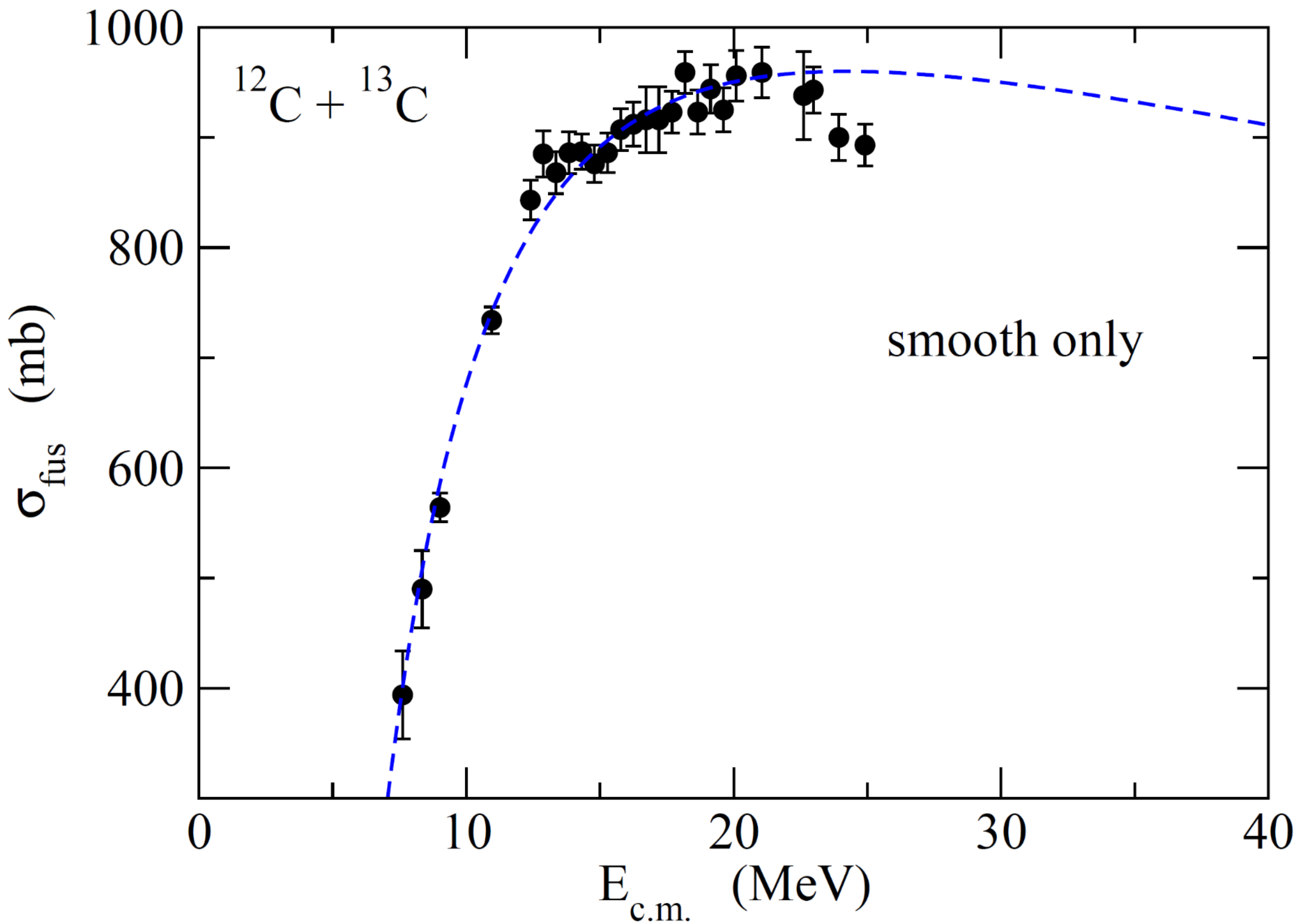
$$\sum_l \to \frac{1}{4}\sum_l \left(1 + (-1)^l\right) + \frac{3}{4}\sum_l \left(1 - (-1)^l\right)$$

$$\sigma_{\rm osc} = \frac{1}{2}\sigma_{\rm osc}({\rm odd} - l)$$

exponential potential with $a = 0.8$ fm

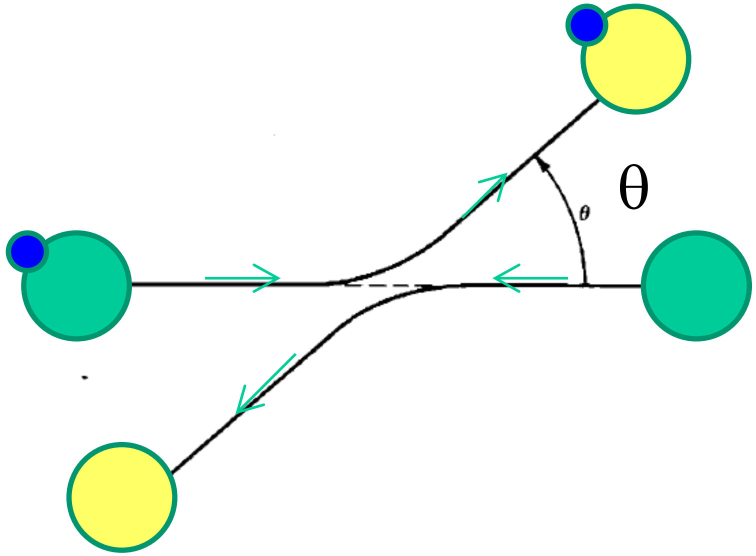


iii) $^{12}C + ^{13}C$



1000
800
600
400
0
10
20
30
40
$^{12}C + ^{13}C$
smooth only
σ_{fus} (mb)
$E_{c.m.}$ (MeV)

role of elastic transfer



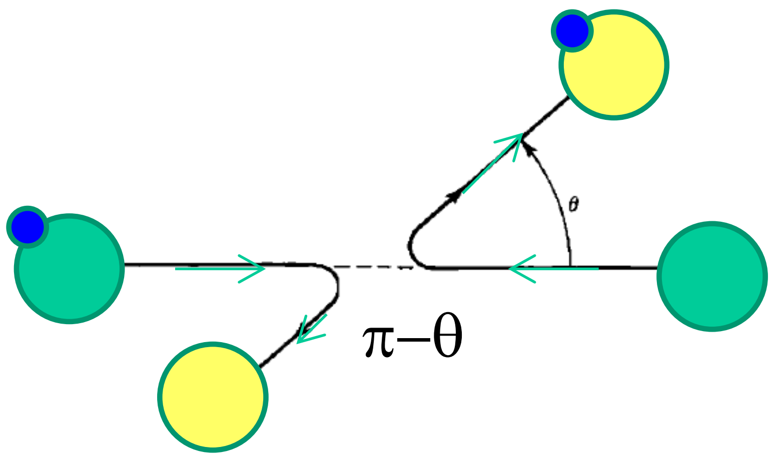
elastic scattering

$f_{\mathrm{el}}(\theta)$

indistinguishable

transfer

$f_{\mathrm{trans}}(\pi - \theta)$



$$f(\theta) \rightarrow f_{\mathrm{el}}(\theta) + f_{\mathrm{trans}}(\pi - \theta)$$

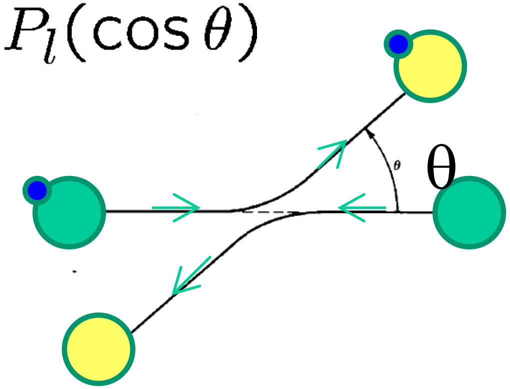
role of elastic transfer

$$f(\theta) \to f_{\text{el}}(\theta) + f_{\text{trans}}(\pi - \theta)$$

$$f_{\text{el}}(\theta) = \sum_l (2l+1) \frac{S_l^{\text{el}} - 1}{2ik} P_l(\cos\theta)$$

$$f_{\text{trans}}(\pi - \theta) = \sum_l (2l+1) \frac{S_l^{\text{trans}}}{2ik} P_l(\cos(\pi - \theta))$$

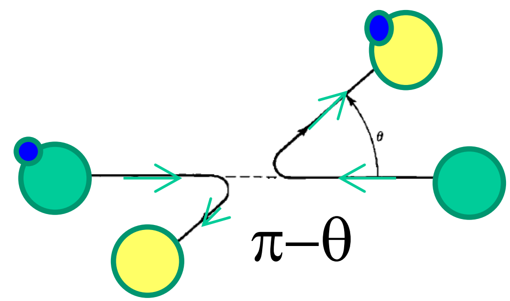
$$= (-)^l P_l(\cos\theta)$$



$$S_l^{\text{eff}} = S_l^{\text{el}} + (-1)^l \, S_l^{\text{trans}}$$

if $\quad S_l^{\text{trans}} \sim \alpha \dfrac{\partial S_l^{\text{el}}}{\partial l}$

$$S_l^{\text{eff}} = S^{\text{el}}(l + (-1)^l \alpha)$$

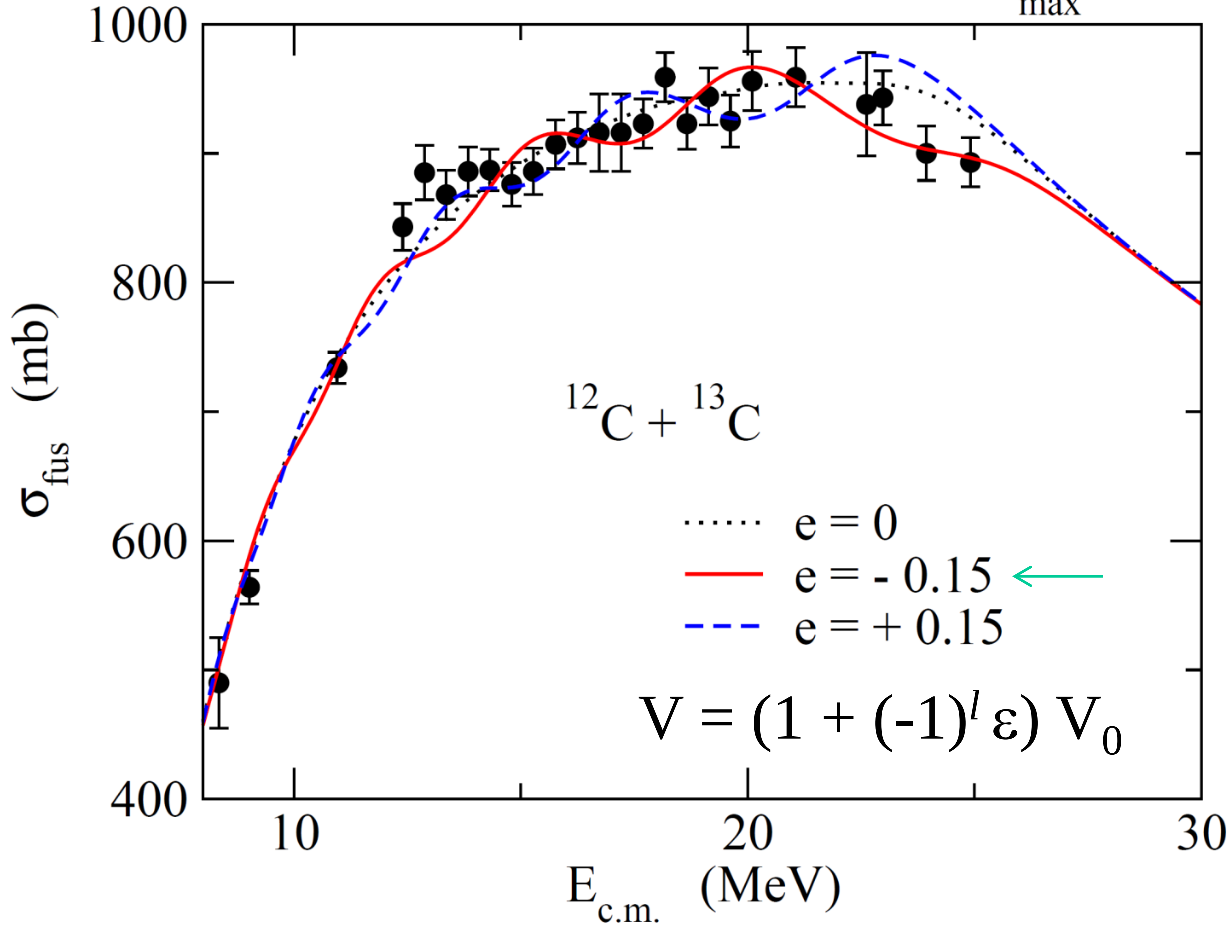


parity-dependent potential

- ✓W. von Oertzen and H.G. Bohlen, Phys. Rep. 19C(‘75) 1
- ✓A. Vitturi and C.H. Dasso, Nucl. Phys. A458 (‘86) 157
- ✓A. Kabir, M.W. Kermode and N. Rowley, Nucl. Phys. A481(‘88) 94

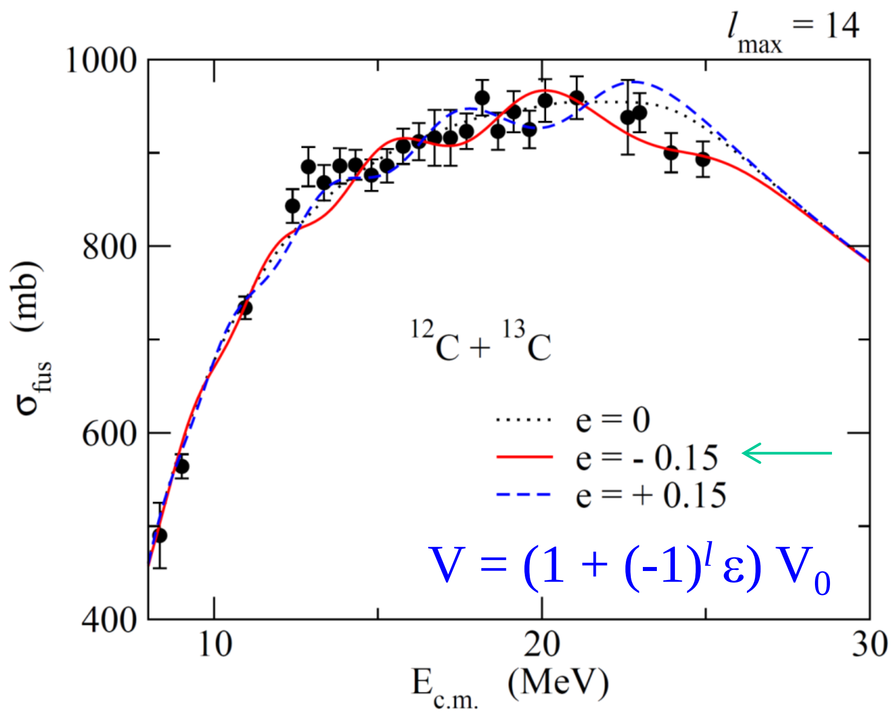
cf. $^{12}C + ^{16}O$

$l_{max} = 14$



exponential potential with a = 0.9 fm

parity-dependent potential



$\varepsilon < 0$

↕

a smaller V

↕

a higer barrier for even-l

cf. sign(V_+ - V-) = εV_0 = - ε

Baye's simple rule: ⟵ RGM with two-center HO shell model

D. Baye, J. Deenen, and Y. Salmon, Nucl. Phys. A289('77) 511
D. Baye, Nucl. Phys. A460 ('86)581

$$\text{sign}(V_+ - V_-) = -(-)^{A_<} \prod_{i:valence} \pi_i$$

(nuclear potential)

for $^{12}C+^{13}C(p_{1/2})$:

Summary

sub-barrier fusion of C+C systems

- Molecular resonances at subbarrier energies

  $^{12}C + ^{12}C$: well pronounced resonance structure

  $^{13}C + ^{13}C$, $^{12}C + ^{13}C$: rather smooth

  ← CN ^{24}Mg: low level density (low Q-value, e-e nucleus)

  cf. Jiang's conjecture

- Fusion oscillations: successive contribution of discrete centrifugal bararier

  $^{12}C(0^+) + ^{12}C(0^+)$, $^{13}C(1/2^-) + ^{13}C(1/2^-)$ } symmetrization of relative wave function

  $^{12}C + ^{13}C$ --- elastic transfer

  cf. $^{14}C + ^{14}C$: R.M. Freeman, C. Beck et al., PRC24 ('81) 2390

- analytic formula for fusion oscillations

  ← parabolic approximation