低エネルギー重イオン反応で視る原子核の形

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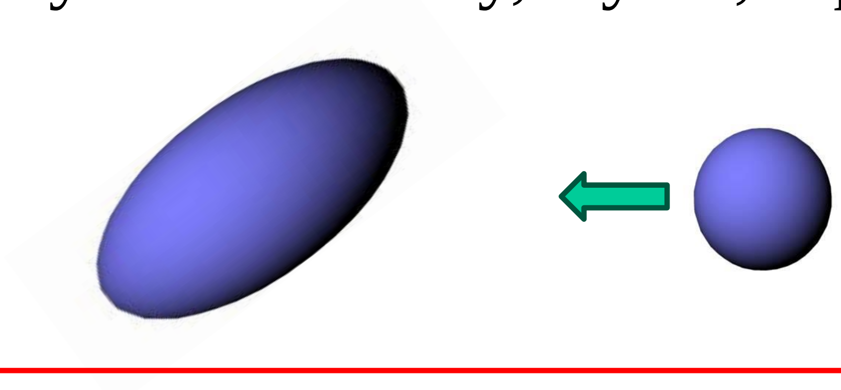


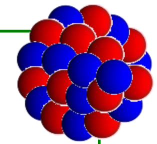
1. Low-energy Nuclear Reactions: overview
2. Role of deformation in sub-barrier fusion reactions
3. Probing nuclear shapes in quasi-elastic scattering
4. Summary

理研ミニWS「RIBFと高エネルギー原子核衝突反応の両面から探る原子核の形状」、理研，2024.9.30

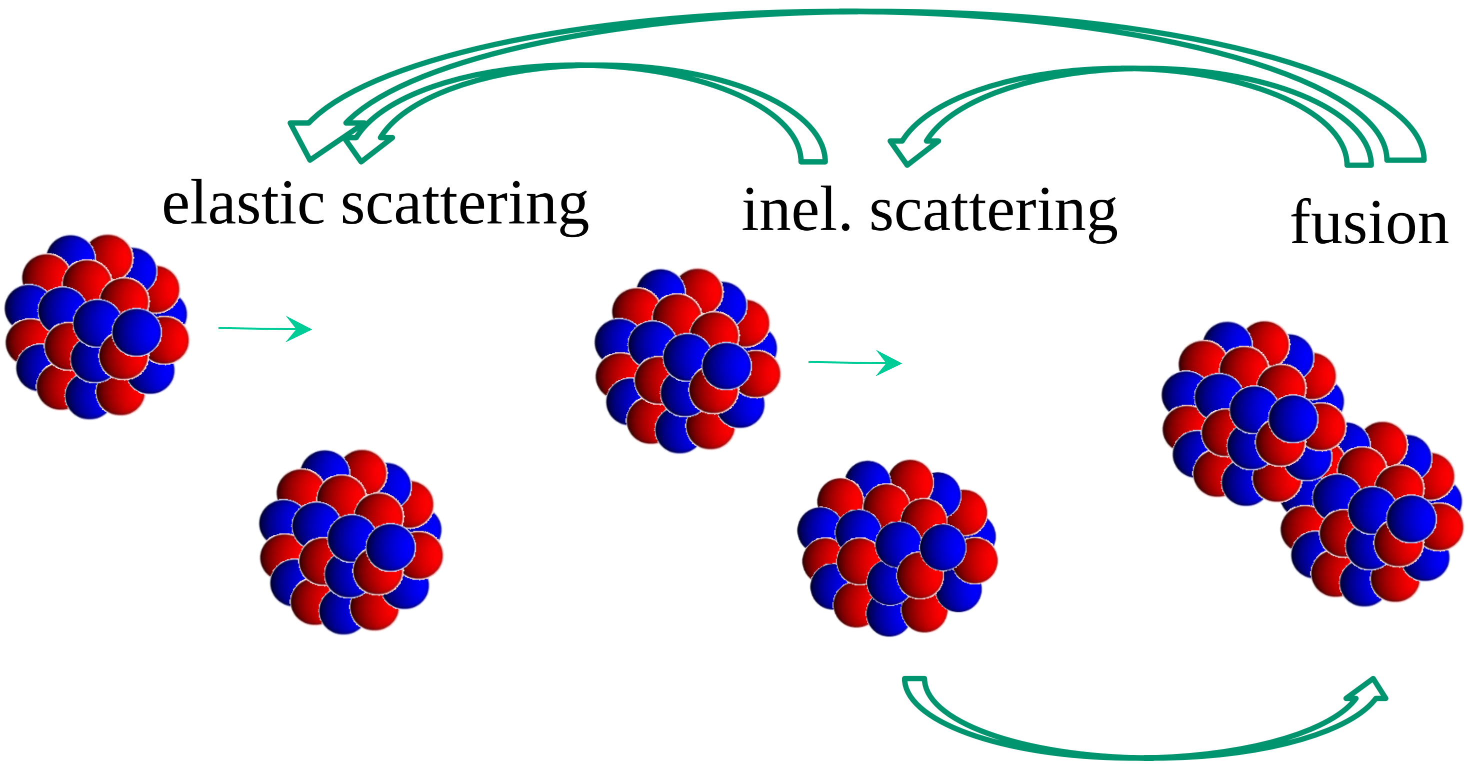
Introduction: low-energy nuclear reactions

nucleus: a composite system

✓ various sort of reactions



- elastic scattering
- inelastic scattering
- transfer rections
- breakup reactions
- fusion reactions



Introduction: low-energy nuclear reactions

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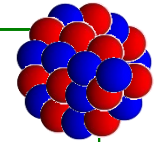
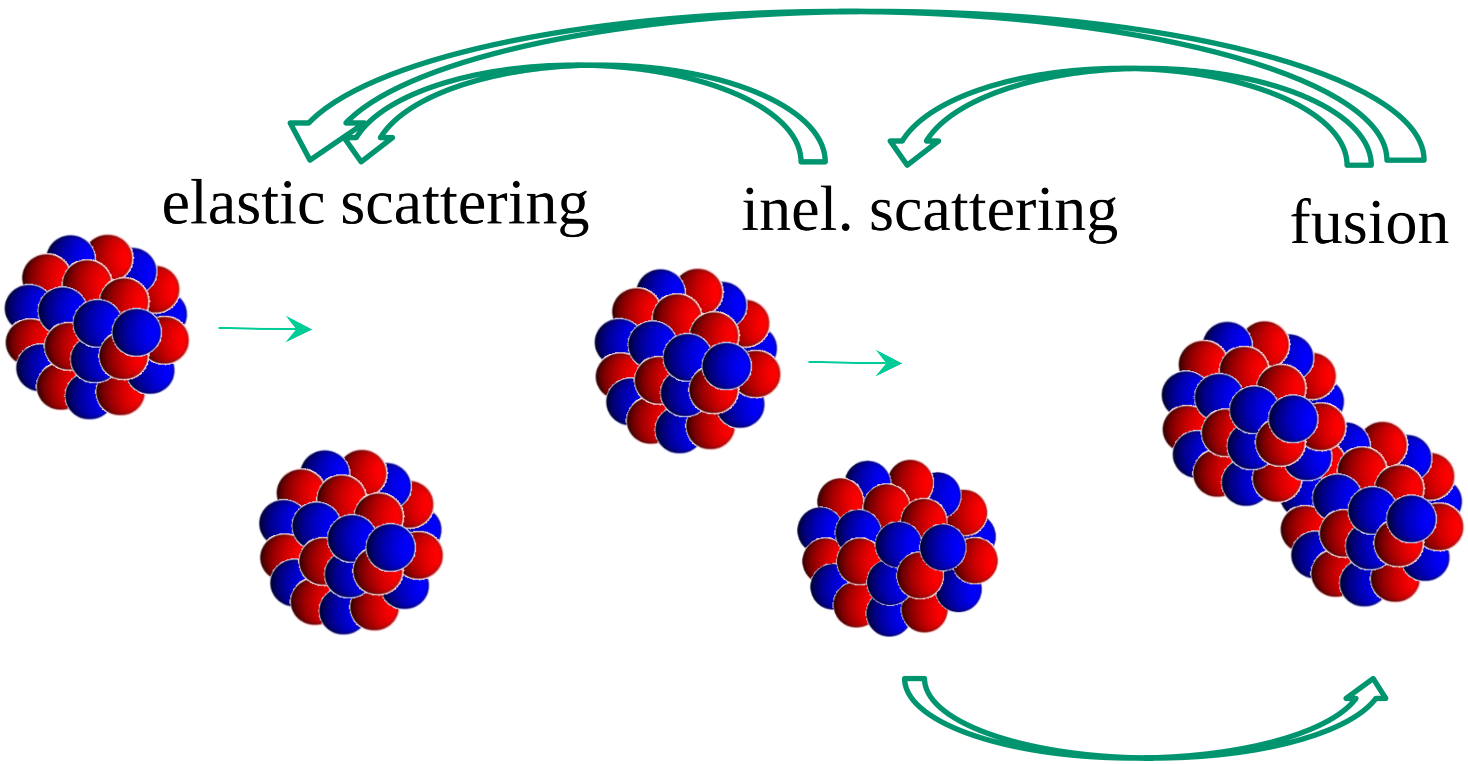


✓ various sort of reactions

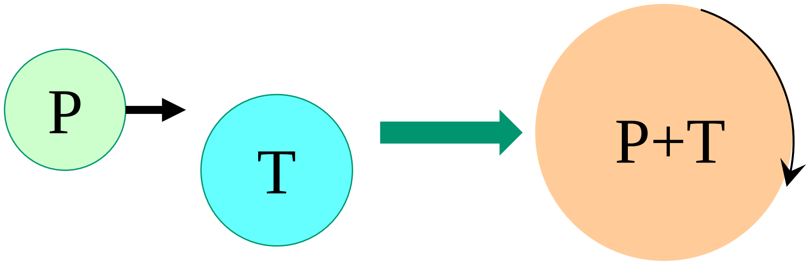
✓ an interplay between nuclear structure and reaction

shapes, excitations, …..

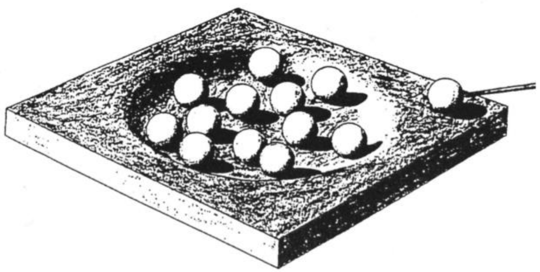
- elastic scattering
- inelastic scattering
- transfer rections
- breakup reactions
- fusion reactions



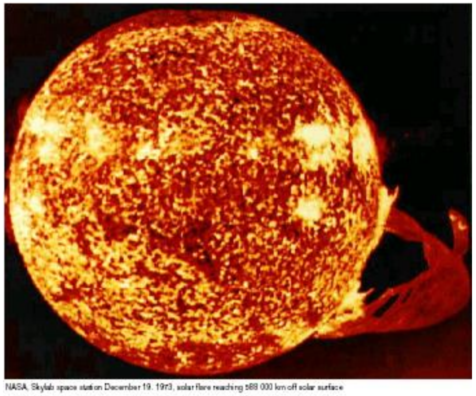
Fusion reactions: compound nucleus formation



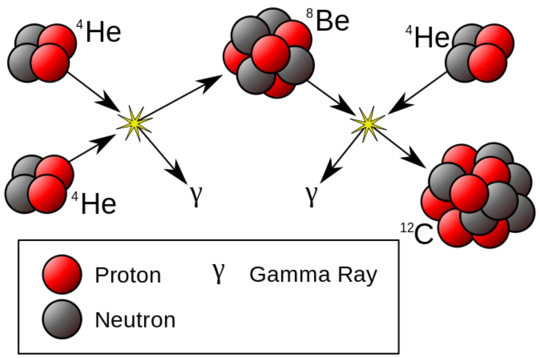
compound nucleus



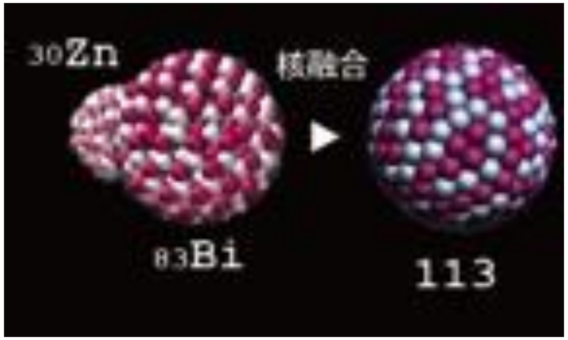
cf. Bohr '36



energy production in stars (Bethe '39)



nucleosynthesis

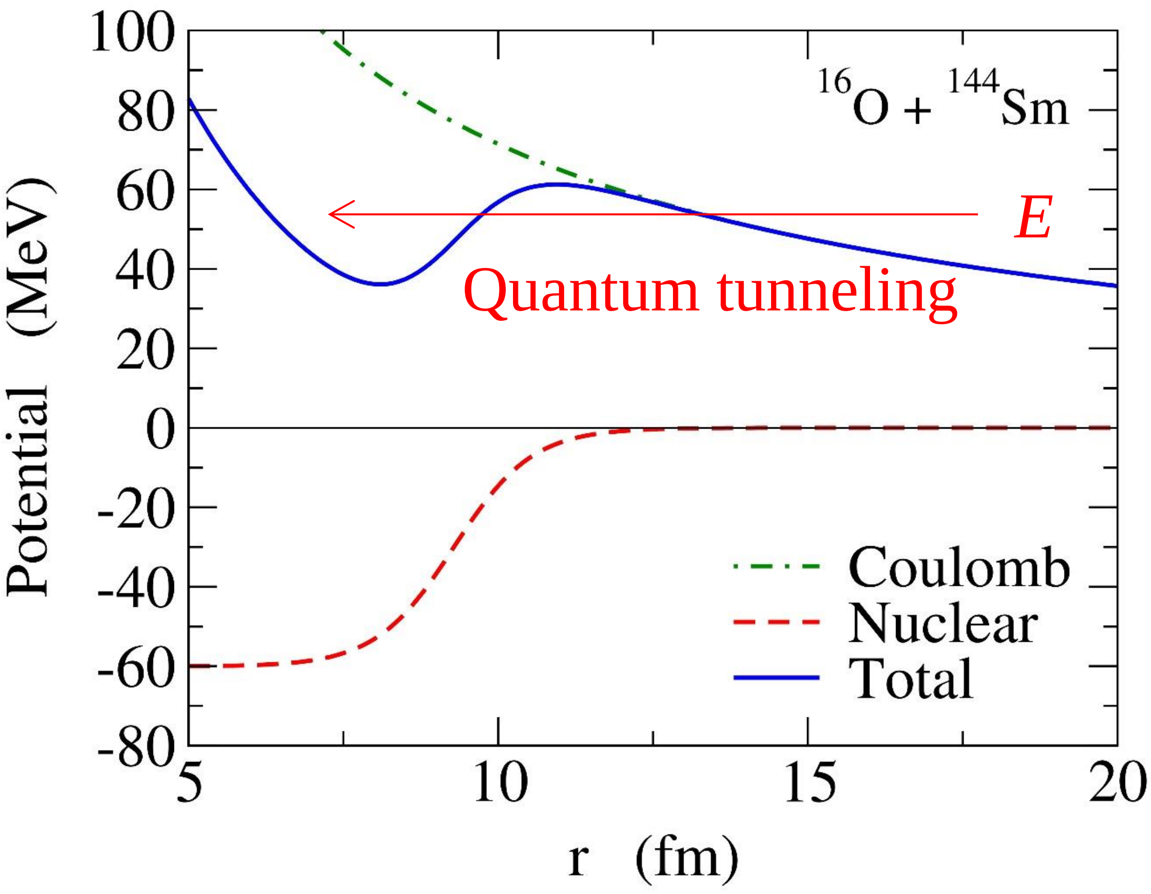


superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: an ultimate goal of nuclear physics

Coulomb barrier



1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

Potential barrier
(Coulomb barrier)

Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

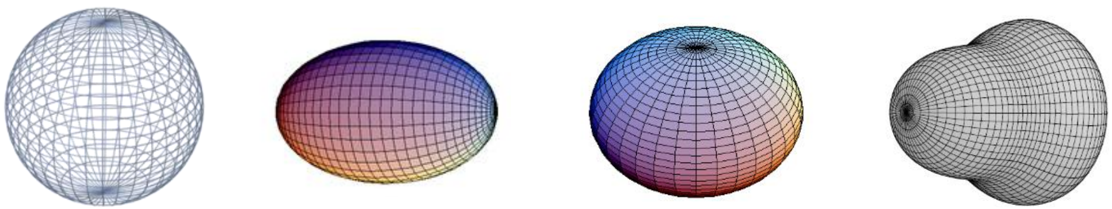
Fusion reactions at energies around the Coulomb barrier

Low-energy heavy-ion fusion reactions and quantum tunneling

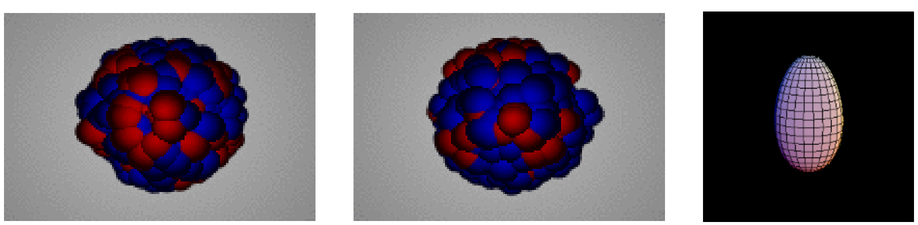
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



- several surface vibrations



several modes and adiabaticities

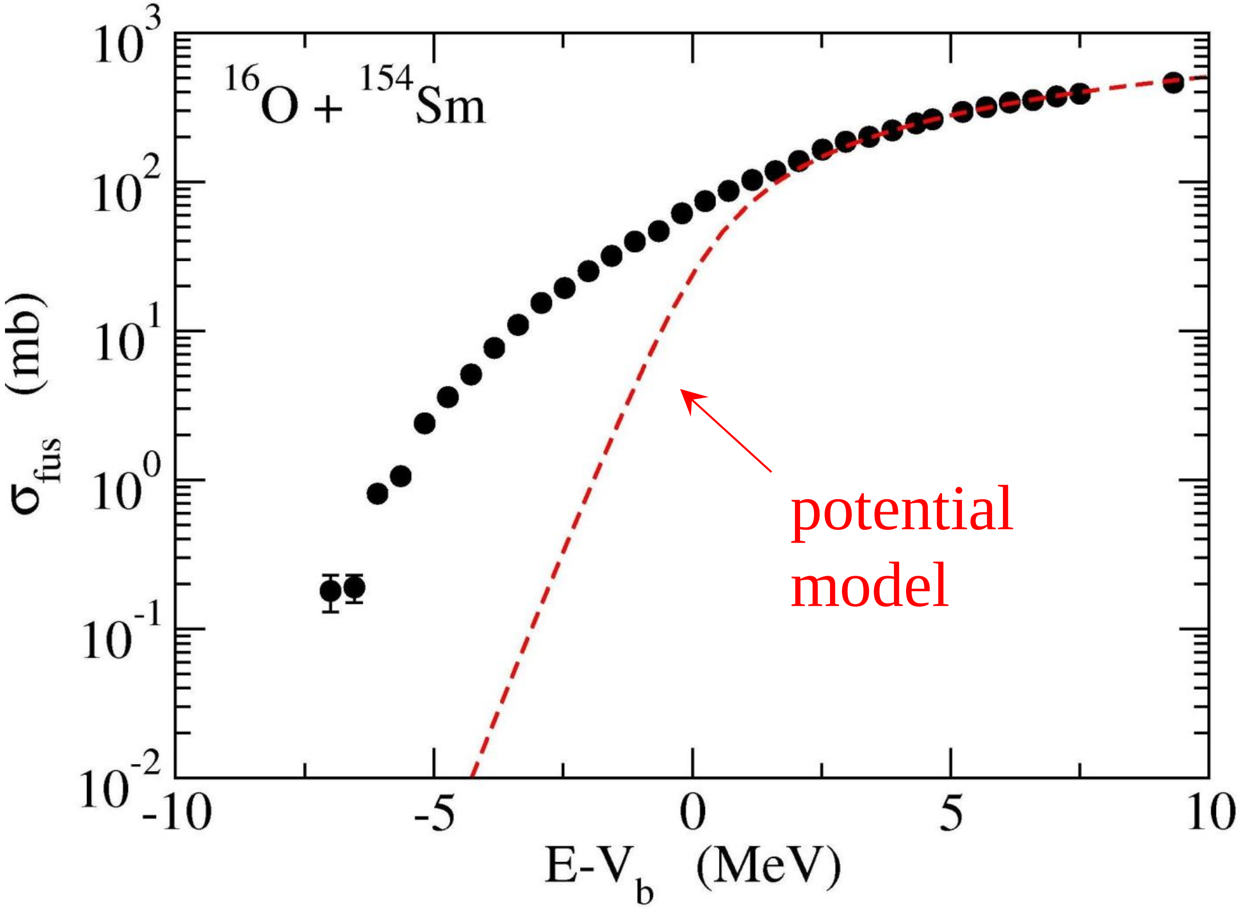
- several types of nucleon transfers

Tunneling probabilities: the exponential E dependence
→ nuclear structure effects are amplified

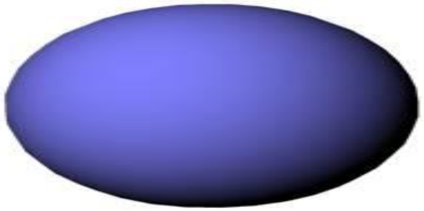
Discovery of large sub-barrier enhancement of σ_{fus} (~80's)

the potential model: inert nuclei (no structure)

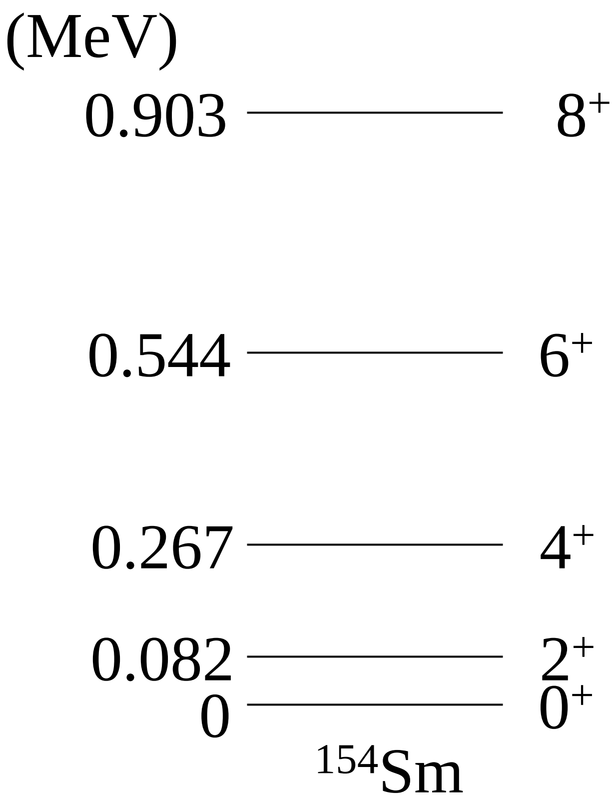
$$\sigma_{\mathrm{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$



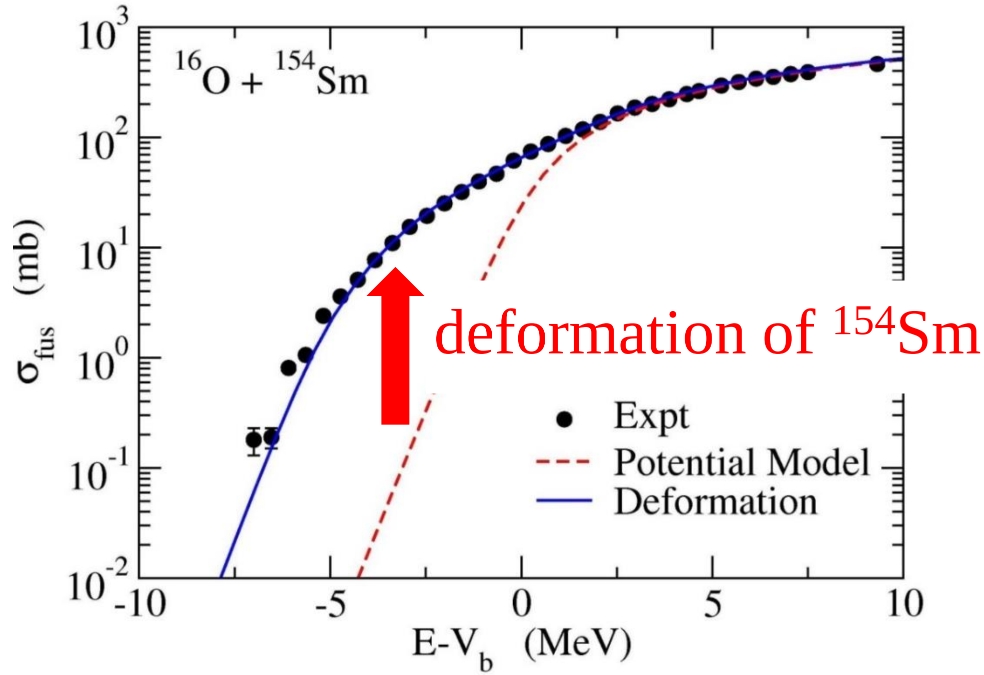
^{154}Sm : a typical deformed nucleus



^{154}Sm

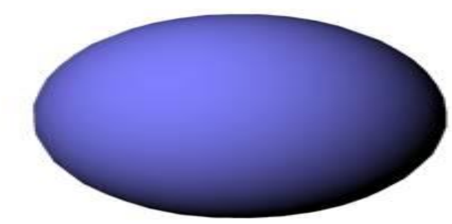


rotational spectrum



K. Hagino and N. Takigawa,
Prog. Theo. Phys.128 (‘12)1061.

Effects of nuclear deformation



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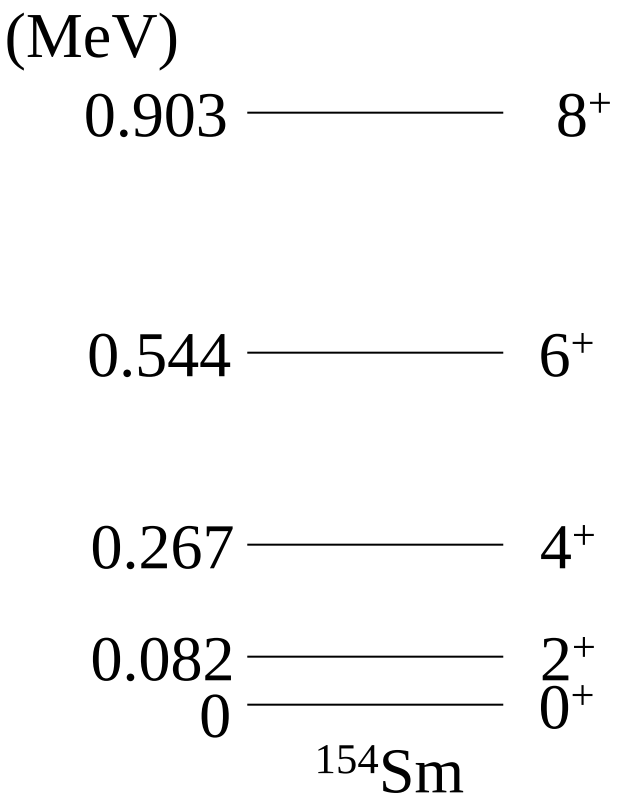


rotational spectrum

回転のエネルギーが小さい

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

↓

慣性モーメント J が大きい

↓

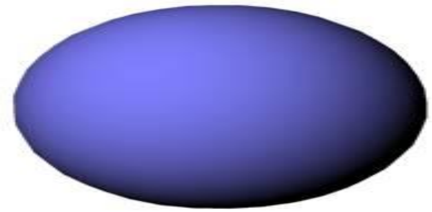
回転は遅い

オリエンテーションの角度を止めて考える

「スナップショット」

Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus



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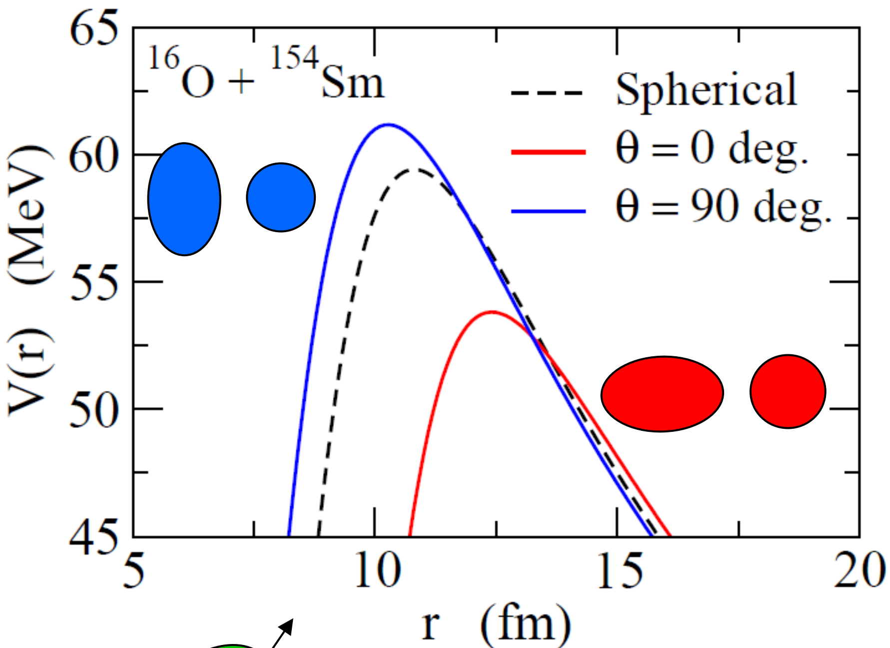

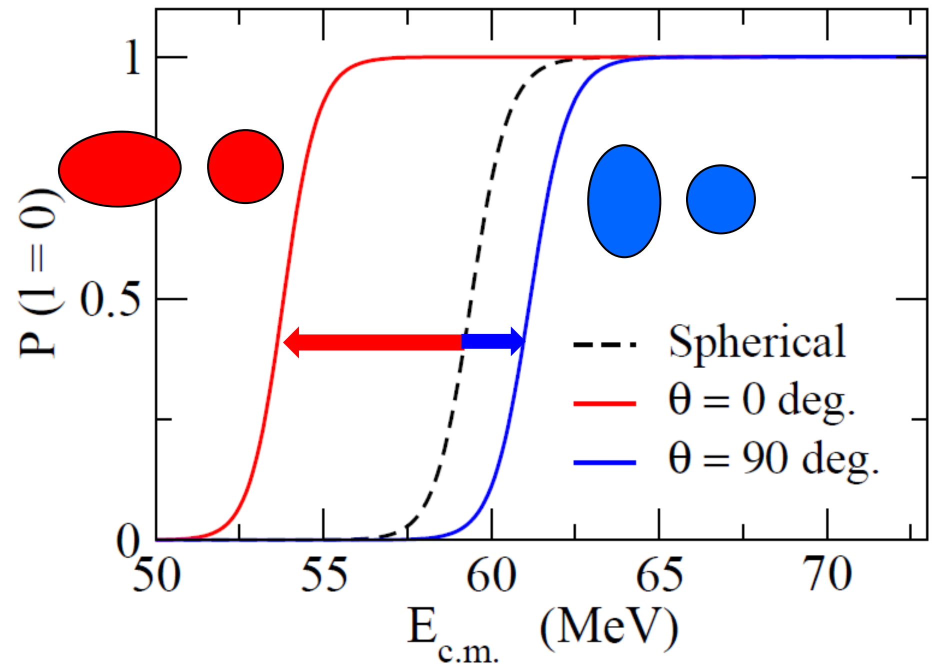

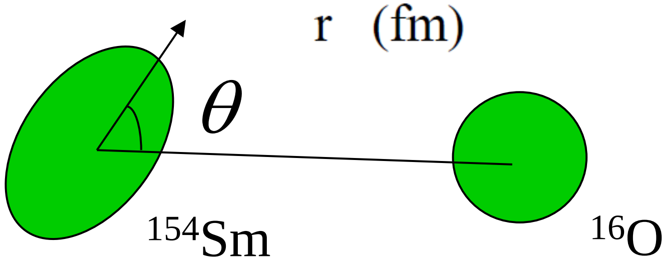

Effects of nuclear deformation

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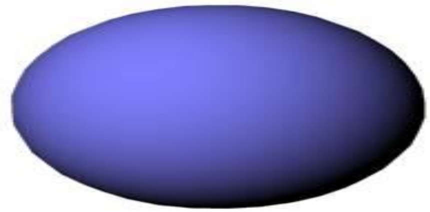
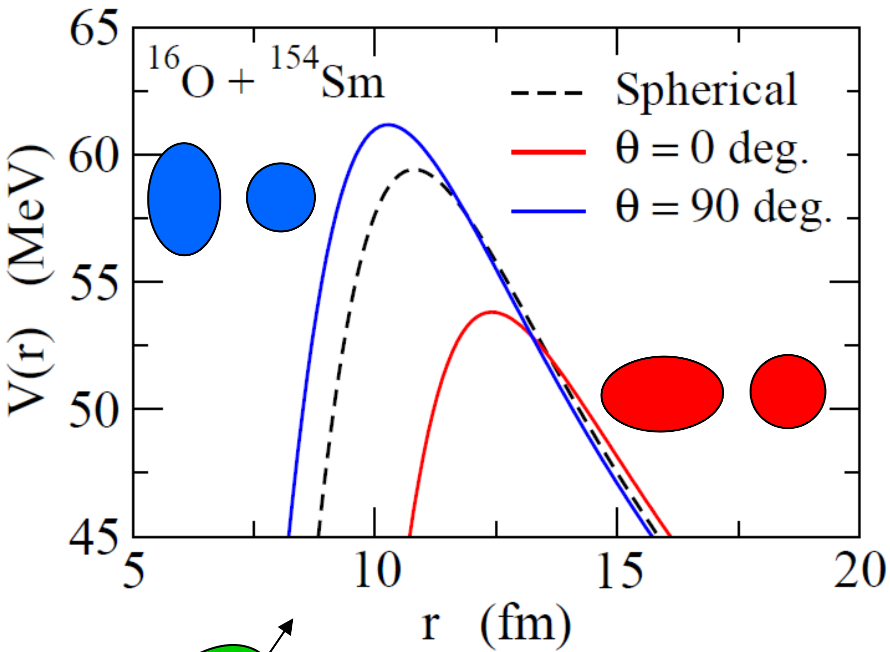
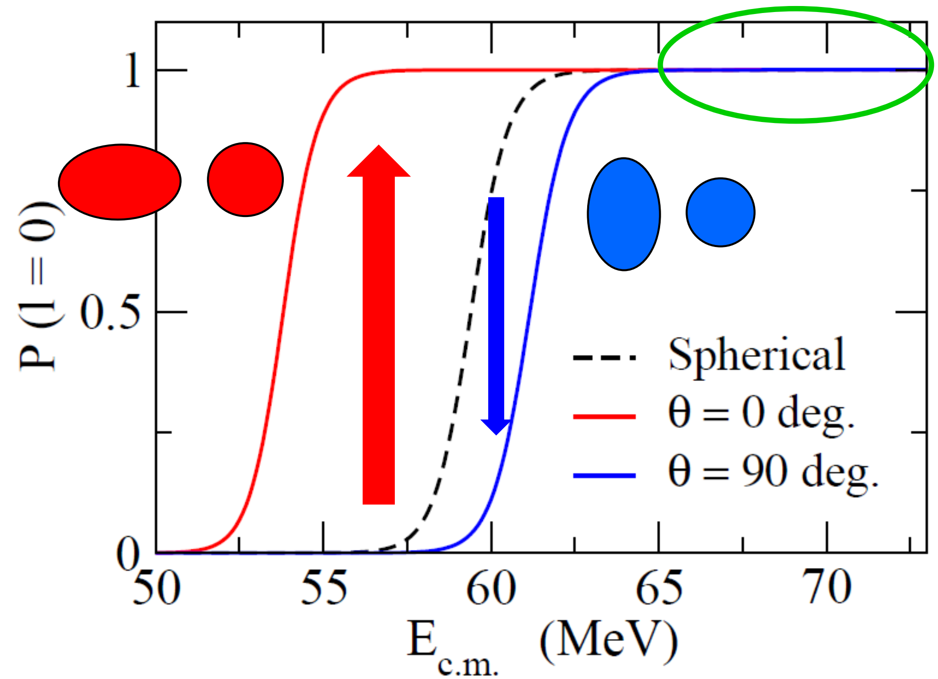
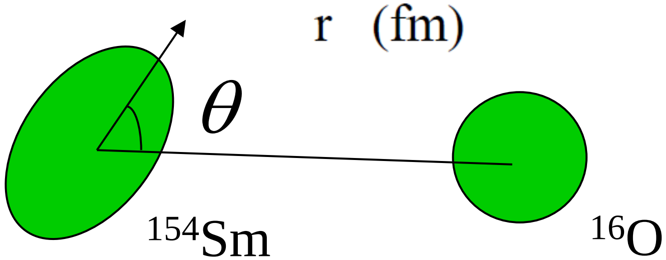


^{154}Sm







Effects of nuclear deformation

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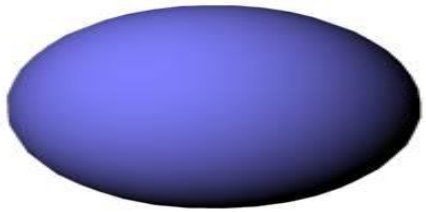
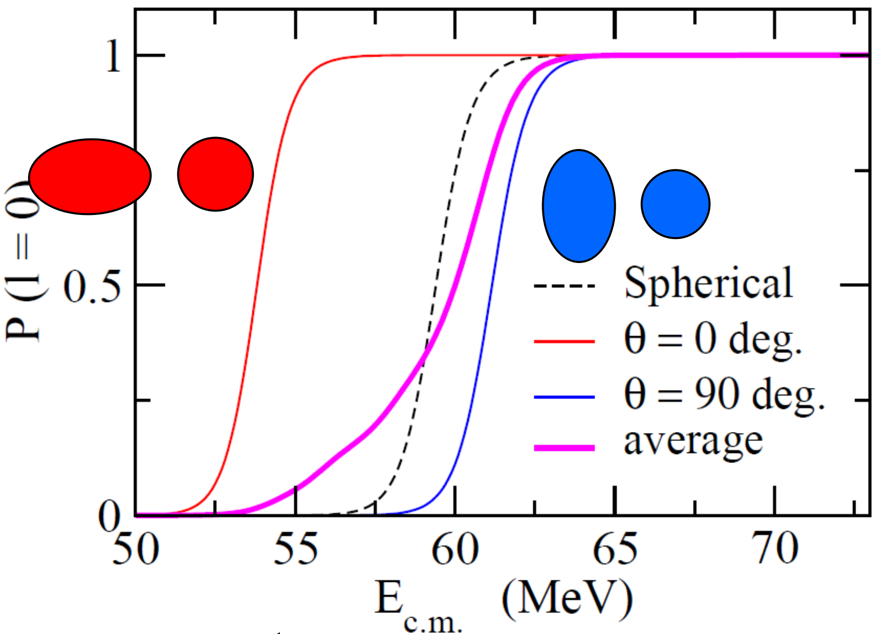
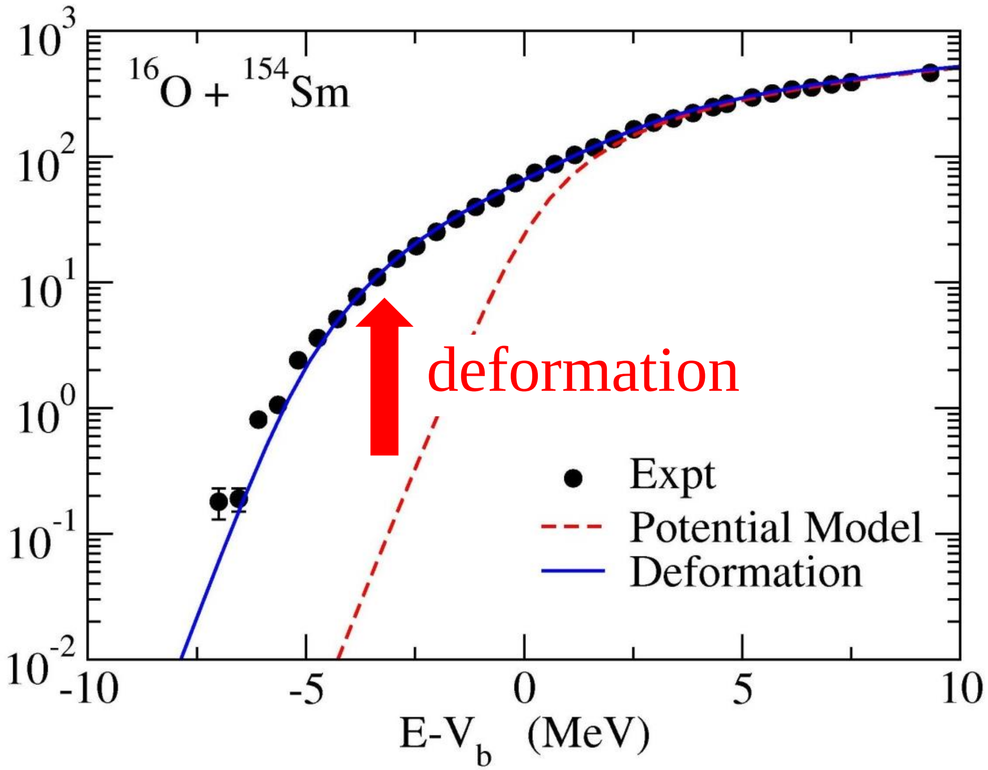
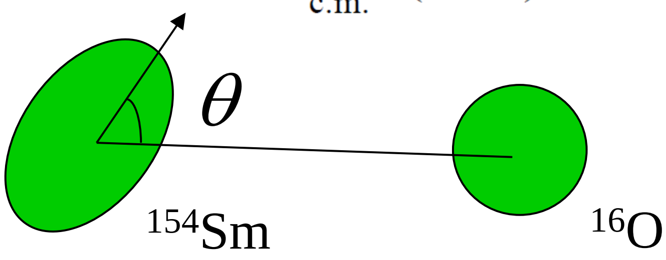




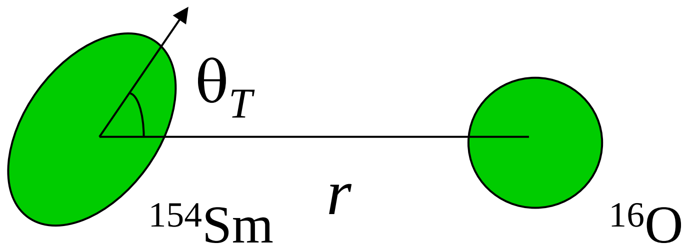




$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

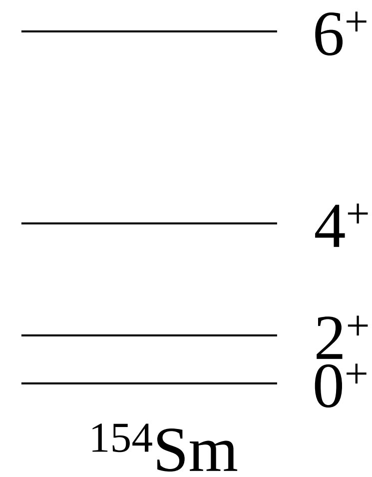
Fusion: strong interplay between nuclear structure and reaction

変形核の反応



核間ポテンシャル

$V(r,\theta_T)$



核融合反応断面積:

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\rm fus}[E; V(r,\theta_T)]$$

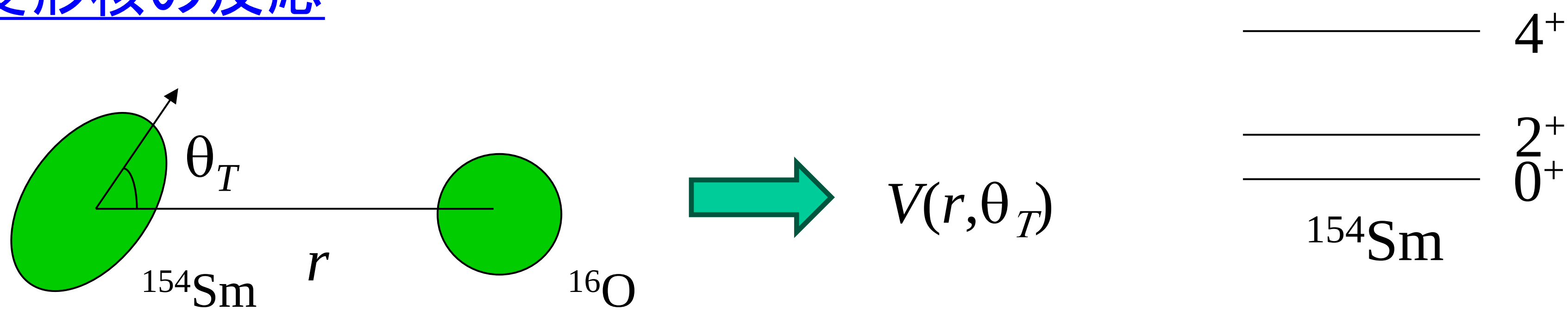
$$= 2\pi\int_{-1}^1 d(\cos\theta_T)\underline{|Y_{00}(\theta_T)|^2}\sigma_{\rm fus}[E; V(r,\theta_T)]$$

基底状態の波動関数

弾性散乱断面積:

$$\frac{d\sigma_{\rm el}}{d\Omega} = |f(\theta)|^2; \qquad f(\theta) = \int_0^1 d(\cos\theta_T) f_{\rm el}[\theta; V(r,\theta_T)]$$

変形核の反応



$$\sigma_{\text{fus}}(E) = 2\pi \int_{-1}^{1} d(\cos\theta_T)\,|Y_{00}(\theta_T)|^2 \sigma_{\text{fus}}[E; V(r,\theta_T)]$$

基底状態の波動関数

時々ある誤解:

「この取り扱いでは、原子核が励起していない」←これは大きな誤解

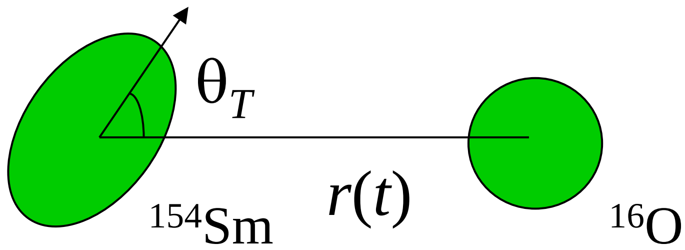
角度 θ_T を固定 ←→ 角運動量状態が完全不確定(不確定性原理)

$$|\theta_T\rangle = \sum_{I=0}^{\infty} \langle \theta_T | Y_{I0}\rangle |Y_{I0}\rangle$$

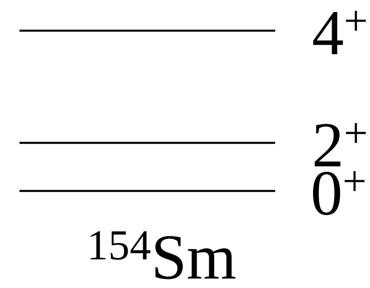
準弾性散乱:

$$\frac{d\sigma_{\text{qel}}}{d\Omega} = \sum_{I=0}^{\infty} \frac{d\sigma_I}{d\Omega} = \int_0^1 d(\cos\theta_T)\,\frac{d\sigma_{\text{el}}}{d\Omega}[V(r,\theta_T)]$$

変形核の反応



$\Rightarrow$ $V(r(t),\theta_T)$



$$H_{\rm rot}(t) = \frac{\boldsymbol{I}^2}{2\mathcal{J}} + V(r(t),\theta_T)$$

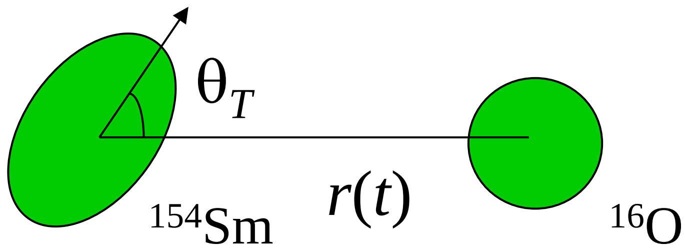
反応が速い
（回転が遅い） →断熱近似→ $H_{\rm rot}$ は θ_T の関数
（$d/d\theta_T$ には依存しない）

$$|\phi(t)\rangle = \hat{T}\exp\left[-i\int dt' H_{\rm rot}(t')\right]|I=0\rangle$$

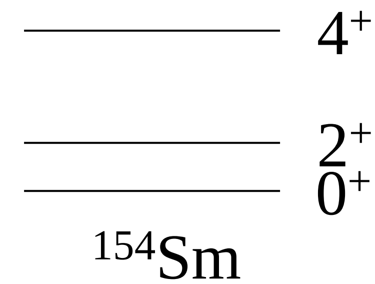
$$\begin{aligned} \to f_{\rm el} &= \langle I=0|\phi(t)\rangle = \langle I=0|\hat{T}\exp\left[-i\int dt' H_{\rm rot}(t')\right]|I=0\rangle \\ &= 2\pi\int d(\cos\theta_T)|\langle\theta_T|I=0\rangle|^2\,\hat{T}\exp\left[-i\int dt' H_{\rm rot}(t',\theta_T)\right] \end{aligned}$$

$= f_{\rm el}(\theta;\theta_T)$

変形核の反応



$V(r(t),\theta_T)$



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$$|\phi(t)\rangle = \hat{T}\exp\left[-i\int dt' H_{\rm rot}(t')\right]|I=0\rangle$$

$$\sum_I |I\rangle\langle I| = 1$$

$$\sigma = \sum_I |\langle I|\hat{u}(t)|I=0\rangle|^2 = 2\pi\int d(\cos\theta_T)|\langle\theta_T|I=0\rangle|^2|\hat{u}(t;\theta_T)|^2$$

$= \sigma(\theta_T)$

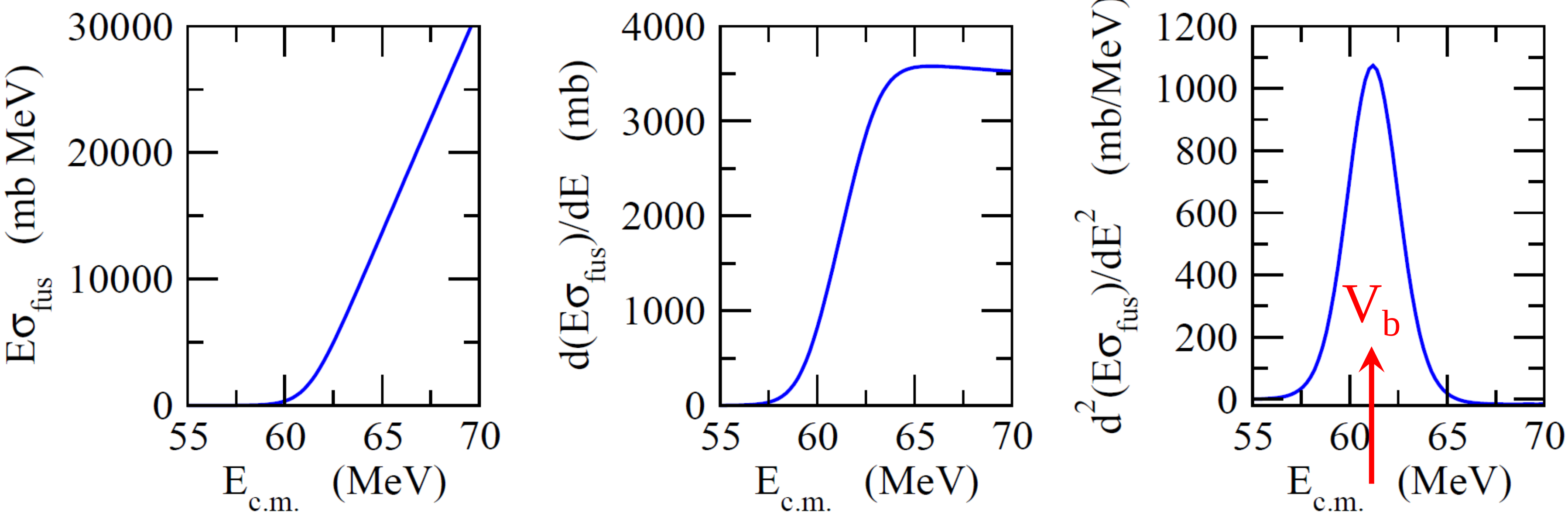
$$\sigma_{\rm fus}(E) \;=\; 2\pi\int_{-1}^{1} d(\cos\theta_T)|Y_{00}(\theta_T)|^2\sigma_{\rm fus}[E;V(r,\theta_T)]$$

Fusion barrier distribution

$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

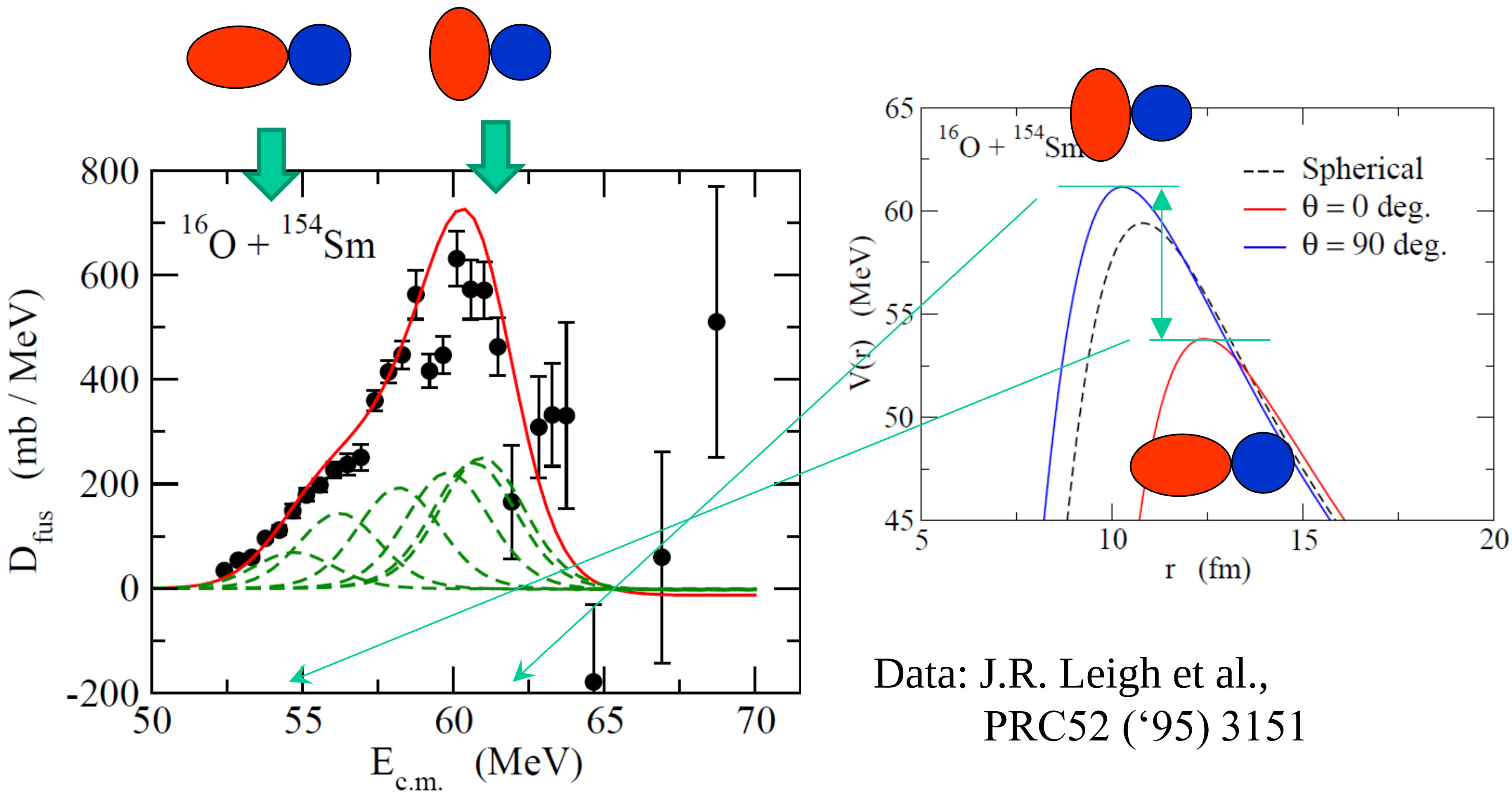
$$E\sigma_{\mathrm{fus}} \qquad \frac{d(E\sigma_{\mathrm{fus}})}{dE} \sim \pi R_b^2\, P_{l=0}(E) \qquad \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2} \sim \pi R_b^2 \cdot dP_{l=0}/dE$$



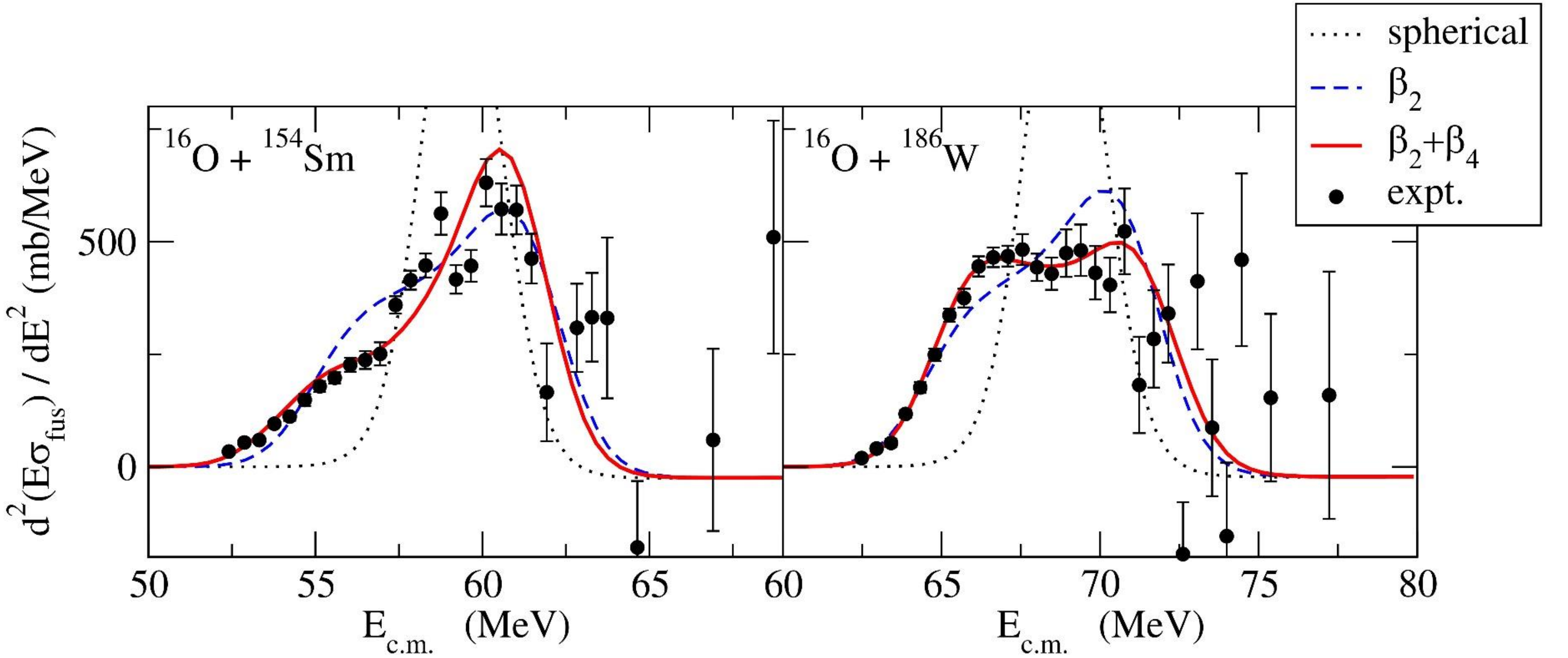
K.H. and N. Takigawa, PTP128 ('12) 1061

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2} \sim \pi R_b^2 \frac{dP_{l=0}}{dE}$$

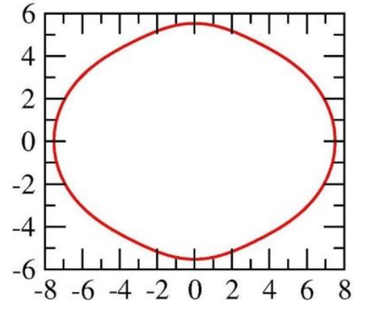


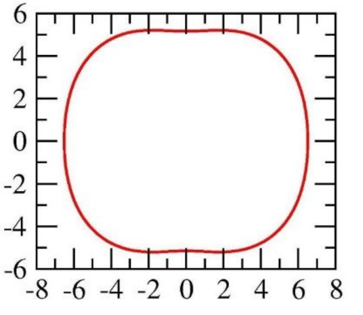
Data: J.R. Leigh et al., PRC52 ('95) 3151

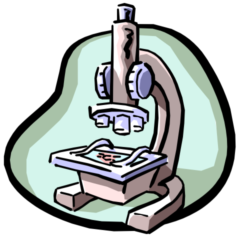


$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$\beta_2 = 0.33$	$\beta_2 = 0.29$
$\beta_4 = +0.05$	$\beta_4 = -0.03$







sensitive to the sign of β_4!

→ Fusion as a quantum tunneling microscope for nuclei

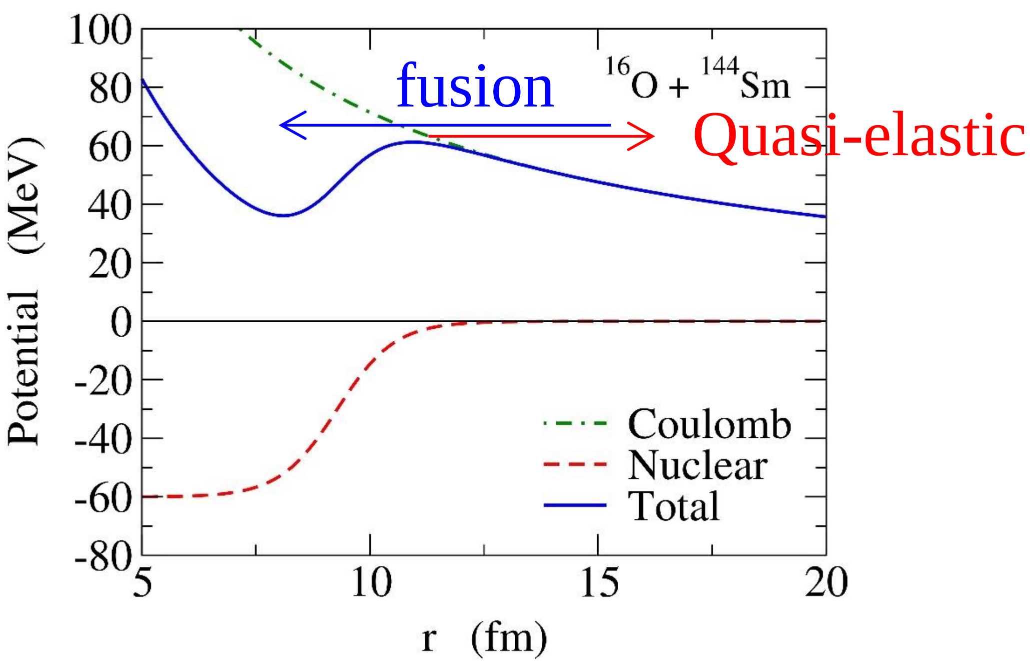
Quasi-elastic barrier distribution

$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

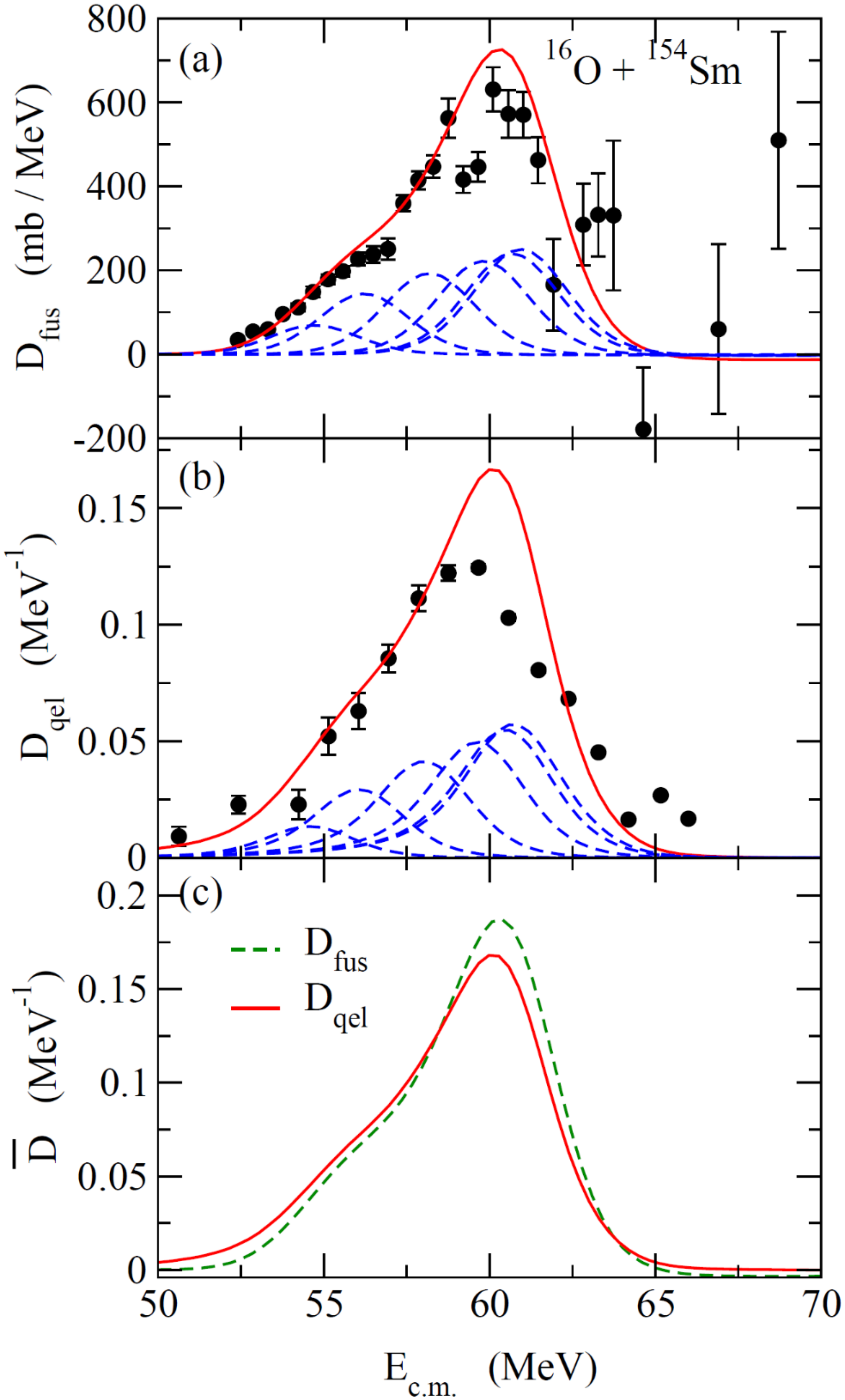
H. Timmers et al., NPA584('95)190

Quasi-elastic scattering:

A sum of all the reaction processes other than fusion
(elastic + inelastic + transfer + ……)



$$P_{l=0}(E) = 1 - R_{l=0}(E) \sim 1 - \frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}$$



D_{fus} and D_{qel}: behave similarly to each other

cf. Application to reactions relevant to SHE

$^{48}Ca + ^{248}Cm \rightarrow {}^{296}_{116}Lv^*$

T. Tanaka et al.,
JPSJ 87 ('18) 014201
PRL124 ('20) 052502

$^{51}V + ^{248}Cm \rightarrow {}^{299}119^*$

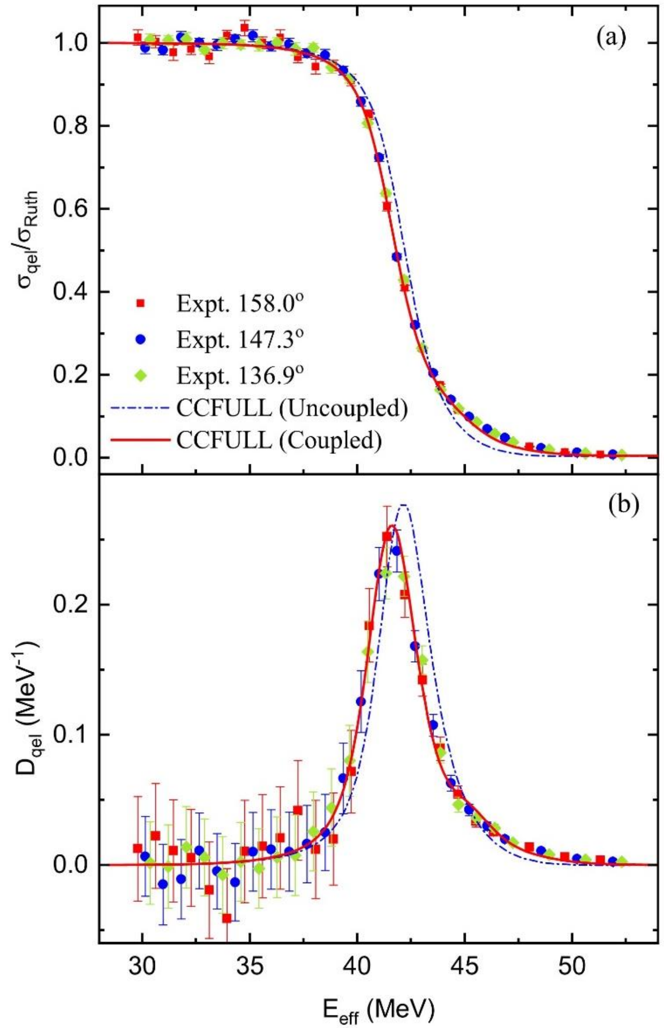
M. Tanaka et al.,
JPSJ 91 ('22) 084201

K.H. and N. Rowley, PRC69('04)054610

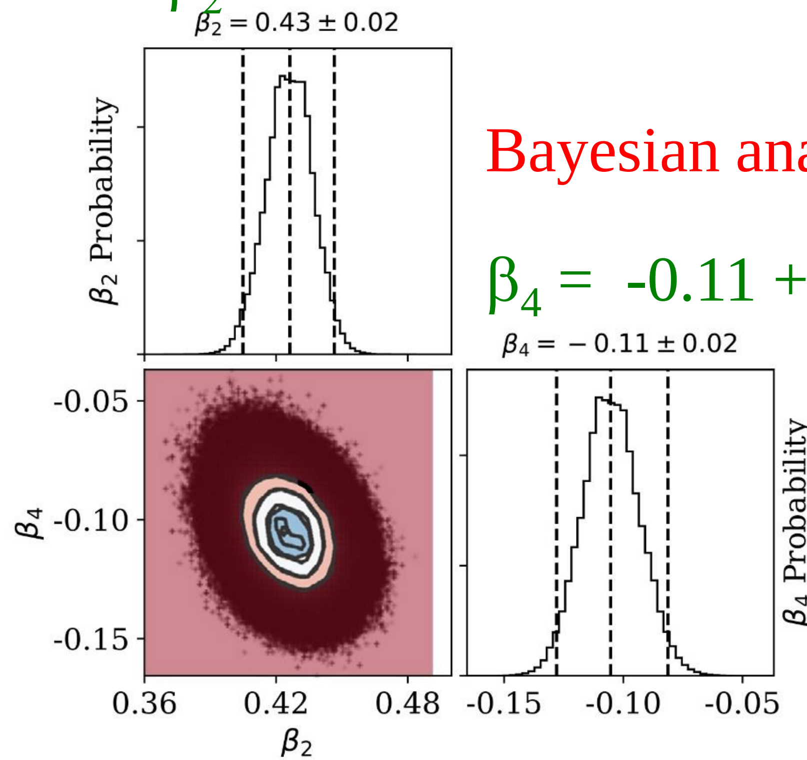
Determination of β_4 of ^{24}Mg with quasi-elastic scattering

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

^{24}Mg + ^{90}Zr



β_2 = 0.43 +/- 0.02



Bayesian analysis

β_4 = -0.11 +/- 0.02

high precision determination of β_4
→ for the first time

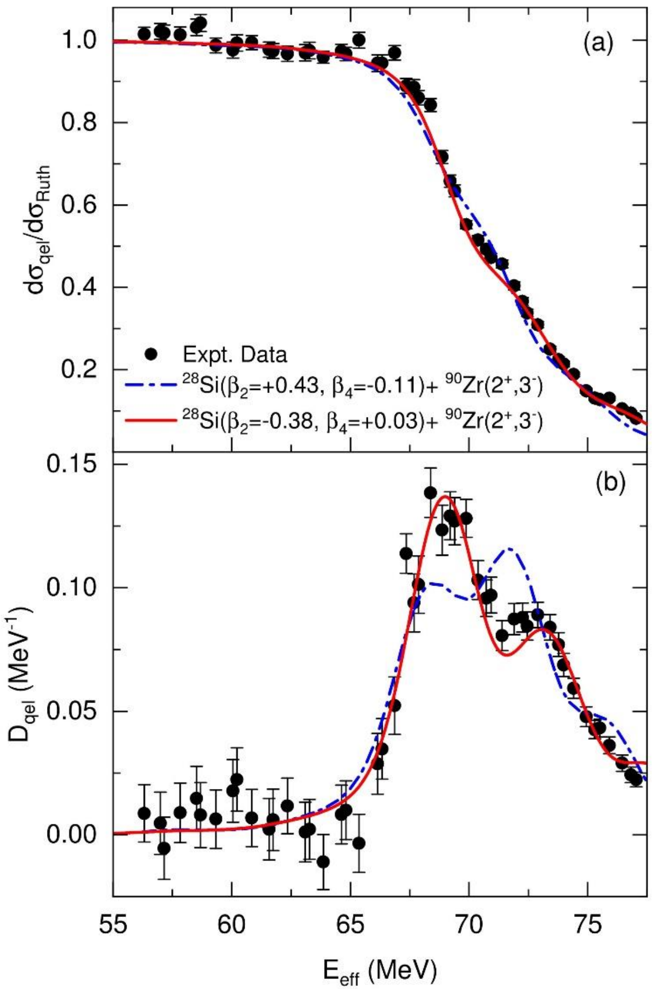
cf. (p,p'): β_4 = -0.05 +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

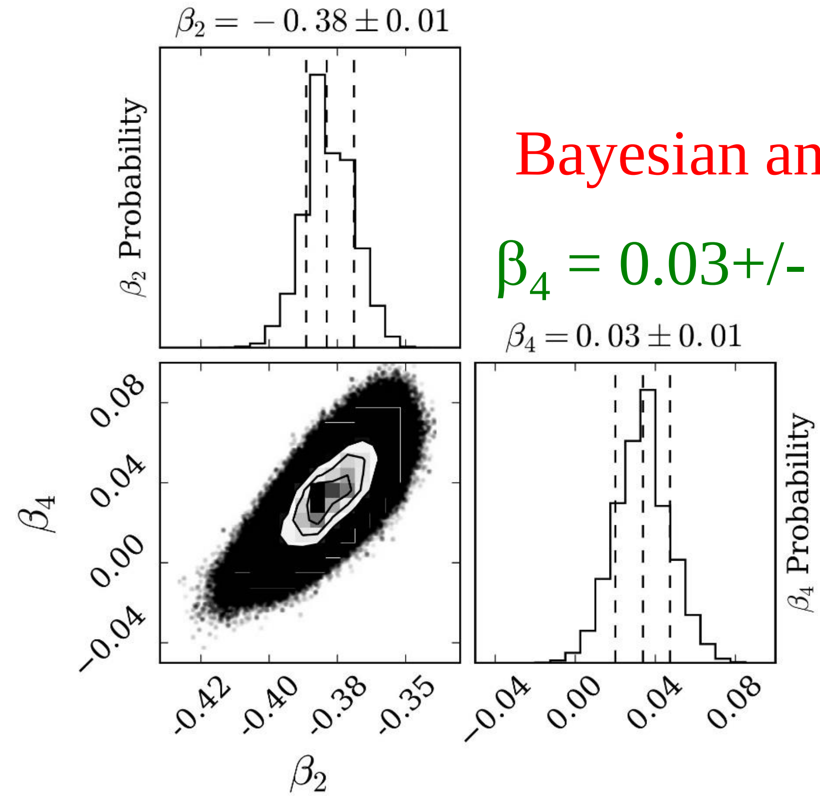
Determination of β_4 of ^{28}Si with quasi-elastic scattering

Y.K. Gupta, V.B. Katariya, G.K. Prajapati, K.H., et al., PLB845, 138120 (2023).

^{28}Si + ^{90}Zr



β_2 = -0.38 +/- 0.01



Bayesian analysis

β_4 = 0.03+/- 0.01

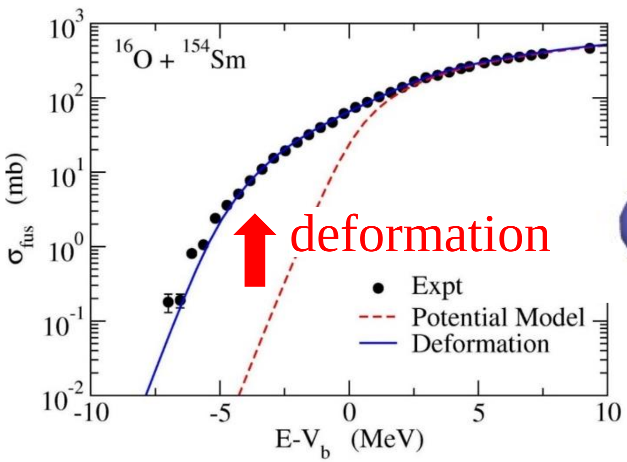
cf. (n,n'): β_4 = 0.20 +/- 0.05

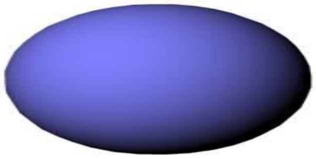
G. Haouat et al., PRC30, 1795 (1984)

Summary

Heavy-ion fusion reactions around the Coulomb barrier

- ✓Strong interplay between nuclear structure and reaction
- ✓Quantum tunneling with various intrinsic degrees of freedom
- ✓Role of deformation in sub-barrier enhancement





↓ amplified

✓ Fusion barrier distribution $D_{\rm fus}(E) = \dfrac{d^2(E\sigma_{\rm fus})}{dE^2}$

✓ Quasi-elastic barrier distribution $D_{\rm qel}(E) = -\dfrac{d}{dE}\left(\dfrac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$

sensitive to the nuclear structure

recent applications to ^{24}Mg, ^{28}Si + ^{90}Zr → determination of β_4