Theoretical modeling of C+C fusion reactions : Open issues

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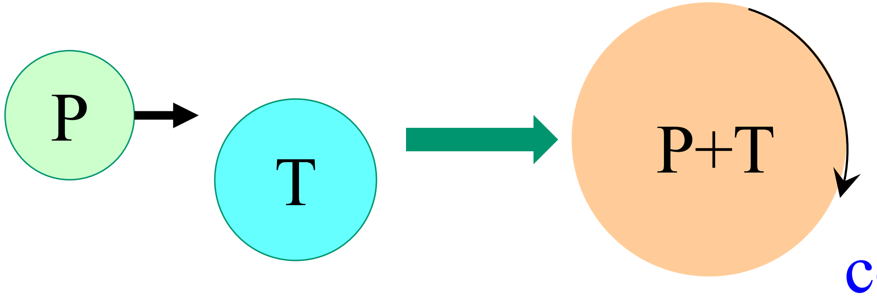
1. Introduction: fusion reactions
2. Summary of theoretical calculations for $^{12}C+^{12}C$ fusion
3. Role of CN resonances
4. Fusion hindrance
5. Summary

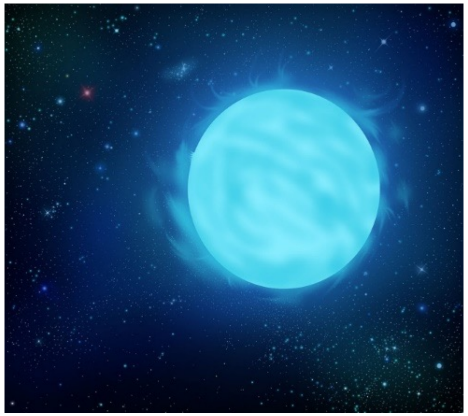
RIKEN workshop on 12C+12C fusion at stellar energies, 2024.11.11-12

Introduction: Fusion reactions



compound nucleus

$^{12}C + ^{12}C$ fusion : a key reaction in nuclear astrophysics



- ✓ Carbon burning in massive stars

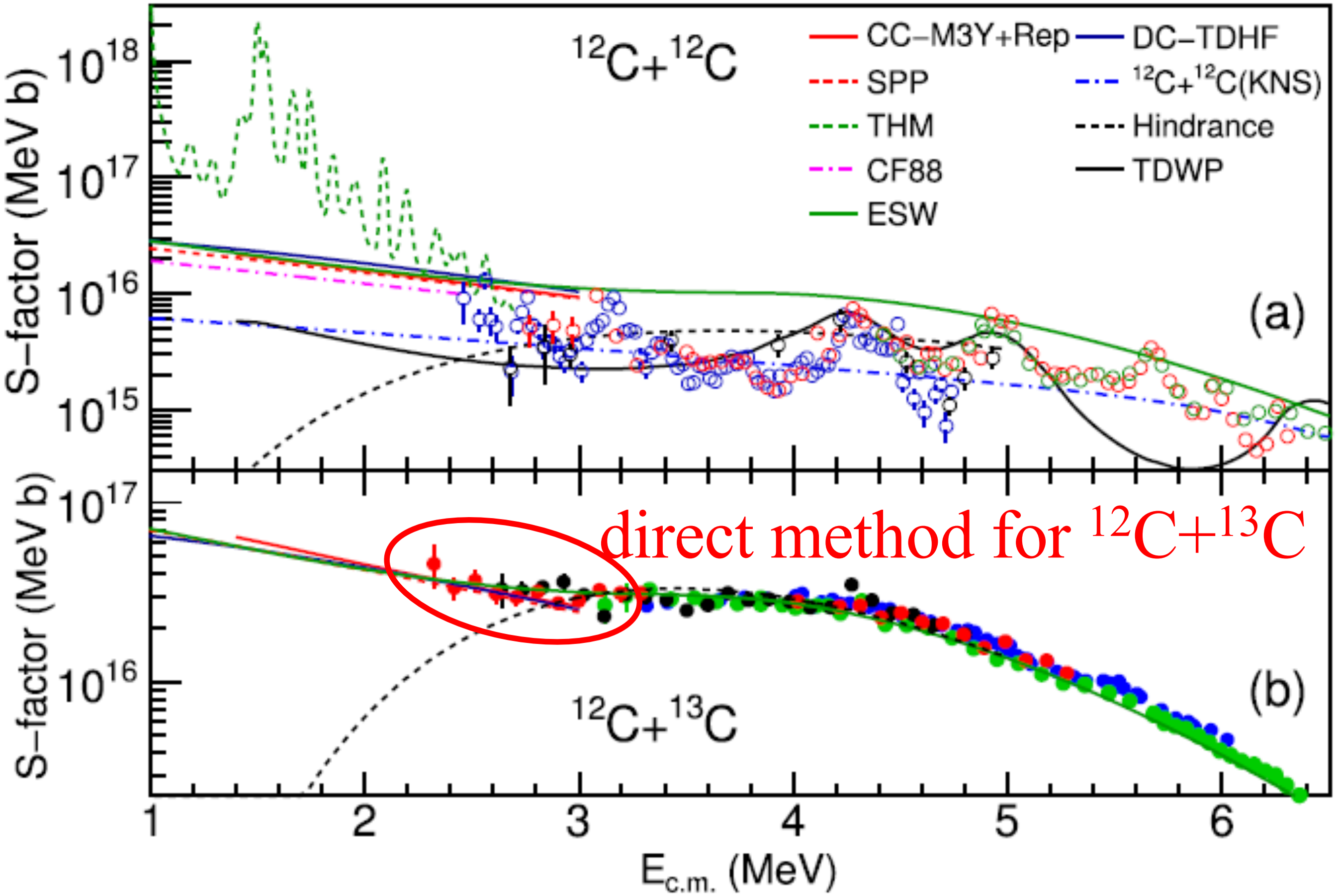
  $^{12}C + ^{12}C \rightarrow \alpha + ^{20}Ne$

  $^{12}C + ^{12}C \rightarrow p + ^{23}Na$
- ✓ Type Ia supernovae
- ✓ X-ray superburst

Fusion and fission: large amplitude motions & many-body tunneling

← microscopic understanding: an ultimate goal of nuclear physics

$^{12}C+^{12}C$ fusion reaction



N.T. Zhang, et al., Phys. Lett. B801 (2020) 135170

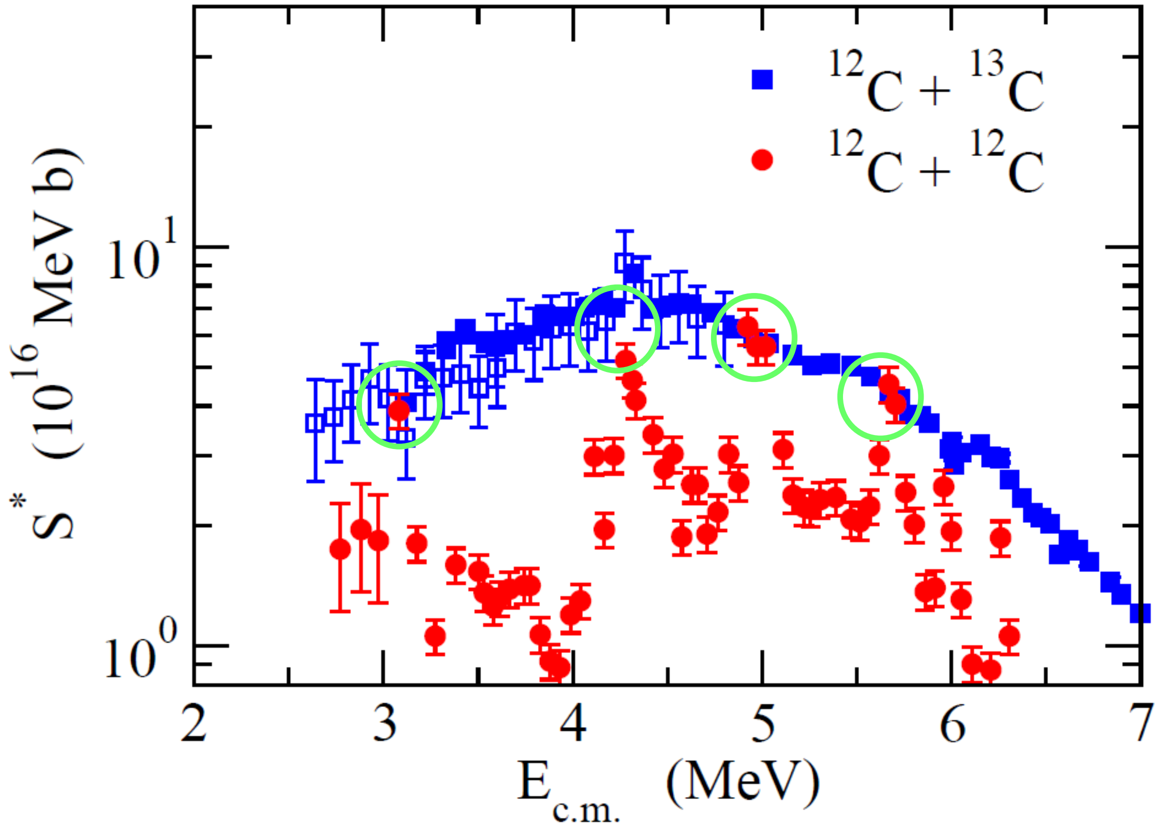
an important issue: can one understand $^{12}C+^{12}C$ and $^{12}C+^{13}C$ in a unified manner?

$^{12}C+^{12}C$ fusion reaction

An open issue:

Does $^{12}C+^{13}C$ provide the upperlimit of $^{12}C+^{12}C$ cross sections?

→ this has to be answered theoretically



off-resonance:
fusion inhibition

on-resonance:
match with $^{12}C+^{13}C$

M. Notani, X.D. Tang et al.,
PRC85('12)014607

Previous theoretical calculations for $^{12}C+^{12}C$

- Coupled-channels approaches

✓ B. Imanishi, PL 27B (‘68) 267, NPA125 (‘69) 33.
✓ Y. Kondo, T. Matsuse, Y. Abe, PTP59 (‘78) 465.
✓ W. Scheid, W. Greiner, R. Lemmer, PRL25 (‘70) 176.
✓ H. Esbensen, X. Tang, and C.L. Jiang, PRC84, 064613 (2011).
✓ M. Assuncao and P. Descouvemont, PLB723, 355 (2013).

- Time-dependent wave packet approach

✓ A. Diaz-Toress and M. Wiesher, PRC97, 055802 (2018).

- TDHF-based approaches

✓ Imaginary-time evolution: A. Bonasera and J.B. Natowitz, PRC102
✓ ATDHF: P.-H. Heenen, PL99B, 298 (1981).
✓ Density-Constrained Hartree-Fock: K. Godbey et al., PRC100

- Anti-symmetrized Molecular Dynamics (AMD) approach

Y. Taniguchi and M. Kimura, PLB823, 136790 (2021).

AMD + R-matrix

Previous theoretical calculations for $^{12}C+^{12}C$

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  Nogami-Imanishi model
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  Band-crossing model
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  Double resonance model
- ✓ H. Esbensen, X. Tang, and C.L. Jiang, PRC84, 064613 (2011).
  Systematic studies for three C+C systems
- ✓ M. Assuncao and P. Descouvemont, PLB723, 355 (2013).
  Role of the Hoyle state

Previous calculations for $^{12}C+^{12}C$

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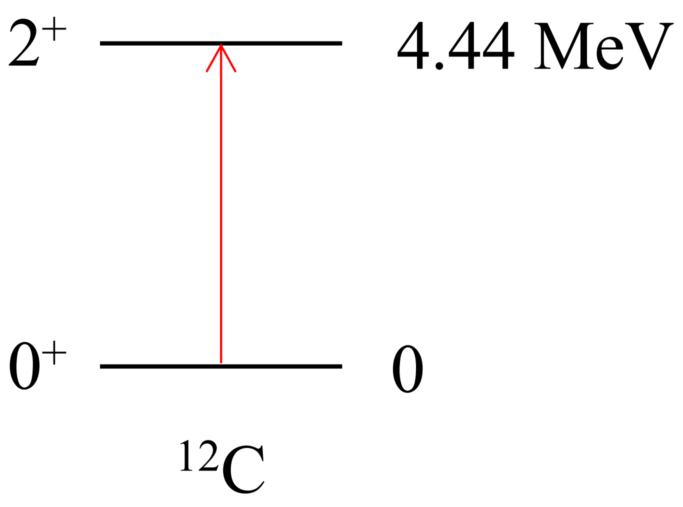
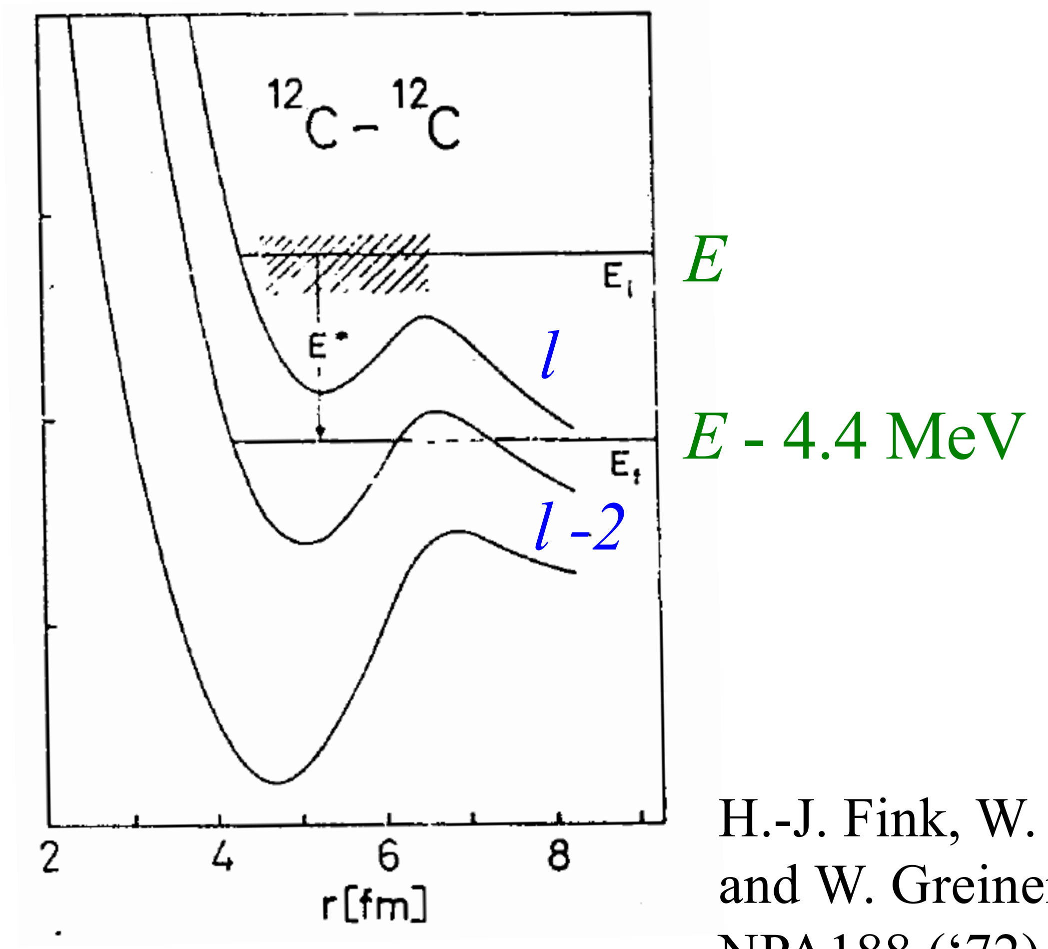
* the basic concept is all same





H.-J. Fink, W. Scheid, and W. Greiner, NPA188 ('72) 259

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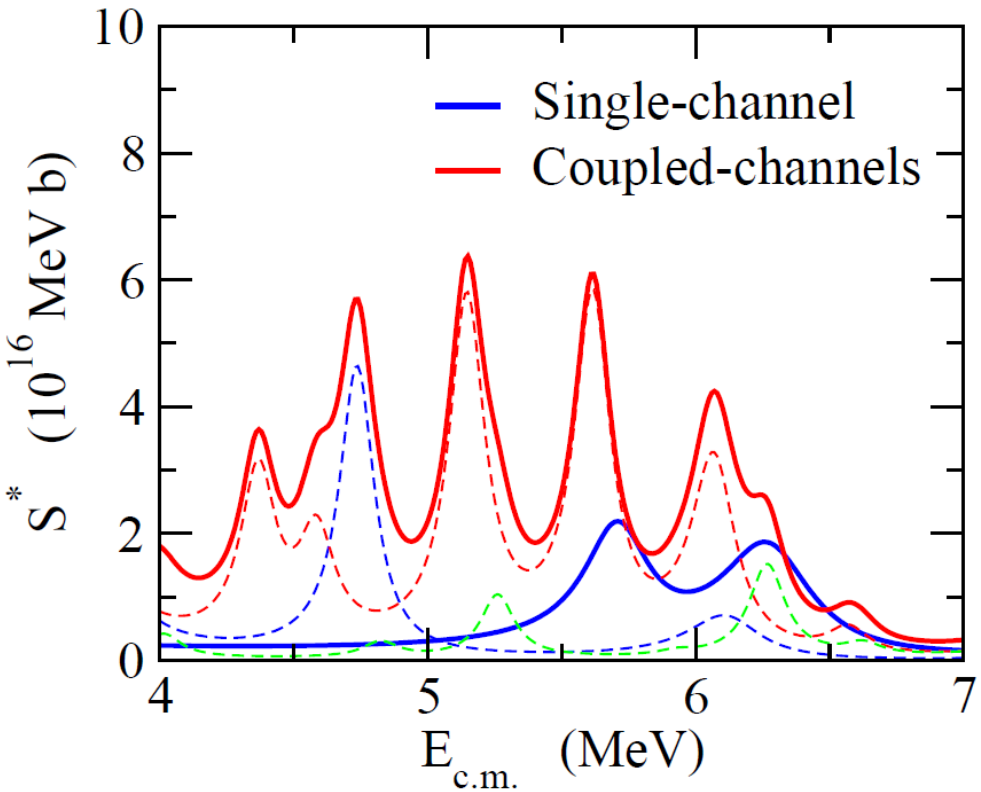
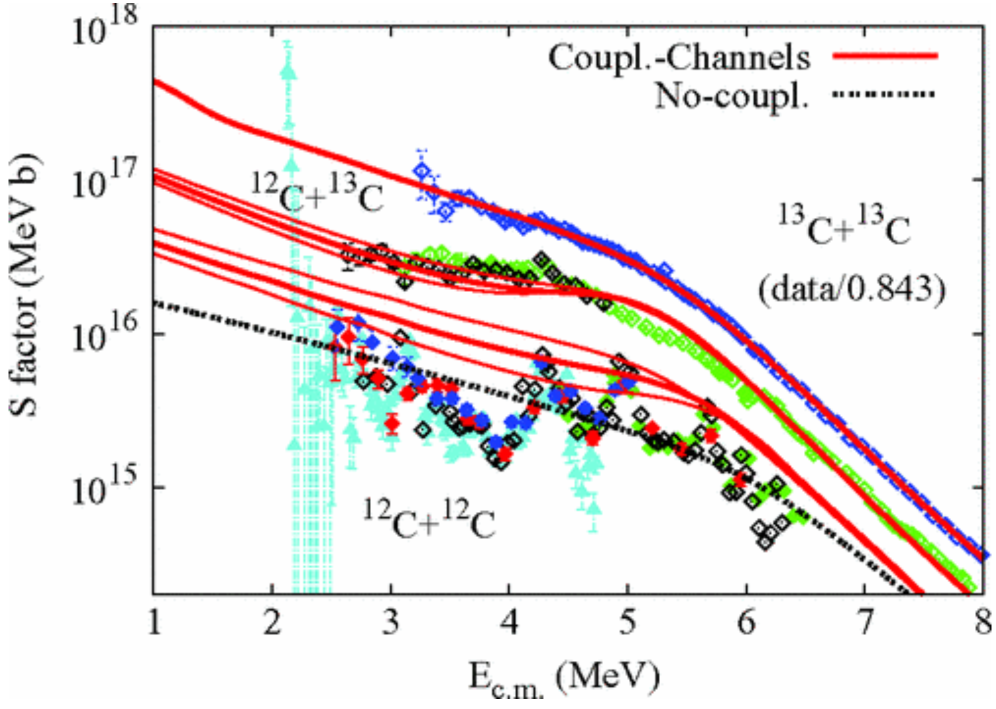


Analogy to Feshbach resonances in cold atoms!

Y. Kondo, T. Matsuse, and Y. Abe, PTP 59 ('78) 465

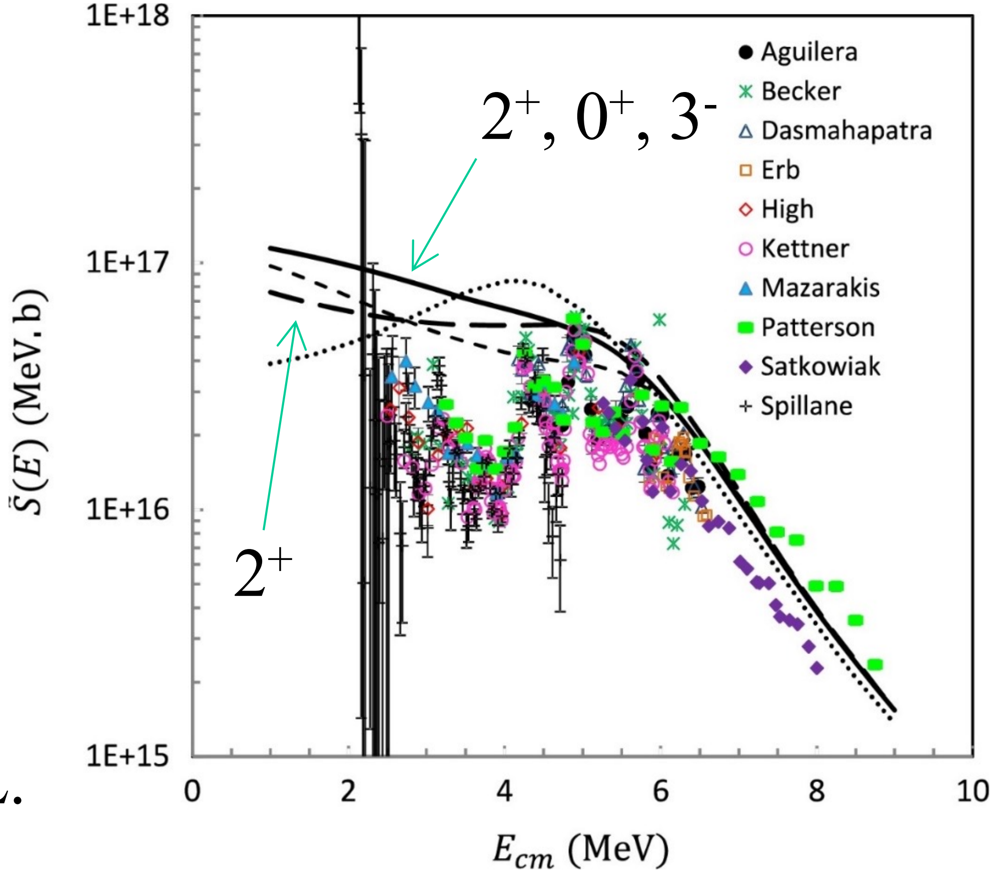
More recent coupled-chanels calculations

Systematic study



H. Esbensen, X. Tang, and C.L. Jiang, PRC84, 064613 (2011).

role of the Hoyle state



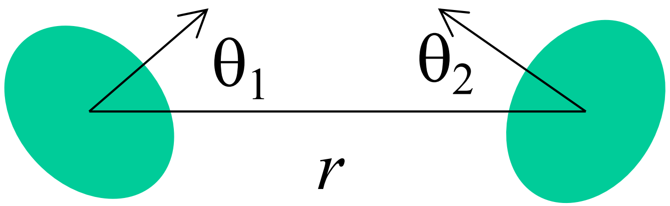
M. Assuncao and P. Descouvemont, PLB723, 355 (2013).

Previous theoretical calculations for $^{12}C+^{12}C$

◻ Time-dependent approach

✓ A. Diaz-Toress and M. Wiesher, PRC97, 055802 (2018).

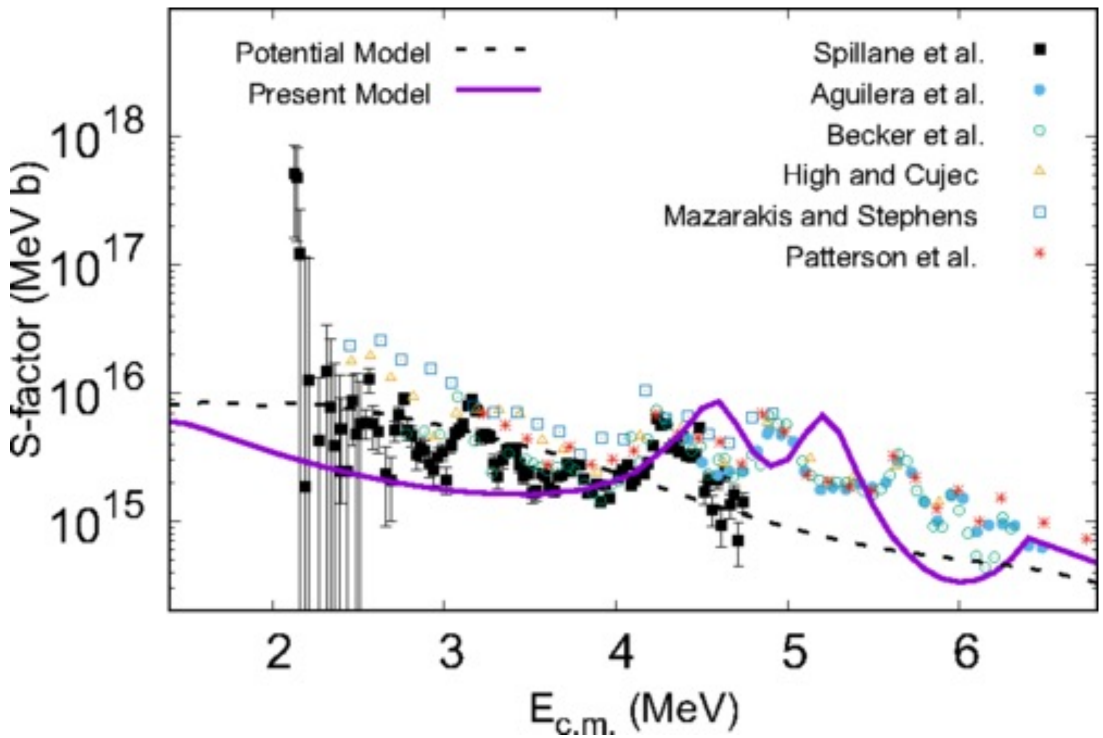
Time-dependent wave packet approach



$$i\hbar\frac{\partial}{\partial t}\psi(r,\theta_1,k_1,\theta_2,k_2;t) = H\psi(r,\theta_1,k_1,\theta_2,k_2;t)$$

$$\psi(r,\theta_1,k_1,\theta_2,k_2;t=0) = \underline{\chi_0(r)}\underline{\psi_0(\theta_1,k_1,\theta_2,k_2)}$$

Gaussian wave packet — rotor



Previous theoretical calculations for $^{12}C+^{12}C$

☐ TDHF-based approaches

✓ P.-H. Heenen, PL99B, 298 (1981).

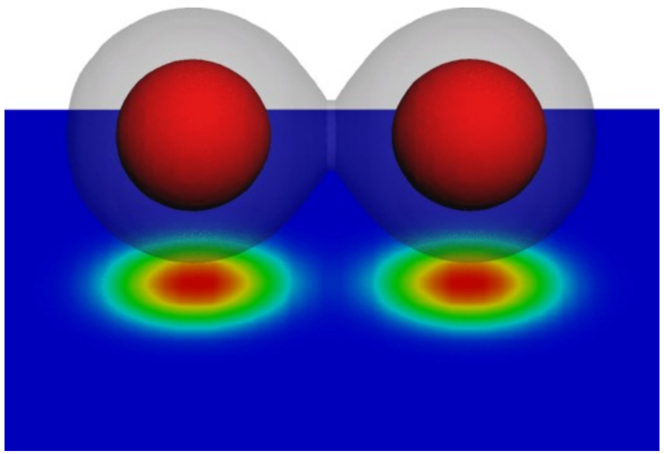
Adiabatic TDHF

✓ K. Godbey, C. Simenel, and A.S. Umar, PRC100, 024619 (2019).

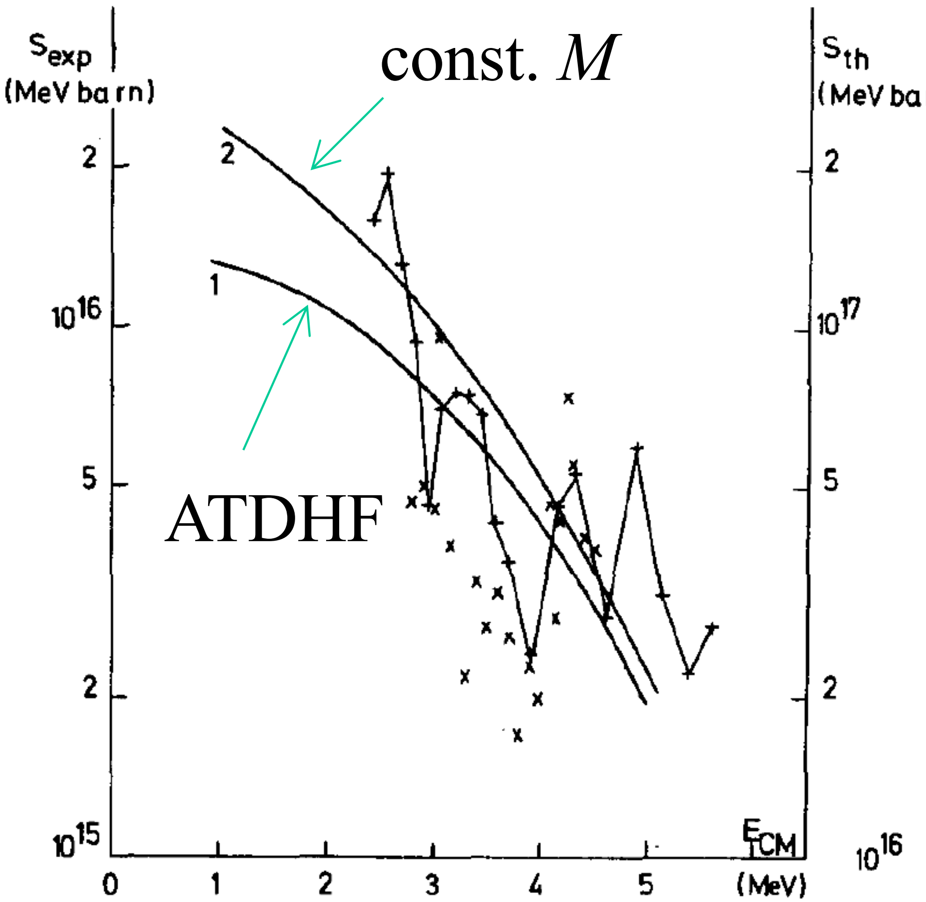
Density-Constrained Hartree-Fock

collective Hamiltonian from TDHF

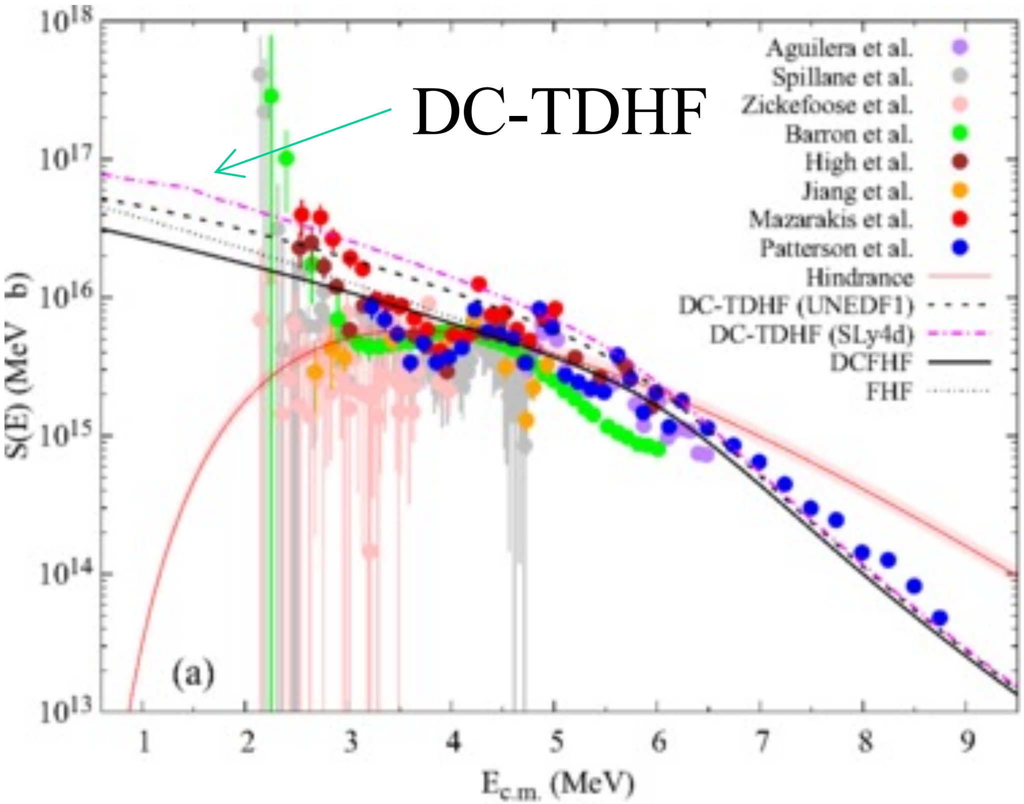
$$H_{\rm coll} = \frac{1}{2}M(R)\dot{R}^2 + V(R) \to P_{\rm fus}(E)$$



ATDHF



DC-TDHF



P.-H. Heenen,
PL99B, 298 (1981)

K. Godbey, C. Simenel, and
A.S. Umar,
PRC100, 024619 (2019).

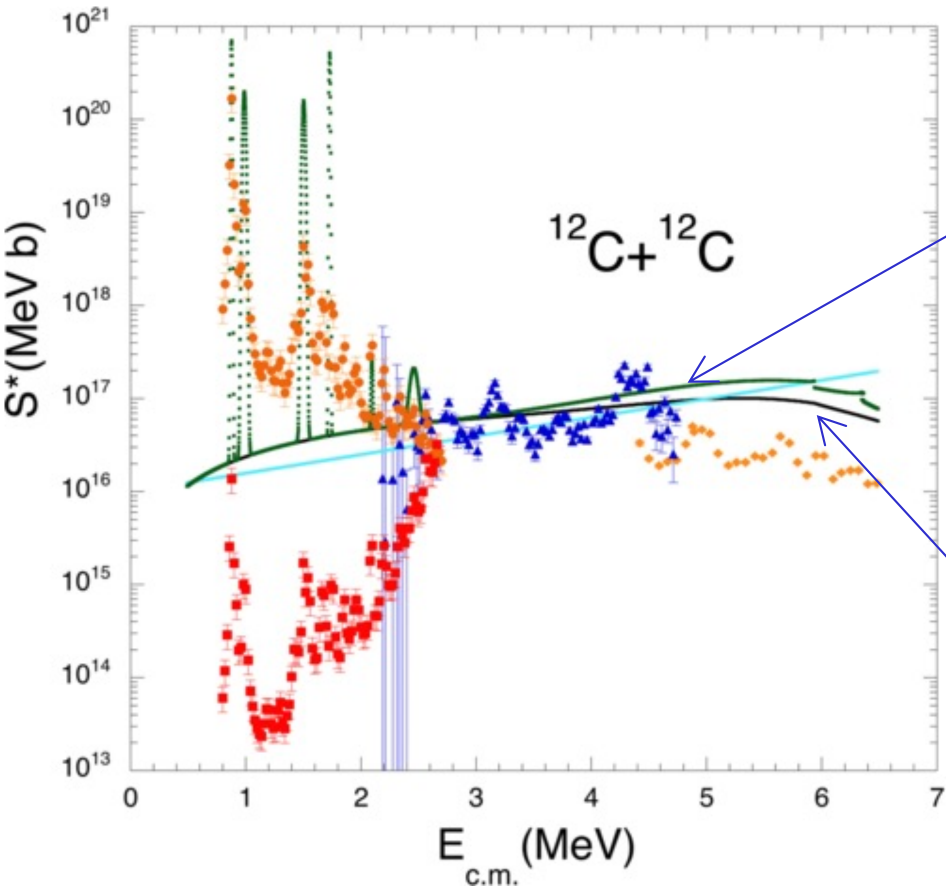
Previous theoretical calculations for $^{12}C+^{12}C$

TDHF-based approaches

✓ A. Bonasera and J.B. Natowitz, PRC102, 061602(R) (2020).

Imaginary-time evolution

$$i\hbar\frac{\partial}{\partial t}\psi_i(\boldsymbol{r},t) = h[\rho]\psi_i(\boldsymbol{r},t) \rightarrow -\hbar\frac{\partial}{\partial \tau}\psi_i(\boldsymbol{r},\tau) = h[\rho]\psi_i(\boldsymbol{r},\tau)$$



im.-time TDHF +resonances (justification?)

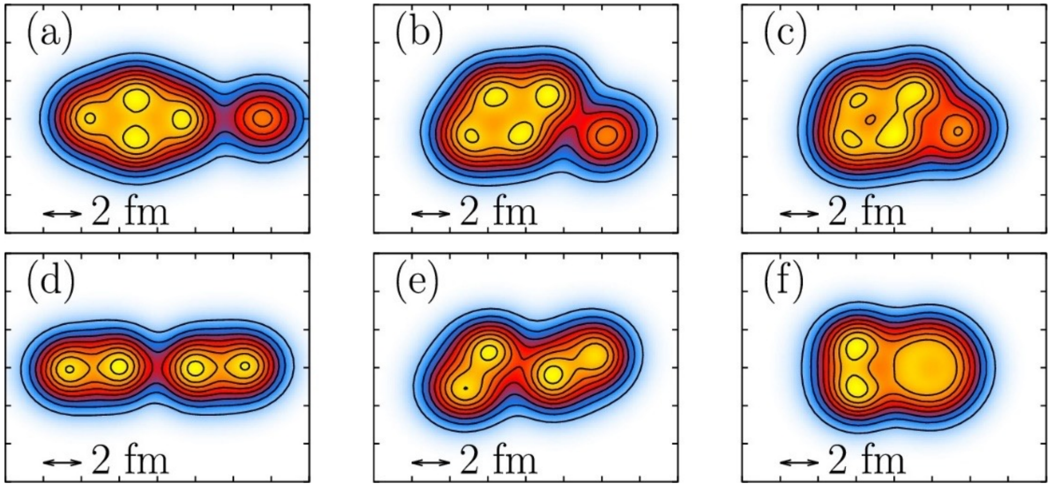
im.-time TDHF

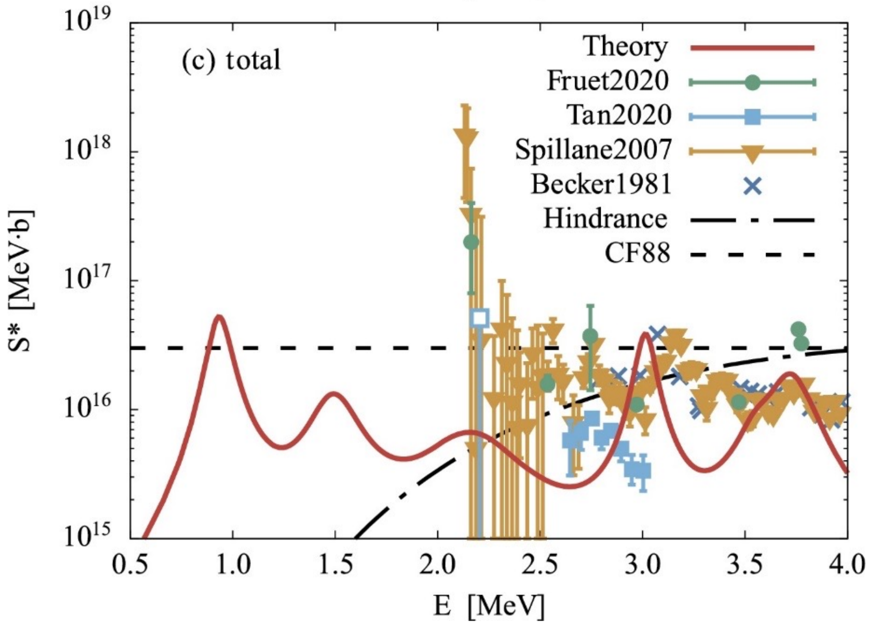
Previous theoretical calculations for $^{12}C+^{12}C$

☐ Anti-symmetrized Molecular Dynamics (AMD) approach

Y. Taniguchi and M. Kimura, PLB823, 136790 (2021).

AMD + R-matrix



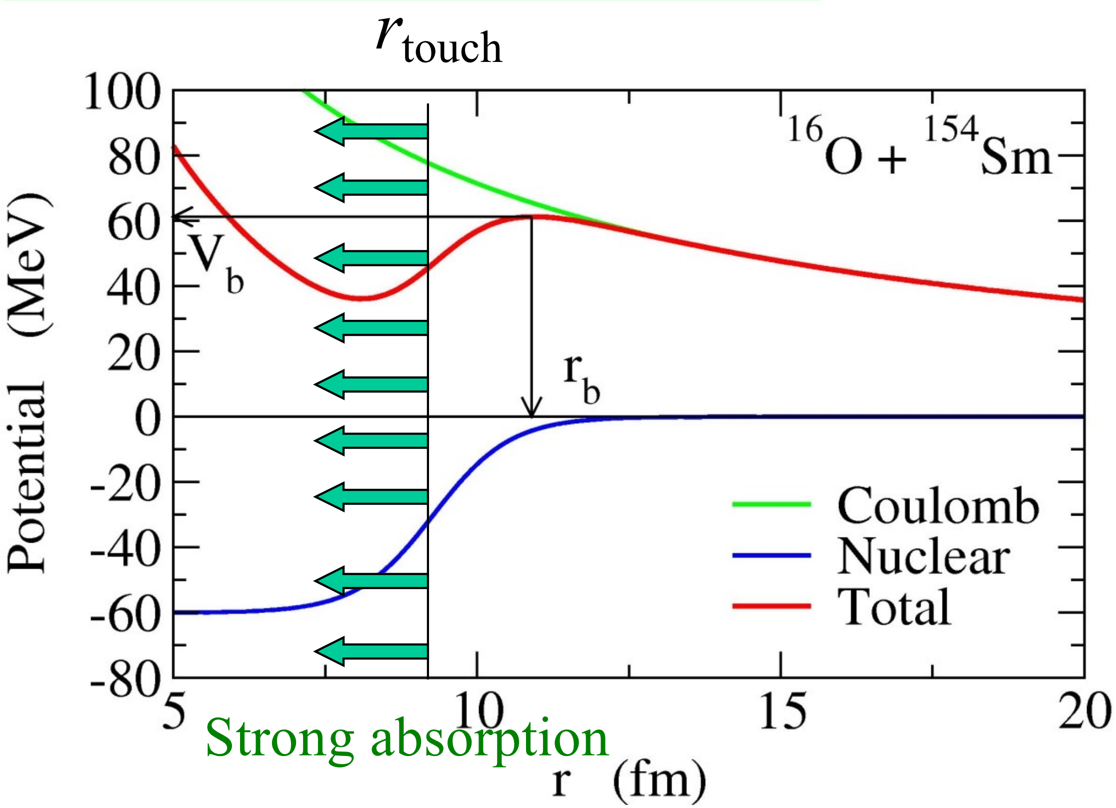


$$\Psi_M^{J\pi} = \sum_{iK} c_{iK} P_{MK}^J \Phi_i^\pi$$

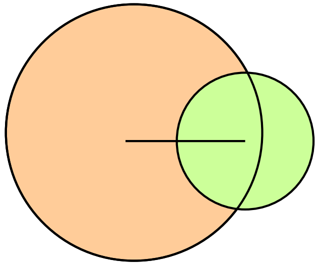
→ R-matrix to match the outer Coulomb wave functions

→ $\sigma_{fus}(E)$

Strong absorption model



the region of large overlap



- High level density (CN)

The relative energy is quickly lost and converted to internal energies

Formation of hot CN (fusion reaction)

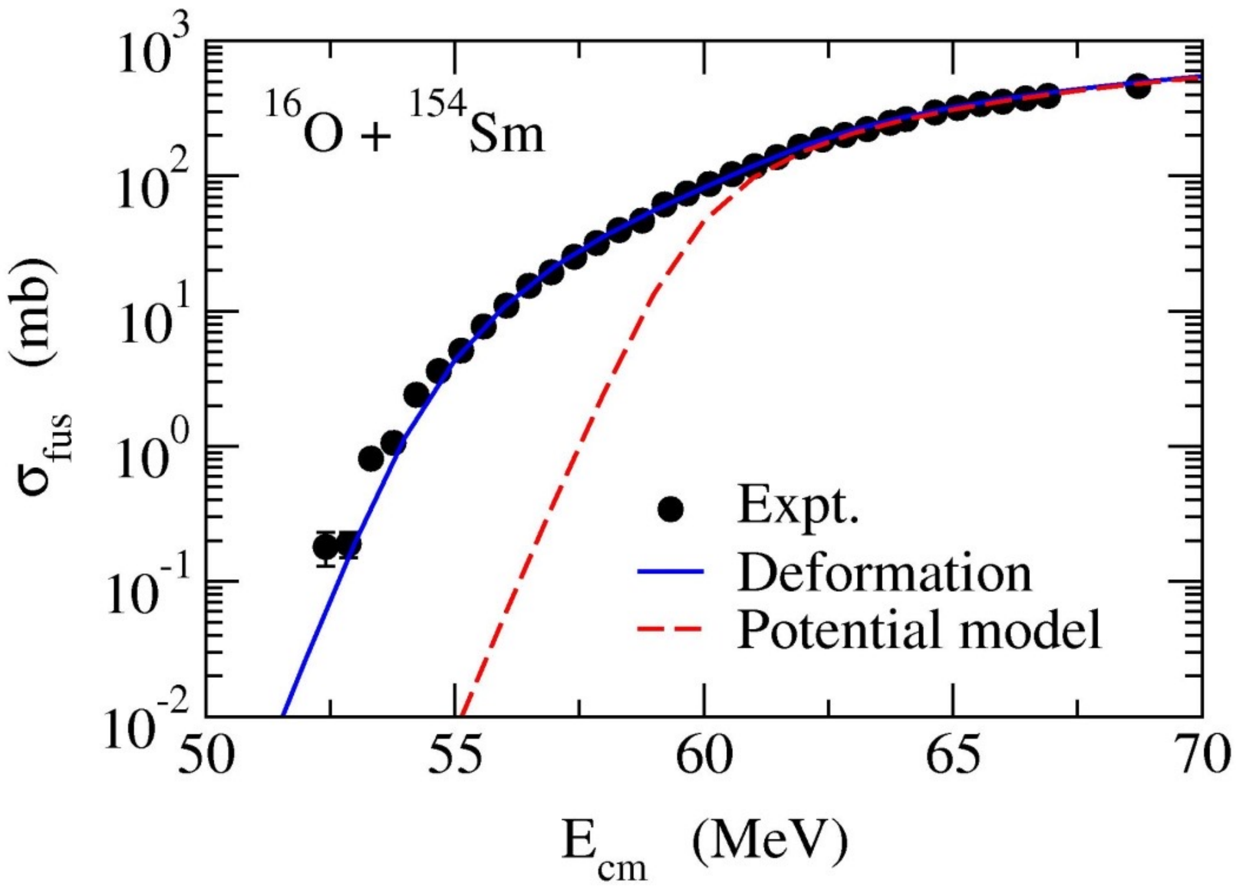
This can be similuated either with

✓ a large imaginary potential at the potential pocket

or with

✓ the incoming wave boundary condition

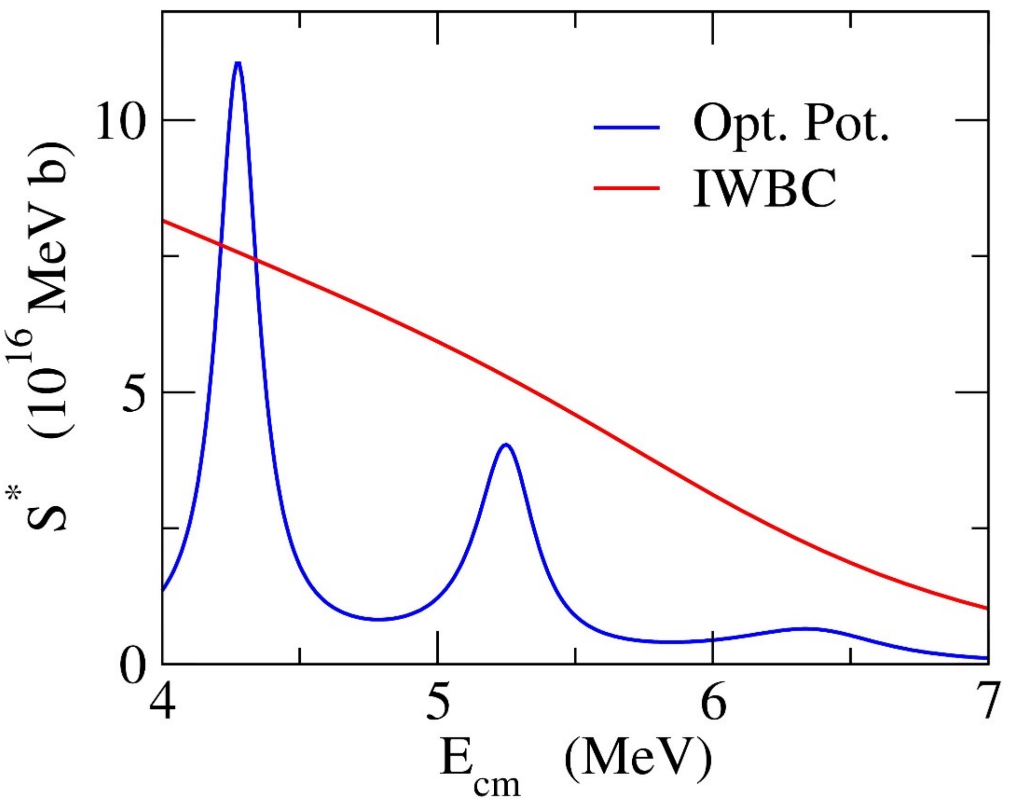
The assumption of strong absorption works good for heavy systems:



K. Hagino and N. Takigawa,
Prog. Theo. Phys.128 (‘12)1061.

….but this may not be the case for light systems such as $^{12}C+^{12}C$

single-channel calculation



V_0=20 MeV, r_0=1.35 fm, a=0.5 fm
W=0.2 MeV, r_w=1.05 fm, a_w=0.2 fm

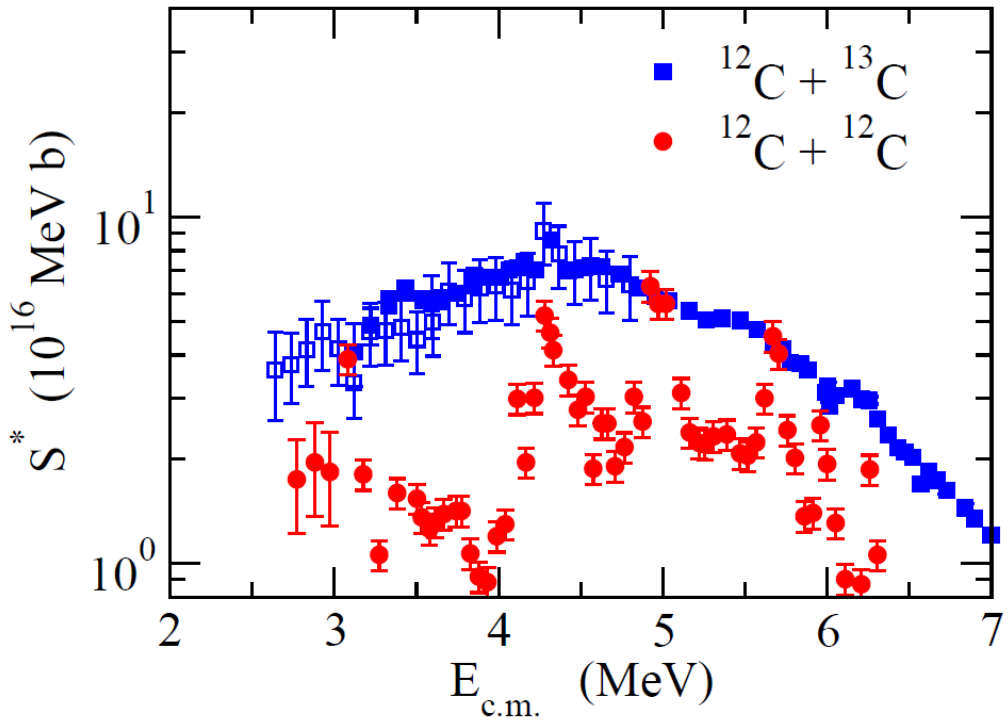
IWBC:

✓ no resonance structure

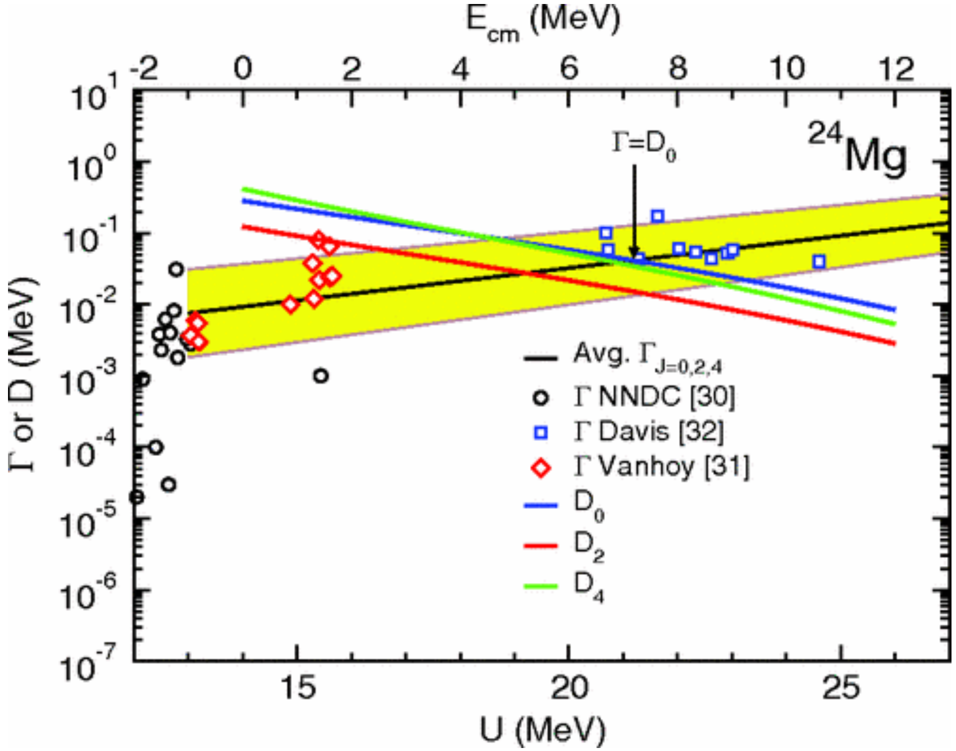
✓ "the upper limit" (but not really) of S-factors

Effects of isolated CN resonances

cross section deficit



C.L. Jiang et al., PRL110, 072701 (2013).



$E_{c.m.} < 7$ MeV

$\Gamma < D \rightarrow$ fusion inhibition

$E_{c.m.} > 7$ MeV

$\Gamma > D \rightarrow P_J \sim 1$

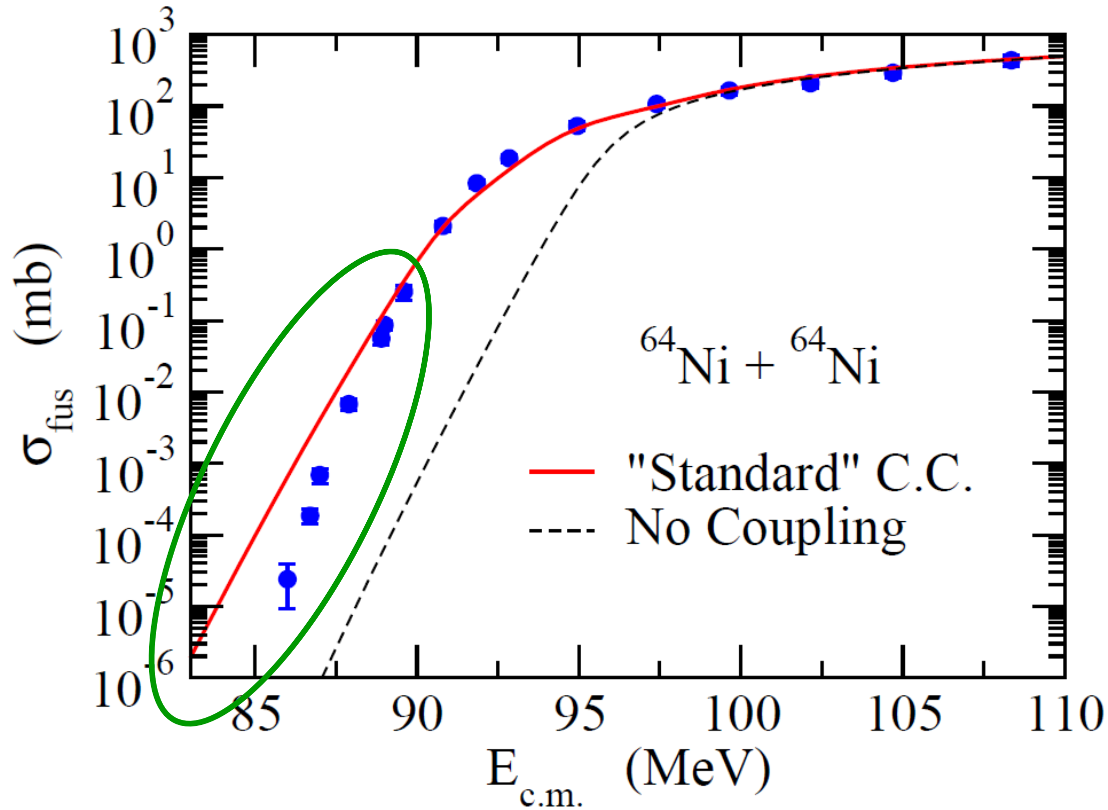
Moldauer factor

$$P_J = 1 - \exp(-2\pi\Gamma_J/D_J)$$

$$\rightarrow \sigma = \sum_J \sigma_{CC}^J P_J$$

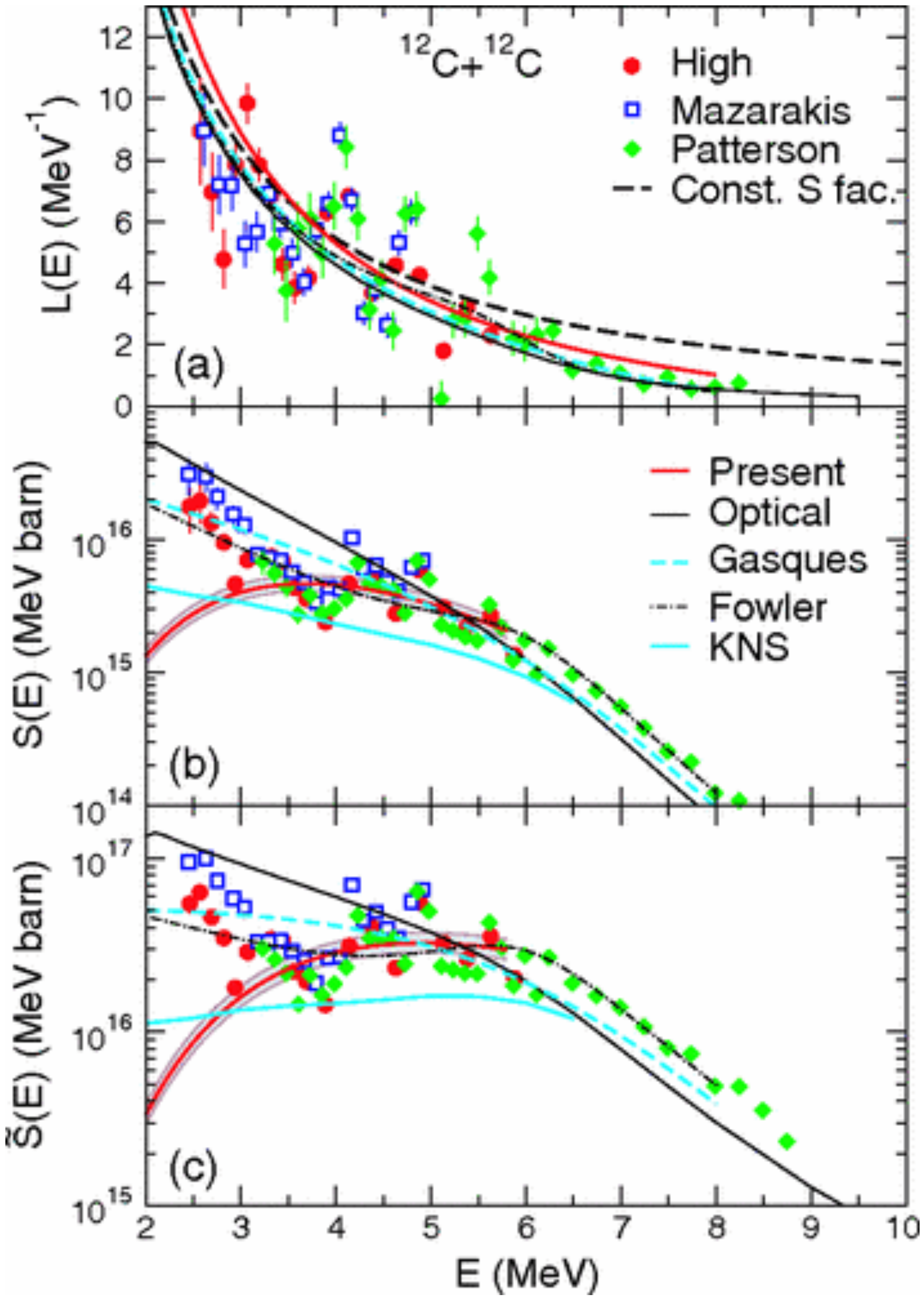
Hindrance

Deep sub-barrier hindrance in medium heavy systems



C.L. Jiang et al., PRL89('02)052701;
PRL93('04)012701

Is there a large hindrance in $^{12}C+^{12}C$ fusion?



C.L. Jiang et al., PRC75, 015803 (2007)

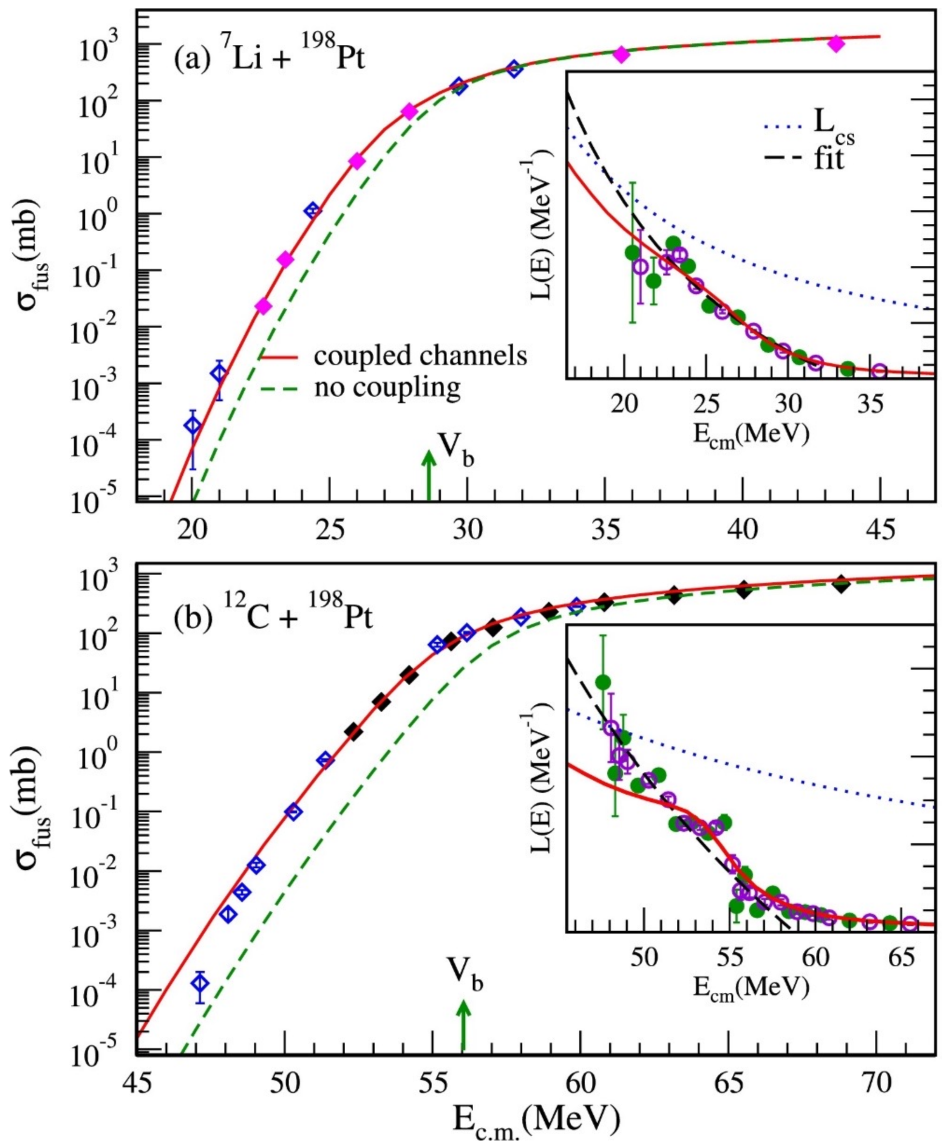
$$L(E) = \frac{d}{dE}\ln(E\sigma)$$

fit with:

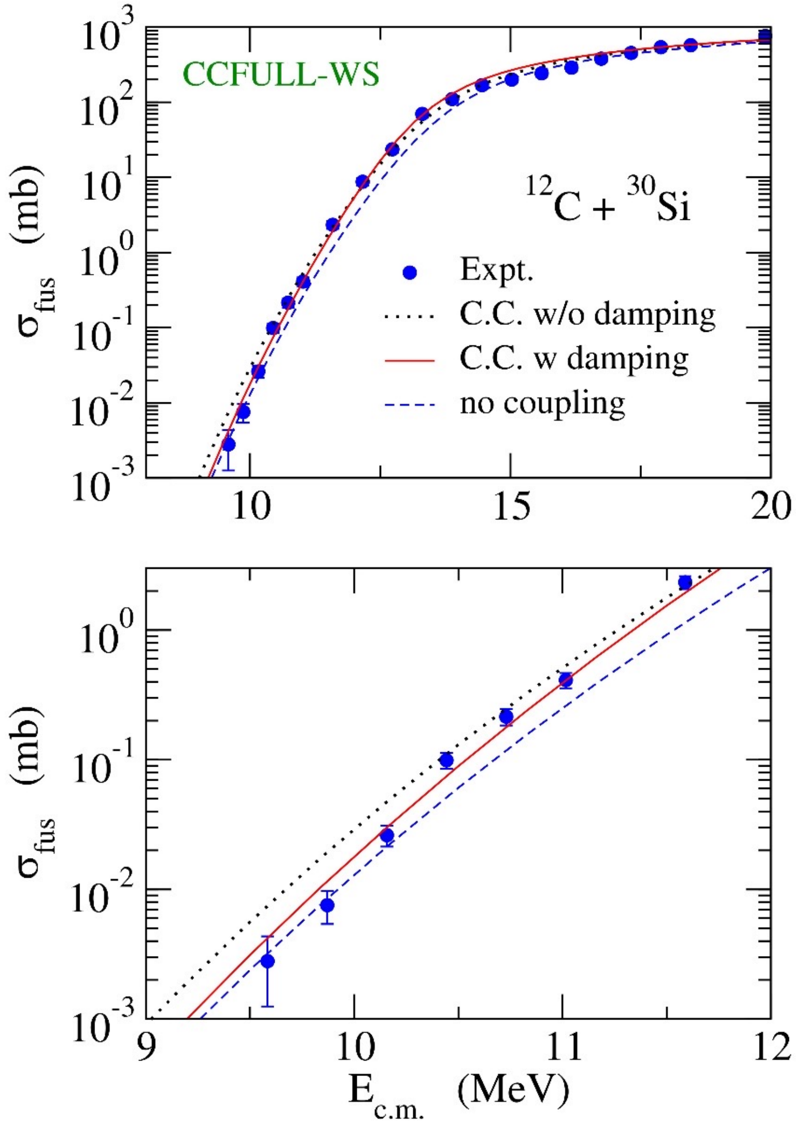
$L(E) = A_0 + B_0/E^n$

→ large hindrance

no (or only tiny) hindrance has been observed in light systems



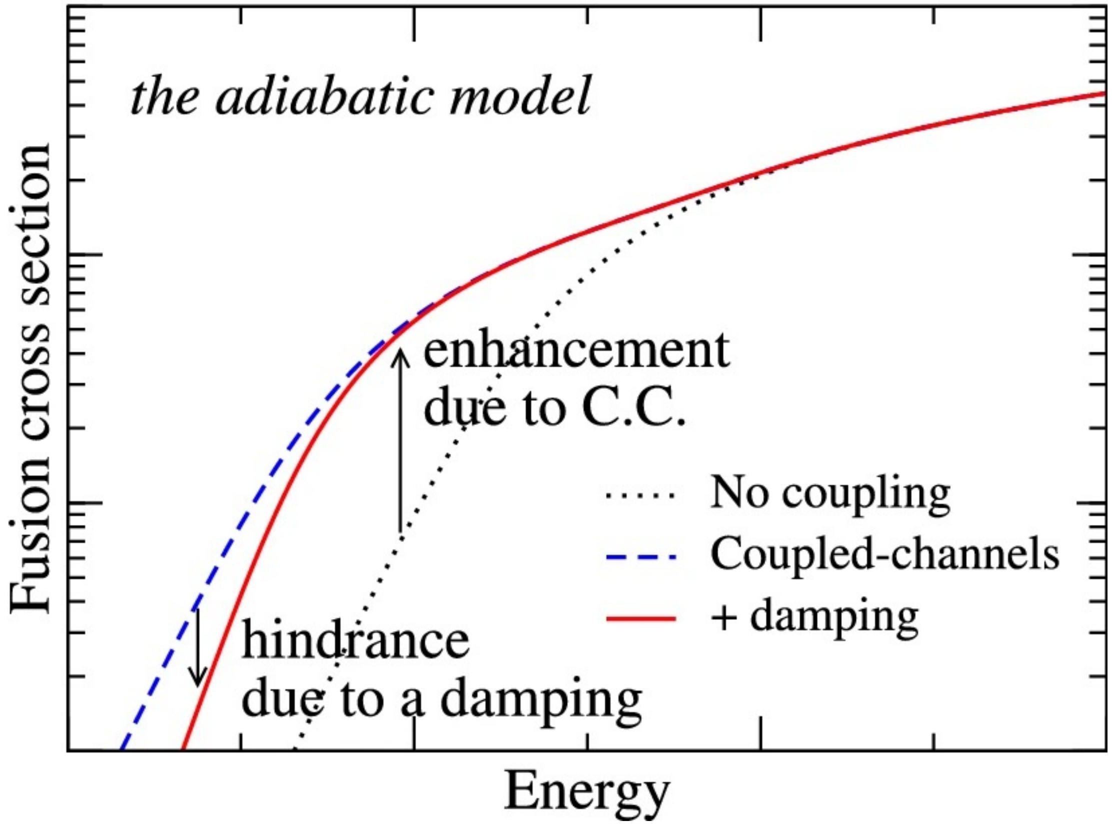
A. Shrivastava et al., PLB755, 332 (2016).



G. Montagnoli et al., PRC97, 024610 (2018).

adiabatic model for fusion hindrance:

T. Ichikawa, K.H., and A. Iwamoto, PRL103('09)202701



C.L. Jiang et al., EPJA57, 235 (2021)

light systems: small enhancement of fusion cross sections

→ small hindrance

Summary

Open questions:

* How can one connect the resonance peaks to CN states?
  - can one have a model which reproduces all the resonance peaks?
  - how can one connect CN states to the reaction dynamics?
  - can one construct a toy model to start with?

* Can one describe $^{12}C+^{12}C$ and $^{12}C+^{13}C$ simultaneously?
  - Does $^{12}C+^{13}C$ provide the upper limit of $^{12}C+^{12}C$ cross sections?
* How much fusion hindrance can one expect for $^{12}C+^{12}C$?
* Electron screening effects?