Role of quantum mechanics in a diffusion process for superheavy elements

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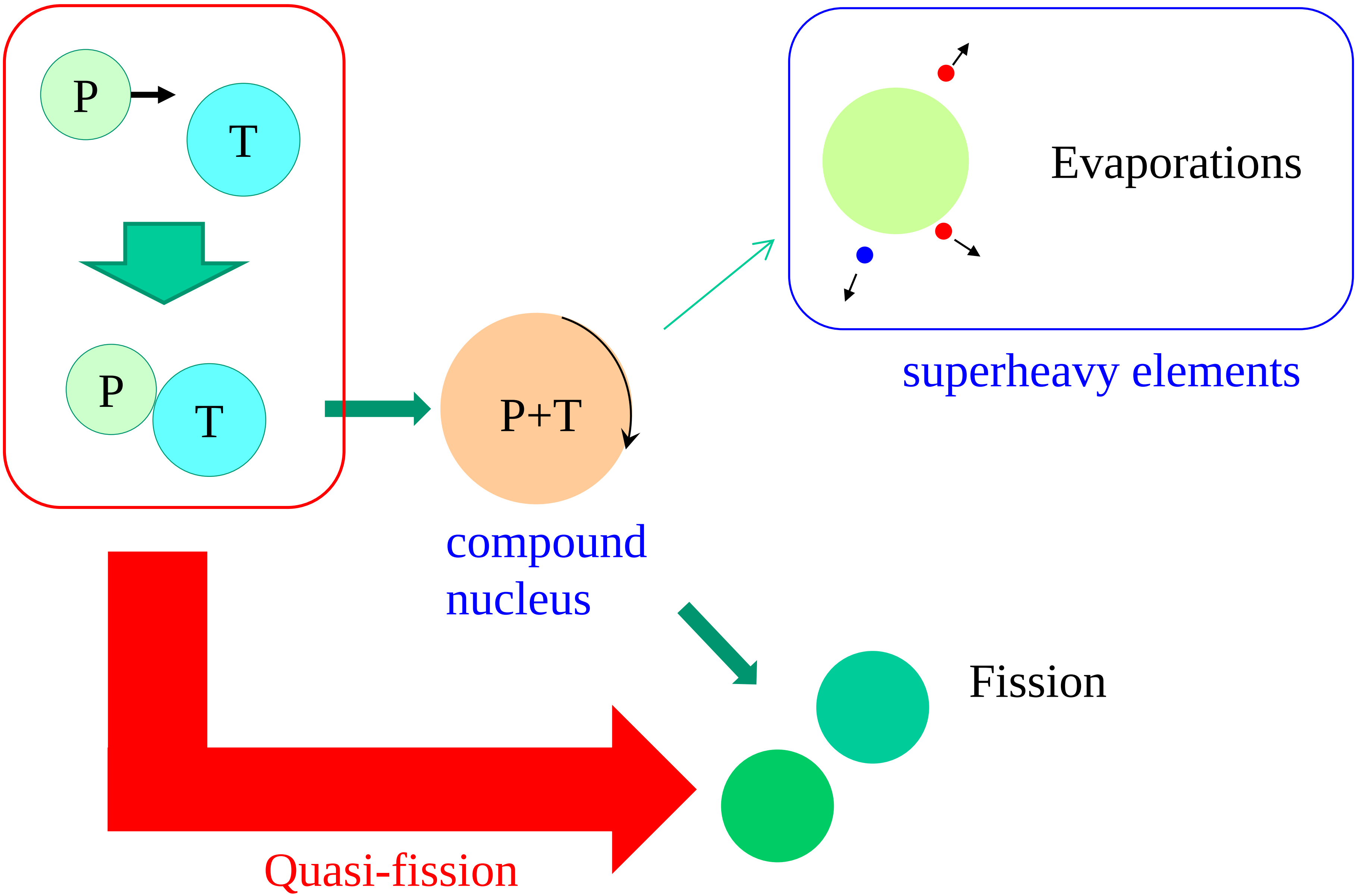
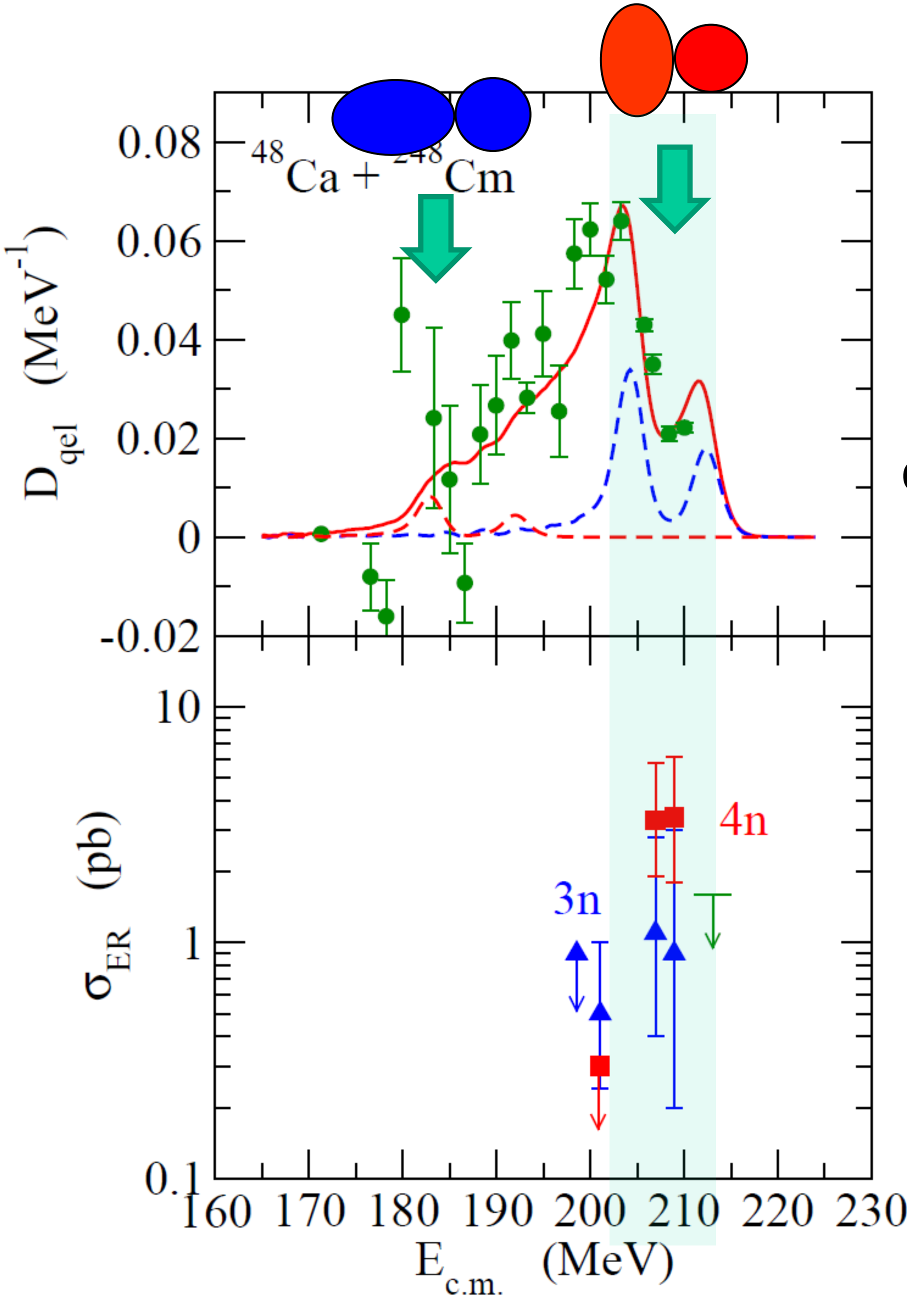
1. Diffusion over the saddle: Langevin approach
2. Generalized Langevin approach and cold fusion
3. Towards quantum Langevin method
4. Summary

The Virtual Superheavy Elements seminars, online, Dec. 8, 2020

Fusion for Superheavy elements



Entrance channel dynamics: capture barrier distribution

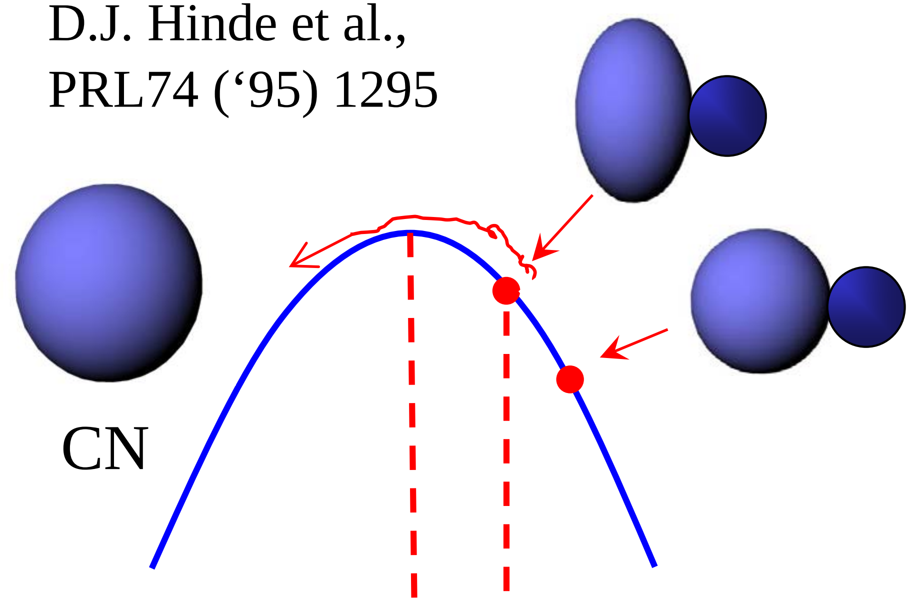


$^{48}Ca + ^{248}Cm \rightarrow {}^{296}_{116}Lv^{*}$

T. Tanaka,..., K.H., et al.,
JPSJ 87 ('18) 014201
PRL124 ('20) 052502

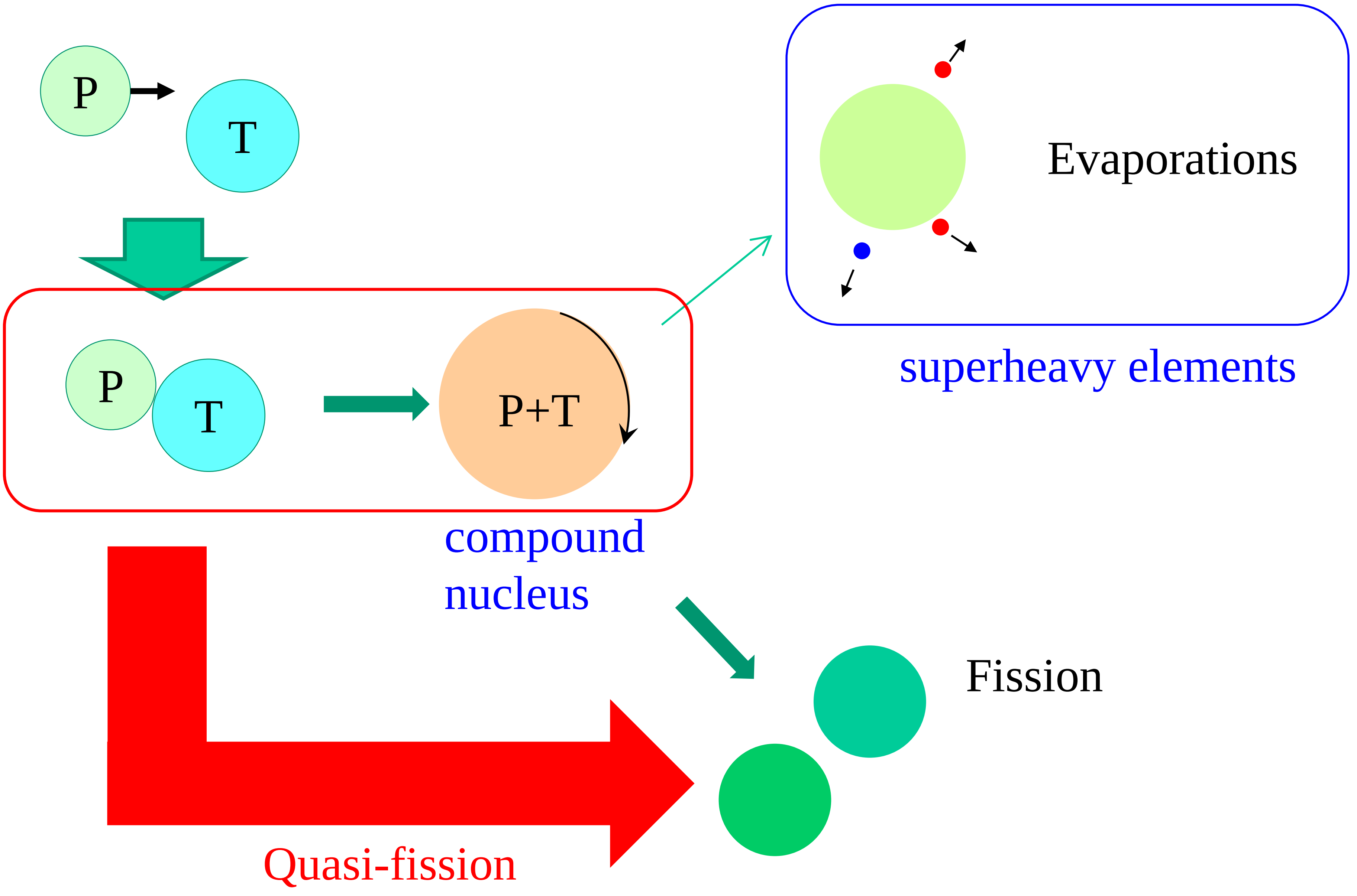
cf. notion of compactness:

D.J. Hinde et al.,
PRL74 ('95) 1295

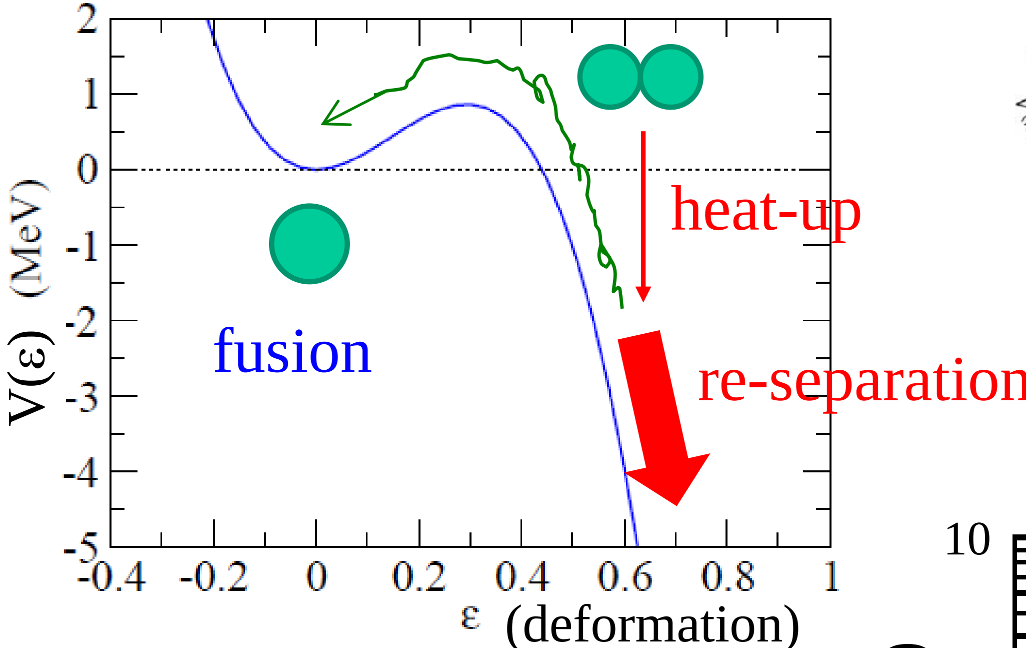


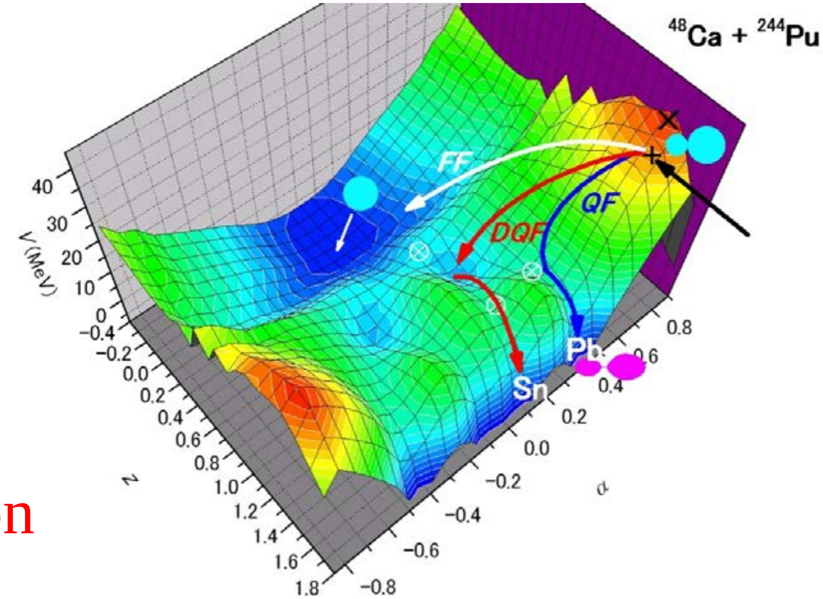
the next talk by Tanaka

Fusion for Superheavy elements



Langevin approach

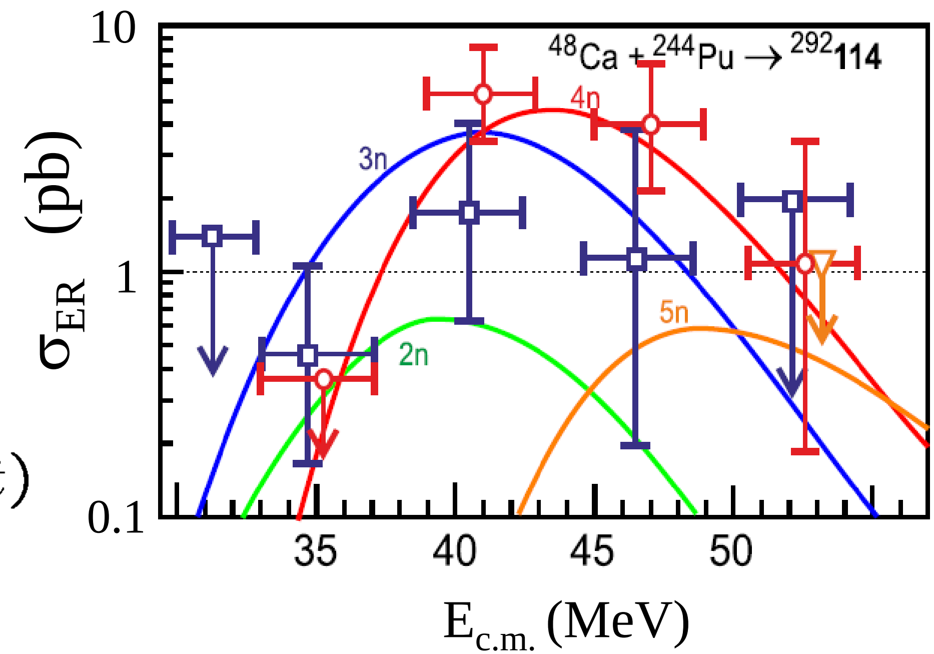




thermal fluctuation

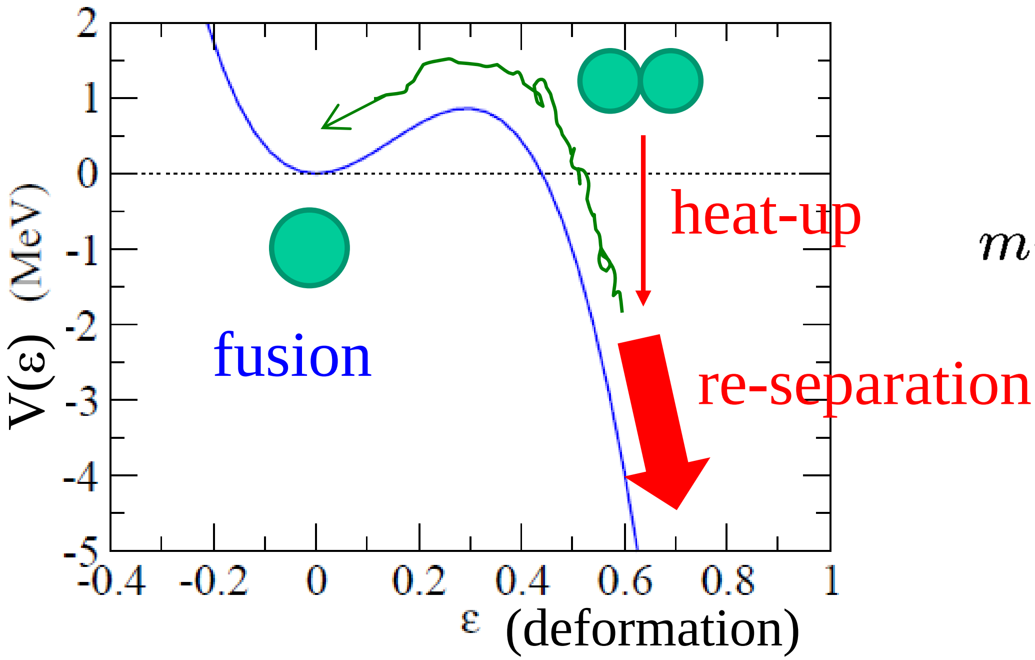
→ *classical* Langevin method

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$



V.I. Zagrebaev and W. Greiner (2015)

Langevin approach



$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

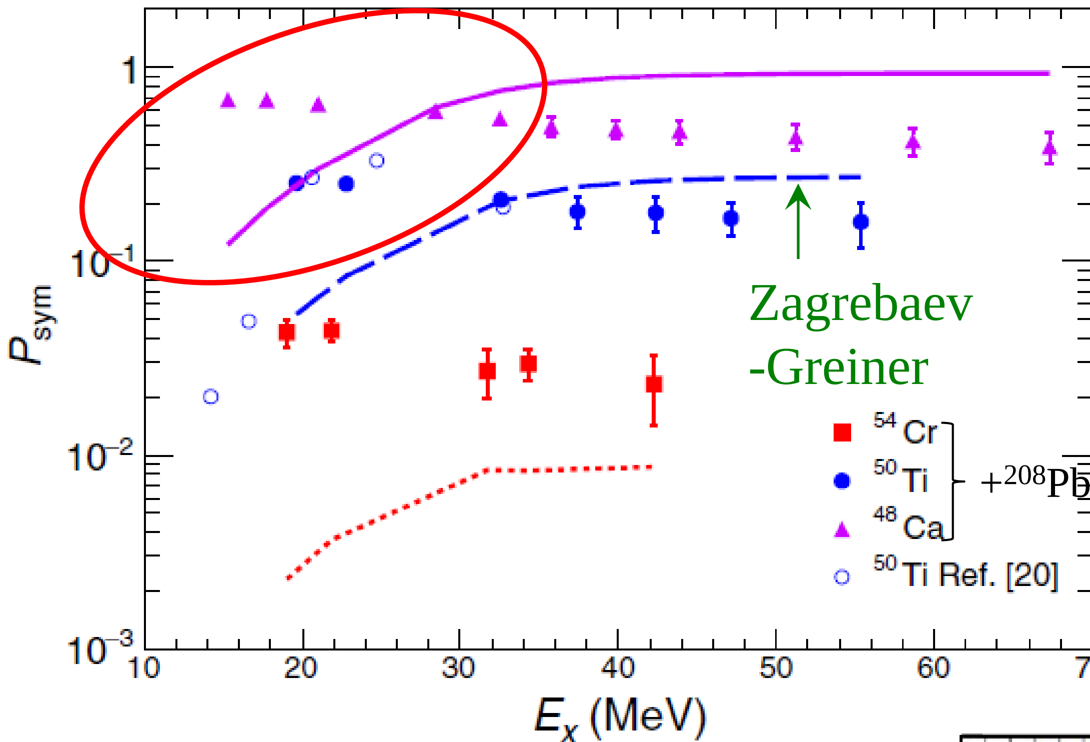
Theoretical issues

- ✓how to thermaize? mechanisms?
- ✓is thermal equilibrium OK?
- ✓Is Markovian approximation OK?
- ✓quantum effects?
- ✓quantal-to-classical transitions (decoherence)?

microscopic models?

open quantum systems

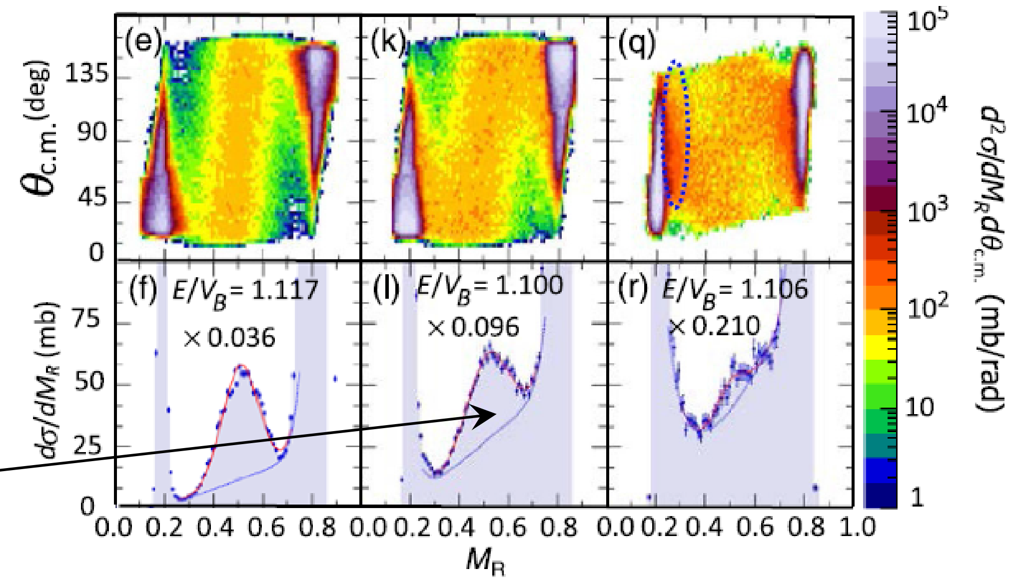
Recent publication by Banerjee et al. (ANU)



K. Banerjee, D.J. Hinde, et al., PRL122, 232503 (2019)

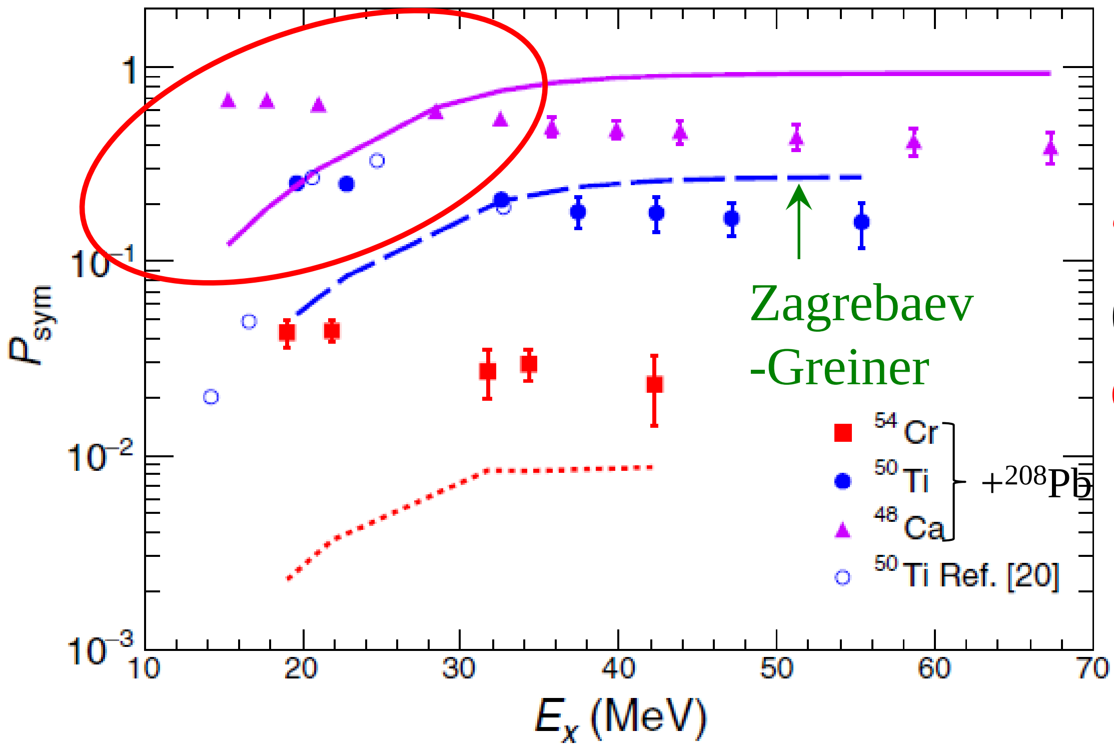
Zagrebaev -Greiner

"cold fusion reactions (involving ^{208}Pb) are not driven by a diffusion process"



$\sigma_{fus\text{-}fiss}$

Recent publication by Banerjee et al. (ANU)



K. Banerjee, D.J. Hinde, et al., PRL122, 232503 (2019)

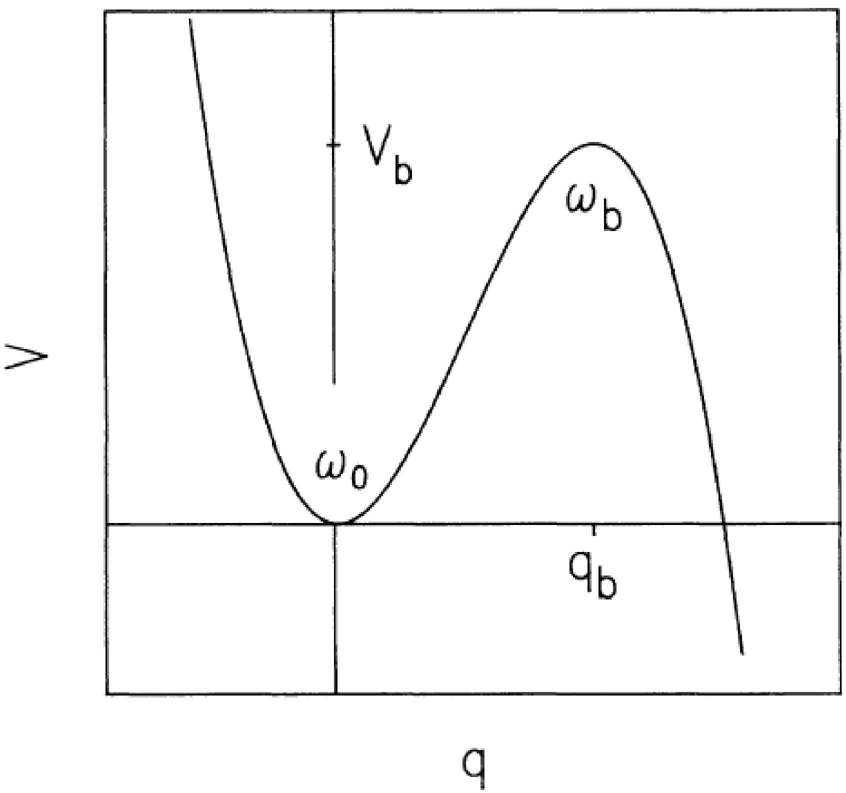
"cold fusion reactions (involving ^{208}Pb) are not driven by a diffusion process"

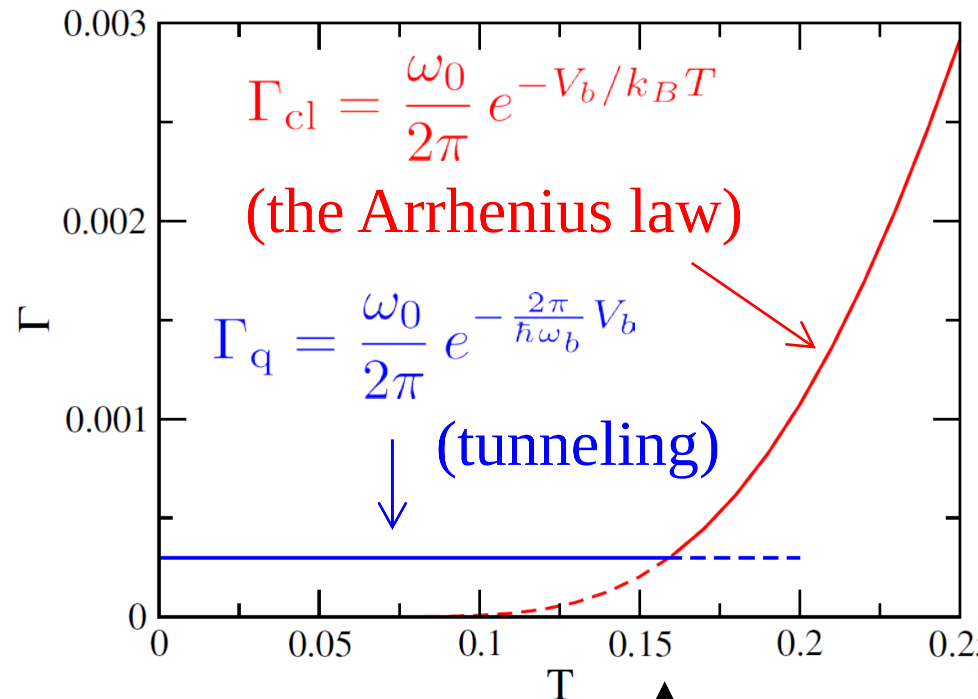
comparisons: to a classical Langevin calculation

$\rightarrow$ quantum effect should be crucial at low E_x

Decay of a metastable state at finite temperatures

cf. induced fission





$$k_BT_c \sim \frac{\hbar\omega_b}{2\pi}$$

H. Grabert, P. Olschowski, and U. Weiss, PRB36, 1931 (1987)

⇨ quantum Langevin for low temperatures?

classical Langevin equation

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

friction

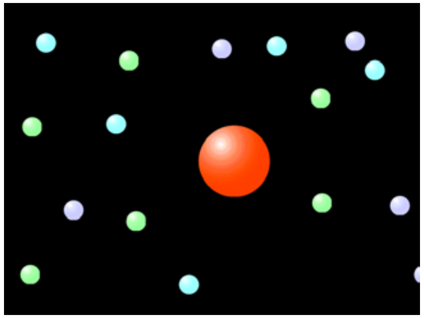
random interaction $\rightarrow \langle R(t)\rangle = 0$

classical: $\boxed{\langle R(t)R(t')\rangle = 2D\,\delta(t-t') \equiv 2D\chi(t-t')}$

$D = \gamma T$ (Einstein relation)

(white noise; no memory)

Brownian motion



interaction of a Brownian particle with atoms

classical Langevin equation

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \underline{\gamma\frac{dq}{dt}} + \underline{R(t)}$$

friction   random interaction $\rightarrow \langle R(t)\rangle = 0$

classical: $\boxed{\langle R(t)R(t')\rangle = 2D\,\delta(t-t') \equiv 2D\chi(t-t')}$

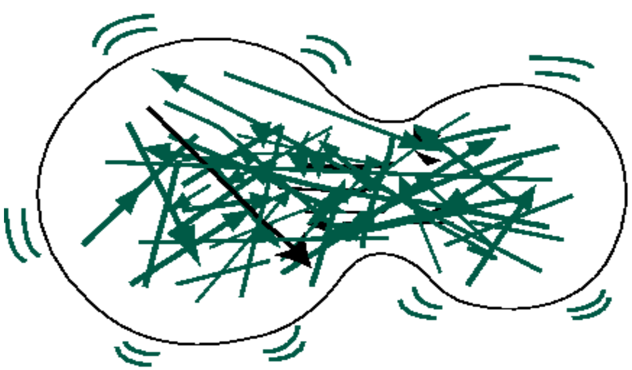
$D = \gamma T$ (Einstein relation)

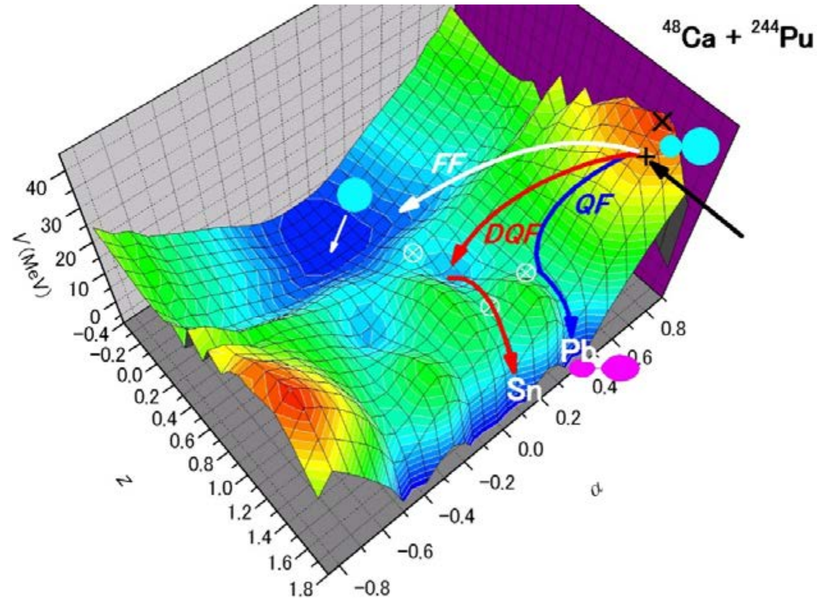
(white noise; no memory)

nuclear reactions:

q = the relative distance etc.

"atoms" = nucleonic d.o.f





quantal Langevin equation

S. Ayik et al., PRC71 (‘05) 054611

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t) \qquad \langle R(t)\rangle = 0$$

classical: $\langle R(t)R(t')\rangle = 2D\,\delta(t-t') \equiv 2D\chi(t-t')$

quantal:

$D = \gamma T$ (Einstein relation)

$$\chi(t-t') = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\underbrace{\frac{\hbar\omega}{2T}\coth\frac{\hbar\omega}{2T}}_{\text{Fermi statics}}\underbrace{\exp\left[-\frac{(\hbar\omega)^2}{2\Delta^2}\right]}_{\text{high energy cut-off}}e^{-i\omega(t-t')}$$

$$\rightarrow \delta(t-t') \;\; (T\rightarrow\infty)$$

quantal Langevin equation

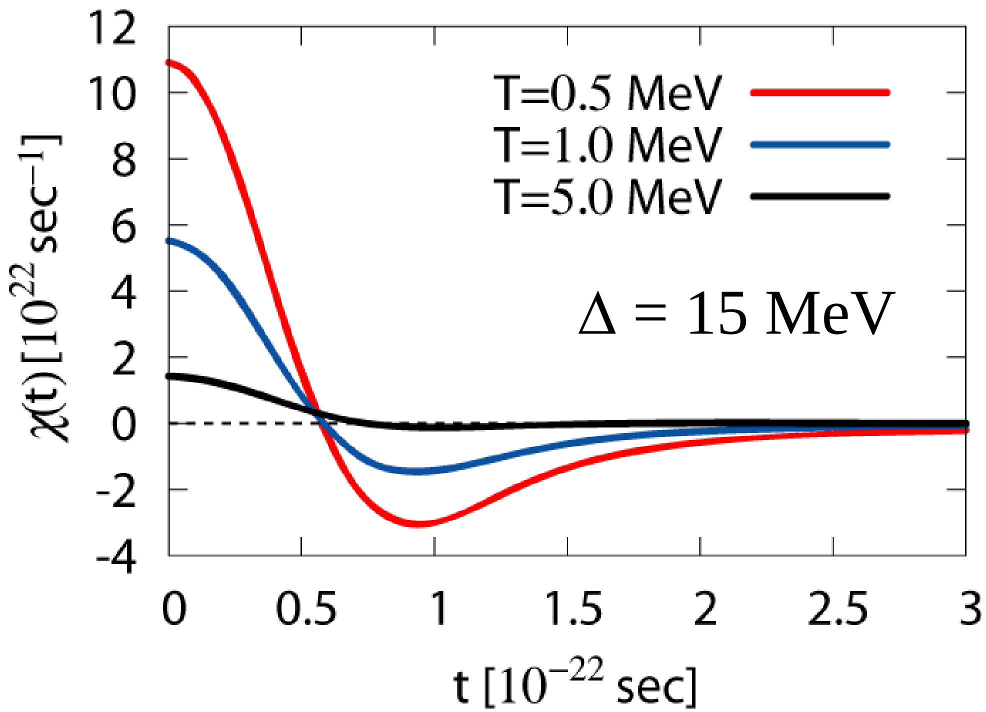
S. Ayik et al., PRC71 ('05) 054611

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t) \qquad \langle R(t)\rangle = 0$$

classical: $\langle R(t)R(t')\rangle = 2D\,\delta(t-t') \equiv 2D\chi(t-t')$

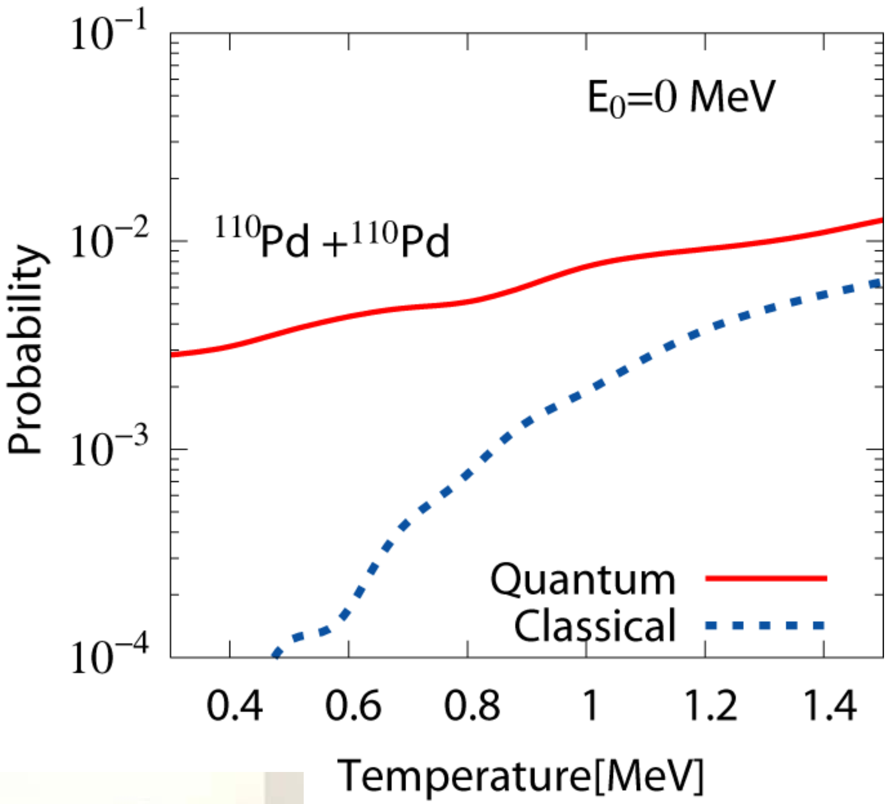
quantal:

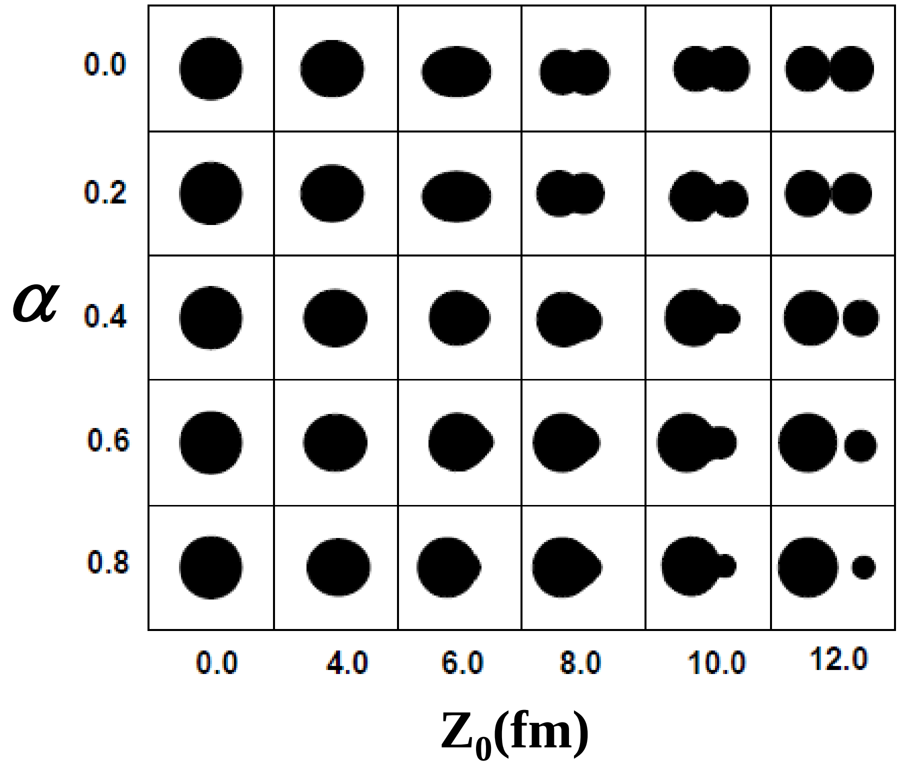
$$\boxed{\chi(t-t') = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\hbar\omega}{2T}\;\coth\frac{\hbar\omega}{2T}\,\exp\left[-\frac{(\hbar\omega)^2}{2\Delta^2}\right]\,e^{-i\omega(t-t')}}$$



a long memory time at low T

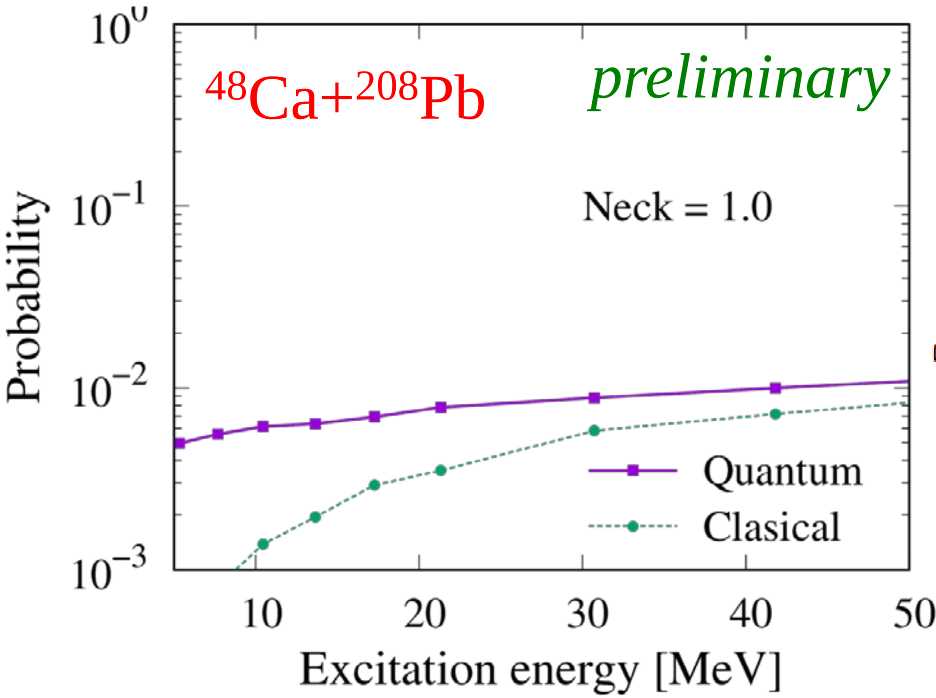
2D Langevin calculations with the rel. coordinate and the mass asym.

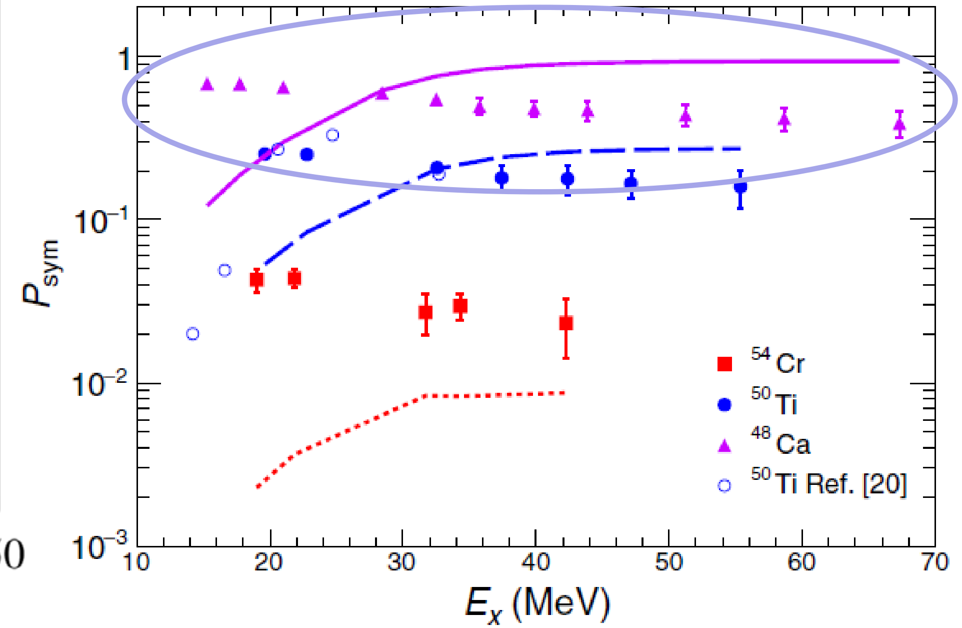




K. Washiyama, Ph. D. thesis, Tohoku University (March, 2007)

2D Langevin calculations with the rel. coordinate and the mass asym.





K. Banerjee, D.J. Hinde, et al.,
PRL122, 232503 (2019)

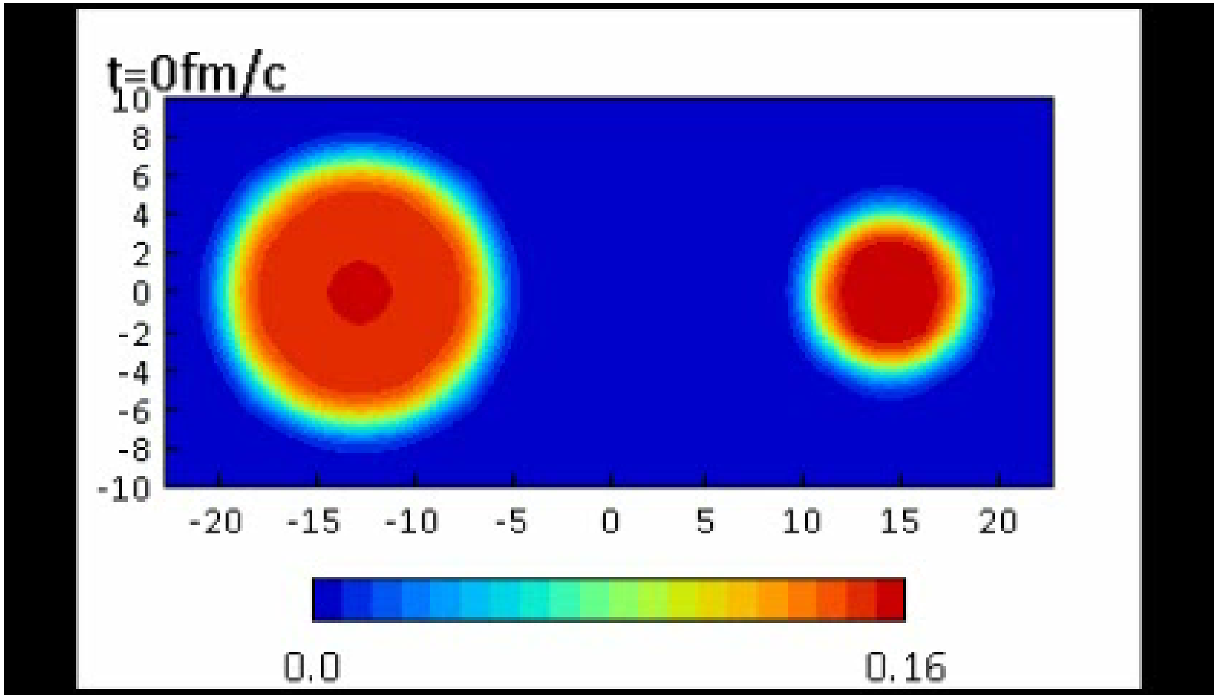
qualitatively good,
but quantitatively bad

→ neck d.o.f?

K. Washiyama and K.H., in preparation

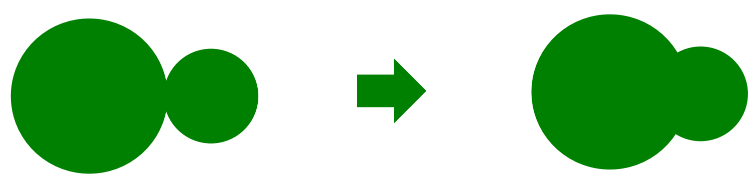
TDHF calculation for $^{48}Ca+^{208}Pb$

E_{cm} = 178 MeV

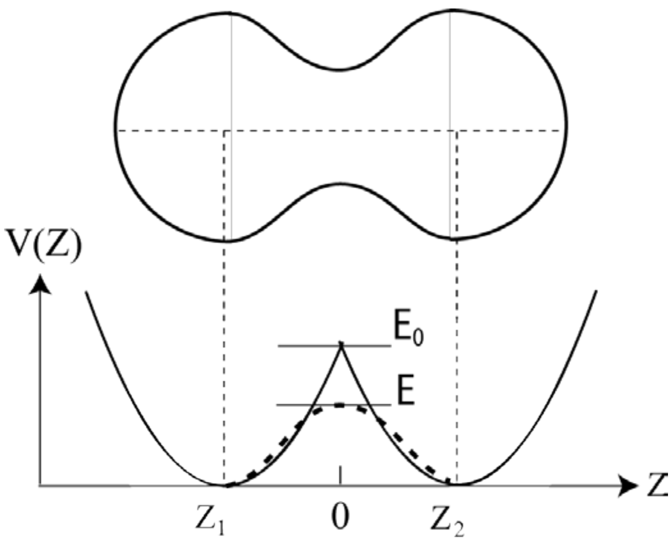


- ✓ Skyrme interaction (SLy4d)
- ✓ TDHF3D code

The neck d.o.f. is fast.



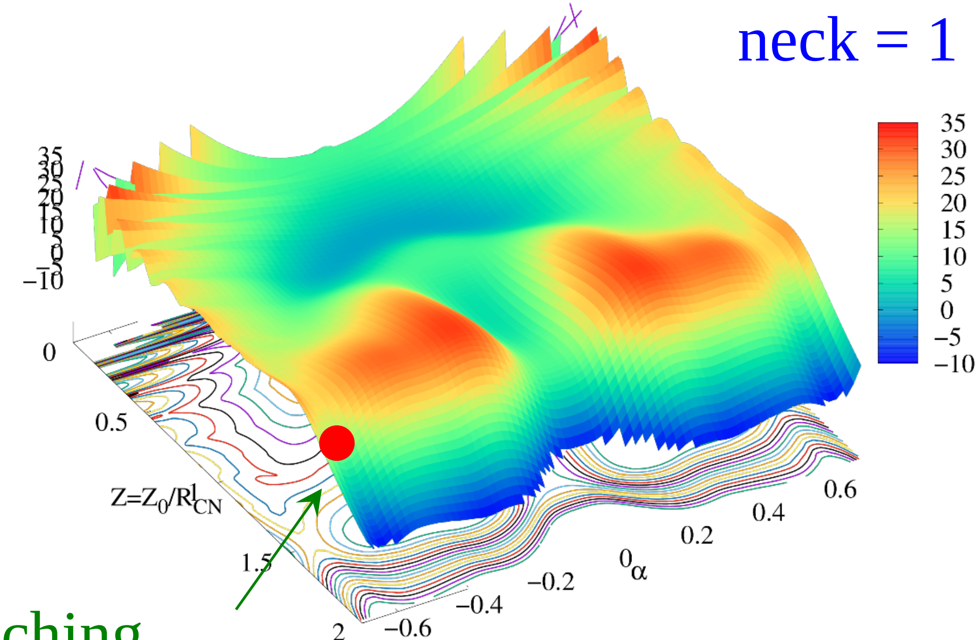
cf. D. Boilley et al., PRC84 (‘11) 054608



Neck = E/E_0

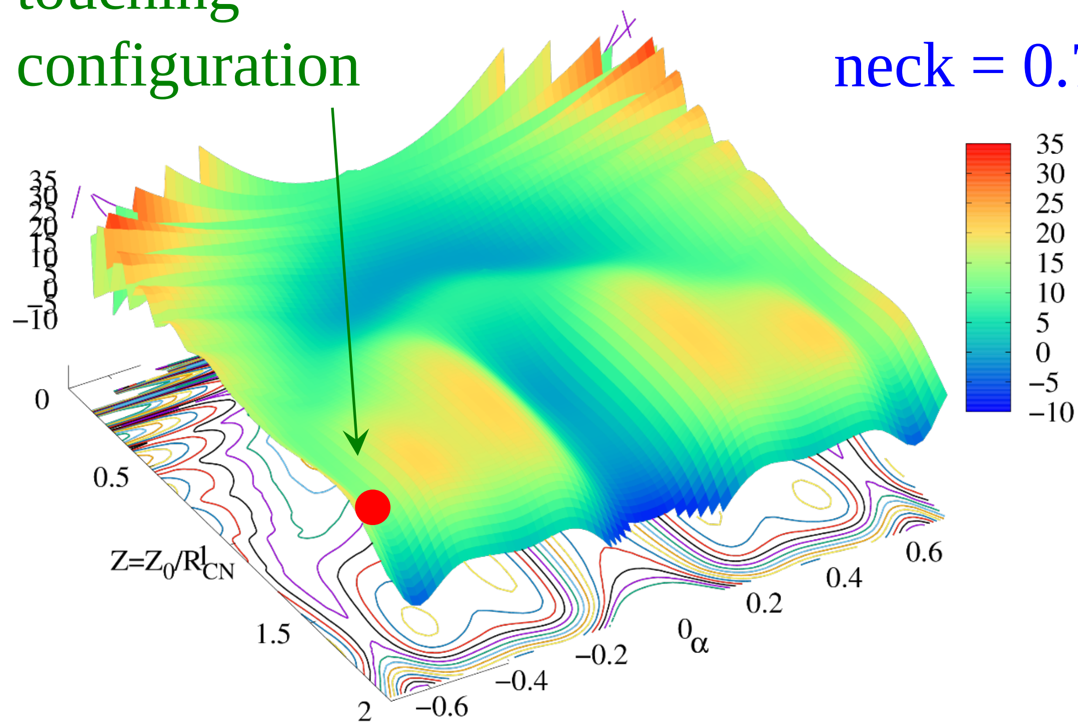
on going work

neck = 1



touching configuration

neck = 0.7



More quantal approach?

generalized Langevin approach: $$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

$$\langle R(t)R(t')\rangle = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\hbar\omega}{2T}\ \coth\frac{\hbar\omega}{2T}\ \exp\left[-\frac{(\hbar\omega)^2}{2\Delta^2}\right]\ e^{-i\omega(t-t')}$$

still classical motion (e.g., no tunneling)

more quantal approach: return to the original Hamiltonian

$$H = \frac{p^2}{2m} + V(q) + \underbrace{\sum_i \hbar\omega_i a_i^\dagger a_i}_{\text{H.O.}} + h(q)\underbrace{\sum_i d_i(a_i + a_i^\dagger)}_{\text{linear coupling}}$$

A.O. Caldeira and A.J. Leggett, Ann. Phys. 149 (‘83) 374

solve *H* quantum mechanically $\longleftrightarrow$ “quantum Langevin”

More quantal approach?

$$H = \frac{p^2}{2m} + V(q) + \sum_i \hbar\omega_i a_i^\dagger a_i + h(q)\sum_i d_i(a_i + a_i^\dagger)$$

solve *H* quantum mechanically $\longleftrightarrow$ "quantum Langevin"

time-dependent coupled-channels equations with an efficient basis

$$\boxed{\Psi_{\mathrm{tot}}(q,t) = \sum_{\{\tilde{n}_k\}} \tilde{\psi}_{\{\tilde{n}_k\}}(q,t)\,|\{\tilde{n}_k\}\rangle}$$

$$|\{\tilde{n}_k\}\rangle = \prod_{k=1}^{K} \frac{1}{\sqrt{\tilde{n}_k!}}\left(b_k^\dagger\right)^{\tilde{n}_k}|0\rangle$$

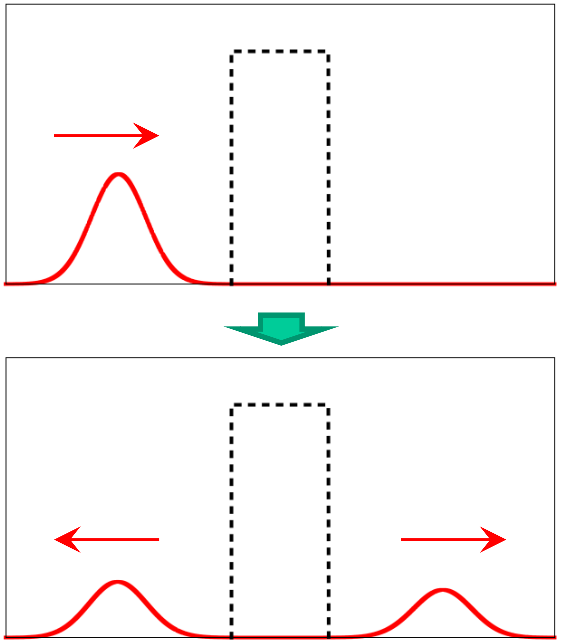
$$b_k^\dagger = \sum_i C_{ki}\, a_i^\dagger$$

M. Tokieda and K. Hagino, Ann. of Phys. 412 (2020) 168005

Front. in Phys. 8 (2020) 8.

Application to heavy-ion fusion reactions

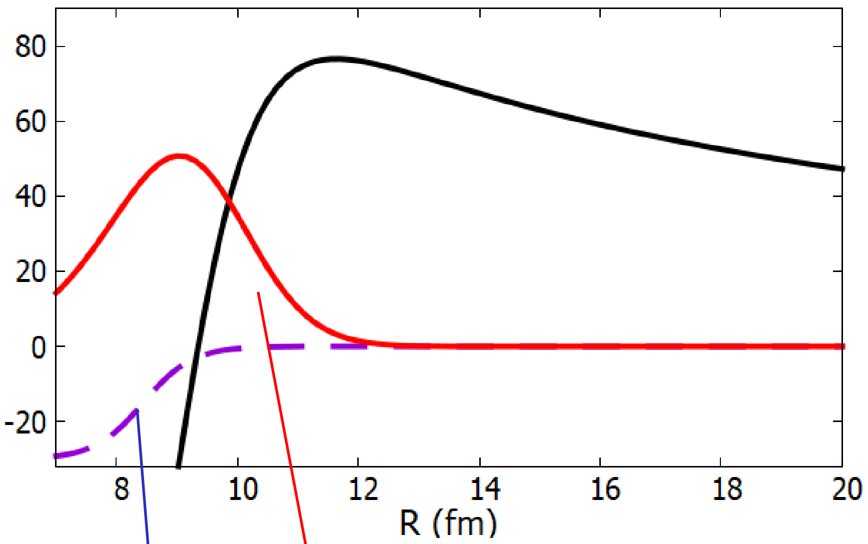
time-dep. wave packet approach



$$R(E) \propto \langle \psi_R(t_f)|\delta(H-E)|\psi_R(t_f)\rangle$$

3D: radial coordinate for each partial wave
(NB. no tangential friction)

$^{16}O+^{208}Pb$



coupling to the bath

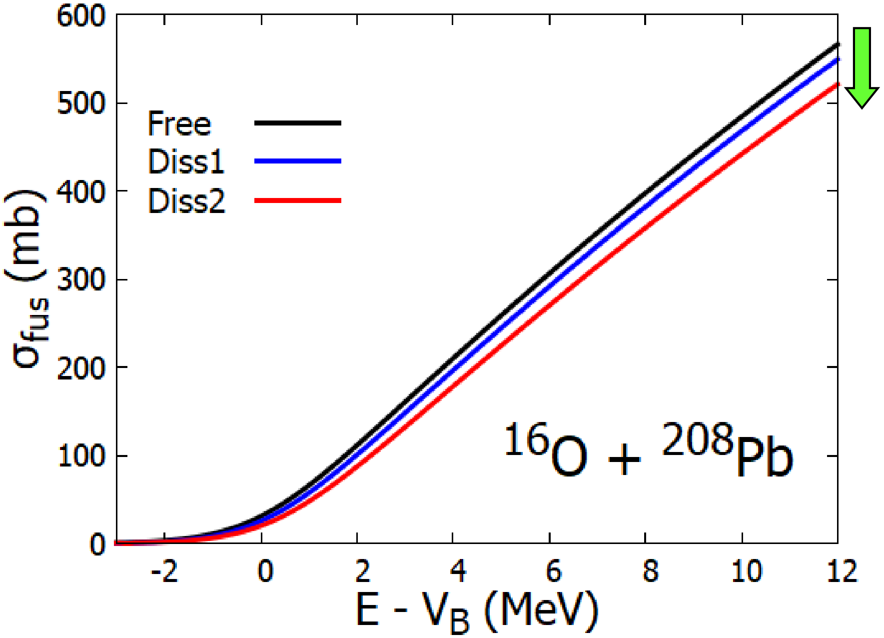
$$h(r) \propto V_N'(r)^2$$

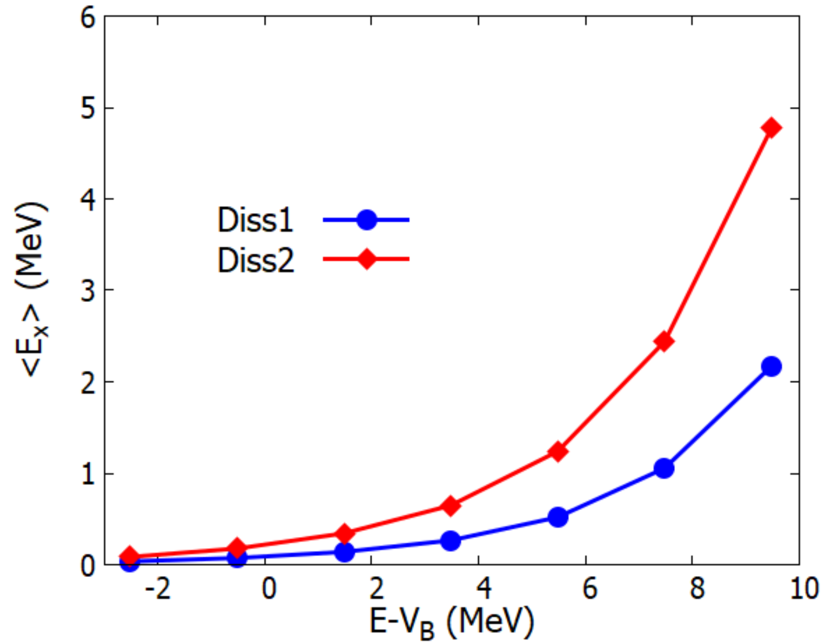
cf. the surface friction model

absorption to simulate fusion

fusion cross sections

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)(1-R_l(E))$$





M. Tokieda, Ph.D. thesis (2021), Tohoku University

fusion cross sections

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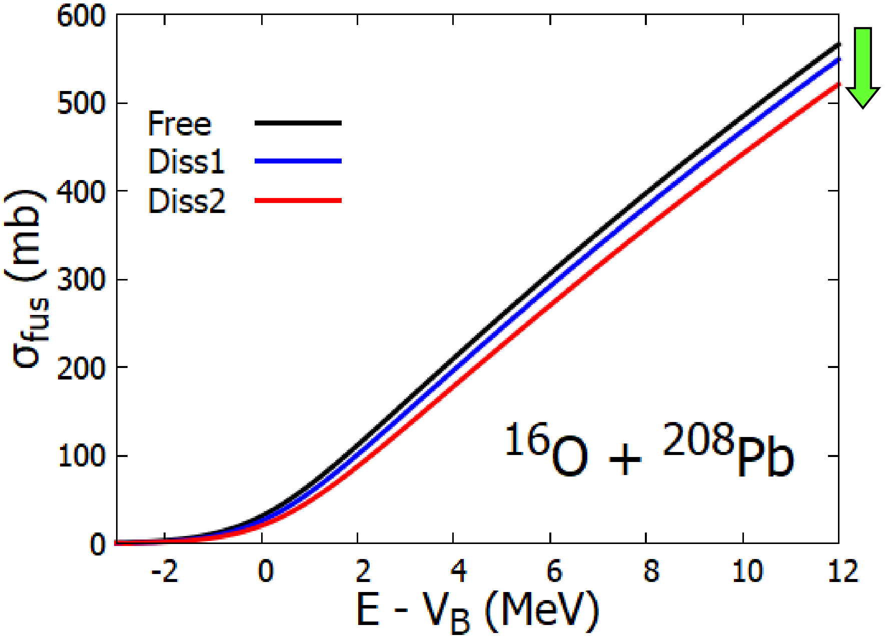

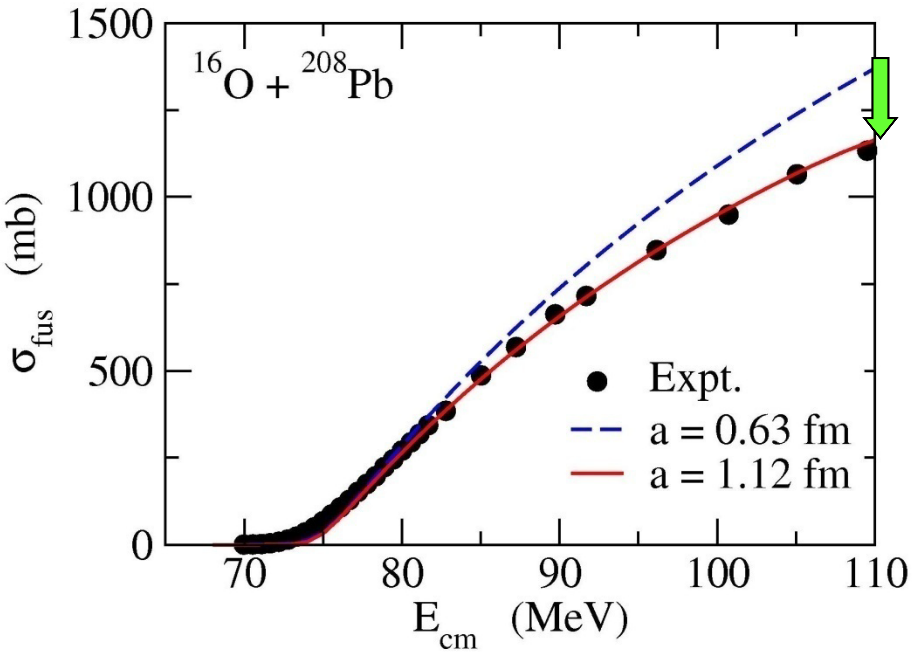

M. Tokieda, Ph.D. thesis (2021), Tohoku University

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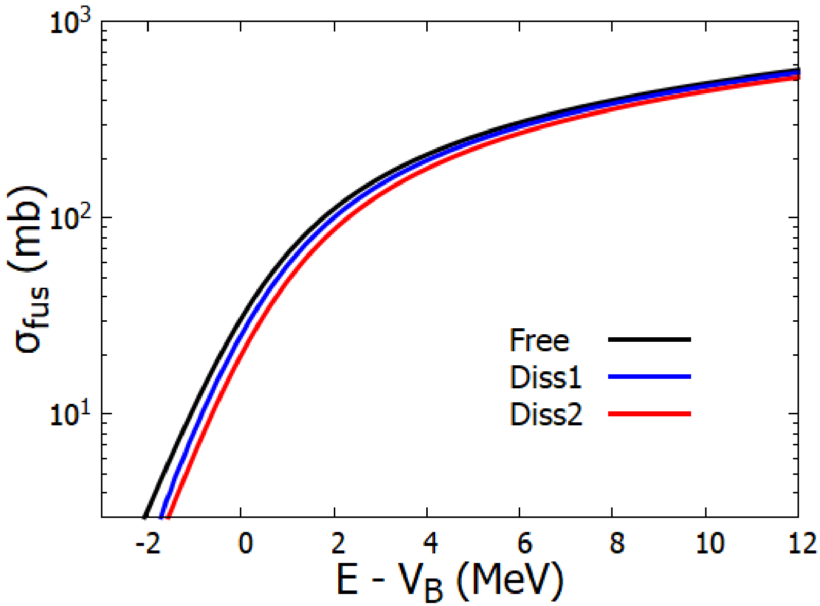
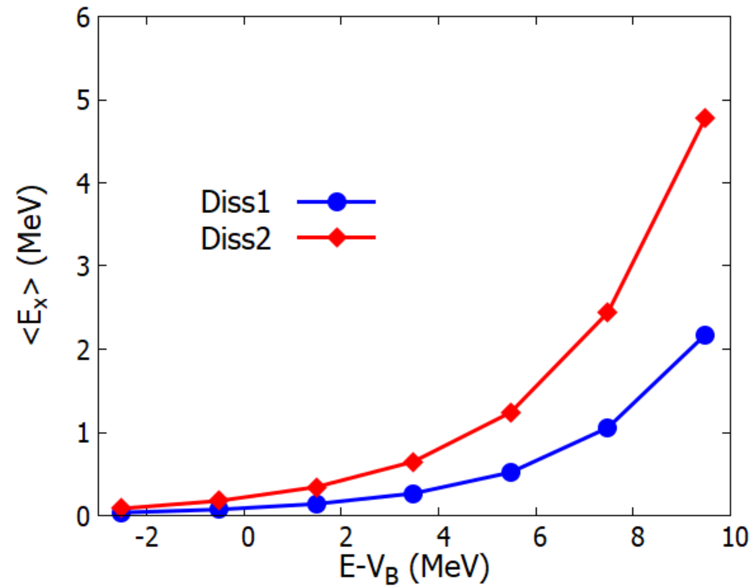
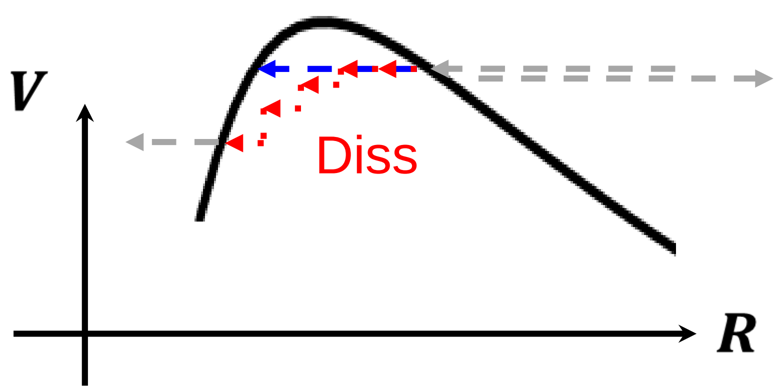




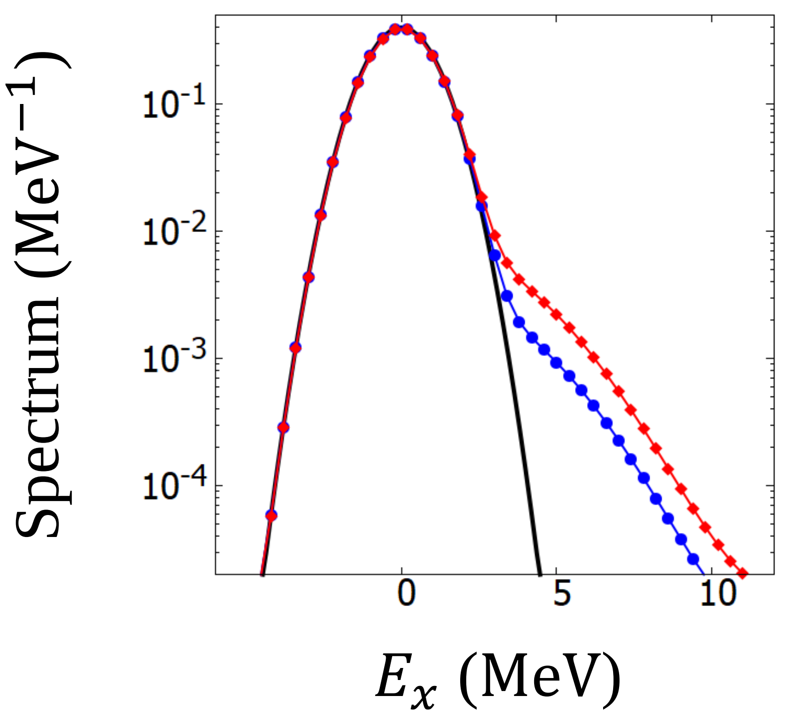
NB. Fusion cross sections: largely influenced even when <E_x> is small

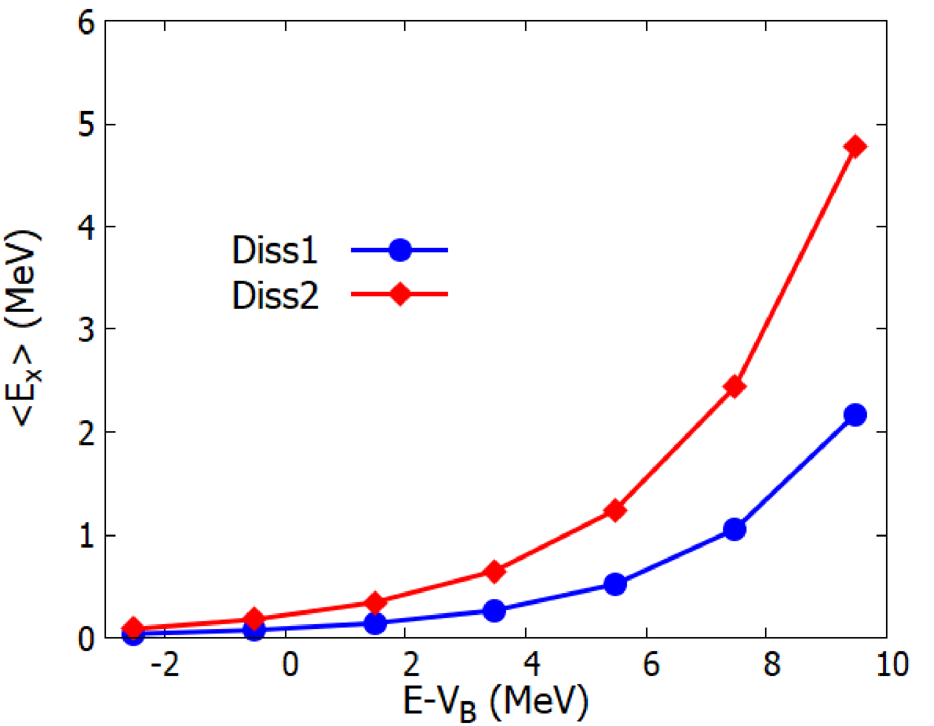


Q-value distribution

$$\langle E_x \rangle = \frac{\langle \psi_R(t_f)|\delta(H_B - E)|\psi_R(t_f)\rangle}{\langle \psi_R(t_f)|\psi_R(t_f)\rangle}$$

$E/V_B \simeq 0.96$



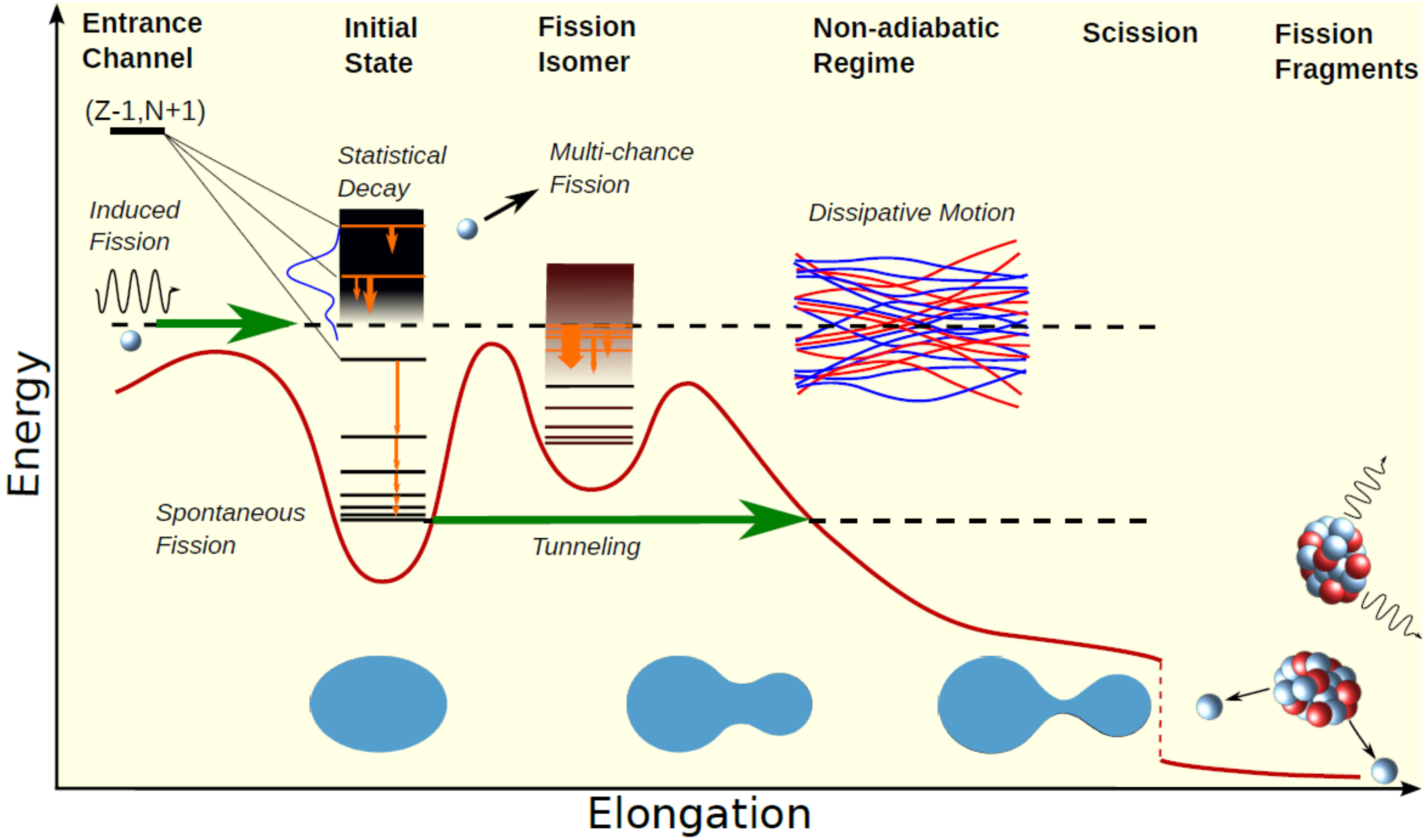


the next step: a comparison to the experimental data

M. Tokieda, Ph.D. thesis (2021), Tohoku University

fission: avery complicated dynamics

a microscopic understanding → far from complete

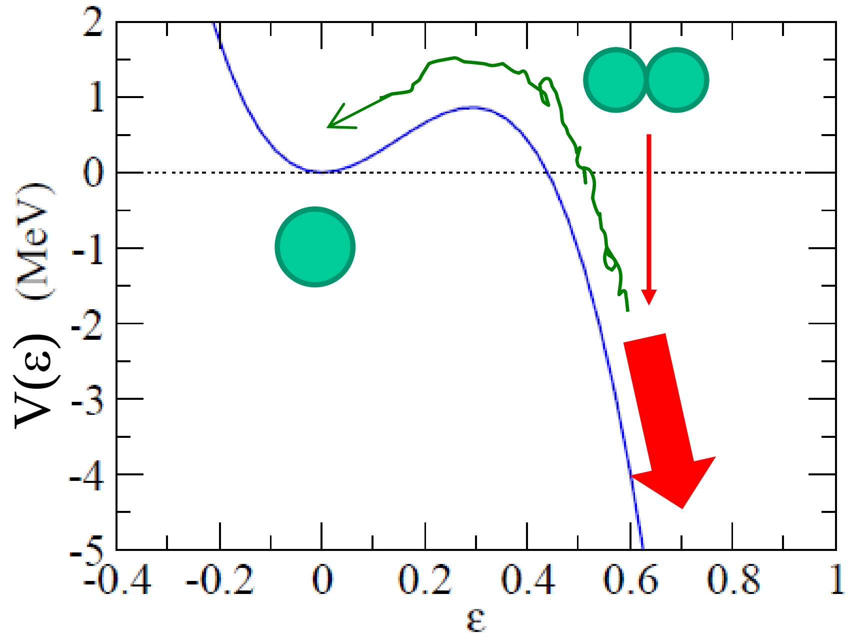


M. Bender et al., J. of Phys. G47, 113002 (2020)

quantum Langevin: a unified description?

Summary

fusion for SHE
→ a very complicated many-body dynamics



the *classicl* Langevin: standard

- quantum extension?

  → maybe important for cold fusion reactions

  - generalized Langevin calculations

    enhanced P_{CN} for ^{48}Ca+^{208}Pb

  - quantum Langevin approach

    CC with Caldeira-Legett Hamiltonian

    friction: hinders subbarrier fusion cross sections

→ a unified description from low-*E* to high-*E*